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Economic Controls Based on Technological Data and Information Theory

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One of the principal results of the theory of linear production models is the characterization of efficient production by interchange ratios of the commodities in the production bill of goods (where the bill of goods includes both inputs and outputs). These interchange ratios are equivalent to a set of prices whose origin is wholly technological in nature. They are all positive and their ratios give marginal rates of substitution of the commodities for efficient production. Such technological interchange ratios can be normalized, through multiplication by some appropriate factor, into a set of prices of the commodities of the bill of goods; and as such they are defined without reference to any market in which exchange takes place. They originate wholly on the supply side as a result of considering the alternatives in efficient production which present technology offers.

If we consider linear production models as effecting a mapping from an activity space $x > 0$ into a commodity space y by means of a matrix operator Γ, where $y = \Gamma x$ then we can isolate the decisions about production in x, the technology in Γ, and the resulting bill of goods in y. (y and x are vectors of different dimensionality in general.) We can now characterize points $\tilde{y}$ as achievable if there is an $x > 0$ which, with technology Γ will produce y; and further we can choose from the set of achievable points some which are efficient in the following sense: that whenever an increase of one of the coordinates of $\tilde{y}$ (the net output of one good) can be achieved only at the cost of a decrease in some other coordinate (the net output of another good) the point (bill of goods) is said to be efficient.

In general there are a large number of efficient points and one of the principal objects of this paper is to present criteria for choosing production points from the set of efficient points, which in turn is a subset of the set of achievable points.

But application of the criterion of efficiency serves to eliminate wasteful modes of production, so it will be necessary to state a criterion for efficiency. The following theorems, stated without proof here, provide such a criterion.*

1. A necessary and sufficient condition that the activity vector x shall lead to an efficient point $y = \Gamma x$ in the commodity space is that there exist a vector p of positive prices such that no activity in the technology permits a positive profit (where profit is defined as $\pi_i = \Sigma a_{ij} p_j$ and a_{ij} are the technological coefficients of the activity) and such that the profit on all activities carried out at a non-zero level shall be zero.

2. In an efficient point in an N dimensional commodity space, at most N-1 linearly independent activities can be carried out simultaneously at positive levels.

3. The elements of the vector p define marginal rates of substitution of the commodities.

4. For a given technology matrix Γ there are various sets of efficient points in general, each set being characterized by its own unique p vector (technological prices). Since the set of achievable points forms a polyhedral cone in the commodity space, and the efficient points are on the faces or facets of this cone, we shall call these various sets of efficient points facets.

We shall denote by x_+ the subvector of x which takes on positive values on a facet. By Γ_+ we shall mean that submatrix of Γ corresponding to x_+.

The problem of choosing an operating point y within any institutional framework is twofold: First an objective must be set up, and secondly economic controls must be instituted to implement the objective. The control system must specify what information is to be communicated and how the principals are to act on this information. What is desired is a set of rules which will provide for originating information about the working of the economy, for the transmission of this information to the principals,

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* For proofs see T. C. Koopmans, "A Mathematical Model of Production," Economics 262, A, B, C.

and for decision making criteria. In setting up such rules technological prices - the vector p - can be used. They may be used as part of the information which is to form the basis of decisions. Even more important, they can serve as guides in setting up the rules so as to insure efficient production, that is, to judge the merits of a control system.

We shall consider economies that buy and sell their bill of goods from and to an exterior economy. For example a private firm is such an economy. A national economy is another example, for it buys raw materials and labor from the population and sells its products to the population. There is a fundamental difference from the control point of view between situations where such segmentation is present, and those where it is not, because segmentation introduces the necessity of communication. Where there is no question of adjusting to an exterior economy the economic problem is one of programming to required objectives and the only question is what is the most economical way of producing a bill of goods with a given technology. There is no exterior population whose desires (in trading) it is necessary to consider. Such a problem (approximately) is the problem of national economic programming in a war when the effect of the programming on the exterior economy - the national population - is of importance secondary to producing the required bill of goods. Programming with given technology is essentially an algebraic problem. However if decentralization of decision or other types of segmentation of the economy are desired, even in programming the information problem becomes critical.

Now linear production models permit a definition of technological prices only for efficient points. Hence if the exterior economy to which the interior one must adjust desired production to be non-efficient these prices would not have much value. The following assumption will be made: Both in the case of the firm and the national economy desires of the exterior economy are tha/such that for any non-efficient point there is always an efficient point that is more desireable. That is, the more of a given product we produce without added expense, the greater the resulting satisfaction. There are special instances in which this assumption does not conform to the real world e.g. demand for diamonds and gold, commodities valued because of their scarcity, but by and large this assumption holds. So we know that the exterior economy wants production on an efficient facet.

Consider first of all an economy consisting of activities where the manager of each activity buys its inputs from and sells its outputs to the exterior economy at prices $p_i(t)$ for the i^{th} commodity in time period t. (If the real world had linear production functions a factory could be considered as a group of activities with a single management for all of them.) First we shall assume that the particular facet that the exterior economy wants the interior economy to operate on is known. Assume further that this facet corresponds to technological prices P_i(P_i can be calculated from Γ_+ in a known manner). We can now introduce a central information agency which has overall knowledge of the interior economy, knows Γ_+ and P_i, and announces P_i to all the managers. The first rule to the activity managers is

1. Operate only if your activity has zero profit with respect to the technological prices P_i, where the profit of the k-th activity is $\bar{\pi}_k = \Sigma\, a_{ik} P_i$ and a_{ik} $i = 1, \ldots N$ are the technological input-output coefficients of the activity.

If $\bar{\pi}_k < 0$ do not produce.

This rule will assure that production takes place efficiently and on the desired facet if P_i are calculated correctly by the central information agency. The technological prices are given as ratios. We shall normalize them by requiring $\sum_i P_i\, y_i(t) = \sum_i p_i(t)\, y_i(t)$ so that the normalizing factor depends on the time period.

Now the problem arises of choosing the efficient point desired by the exterior economy if the interior economy is operating according to some rule. This desired point will vary with the rule followed by the interior economy. If the interior economy is the government of a welfare state its rule will be to make zero profits. If the interior economy is a private firm its rule will be to maximize profits, for example. The resulting desired operating points (desired operating point = equilibrium operating point) will very with the rule. We are not concerned with inputs limited by nature because the burden of supply is placed on the exterior economy, and any limitations of nature will be reflected in the trading prices $p_i(t)$. As an input becomes increasingly unavailable the price $p_i(t)$ will increase very rapidly. We are not dealing with free goods and every commodity must be paid for.

We shall sharpen the idea of the institutional setting by setting up costless sales and procurement managerships. The L-th sales manager of the i-th output receives the total output from all activities at the beginning of the t-th period. The j-th procurement manager receives total orders for the j-th input at the beginning of the t-1-th period. The sales managers use the following rules:

1) Set current market prices (with the external economy) to sell all of current delivery. On the basis of daily sales and remaining stock raise or lower current prices so that everyone who wants to pay the current price can buy your product and there will be none remaining.

2) At the end of the t-th period compute an average $p_i(t)$ for the commodity and announce these prices.

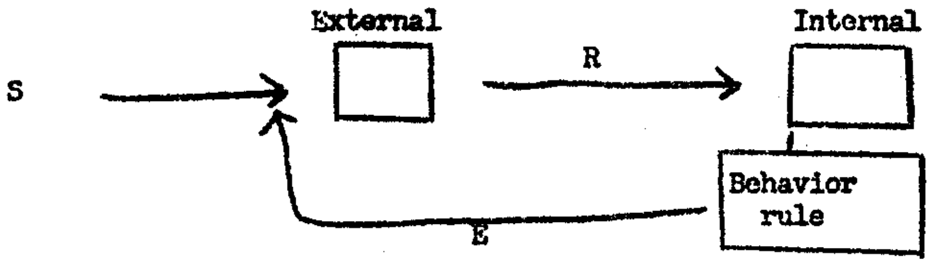
The procurement managers follow similar rules:

1) Buy the inputs at a market price - gauged on the basis of daily purchases and remaining unfilled orders - such that everyone in the external economy who will sell for this price has his input bought, and all the orders will be filled.

2) Announce the average prices to the activity managers.

Thus the activity managers get $p_i(t-1)$ at the beginning of the t-th period.

In order to formulate rules for the activity managers that will move production to the desired operating point it is necessary that they be able to predict the reaction of the prices $p_i(t)$ in exchange with the external economy, which follow their own moves. This necessity for approximate prediction is a general concept of communication theory, when applied to an information feedback system. This can be demonstrated as follows:



S = exogenous stimulus

R = reaction of entity to be controlled

E = endogenous stimulus (feedback)

In our case S is the "force" that determines the prices $p_i(t)$, that is the desires of the external economy. R is $p_i(t)$ and E is the change in activity levels on the facet which results in a new operating point.

Let us define equilibrium in an information system as a situation in which, when S is any stimulus from a set $\bar{S}$, then for any stimulus S $\quad \bar{\bar{S}}$, S and R will satisfy the following inequality symbolically

$\left| R - R_o \right| < \eta (S, t)$ where R_o is a constant and η becomes arbitrarily small as t increases: S can be a matrix of stimuli or a series of stimuli as well as an individual stimulus. S will not necessarily disappear as $t \rightarrow \infty$ and will in general approach S_o. S_o, R_o are then called equilibrium values of stimulus and reaction.

Now in general $R = R(S)$ and with feedback

$R = R(S + E)$ and for equilibrium $R_o = R(S_o + E)$

If prediction is possible $E = f(R)$, otherwise E is a random variate. When $E = f(R)$ the equilibrium condition is $R_o = R\left[S_o + f(R_o)\right]$ and there is a mapping from R_o to S_o. I conjecture that if E is not a function of R but a random variate there will be no equilibrium in the information system. Consider, for example, a system in which $E = K_1 R$ and $R = K_2 S'$ where $S' = S + E$. Then for equilibrium

$$R_o = K_2 \left[S_o + K_1 R_o\right] \qquad \text{or} \qquad R_o = \frac{K_2}{1-K_1 K_2} S_o$$

In such an explicit system we can also ask about the stability of the equilibrium.

$$\Delta R_o = \frac{K_2}{1-K_1 K_2} \Delta S_o \quad .$$

If K_2 is small and k_1 large (small reaction and large feedback). $\Delta R_o \sim \frac{1}{K_1} \Delta S_o$ so that the equilibrium is stable in the sense that deviations from equilibrium are minimized. On the other hand if $K_1 = \frac{1}{K_2}$ ΔR_o will be infinite and the equilibrium is completely unstable. So it is not even sufficient to guarantee that $E = f(R)$ for equilibrium but it must be a particular type of function.

Now suppose E is a random variable. Then consider the information in successive time periods.

$R(0) = K \; S(0)$ $\qquad$ S(0) a constant stimulus, and the game is to change $R(n)$ to a desired $R(n) = Rd$

$$R(1) = K \left[S(0) + E(1)\right]$$

$$R(2) = K \left[S(0) + E(1) + E(2)\right]$$

$$R(n) = K \left[S(0) + \sum_i E(i)\right]$$

and unless $\Sigma E(i)$ converges R(n) will not converge. Unless $\Sigma E(\infty)$ converges to $\left[\Sigma E(i)\right]^*$ where $\left[\Sigma E(i)\right]^*$ satisfies $Rd = K \left[S_o + \left[\Sigma E(i)\right]^*\right]$ the information system is ineffective. So ordinarily a random feedback response does not lead to equilibrium (convergent R(n)) and it is even more improbable to obtain the desired equilibrium. Hence for this simple model $R = KS$ we have shown that approximate prediction $E = f(R)$ is necessary. A general information system requires $E(t) = f\left[R(t)\right]$ not $f\left[R(t-1)\right]$ so it is really prediction, and not just a past response that is necessary. What sort of prediction will we have? We will use the preinformation that the external economy demand curves are of the type

$$p_i(t) - p_i(t-1) = c_i(t) \Delta y_i(t) \quad \text{for sufficiently small } \Delta y_i(t).$$

If, for instance, p is a continuous function

$$p_i(t) = p_i \left[p_1(t) \dots p_{i-1}(t), p_{i+1}(t) \dots p_N(t), y_i(t), t \right] \text{ or alternatively}$$

$$p_i(t) = p_i \left[y_1(t) \dots y_i(t) \dots y_N(t), t \right]$$ we can expand in a Taylor series in $y_i(t) = y_i(t-1) + \Delta y_i(t)$. Now we shall assume that $p_i(t)$ is more sensitive to $y_i(t)$ than to the other quantities so that

$$p_i(t) = p_i(t-1) + c_i(t) \Delta y_i(t) + c_i^{(2)}(t) \left[\Delta y_i(t) \right]^2 + \Sigma K_j^{(2)}(t) \left[\Delta y_j(t) \right]^2 \qquad \text{i.e., that}$$

$p_i(t) - p_i(t-1)$ is in first order dependent only on $\Delta y_i(t)$ and dependent on the other increments only in higher order terms. If we choose sufficiently small $\Delta y_i(t)$'s we can write

$$p_i(t) - p_i(t-1) = c_i(t) \Delta y_i(t)$$

Since Γ_+ has been chosen for this model we know

$$y(t) = \Gamma_+ X(t)$$

We now have $p_i(t) - p_i(t-1) = c_i(t) \Delta y_i(t)$.

Let c(t) be a diagonal matrix with $c_i(t)$ as the ii-th element. Then

$$\Delta P(t) = P(t) - P(t-1) = c(t) \Delta y(t) \text{ in vector notation.}$$

We shall also write $X_i(t) = D_i(t) X_i(t-1)$ where $D_i(t)$ represents the rule used in shifting the activity levels. $D_i(t)$ may depend on any data known up to the t-th period. Letting $D_i(t)$ be the ii-th element of a diagonal matrix. D(t) we can write in vector notation

$X(t) = D(t) X(t-1)$ where the operator D(t) can now be called the t-th period rule operator.

For the resulting change in operating point y we have

$$\begin{aligned} \Delta y(t) = y(t) - y(t-1) &= \Gamma_+ X(t) - \Gamma_+ X(t-1) \\ &= \Gamma_+ \left[X(t) - X(t-1) \right] \\ &= \Gamma_+ \left[D(t) - 1 \right] X(t-1) \end{aligned}$$

Since $\Delta P(t) = c(t) \, \Delta y(t)$

$$\Delta P(t) = c(t) \, \Gamma_+ \left[D(t) - 1\right] X(t - 1)$$ and by successive applications

$$\Delta P(t) = c(t) \, \Gamma_+ \left[D(t) - 1\right] \left[D(t - 1) - 1\right] \ldots \left[D(1) - 1\right] X(0)$$

The conditions that a suitable rule $D(t)$ must satisfy are those of equilibrium of the information system; that is,

1) $D(t) \longrightarrow 1$ for large t, preferably that $D(t) \longrightarrow 1$ "fast"

2) $\Delta P(t) \longrightarrow 0$ for large t, preferably that $\Delta P(t) \longrightarrow 0$ "fast"

3) $D(t)$ must satisfy the requirements (objectives) of the interior economy.

Unless the demand curves have a very rapidly varying slope (actually an infinite slope) with respect to their principal variables $c(t)$ will be finite. Furthermore unless the slopes change rapidly from one time period to the next they may be considered, in first approximation, constant. If $D(t)$ ever equals 1 the system becomes static. And if condition 1) is satisfied condition 2) will be satisfied.

Let us look at some rules $D(t)$. First consider the zero profit rule for a national economy. A possible rule is

$$D_i(t) = 1 + \frac{\sum a_{ij} \, p_j(t - 1}{N_i} = 1 + \frac{\pi_i(t - 1)}{N_i}$$

$= 1 +$ a quantity proportional to profit or loss in last period.

The normalizing factor is chosen to meet the requirement that $\Delta y_i(t)$ be small.

If $\Delta P_i(t) = c_i(t) \, \Delta y_i(t)$ has the usual characteristics of demand and supply curves in which $c_i(t)$ is negative in sign for demand curves and positive in sign for supply curves for the exterior economy, the process converges. In Hicks terms this means that the substitution effect of a price change predominates over the income effect, so that even in the case of "inferior goods" on the demand curves $c_i(t)$ is negative. The situation is more equivocal on the supply side, where $c_i(t)$ may be negative if the income effect predominates. In examining the above rule we shall assume the substitution effect is greater in all cases in the range of prices and outputs considered, realizing that otherwise the rule fails.

If an activity makes a profit the rule D(t) will increase the activity level and more of its generally profitable commodities will be produced. Then prices will go down and $\frac{\pi_i(t-1)}{N_i}$ will decrease. Similarly if the activity makes a loss the activity level will be decreased and less of its generally unprofitable commodities will be produced. Their prices will go up (or on supply side, their prices will go down) so that $\frac{\pi_i(t-1)}{N_i}$ will increase. In either case $D_i(t) \longrightarrow 1$ and

$\Delta P(t) = c(t) \Gamma_+ \left[\frac{\text{Profit}}{N_i}\text{ matrix}\right]_t \left[\frac{\text{Profit}}{N_i}\text{ matrix}\right]_{t-1} \cdots$ X(0) will

approach zero. The third condition, that of satisfying the internal economy objective is satisfied since the rule acts to decrease profits. Zero profits at any time period will make the situation static.

This analysis is only qualitative, i.e., it doesn't tell us how fast the rule will make the information system converge to equilibrium. This can only be calculated if the $c_i(t)$ and the initial operating point $y(0) = \Gamma_+ X(0)$ and initial prices $P_i(0)$ are given. However, it may be possible to set up statistical criteria for judging the merits of various rules (how fast they will bring convergence) without knowing these.

Consider now a private firm whose objective in dealing with the exterior economy is profit maximization. A rule D(t) which can be considered is the following

$$D_i(t) = 1 + N_i \frac{\pi_i(t-1) - \pi_i(t-2)}{\pi_i(t-1)} \quad \text{where}$$

N_i is again a normalizing factor, perhaps time dependent, which assures small Δ y's. Large profits make for small increases in the activity levels, unless the rate of increase of profits is large. We consider four cases

Large profits and large profit increase (decrease) $\sim$ medium activity expansion (contraction)

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It is obvious that more sensitive rules can be formulated. For example

$$D_i(t) = 1 + N_i \frac{\left[\pi_i(t-1) - \pi_i(t-2)\right]^2}{\pi_i(t-1)}$$ or less sensitive rules

$$D_i(t) = 1 + N_i \left[\frac{\pi_i(t-1) - \pi_i(t-2)}{\pi_i(t-1)}\right]^2$$

Making the same assumptions about the supply and demand curves of the external economy as before, one can show in a similar way that this rule will make the process converge. Again, however, it is necessary to know $c_i(t)$, $p_i(0)$, $y_i(0)$ to make statements about the rapidity of the convergence. Up to now we have assumed a set of technological prices known to the activity managers, i.e., it was known on which facet the internal economy was to operate, and it operated efficiently with N-1 activities at most at positive levels. A more general problem is the following: If the internal economy does not know on which facet to operate and begins operating inefficiently (with more than N-1 activities, and without a set of positive interchange ratios), specify a rule that will lead to efficient production, and seek out the facet preferred by the external economy. This is analogous to pure competition where the activities receive no information but the market prices.

Notice that the procedure given before for shifting the X vector and with it the bill of goods did not involve the technological prices so it can be retained. If we choose N-1 or fewer activities with positive technological prices we are assured of efficient production. Further more we can adjust the X vector so as to best satisfy the external economy on this facet. But how do we know what is the preferred facet. We will assume that the activities which, after an infinite number of adjustments, have collectively the most profit, are preferred. By the assumption about the indifference maps we know that the exterior economy prefers a situation with N-1 or fewer activities operating. So if we simply use the same rules and decrease or increase the activity levels till

N-1 or fewer activities have profits much larger than the other activities (which operate at very small or virtual levels by this time) we will be on the preferred facet. If the objective is zero profits the operating activities will have near zero profits; the virtual profits of the other activities will be more negative. If the objective is to maximize profits the market prices $p_i(t)$ will give non-zero profits $\pi_i(t)$ for the operating activities which are "much" larger than the non-operating virtual profits.

Now however, the problem becomes one of statistical inference. We need a criterion for which activities give small profits because they should not be operated on the desired facet, and which give small profits as a result of divergences from the preferred operating point on the preferred facet. After a large number of adjustments i.e., for large t, the problem disappears in the case of zero profits because the divergences between the zero profit activities and the negative profit activities continues to grow with t, if the information system tends to equilibrium. (Even here it is desireable to separate efficient from non-efficient activities for smaller t.) But in the case of profit maximization perhaps 2N activities will yield positive profits, and since we don't know how much profit each of the efficient activities will yield if operated on the preferred point, we don't know whether to choose the N-1 activities with largest profits, or the N-10 with larger profits, or even whether the 2N-th in rank of profits may not be one of the efficient activities at the preferred operating point.

The problem in statistical inference is the following: Given a set of variables with a joint probability distribution (the profits) which are linear forms of the $p_i(t)$, $\pi_j(t) = \Sigma\, a_{ij}\, p_i(t)$ and the value of $\pi_j(t)$ depends on the $p_j(t)$ which are jointly determined. $P_j(t) = \tilde{p}_j + E_j(t)$ where $\tilde{p}_j$ is the market price at the preferred operating point y and $E_j(t)$ is a disturbance due to the initial conditions and the way the rule D(t) operates. Consider first a rule whose objective is zero profits.

Then $\tilde{p}_j = P_j$ the technological prices. For the activities on the facet $\pi_i(t) = \Sigma a_{ji} E_j(t)$ and for large t $E_j \longrightarrow 0$ so $\pi_i \rightarrow 0$ as $t \rightarrow \infty$. But for those not on the preferred facet $\pi_i \rightarrow \Sigma a_{ji} P_j < 0$. However, when $\pi_i(t) < 0$ we must decide if $\pi_i(t) = \Sigma a_{ji} P_j(t)$ has its value due to P_j part or due to $E_j(t)$ part. We can set up the null hypothesis

$$\pi_i(t) = 0 \qquad t \longrightarrow \infty \qquad \text{for } i \in \text{ facet activities}$$

and other $\pi_j(t) \quad 0 \qquad$ for $t \longrightarrow \infty$

which we test given $\left\{\pi_i(t)\right\}$ for small t. (If the $\pi_i(t)$ converge rapidly this is not a pressing problem.) If we know the distribution of the $E_j(t)$ we can set up fiducial limits and calculate the power of the sampling test inherent in successive applications of the rule D(t).

When the objective is profit maximization the problem can be formulated this way: Given the ordered set of profits $\left\{\pi_i(t) \text{ ordered}\right\}$ e.g.

$$\pi_7(t) = 150$$

$$\pi_{225}(t) = 148.5$$

$$\vdots$$

$$\pi_{90}(t) = -500$$

set up the null hypothesis that for $t = \infty$ $\sum_{S} \pi_j(t) > \sum_{T} \pi_j(t)$ where S is the given subset of activities considered (less than N-1 in number) and T is any other subset of N-1 or fewer activities. Again if we know the distribution of the $E_j(t)$ we can set up fiducial limits and calculate the probability of accepting each null hypothesis if it is false, and of rejecting it if it is true, on the basis of the observations $\left\{\pi_i(t)\right\}$.

This inference problem inherent in such a system without technological prices given as information, can be rephrased as follows:

Given $\{\pi_i(t)\}$ find a rule R such that R $\pi_i(t)$ rejects some $\{i\}$ and accepts others in such a way that $\{i\}$, the accepted activities, converge to a fixed set under repeated application of the rule.

$$R \cdot R \cdot R \cdot R = R^n \{\pi_i(t)\} = \{i\} \quad \text{and}$$
$$R(R^n \{\pi_i(t)\}) = \{i\}$$

The key to the problem of course is to relate the joint distribution of $E_j(t)$ to the rule.

This complicated inference problem arises whenever it is necessary to find the desired facet. In a large private firm, it may be presumed that the desired facet is prescribed. In a planned national economy the facet may also be prescribed, thus obviating the inference problem. In a planned national economy like Lange and Lerner's, where the facet is prescribed but the objective is zero profits the inference problem is important if D(t) does not converge quickly. But in a competitive economy with the objective of maximum profits for each of its segments in its relations with the other segments the inference problem is the most difficult, because even a rule that leads to rapid convergence may not spread the $\pi_i(t)$ sufficiently to make obvious which subset of the activities S has a maximum $\sum_S \pi_i(t)$ for t large.

II. Costs and Controls

Thus far we have not considered the usual controls at all - unit costs. This is because the notion of a bill of goods profit or an activity profit is more general than the concept of profit in a particular commodity. It is understandable that unit costs are used exclusively in accounting practice, because it is only necessary to compare the unit cost with selling price to find out if the particular commodity is profitable. But often this gives misleading information because it is non-operational information. The purpose of information devices - controls - is to change the operating point y in accordance with some predetermined aim. While bill of goods information can always be used for this purpose - since you can vary the level of the activity as a whole - it is not always possible to change production of just one commodity, and it is useless to think of one commodity at a time. If we know that item A is making a loss but cannot change production of this item without changing production of thor items, this knowledge - the unit cost - is useless for control purposes. This simple fact is often ignored in accounting practice in favor of the touchstone of a unit cost. A simple example is a factory which can carry on only one activity (in the linear production model sense). Here unit costs, by whatever devious processes arrived at, are useless. An index must be found for the activity as a whole since only the activity level can be varied. If information is not capable of being operated on, it is useless for control purposes.

Thus if $z \leq N-1$ activities are in the system no more than $z + 1$ unit costs can be useful, for this is the maximum number of commodities whose outputs can be varied. That is, if $y = \Gamma_z X_z$ we can eliminate the x's - taking due cognizance of the condition $X > 0$ by limiting the range of y - by substituting the x's from the first z equations into the other equations so that

$$\begin{aligned} y_{z+1} &\sim y_1 \cdots y_z \qquad \text{Linearly} \\ y_{z+2} &\sim y_1 \cdots y_z \\ y_N &\sim y_1 \cdots y_z \end{aligned}$$

Specifying $z + 1$ unit costs fixes all the y's. z unit costs specify $y_1 : y_2 : \ldots y_z$ according to some rule of operation (profit maximization for instance) up to a scalar factor. Specifying another unit cost fixes this scalar factor, and hence all the y's are determined by $z + 1$ unit costs.

If N-1 activities are operated on an efficient facet, unit costs of production became uniquely defined from an operational point of view. One method of obtaining these costs so as to adjust all the commodities to a desired operating point is the following method, which is selected because its asymmetrical treatment of inputs and outputs is analogous to the accounting practice of ascribing unit costs to outputs - as opposed to the symmetrical technological prices discussed earlier, which is most useful when the bill of goods (the activity) is considered as a whole.

Consider Γ_+. Multiply each of the input rows across by $P_I(t)$, its market price. Call this new matrix $\bar{\Gamma}_+$. Now since $\det\left[y\ \Gamma_+\right] = 0$ we get (in analogy to the method of obtaining the technological prices $\sum_I c_I M_I = \sum_0 c_o y_o$ where 0 refers to outputs, I to inputs, and $M_I = -p_I(t)\ y_I(t)$ = expenditures on input I in t-th period. The c_o are in the ratio of the unit costs of outputs and the c_I will be given only a normalizing interpretation. If we define a "center of mass" of the M's by $\bar{c} \sum_I M_I = \sum_I c_I M_I$ then

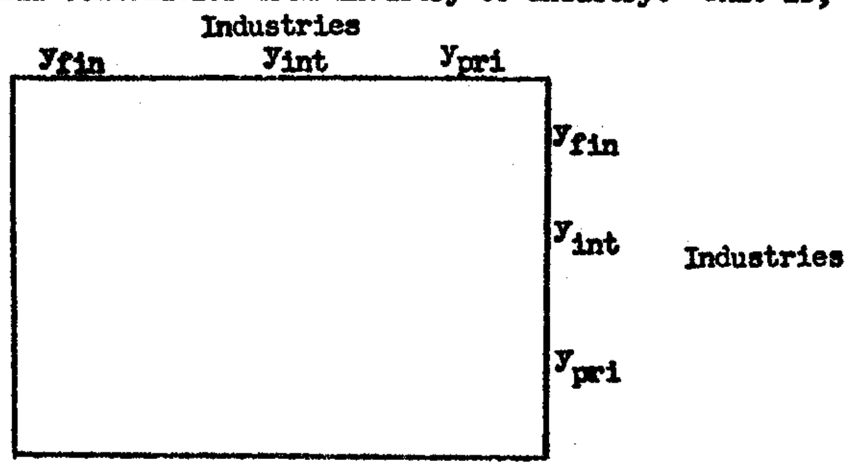
$$\sum_I M_I = \sum_o \frac{c_o}{\bar{c}}\ y_o.$$

This equation shows that the total money cost of production is divided among the y_o by the weights i.e. the unit money costs $c_o/\bar{c}$. These unit costs, defined for efficient production, depend on the facet and on the particular inputs and their prices. They satisfy the usual definition of unit costs in a linear production system

$$\frac{\partial}{\partial y_o}\left(\sum_I M_I\right) = c_o/\bar{c} \quad = \text{unit cost of 0-th product.}$$

It is possible, however, to define unit costs operationally in the ordinary sense even for a single activity, or even for non-efficient production. This can be achieved for a particular kind of production situation which appears to be very common; one assumes that a production situation resulting in an activity can be broken down into a number of intermediate activities. As an empirical observation many activities found in industrial situations are continuously variable, for practical purposes. That is, they can be built up from more fundamental activities. After all, if the activities to be considered are few enough in number for practical calculations, even in accounting, they must be composite activities, so the idea of a hierarchy of activities implied in this method should not be strange.

A shop producing steel tools can be subdivided into a materials handling activity, a rough shaping activity, a die activity, a machining activity, a finishing activity, an assembly activity, and a packing activity. Some of these headings really represent several activities because of the several types of products. Such a breakdown can be adequately represented in an open Leontif model. Each activity or department can be considered one of Leontif's industries, and we consider the transfer of primary, intermediate, and final commodities from industry to industry. That is,



to each activity (or department) we associate one of y_{fin}, y_{int}, y_{pri}. The choice of y_{int} depends on the department chosen (we are speaking strictly from the point of view of practical calculation now) while y_{fin} and y_{pri} will necessarily be in some department. Let X's represent the operating levels (or activity levels) of the departments. It is assumed that these departments exist in the organizational structure of the shop, in the sense that their expenditures can be traced. These expenditures, or budgets, might be used to measure the operating levels X. Then, as in any open Leontif system

$$X_1 = A_{11} y_1 + A_{12} y_2 + \ldots + A_{1N} y_N$$
$$X_2 = A_{21} y_1 + A_{22} y_2 + \ldots$$
$$- - - -$$
$$X_N = A_{N1} y_1 + A_{N2} y_2 + \ldots + A_{NN} y_N$$

Now we set the $y_I = 0$ and since $\sum X_i$ = total production cost, $\sum_i A_{ij}$ = cost of production of output j, for $\sum_i A_{ij} y_j$ = that part of the shop's output which is due to final output of y_j units of commodity j. Thus the possibly of carrying on a further segmentation of any activity due to its complex nature of production makes possible the attributing of unit costs which are operational in the sense that the operating levels, the budgets, can be varied to vary the final hierarchy activity and the final bill of goods.

In retrospect this paper seems to raise problems instead of solving them, but it presents a point of view on economic control based on technological data and on information theory which may be valuable.