Matching with Transfers
2015 Koopmans Lecture, Yale University
Part 2: Empirical applications

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1. Empirical implementation
2. The US education puzzle
   - One-dimensional version: CSW (2014)
   - Two-dimensional version: Low (2014)
   - Matching patterns and behavior: CCM 2015
Roadmap

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2. The US education puzzle
   - One-dimensional version: CSW (2014)
   - Two-dimensional version: Low (2014)
   - Matching patterns and behavior: CCM 2015
Empirical implementation: which data?

- Basic question: what do we observe?
  → various possibilities:

  - Matching patterns only
  - Matching patterns and (information on) total surplus
  - Matching patterns and transfers

Basic issue: reconcile the somewhat mechanical predictions of theory and the fuzziness of actual data

For instance, with supermodular surplus, matching should be exactly assortative ...

which we never observe

Two solutions:

- Frictions (search,...)

Shimer and Smith, Robin and Jacquemet, Goussé, ...

Unobservable heterogeneity: some matching traits are unobservable (by the econometrician)

Here: second path
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Initial remark:

Matching models cannot be identified from matching patterns only

- Simple example: assume one dimensional matching, with supermodular surplus. Then:
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- Therefore: specific stochastic structures are
  - indispensable
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- ... unless we can observe more than only matching patterns!
Empirical implementation 1: matching patterns only

- Agent belong to a (small) number of categories: $i \in I, j \in J$

Basic insight: unobserved characteristics (heterogeneity)

Gain $g_{IJ}^{ij}$ generated by the match $i \in I, j \in J$:

$$g_{IJ}^{ij} = Z_{IJ} + \varepsilon_{IJ}^{ij}$$

where $I = 0, J = 0$ for singles, and $\varepsilon_{IJ}^{ij}$ random shock with mean zero.

Therefore: dual variables $(u_i, v_j)$ also random (endogenous)

What do we know about the distribution of the dual variables? Not much!

Alternative approach: use the stability inequalities $u_i + v_j \geq g_{IJ}^{ij}$ for any $(i, j)$

Large number (one inequality per potential couple)
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- Alternative approach: use the stability inequalities
  \[ u_i + v_j \geq g_{ij}^{IJ} \quad \text{for any } (i, j) \]
  \( \rightarrow \) large number (one inequality *per potential couple*)
Empirical implementation

- Crucial identifying assumption (Dagsvik 2000, Choo-Siow 2006)

**Assumption S (separability):** the idiosyncratic component \( \varepsilon_{ij} \) is additively separable:

\[
\varepsilon_{ij}^{IJ} = \alpha_i^{IJ} + \beta_j^{IJ} \tag{S}
\]

Interpretations:
- Idiosyncratic preferences for an educated partner
- Idiosyncratic attractiveness for an educated partner

Then:
- Theorem
- Under S, there exists \( U^{IJ} \) and \( V^{IJ} \) such that \( U^{IJ} + V^{IJ} = Z^{IJ} \) and for any match \((i_2^I, j_2^J)\)

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u_i^{ij} = U^{IJ} + \alpha_i^{IJ} \quad v_j^{ij} = V^{IJ} + \beta_j^{IJ}
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  - Or: idiosyncratic attractiveness for an educated partner
  - Only the spouse’ s category matters
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\begin{align*}
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- What's wrong without separability (i.e. $\varepsilon_{ij}$)? → Many issues
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  - Bounded support: degenerate stochastic structure (limit)

More generally:
- the frictionless assumption hard to justify with many agents
  - but not with a small number of categories!

Lastly, parcimony!
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Theorem

A NSC for \( i \in I \) being matched with a spouse in \( J \) is:

\[
U^{IJ} + \alpha_{iJ}^{IJ} \geq U^{I0} + \alpha_{i0}^{I0}
\]
\[
U^{IJ} + \alpha_{iJ}^{IJ} \geq U^{IK} + \alpha_{iK}^{IK} \text{ for all } K
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- In practice (Choo-Siow approach):

  - take singlehood as a benchmark (interpretation!)
  - assume the $\alpha^{IJ}_i$ are extreme value distributed
  - then $2$ logits (one for each gender and education)
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  - then $2 \times K$ logits (one for each gender and education) $\rightarrow U^{IJ}, V^{IJ}$
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  - assume the \( \alpha_{ij} \) are extreme value distributed
  - then \( 2 \times K \) logits (one for each gender and education) → \( U_{IJ}, V_{IJ} \)
  - and expected utility:

\[
\bar{u}^I = E \left[ \max_j (U_{IJ} + \alpha_{ij}) \right] = \ln \left( \sum_j \exp U_{IJ} + 1 \right) = -\ln \left( a_{i0}^{I0} \right)
\]
Empirical implementation (cont.)
Generalization: ‘Cupid’ framework (Galichon-Salanie 2014)

- Relax the extreme value assumption
  → the $\alpha$s and $\beta$s follow any distribution
Empirical implementation (cont.)

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- Relax the extreme value assumption
  → the $\alpha$s and $\beta$s follow any distribution

- Define the function $G_I$ by:

  $$G_I \left( U^{I\emptyset}, \ldots, U^{IK} \right) = E \left[ \max_{J=\emptyset,1,\ldots,K} \left( U^{IJ} + \alpha_i^J \right) \right]$$

  which can be computed if the distribution of the $\alpha$s is known. Then $G_I$ is increasing, convex and envelope theorem: $\partial G_I / \partial U^{IJ}$ is the probability that $i \in I$ marries someone in $J$
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*Legendre-Fenchel transform* (conjugate) of $G_I:$

$$G_I^* \left( \gamma^0, \ldots, \gamma^L \right) = \max_{U^0, \ldots, U^K} \left( \sum \gamma^L U^L - G_I \left( U^0, \ldots, U^K \right) \right)$$

Then $G_I^*$ is convex, and envelope theorem: $\partial G_I^* / \partial \gamma^J = U^{IJ}$
Empirical implementation (cont.)

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- *Legendre-Fenchel transform* (conjugate) of $G_I$:

$$G_I^* \left( \gamma^0, \ldots, \gamma^L \right) = \max_{U^0,\ldots,U^K} \left( \sum \gamma^L U^L - G_I \left( U^0, \ldots, U^K \right) \right)$$

Then $G_I^*$ is convex, and envelope theorem: $\partial G_I^* / \partial \gamma^J = U^{IJ}$

- $G^* \left( \gamma^I \right)$ is called the *generalized entropy* of the corresponding discrete choice problem.
Empirical implementation

- What can we identify?

Basic CS model:
- Severe parametric restrictions (distribution of $\alpha$s and $\beta$s known, no heteroskedasticity,...)
- Even then, the model is exactly identified
- In particular, no testable restriction

Can we improve testability?
- One solution: 'multi-markets' (cf. the IO literature). Ex: CSW requires invariance of (part of) the surplus...
- ...for instance the 'supermodular core' (preferences for assortativeness)

Alternatively, more information is needed

$Z_{IJ(t)} = \zeta_{I(t)} + \xi_{J(t)} + Z_{IJ(0)}$
- ...or at least some restrictions on its variations (e.g. linear trend):
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- Basic insight
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Here, pairwise surplus (as a function of traits)

Where can such information come from?

Answer: from observed behavior

Structure:

Start with given preferences, satisfying TU

Once a couple is formed, they maximize total utility

... observed behavior (e.g. labor supply) allows to identify preferences... therefore the surplus

In practice:

either double set of logit regressions, plus constraints across equations

or simulated moments...

... especially since simulating the model is easy (linear optimization)
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Empirical implementation 3: matching patterns and transfers

- Basic reference: *hedonic models*
- Strong, non parametric identification results
- See f.i. Ekeland, Heckman and Nesheim (2004), Heckman, Matzkin and Nesheim (2010), Chernozhukov, Galichon and Henry (2014) and Nesheim (2013)
Roadmap

1. Empirical implementation

2. The US education puzzle
   - One-dimensional version: CSW (2014)
   - Two-dimensional version: Low (2014)
   - Matching patterns and behavior: CCM 2015

The demand for education puzzle

- **Motivation:** remarkable increase in female education, labor supply, incomes during the last decades.
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Two questions:

- Impact on intrahousehold allocation?
- How can the asymmetry between genders be explained?

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- First question: just compute the dual variables!
- Second question: ‘marital college premium’
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![Graph showing education trends over time]

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The marital college premium (CIW AER 2009)

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Marriage-market benefits (the ‘marital college premium’):
- have been largely neglected
- their evolution markedly differs across genders
- may influence investment behavior
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But a structural model is needed!
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$$g_{ij,t} = Z_{ij} + \alpha_{i,t} + \beta_{j,t}$$

where $\alpha, \beta$ extreme value distributed
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where \( \alpha, \beta \) extreme value distributed

Identifying assumption:

either

\[
Z_{t}^{IJ} = \zeta_{t}^{I} + \zeta_{t}^{J} + Z_{0}^{IJ}
\] (1)

or

\[
Z_{t}^{IJ} = \zeta_{t}^{I} + \zeta_{t}^{J} + \left( Z_{0}^{IJ} + \delta^{IJ} \times t \right)
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**Interpretation:**

Non parametric trends \( \zeta^I, \xi^J \) affecting the surplus but not the supermodularity

(1): 'preferences for assortativeness' do not change!

(2): 'preferences for assortativeness' follow linear trends
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or

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- (1): ‘preferences for assortativeness’ do not change → **testable**
Idea: structural model holds for different cohorts $t = 1, \ldots, T$ with varying class compositions.

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Identifying assumption:

- either
  $$Z_{t}^{IJ} = \zeta_{t}^{I} + \xi_{t}^{J} + Z_{0}^{IJ}$$  \hspace{1cm} (1)
  
  Interpretation:
  - Non parametric trends $\zeta^{I}, \xi^{J}$ affecting the surplus but not the supermodularity
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- or
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  Interpretation:
  - (2): ‘preferences for assortativeness’ follow linear trends $\delta^{IJ}$
What do raw data say?
Comparing educations within white couples

- Husband more educated
- Same education
- Husband less educated
Comparing educations within black couples

<table>
<thead>
<tr>
<th>Year of birth of husband</th>
<th>Husband more educated</th>
<th>Same education</th>
<th>Husband less educated</th>
</tr>
</thead>
<tbody>
<tr>
<td>1940</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1945</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1950</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1955</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1960</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Marriage patterns of white men

Born 1940−1942

- men: CG+
- men: CG
- men: SC
- men: HSG
- men: HSD

Born 1960−1962

- men: CG+
- men: CG
- men: SC
- men: HSG
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Proportion
Marriage patterns of white women

<table>
<thead>
<tr>
<th></th>
<th>Born 1941−1943</th>
<th>Born 1961−1963</th>
</tr>
</thead>
<tbody>
<tr>
<td>women: CG+</td>
<td></td>
<td></td>
</tr>
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Marriage patterns of black men

- **HSD**
- **HSG**
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- **CG**
- **CG+**

**Born 1940–1942**

**Born 1960–1962**

- men: CG+ (proportions)
- men: CG (proportions)
- men: SC (proportions)
- men: HSG (proportions)
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Marriage patterns of black women

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Marriage patterns of black women
Results: preferences for assortativeness

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<th>HSG</th>
<th>SC</th>
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<tr>
<td><strong>HSD</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Women</td>
<td>0.0118***</td>
<td>0.0067***</td>
<td>0.0146***</td>
<td>-0.0023</td>
<td>-0.0366***</td>
</tr>
<tr>
<td></td>
<td>(0.0015)</td>
<td>(0.0012)</td>
<td>(0.0018)</td>
<td>(0.0017)</td>
<td>(0.0017)</td>
</tr>
<tr>
<td><strong>HSG</strong></td>
<td>-0.0237***</td>
<td>0.0024</td>
<td>0.011***</td>
<td>-0.0009</td>
<td>-0.01***</td>
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<td><strong>SC</strong></td>
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<td>-0.001</td>
<td>0.0056***</td>
<td>0.004***</td>
<td>0.0001</td>
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</tr>
<tr>
<td><strong>CG+</strong></td>
<td>0.0436***</td>
<td>0.0055***</td>
<td>-0.0087***</td>
<td>-0.0059***</td>
<td>0.0149*</td>
</tr>
<tr>
<td></td>
<td>(0.0004)</td>
<td>(0.0006)</td>
<td>(0.0008)</td>
<td>(0.001)</td>
<td>(0.0017)</td>
</tr>
</tbody>
</table>

Table: Slopes - linear extension
Results: college premium

Figure 12: The marital college premium
Roadmap

1. Empirical implementation
2. *The US education puzzle*
   - One-dimensional version: CSW (2014)
   - *Two-dimensional version: Low (2014)*
   - Matching patterns and behavior: CCM 2015
Reproductive capital and women’s demand for higher education

Source: Corinne Low’s dissertation (2014)

- Basic remark: sharp decline in female fertility between 35 and 45
Reproductive capital and women’s demand for higher education

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- Basic remark: sharp decline in female fertility between 35 and 45
- Consequence: matching patterns and age

Consider the choice between entering the MM after college or delaying, in order to acquire a 'college +'degree

Pros and cons of delaying:
- Pro: higher education, higher wage, etc.
- Con: delayed entry, loss of 'reproductive capital'

Impact on marital prospects?

P.A. Chiappori (Columbia University)
Matching with Transfers
Yale, November 2015
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- Impact on marital prospects?
Model

- Two commodities, private consumption and child expenditures; utility:
  \[ u_i = c_i (Q + 1), \ i = h, w \]

and budget constraint (\( y_i \) denotes \( i \)'s income)

\[ c_h + c_w + Q = y_h + y_w \]
Two commodities, private consumption and child expenditures; utility:

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and budget constraint (\( y_i \) denotes \( i \)’s income)

\[ c_h + c_w + Q = y_h + y_w \]

Transferable utility: any efficient allocation maximizes \( u_h + u_w \); therefore surplus with a child

\[ s(y_h, y_w) = \frac{(y_h + y_w + 1)^2}{4} \]

and without a child \( (Q = 0) \)

\[ s(y_h, y_w) = y_h + y_w \]

therefore, if \( \pi \) probability of a child:

\[ s(y_h, y_w) = \pi \frac{(y_h + y_w + 1)^2}{4} + (1 - \pi)(y_h + y_w) \]
Men: differ in income $\rightarrow y_h$ uniform on $[1, Y]$
Men: differ in income \( \rightarrow y_h \) uniform on \([1, Y]\)

Women: more complex

\[
y_w = \lambda s \text{ if invest (with } \lambda > 1) \\
y_w = s \text{ if not}
\]

Therefore: once investment decisions have been made,
bidimensional matching model, and three questions:

- who marries whom?
- how is the surplus distributed?
- what is the impact on (ex ante) investment?
Populations

- Men: differ in income $\rightarrow y_h$ uniform on $[1, Y]$
- Women: more complex
  - differ in skills $\rightarrow s$ uniform on $[0, S]$
Populations

- Men: differ in income $\rightarrow y_h$ uniform on $[1, Y]$
- Women: more complex
  - differ in skills $\rightarrow s$ uniform on $[0, S]$
  - may choose to invest $\rightarrow$ income:
    \[
    y_w = \lambda s \text{ if invest (with } \lambda > 1) \\
    y_w = s \text{ if not}
    \]
    but investment implies fertility loss $\pi = p$ if invest $\pi > p$ if not

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Therefore: *once investment decisions have been made*, bidimensional matching model, and three questions:

- who marries whom?
- how is the surplus distributed?
- what is the impact on (ex ante) investment?
Resolution

- Assumption: investment decision such that there exists some $\bar{s}$ such that

  \[ \text{invest iff } s \geq \bar{s} \]

Then:
Resolution

- **Assumption:** investment decision such that there exists some \( \bar{s} \) such that
  
  \[
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  Then:

- There exists a stable match (conditional on education); generically unique
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Then:
- There exists a stable match (conditional on education); generically unique
- For given fertility, assortative matching on income
Resolution

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  Then:
  - There exists a stable match (conditional on education); generically unique
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  - Matching and fertility: three possible regimes (plus intermediate randomization)
Resolution

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Then:
- There exists a stable match (conditional on education); generically unique
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- Matching and fertility: three possible regimes (plus intermediate randomization)
  - Regime 1: negative assortative matching (can be discarded)
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  - Matching and fertility: three possible regimes (plus intermediate randomization)
    - Regime 1: negative assortative matching (can be discarded)
    - Regime 2: non monotonic matching
shown in Figure 1.4.

*Figure 1.4: Non-monotonic equilibrium match*

Let $x$ and $z$ represent the lower and upper ends of the second segment of men, and $r$ and $t$ represent the lower and upper cutoffs for women. Poor men, from 1 to $x$, marry low-skill, fertile women (matching assortatively). On the other side of the threshold, the richest group of women matches assortatively with the middle group of men, from $x$ to $z$. But, the richest men, from $z$ to $Y$, marry the “best of the rest”—the more high-skilled women among those who have not invested and are thus still fertile.\(^5\)

This general form allows for the match to be non-monotonic, as depicted, or collapse to positive assortative matching, when $r^* = t$ (and thus segment 2 in Figure 1.4 has zero mass),

\(^5\)The matching functions in this uniform case are linear, but in the general case, their form will be determined by the distribution so that the number of women above any point on each “segment” exactly matches the number of men above that point.
Resolution

- Assumption: investment decision such that there exists some $\bar{s}$ such that

  $$\text{invest \ iff \ } s \geq \bar{s}$$

  Then:
  - There exists a stable match (conditional on education); generically unique
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  - Matching and fertility: three possible regimes (plus intermediate randomization)
    - Regime 1: negative assortative matching (can be discarded)
    - Regime 2: non monotonic matching
    - Regime 3: positive assortative matching
or her spouse to maximize his or her own payoff, under the constraint that the spouse will accept that match.

Let $v_i(s), i \in \{1, 2, 3\}$ represent the value function of a woman of skill $s$ matching in segment $i$, and $u_i(y), i \in \{1, 2, 3\}$ the value function of a man of income $y$ matching in segment $i$.

Note that for any individuals of skill $s$ and income $y$, $u_i(y) + v_i(s) \geq T_i(y, s)$. For married individuals, this holds with equality, and we can solve for the slope of the value function:

$$u_i(y) = \max_s\{T_i(y, s) - v_i(s)\} \Rightarrow v'_i(s) = \frac{\partial T_i(y, s)}{\partial s}$$

and

$$v_i(s) = \max_y\{T_i(y, s) - u_i(y)\} \Rightarrow u'_i(y) = \frac{\partial T_i(y, s)}{\partial y}$$
Resolution

- Assumption: investment decision such that there exists some $\bar{s}$ such that

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  Then:

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  - Regime 1: negative assortative matching (can be discarded)
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  - Regime 3: positive assortative matching
- Which regime? Depends on the parameters. In particular:
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Which regime? Depends on the parameters. In particular:

- If $\lambda$ small and $P/p$ large, regime 2
Assumption: investment decision such that there exists some $\bar{s}$ such that

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Which regime? Depends on the parameters. In particular:

- If $\lambda$ small and $P/p$ large, regime 2
- If $\lambda$ large and $P/p$ not too large, regime 3
Empirical predictions

Basic intuition: we have moved from ‘\( \lambda \) small, \( P/p \) large’ to ‘\( \lambda \) large, \( P/p \) not too large’
Why?

- Increase in \( \lambda \): dramatic increase in ‘college + premium’
Empirical predictions

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Why?

- Increase in \( \lambda \): dramatic increase in ‘college + premium’
- Decrease in \( P/p \): two factors

- Progress in assisted reproduction (much more important): dramatic change in desired family size

Consequence: according to the model:

- Before the 80s: college + women marry ‘below’ college graduate
- After the 80s: college + women marry ‘above’ college graduate

What about data?
Empirical predictions

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Consequence: according to the model:
- Before the 80s: college + women marry ‘below’ college graduate
- After the 80s: college + women marry ‘above’ college graduate
Notes: “Don’t know/refused” responses not shown. Respondents were asked: “What is the ideal number of children for a family to have?”

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What about data?
Higher education only recently offers a “marriage premium”

Spousal income by wife’s education level, white women 41-50
Roadmap

1. Empirical implementation
2. *The US education puzzle*
   - One-dimensional version: CSW (2014)
   - Two-dimensional version: Low (2014)
   - *Matching patterns and behavior: CCM 2015*
The basic motivation for this project is to understand how policy affects individual life-cycle decisions.

Long term effects will change education choices and the marriage market.

In turn this will have effects on labor supply and will have intergenerational impacts.

Two fundamental, Beckerian insights: Notion of Human Capital and Matching as an equilibrium phenomenon.
Basic features:

- Agents invest in education *before entering the matching game*
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- Human Capital: education + random dynamics
Basic features:

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Basic features:

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- Human Capital: education $+$ random dynamics.
- At any moment, Human Capital stock determines the wage.
- Risk: shocks affecting HC and wages, multiplicative.
- Efficient risk sharing within the household, efficient labor supply.
- Preferences: leisure, one private and one public good.
Basic features:

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- Preferences: leisure, one private and one public good
- TU context
1. Agents invest in education; heterogeneous costs
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2. Agents enter the MM with their education level $H$; matching takes place; full commitment
Agents invest in education; heterogeneous costs

Agents enter the MM with their education level $H$; matching takes place; full commitment

Life cycle labor supply $\rightarrow T$ subperiods; at each subperiod:

1. Agents invest in education; heterogeneous costs
2. Agents enter the MM with their education level $H$; matching takes place; full commitment
3. Life cycle labor supply $\rightarrow T$ subperiods; at each subperiod:
Agents invest in education; heterogeneous costs

Agents enter the MM with their education level $H$; matching takes place; full commitment

Life cycle labor supply $\rightarrow T$ subperiods; at each subperiod:

- Shocks are realized:

$$\ln w_{i,t} = \ln W_t + \ln H_i + \ln(e_{i,t}), \quad i = 1, 2$$
1. Agents invest in education; heterogeneous costs
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3. Life cycle labor supply $\rightarrow T$ subperiods; at each subperiod:
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     \[
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   - $\rightarrow$ agents supply labor and consume
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   - Note that shocks can be permanent ...
Agents invest in education; heterogeneous costs

Agents enter the MM with their education level $H$; matching takes place; full commitment

Life cycle labor supply $\rightarrow T$ subperiods; at each subperiod:

- Shocks are realized:
  \[
  \ln w_{i,t} = \ln W_t + \ln H_i + \ln (e_{i,t}), \quad i = 1, 2
  \]

  - agents supply labor and consume
  - Note that shocks can be permanent ...
  - ... including initial productivity (or HC) shock
Backwards:

- Start with periods 3
Solution

Backwards:

- Start with periods 3
  - Collective, life cycle LS model

\[ u_i(Q_t, C_{i,t}, L_{i,t}) = \ln(C_{i,t}Q_t + \alpha_i(\text{age}, g, s)L_{i,t}Q_t) \]

Under TU household utility standard, unitary model defines total expected surplus at the household level. Intra-household allocation not determined. Then period 2: determines matching patterns (who marries whom by education) (future, contingent) intra-household allocation! Ultimately, the returns to education. Finally period 1: education decisions.
Solution

Backwards:

- Start with periods 3
  - Collective, life cycle LS model
    
    \[ u_i(Q_t, C_{i,t}, L_{i,t}) = \ln(C_{i,t}Q_t + \alpha_i(\text{age, g, s})L_{i,t}Q_t) \]

- Under TU → household utility → standard, unitary model
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  - Matching patterns (who marries whom by education)
  - (Future, contingent) intra-household allocation
Solution

Backwards:

- Start with periods 3
  - Collective, life cycle LS model
    \[ u_i(Q_t, C_{i,t}, L_{i,t}) = \ln(C_{i,t}Q_t + \alpha_{i}(age, g, s)L_{i,t}Q_t) \]
  - Under TU → household utility → standard, unitary model
    - Defines total expected surplus at the household level
    - Intra-household allocation not determined

- Then period 2: determines
  - Matching patterns (who marries whom by education)
  - (Future, contingent) intra-household allocation
  - → ultimately, the returns to education
Solution

Backwards:

- Start with periods 3
  - Collective, life cycle LS model
    \[ u_i (Q_t, C_{i,t}, L_{i,t}) = \ln (C_{i,t} Q_t + \alpha_i(\text{age}, g, s)L_{i,t} Q_t) \]
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    \[ \rightarrow \text{ultimately, the returns to education} \]

- Finally period 1: education decisions
Basic idea: simulated moments
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- Choose some parameters
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- Choose some parameters
- Simulate the model
Basic idea: simulated moments

- Choose some parameters
- Simulate the model
- Iterate to fit a set of moments
Estimation

- Basic idea: simulated moments
  - Choose some parameters
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- Problem: very hard
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Problem: very hard

- Stage 3: dynamic, stochastic LS model
Estimation

- Basic idea: simulated moments
  - Choose some parameters
  - Simulate the model
  - Iterate to fit a set of moments

- Problem: very hard
  - Stage 3: dynamic, stochastic LS model
  - Stage 2: matching model (with the surplus estimated from stage 3)
Estimation

- Basic idea: simulated moments
  - Choose some parameters
  - Simulate the model
  - Iterate to fit a set of moments

- Problem: very hard
  - Stage 3: dynamic, stochastic LS model
  - Stage 2: matching model (with the surplus estimated from stage 3)
  - Stage 1: *Rational expectations* → *fixed point in a functional space*
Estimation

- Basic idea: simulated moments
  - Choose some parameters
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- Problem: very hard
  - Stage 3: dynamic, stochastic LS model
  - Stage 2: matching model (with the surplus estimated from stage 3)
  - Stage 1: *Rational expectations* → *fixed point in a functional space*

- Simplification: use the ‘fictitious game’
Pre-matching investment

- Two-stage model:
Pre-matching investment

- Two-stage model:
  - Stage one: agents choose a level of human capital at some cost → *non cooperative*
Two-stage model:

- Stage one: agents choose a level of human capital at some cost → non cooperative
- Stage two: matching game on HC + other characteristics
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- Resolution: backwards
Pre-matching investment

- Two-stage model:
  - Stage one: agents choose a level of human capital at some cost → non-cooperative
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- Resolution: backwards
  - Stage 2: stability give $U$, $V$ as functions of HC
Pre-matching investment

Two-stage model:
- Stage one: agents choose a level of human capital at some cost → non cooperative
- Stage two: matching game on HC + other characteristics

Resolution: backwards
- Stage 2: stability give $U, V$ as functions of HC
- Stage 1: agents choose HC to maximize utility - cost
Main result (Cole Mailath Postlewaite 2001, Nöldeke Samuelson 2015)

- Same framework
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- Same framework
- Fictitious game:

Stage one: agents match (on their cost and any other predetermined parameters)

Stage two: jointly choose HC investment to maximize joint surplus

Main result: The stable matching of the fictitious game is always an equilibrium of the initial, two-stage game. However, other equilibria may exist ('coordination failures').

Important empirical application: The two stage game is complex, because of its rational expectation structure (fixed point in a functional space). The fictitious game is much easier to simulate (matching linear programming).
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  - The fictitious game is much easier to simulate (matching → linear programming)
Roadmap

1. Empirical implementation
2. The US education puzzle
   - One-dimensional version: CSW (2014)
   - Two-dimensional version: Low (2014)
   - Matching patterns and behavior: CCM 2015
Basic insights

- Two types of skills: manual and cognitive \(\rightarrow\) workers and jobs (2 \times 2 matching)
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- → Sorting improves along the cognitive dimension but deteriorates along the manual dimension
Basic insights

- Two types of skills: manual and cognitive $\rightarrow$ workers and jobs ($2 \times 2$ matching)
- Task-biased technological change increases the level of complementarities between cognitive skills and skill demands (relative to those in the manual dimension)
- $\rightarrow$ Sorting improves along the cognitive dimension but deteriorates along the manual dimension
- $\rightarrow$ Wages more convex in cognitive but less convex in manual skills
Basic insights

- Two types of skills: manual and cognitive → workers and jobs ($2 \times 2$ matching)
- Task-biased technological change increases the level of complementarities between cognitive skills and skill demands (relative to those in the manual dimension)
- → Sorting improves along the cognitive dimension but deteriorates along the manual dimension
- → Wages more convex in cognitive but less convex in manual skills
- → Increased wage inequality along the cognitive dimension, compressed inequality in the manual dimension.
Model:

$$\pi_{ij} = F_C (x^j_C, y^i_C) + F_M (x^j_M, y^i_M)$$
Job matching by skills (Lindenlaub 2014)

- Model:
  \[ \pi_{ij} = F_C (x_C^i, y_C^i) + F_M (x_M^i, y_M^i) \]

- Matching: if pure,
  \[ y_C = \Phi_C (x_C, x_M) \]
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Job matching by skills (Lindenlaub 2014)

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  \[
  \pi_{ij} = F_C(x^i_C, y^i_C) + F_M(x^i_M, y^i_M)
  \]

- **Matching:** if pure,
  \[
  y_C = \Phi_C(x_C, x_M) \\
  y_M = \Phi_M(x_C, x_M)
  \]

- **PAM:** \(\partial \Phi_C / \partial x_C > 0, \partial \Phi_M / \partial x_M > 0, \text{Det} > 0\)

- **Theorem:** if
  \[
  \partial^2 F_C / \partial x^i_C \partial y^i_C > 0 \text{ and } \partial^2 F_M / \partial x^i_M \partial y^i_M > 0
  \]

  then PAM
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  \[ \partial^2 F_C / \partial x_C^i \partial y_C^i > 0 \text{ and } \partial^2 F_M / \partial x_M^i \partial y_M^i > 0 \]
  then PAM

- Then Quadratic-Gaussian model
Conclusion

1. Frictionless matching: a powerful and tractable tool for theoretical analysis, especially when not interested in frictions

2. Crucial property: intramatch allocation of surplus derived from equilibrium conditions

3. Applied theory: many applications (abortion, female education, divorce laws, children, ...)

4. Can be taken to data; structural econometric model, over identified

5. Multidimensional versions: index (COQD 2010), general (CMcCP 2015)

6. Extensions
   - ITU: theory; empirical applications still to be developed (but: Galichon-Kominers-Weber 2015)
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- ITU: theory; empirical applications still to be developed (but: Galichon-Kominers-Weber 2015)
- Joint estimation of surplus and matching (→ ‘consistency’!); for instance domestic production
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Extensions:
- ITU: theory; empirical applications still to be developed (but: Galichon-Kominers-Weber 2015)
- Joint estimation of surplus and matching (→ ‘consistency’!); for instance domestic production
- Dynamics: divorce, etc.