A Theory of Asset Prices based on Heterogeneous Information and Limits to Arbitrage

Elias Albagli – USC Marhsall

Christian Hellwig – Toulouse School of Economics

Aleh Tsyvinski – Yale University

September 20, 2011
Motivation

- This paper: asset pricing theory based on heterogeneous info and limited arbitrage

  - **Parsimonious**: all results derive from these 2 elements
  - **General**: tractability allows analysis of wide class of securities

- Central message: **systematic departure** of prices from fundamentals

  - Beliefs are heterogeneous: private signal + price
  - Price = expectation of **marginal** trader
  - Noisy info aggregation ⇔ prices ≠ expected dividends (cond. on public info)
    - Price over/undervaluation (depending on payoff structure)
    - Price volatility can exceed realized dividend volatility

- **Variety of applications**:
  - M&M capital structure irrelevance
  - Excess volatility of stock returns
  - Price reaction to public announcements
Relation with Literature

1. **Information aggregation**
   - Grossman and Stiglitz (AER 80); Hellwig (JET 80); Diamond and Verrecchia (JFE 81); Wang (REStud 93).

2. **Heterogeneous Beliefs and Bubbles**
   - Harrison and Kreps (QJE 78); Scheinkman and Xiong (JPE 03); Abreu and Brunnermeier (ECT 03).

3. **Finance puzzles**
   - M&M Capital structure irrelevance: Myers (JF 84); Myers and Majluf (JFE 84).
   - Excess return volatility: Shiller (AER 81), Campbell and Shiller (RFS 88), Cochrane (RFS 92).
   - Stock price under/overreaction: Barberis et al. (JFE 98); Daniel et al. (JF 98); Hong and Stein (JF 99).
Outline of Talk

1. Setup
2. Information Aggregation Wedge
3. Applications
4. Robustness
Setup
Environment

- Single risky asset in unit supply
- Pays $\pi(\theta)$;
  - Fundamental: $\theta \sim N(0, \sigma^2_\theta)$
  - Dividend function: $\pi'(\cdot) > 0$, otherwise unrestricted
- Two dates:
  - Trading in financial market ($t = 0$)
  - Payoffs realized ($t = 1$)
Financial Market: \( t = 0 \)

- Informed traders: \( i \in [0, 1] \)
  - Risk-neutral
  - Limits to arbitrage: Can buy at most 1 share, and cannot short-sell
  - Observe private signal \( x_i \sim N(\theta, \beta^{-1}) \), share price \( P \)
  - Buy \((d_i = 1)/\) don’t buy \((d_i = 0)\):
    \[
    d(x, P) = \begin{cases} 
    1 & \text{if } \mathbb{E}[\pi(\theta) \mid x_i, P] \geq P \\
    0 & \text{otherwise}
    \end{cases}
    \]

- Aggregate informed demand: \( D(\theta, P) = \int d(x, P) d\Phi(\sqrt{\beta}(x - \theta)) \)

- Noisy demand: \( \Phi(u); \ u \sim N(0, \sigma_u^2) \)
Equilibrium Definition

A Perfect Bayesian Equilibrium (PBE) consists of

1. Price function $P(\theta, u)$

2. Informed traders’ demands $d(x, P)$

3. Posterior beliefs $H(\theta|x, P)$ for informed traders s.t.,
   
   (i) Informed traders demands are optimal (given beliefs)
   (ii) The asset market clears
   (iii) Posterior beliefs satisfy Bayes’ rule
Trader Optimality: Threshold Strategy

- Expected dividend, and demand $d(x, P)$: monotone in $x$

  - Trading strategy: signal threshold $\hat{x}(P)$
    
    $$d(x, P) = \begin{cases} 
      1 & \text{if } x_i > \hat{x}(P) \\
      0 & \text{if } x_i < \hat{x}(P) \\
      \in (0, 1) & \text{if } x_i = \hat{x}(P) 
    \end{cases}$$

- Price = dividend expectation of marginal trader ($x_i = \hat{x}(P)$)

  $$P = \mathbb{E}[\pi(\theta) | x_i = \hat{x}(P), P] = \int \pi(\theta) dH(\theta | \hat{x}(P), P)$$
Market Clearing

\[ D(\theta, P) + \Phi(u) = 1; \]

\[ \Phi(\sqrt{\beta}(\hat{x}(P) - \theta)) = \Phi(u) \]

\[ \hat{x}(P) = \theta + 1/\sqrt{\beta} \cdot u \equiv z \]

- \( P \): aggregates private info
  - \( P \) informationally equivalent to \( \hat{x}(P) = z \)

- \( z \): endogenous public signal
  - Increasing in fundamental \( \theta \), noisy demand \( u \)
  - \( \theta|z \sim N(z, \sigma_u^2/\beta) \); \( \beta/\sigma_u^2 \) precision of \( z \)
  - \( \beta \): private info precision; \( \sigma_u^2 \): noisy demand variance
Proposition: Asset Market Equilibrium

- Unique equilibrium: price $P_\pi(z)$ and traders' threshold $\hat{x}(p) = z = P_\pi^{-1}(p)$,

\[
P_\pi(z) = \int \pi(\theta) d\Phi \left( \sqrt{\sigma_\theta^{-2} + \beta + \beta \sigma_u^{-2}} \left( \theta - \frac{\beta + \beta \sigma_u^{-2}}{\sigma_\theta^{-2} + \beta + \beta \sigma_u^{-2}} z \right) \right) \\
= \int \pi(\gamma_P z + \sigma_\theta \sqrt{1 - \gamma_P u}) \phi(u) du
\]

- Marginal trader pricing share conditions on private signal $x_i = z$; public signal $z$

- Bayesian weight $\gamma_P$ on signal $z$; residual uncertainty $= 1 - \gamma_P$

- Expected dividend, conditional on public signal $z$ only

\[
V_\pi(z) = \int \pi(\theta) d\Phi \left( \sqrt{\sigma_\theta^{-2} + \beta \sigma_u^{-2}} \left( \theta - \frac{\beta \sigma_u^{-2}}{\sigma_\theta^{-2} + \beta \sigma_u^{-2}} z \right) \right) \\
= \int \pi(\gamma_V z + \sigma_\theta \sqrt{1 - \gamma_V u}) \phi(u) du
\]

- Bayesian weight $\gamma_V (< \gamma_P)$ on signal $z$; residual uncertainty $= 1 - \gamma_V$
Information Aggregation Wedge
Information Aggregation Wedge

- Information aggregation wedge: \( W_\pi(z) \equiv P_\pi(z) - V_\pi(z) \)

  - Marginal trader puts higher weight on market signal \( z \) than “outsider” who only observes the price \((\gamma_P > \gamma_V)\)

\[
P_\pi(z) = \int \pi(\theta) d\Phi \left( \sqrt{\sigma_\theta^{-2} + \beta + \beta\sigma_u^{-2}} \left( \theta - \frac{\beta + \beta\sigma_u^{-2}}{\sigma_\theta^{-2} + \beta + \beta\sigma_u^{-2}} z \right) \right)
\]

\[
V_\pi(z) = \int \pi(\theta) d\Phi \left( \sqrt{\sigma_\theta^{-2} + \beta\sigma_u^{-2}} \left( \theta - \frac{\beta\sigma_u^{-2}}{\sigma_\theta^{-2} + \beta\sigma_u^{-2}} z \right) \right)
\]

- But \( V_\pi(z) \) is the **correct metric** for valuing the unconditional dividend:

\[
\implies \mathbb{E}[\pi(\theta)] = \mathbb{E}[V_\pi(z)]
\]
Information Wedge in Linear Case

Price more responsive to $z$ than expected dividend
Intuition: Shift in Marginal Trader’s Identity

- Key intuition: for each realization of \( z \), marginal trader is a different agent
- Higher \( z \) (due to \( \theta \), and/or \( u \)) has two effects on beliefs
  - Higher \( \theta \) shifts up distribution of signals \( x' \)’s: higher demand → higher \( \hat{x}(P) \)
  - Higher \( u \) lowers net supply → higher \( \hat{x}(P) \) to deter buying by informed
- Expectations of new marginal trader pricing shares raised through both effects
  - Higher expectations due to market signal (just like anyone else)
  - Higher expectations due to shift in identity (this is the extra kick)

⇒ “Double weighting” of market info \( z \) is rational (Bayesian updating)
Unconditional Wedge

Lemma (unconditional wedge): The unconditional wedge is given by

\[ W_\pi (\sigma_P) \equiv \mathbb{E}[W(z)] = \int_0^\infty (\pi' (\theta) - \pi' (-\theta)) \left( \Phi \left( \frac{\theta}{\sigma_\theta} \right) - \Phi \left( \frac{\theta}{\sigma_P} \right) \right) d\theta, \]

- **Sign**: related to curvature of \( \pi(\cdot) \)
- **Magnitude** given by informational frictions \( \sigma_P^2 \)
  - Marginal trader’s posterior \( \theta|z \): \( \sim N(\gamma_P z, (1 - \gamma_P)\sigma_\theta^2) \)
  - Prior: \( z \sim N(0, \sigma_\theta^2/\gamma_V) \)
  - Compounded distribution: \( \theta \sim N(0, (1 - \gamma_P)\sigma_\theta^2 + \gamma_P^2\sigma_\theta^2/\gamma_V) \)
    - \( \sigma_P^2 > \sigma_\theta^2 \)

- Marginal trader overweights tails of \( \theta \) distribution (overreacts to \( z \))
  - Pricing of shares as if \( \theta \sim N(0, \sigma_P^2) \), rather than \( \sim N(0, \sigma_\theta^2) \) (fatter tails)
Results: Over/Under-Valuation

- **Definition (risk type):** a dividend function $\pi(\theta)$, $\forall \theta > 0$,
  
  (i) Has symmetric risks if $\pi'(\theta) = \pi'(-\theta)$,
  
  (ii) Has upside risks if $\pi'(\theta) \geq \pi'(-\theta)$,
  
  (iii) Has downside risks if $\pi'(\theta) \leq \pi'(-\theta)$,
  
  (iv) If $\pi'_1(\theta) - \pi'_1(-\theta) \leq \pi'_2(\theta) - \pi'_2(-\theta)$,
      
      $\Rightarrow \pi_1(\cdot)$ has more downside (less upside) risk than $\pi_2(\cdot)$

- **Theorem (value bias):**
  
  (i) If $\pi(\cdot)$ has symmetric risk, $W_\pi(\sigma_P) = 0$
  
  (ii) If $\pi(\cdot)$ has upside risk, $W_\pi(\sigma_P) > 0$
  
  (iii) If $\pi(\cdot)$ has downside risk, $W_\pi(\sigma_P) < 0$
  
  (iv) $||W_\pi(\sigma_P)||$ increasing in info frictions $\sigma_P$
  
  (v) If $\pi_1(\cdot)$ has more downside (less upside) risk than $\pi_2(\cdot)$,
      
      $\Rightarrow W_{\pi_2}(\sigma_P) - W_{\pi_1}(\sigma_P)$ increasing in $\sigma_P$
Risk Types and Information Wedges

Symmetric Risk

\[ E[P(z)] = E[V(z)] \]

Expected wedge = 0 (as in CARA-normal)
Risk Types and Information Wedges

Expected wedge > 0: Overpriced security (on expectation)
Risk Types and Information Wedges

Expected wedge < 0: Underpriced security (on expectation)
Formal Results: Volatility

Theorem (excess variability):

For any payoff function $\pi(\cdot)$ with symmetric, upside or downside risks,

(i) $\mathbb{E}((\pi(\theta) - \pi(0))^2) > \mathbb{E}((V_\pi(z) - V_\pi(0))^2)$

(ii) $\mathbb{E}((P_\pi(z) - P_\pi(0))^2) > \mathbb{E}((V_\pi(z) - V_\pi(0))^2)$

- Prices more volatile than expected dividends

(ii) $\mathbb{E}((P_\pi(z) - P_\pi(0))^2) > \mathbb{E}((\pi(\theta) - \pi(0))^2)$, if $\sigma_u^2$ and/or $\beta$ high enough

- Prices more volatile than realized dividends, in the absence of a SDF

- Compare with West (Ect, 1988):

  $\Rightarrow$ variability of posterior expectation $<$ variability of realized dividends

  $\Rightarrow$ our model: change in identity delivers the extra volatility
Recap: Key Results thus far

- **Parsimonious model of info aggregation**
  - Applies to arbitrary (monotone) payoff functions
  - Tractability arises from risk-neutral setup with limited arbitrage

- **Main result: Information aggregation wedge**
  - Prices overreact to market info due to *identity* effect
  - Leads to average over/undervaluation (depending on curvature of $\pi(\cdot)$)
  - Leads to excess volatility of prices
Applications
Application 1: M&M Dividend Split Irrelevance

- Suppose dividend is split in 2 and sold in separate markets
  - $\pi(\cdot) = \pi_1(\cdot) + \pi_2(\cdot)$
  - $\pi_1(\cdot)$ has downside risk, $\pi_2(\cdot)$ has upside risk

- Market characteristics
  - Informed traders active in one market only, observe $x_{i,j} \sim \mathcal{N}(\theta, \beta_j^{-1})$
  - Noisy demands:
    $$
    \begin{pmatrix}
    u_1 \\
    u_2
    \end{pmatrix}
    \sim
    \mathcal{N}
    \left(
    \begin{pmatrix}
    0 \\
    0
    \end{pmatrix},
    \begin{pmatrix}
    \sigma_{u,1}^2 & \rho \sigma_{u,1} \sigma_{u,2} \\
    \rho \sigma_{u,1} \sigma_{u,2} & \sigma_{u,2}^2
    \end{pmatrix}
    \right)
    $$

- Consider informationally segmented markets
  - Traders in mkt $j$ don’t observe price $P_{-j}$
  - Results also hold in info connected markets (see paper)
  - Market characterized fully by info frictions $\sigma_{P,j}$
Application 1: M&M Dividend Split Irrelevance

Proposition:

(i) Seller’s revenue is independent of split iff $\sigma_{P,1} = \sigma_{P,2}$

$\Rightarrow$ Markets have identical information frictions

(ii) Total expected revenue from $\pi(\cdot)$ maximized by following split:

- $\pi^*_1(\theta) = \min \{ \pi(\theta), \pi(0) \}$, and $\pi^*_2(\theta) = \max \{ \pi(\theta) - \pi(0), 0 \}$
- Assign $\pi^*_1$ to investor pool with lower informational friction ($\sigma_{P,1}$)

Intuition

- $\pi^*_1$ has more downs. risk than any other $\pi_1$,
- $\pi^*_2$ has more ups. risk than any other $\pi_2$,

$\Rightarrow$ Any alternative split $\{\pi_1, \pi_2\}$ transfers ups. risk from $\sigma_{P,2}$ to $\sigma_{P,1}$ investors

$\Rightarrow$ ...resulting in a net loss of revenue (lower overall wedge)
Splitting Cash Flows for Arbitrary $\pi(\theta)$

$\Rightarrow \pi_1^*$ has max. downside risk; $\pi_2^*$ max. upside risk
Application 2: Dynamic Trading

- Dynamic extension:
  - Dividend each period: \( \pi(\theta_t) \), \( \theta_t \) i.i.d.
  - Traders infinitely lived, discount future at fixed rate \( \delta \in (0, 1) \)

- Price and expected dividend satisfy recursive expression:

\[
P_\pi(z_t) = \mathbb{E}(\pi(\theta_t) + \delta P_\pi(z_{t+1})|x = z_t, z_t)
\]
\[
V_\pi(z_t) = \mathbb{E}(\pi(\theta_t) + \delta V_\pi(z_{t+1})|z_t)
\]

- And so does the wedge:

\[
W_\pi(z_t) = w_\pi(z_t) + \delta \mathbb{E}(W_\pi(z)) = w_\pi(z_t) + \frac{\delta}{1 - \delta} \mathbb{E}(w_\pi(z)),
\]

where \( w_\pi(z_t) = \mathbb{E}(\pi(\theta_t)|x = z_t, z_t) - \mathbb{E}(\pi(\theta_t)|z_t) \)
Application 2: Dynamic Trading

Proposition (Dynamic Wedge):
Suppose that $\pi(\cdot)$ is bounded below, increasing, and convex:

- For any $\sigma_P > \sigma_\theta$, $\exists \hat{\delta} < 1$ s.t. $\forall \delta > \hat{\delta}$, $W(z_t) > 0$, for all $z_t$.

- Dynamic model implies:
  - Future expected wedges raise current share price (if $\pi(\cdot)$ has upside risk)
  - For high enough $\delta$, share might always be overpriced
Application 3: Public Disclosures

▶ How does exogenous public info about $\theta$ affect wedge?

▶ Let $y \sim N(\theta, \alpha^{-1})$ be a public disclosure on $\theta$

▶ Same eq. characterization as before, but with extra info

$$P_\pi(y, z) = \int \pi(\theta) d\Phi \left( \sqrt{\frac{1}{\sigma_\theta^2} + \alpha + \beta + \frac{1}{\sigma_u^2}} \left( \theta - \frac{\alpha y + (\beta + \frac{1}{\sigma_u^2} z)}{\frac{1}{\sigma_\theta^2} + \alpha + \beta + \frac{1}{\sigma_u^2}} \right) \right)$$

$$V_\pi(y, z) = \int \pi(\theta) d\Phi \left( \sqrt{\frac{1}{\sigma_\theta^2} + \alpha + \frac{1}{\sigma_u^2}} \left( \theta - \frac{\alpha y + \beta \frac{1}{\sigma_u^2} z}{\frac{1}{\sigma_\theta^2} + \alpha + \beta \frac{1}{\sigma_u^2}} \right) \right)$$

▶ Public info crowds out impact of $z$ on price and expected dividend

▶ In the limit $\alpha \to \infty$, wedge dissapears

▶ But for finite levels of precision $\alpha$, impacts are more subtle...
Application 3: Public Disclosures

Proposition (Public Disclosures): Consider linear dividend \( \pi(\cdot) \) (holds more generally)

(i) \( \text{Var} \left( V_\pi (y, z) \right) \) increasing in \( \alpha \)
   - Standard Blackwell

(ii) For \( \sigma_u^{-2} \geq 2 \), \( \text{Var} \left( P_\pi (y, z) \right) \) increasing in \( \alpha \);
    Otherwise, \( \text{Var} \left( P_\pi (y, z) \right) \) increasing in \( \alpha \) iff \( \alpha \geq \alpha' \)
    - If noisy demand too volatile, \( \alpha \) reduces price overreaction to \( z \)
    - But for large enough \( \alpha \), price vol increasing (more responsive to \( \theta \))

(iii) \( \text{Var} \left( W (y, z) \right) \) is decreasing in \( \alpha \) iff \( \alpha \geq \alpha'' \) (and increasing otherwise),
    - For low \( \alpha \), an increase reduces impact of \( z \) on \( V_\pi (y, z) \) more than on \( P_\pi (y, z) \)
      \( \Rightarrow \) Larger wedge
    - But for large enough \( \alpha \), both \( V_\pi (y, z) \) and \( P_\pi (y, z) \) hardly respond to \( z \)
      \( \Rightarrow \) Wedge vanishes
Robustness
Robustness

- Alternative distributional assumptions: let
  - $\theta \sim$ on arbitrary smooth prior on $\mathbb{R}$, $x_i \sim$ iid cdf $F(\cdot|\theta)$ satisfying MLRP,
  - Noisy demand $D \sim$ according to cdf $G(\cdot)$ on $[0, 1]$
  - Can always characterize wedge in this environment

- Price impact of information
  - Let noisy demand be elastic: $D(u, P) = \Phi(u + \eta(\mathbb{E}(\pi(\theta)|P) - P))$
  - Wedge is inversely related to elasticity $\eta$
  - Noise traders arbitrage away the wedge

- Wedge in CARA-normal setup (noisy REE)
  - Can only solve in the linear case: $\pi'(\cdot) = k > 0$
  - Wedge has two components
    - A constant reflecting discount (premium) for average shares held
    - A symmetric information aggregation wedge
  - Unconditional returns driven by the average compensation for risk
Conclusions

- Tractable noisy REE framework
  - Heterogeneous beliefs, risk neutrality and limited arbitrage
  - Useful to analyze more general payoff structures

- Key result: **information aggregation wedge**
  - Prices overreact to market information
  - Generates excess price/return volatility
  - Generates over/undervaluation on average (depending on shape of payoffs)

- Applications in finance
  - M&M capital structure irrelevance
  - Excess volatility puzzle
  - Impact of public disclosures