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PRODUCTION UNDER INCOMPLETE INFORMATION (*)

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0. Plan of this paper:

1. one-person single-period planning of production under incomplete information
2. multi-person multi-period planning and organization of production under incomplete information.

1.1 Let $\underline{a}$ denote the complete description of what happens at (and to) the plant during the production period and denote by $\underline{y}$ a point in the commodity space. Further, let α denote "decision variables," i.e., the factors over which the planner (production manager) has control.

We assume

$$y = \varphi(a). \tag{1}$$

<u>Complete information</u> is said to exist when

$$a = \psi(\alpha), \tag{2}$$

so that

$$y = \chi(\alpha) \tag{3}$$

and the decision variables determine uniquely the point y in the commodity space.

When two or more points in the y-space correspond to a given value of α <u>incomplete information</u> is said to exist. This amounts to saying that under incomplete information (2) no longer holds. We shall find it convenient to maintain (1) even under incomplete information, but (3) loses validity when (2) does.

(*) The author has greatly profited from discussions with C. Hildreth and S. Reiter concerning some of the problems discussed in this note.

Under incomplete information (2) is replaced by the postulate that, given α, $\underline{a}$ has a probability distribution $F = F(a \mid \alpha)$. However, at the time of planning the planner may only know that

$$F \in \mathcal{F} \tag{2bis}$$

where $\mathfrak{F}$ is a class (a proper subset) of F's. (1) and (2bis) imply

(3bis) $$G \in \mathfrak{G}$$

where $G \equiv G(y|\alpha)$.

1.2 In the linear programming models $\underline{a}$ is represented by the non-negative vector ("activity level") $\underline{x}$, while φ in (1) is linear homogeneous. (1) is then written as

(4) $$y = \Gamma x.$$

When dealing with models of this type we shall denote the decision variable (vector) by ξ. The dimensionality of ξ is limited by that of $\underline{x}$ as well as by the restrictions on $\underline{x}$.

1.3 Under the assumptions made the problem of model construction under incomplete information is reduced to deciding on the manner in which $\mathfrak{F}$ depends on ξ.

An example of interest is given by

(5) $$x = (x^{(1)}, x^{(2)})$$

with

(6) $$x^{(1)} = \xi,$$

while $x^{(2)}$ has a distribution independent of ξ and known to the planner at the time of planning.

We write

(7) $$y = \Gamma x = \Gamma^{(1)} x^{(1)} + \Gamma^{(2)} x^{(2)} = \Gamma^{(1)} \xi + \Gamma^{(2)} x^{(2)}$$

so that

(8) $$E(y|\xi) = \Gamma^{(1)} \xi + \Gamma^{(2)} E x^{(2)}$$

Here, in general, $E(y|\xi)$ will not be a linear homogeneous function of ξ.

Other models have been constructed and their properties studied to some extent.

1.4 Under ~~the~~ incomplete information the planner's utility function depends not on $\underline{y}$ but on the family $\mathfrak{G}$ of $G(y|\xi)$. Hence efficiency is expressed ~~xxxx~~ in terms of functions of $\mathfrak{G}$ and an efficient ξ is selected accordingly. (In particular, when $\mathfrak{G}$ contains one element only, efficiency may be expressed in temrs of maximizing certain functionals, e.g., moments of G. When $\mathcal{G}$ contains two or more elements some principle such as the minimax may have to be applied to these functionals.)

The importance of the fact that utility is a function of the probabilities lies in the fact that, in terms of probabilities viewed as "outputs," adjoining independently to a given plant its "carbon copy" does not result in doubling the "output." (Thus linear homogeneity becomes untenable in a way that is more fundamental than in (8); the latter is essentially a stochastic limitational factor.)

1.5 In the case discussed under 1.3 the planner's utility function depends on ξ and the distribution of $x^{(2)}$. If we generalize the model by assuming that the distribution of $x^{(2)}$ is only known to belong to a specified class $\mathcal{G}$, we have a situation that covers the case of a two-person game. The class $\mathcal{G}$ corresponds to the set of all possible strategies of the "opponent." The analogy with Wald's model of decision making is evident.

The two-person representation of a one-person planning under incomplete information leads to the subject matter of the second part of this note, viz. production planning by two or more persons.

2.1 Production planning, even with complete information available, may exceed the intellectual capacity of one person. One individual may lack the needed combination of skills or his speed with regard to the intake of information, etc., may be inadequate for the problem at hand.

For such reasons production planning is often done by several persons who may divide the planning functions in a variety of ways. A simple example is given as follows. There is one chief planner A and several junior planners $B_1, B_2, \ldots$ On the 28th of each month A decides on the general "strategy" for the following month while each of the B_i has a sphere of activity where he will be making independent daily decisions ("moves") in accordance with the strategy. Collecting information for A may among the functions to be performed by the B_i's.

2.2 A case of particular interest is that of incomplete information. That is, the daily moves to be made during the given month by the junior planners will be made in the light of information not available on the 28th of the preceding month when the strategy was decided upon. Hence the rules of the strategy must be conditional or sequential. (Under complete information this might also happen if the time available for deciding on next month's moves is too short in relation to the time it takes to think things through.)

2.3 In general, we assume imperfect communication among the junior planners. (The communication between A and the B_i's is also imperfect as indicated by the earlier description.) The reason is again in the intellectual limitations of the planners as well as in the imperfections of the communication media. (The points made would be valid if automatic computing machines were substituted for each of the planners.) We must expect that this decentralization will lead to losses. If perfect infinitely rapid communication among the planners were possible, the total output would be greater, or at worst as high as, under imperfect communication. (Imperfect communication means not only delay, but distortion, disturbances, etc.)

The imperfect character of communication leads to loss of information and would create conditions of incomplete information in an organization where complete information would otherwise have been obtained.

2.4 The "strategy" consists of "rules of behavior" S_i for each B_i respectively. One type of rule is to provide principles of ordering of moves which are accessible to each B_i, together with the requirement that the ~~xxxxx~~ move maximal with regard to the ordering be selected. Profit maximization at accounting (non-market) prices provides an example of such a rule. Rules of this type are used in models of the socialist economy.

The rules may turn out to be such as to make competitors out of the B_i's. The theory of the optimal features of an (idealized) perfectly competitive economy can be classified under this heading.

2.5 If the junior planners, instead of being obedient automata, are endowed with independent utility functions, and if the enforcement of the rules of behavior is impossible or undesirable, it may be necessary to give the B_i an incentive to follow the rules.

For instance, in the case of a profit maximization rule the reward of each of the B_i might be a monotone function of the profits to be maximized ("profit-sharing").

2.6 On the basis of the preceding considerations it is possible, in principle at least, to find the optimal policies for a plant a specified type of external environment, as well as the optimal organizational structure, given the technological and intellectual limitations (the latter covers processing and transmittal of information.) A theory of the optimal size of the plant could be constructed on this basis. (Let there exist a "basic plant" P and suppose that it is possible to duplicate it exactly as many times as desired, within the range of interest. Assuming that the P's can be operated independently, we can always get double output from double input by using two P's instead of one. But intercommunication between two P's might conceivably yield returns higher than to scale. The highest number of P's which it is worth-while to operate in a hook-up, i.e., with intercommunication, yields, under simplified assumptions, the optimal size of the plant. The firm, as distinct from the plant, might well ~~xx~~ embrace several plants.)