

Models of International Trade and the Effects of Depreciation

by

Arnold C. Harberger¹

1. With the work of applying Keynesian technique to problems of international trade by now well advanced², it seems appropriate to investigate one of its major limitations: namely, its neglect of price relations. Such an investigation can also be considered as an attempt to generalize the classical supply-and-demand analysis to take account of income relations. A recent paper by J. Tinbergen³ has broken the ground for a development of this type by suggesting models of international trade in which both prices and incomes appear as explicit variables. Tinbergen, however, does not consider the implications which acceptance of this type of model would have in regard to current notions of the effects of chosen disturbances on variables considered to be important.

The purpose of ^{the} present paper is to show how existing ideas as to the effects of depreciation on the balance of trade and on income and employment must be revised if models of this generalized type are considered preferable

1. I wish to express my indebtedness to L. A. Metzler and K. J. Arrow for helpful comments and suggestions.

2. The literature in this area is abundant. Only some of the outstanding early work is cited here: J. J. Polak, "The International Propagation of Business Cycles", Review of Economic Studies, Vol. VI, (1939), pp. 79-99. L. A. Metzler, "Underemployment Equilibrium in International Trade," Econometrica, X (1942), pp. 97-112. "The Transfer Problem Reconsidered," Journal of Political Economy, L (1942), pp. 397-414. F. Machlup, International Trade and the National Income Multiplier, (Philadelphia, 1944).

3. J. Tinbergen, "Models of International Trade," Giornale degli Economisti, VII (1948) pp. 627 ff.

to those currently in use. This application is particularly relevant because the traditional discussion of the effects of depreciation on the balance of trade is framed in terms of a "classical" demand-and-supply model (in which incomes are assumed constant), while the Keynesian discussion of the effects of depreciation on employment assumes that the classical conditions for improvement in the trade balance are directly applicable to a model in which incomes are variable and prices are assumed constant. It will be shown below that the transition between the classical and Keynesian cases is not without pitfalls. Specifically, it will be shown that, within the framework of the model here suggested, the traditional conditions for improvement in the balance of trade are not applicable when income is variable, and further, that the conditions for increased employment through depreciation are not the same as the conditions for improvement in the balance of trade.

2. The model which provides the basis for the current theory of the effects of depreciation consists simply of supply of export and demand for import equations in which the only variables are the quantities and absolute prices of the goods involved. Thus, stating the price of each good in the currency of the country in which it is produced, and choosing units of commodities and units of currency so that at the point where depreciation is contemplated p_1 , p_2 , and $k = 1$ (where k is the exchange rate), we have

$$\left. \begin{aligned} x_1 &= \bar{T}_1 + V_1 p_1 = \bar{\delta}_2 + \theta_2 p_1 k \\ x_2 &= \bar{\delta}_1 + \theta_1 \frac{p_2}{k} = \bar{T}_2 + V_2 p_2 \end{aligned} \right\} \text{Supply } (\bar{T}, V) = \text{Demand } (\bar{\delta}_1, \theta)$$

Differentiating with respect to k ,

$$\frac{dp_1}{dk} = \theta_2 ; \quad \frac{dx_1}{dk} = \frac{V_1 \theta_2}{V_1 - \theta_2} ; \quad \frac{d(p_1 x)}{dk} = \frac{x_1 \theta_2 + V_1 \theta_2}{V_1 - \theta_2}$$

$$\frac{dp_2}{dk} = \frac{-\theta_1}{\sqrt{2} - \theta_1}, \quad \frac{dx_2}{dk} = -\frac{\sqrt{2} \theta_1}{\sqrt{2} - \theta_1}; \quad \frac{d(p_2 x_2)}{dk} = \frac{-x_1 \theta_1 - \sqrt{2} \theta_1}{\sqrt{2} - \theta_1}$$

The relation between slopes and elasticities, together with the fact that we have chosen our units so as to make $p_1, p_2 = 1$, give us the following elasticities of supply and demand.

$$\text{Define } \eta_1 = \frac{\theta_1}{x_2} \quad \eta_2 = \frac{\theta_2}{x_1} \quad \omega_1 = \frac{\sqrt{1}}{x_1} \quad \omega_2 = \frac{\sqrt{2}}{x_2}$$

Since the balance of trade of country 1 can be written (in terms of the currency of country 2) as $B_1 = x_1 p_1 k - x_2 p_2$, the change in the balance of trade as a result of depreciation is

$$\frac{dB_1}{dk} = x_1 + x_1 \frac{\eta_2 (1 + \omega_1)}{\omega_1 - \eta_2} + x_2 \frac{\eta_1 (1 + \omega_2)}{\omega_2 - \eta_1}$$

We shall assume throughout this paper that trade was originally balanced i.e.,

$x_1 = x_2$ ⁴, so that our expression becomes

$$\frac{dB_1}{dk} = k_1 \left[1 + \frac{\eta_2 (1 + \omega_1)}{\omega_1 - \eta_2} + \frac{\eta_1 (1 + \omega_2)}{\omega_2 - \eta_1} \right]$$

Since we assume country 1 to depreciate, and since in our notation k is that number by which the price of a good in country 1 must be multiplied to yield its price in country 2, improvement in the balance of trade for country 1 requires that $\frac{dB_1}{dk}$ be < 0 . If elasticities of supply are assumed to be ≥ 0 , and elasticities of demand ≤ 0 , it can be shown that, in the absence of knowledge about the elasticities of supply, we can be sure that improvement in the

4. This assumption can be avoided by inserting a term representing the ratio of $\frac{x_2}{x_1}$ at the start. But if trade is not originally balanced, the conditions for improvement in terms of home currency are not the same as those for improvement in terms of foreign currency. Cf. A. O. Hirschman, "Devaluation and the Trade Balance: A Note", Review of Economics and Statistics, February, 1949, pp. 50-53.

balance of trade of country 1 will result only if the sum of the elasticities of demand is < -1 .⁵

This, the current theory concerning the effects of depreciation, has been clearly elaborated by Joan Robinson. Mrs. Robinson and others, however, go on to utilize the above results in considering whether depreciation will help to relieve unemployment. They thereby make two assumptions:

a) that the condition for improvement in the balance of trade is not changed when income variables are introduced into the model, and

b) that the condition for improved employment is the same as the condition for improvement in the balance of trade.⁶

5. i.e., our condition can be restated: $[1 - \frac{|\eta_2|(1+\omega_1)}{\omega_1 + |\eta_2|} - \frac{|\eta_1|(1+\omega_2)}{\omega_2 + |\eta_1|}] > 0$.

Now if either $|\eta| > 1$, the term in which that $|\eta|$ enters will be > 1 , and the condition will be fulfilled regardless of the value of the other η . If both $|\eta| < 1$, then each $\frac{|\eta|(1+\omega)}{\omega + |\eta|} > |\eta|$, and our most stringent condition becomes $1 - |\eta_2| - |\eta_1| < 0$, which coincides with the condition when both elasticities of supply are infinite.

6. Cf. Joan Robinson, Essays in the Theory of Employment, (Basil Blackwell: Oxford, 1947) 2nd Edition, p. 136: "A positive balance of trade is equivalent to investment, from the point of view of the home country, and it has the same influence as investment upon the level of effective demand in the home country." Cf. also A.J. Brown, "Trade Balances and Exchange Stability," Oxford Economic Papers, No. 6 (April, 1942), p. 74: "If the trade balance, acting through the multiplier, affects the levels of activity in the two countries, the critical values of the elasticities necessary to make equilibrium stable are not affected, but the trade balance is made less sensitive to changes in the rate of exchange than when levels of activity are fixed." Brown's mathematical proof of the above statement (loc. cit., pp. 64-5) is based on assumption b) and on this assumption is not faulty, but we shall show below that this assumption does not follow from the relationships generally postulated in analysis of this type.

3. The plan of this paper is to investigate the conditions for improvement in the balance of trade and for increase in home output, as a result of depreciation, in terms of models which include both prices and incomes as variables. It is assumed in the construction of these models that each country produces only one good, and that its choice as to how much of that good it will consume and how much it will trade depends upon its current production of that good and upon the real terms of trade (i.e., demands for home goods and for imports are functions of real income [production] and relative prices in terms of home currency.) Equilibrium conditions are provided by the requirement that total production of each good equal its total consumption. Four variants of the model will be considered:

- 1) The Keynesian case, characterized by infinitely elastic supply curves of products.
- 2) The classical case, characterized by Say's law and completely inelastic supply curves of products.
- 3) The quasi-classical or modern case, characterized by the maintenance in each country of full employment and a stable price level.
- 4) The general case, in which prices in each country are functions of production, and from which the previous three cases can be derived under special assumptions.

In the discussion of all of these cases, units of goods and currency are so chosen that at the point of depreciation p_1, p_2 , and $k = 1$.

4. The following model describes the Keynesian case:

Equilibrium Conditions (1) $y_1 = c_1 + x_1$; $y_2 = c_2 + x_2$

Demand for Home goods (2) $c_1 = \alpha_1 + \beta_1 y_1 + d_1 \frac{p_1 k}{p_2}$; $c_2 = \alpha_2 + \beta_2 y_2 + d_2 \frac{p_2}{p_1}$

y = income

x_i = exports of i

c = cons of home goods

quantities

$$\begin{aligned} y_1 P_1 &= c_1 P_1 + x_2 P_2 \\ y_2 P_2 &= c_2 P_2 + x_1 P_1 \end{aligned}$$

$= k$

Demand for Imports (3) $x_2 = \delta_1 + \epsilon_1 y_1 + \theta_1 \frac{P_2}{P_1 k}; x_1 = \delta_2 + \epsilon_2 y_2 + \theta_2 \frac{P_1 k}{P_2}$

Supply of Products (4) $P_1 = 1$; $P_2 = 1$

It will be seen that, as before, prices are expressed in the currency of the country in which the good in question is produced, and that k is that number by which the price of a good in terms of currency 1 must be multiplied to yield its price in terms of currency 2. Again $B_1 = x_1 P_1 k - p_2 x_2$ (in terms of foreign currency), and

$$\frac{dB_1}{dk} = x_1 + \theta_2 + \theta_1 + \epsilon_2 \frac{dy_2}{dk} - \epsilon_1 \frac{dy_1}{dk}$$

$$\frac{dy_1}{dk} = \beta_1 \frac{dy_1}{dk} + \gamma_1 + \theta_2 + \epsilon_2 \frac{dy_2}{dk}; \frac{dy_2}{dk} = \beta_2 \frac{dy_2}{dk} - \gamma_2 - \theta_1 + \epsilon_1 \frac{dy_1}{dk}$$

Now we can define $P_1 = \gamma_1 + \theta_2; P_2 = \gamma_2 + \theta_1$; P here being the price slope of the total demand function for the goods of the country in question (the aggregate of the domestic and foreign demand functions).

Thus $\frac{dy_1}{dk} (1 - \beta_1) = P_1 + \epsilon_2 \frac{dy_2}{dk}$, and $\frac{dy_2}{dk} (1 - \beta_2) = -P_2 + \epsilon_1 \frac{dy_1}{dk}$

Solving, we find that

$$\epsilon_2 dy_2 - \epsilon_1 dy_1 = \frac{-\epsilon_2 P_2 (1 - \beta_1) + \epsilon_2 \epsilon_1 P_1 - \epsilon_1 P_1 (1 - \beta_2) + \epsilon_2 \epsilon_1 P_2 dk}{(1 - \beta_1)(1 - \beta_2) - \epsilon_1 \epsilon_2} dk?$$

If we divide both the numerator and denominator by $(1 - \beta_1)(1 - \beta_2)$, and define

$$\pi_1 = \frac{\epsilon_1}{1 - \beta_1}, \pi_2 = \frac{\epsilon_2}{1 - \beta_2}$$

then $\epsilon_2 dy_2 - \epsilon_1 dy_1 = \frac{P_1 (\pi_1 - \pi_1 \pi_2) + P_2 (\pi_2 - \pi_1 \pi_2)}{1 - \pi_1 \pi_2} dk?$

If now we define $\sigma_1 = \frac{P_1}{y_1}, \sigma_2 = \frac{P_2}{y_2}$ (σ representing the elasticity of total demand for the product of the country in question), and if we let $r_1 = \frac{y_1}{x_1}; r_2 = \frac{y_2}{x_2}$,

and if trade is originally balanced, then the change in the balance of trade of country 1 as a result of depreciation can be written:

$$\frac{dB_1}{dk} = x_1 \left[1 + \eta_1 + \eta_2 - \left\{ \frac{r_1 \sigma_1 (\pi_1 - \pi_1 \pi_2) + r_2 \sigma_2 (\pi_2 - \pi_1 \pi_2)}{1 - \pi_1 \pi_2} \right\} \right]$$

For improvement in the balance of trade to result, $\frac{dB_1}{dk}$ must be < 0 . The above equation shows that the traditional condition for improvement (that $(1 + \eta_1 + \eta_2)$ be < 0 , is not the appropriate one in this case. It can further be shown that the presumptive sign of the additional term is positive, so that the critical value of the sum of the elasticities (absolute values) is something greater than 1, and quite possibly significantly greater.⁷

5. The "classical assumption of no "money illusion" on the part of buyers and sellers of labor leads us to a completely inelastic supply curve of the national product. Thus we replace equations (4) in the above system by equations (4a):

$$(4a) \quad y_1 = \psi_1 \quad ; \quad y_2 = \psi_2$$

Since now, however, P_1 , P_2 , and k enter only as a ratio, our system is over-determined, with 8 equations and 7 unknowns. But if we add the additional "classical" requirement that all income be spent, we can eliminate one of the two demand equations for each country by subtracting the remaining demand equation from total income, reducing the number of equations to 6. The seventh equation is provided by the requirement of balance of payments equilibrium, which also follows from the assumption of Say's law. Thus $\psi_1 P_1 = c_1 P_1 + x_1 P_1$; $\psi_2 P_2 = c_2 P_2 + \frac{x_2 P_2}{k}$; so that

$$(5a) \quad x_1 P_1 = x_2 \frac{P_2}{k} .$$

7. It is assumed that both countries are "stable in isolation", i.e., that they have positive propensities to save, so that $\pi_1 (= \frac{\epsilon_1}{1-\beta_1})$ and $\pi_2 (= \frac{\epsilon_2}{1-\beta_2}) < 1$. This means that $(\pi_1 - \pi_1 \pi_2)$, $(\pi_2 - \pi_1 \pi_2)$ and $(1 - \pi_1 \pi_2)$ are all > 0 . Since r_1 and r_2 are necessarily > 0 , the sign of the bracketed expression is the same as the sign of σ_1 and σ_2 (elasticities of total demand), which are assumed to be < 0 . (A qualification to this assumption appears in the Appendix). Since the bracketed expression appears with a negative sign, its total contribution to $\frac{dB_1}{dk}$ is > 0 .

It can easily be seen that if this system has a unique solution, it will be independent of k , since any change in k will be accompanied by a countervailing change in the ratio $\frac{p_1}{p_2}$. Stated alternatively, since equation (5a) requires that both the equilibrium existing before and the equilibrium existing after a change in k have as a property that $B_1 = x_1 p_1 k - x_2 p_2 = 0$, it follows that $\frac{dB_1}{dk} = 0$.

The results of using a classical microeconomic model on the one hand and using a classical macroeconomic model on the other are thus incompatible. In the macroeconomic case just considered, not only is the criterion that $1 + \eta_1 + \eta_2 < 0$ inapplicable for improvement in the balance of trade as a result of depreciation; there is in fact no way by which the balance of trade can be improved.

6. This conclusion is not distressing, since it depends on the unrealistic assumption of Say's law. Furthermore, the attempt on the part of modern governments to maintain full employment and a stable price level would appear to justify us in constructing what I have called the quasi-classical case, in which the classical assumption of fixed national production is maintained while the assumption of Say's law is abandoned. Our model then becomes:

$$\begin{aligned}
 (1a) \quad & g_1 = \psi_1 - c_1 - x_1 \quad ; \quad g_2 = \psi_2 - c_2 - x_2 \\
 (2) \quad & c_1 = \alpha_1 + \beta_1 y_1 + \gamma_1 \frac{p_1 k}{p_2} \quad ; \quad c_2 = \alpha_2 + \beta_2 y_2 + \delta_2 \frac{p_2}{p_1 k} \\
 (3) \quad & x_2 = \delta_1 + \epsilon_1 y_1 + \theta_1 \frac{p_2}{p_1 k} \quad ; \quad x_1 = \delta_2 + \epsilon_2 y_2 + \theta_2 \frac{p_1 k}{p_2} \\
 (4) \quad & p_1 = 1 \quad ; \quad p_2 = 1 \\
 (4a) \quad & y_1 = \psi_1 \quad ; \quad y_2 = \psi_2
 \end{aligned}$$

8. This is the conclusion reached by Tinbergen, loc. cit.

This can be visualized by conceiving of each government declaring a commodity standard in the good produced in its country. This would automatically insure constancy of the full employment income and of the price level. Any deficiency in quantity demanded at $p = 1$ would be filled by the government adding to stocks; any deficiency in quantity supplied (i.e., any excess of quantity demanded over full employment output at $p = 1$) would be filled by the government out of its (presumably unlimited) stocks.

In this case the criterion for improvement in the balance of trade as a result of depreciation is the same as that which results from the use of the traditional supply-and-demand model. This can be easily shown, since equations (3) are independent of equations (1a) and (2), so that $\frac{dx_1}{dk} = \theta_2$, $\frac{dx_2}{dk} = -\theta_1$, and if $x_1 = x_2$ at the start, $\frac{dB_1}{dk} = x_1(1 + \eta_1 + \eta_2)$. Thus if governments accept the Keynesian challenge and succeed in maintaining full employment by internal measures, heeding the Keynesian warning against "beggar-my-neighbor" trade policies, their actions vis-à-vis balance of trade problems can again be governed by the traditional ("classical") statement of the critical value for the elasticities of demand for imports.

7. We can now proceed to generalize the models we have been considering. Our general model consists of equations (1), (2), and (3), with equations (4) or (4a) replaced by

$$(4b) \quad p_1 = \mu_1 + \lambda_1 y_1 ; \quad p_2 = \mu_2 + \lambda_2 y_2$$

If we set $\frac{p_1 k}{p_2} = \omega'$ (Yntema's "net monetary factor") and again choose units so

so that at the start $p_1, p_2, k = 1$, we can consider equations (1), (2), and (3) alone. Differentiating now with respect to ω' , we find $\frac{\partial B_1}{\partial \omega'}$ to be the same

expression as we obtained for $\frac{dB_1}{dk}$ in the Keynesian case above. Thus for our general case

$$\frac{dB_1}{dk} = x_1 \left[1 + \eta_1 + \eta_2 - \left\{ \frac{r_1 \beta_1 (\pi_1 - \pi_1 \pi_2) + r_2 \beta_2 (\pi_2 - \pi_1 \pi_2)}{1 - \pi_1 \pi_2} \right\} \right] \frac{dw}{dk}$$

Thus so long as $\frac{dw}{dk} > 0$, the conditions for improvement in the balance of trade are exactly the same as those we obtained for the Keynesian case. Variations in $\frac{dw}{dk}$ ($\frac{dw}{dk} > 0$) will effect the magnitude but not the sign of the change in the trade balance.

The question arises, however, as to whether the sign of $\frac{dw}{dk}$ must be positive. A priori, we would expect a fall in k to lead to a rise in p_1 and a fall in p_2 , but we would also expect that the effect on $\frac{p_1}{p_2}$ would not be so great as to make $\frac{p_1 k}{p_2}$ move in the opposite direction from k . The exact conditions for this to be true are formulated below:

$$\begin{aligned} \frac{dw}{dk} &= 1 + \frac{dp_1}{dk} - \frac{dp_2}{dk} = 1 + \left(\lambda_1 \frac{\partial y_1}{\partial w} - \lambda_2 \frac{\partial y_2}{\partial w} \right) \frac{dw}{dk} \\ &= \frac{1}{1 + \lambda_2 \frac{\partial y_2}{\partial w} - \lambda_1 \frac{\partial y_1}{\partial w}} \end{aligned}$$

$$\begin{aligned} \text{Sign of } \frac{dw}{dk} &= \text{sign of } 1 + \lambda_2 \frac{\partial y_2}{\partial w} - \lambda_1 \frac{\partial y_1}{\partial w} \\ &= \text{sign of } \left[\frac{\lambda_2}{1 - \beta_2} (P_2 - \pi_1 P_1) + \frac{\lambda_1}{1 - \beta_1} (P_1 - \pi_2 P_2) \right] \end{aligned}$$

From this we can see that if either $\frac{\lambda_1}{1 - \beta_1} = \frac{\lambda_2}{1 - \beta_2}$ or $P_1 = P_2$, $\frac{dw}{dk}$ must be > 0 .

But beyond this no definite statements can be made, except that it is impossible for $(P_2 - \pi_1 P_1)$ and $(P_1 - \pi_2 P_2)$ both to be > 0 , under our assumptions that $P < 0$, $0 < \pi < 1$. If now one of these terms is > 0 , we can conceive of a case in which its $\frac{\lambda}{1 - \beta}$ coefficient is large (say infinite) and in which the

other $\frac{\lambda}{1-\beta}$ coefficient is small (say 0), so that we can definitely state that it is not impossible for $\frac{dw}{dk}$ to be > 0 under "reasonable" assumptions. If this were the case, the elasticity conditions stated above would have to be reversed, so that a small (absolute) sum of elasticities would lead to improvement, and a large sum to deterioration of the balance of trade. It is probably unlikely that such cases are frequent in practice, however.

It should be apparent that the models considered earlier are special cases of the present one, the Keynesian case arising when $\lambda_1 = \lambda_2 = 0$, so that $\frac{dw}{dk} = 1$, the classical case arising when $\frac{dw}{dk} = 0$ and π_1 and π_2 are (effectively) $= 0$.

8. To consider the effect of depreciation on employment, we need only look at the Keynesian and "general" cases, since in the other two cases employment (production) is determined. In both of these cases, (so long as in the second case $\frac{dw}{dk} > 0$), the sign of the change in employment is determined by the sign of

$$\frac{dy_1}{dw} - \frac{1}{1-\beta_1} (P_1 - \pi_2 P_2)$$

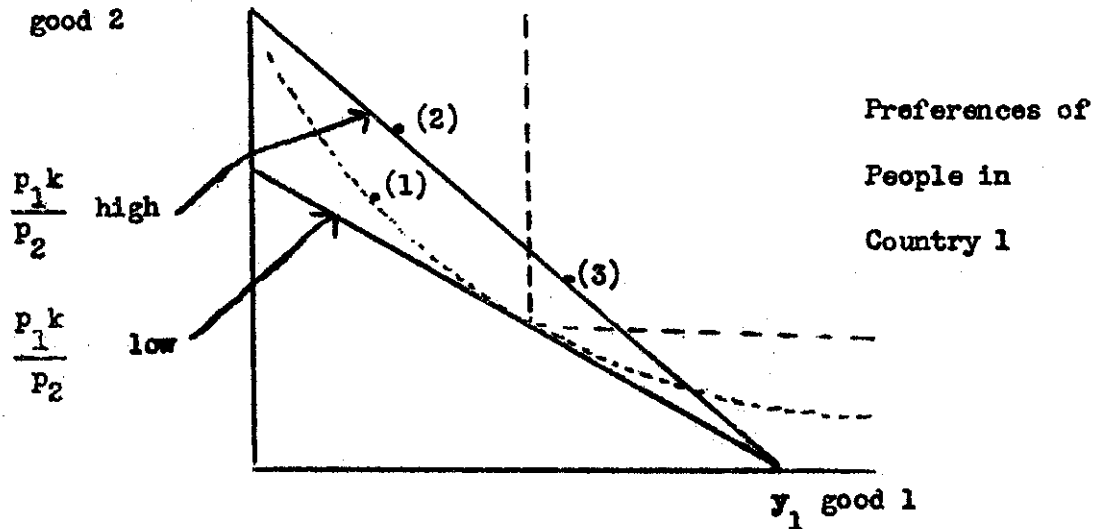
If this expression is < 0 , employment will increase as a result of depreciation, and conversely.

It is to be noted that this condition is not identical with the condition for improvement in the balance of trade. It is quite conceivable that depreciation could lead to an increase in employment and to a deterioration of the balance of trade. Such a case would occur, for example, if the elasticities of demand for home goods and for imports were identical as between the two countries, and if the elasticities of demand for imports were sufficiently small. The converse, an improvement in the balance of trade, together with a decrease in employment,

is also possible, as is shown by the case where the price slope of the total demand function for foreign goods is sufficiently greater than the price slope of the total demand function for home goods, while the sum of the two elasticities of demand for imports is large. The traditional assumption of identity between these two conditions thus appears to have been unwarranted.

APPENDIX

It has been assumed throughout this argument that the signs of all demand elasticities are negative. However, the theory of consumer choice indicates that they need not be. This can be illustrated on the following indifference diagram.



Since γ_1 and θ_1 are defined as measuring by how much consumption of home goods and imports, respectively, will change if $\frac{p_1^k}{p_2}$ changes, with national production constant, we are justified in making the two price lines converge at y_1 . Now it can easily be seen that, though the substitution effect will always lead toward γ_1 , $\theta_1 < 0$ (point ①), the income effect will do the same only in the case where the home produced good is inferior (point ②). In the "normal" case in which neither good is inferior, the income effect will lead toward $\theta_1 < 0$ and $\gamma_1 > 0$ (point ③).

If we assume neither good inferior, our presumptive conclusions remain intact if a) the substitution effect outweighs the income effect on γ_1 or b) if, even failing a), γ_1 , though positive is not greater in absolute value than θ_2 .

Condition b) is explained by the fact that in the conditions derived above γ enters only as an element in $\rho_1 (= \gamma_1 + \theta_2)$. The sign of γ_1 is thus by itself of little importance.

It is nevertheless conceivable that condition b) might not hold. If this were the case, the critical value of the sum of the elasticities of demand would be lowered. The effects on $\frac{\partial y_1}{\partial w}$ are dual: $\rho_1 > 0$ would make an increase in income more difficult, while $\rho_2 > 0$ would make it easier to achieve by depreciation. The effects on $\frac{dw}{dk}$ are more complex: Each ρ enters twice (with opposite signs) in the expression for $\frac{dw}{dk}$, multiplied in each case by different coefficients. It can be said, however, that if both ρ_1 and ρ_2 are > 0 it will generally be more likely that $\frac{dw}{dk}$ will be < 0 than if ρ_1 and ρ_2 had their presumptive signs.