Inventory Fluctuations in Flaxseed and Linseed Oil,

1926-1939

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1. Introduction

The primary purpose of this paper is the construction of economic relations which comprise a theory underlying inventory behavior of particular commodities and the estimation of certain of these relations.

Since the decisions of economic units with respect to inventories and other economic variables subject to their control are in some sense joint decisions, it therefore seems appropriate to specify a set of simultaneous economic and technical relationships as the joint explanation of observations on the several variables. The generation of the observations is then explained by assuming the set of relations comprise a system of stochastic equations. It is this kind of theoretical model which provides the framework for this study.

Of the choice of flaxseed and linseed oil as the commodities studied, it was thought their highly specialized uses would enable attention to be focused on comparatively few groups of economic units and thus require the specification of fewer behavior equations than would be the case had commodities of more general uses been selected. It was further believed that the markets in which these commodities were traded were such that some of the traditional notions regarding market behavior, e.g., competitive buying and selling in some cases, price fixing in others, would be of aid in the specification of a model.

The inventory data which are available are end-of-quarter inventories of linseed oil held by linseed oil producers and consumers and end-of-quarter

1. I wish to acknowledge the many helpful suggestions and criticisms of Professors Kenneth Arrow and Clifford Hildreth.
flaxseed inventories of linseed oil producers. Inventory equations for these commodities, which are the equations to be estimated in this study, will be adapted to the use of this data.
2. A Summary of the Production, Consumption, and Trade of Flaxseed, Linseed Oil, and Linseed Cake during the Period.

2.1 Flaxseed

The two important industrial raw materials furnished by the flax plant are flaxseed and flax fiber. However, the seed and fiber are not jointly produced; generally a different variety of plant is grown for seed than for fiber. Flax fiber is spun into yarn used in linen manufacture. The only important industrial use of flaxseed is in the production of linseed oil and linseed cake.

Argentina, the Soviet Union, and British India produce about eighty percent of total world flaxseed production. Argentina alone produces about half of world production, exports about eighty percent of flaxseed exported. The United States, which typically imports about half of its consumption requirements, obtains practically all its imports from South America, about eighty percent of its imports originating in Argentina. In some years small quantities have been imported from Canada or India.

The Argentina harvest occurs in December, and the new crop is available for exporting in the first quarter of the calendar year. Little flaxseed is retained in Argentina for consumption, and little seed is carried over from crop-year to crop-year. The Indian harvest occurs in early spring and is available for exporting in the second quarter of the year.


Domestic as well as Canadian seed is planted in the spring, harvested in August, and the bulk of it marketed before the end of the year.\(^1\) About ninety-five per cent of domestic flaxseed is grown in the North Central United States, although in the late thirties California became an important producing state.\(^2\) In this region flaxseed competes most strongly with spring wheat and to a lesser extent with other grains (corn, oats, barley, etc.) for the farmers' acreage at planting time.\(^3\) The comparative price of flaxseed and wheat is regarded as an important influence in the amount of flaxseed acreage planted.\(^4\) Again, the comparative price is important in the harvesting and marketing of flaxseed. Relatively high flaxseed prices induce farmers to give more attention to the flaxseed harvest, to market it more readily, and to draw down more heavily on their winter storage of seed for shipment to market.\(^5\)

Practically all the domestic crop of flaxseed is traded on the Minneapolis Grain Exchange.\(^6\) A large amount of purchases of Argentine and Canadian seed are made directly on the Buenos Aires and Winnipeg exchanges for in the United States by their own buyers or by their agents. These purchases are then shipped directly to the plants of consumers. Also flaxseed is shipped on consignment to dealers on the East Coast by Argentine exporters.

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for subsequent sales to consumers. Because of relatively inexpensive shipment on the Great Lakes, Canadian exports of seed usually move toward consumers in the Eastern Great Lakes region rather than toward the area of domestic supply. This is particularly the case at the time of the domestic harvest, when the seasonally depressed Minneapolis price renders the flaxseed tariff prohibitive to importation to that area. Argentine seed is usually consumed by plants on or near the East Coast, and domestic seed in the Midwest. However in times of domestic seed shortages, Argentine seed is consumed in the Midwest, and in times of heavy domestic production, domestic seed reaches the East Coast.

For a given Argentine price, the excess of the Minneapolis over the Argentine price depends on the flaxseed tariff, the transportation costs of flaxseed and flaxseed products, available flaxseed supplies in the Midwest, and the distribution of demand for flaxseed products. The excess is usually less than the amount of the tariff and flaxseed shipment costs. The typical firms of the linseed oil industry have plants at the principal sources of seed supply. When the Minneapolis price becomes too high relative to the price of Argentine seed, production can be shifted from the Midwest to the East, thereby weakening the Minneapolis price. If the Minneapolis price is relatively low, production can be shifted best, thereby strengthening the Minneapolis price. Furthermore the cost of the tariff is reduced to East Coast plants since those which export products of imported seed obtain a drawback on the tariff. Because of the good market for linseed cake in Europe and the great distance to the principal American markets for cake,


3. Tariff Commission, Flaxseed, p. 31f.

i.e. the Midwest, those plants export practically all cake produced from imported seed taking full advantage of the drawback. However, apparently the drawback has not enabled them to compete in foreign oil markets: exports of oil have been negligible.

The relation between prices of Argentine and domestic seed is of obvious importance in determining imports of flaxseed by American consumers. Argentine and domestic seed may be regarded as substitutable factors of production, the extent to which each is utilized depending on their comparative prices. Because of the importance of their American customers it might be expected that seed offerings of Argentine suppliers, particularly those who export flaxseed on consignment to American flaxseed dealers, are influenced by the relation between Argentine and United States prices: for example, a relatively high price in the United States, indicative of domestic scarcity, may induce Argentine suppliers to hold their seed for higher Argentine prices or to ship seed to the United States for eventual sale.

2.2. Linseed Oil

During the period under study practically all linseed oil consumed in the United States has been domestically produced. Apparently the compensatory tariff on linseed oil has been prohibitive to imports of oil.

The major means of producing linseed oil and cake from flaxseed in the United States has been the hydraulic press method. Crushing flaxseed under hydraulic pressure yields about one-third its weight in linseed oil,


2. Hereafter, the Minneapolis price of seed will be considered the domestic or U. S. price.

3. But also on those features which tend to make location of plants of importance. *Supra*, p. 5.


variation in this yield being primarily due to variation in the oil content of seed grown under different climatic conditions.\textsuperscript{1} The remainder of the "crush" is the by-product linseed cake. In 1938-1939, linseed oil producers began changing over to a new type of equipment, but this change was not completed for four or five years.\textsuperscript{2}

The firms of the linseed oil industry have engaged in the production of other vegetable oils. Two factors have tended to minimize the competition of these other oils with linseed oil for the means of production: (1) differences in sources of supply of the oil-bearing materials, (2) technical specialization of plant.\textsuperscript{3} Linseed oil production has been adjusted more or less to the seasonality in the demands for flaxseed products. This seasonality has been a long-term phenomenon; apparently the economics of production have not justified more flexibility in linseed oil plant.

While there is an organized market for trading in flaxseed, linseed oil consumers contract directly with producers for deliveries over a three-to-six month period for a price set at the time of contracting and are billed for the oil upon delivery. Approximately ninety per-cent of linseed oil sales are handled in this manner.\textsuperscript{4,5}

An attempt to organize a futures market in linseed oil failed because of lack of interest: "... the custom of hedging in flaxseed was evidently too strongly entrenched (referring to linseed oil producers), and the users of linseed oil apparently did such hedging as they found necessary in terms of future sales of their finished products."\textsuperscript{6}

\begin{itemize}
  \item \textsuperscript{1} U. S. Congress, \textit{Flaxseed Prices}, p. 18.
  \item \textsuperscript{2} Letter from J. D. Craig, December 16, 1949.
  \item \textsuperscript{3} Ibid.
  \item \textsuperscript{4} Letter from J. W. Moore, October 29, 1949.
  \item \textsuperscript{5} Letter from J. D. Craig, Spencer Kellogg and Sons, Inc., November 2, 1948.
  \item \textsuperscript{6} Ibid.
\end{itemize}
In the United States, as abroad, linseed oil is used chiefly as a drying oil. Because drying oils are used chiefly in paints and varnishes, building activity is regarded as the most important stimulus to the world wide demand for linseed oil and thus for flaxseed.\(^1\) Other important industries whose activity affects linseed oil demand are the automobile, furniture, linoleum, oiled-clothing, and rubber-tire industries.\(^2,3\)

The next most important drying oils are tung and perilla oils, the domestic production of which has been negligible. The United States has been dependent on imports of these oils from China and Japan. They have specialized uses as drying oils to which linseed oil is not well adapted and vice versa. The wide fluctuations of their relative prices supports to some extent the lack of a high degree of interchangeability of these oils. Annual consumption of linseed oil declined greatly in 1930-1934, primarily due to the depressed state of activity in consuming industries but also due to some substitution of tung and perilla oils for linseed oil, a substitution accompanied by lower tung and perilla oil prices relative to linseed oil price. An excise tax placed on perilla oil curtailed its consumption after 1936. After 1934, tung oil price again became high relative to linseed oil but tung oil consumption remained relatively high. Changes in drying oil technology and increased demand for drying oil products for which tung oil was better suited are of importance here.\(^4\)

### 2.3. Linseed Cake

The by-product of linseed oil production linseed cake, when ground into

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3. Cf. OPRR, March 11, 1932, p. 35.
meal, is used as a livestock feed. Besides the export market, there is a large market for this product among feed mixers and distributors in the Midwest.

The price of linseed cake is tied closely to the prices of other oil cake used as feed. Generally these prices follow those of feed grains because of substitutability among all these feed materials. Domestically all forms of oil cake constitute less than three per-cent of all feed materials.¹ Abroad an analogous situation would presumably exist, although Europe is a deficit feed-producing area while the United States is a surplus feed producer.

¹ Ibid, p. 29.
3. Factors Affecting Inventory Behavior

3.1. The Seasonal Pattern in Flaxseed and Linseed Oil Stocks.

There is a marked seasonal pattern in linseed oil stocks and consumption. Peaks in linseed oil stocks regularly occur at the end of the first quarter; peaks in consumption, during the following quarter. Troughs in these series generally follow the peaks by two quarters. Peaks occur in flaxseed stocks of consumers, i.e., linseed oil producers, at the end of the fourth quarter, during which the bulk of the domestic harvest is marketed. The seasonal pattern in flaxseed stocks however is less regular, presumably reflecting fluctuations in domestic flaxseed production, which is highly irregular, particularly during those years when droughts caused many crop failures. A seasonal pattern is even less clearly defined in linseed oil production (or flaxseed consumption, which amounts to the same thing). Generally, peaks in production occur in the fourth or first quarters with troughs tending to follow by two quarters. There are no clear cut tendencies for flaxseed stocks to lead or lag flaxseed consumption as in the case of oil stocks and oil consumption.

The seasonal pattern in linseed oil consumption is of course a reflection of the seasonality in the demand for linseed oil products. The heaviest demand for linseed oil products, particularly paints and varnishes, occurs in late Spring and early Summer, which is also the season of heavy building activity. Building activity is not the only element of seasonality; there is normally considerable repainting and maintenance work done after the end of Winter in industry, on farms, and in urban housing.

The tendency of linseed oil stocks to lead oil consumption lends support to a conjecture that stocks are built up in anticipation of seasonal oil requirements. Consumers tend to carry a month's supply of linseed oil on hand.1

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1. Cf. OPTR, November 25, 1929, p. 32 and April 4, 1932, p. 35.
2. Cf. OPTR, December 31, 1928, p. 28.
and there is much to suggest that in normal times consumers carry the bulk of linseed oil stocks while producers produce oil largely on a "to order" basis.

"The big storage facilities are for handling flaxseed and it might be said that the linseed oil inventories (of producers) are actually carried in the form of flaxseed, the seed being converted into oil at a rate sufficient to meet the current demand for it...."

"There is tremendous elevator capacity to handle the seed supplies and of course seed can be stored more advantageously and more economically than either the oil or the by-product oil meal." ¹

If the simple behavior ascribed to consumers and producers in the above were the complete explanation of their stockholding behavior and if their anticipations of oil needs were accurate, a fairly simple and stable relation between observed linseed oil stocks and consumption should be detectable in the data. Such is not the case. Consumers must decide on a level of oil stocks to hold on the basis of anticipated or planned consumption, anticipations which are subject to disappointment and plans which may be changed. Also, it is reasonable that circumstances which lead consumers to contract with producers for future oil supplies covering a greater or less proportion of anticipated oil requirements may also lead them to increase or decrease the proportion of oil actually on hand.² For example, if heavy oil requirements and light supplies are anticipated, consumers may protect themselves against rising prices by contracting for large amounts of "forward" oil and at the same time accumulating stocks at their plants against the possibility of delivery delays, etc.³ Frequently both producers and consumers anticipate heavy oil requirements, but consumers may feel that producers' quotations on

¹. Letter from J. D. Craig, November 2, 1948.
². Cf. OFER, March 10, 1930, p. 32.
forward oil are out of line with present prices and future values. Consumers may purchase oil for immediate shipment to build up stocks but settle few contracts with producers for forward oil.\(^1\)

Although oil producers may wish to minimize the oil stocks they carry by adjusting production to shipments of oil they have contracted to deliver, frequently they do not or cannot achieve this balance even over fairly long periods. Consumers may be delinquent in giving shipping instructions on oil they have contracted for when they have overestimated their needs or order additional supplies for quick shipment when they have underestimated them.\(^2\) The depression years provide many instances of failures to fulfill contracts, tardy output adjustments and unintended accumulation of oil stocks by producers.

A further source of unbalance between oil production and oil shipments occurs when producers adjust production in order to accumulate supplies of cake and meal. In the Midwest the heaviest demand for commercial feeds occurs in the Winter months when natural pasturage is unavailable.\(^3\) Although shipments of oil are seasonally expanding at this time, it cannot be expected that shipments of oil and cake or meal are well correlated.\(^4\) A priori the lower value of cake and meal\(^5\) (about thirty per-cent of the total value of flaxseed products in this country) and the far greater sensitivity of lin-

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1. Cf. OPDR, July 1, 1929, p. 33 and August 19, 1929, p. 32.
3. Cf. OPDR, December 30, 1929, p. 32.
4. The lack of time series on producers' oil and meal stocks and shipments precludes quantitative information on the extent to which stocks of one of the joint products are accumulated (for the sake) of supplying the market for the other. A high price for meal, which may induce production increases not justified by oil shipments, may allow producers to offer oil at lower prices, but this may fail to provide stimulation to oil sales sufficient to balance shipments to the new rate of production, particularly during periods of greatly depressed activity in consuming industries.
5. U. S. Senate, Flaxseed Prices, p. 2.
seed oil price to fluctuations in supply would predicate a closer adjustment of production to linseed oil shipments. The fact that linseed oil stocks do tend to lead instead of lag oil consumption is some evidence that producers do make fairly prompt adjustments in output when unsold oil accumulates.

Thus, in summary, the extent to which oil consumers correctly anticipate their needs and producers adjust their production to oil shipments determines the regularity with which linseed oil stocks lead consumption.

The lack of a well defined seasonal relationship between flaxseed stocks and linseed oil production does not mean that oil producers are not guided by the same motivations in accumulating and liquidating stocks of their raw material as in the case of oil consumers. Mention has already been made that producers carry their "oil inventories in the form of flaxseed." Variability in the domestic harvest and in the availability of imports provide many uncertainties in the supply of flaxseed to oil producers. If they are to contract to supply their customers at fixed prices over long periods, they must exercise some control over the cost of the required raw material. The practice of hedging sales of forward oil by accumulating stocks of flaxseed is acknowledged throughout the industry.

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1. Supra, p. 9. The demand for meal is far more elastic than the demand for oil. Market reports suggest that producers have a great deal more freedom to adjust oil prices and that the supply price of oil tends to be determined for given prices of meal and flaxseed. (Cf. OPFR, June 10, 1929, p. 32, December 16, 1929, p. 32, February 3, 1930, p. 30, March 17, 1930, p. 30, May 19, 1930, p. 32, and June 23, 1930, p. 32).

Examples of downward output adjustments when unsold oil accumulates in spite of good meal demand appear more frequently than the contrary case. (As illustration, Cf. OPFR, December 20, 1937, p. 36 and May 16, 1938, p. 38).

The working hypothesis of this study is that producers in making their long-period production plans are influenced chiefly by their anticipations of oil needs.

2. OPFR, December 1, 1929, p. 31, October 8, 1928, p. 29, August 18, 1930, p. 32, and January 24, 1938, p. 32.
Unintended accumulation or liquidation of oil stocks by oil producers is a factor explaining the lack of a definite lead or lag of flaxseed stocks with respect to oil production. Although oil producers may accumulate flaxseed stocks in anticipation of production requirements, downward production adjustments in response to unexpected reductions in shipments tend to cause seed stocks to lag production while an opposite situation produces a lead.

There is however a source of seasonality in flaxseed stocks independent of the seasonality in the demand for linseed oil. The tendency for seed stocks to be built up in the fourth quarter reflects the fact that the bulk of the domestic harvest appears on the market at this time. After this, domestic flaxseed is not available on the market in comparable quantities until the next harvest. Furthermore, if the domestic harvest is insufficient for the requirements of Midwestern plants, the comparatively high cost of shipping flaxseed from the East by rail necessitates the accumulation of Argentine seed by these plants for their entire period of Winter operations before the close of navigation on the Lakes.¹ Similarly, Eastern plants, if they are to utilize any of the domestic crop, must also accumulate stocks of domestic seed at this time. Thus, as the bulk of the domestic harvest is marketed, stocks, anticipatory of seed requirements for periods longer than a quarter, tend to be highest at this time regardless of the seasonal fluctuations in seed consumption for any given year.

This tendency was of less importance in flaxseed stocks during the thirties. Linseed oil consumption was generally lower in the thirties than in the twenties with the result that linseed oil production was shifted almost entirely to Eastern plants in years of short supplies in the Midwest.² When

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1. OPDR, October 6, 1924, p. 33. Lake navigation closes about December 1.
2. OPDR, May 9, 1932, p. 34 and July 26, 1937, p. 44.

Not only was there a general downward trend in domestic and Canadian flaxseed cultivation, but the period was also marked by severe crop failures.
shifts in production to the East occur or when linseed oil plants are largely dependent on import supplies of flaxseed, the seasonality in the domestic harvest is less influential in flaxseed stocks. Seasonality is of course much less important in the supply of imports.

The importance attributed to oil producers' and consumers' anticipations in the foregoing requires an investigation of variables used by firms in predicting market developments. But first it will be useful to consider a theoretical inventory demand equation in which these influences can be fitted.

3.2. A Theoretical Inventory Demand Equation.

A frequently encountered notion in the theory of inventories is that of the firm's pipeline stock. The pipeline stock is that proportion of raw material stocks to output or of stocks of final product to sales made necessary by differences among the rate of raw material receipts, the rate of output, and the rate of shipments of final product. However it would be mistaken to assume that such proportions are not the result of an optimization process. It might be technically feasible but prohibitively costly to equate these rates. The costs of storage and of departures from an efficient rate of production and lags between purchases and receipts of commodities are influences underlying the decision as to the proportion of stocks carried to output or sales. While there is no reason why pipeline stocks should not be a variable proportion of output or sales, a linear relation is usually assumed as an approximation for purposes of estimation.

The linear relation appears to be particularly useful if those speculative influences which tend to increase or decrease the desired proportion of stocks to planned or anticipated output or sales are excluded from the explanation of pipeline stocks, e.g., the responses of producers to anticipations of future decreases or increases in the supply of raw materials. While Keynes' definition
of working capital\textsuperscript{1} includes stocks carried "to avoid risks of interruption of process" due to seasonal fluctuations in supply (prosumably these must be anticipated by producers), a decision, say, to increase the proportion of stocks carried because of anticipated shortages is as speculative in character as one to carry stocks for purposes of realizing "speculative" profits on liquidation. Anticipated scarcities and anticipated price rises rationalize the same thing.

The above considerations can be formulated in the following simple expression:

\begin{equation}
    H_t^D = \beta_1 \Delta x_{t+1} + \beta_2 \Delta p_{t+1}, \quad \beta_1, \beta_2 > 0
\end{equation}

where

- $H_t^D$ = desired stocks to be held at end of period $t$
- $\Delta x_{t+1}$ = consumption (or sales, if the stock is that of a final product) in period $t + 1$ as anticipated during $t$.
- $\Delta p_{t+1}$ = change in price of commodity from period $t + 1$ to $t$ as anticipated during $t$, i.e., $\Delta p_{t+1} = p_{t+1} - p_t$.

The term $\beta_1 \Delta x_{t+1}$ represents the pipeline component and $\beta_2 \Delta p_{t+1}$ the speculative component of desired stocks.

The following is a simple theory of anticipated price changes: a producer will predict a change in price if at the current price future demand and supply (as he anticipates them) are unequal.\textsuperscript{2} In particular,


\textsuperscript{2} Future supply and demand conditions of a durable commodity as evaluated by the market tend to be discounted in current market prices. However there is no reason why the market's evaluation of these conditions should be that of the individual buyer or seller.
(3.2.2) \[ a_n \Delta p_{t+1} \leq 0 \text{ if } a_n \Delta d_{t+1} \leq a_n S_{t+1} \]

where

\[ D_{t+1} = \text{quantity demanded in } t + 1 \]

\[ S_{t+1} = \text{quantity supplied in } t + 1. \]

A simple function with the properties of (3.2.2) is

(3.2.3) \[ a_n \Delta p_{t+1} = J(a_n \Delta d_{t+1} - a_n S_{t+1}) \]

where \( J \) is a positive real number.

The actual stocks a producer has on hand at a given time rarely represent the amount he planned to hold in accordance with (3.2.1). And it may be impossible or at any rate very costly to change such currently existing rates of output, receipts of raw material, and/or shipments of final product as were determined by decisions based on prior information, although unplanned accumulation or liquidation of stocks may be the result.

"Unless a manufacturer, for example, produces exclusively to order, he must base his decisions about the quantity of goods to purchase and to produce in large part on a guess about the quantity he will sell.... If he has overestimated demand, he may add more goods to stock than he had planned. Or by reducing prices or offering other inducements to customers, or by advertising, he may achieve his anticipated rate of sales.... In any event, the quantity of goods a manufacturer sells during a given period is unlikely to be identical with the quantity he expected to sell at the beginning of the period. And since he will not be able to make a full adjustment in his receipts of purchased goods and in his own rate of production, the amount of goods he holds in stock at the end of any period is unlikely to be identical with the quantity he planned to hold."

The above passage suggests that unplanned inventory accumulation or liquidation during a period represents a systematic component of observed inventories at the end of the period, say the linear function

(3.2.4) \[ \Delta h_t^u = \beta_0 (a_{t-1} x_t - x_t), \quad 0 < \beta_0 < 1 \]

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where
\[ \Delta H^D_t = H_t^D - H_t^D \]
\[ H_t = \text{observed stocks at end of } t \]
\[ I_t = \text{actual consumption (or sales) during } t. \]

The possibility of some adjustment in the receipts of raw material to its current consumption (or in production to current sales or shipments of final product) is not precluded in the above: unplanned accumulation or liquidation of stocks of raw material (or of final product) is represented as a proportion of the difference between planned and actual consumption (or planned and actual sales).

According to the theory outlined above, observed inventories of a producer at the end of a given period then may be presumed to satisfy the relation
\[ (3.2.5) \quad H_t = H_t^D + \Delta H^D_t \]
\[ = \beta_0 (a_{t-1} I_t - I) + \beta_1 a_t I_{t+1} + \beta_2 a_t \Delta P_{t+1}. \]

The statistical investigation of such a theory requires the choice of functions of observable variables to substitute for the unobservable variables, i.e., the anticipations of future values of variables.

3.3 The Demands for Flaxseed and Linseed Oil Inventories

The adaptation of equation (3.2.5) to inventory demand equations for flaxseed and linseed oil will be simplified if a certain amount of aggregation is presumed. ¹

¹ The aggregation of microeconomic equations, even if assumed linear, presents problems, particularly if the equations contain variables specific to individual firms with coefficients also dependent on individual firms. If this possibility in the present case may be assumed away, and in particular, if the anticipations of individual firms are assumed to be functions of "market" variables, e.g., prices, aggregate consumer stocks, etc., aggregation becomes a trivial problem.
It is assumed that in the current quarter oil consumers determine a level of oil stocks to be held at the end of the quarter in anticipation of the following quarter's consumption needs. Further, that the influence of anticipated supply conditions in the oil market on the planned level of oil stocks of consumers may be represented as a linear function of their anticipation of the change in price from the following to the current quarter.

The planned level of consumers' oil stocks is determined by

\[(3.3.1) \quad h^c_L(t) = \gamma_1 a^c_t C^c_L(t + 1) + \gamma_2 a^c_t \Delta p^c_L(t + 1)\]

where

\[h^c_L(t) = \text{stocks oil consumers plan to hold at end of } t^{\text{th}} \text{ quarter},\]

\[a^c_t C^c_L(t + 1) = \text{oil consumption during quarter } t + 1 \text{ anticipated by consumers during } t,\]

\[a^c_t \Delta p^c_L(t + 1) = \text{change in oil price between quarters anticipated by consumers during } t.\]

It is also assumed that consumers have in the previous quarter estimated their consumption requirements and have made contracts with producers for shipments of oil for the current quarter. But during the current quarter consumers may revise their plans for oil consumption. Differences between consumption previously planned and actual consumption are assumed partly met by (unplanned) adjustments in their oil inventories and partly met by orders for oil shipments revised from those previously planned, i.e.

\[(3.3.2) \quad H^c_L - h^c_L(t) = \gamma_o [a^c_{t-1} C^c_L(t) - C^c_L(t)]\]

\[(3.3.3) \quad O^c_L(t-1) - S^c_L(t) = (1 - \gamma_o) [a^c_{t-1} C^c_L(t) - C^c_L(t)]\]

\[0 < \gamma_o < 1\]
where

\[ H^C(t) = \text{oil stocks held by consumers at end of } t^{\text{th}} \text{ quarter.} \]
\[ O^C_L(t-1) = \text{oil shipments during current quarter contracted for as of the beginning of } t^{\text{th}} \text{ quarter.} \]
\[ S^C_L(t) = \text{oil shipments to consumers during } t. \]

The consumers' demand for oil stocks is the sum of (3.3.1) and (3.3.2):

\[
H^C_L(t) = \gamma_0 [an^C_{t-1} C^C_L(t) - C^C_L(t)] + \gamma_1 \frac{an^C}{t} C^C_L(t + 1) \\
+ \gamma_2 \frac{an^C}{t} \Delta p^C_L(t).
\]

It is assumed that oil producers act to maintain a stock of oil on hand sufficient to meet anticipated shipments to consumers. However, the planned proportion of oil stocks to these anticipated shipments may increase as anticipations of a rising meal market induce them to accumulate a supply of meal. Anticipations of a falling meal market presumably induce the opposite behavior.

The planned level of producers' oil stocks is determined by

\[
h^P_L(t) = \varepsilon_1 \frac{an^P}{t} S^P_L(t + 1) + \varepsilon_2 \frac{an^P}{t} \Delta p^P_L(t + 1)
\]

where

\[ h^P_L(t) = \text{stocks oil producers plan to hold at end of } t^{\text{th}} \text{ quarter.} \]
\[ an^P_L S^P_L(t + 1) = \text{oil shipments to consumers in } t + 1 \text{ anticipated by producers during } t. \]
\[ an^P_L \Delta p^P_L(t + 1) = \text{change in meal price between quarters anticipated by producers during } t. \]

When oil consumers fail to order out all the oil they have contracted for or close contracts more promptly than planned, or order additional supplies for immediate shipment, it is assumed that part of the difference between actual shipments and the shipments planned in existing contracts results in an unplanned accumulation or liquidation of producers' oil.
stocks, i.e.,

\[ H^p_L(t) - h^p_L(t) = \varepsilon_0 \left[ q_L(t - 1) - s_L(t) \right] \quad 0 < \varepsilon_0 < 1 \]

where

\[ H^p_L = \text{oil stocks held by producers at end of } t^{th} \text{ quarter.} \]

The producers' demand for oil stocks is the sum of (3.3.5) and (3.3.6):

\[ H^p_L(t) = \varepsilon_0 \left[ q_L(t - 1) - s_L(t) \right] + \varepsilon_1 \alpha_t^p s_L(t + 1) \]

\[ + \varepsilon_2 \alpha_t^p \Delta p_M(t + 1) \]

An analogous relation to (3.3.1) is assumed to determine oil producers' end-of-quarter flaxseed stocks:

\[ h^F_S = \int_1^r \alpha_t^F L(t + 1) + \int_2^r \alpha_t^F \Delta p_S(t + 1) \]

where

\[ h^F_S = \text{flaxseed stocks oil producers plan to hold at end of } t^{th} \text{ quarter.} \]

\[ \alpha_t^F L(t + 1) = \text{oil production during } t + 1 \text{ anticipated by producers during } t. \]

\[ \alpha_t^F \Delta p_S(t + 1) = \text{change in domestic flaxseed price between quarters anticipated by oil producers during } t. \]

Unplanned adjustments of production in response to consumers' revisions of orders for oil shipments are assumed to result partially in unplanned adjustments in flaxseed stocks, i.e.,

\[ H^F_S(t) - h^F_S(t) = \int_0^r \left[ q_L(t - 1) - s_L(t) \right] \]

where

\[ H^F_S(t) = \text{flaxseed stocks held by oil producers at the end of } t^{th} \text{ quarter.} \]
Thus the producers' demand for flaxseed stocks is:

\[
H^P_S(t) = \int_1^\infty [a(t - 1) - S(t)] + \int_1^\infty (t + 1) an^P_t S(t + 1) + \int_2^\infty an^P_t \Delta p_S(t + 1).
\]

In estimating future linseed oil requirements, both linseed oil producers and consumers are influenced by the state of building activity and by general conditions in industry. Reports on the flaxseed and linseed oil markets make an extensive analysis of the F. W. Dodge series of construction contracts awarded, noting declines or increases in this series as indicative of probable developments in linseed oil consumption.\(^1\) Activity in a more general industrial sector is also important.\(^2\) An index of industrial production in an equation explaining anticipated linseed oil consumption can be justified in this connection\(^3\) as well as in connection with such stimulation to the consumer demand for linseed oil products as may stem from national income. Prices of linseed oil, of linseed oil products, and of linseed oil substitutes are of importance in the determination of linseed oil consumption. However it is likely that short period changes in consumption are best forecast by fluctuations in the variables above and by the regular seasonal tendencies in the use of linseed oil products.\(^4\)

Thus a reasonable hypothesis about consumption requirements anticipated\(^5\)

---

2. Supra, p. 3.
3. Cf. OPDR, August 19, 1929, p. 32, November 17, 1929, p. 34, and October 20, 1930, p. 36.
4. Cf. OPDR, April 4, 1932, p. 35.
5. To economize on notation, the convention of specifying anticipations to particular market groups, e.g., producers or consumers, will be dropped.
by consumers and producers is:

\[(3.3.11) \quad c_L(t + 1) = \eta_1 B(t) + \eta_2 Y(t)\]

where

- \(c_L\) = quarterly linseed oil consumption.
- \(B\) = construction contracts awarded during quarter in 1000's of square feet of floor area.
- \(Y\) = index of industrial production during quarter (1926 = 100).

By and large, oil producers may be assumed to know the amount of oil stocks in the hands of consumers during the quarter. Thus a reasonable hypothesis about producers' anticipations of oil shipments is:

1. The following least square regressions are of interest here:
   
   (a) \(c_L = 1.01 B_{-1} + 753.50 Y_{-1} + 26616.22 \quad (R = .7334)\)
   
   (b) \(c_L = 1.43 B + 430.28 Y + 40755.13 \quad (R = .8887)\)
   
   (c) \(c_L = .98 B_{-1} + 1015.94 Y + 6367.07 \quad (R = .8068)\).

Average values for these variables for 1926-1939 are:

\[
\begin{align*}
\bar{c}_L &= 138,796 \\
\bar{B} &= 39,832 \\
\bar{Y} &= 95
\end{align*}
\]

2. The use of a constant term which depends upon the particular quarter of the year in which anticipations are formed might be justified in the equation to explain the "regular" seasonality of consumption. (Supra, p. 10) However, in estimating these equations it was thought that seasonality in the time series on the variables \(B\) and \(Y\) would be sufficient.

3. The use of quarterly values of predicting variables in an equation explaining anticipations formed during the quarter may be justified as follows: while there are perhaps lags of one to two weeks or longer in the transmittal of market information, at the end of a given quarter interested groups do know for all practical purposes the quarterly value of a market variable and have continuously revised anticipations concerning succeeding periods with the acquisition of the more complete information.

4. For more complete description of data, see Appendix.
(3.3.12) \[ a_n S_L(t+1) = \theta_1 a_n C_L(t+1) + \theta_2 H^C_L(t). \]

The size of total oil stocks\(^1\) would, besides anticipations of oil consumption, presumably influence production plans of producers:

(3.3.13) \[ a_n L(t+1) = \lambda_1 a_n C_L(t+1) + \lambda_2 H_L(t). \]

where

\[ H_L(t) = H^C_L(t) + H^P_L(t) \]

It is assumed that a fall or rise in the domestic price of flaxseed will be anticipated accordingly as a surplus or deficit between available supplies and flaxseed consumption requirements is anticipated, i.e.,

(3.3.14) \[ a_n \Delta P_S(t+1) = \kappa [a_n C_S(t+1) - a_n P_S(t+1)] - H_S(t) - a_n I_S(t+1) \]

where

(3.3.15) \[ a_n C_S(t+1) = \lambda_1 a_n L(t+1) \]

(3.3.16) \[ a_n P_S(t+1) = \lambda_2 A_S(t) \]

(3.3.17) \[ a_n I_S(t+1) = \lambda_3 [P_S(t) - P^A_S(t)] \]

and where

\[ C_S = \text{quarterly flaxseed consumption} \]
\[ P_S = \text{quarterly domestic flaxseed production} \]
\[ H_S = \text{total end-of-quarter flaxseed stocks} \]
\[ I_S = \text{quarterly flaxseed imports} \]
\[ A_S = \text{acreage planted in flaxseed during quarter} \]
\[ p_S = \text{quarterly domestic price of flaxseed} \]
\[ p^A_S = \text{quarterly Argentine price of flaxseed.} \]

Equation (3.3.15) stems from the technical relation between flaxseed input and linseed oil output. \( \lambda_1 \) is used to express linseed oil in terms of its flaxseed equivalent.

\(^1\) Infra, p. 26, n. 1.
Equation (3.3.16) states that anticipations of flaxseed production depend only on the acreage planted in the preceding quarter.

Equation (3.3.17) is a drastic simplification. The many influences, besides price, conditioning the supply of imported seed that are very likely known to the large oil producers, who deal directly in the Buenos Aires flaxseed market, are ignored. As it stands, a reasonable interpretation of the spread between domestic and foreign prices, which is indicative of domestic scarcities or surpluses, would be that it indicates to oil producers the likely extent to which they will be dependent on imports for future supplies of seed.

The behavior of producers with respect to oil prices leads to an assumption:

\[(3.3.18) \quad a_n \Delta p_L(t) = \mu_1 a_n \Delta p_S(t + 1) + \mu_2 a_n \Delta p_M(t + 1)\]

where

\[(3.3.19) \quad a_n \Delta p_M(t + 1) = p_M(t) + 4p_F(t)\]

and where

- $p_M$ = quarterly price of meal.\(^1\)
- $p_F$ = quarterly price of livestock feeds.

Equation (3.3.19) bears the following interpretation: if current meal prices are out of line with their normal relation to prices of feeds, the change in meal price will tend to a correction.\(^2\)

---

1. A fixed differential between linseed cake and meal prices is also assumed.

2. Typical reactions of producers to information on feed prices are illustrated in OPDR, July 22, 1929, p. 32 and September 9, 1929, p. 34.
By substituting the right-hand members of (3.3.4) into equation (3.3.12) and of (3.3.11) - (3.3.19) into equations (3.3.4), (3.3.7), and (3.3.10) and by adding equations (3.3.4) and (3.3.7), demand equations for total linseed oil stocks¹ and oil producers' flaxseed stocks may be written (ignoring the restrictions on coefficients):

\[(3.3.20) \quad H_L(t) = \xi_1 B(t-1) + \xi_2 Y(t-1) + \xi_3 C_L(t) + \xi_4 B(t) + \xi_5 Y(t) + \xi_6 H_S(t) + \xi_7 A_S(t) + \xi_8 \left[ p_S(t) - p^A_S(t) \right] + \xi_9 p_M(t) + \xi_{10} p_F(t).\]

\[(3.3.21) \quad H^P_S = \gamma_1 B(t-1) + \gamma_2 Y(t-1) + \gamma_3 C_L(t) + \gamma_4 B(t) + \gamma_5 Y(t) + \gamma_6 H_L(t) + \gamma_7 H_S(t) + \gamma_8 A_S(t) + \gamma_9 \left[ p_S(t) - p^A_S(t) \right].\]

In estimating the inventory demand equations, the variable \([p_S(t) - p^A_S(t)]\) will be replaced by \(\frac{p^A_S}{p_S}(t)\) and the variables \(p_M(t)\) and \(p_F(t)\) by \(\frac{p_F}{p_M}(t)\). The usual trend terms, constants, and random variables will be added.

¹ There are a few independent distributors who sell oil to the small consumers. However oil held by distributors is largely held on consignment from producers and is included in the latters' reports of "warehouse stocks." (Letter from J. D. Craic, November 2, 1948.)
h. The System of Equations

There are fifteen (non-lagged) endogenous variables in the system of equations:

\( H_L \) = total U. S. factory and warehouse stocks of linseed oil in 1000's of lbs. at end of quarter.

\( C_L \) = U. S. linseed oil consumption in 1000's of lbs. during quarter.

\( H_S \) = total U. S. flaxseed stocks in tons at end of quarter.

\( H_S^P \) = total U. S. factory and warehouse flaxseed stocks in tons held by linseed oil producers at end of quarter.

\( A_S \) = annual flaxseed acreage in 1000's of acres seeded in U. S.

\( L \) = U. S. linseed oil production in 1000's of lbs. during quarter.

\( C_S \) = U. S. flaxseed consumption in tons during quarter.

\( I_S \) = U. S. flaxseed imports in tons during quarter.

\( S_S \) = domestic flaxseed in tons supplied by U. S. suppliers to linseed oil producers during quarter.

\( P_S \) = annual U. S. flaxseed production in tons.

\( \frac{P^A}{P_S} \) = quarterly Buenos Aires price (in dollars) per bushel of flaxseed deflated by quarterly Minneapolis price (1926 = 100)

\( T/P_S \) = U. S. tariff per bushel of flaxseed deflated by quarterly Minneapolis flaxseed price (1926 = 100)

\( P_F/P_M \) = quarterly index of livestock feed prices deflated by quarterly Minneapolis price of linseed meal per ton (1926 = 100)

\( P_S/P_G \) = quarterly Minneapolis flaxseed price deflated by quarterly index of grain prices (1926 = 100)

\( Y \) = quarterly index of industrial production (1923-1925 = 100).

Exogenous variables and endogenous variables which appear only as lagged are:
\( B \) = residential and non-residential construction contracts awarded
in 1000's of square feet of floor area in 37 states during quarter.

\( B^w \) = quarterly index of building activity in Argentina, France, Germany,
the Netherlands, and U. K. (1929 = 100)

\( P^A_S \) = annual Argentine production of flaxseed in tons

\( P^I_S \) = annual Indian production of flaxseed in tons

\( H^P_S \) = U. S. flaxseed stocks in tons held by non-consumers at end of
quarter.

\( A^s_S \) = annual flaxseed acreage harvested in 1000's of acres in U. S.

\( \tau \) = time, numbering each quarter successively.

\( \xi_{tt} \) = Kronecker symbol

The system of equations is specified as follows:

\( (h.1) \) Demand for linseed oil inventories by linseed oil producers and
consumers

\[
H^P_L(t) = \alpha_{11} B(t - 1) + \alpha_{12} Y(t - 1) + \alpha_{13} C_L(t) + \alpha_{14} B(t) \\
+ \alpha_{15} Y(t) + \alpha_{16} H^L(t) + \alpha_{17} A_S + \alpha_{18} \frac{P^A}{P^F_S}(t) \\
+ \alpha_{19} \frac{P^F}{P^M}(t) + \alpha_{1,10} \tau \\
+ \alpha_{1,11} + U_1(t)
\]

\( (h.2) \) Demand for flaxseed inventories by linseed oil producers

\[
H^P_S(t) = \alpha_{21} B(t - 1) + \alpha_{22} Y(t - 1) + \alpha_{23} C_L(t) \\
+ \alpha_{24} B(t) + \alpha_{25} Y(t) + \alpha_{26} H^L(t) \\
+ \alpha_{27} \frac{H^L_S(t)}{A_S} + \alpha_{28} A_S \\
+ \alpha_{29} \frac{P^A}{P^F_S}(t) + \alpha_{2,10} \tau + \alpha_{2,11} + U_2(t)
\]

1. \( t = 1, 2, 3, 4 \). When \( t = 1 \) in a given year, \( t - 1 = 4 \) in the preceding
year. The time designation for those variables which have only annual
values has been omitted. A subscript designates lags of annual variables.
The use of the Kronecker symbol in equations should be noted. This
insures that certain variables, e.g., acreage seeded and flaxseed produced, assume
zero values in appropriate quarters. Also, see p 31, n. 3.
(4.3) Linseed oil consumption demand

\[ C_L(t) = \alpha_3 \bar{B} + \alpha_3 \bar{Y} + \alpha_3 \bar{Z} + \alpha_3 \bar{U}_3(t) \]

(4.4) Linseed oil production (a technical relation)

\[ L(t) = \alpha_4 \bar{C}_S(t) + \alpha_5 \bar{Z} + \alpha_5 \bar{U}_4(t) \]

(4.5) Demand for imports of flaxseed

\[ I_S(t) = \alpha_6 \bar{P}_S(t) + \alpha_5 \bar{S}_S(t) + \alpha_6 \bar{T} + \alpha_6 \bar{U}_5(t) \]

(4.6) Supply of flaxseed imports

\[ I_S(t) = \alpha_6 \bar{P}_S(t) + \alpha_5 \bar{S}_S(t) + \alpha_6 \bar{T} + \alpha_6 \bar{U}_5(t) \]

(4.7) Domestic flaxseed supplied to linseed oil producers

\[ S_S(t) = \alpha_7 \bar{P}_S(t) + \alpha_7 \bar{S}_S(t) + \alpha_7 \bar{T} + \alpha_7 \bar{U}_7(t) \]

(4.8) Acreage seeded in flaxseed in U. S.

\[ A_S = \alpha_8 \bar{A}_S + \alpha_8 \bar{A}_{S-1} + \alpha_8 \bar{A}_S + \alpha_8 \bar{A}_S \frac{P_S(t)}{P_G} \]

(4.9) Flaxseed produced in U. S. (a behavior relation)

\[ P_S = \alpha_9 \bar{P}_S(t) + \alpha_9 \bar{P}_S(t) + \alpha_9 \bar{P}_S(t) \]

\[ + \alpha_9 \bar{P}_S(t) + \alpha_9 \bar{P}_S(t) + \alpha_9 \bar{P}_S(t) \]
\( L(t) - G_L(t) - H^L_L(t) + H^L_L(t - 1) \equiv 0 \)
\( S_S(t) - S^P_S + H^P_S(t) - H^P_S(t - 1) \equiv 0 \)
\( S_S(t) + I_S(t) - G_S(t) - H^P_S(t) + H^P_S(t - 1) \equiv 0 \).

The variables \( \xi_1, \ldots, \xi_p \) are the random components in the equations.\(^1\)

Interpretations of equations (4.1)-(4.4) have already been anticipated in the preceding chapter. There, an oil producers' demand for linseed oil stocks, a price-fixing equation, and a fixed relation between linseed oil production and flaxseed consumption were presumed. Independent behavior equations relating to linseed oil production and flaxseed consumption decisions of oil producers would be inconsistent with the theory presented and thus are not specified in the system (4.1)-(4.12).

The implied decision of producers with respect to linseed oil production and the decision with respect to flaxseed stocks determine their total purchases of seed. Since the large oil producers both purchase seed domestically and abroad, it has been assumed in equation (4.3) that for a given quantity of total purchases the amount purchased abroad for importing depends on the comparative costs of domestic and foreign seed. Since the full amount of the tariff is not reflected in the cost of using imported seed (because of the drawback), the tariff term has a separate coefficient.\(^2,\(^3\) Such influences as transportation costs and probable discontinuities in the equation resulting

\(^1\) It is conventional to assume they have zero means and a multivariate normal distribution independent of time and of exogenous and lagged endogenous variables.

\(^2\) Supra, pp. 5f.

\(^3\) Actually the proportion of the tariff drawn back per bushel of imported seed when the linseed cake equivalent is exported is the ratio of the value of the cake equivalent to the combined values of the oil and cake equivalents of a bushel of seed. This proportion has tended to remain roughly .3. (Cf. U. S. Congress, Flaxseed Prices, p. 30.)
from the geographical distribution of linseed oil plants and of the demands for flaxseed products are ignored.

The "foreign surplus" equation of text-book theory, i.e. the residual of foreign demand from foreign supply, furnished the model for equation (4.6). The use of the price ratio $\frac{P_A}{P_S}$ in the equation receives justification from earlier remarks.\(^1\) Other supply factors are annual Argentine and Indian production. To force more information out of this meager data annual Argentine production is regarded as four different variables, each having zero values in three out of four quarters of the year.\(^2\) Other things being equal, the use of this device implies typical proportions of Argentine production available on the market for each of the four quarters of the year. This procedure (of doubtful merit in any case) is not followed with respect to Indian production. Thus in effect, a given year's Indian production is presumed to have equal effect on import supplies to the U.S. in each quarter.\(^3\) Demand factors in the equation include current and lagged indices of foreign building activity.\(^4\)

Equation (4.7) states that for given values of their initial stocks of flaxseed and for a given value of current production the flaxseed offered by domestic suppliers, e.g. farmers, elevators, depends on the current and lagged ratios of flaxseed price to grain price.\(^5\) The use of the lagged ratios

---

1. Supra, p. 6.

2. In particular, it was desired to lend an element of seasonality to the supply of imports.

3. The value of a given year's Argentine production first appears in the equation when $t = 1$ in that year. The same value appears then in each of three succeeding quarters. The value of a given year's Indian production first appears when $t = 2$ of that year. The same value then appears in each of three succeeding quarters.

4. The weighting of building activity indices in individual countries was suggested in a 1939 B.A.E. study of flaxseed prices. A fairly good fit for a multiple regression of flaxseed price on an index of world building activity and world flaxseed production was obtained. (Cf. U.S. Congress, Flaxseed Prices, pp. 44, 47.)

5. Supra, p. 4.
has the usual rationale, namely, the influence of past decisions in currently observed behavior.

These same ratios as well as a scale factor influence farmers' flaxseed acreage decisions at the time of planting, equation (4.8). Acreage harvested in the previous year is used because of some evidence that acreage abandonments in a given year discourage planting in the succeeding year. This was particularly evident in the years following drought years in the thirties. A closer correspondence between acreage planted in one year and that harvested in the preceding year than between the acreage planted in two successive years.

The influence of a weather variable in the acreage decision as well as in the flaxseed harvesting decision, equation (4.9) is needed, but the choice of statistical data to use as observations on such a variable would be highly arbitrary. Weather as well as the comparative prices of crops influence the farmers' abandonments of cultivated acreage and the diligence of their efforts in harvesting particular crops.

The nine behavior and technical relations, equations (4.1)-(4.9) with the equilibrium conditions, identities (4.10)-(4.12), permit a unique solution of the first twelve of the endogenous variables listed on page twenty-seven in terms of the remaining ones. Three of the endogenous variables \( P_F, P_S, P_{EF}, P_F, P_G \), and \( P_I \) remain undetermined by the system. Completing the system would involve the examination of market relationships outside the area of this study, although presumably knowledge of additional variables appearing in such relations would increase the efficiency of the estimates of equations (4.1) and (4.2).

---

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<tr>
<th>Eqn. No.</th>
<th>Variable</th>
<th>Observed Mean</th>
<th>Coefficient of Variable</th>
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<th>Col. (3) X Col. (4b)</th>
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<td>(2)</td>
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<td>(4a)</td>
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### Table II

**Calculated Residuals Equation (4.1)**

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<th>L.S. Estimate of Equation</th>
<th>Year</th>
<th>Qtr.</th>
<th>L.I. Estimate of Equation</th>
<th>L.S. Estimate of Equation</th>
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