THEORY OF THE FIRM: SOME SUGGESTIONS FOR REVISION, II

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Introduction

In a previous paper we suggested directions for revision of the theory of the firm at the "atomic" level were offered. The present paper offers some additional suggestions for exploring the internal or sub atomic layer of firm operations.

The ordinary theory of the firm, at least as found in economic textbooks, perhaps hints at the division made here. In the main, however, the entrepreneur-promoter, as he may be called, is viewed as having access, at a cost, to whatever resources he thinks he needs. He possesses a set of risk-preference functions and surveys his economic horizon between industries and firms and chooses his (relatively) most profitable opportunity as defined by his risk-preference map. This represents his profit maximizing activity qua entrepreneur-promoter.

Having chosen, on the basis of his estimates, the relatively most profitable opportunity, the entrepreneur-promoter proceeds to the next phase, operations. He chooses a person, (perhaps himself) to act as operating chief of the enterprise.


2/ But of, the somewhat stronger (and related) statement of George Stigler, "An additional problem...arises out of the relationship between the control unit (the firm) and the production unit (the plant). Almost universally the firm and the plant are assumed to be in a one-to-one ratio, and certainly no general theory of the relationship between the two has yet been evolved." From "Production and Distribution in the Short Run," Journal of Political Economy, Vol. 47, June 1939, reproduced in Readings in the Theory of Income Distribution, William Fellner and Bernard F. Haley, eds. (Philadelphia: The Blakiston Co., 1946.) p. 122

3/ Of course, if the risk-preference functions and the (estimated) profit functions are inconsistent the person is not an entrepreneur-promoter. He does not "invest," or, more strictly, promote.
Whether he chooses another or himself is not material; functionally defined, he now becomes an entrepreneur-operator. For at this stage, the commencement and execution of operations, he must understand how to choose men, as managers, and be able to control them. Without these abilities the business, except under fortuitous circumstances, must fail and/or the entrepreneur lose control.

In the operating phase, the entrepreneur-operator is viewed as being confronted with supply functions of factors, with fixed or varying prices, and certain

4/ In some, perhaps in many, lines of business, success may also depend upon technical knowledge. It is difficult to believe that in any line of endeavor—that a person may long be successful or retain control without knowledge of men, situations, and control techniques. It should also be noticed that successful maintenance of control requires, generally, that the entrepreneur also have some choice of other men in addition to the chief operating official. In particular, he must be concerned about the choice of those persons whose reports are to be used to control the chief operating official.

This is not the place to dilate on the subject of control. The following diagram may, perhaps, afford a complete—if rough—notion of what is intended. A situation may be said to be in control insofar as the elements and circuit in the following diagram are fulfilled:

![Diagram](image)

Definition and installation of standards run through the whole process. In the reporting system the word standards has a double meaning: (1) a means of judging the adequacy of the reports and (2) a means of interpreting the reports. In a complex set of cost reports, for example, it is of little use to inform an executive that an item has cost $5 to produce in a particular period without providing an acceptable standard against which this figure may be judged. This is, of course, one of the functions of standard costs. An adequate accounting system must have some such standards built directly into the reports.
technological constraints. Given the demand conditions the entrepreneur-operator then seeks to maximize the difference between receipts and costs--although, to be sure, he may utilize part of his resources to alter demand (or supply) conditions, as well as technology.\textsuperscript{5/}

Powerful techniques of economic analysis have been brought to bear on these problems and their theoretical implications exhaustively explored under varying conditions.\textsuperscript{6/} The process need not be repeated here.\textsuperscript{7/} It is sufficient to note that, say, in isoquant or product substitution analysis the entrepreneur is viewed as operating through agents with fixed budgets. On the basis of specified criteria which determine their behavior,\textsuperscript{8/} these officials then choose between factors or products. Whether the analysis may be said to contain the germ of a theory of agency, it is at least not incompatible with such a theory. The choice proceeds, on the assumption of profit maximization, to the point where the products or factors under consideration are perfect substitutes. Equilibrium, stable or unstable, is then said to maintain. If the equilibrium is stable, the agent cannot undertake further action without worsening his (the firm's or department's) position in terms of the criteria by which his conduct is judged.

It may be worthwhile exploring the possibility of extending this theory of agency. After all, the firm is an organization. As such it requires direction and

\textsuperscript{5/} In any event, the entrepreneur's economic horizon may generally be expected to change when he moves from pure promotional to operating activity.

\textsuperscript{6/} See, for example, Sune Carlson, A Study on the Pure Theory of Production, Stockholm Economic Studies Reprint No. 5 (London: P.S. King & Sons, Ltd., 1939.)

\textsuperscript{7/} For one of the more cogent criticisms, which nevertheless, builds on this apparatus, see Albert Ailord Hart, Anticipations, Uncertainty, and Dynamic Planning (Chicago: University of Chicago Studies in Business Administration, Vol. XI, No. 1, 1940). Professor Hart attempts to dynamicize the usual model by introducing uncertainty. He does this, however, only at the atomic level.

\textsuperscript{8/} Profit maximization
control. Rather than operating directly on factors which are perfect substitutes, the entrepreneur-promoter, certainly, and the entrepreneur-operator, probably, operate through a variety of agents: general managers, superintendents, foremen, and so on.

When operations occur through agents, control mechanisms must be installed. Viewed downward, through the eyes of top management, such devices are essential if defined objectives are to be secured and if redefinition is to be made when necessary. Viewed upward, through the eyes of the down-the-line management, such devices are essential both to obtain direction and to gauge the satisfactoriness with which they are performing appointed tasks. It is manifestly wasteful, if not impossible, for top management to undertake the careful scrutiny of monthly cost reports, say, on the thousands of items which may be produced in a modern large-scale operation. The purpose of the control mechanism then is to see that the tasks are performed, for the most part, in a satisfactory manner without the need for detailed surveillance and review by top management. This is the meaning of the statement that "a good reporting system must itself get the bulk of the job done."  

Vide supra, footnote 4.

9a/ See the interesting statement by P.D. Newbury, formerly senior vice president of Westinghouse Electric Corporation in "A Businessman's Reaction to the Theory of Monopolistic Competition" (mimeo, 28 pp.): "Marginal analysis deals with unit costs and unit revenue. Even in the case of relatively small single manufacturing plants, a large number of different kinds and sizes of units are manufactured. In large-scale manufacturing a corporation may manufacture in a single plant hundreds or thousands of different units each with its own cost and unit price."

"The top management of a plant or corporation usually deals with total revenue, total costs, and total profit, e.g., plant, company, or consolidated income-and-expense statements. Policy decisions concerning production volumes, employment, control of overhead expense, and similar problems, are based on total operating figures—on financial accounting statements, and not on detail cost accounting figures."
How is this task accomplished? Needless to say, business firms have evolved a variety of techniques for the solution of this problem. In terms of organization, there are the staff and line devices, with the former utilized as a check over the latter either through organizational, functional, or other arrangements. In terms of techniques, there are various devices such as financial, cost, and production budgets, accounting reports, etc. Special studies and investigations, audits—including internal audits—and the whole paraphernalia of informal reports must be taken into account in any complete description of these control mechanisms.

It is not the intention of the present paper to explore all of these varieties of controls. Of the formal devices, certainly one of the most persuasive and important in modern business enterprise is that supplied by accounting. In the

10/ E.g., The plant accountant may be organizationally responsible to the plant superintendent and functionally responsible to the controller—as a means of sharing his independence and assuring validity to his reports.

previous article an attempt was made to explore, at least in part, the utilization
of financial accounting at the atomic level. Here, attention will be devoted pri-
marily to the other broad phase of accounting as evidenced by the cost records and
reports. Of particular interest in this connection are the techniques of standard
costing. For standard costs form a crucial point at which a variety of these con-
trols are brought together. They form the basis, through e.g., production and sales
projection, of budgeting. They are used to control actual or historical costs.12/
Since the standards frequently reflect the results of time, motion, and method study
they furnish top management an "objective" basis of judging the actual against the
possible or desirable.13/ Moreover, the juxtaposition of actual and standard costs

12/ It is not always realized that it is standard rather than actual costs which
provide the basis of price decisions—e.g., in "cost plus" pricing—when a standard
cost system is in operation. Standard costs are, of course, used to control actual
costs. But too much emphasis should not be placed on this point. The reverse is
also true. Control depends on two way causation. If actual did not also control
standard, management would have no control over its staff agencies—e.g., its
management engineering department.

13/ It is not possible here to proceed into an analysis of the different types
of standard cost systems such as basic and current standards, complete and incomplete
systems, commodity and activity standards, etc. The interested reader may consult
any of the usual cost accounting texts or, for a summary analysis, see the article,
"The Basic Theory of Standard Costs" by Walter B. McFarland in The Accounting Review
XIV, No. 2, June, 1939, pp. 151-158. The following definitions (adopted from a study
of cost practices in American industry conducted by Martin Black and Harold Eversole
and contained in a memorandum to Paul Orren, Deputy Administrator for Accounting,
Office of Price Administration, Washington, D. C., April 8, 1945) contain the main points:

"1. A Job Cost System is one which collects separately each
element of cost for each job or order worked on in the plant.

"2. A Process Cost System is one in which costs and quantities
are collected by departments, and the departmental costs
are reduced to a cost per unit of production...

"3. A Standard Cost System provides a predetermined cost for each
operation, for each unit of finished product. Standard costs
are not actual current costs but with proper adjustments for
labor, material, and burden variances may be converted to
actual cost. The predetermined cost is intended to represent
the cost of direct material, direct labor and manufacturing
burden normally required under efficient conditions at normal
capacity to process a unit of product..."
and the listing of the variances—perhaps in index number form—allows top management to focus attention on trouble spots. It permits a ranking of trouble spots and hence allows the systematic ordering of scarce management time, including ranking of cost components by organization, function, object, etc. For lower echelons, the standards provide an articulate guide by which they may judge their conduct, for it specifies in advance the standards by which their conduct will be judged. Indeed, it may not be too much to say that one of the most important functions of the standards is to dissipate internal uncertainty in the firm and allow full delegation with control retention. Through the specification of standards both lower and top echelon management know what is expected in the way of cost performance and may judge (and adjust) actual performance in this light.

A developed theory of the firm, it would seem, would take into account these agency and control devices. Some such reorientation and extension of the existing theory of the firm would seem desirable if even the prospect of contact at this level is to be made with what Haavelmo has called "the laboratory of

"4. An Estimating Cost System is one in which the cost of production is estimated in advance of the actual production. In an estimating cost system, there is no attempt to establish cost standards as is the case in a standard cost system. These costs may be based upon past actual costs, but are tempered by opinions of future actual costs..."

It is of interest to note the frequency with which the British Working Parties, as a result of their surveys of American industry, have recommended the adoption of standard cost techniques. See, for example, Hosiery Board of Trade Working Party Reports (London: His Majesty's Stationery Office, 1946) pp. 91-92:

"It is not enough to know that the cost of an article in one period is more or less than in another; it is necessary to know the extent to which the actual differs from the standard and the reasons for the differences. If for reasons within the control of the manufacturer, the differences will provide the subject for immediate inquiry in the right quarter..."

14/ And, per contra, spots where benefits may be captured and built into the process.

15/ Obtaining the consent of both parties to the standard is, of course, a difficult and important part of the operation.

nature."\textsuperscript{17} Scitovsky\textsuperscript{18} is perhaps overly harsh but there is, nevertheless, truth in his statement that "between economic theory and business practice there is a deep and as yet unbridged gulf. The economist evolved a theory of how the rational businessman maximizes his profits; but this theory, however unassailable it may be logically, does not fit the facts of business practice very well. For this shortcoming economists used to lay the blame on the businessman saying either that his behavior was not rational, or that he did not aim at maximizing profits. Only recently has it begun to be realized (since businessmen failed to reform themselves) that the economist may be the one to be blamed: he may have oversimplified his theory, and that he should learn something of the businessman's trade before theorizing about it." Of course, one would hope that in learning something of the businessman's trade one would hope that the economist would continue to theorize. Wholesale abandonment of theory would be unwise. The point of contact with the laboratory of nature, as Haavelmo has made abundantly clear, is to provide a systematic flow of problems into the discipline, and thus fructify both theory and practice.\textsuperscript{19}

\textsuperscript{17} This should not be interpreted to mean that economists have had no contact with business firm operations. Such is obviously not the case. In addition to contacts of a "practical nature" for the solution of particular business problems there have been numerous studies of a more abstract and rigorous sort which have attempted to bring to bear on business problems more powerful techniques of disciplined economic analysis. In the field of cost analysis, for example, there have been the numerous studies pioneered by Professor Joel Dean and others. (A summary of many of these studies may be found in the recently reissued volume, Cost Behavior and Price Policy (New York: National Bureau of Economic Research, 1943.) See especially Chapter V, "Costs and Rate of Output: Empirical Studies of Cost Functions."

\textsuperscript{18} "Recent Publications on Costs," The Accounting Review XVIII, No. 1, January, 1943, p. 72.

\textsuperscript{19} It would probably be too much to hope, in the near future, for the type of fruitful and complete contact that has now been made by the statistician—to say nothing of the interchange that has long been common in other disciplines such as engineering and physics, or biology and medicine. One of the great accomplishments of Keynes lay in this direction. The type of theory evolved by this school of thought—whatever its other merits—has brought economists into broad contact with whole categories of government problems on a continuing basis. This fact may be easily verified by reference to standard economic journals. It is now common to find economists retained in government capacities on a professional basis making contributions to this literature as a result of their official duties.
It is impossible to explore here the gamut of these agency and control mechanisms. Nevertheless, a few tentative suggestions may be offered. Of primary interest will be the exploration of cost behavior. A few simplified models involving the behavior of actual costs under standard cost controls will be utilized as a means of illustrating the possible extensions of theory that may be made in this direction.

The Planning Cost Curve

Before proceeding further, it might be well to emphasize again the need for conserving theory. It would be surprising if it were not necessary in economics—as in other disciplines—for the "practitioner" to undertake forgetive adaptations of theory to the solution of particular problems. It would be equally surprising (and wasteful) if each problem has to be approached de nouveau, without benefit of theory. "Economics" may be, in Professor Viner's tart definition, "what economics do." Nevertheless, it is economics, and to that extent must be either theoretical or deficient. In any event, much of the theory of the firm has proved, or can prove, useful to economists or others in the solution of business problems.

Perhaps nowhere is this more clear than in the case of the so-called planning or interplant cost curve, which it will be remembered, is the envelope to the whole family of particular cost curves which contain fixed as well as variable costs. Whenever the major assumptions of this curve—fixed technological coefficients, production of a single product, certainty, planning for optimum points, and no previous scale commitments—are met, the type of curve posited in theory can be expected to obtain

20/ Only the degree of certainty necessary for engineering estimates need be assumed.

21/ I.e., freedom of movement toward the optimum solution, under given technological conditions, is not prevented by previous condition of operation. This need not imply complete disappearance of existing structure and layout. An operating company may, for example, build a new plant in a different locality, possess ample space for new machines, etc.
in reasonably recognizable form. That such phenomena as diminishing returns are taken into account and have proved useful constructs, say in equipment design and layout can be verified by reference to any of numerous engineering texts.\textsuperscript{22/}

The types of delegation and control discussed earlier in the paper have little place in business operations when problems of plant expansion or the purchase of new and expensive items of equipment are involved. This is likely to be a closely supervised top management task. (Cost controls, such as standard costs, which affect cost behavior are largely out of the question for intermittent equipment purchases or facilities modification.) As a matter of fact, financial and (atomic) control considerations of the type discussed in the previous paper may prove preponderant and further obscure the workings of the process.

\textsuperscript{22/} See, for example, Eugene L. Grant, Engineering Economics (New York: The Ronald Press, 1936) Ch. 11 ff.; Clarence E. Bullinger, Engineering Economic Analysis (New York: McGraw-Hill Book Co., Inc., 1942) Ch. 9 ff.; J. K. Finch, \textit{An Introduction to the Economics of Civil Engineering} (New York: Columbia University Press, 1942,) Ch. IV ff. Numerous "empirical" curves and formulae from various construction projects may be found in the latter volume.

An interesting example of the use of such optimum estimates may be found in the design of cyclotron magnets. See Martyn H. Foss, \textit{Design of Cyclotron Magnets} (Published by the Department of Physics, Carnegie Institute of Technology under the Joint Program of the Office of Naval Research and the Atomic Energy Commission, 1948). After analyzing each component separately, Dr. Foss combines the cost of steel, the cost of copper, and the cost of energy (operating cost) to determine the optimum ratio of the outside to the inside coil radius of the magnet. The parabolic function thus formed is differentiated to determine the optimum point, although for other reasons, such as convenience in use, the optimum point was not chosen. This alternation in choice could be made without much sacrifice since, as Dr. Foss notes, "the variation is small over a wide range of diameters near the minimum."

Other cyclotrons have, of course, been designed with less care. Admitting the gain in efficiency some of the personnel connected with the Carnegie Tech synchrocyclotron seem to feel that perhaps too much care was used—with consequent delay. The less carefully designed models are in operation while Tech's is still in the process of construction. It is possible that a businessman facing aggressive competition and potential loss of market might have proved less patient.

Of course, if marked changes in size are required the assumption of constant technological coefficients loses its validity. If the synchro-cyclotron were built beyond a certain size, for example, it would then be necessary to redesign the building in which it is to be housed.
Even when these considerations are not present the process may be obscured. Multiple commodity production or other types of "flexibility" may, as Stigler notes, result in a marked flattening of the curve by raising it throughout the region of the optimum point. Lack of certainty and the relaxation of the assumption of planning for point optima may also modify the curve. There has been little investigation of the behavior of these curves away from the region of optima points yielded by the envelope. Yet it is clear that range, rather than point, planning must move the curve away from this long run planning curve.


24/ E.g., suppose an entrepreneur with ability to adjust to optimum is unable to do so because of inability to forecast his exact demand conditions. He may then choose, say, a linear curve over a range of outputs from q = a to q = b. This will allow him to adjust in minimum time from one output level to another over the range. After operating experience is gained he may seek to adjust within the range. (In Stigler's terms, he exchanges "flexibility" for "adaptability.") Suppose he attempts to minimize his average costs, assuming, his minimum and maximum capacity costs at a and b fixed; it may then be shown that his cost function will take the form of a catenary. He seeks to adjust and minimize his average cost

\[ c = \frac{\int_a^b c \sqrt{1 + \left( \frac{dc}{dq} \right)^2} \, dq}{\int_a^b \sqrt{1 + \left( \frac{dc}{dq} \right)^2} \, dq} \]

where \( c \) represents cost, \( \bar{c} \) average cost, and \( q \), quantity. (This is, of course, the formula for the centroid or average value of the ordinates of the cost function which is assumed to be continuous. Cf. Granville, Smith and Lougley, Elements of the Differential and Integral Calculus, p. 335.) This may be put in more convenient form and minimized at

\[ \bar{c} = \frac{\int_a^b \sqrt{1 + \left( \frac{dc}{dq} \right)^2} \, dq}{\int_a^b \sqrt{1 + \left( \frac{dc}{dq} \right)^2} \, dq} \]

and minimized at

\[ \bar{c} = \frac{b}{a} \left( \frac{q-a}{b} + \frac{q-a}{b} \right) \]
Suppose, for example, that an entrepreneur, not certain of his precise output, decides rather that he wishes to plan for production over a range. The precise path he will choose will depend upon various considerations of which cost minimization is only one. He may, for example, decide to obtain the maximum flexibility in the sense that he wishes to be able to move from one output level to another in minimum time, at least until he gains sufficient experience to determine his most relevant range of output. Having gained experience he may then seek to minimize costs within the range. He may seek to minimize costs at the outset over the range on the basis

See R. G. D. Allen, Mathematical Analysis for Economists, pp. 551 - 552. (Corner conditions are neglected by Allen. For a treatment of this subject see G. A. Bliss, Calculus of Variations, Ch. IV. From economic considerations it would seem that these corner conditions, which yield an absolute minimum, would lie along the optimum curve—i.e., a series of point optima. Bliss also shows, pp. 122 ff., for the one variable end point problem—e.g., where the entrepreneur seeks to increase capacity beyond the range while minimizing his costs, the path followed will also be a catenary. By analogy with Newton's problem what is sought is the shape of a body which will meet with the minimum resistance—as measured by cost—when moved in the direction of its axis through a resisting medium.)

Probably of more interest is the time path of investment which would be followed by the entrepreneur with a given rate of interest. This problem is too broad to pursue here but it should be noted that the fact that investment is likely to occur discontinuously offers no particular difficulties, for the introduction of the discount factor will generally convert a discontinuous to a continuous function. Suppose, for example, that an investment path is being traced out by a given entrepreneur at a rate \( F'(t) \). For given intervals no investment occurs and \( F'(t) = 0 \); a finite increase then occurs and the firm proceeds at a new level. The total discounted investment is then given by

\[
\int_0^T e^{-rt} F'(t) \, dt
\]

where \( r \) is the rate of interest, assumed constant. If we let \( T \to \infty \)—e.g., a going of continuing concern—we then have an ordinary Laplace transform of the derivative and each discontinuity appears in the equation as a constant times the discount factor. (Of course \( F(t) \) must be sectionally continuous and of exponential order—i.e., the discounted rate of investment must be finite, cf. R. V. Churchill, Modern Operational Mathematics in Engineering, Ch. 1). For one such jump we would obtain

\[
r f(r) - F(0^+) = \left[ F(t_o + 0) - F(t_o - 0) \right] e^{-rt_o}
\]

where the quantity in brackets is the jump. The solution in the form of the integral of a discontinuous function, is thus seen to be continuous.
of stochastic estimates in which probability weights are assigned to each amount of costs. Other possibilities suggest themselves. In some cases it may be deemed wise to avoid overextension. In others it may be the case that "it is better to be caught with excess capacity in a depression than with shortage of capacity in a boom" because of fear of customer loss to competitors. And so on.

The possibility of point optimum adjustment is likely to be vitiated further when multiple commodities are under consideration. Even in the planning stage it is not likely that the entrepreneur will spend much time calculating or reviewing cost estimates based on the assumption of mutatis mutandis adjustments in each and every one of numerous commodity lines and production departments. It is doubtful, given the state of development of cost accounting and allowing for projections into untried areas, that it would even be possible to form reasonable judgements as to what these optimum adjustments should be. Finally, as noted at the outset of this section, financial and control considerations of the type discussed in the preceding paper are likely to enter and qualify all judgments and estimates in this area.

Cost Behavior

A. Single Commodities

How will costs, as distinct from cost estimates, behave after installation is made and operations under way? Suppose, for example, that the "true" cost function—or technical economic cost function as we shall call it—is non linear, say, of the parabolic type posited by theory. It need not be the case that the actual, or observed, costs will appear in this form. The observed (actual costs will depend not only on the character of the technical-economic cost function but also on the behavior of production officials who guide the production equipment and factors of production. The behavior of these production officials will in turn depend on the control mechanisms—standard costs, budgets, etc.—to which these officials are subject. It may be the case that linearity of the controls will in turn produce linear observations despite the fact that the "true"
or technical-economic cost relation is non linear.25/

This may be shown with the use of a few simple mathematical models already familiar to economists.26/

25/ Most cost controls, and particularly standard costs, are linear, perhaps, because of ignorance, perhaps because of simplicity, perhaps for other reasons. As a matter of practice, I doubt whether much can be done with non linear controls in planning a complex production operation. Even in theory, non linear algebras are exceedingly difficult to work with and the theory is incomplete, except for particular cases. Of course, it is possible to transform non linear into linear sets of equations. But this is of little practical consequence because the transforms are not likely to find counterparts in the real world and hence will not provide mechanisms for control manipulations. (In a sense, nonexistence in the "real" world of these transforms is what is meant by "theoretical" as distinct from "practical" solutions.) Nevertheless, it may be worthwhile for "practical" as well as "theoretical" purposes to know the "true" character of the technical-economic cost functions, even when they are non linear. Such information may prove of particular value when questions of adjustments of the controls are at issue.

26/ The need for dynamic models and the inapplicability of empirical statistical investigations utilizing ordinary regression techniques without taking into account behavior patterns may be seen from the following simple model.
As a first approximation it may be assumed that a production official is in charge of the output of a single commodity. His behavior may be given in terms of an equation similar to that utilized for analysis of firm behavior at the atomic level:

\[ q(t + 1) = f(\hat{c} - c_t) \]

where

- \( q(t + 1) \) = output in period \((t + 1)\)
- \( \hat{c} = \hat{c}(q) \) = standard cost for the output of period \(t\)
- \( c_t = c_t(q) \) = actual cost for the output at period \(t\)

Stated in words, the rate of output at period \((t + 1)\) is a function of the variation of actual from standard cost in period \(t\).

The properties of \( f \) will be determined by postulates about the behavior of the official responsible for production. It may be assumed that this official is guided by two sets of controls: (1) budgeted production and (2) standard costs. He must then reconcile his behavior between these two (sometimes) conflicting criteria.

In the above diagram are given three cost functions, \( c_1 q_1, c_2 q_2, c_3 q_3 \). Suppose at the outset the entrepreneur is operating at point \(B\) on \( c_1 q_1 \) with output at \(b\). He wishes to reduce output, over some finite interval of time, to \(a\) and continue production on the "optimum" curve \( c_1 q_1 \). The pattern of observed costs through time will depend on the speed with which he wishes to adjust. One of an infinitude of time adjustment paths is illustrated above. From operating on \( c_1 q_1 \) he then operates on \( c_2 q_2 \), then on \( c_3 q_3 \), and finally on \( c_1 q_1 \) through the path \( BDEFGA \). The only two relevant points for testing the economic hypothesis of the shape of the cost function are points \(A\) and \(B\). All other points should be cast aside. From a statistical point of view this represents an inefficient use of data. Actually it is more likely that points \(A\) and \(B\) would be eliminated for statistical purposes because of their extreme character.

No statistical errors were introduced in the above model, as would be implied in a statistical fit of a time series. With the introduction of statistical errors the case becomes even more complicated. The idealized form of this model illustrates another point: the ascription of statistical errors to what is actually purposive behavior. This is a point of some significance which has on occasion been overlooked in statistical analyses of human behavior.
On the production side, so long as he is above a fairly easily obtained minimum, the official is subject to no sanctions of a severe character. Through incentive schemes and possible promotions, however, he is encouraged to expand the rate of output. On the cost side he is subject to sanctions, generally, only if he is over standard by a marked amount at a particular time, or if he persists in exceeding standard from period to period. Since his department (or organization unit) budget consists of a projection of his standard costs the sanctions on the cost side are further reinforced when persistent or marked "overages" occur. Sometime before the end of the budget period he will be forced to request appropriation of additional resources if he is to remain in operation. (Of course, this may also occur when costs are below standard but production runs higher than the budget. In such cases he need have little fear, for he is possessed of a "legitimate" reason: a good production and cost record.)

With these factors in mind the following behavior pattern may be assumed: When costs are above standard, \( \Delta c = \bar{c} - c_t < 0 \), the production official will retard the rate of expansion of production (if technical economic cost increases at a faster rate than standard) and may go so far as to reduce production absolutely. When costs are below standard, \( \Delta c = \bar{c} - c_t > 0 \), the official will strive to expand production, perhaps at an accelerated rate.

There are two cases, \( f > 0 \) and \( f < 0 \), which, under appropriate conditions are stable.\(^{27}\) The first yields asymptotic and the second yields oscillatory stability. With a linear standard cost function, actual observations in the oscillatory case might appear as follows:

\(^{27}\) Cf. the preceding article, "Theory of the Firm: Some Suggestions for Revisions," American Economic Review. This analysis need not be repeated here. Unstable cases may be of interest but are unlikely long to maintain in such simple cases as single commodity production because of their upsetting effect on planning mechanisms. In such cases the controls are likely to be revised after not too long a lapse of time.
Diagram I

$\tilde{c}(q)$ represents the linear standard cost function and ordinate at $a, b, c, d, e$ represent actual cost observations. It should be noted that what appear as random deviations about the linear cost function are the reflection of purposive behavior on the part of the production official. The behavior is quite rational in the sense that the production official deliberately adjusts himself to the control mechanism and technical economic conditions which he confronts.\footnote{The question of constancy of the technological coefficients is present here. Since part of the assumed technology is managerial "know how" and application, these coefficients, in the usual sense, are varying. Furthermore, (see supra footnote 25) the technological coefficients depend upon the time speed of adjustment to output variations. Both of these variations are taken into account in the above model.}
justs his technical economic means to the purposive ends of the control system.

To explore asymptotic behavior, equation (1) may be put in altered form.

Using the notation of Professor Samuelson, we write,

\[ \frac{dq}{dt} = \dot{q} = H \left[ \bar{c}(q, \sigma) - C(q) \right] \]

where, as before, \( \bar{c} \) represents standard costs (a function of quantity, \( q \), and a shift factor, \( \sigma \)) and \( C \) represents technical economic costs. Again it is assumed that standard cost is linear while technical economic cost may be non-linear. (The assumption of linear standard costs is formally not necessary for qualitative behavior analysis, but is convenient and "realistic.")

Equation (2) may be expanded in series and solved to yield the approximate solution:

\[ q(t) = q^0 + e^{\lambda(\bar{c}_q^0 - c_q^0)t} \]

where \( q^0 \) is the initial quantity, \( \lambda \) is a positive constant and \( \bar{c}_q^0 \) and \( c_q^0 \) are the derivatives of standard and technical economic cost at time \( 0 \). In this case production expands exponentially as illustrated in the following diagram. Stability requires that marginal technical economic cost exceed marginal standard cost. The condition of stability may, of course, be fulfilled with linear technical economic cost function. A fortiori these stability conditions will (ultimately) be fulfilled by the type of cost function posited by economic theory--i.e., any function whose first and second derivatives are positive.

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29/ Foundations of Economic Analysis (Cambridge: Harvard University Press, 1947) Ch. IX. It is also assumed that \( H(0) = 0 \) and \( H'(0) = 0 \); i.e., attention is restricted to deviations of standard from actual cost and an excess of standard over actual cost results in an increased rate of output.
In the above diagram the straight line represents standard cost. Observed cost behavior is represented by the curved lines which approach the standard cost function asymptotically. (The broken lines, \( c_1(q) \) and \( c_2(q) \), represent technical economic cost functions on which operations are conducted at two different time points.)

In practice, under reasonably effective control conditions, a series of discrete observations would tend to cluster about the linear standard cost function.

Application of statistical regression techniques without taking into account the control apparatus and resulting production behavior patterns might thus serve to
confirm only that the controls were effective on the cost side. To test economic hypotheses in this area may require something more than "statistical goodness of fit." Careful formulation of the hypotheses to be tested and adaptation of statistical methods to these hypotheses will be required. This implies that factors which significantly affect cost behavior must be taken into account. But cost behavior is affected by production behavior. And the latter is guided by the behavior of the official responsible for production, reacting against the constraints of the technical-economic cost function. It is clear that the behavior of costs can no more be abstracted from the behavior of the production official than the latter's behavior can be abstracted from the control milieu in which he operates and the criteria by which he is judged.

The matter may be stated alternatively. Statistically determined and observed linear cost behavior is not incompatible with the hypothesis of non linear technical economic cost functions, as posited by economic theory. Statistical "goodness of fit" is not, alone, as Haavelmo has shown, the test. The test lies, rather,

30/ It is perhaps of interest to note that one of the few cases of statistical investigation of the behavior of business costs which yielded non linearity and rising marginal costs appears in Roswell Whitman's study of department store cost functions. See American Economic Review, Supplement, XXX (March, 1940), pp. 401 ff. For a variety of reasons it is not usually possible to employ standard cost controls to retailing in department stores, at least in the usually narrow and restricted sense of standard costs.

The "empirical" studies of OPA and the War Department, conducted as part of their price analyses, seemed, in many cases, to display rising costs of a non linear character with markedly expanded output. Under the exigencies of war production many manufacturers abandoned or disregarded their standard cost systems, along with other controls. (The employees balked at being judged by these standards when large expansions in production budgets were imposed on them.) Much broader controls, such as cost models, etc., were utilized--insofar as any semblance of control was retained. See E. L. Kohler and W. W. Cooper, "Costs, Prices and Profits: Accounting in the War Program," The Accounting Review, July 1945, pp. 1-42.


\[
\begin{align*}
(1a) & \quad q = H \left[ C(q) \right] \\
(1b) & \quad c = C(q, \alpha) \\
(1c) & \quad \dot{c} = \frac{\partial c}{\partial q}(q)
\end{align*}
\]
in prediction. If behavior is altered, say, by adjustment of the control mechanism—if, for example, standard costs are adjusted through the shift factor \( \alpha \)—the question is whether, within prescribed limits of error, the new level of costs may be predicted (e.g., from production budgets). Such an approach is, however, more complicated than statistically determining "goodness of fit." Failure of the test leaves no easy, automatic solution such as increasing the power of the curve to be used. Such failure leaves unresolved the question of which part of the model is at fault. It may be the assumed behavior pattern; it may be the assumed technical economic cost function; or

\[
q(t) = q^0 + \text{const.} \sum_{q} (q^0, \alpha^0) - c_q(q^0) t \quad e^{\alpha (q^0, \alpha^0) (\frac{\alpha - \alpha^0}{q - q^0}) t}
\]

Diagramatically this may be portrayed as follows:

(See Diagram II-A below)
the fault may lie elsewhere—e.g., in the statistical assumptions. Similar doubts may be entertained even when the prediction is successful. The problem is further complicated by the fact that behavior patterns are likely to be altered whenever controls are adjusted. 32/

Cost Behavior

B. Multiple Commodities

Attention has been restricted to stable cases in the preceding analysis. As in other fields of economics, instability may not be of major interest. For the firm producing a single commodity, controls lie ready at hand to prevent instability. Plans may be laid and production and costs guided with relative ease and simplicity. (This, of course, does not preclude the necessity or desirability of altering plans.) The consequences of cost (or production) departures and control adjustments are immediate, not remote, and may, for the most part, be detected with little difficulty.

The discontinuous line represents standard costs and the solid curved lines represent cost behavior before, and dotted curved lines represent cost behavior after the shift—which may be assumed to have been made, for example, to accelerate production per unit time and thus overcome retardation introduced at the previous control level. Of particular interest in this connection are the remarks on prediction and identification in the articles cited in this note.

32/ Behavior patterns are likely to be affected by a host of "institutional" factors, many of which may not be measurable—at least directly and easily. For statistical manipulation it is, of course, necessary that some means of measurement be found if the items are to enter directly into the analysis. In any event it would seem wise to conduct the "hard core" of the analysis in these terms and enter the qualifications at a later stage. For this reason attention has been focused here on the directly measurable parts of the control mechanism, such as standard costs.
Under conditions of multiple commodity production, the matter is less simple. Instability may easily become an important and intractable problem. Its sources may be concealed and the consequences of adjustment in a particular case may be remote and difficult to trace. In a particular department things may not be proceeding in accordance with plan; production budgets may not be met, or production may be in excess of the budgeted amount; costs may fail to meet standards, etc. Things are out of control. Adjustment of controls in the affected department may prove unavailing, or worse. The trouble may lie elsewhere, perhaps in seemingly removed processes or departments which are related vertically, horizontally, or overhead. Adjustment of controls and alteration of behavior patterns may compound the problems not only in the affected but in related departments. In multiple commodity operations independence and neutrality are likely to be the exceptions. Complementarity and substitutability, cooperancy and rivalry are likely to be the rule. Costs and facilities are likely to be common and joint.

Problems arising from these sources have proved to be some of the most difficult and baffling which industry has faced. Perhaps the commonest solutions have been through so-called administrative layering. This may take innumerable forms. If schedules are not being met and cost behavior is unsatisfactory a production planning and control department, for example, may be installed, perhaps in a staff capacity.

33/ "Stability may be variously defined." Cf. V. A. Kostitzin, Mathematical Biology (London: George G. Harrop & Co., Ltd., 1939) p. 35. "We may speak of stability when small variations in the coefficients are followed by only small variations in the results... We may speak of stability when in the neighborhood of a stationary state small variations in initial values lead to solutions tending, when \( t \to \infty \), towards the equilibrium or prescribed values." Although Kostitzin's first concept of stability would probably be more useful for most economic investigations it is the latter, and more exact definition which will be used here. Thus, if upon disturbance the variable fails to return to the prescribed point or path its behavior will be described as unstable. For further discussion of the notion of stability see Paul Samuelson, op. cit. Ch. IX, ff.

34/ See supra footnote 2/


36/ See Alfred Marshall, Industry and Trade (London: Macmillan & Co., Ltd., 1932.) p. 193: "When two things, say locomotives and stationary engines, are made in the same
Once installed, forces drawing it into the line are almost irresistible. Even when this temptation is resisted, the necessity for utilizing chains of command interrupts communication. More may be done to retard than to facilitate production and cost control. Under pressure, the formal may slide into the informal organization with all that this implies for dissipation of continuing purpose and efficiency.37/

The problem is even more complex viewed in entirety. In a modern large-scale organization production of a vast variety of commodities, ranging perhaps to many thousands in number, may be under way. This may necessitate the installation of a number of different production control organizations. Conflicts may thus arise between the different control organizations as well as between the control organs and the line departments. Committees or other forms of organization may then have to be superimposed on this overlay in order to avoid "backfire" of the delegation and overwhelming top management with the routine of report interpretation and dispute settlement. Given the factorial and exponential38/ increase of contacts and possibilities of conflict, even such layering has not always proved successful in avoiding these increased burdens. Lengthening of the executive arm by appointment of staff assistants may help, but it adds problems of its own.

Solution of these problems by such organization devices can hardly be said to be completely satisfactory. The basic lack is information. And this lack is re-

works, and in a great measure by the same labour and plant, it is often said that their costs are 'joint'; but, this term has a special historical association with groups of things, such as wheat and straw, which cannot be produced separately; and it seems better to speak of such groups as having 'common' or 'allied' costs."

37/ See the references cited in supra footnote 9/.

enforced by absence of techniques with which to obtain the necessary information. Perhaps with the development of some of the newer statistical techniques, the economist may be able to bring to bear the discipline of his theory in the solution of these problems. In the process, contact may perhaps also be made with that "laboratory of nature" which has proved so fruitful in other disciplines.

Although vigorous prosecution of this line of economic inquiry may not be of great value without recourse to empirical investigations, still it may be worthwhile again to suggest what is involved by the use of a simple model already familiar to economists from investigations in other areas. Since administrative and accounting procedures attempt to "fix responsibility," standard costs are generally constructed for only a single commodity, production order or department. As before, the standard cost function may therefore be written:

$$c_k = c_k(q, \gamma) \quad k = 1, 2, \ldots, n.$$  

The technical economic cost functions must, however, be altered to include all commodities under consideration. This function becomes:

$$c_k = c_k(q_1, q_2, \ldots, q_n) \quad k = 1, 2, \ldots, n.$$  

The behavior equations then take the form:

$$q_k = h_k \left\{ c_k(q, \gamma) - c_k(q_1, q_2, \ldots, q_n) \right\} \quad k = 1, 2, \ldots, n,$$

with the properties assumed for the single commodity case. Even though the standard cost control is constructed with reference to a single commodity or department, the production official adjusts his behavior (perhaps unconsciously), through the technical-economic cost function, to activity in other departments or commodities.

-- 39/ E.g., the simultaneous equation approach as developed by Haavelmo and others. See the references cited in footnote 31.

Expanding in Taylor series, an approximate solution to equation (6) becomes:

\[
q_k(t) = \sum_{i=1}^{n} \lambda_i t
\]

where the \(\lambda_i\) are determined from the characteristic determinant:

\[
\begin{vmatrix}
m_1 H_1' (c_{11} - c_{11}) - \lambda & m_1 H_1' c_{12} & \cdots & m_1 H_1' c_{1n} \\
m_2 H_2' c_{21} & m_2 H_2' (c_{22} - c_{22}) - \lambda & \cdots & m_2 H_2' c_{2n} \\
m_n H_n' c_{n1} & m_n H_n' c_{n2} & \cdots & m_n H_n' (c_{nn} - c_{nn}) - \lambda \\
\end{vmatrix} = 0
\]

The formal properties of such systems have been examined elsewhere. Lange and Metzler have shown that the constants \(m_i\) and \(H_i'\) represent speeds of adjustment. \((H_i'\) is an index of cost flexibility and \(m_i\) a measure of the force brought to bear on cost departures.) When standard and actual costs are equal in every department the \(\lambda\) become zero.

In this sense, equilibrium may be said to obtain. But, if any cost is out of line the attempt to bring it back to standard will affect the costs (and production) of every related commodity or department. Of course, the system may ultimately return to equilibrium, and in some cases it may be best to leave conditions alone despite danger signals from the control apparatus. But ultimate solutions may hardly prove satisfactory in a business enterprise. In any event the acquisition of appropriate information should make it possible to effect necessary adjustments more quickly and intelligently by isolating the cause or causes of the trouble and facilitating the projection of consequences. It is [41] In the reference cited supra footnote 40.

[42] Not only because of their cost effects on commodities but also because the continued existence of maladjustments may do irreparable harm to the organization itself.

[43] If attention be restricted to two commodities, the point involved may be illustrated graphically, as in the following diagrams:

(See the diagrams on the following page.)
Diagram A shows the cost behavior in Department A and diagram B shows cost behavior in Department B. As production is accelerated in B it forces costs further above standard in A. Retardation begins to occur in A which results in "cost economies" and acceleration in B. The methods department and accounting department then undertake a revision of standards in A from $\tilde{c}_1$ to $\tilde{c}_2$. The alteration in cost and production behavior in A then force costs away from standards in B. (The altered behavior pattern is indicated by the dotted lines.) This may lead to continuing adjustments from B to A, and so on. Ultimately the problem may be solved by cut and try methods, but continuing instabilities may, in the meantime, disrupt production and budget schedules. Statistical investigation may have suggested the alteration indicated by $\tilde{c}_3$ with a satisfactory solution in both departments. (This involves both a shift and change in slope of the control curve in order to encourage output of A, for its beneficial effect on B, up to a point, and then discourage further increases in the rate of production.) Even where this cannot be done, such studies may well serve to point toward repercussions which ought to be taken into account before undertaking adjustment in A.

The latter case, isolation of repercussions and guidance to potential trouble spots, is the more probable. Statistical method is not likely to provide an automatic substitute for managerial judgment. Time study, method and motion study will undoubtedly continue to be necessary and beneficial. But many mistakes can perhaps be avoided and more orderly scheduling of work obtained through the use of more careful analyses of "hidden" factors. In so far as statistical-economic methods can achieve this it can make an important contribution—even failing "exact" solutions. As such it may effect a logical extension of the regular reporting-control system.
worth noting that adjustment of the standards will, in general, again affect the system beyond the department or commodity for which adjustment is made. Repeated adjustments certainly create problems of transition. In addition, by imposing repeated shocks on the system, they may engender additional instabilities and, perhaps, render the whole system inoperative. More than one case has occurred in which the entire control mechanism has been discredited and cast aside by such activities.\footnote{An interesting example may be found in the Harvard School of Business case study of the expensive standard cost installations of the Doehler Die Casting Company, each of which was installed, adjusted repeatedly, and abandoned.}

**Conclusion**

In a previous article\footnote{Reference cited in supra footnote 1.} an attempt was made to view the business firm from top level management. Emphasis was on the atomic level of firm operations and policy. The present article offers some suggestions for viewing the firm from the standpoint of down-the-line management—the subatomic level. Of course these are matters of emphasis only. A complete theory of the firm would work out rigorously the interconnection between the two levels and explore the formal properties of such a system. This would involve the solution of many problems such as aggregation problems, which have occupied the attention of economists in other areas of research. Problems of administrative as well as economic theory would have to be encompassed in a meaningful theory. In addition to the static theory of organization and control, \footnote{A branch of administrative theory still in its embryonic stage despite the work of such writers as Simon and Barnard, Cf. Supra footnote 11.} administrative dynamics would have to be developed.

Similar comments might be offered for reporting and control mechanisms.
Perhaps something might be done by the introduction of notions of "feedbacks" and servomechanical theory as developed in the natural sciences.\footnote{\textit{E.g.}, robot designs, thermostats, etc., in which the flow of information or feedback, controls the operation of the system and hence is both powered by and directs the system. See Hubert M. James, Nathaniel B. Nichols, Ralph S. Phillips, \textit{The Theory of Servomechanisms} (New York: McGraw-Hill Book Co., Inc., 1947), p. 12: "A servo system is a combination of elements for the control of a source of power in which the output of the system is fed back for comparison with the input and the difference between these quantities is used in controlling the power."} Despite the powerfully suggestive analogies of this theory for administration and economics, and despite the recent extension of this theory in the new discipline of cybernetics developed by Professor Wiener,\footnote{\textit{Cybernetics} (New York: John Wiley \& Sons, 1948.)} much work must be undertaken before it can be applied to business firm theory in fruitful form. In the end, the scope and complexity of top management decisions, in particular, may prove too difficult for such applications.\footnote{See next page.}

However one may feel about the possibility of theoretical extension by such devices, a great deal more must become available in empirical evidence before clarity and meaningfulness in formulation can be achieved. In the present state of knowledge it has hardly seemed worth while pushing beyond a few simple models. It is not that empirical evidence is completely lacking. The difficulty lies in the fact that the evidence is unsystematic and unavailable in a form suitable for theoretical analysis. Economists who have brought their talents to bear on applied problems seem to have a tendency to abandon their theory in the face of problems. Perhaps this is the fault of theory; perhaps the fault lies elsewhere. Whatever may have been the accomplishments of such economists in the solution of practical problems seems to be more a tribute to their talent than their discipline. In so far as the comments offered here are correct, economics has been deprived of the powerful discipline and impetus of systematic problem presentation and suggestion from an important segment of the economic universe—
business firm. Under such circumstances it seems worth while exploring the possibility of revising theory to determine whether it may be put in more suitable form.

49/ See the remarks of Professor Wiener, ibid., p. 34: "The human sciences are very poor testing-grounds for a new mathematical technique... Moreover, in the absence of reasonably safe routine numerical techniques, the element of the judgment of the expert in determining the estimates to be made of sociological, anthropological, and economic quantities, is so great that it is no field for a newcomer who has not yet had the bulk of experience which goes to make up the expert... The modern theory of small samples, once it goes beyond the determination of its own specially defined parameters and becomes a method for positive statistical inferences, does not inspire me with any confidence unless it is applied by a statistician by whom the main elements of the dynamics of the situation are either explicitly known or implicitly felt."