COWLES COMMISSION DISCUSSION PAPERS, ECONOMICS: 228

Comments on the Input-output Technique*
(Prepared for a Committee to appraise this technique)

by T. C. Koopmans

October 4, 1948

In these brief and hurriedly prepared comments the input-output technique is understood to be a technique of programming the use of resources for a stated objective, by means of a mathematical model specifying a number of productive activities, each of which is characterized by constant ratios between the quantities of the commodities (goods and services) absorbed and produced by it.

The comments are arranged under the headings:

A. Generality and limitations of the model.

B. Principles of classification.

C. Substitution and equivalence ratios.

D. Decentralization of programming decisions.

E. Formulation of the objective.

Reference will be made to the list of 13 numbered problem areas that has been circulated by the Committee.

A. Generality and limitations of the model (Problem 2)

If no limit is placed on the number of activities and the fineness of classification of commodities, the fixed input-output ratio assumption is extremely general and valid over a wide area. The basic assumption involved is that it is possible to duplicate, triplicate, etc. any activity

* For a mathematical formulation of the theory underlying these comments see "Systems of linear production functions," C.C. Discussion Paper No. 215. For an application of linear programming techniques to the transportation industry, see "Optimum utilization of the transportation system," C.C. Discussion Paper No. 214.

that is capable of execution in unit amount. This is not in conflict with the phenomenon of decreasing returns noted particularly with respect to agricultural production. That phenomenon is not due to any basic non-linearity or non-reproduceability of productive activities. It is due to limits to the available amounts of basic factors (e.g. land) in the qualities required for the most efficient of the possible productive processes. Such limits can be introduced explicitly in the analysis.

Situations (also frequent in agriculture) where one factor can be continuously substituted for another can be incorporated in the model by choosing a discrete set of combination ratios (with sufficiently small discrepancies between any two adjoining ratios) and letting each such ratio define another productive activity.

The following are limitations which one can attempt to meet by refinements of the model:

(a) Indivisibilities. In its usual form the model specifies also that the amount of each activity can be halved, divided by 10, etc., without changing the input-output ratios. This is not true for highly mechanized production. Neither is it true for situations such as occur in transportation where an entire road is needed before a single truck or train can move. To meet these situations, it is necessary to introduce activities which can be carried on only at amounts which are an integral multiple of a given unit.

(b) Uncertainty. The inventories required to meet contingencies such as interruption of production through mechanical failure, weather conditions, etc. are less than proportional to the scale of operations.

Further departure from fixed input-output ratios, and indeed from any definite and unique functional relationship, may and will occur as a result of aggregation of activities and commodities (problem 3, see B below).

B. Principles of classification (Problem 3)

The following seem to be relevant factors affecting the desirable degree of aggregation.

Favoring disaggregation:

(a) The objective may be formulated as a bill of goods and services of many dimensions. The amount of detail in the bill of goods sets a limit to the extent of aggregation of commodities and activities.

(b) The further aggregation is carried, the greater departure must be expected from fixed input-output ratios. If for instance more steel can be had promptly only at greater than average cost, this can be expressed in a fixed-ratio model only by defining two or three different steel-producing activities of successively decreasing efficiency, each activity being circumscribed by its own given limit to promptly available capacity.

Favoring aggregation:

(c) The cost and time involved in construction of very elaborate models and in the data collection and the computation for such models.

A good compromise may be helped through techniques of decentralization of decisions suggested under D below.

C. Substitution and equivalence ratios (Problem 8)

Substitution and choice can be studied in the models under consideration with greater generality than is possible by the use of the "smooth" production functions of economic literature. It is, however, not studied through continuous change in the ratios in which factors are combined in a productive process. It is studied instead by simultaneous increase in the amount of one (or more) activity (-ties) and decrease in the amount of another (others). It has been argued under A that this includes the "smooth" substitution case to a sufficient degree of approximation. In addition, it includes a type of discrete substitution, prevalent particularly in industry, which has been insufficiently studied in economic literature. Often an entirely different material is substituted, or an entirely different industrial process, for one previously used.

For a fixed-ratio model to permit the study of substitution and choice, it is necessary that— unlike the models studied in Leontief's published work— the number of activities incorporated in the model exceed the number of items in the bill-of-goods. This point can be illustrated by the following example of a sheep-raising economy with one input commodity (labor), a bill-of-goods consisting of two commodities (mutton and wool) and three productive activities (three breeds of sheep). Let us assume that the input-output coefficients for the unit of each activity (negative coefficients denote input) are:

	activity		
	(1)	(2)	(3)
wool	1	1.2	0.5
mutton	1	0.5	1.2
labor	-1	-1	-1

To produce given amounts of wool and mutton, choice can be exercised to select that combination of the three activities which minimizes the labor required. It is for instance clear that it cannot be economical to engage simultaneously in activities (2) and (3), because two equal one-unit amounts of (2) and (3) are inferior, in the product derived from the same amount of labor, to two units of activity (1) -- (quantities 1.7 of both mutton and wool as compared with 2). Further analysis shows that any bill-of-goods in which the desired amount of wool exceeds (in the units used) the desired amount of mutton (by not more than 140%) is most efficiently met by the appropriate combination of activities (1) and (2). Similarly, any bill-of-goods in which mutton exceeds wool (within the same limit) is met by a combination of (1) and (3).

In the following example the first case applies:

	(1)	(2)	(3)	total
amount of activity	1	1	0	
output of wool	1	1.2	0	2.2
output of mutton	1	0.5	0	1.5
input of labor	-1	-1	0	-2

The same quantities of wool and mutton can not be produced with less labor by some other combination of activities.

The next question involving substitution of activities is this: If we can only spend the same amount of labor and wish now to produce 1.6 units of mutton instead of 1.5, how much wool must of necessity be sacrificed? The answer is given by the following table:

	(1)	(2)	(3)	total
amount of activity	1.2	0.8	0	
output of wool	1.2	0.96	0	2.16
output of mutton	1.2	0.4	0	1.6
input of labor	-1.2	-0.8	0	-2

It follows that .04 units of wool are sacrificed to gain 0.10 units of mutton. Analysis shows that this equivalence ratio of $\frac{4}{10}$ units of wool to one unit of mutton is a constant applicable to all variations in the bill of goods which leave the proportion of wool to mutton in the new (changed) total bill-of-goods within the range (1/1 to 1.2/0.5) capable of efficient realization by a combination of activities (1) and (2).

Similar equivalence ratios can be worked out between labor and wool, labor and mutton, for each efficient combination of activities. The following table summarizes the ratios:

One additional unit of

wool	mutton	labor	is equivalent to additional units of	
1	0.4	-1.4	wool	if both the original and the new bill-of-goods are attainable by combining activities (1) and (2)
2.5	1	-3.5	mutton	
$-\frac{5}{7}$	$-\frac{2}{7}$	1	labor	
1	2.5	-3.5	wool	if both the original and the new bill-of-goods are attainable by combining activities (1) and (3).
0.4	1	-1.4	mutton	
$-\frac{2}{7}$	$-\frac{5}{7}$	1	labor	

In more elaborate models, similar equivalence ratios between variations of the items in the efficiently produced bill-of-goods can be established, which are applicable within a range corresponding to all efficient combinations of a given set of activities. These ratios can be extended to intermediate products that do not appear in the bill-of-goods and to basic factors of production which are limited in quantity (like labor in the example).

D. Decentralization of programming decisions (Relevant to problem 3)

The equivalence ratios can be regarded as accounting prices (in the sense of Meade, Lange, Lerner) which can be used to guide, and ensure compatibility of, allocative decisions taken in different administrative units. One possibility is to construct a model with a considerable degree of aggregation for the purpose of evaluating equivalence ratios for the main raw materials, types of labor, and classes of products, for each of a number of future periods. These equivalence ratios are then given as a frame of reference to a number of specialized programming units that work out more detailed programs, each dealing with a particular sector of the entire program. In this way one avoids the difficulties of channelling an enormous mass of detailed technical information through one single programming unit.

A very important particular case of this technique of decentralization is that in which the equivalence ratios are expressed by money prices in a free market and the specialized programming units are firms pursuing a policy of profit maximization which regards the prices as given.

E. Formulation of the objective

In preceding comments the objective has been taken as the production of a given bill-of-goods at a minimum cost in some sense. This formulation is not general enough to meet realistic situations. In most cases there is not one single unit of cost, such as money or labor, but a number of basic and unalterable limitations to the size of the productive effort, such as the size of the population, the total area of various grades of land, the initial amounts of capital equipment of various types, and the technically maximum rate of depletion of various mineral resources. It cannot be foreseen without elaborate calculations whether a given bill-of-goods is capable of production within these limitations. An ideal objective would be a complete description of the set of alternative bills- of-goods that can be produced efficiently within these limitations. This would fully inform the policy-making authority or body of the alternatives before it. However, it would seem that such a set of alternatives is too vast to be capable either of being encompassed by a single or collective mind or of being determined and described with conceivable computation machinery.

A narrower objective that has been proposed is the maximization of a linear function of the total net outputs of a number of desired commodities. This objective implies that the programming unit sets "subjective" prices (valuations, expressed by the coefficients of the linear function) on the desired commodities, and welcomes any rearrangement of activities (within the basic limitations) that increases the total value of the commodities produced, as evaluated in these "subjective" units. Such rearrangements remain possible until the technological equivalence ratios discussed under C correspond to the "subjective" valuations.

This formulation is still too narrow. It implies that we can say how many pounds of butter are to be equivalent to one gun before we know how much butter and how many guns will be produced in a program based on that indication. This is at variance with the law of decreasing marginal utility (or decreasing marginal rate of substitution).

An intermediate possibility is an experimental procedure whereby "subjective" prices are tentatively specified as a first step, and an optimal program in that valuation system is computed. This yields one feasible program that contains no internal inefficiencies. The corresponding equivalence ratios suggest in which way the program can be modified by limited amounts without losing either feasibility or internal efficiency. If larger variation is called for, this can be effected, as a second step, by revising the "subjective" prices, setting down the prices of commodities of which the first program contains relatively "too much," setting up the prices of those represented "too little." Such successive revisions can be repeated until a satisfactory program is obtained.