Inequality in Human Capital and Endogenous Credit
Constraints*

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Abstract

This paper studies the determinants of inequality in human capital with particular emphasis on the role of the credit constraints. We develop and estimate a model in which individuals are subject to uninsured human capital risks and invest in education, acquire work experience, accumulate assets, and smooth consumption. Agents can borrow up to a model-determined limit, which we explicitly derive from a private lending market natural borrowing limit and government student loan programs. We also quantify the effects of cognitive ability, noncognitive ability, parental education, and parental wealth on educational attainment, work experience, and consumption. We conduct counterfactual experiments with respect to tuition subsidy and enhanced student loan limits and evaluate their effects on educational attainment and inequality.

Keywords: Human Capital, Credit Constraints, Education, Wealth

JEL codes: I1, I2, J2

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1 Introduction

Evidence on the importance of credit constraints on human capital formation is mixed. As noted in Lochner and Monge-Naranjo (2016), the early literature found little evidence for them. The recent literature – based on more recent data – shows much stronger evidence of credit constraints. This evolution of the evidence is not a matter of choice of estimation methods, but appears to be a real empirical phenomenon first noted in Belley and Lochner (2007).

This paper develops and estimates a dynamic model of schooling and work experience in which agents are subject to uninsured human capital risks, and face restrictions on their borrowing possibilities. Previous empirical research on this topic fixes lending limits at ad hoc values, or else introduces additional free parameters to model credit limits. In our analysis, agents can borrow up to model determined limits derived from an analysis of private lending with a natural limit combined with access to government student loan programs. No free parameters are introduced. Table 1 summarizes the leading papers in the structural literature and places this paper in context.

We further extend the existing literature by analyzing how cognitive and noncognitive ability affect choices through: (i) psychic costs of working and schooling; (ii) the technology of human capital production; and (iii) the discount factor. Following Cunha and Heckman (2008) and Cunha, Heckman, and Schennach (2010), we allow our measures of abilities to be fallible.

We use our estimated model to understand the sources of inequality in education, earnings, and consumption over the life cycle. We consider how parental characteristics and transfers, as well as credit markets, affect choices and outcomes. We find strong effects of adolescent endowments of cognitive and noncognitive ability on human capital development. Tuition costs and family transfers to children play important roles in explaining differences in life outcomes due to human capital investments.

Credit constrained agents fall into two groups: (a) those with poor initial endowments
and family background who acquire little human capital and have low wage levels and low life cycle wage growth, and (b) the very able and those from good family backgrounds who have high levels of human capital, high wage levels, and high life cycle wage growth. The first group is constrained throughout the life cycle because of low endowments. The second group is initially constrained because, while it has high levels of life cycle wealth, it cannot smooth consumption over the early stages of the life cycle.

There is a bimodal, two-humped, profile of the constrained with respect to endowments and abilities at the early stages of the life cycle. As income is harvested over the life cycle, the second hump (associated with the second group) diminishes. The first group remains roughly stable over its lifetime.

The rest of the paper is organized as follows. Section 2 presents our model. Section 3 describes the data and conducts basic regression analysis that summarizes the empirical regularities in the data. Section 4 presents our empirical strategy for estimating the model. Section 5 discusses our estimates. Section 6 conducts counterfactual simulations. Section 7 concludes.
## Table 1: Structural Models of Educational Choice and Credit Constraints

<table>
<thead>
<tr>
<th>Model</th>
<th>Human Capital Investment</th>
<th>Labor Supply</th>
<th>Government Student Loans</th>
<th>Private Loan Limit</th>
<th>CRRA Risk Aversion</th>
<th>Parental Influence</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Keane and Wolpin (2001)</td>
<td>Education and work experience</td>
<td>Yes</td>
<td>No</td>
<td>Borrowing limits not observed; proxied by a function of age and human capital; the parameters of the borrowing limit are estimated</td>
<td>$\gamma = 0.4826$</td>
<td>Parental transfer is a function of parental education and individuals choices</td>
<td>NLSY79 (1979-1992)</td>
</tr>
<tr>
<td>Navarro (2011)</td>
<td>Education</td>
<td>No</td>
<td>No</td>
<td>Models an endogenous natural borrowing limit based on education and inelastic labor supply, due to borrowers' limited repayment ability in the presence of uninsurable wage risk; however, in estimation, the borrowing limit is set equal to the lowest level of assets observed in samples in each period independently of the model</td>
<td>$\gamma = 0.82$</td>
<td>No</td>
<td>NLSY79 &amp; PSID</td>
</tr>
<tr>
<td>Lochner and Monge-Naranjo (2011)</td>
<td>Education and work experience</td>
<td>No</td>
<td>Yes</td>
<td>Endogenous credit limit based on borrowers' cost of default (including temporary exclusion from credit market and wage garnishments), due to private lenders' limited ability to punish default; parameters on the cost of default are calibrated outside the model</td>
<td>Set $\gamma = 2$</td>
<td>No</td>
<td>NLSY79 (1979-2006)</td>
</tr>
<tr>
<td>Johnson (2013)</td>
<td>Education and work experience</td>
<td>Yes</td>
<td>Yes</td>
<td>Borrowing limits not observed; proxied by a function of age and human capital; the parameters of the borrowing limit proxy equation are estimated</td>
<td>Set $\gamma = 2$</td>
<td>Parental transfers is a function of parental income and child choices</td>
<td>NLSY97 (1997 to 2007)</td>
</tr>
<tr>
<td>Abbott, Gallipoli, Meghir, Violante (2016)</td>
<td>Education</td>
<td>Yes</td>
<td>Yes</td>
<td>No borrowing for high-school students; exogenous fixed debt limits for workers in the work-stage and college students whose parents are wealthy, respectively; borrowing limits are calibrated to match the fraction of households with zero or negative net worth and the aggregate private loans/GSL ratio, respectively</td>
<td>Set $\gamma = 2$</td>
<td>Parental transfers explicitly modeled</td>
<td>Multiple data, including NLSY79, NLSY97</td>
</tr>
<tr>
<td>Blundell, Costas Dias, Meghir, Shaw (2016)</td>
<td>Education and work experience</td>
<td>Yes</td>
<td>Yes</td>
<td>No borrowing permitted</td>
<td>Set $\gamma = 1.56$</td>
<td>Parental income and background factors affect youth's psychic cost of schooling</td>
<td>BHPS (1991 to 2008)</td>
</tr>
<tr>
<td>This paper</td>
<td>Education and work experience</td>
<td>Yes</td>
<td>Yes</td>
<td>Model-determined natural borrowing limit based on education and labor supply decisions, due to borrowers' limited repayment ability in the presence of uninsurable wage risk; no new auxiliary parameters for borrowing limit are added in estimation, unlike many previous papers</td>
<td>Set $\gamma = 2$</td>
<td>Parental transfer is a function of parental education and net worth, and individual choices; parental education affects youth's psychic cost of schooling</td>
<td>NLSY97 (1997-2013)</td>
</tr>
</tbody>
</table>


This section presents our model specification, solution concepts, initial conditions, and measurement system.

2.1 Model Specification

We first present our model specification.

2.1.1 Choice Set

At each age $t \in \{t_0, \ldots, T\}$ an individual makes decisions on: (i) consumption $c_t$ and savings $s_{t+1}$, (ii) whether to go to school $d_{e,t} \in \{0, 1\}$, and (iii) employment $d_{k,t} \in \{0, 0.5, 1\}$, where $d_{k,t} = 0, d_{k,t} = 0.5$ and $d_{k,t} = 1$ indicate not working, part-time working, and full-time working, respectively. An individual cannot go to school and work full-time at the same time, i.e. $d_{e,t} + d_{k,t} < 2$. The cost of schooling includes monetary costs (tuition and fees), psychic costs, and foregone earnings. However, individuals can work part-time while in school.

2.1.2 State Variables

At each age $t$, an individual is characterized by a vector of predetermined state variables that shape preferences, production technology, and outcomes:

\[ \Omega_t := (t, \theta, e_t, k_t, s_t, d_{e,t-1}, e_p, s_p) \]

where $\theta$ is a vector that summarizes individual components of unobserved heterogeneity (unobserved cognitive ability and noncognitive ability), $e_t$ is the individual’s years of schooling at $t$, $k_t$ is accumulated work experience, $s_t$ is net worth determined at the end of period

\[ ^1\text{Data on educational decisions after age 27 are not available.} \]
t − 1, \( d_{e,t−1} \) is the previous schooling status, \( e_p \) is the parental educational level, and \( s_p \) is parental net worth.\(^2\) The information set, that includes all the predetermined state variables and realized idiosyncratic shocks at age \( t \) (\( \epsilon_t \)), can be written as \( \Omega_t := \{\Omega_t, \epsilon_t\} \).

### 2.1.3 Preferences

An individual has well-defined preferences over his consumption \( c_t \) and choices on schooling and working \( (d_{e,t}, d_{k,t}) \):

\[
U(c_t, d_{e,t}, d_{k,t}; \Omega_t) = u_c(c_t; \Omega_t) + u_e(\Omega_t) \cdot d_{e,t} + u_k(d_{k,t}, \Omega_t) + \phi_{k,e} d_{e,t} 1(d_{k,t} = 0.5). \tag{2}
\]

Psychic costs associated with schooling \( d_{e,t} = 1 \) are captured by the function \( u_e(\Omega_t) \). \( u_k(d_{k,t}, \Omega_t) \) captures the psychic benefits/cost associated with labor supply decision \( d_{k,t} \). \( \phi_{k,e} \) is the preference parameter associated with part-time working while in school. Agents discount future returns using a subjective discount factor \( \exp(-\rho(\theta)) \), where \( \rho(\theta) > 0 \) is the subjective discount rate.

### 2.1.4 Human Capital Production and Wage Equations

Human capital at age \( t \) (measured in labor efficiency units), \( \psi_t \in \mathbb{R}_{++} \), is produced according to the following function:

\[
\psi_t = F^\psi(e_t, k_t, \theta, \epsilon_{w,t}) \tag{3}
\]

where \( \epsilon_{w,t} \in [\xi_w, \xi_w] \) is an idiosyncratic shock to the human capital stock at age \( t \). Equation (3) explicitly allows for two types of human capital: education and work experience, both of which are consequences of previous decisions. Equation (3) allows productivity in the labor market to depend on cognitive and noncognitive abilities. An individual’s hourly wage offer depends on whether the individual works part-time or full-time and whether the individual

\(^2\)Individuals’ unobserved heterogeneity \( \theta \) and parental education and wealth \( (e_p, s_p) \) are measured at the initial model period, i.e., age 17.
is enrolled in school:

\[ w_t = \psi_t \cdot F^w(d_{k,t}, d_{e,t}) \]  

(4)

where \( F^w(d_{k,t}, d_{e,t}) \) is the rental price for each unit of human capital. We allow the rental price of human capital to be different between a part-time job and a full-time job. We also allow the part-time wage rate to be different depending on whether the individual is enrolled in school (see Johnson, 2013). We normalize the rental price of human capital for full-time job to be one (i.e., \( F^w(1, 0, 0) = 1 \)). An individual’s full-time wage offer equals to his human capital level: \( w_t = \psi_t = F^\psi(e_t, k_t, \theta, \epsilon_{w,t}) \) if \( d_{k,t} = 1 \).

Accumulation of work experience evolves in the following way:

\[ k_{t+1} = k_t + d_{k,t} - \delta_k k_t \cdot 1(d_{k,t} = 0) := F^k(k_t, d_{k,t}) \]  

(5)

where \( \delta_k \) is the depreciation rate of work experience when the individual does not work. Education level at \( t + 1 \), measured by years of schooling, is:

\[ e_{t+1} = e_t + d_{e,t} \]  

(6)

**2.1.5 Financial Market Frictions and Endogenous Credit Constraints**

To finance education and consumption, individuals can borrow at an exogenous borrowing interest rate \( r_b \). An individual can also accumulate physical assets by the means of a riskless asset with the rate of return \( r_l \).\(^3\) To capture an important feature of imperfect capital markets, we allow the lending rate to be smaller than the borrowing rate, i.e., \( r_l < r_b \).\(^4\)

The smallest amount of net worth \( s_{t+1} \) that an agent can choose at the end of period \( t \) is captured by a (potentially negative) lower bound \( s_{t+1} \in \mathbb{R}^- \), which is determined by

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\(^3\)We abstract away from portfolio choices.

\(^4\)We only keep track of an agent’s net worth so there is no need to separately keep track of an agent’s debt and asset. Furthermore, when \( r_l < r_b \), an individual never finds it optimal to be both a borrower and a lender.
both the private loan market borrowing limit and the maximum credit from the government student loan programs as follows:

\[ s_{t+1} \geq s_{t+1} := -\max\{d_{e,t} \cdot L^g(d_{e,t} + e_t), L^s_t(e_{t+1}, k_{t+1}, \theta)\} \]  

(7)

where \( L^g(d_{e,t} + e_t) \in \mathbb{R}^+ \) is the maximum government student loan credit for schooling level \((e_t + d_{e,t})\) if the individuals choose to enroll in school \((d_{e,t} = 1)\), and \( L^s_t(e_{t+1}, k_{t+1}, \theta) \in \mathbb{R}^+ \) is the natural borrowing limit of an individual in the private debt market. \( L^s_t(e_{t+1}, k_{t+1}, \theta) \) is determined by the maximum loan that the individual can pay back with probability one at the end of his decision period \( T \), i.e., \( L^s_T = 0 \). We discuss the formulation of the endogenous natural borrowing limit \( L^s_T(\cdot) \) in Section 2.2 below.

2.1.6 Budget Constraint and Transfer Functions

To finance a youth’s college tuition and fees, parents may provide monetary transfers \( tr_{p,t} \geq 0 \). Parental monetary transfers depend on the parents’ characteristics including education \((e_p)\) and net worth \((s_p)\) and the youth’s own schooling and working decisions \((d_{e,t}, d_{k,t})\) as well as the youth’s own education level and age:\(^5\)

\[ tr_{p,t} = tr_p(e_p, s_p, d_{e,t}, d_{k,t}, e_t, t). \]  

(8)

Examples of parental monetary transfers include college financial gifts if the youth chooses to attend college. The parental transfer rule is taken as given. This captures paternalism and tied transfers on the part of the parents, which is consistent with the findings of previous research (see, e.g., Keane and Wolpin, 2001 and Johnson, 2013).

Defining \( r(s_t) := r_l 1(s_t > 0) + r_b 1(s_t < 0) \), the budget constraint for an individual who

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\(^5\)This is an extension of the parental transfer function in Keane and Wolpin (2001).
chooses to attend college (i.e., \( d_{e,t} \cdot 1(e_t + d_{e,t} > 13) = 1 \)) is:

\[
    c_t + (tc(e_t + d_{e,t}) - gr(e_t + d_{e,t}, s_p)) + s_{t+1} = (1 + r(s_t)) \cdot s_t + w_t \cdot h \cdot d_{k,t} + tr_{p,t}
\]

\[
    c_t \geq rc(e_t + d_{e,t})
\]

where \( h \) is full-time hours of work, \( tc(e_t + d_{e,t}) \) is the amount of college tuition and fees, \( gr(e_t + d_{e,t}, s_p) \) is the amount of grants and scholarships which depend on schooling level and parental wealth, and \( rc(e_t + d_{e,t}) \) denotes the cost of college room and board.

The budget constraint for an individual who is not currently enrolled in college (i.e., \( d_{e,t} \cdot 1(e_t + d_{e,t} > 13) = 0 \)) is:

\[
    c_t + s_{t+1} = (1 + r(s_t)) \cdot s_t + w_t \cdot h \cdot d_{k,t} + tr_{p,t} + tr_{c,t} + tr_{g,t}
\]

\[
    c_t \geq c_{min}
\]

where \( tr_{c,t} \geq 0 \) is the direct consumption subsidy from the parents to their dependent child in the forms of shared housing and meals. \( tr_{g,t} \geq 0 \) is the amount of government transfers, which consist of unemployment benefits and means-tested transfers that guarantees a minimum consumption floor \( c_{min} \). Treating \( c_{min} \) as a subsistence level of consumption, we require \( c_t \geq c_{min} \).

### 2.2 Model Solution

The value function \( V_t(\cdot) \) for \( t = 1, \ldots T \) is characterized by the following Bellman equation:

\[
    V_t(\Omega_t) = \max_{d_{e,t},d_{k,t},s_{t+1}} \{ U(c_t, d_{e,t}, d_{k,t}; \Omega_t) + \exp(-\rho(\theta)) \mathbb{E}(V_{t+1}(\Omega_{t+1})|\Omega_t, e_{t+1}, s_{t+1}, k_{t+1}, d_{e,t}) \}
\]

subject to restrictions imposed by wage functions and human capital accumulation functions (Equations (3)-(6)), borrowing constraints (Equation (7)), and the state-contingent budget constraints (Equation (9)-(11)).
The model is solved through numerical backward recursion of the Bellman equation assuming a terminal value function when the agent reaches age \( T + 1 \). Ideally we would like to choose a very large age \( T + 1 \). However, we would also like to avoid the computational burden of having to solve the model over long horizons. We set the terminal age to be \( T + 1 = 51 \) so that individuals decisions during their 20s are not sensitive to the functional form specification of the terminal value function, and at the same time the computational burden is also manageable.\(^6\)

### 2.2.1 Natural Borrowing Limit

At age \( t \), the smallest possible full-time wage earnings an individual receives is \( F^\psi(e_t, k_t, \theta, \xi_w) \cdot h \), where \( \xi_w \) is the worst possible productivity shock and \( h \) is full-time hours of work. The individual receives zero wage income if he does not work.

To illustrate our approach, consider an extreme case where individuals supply their labor inelastically from period \( t \) onwards, i.e, \( d_{k,\tau} = 1 \) for all \( \tau \geq t \). The natural borrowing limit in the private loan market in period \( t - 1 \) in this extreme case is:

\[
L^s_{t-1}(e, k_t, \theta) = \frac{L^s_t(e, k_t + 1, \theta) + \max\{0, F^\psi(e_t, k_t, \theta, \xi_w) \cdot h - c_{min}\}}{1 + r_b}.
\]

Navarro (2011) develops a version of this constraint, but does not use it in estimating his model.\(^7\)

When employment decisions are endogenous, the formulation of the natural borrowing limit is more involved. For an individual who does not work at \( t \), the natural borrowing limit at period \( t - 1 \) (suppressing arguments), is \( L^s_{t-1} = L^s_t/(1 + r_b) \). At age \( t \) the individual carries debt \( s_{t+1} = -L^s_t \leq 0 \) and consumes government transfers \( c_t^u = tr_{g,t} \geq c_{min} \). Let \( C^e_t \) be the compensation that makes an individual indifferent between working and not working.

---

\(^6\)In comparison with previous studies, Keane and Wolpin (2001) approximate a terminal value function at age 31. Johnson (2013) approximates the terminal value function at age 40.

\(^7\)Navarro (2011) uses \( L^s_{t-1}(\cdot) = (L^s_t(\cdot) + F^\psi(e, \theta, \xi_w) \cdot h)/(1 + r_b) \).
We implicitly define $C_t^{ev}$ as follows:

$$u_c(C_t^{ev}; \Omega_t) + u_k(d_{k,t} = 1, \Omega_t)$$

$$+ \exp(-\rho(\theta))E(V_{t+1}(\Omega_{t+1})|\Omega_t, e, s_{t+1} = -\bar{T}_t^s, k_{t+1} = F^k(k_t, d_{k,t} = 1))$$

$$= u_c(c_t^{w}; \Omega_t) + u_k(d_{k,t} = 0, \Omega_t)$$

$$+ \exp(-\rho(\theta))E(V_{t+1}(\Omega_{t+1})|\Omega_t, e, s_{t+1} = -\bar{T}_t^s, k_{t+1} = F^k(k_t, d_{k,t} = 0))$$

If $C_t^{ev} < c_{min}$, then we set $C_t^{ev} = c_{min}$. As seen in Equation (13), $C_{t+1}^{ev}$ depends on the sustainable consumption level, unemployment benefits, the individual’s psychic cost of working, and the future productivity gains of increased work experience.

Under the most unfavorable possible income shocks, if the wage earnings is higher than the consumption equivalence value, i.e., $F_t^\psi(e, k_t, \theta, \epsilon_w) \cdot h \geq C_t^{ev}(e, k_t, \theta)$, the individual works $d_{k,t} = 1$ and the maximum amount of debt that he can pay back at age $t$ is $F_t^\psi(e, k_t, \theta, \epsilon_w) \cdot h - C_t^{ev}(e, k_t, \theta)$. This is the surplus of employment in terms of consumption value under the most unfavorable productivity shock. Using these notions, an individual’s natural borrowing limit defined in this paper is:

$$L_{t-1}^s(e, k_t, \theta) = \frac{L_t^s(e, k_{t+1}, \theta) + \max\{0, F_t^\psi(e, k_t, \theta, \epsilon_w) \cdot h - C_t^{ev}(e, k_t, \theta)\}}{1 + r_b}$$

$$k_{t+1} = F^k(k_t, d_{k,t}), \quad d_{k,t} = 1(F_t^\psi(e, k_t, \theta, \epsilon_w) \cdot h - C_t^{ev}(e, k_t, \theta) \geq 0).$$

At terminal age $T$, $L_T = 0$, we calculate $C_T^{ev}$ using Equation (13). We then calculate the natural borrowing limit $\bar{L}_{T-1}$ at $T - 1$ based on Equations (14) and (15). Therefore, Equations (13)-(15) enable us to calculate the natural borrowing limit recursively for any age.

Default is not allowed for student loans. If $\bar{L}_t^g > \bar{L}_t^s$, an individual may borrow more than his natural borrowing limit allows, i.e., $s_{t+1} < -\bar{T}_t^s$. In such a case, the individual cannot borrow from private lending market, and the individual has the option to carry his

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8See Lochner and Monge-Naranjo (2016) for evidence supporting this assumption.
Our analysis differs from Navarro’s in four ways: (1) we allow for an elastic labor supply response to shocks in our sequence of credit constraints (he assumes labor is inelastically supplied); (2) we actually use the recursive form of the constraints in (13)-(15) in estimating our model (Navarro assumes that the constraint in each period is the minimum asset holding in his data); (3) he does not account for consumption floors or floor aspect of \( \max\{0, F_t^\psi(\cdot) h - C^\psi_t(\cdot)\} \); and (4) he does not explicitly specify the minimum shock that he imposes in his model. Our analysis differs from the approach of Keane and Wolpin (2001) and Johnson (2013) by not introducing any additional free parameters to proxy unmeasured credit constraints. This is a more stringent approach to estimation. Finally, unlike other approaches in the literature, we do not specify \textit{ad hoc} fixed credit limits or calibrate the model to fit asset distributions. Table 1 summarizes the literature and our distinct approach.

2.2.2 Optimal Decisions

The envelop condition implies

\[
\frac{\partial V_t}{\partial s_t} = \lambda_{b,t}(1 + r(s_t)), \quad \text{if } s_t \neq 0, \quad (16)
\]

where \( r(s_t) = r_l 1(s_t > 0) + r_b 1(s_t < 0) \) and \( \lambda_{b,t} \) is the Lagrangian multiplier of the budget constraint.

First-order conditions with respect to \( c_t > 0 \) and \( s_{t+1} \neq 0, t < T \) are:

\[
\frac{\partial u_c(c_t; \Omega_t)}{\partial c_t} = \lambda_{b,t}
\]

\[
\exp(-\rho(\theta)) \left( \frac{\partial E V_{t+1}}{\partial s_{t+1}} \right) + \lambda_{s,t} = \lambda_{b,t} \quad (18)
\]

where \( \lambda_{s,t} \) is the Lagrangian multiplier of the borrowing constraint. If \( \lambda_{s,t} > 0 \), the borrowing constraint binds, i.e., \( s_{t+1} = S_{t+1} \). If \( \lambda_{s,t} = 0 \), the borrowing constraint does not bind and the individual is able to smooth consumption between age \( t \) and age \( t + 1 \).
Individuals value education and work experience not only because they improve productivity and thus earnings, but also because they increase the natural borrowing limit and thus provide insurance values for consumption against adverse wage shocks. The first order conditions are consistent with agent rationality associated with the employment choices.

2.3 Initial Distribution and Our Measurement System

The model is completed by defining the initial conditions and a set of measurement equations that relate proxied cognitive and noncognitive endowments to a set of observed measures. Individuals start life as autonomous agents at age 17 ($t_0 = 17$). The components of the age 17 information set, $\Omega_{17}$ are:

$$\Omega_{17} := (17, \theta_c, \theta_n, k_{17}, e_{17}, s_{17}, d_{e,16}, e_p, s_p).$$

The initial condition at age 17 that can be determined from sample information are:

$$\Omega_{17}^{\text{observed}} := (17, k_{17}, e_{17}, s_{17}, d_{e,16}, e_p, s_p).$$

We proxy $\theta$ but do not directly observe it.

The joint distribution of unobserved ability at initial age 17, conditional on parental background at 17 ($X_{17}$) is given by:

$$\left(\begin{array}{c}
\theta_c \\
\theta_n
\end{array}\right) | X_{17} \sim N \left( \left( \begin{array}{c}
\mu_c(e_p, s_p) \\
\mu_n(e_p, s_p)
\end{array} \right), \left( \begin{array}{cc}
\sigma^2_c \\
\sigma^2_c & \sigma^2_n
\end{array} \right) \right)$$

where $\mu_j(e_p, s_p) = \mu_{j,e,1}1(e_p = 12) + \mu_{j,e,2}1(e_p > 12 & e_p < 16) + \mu_{j,e,3}1(e_p \geq 16) + \mu_{j,s,1}1(s_p = 2) + \mu_{j,s,3}1(s_p = 3)$, for $j = c, n$. Thus we allow the initial distribution to differ by parents’ wealth and education, to capture early parental investment due to parents’

---

9Education, lagged school attendance, parental education, and parental wealth ($e_{17}, d_{e,16}, e_p, s_p$) are observed in our sample. We also set the accumulated years of working experience and net worth to be zero ($k_{17} = 0, s_{17} = 0$).
financial resources, knowledge, or preferences.

We lack direct measurements of cognitive and noncognitive endowments. Instead, we observe a set of measurement equations for $\theta$. Specifically, we assume that at age 17 there exist two sets of dedicated measurement equations for $(\theta_c, \theta_n)$ given by Equations (19) and (20), respectively:

$$Z_{c,j}^* = \mu_{z,c,j} + \alpha_{z,c,j} \theta_c + \epsilon_{z,c,j}, \quad j \in \{1, \ldots, J_c\} \tag{19}$$

$$Z_{n,j}^* = \mu_{z,n,j} + \alpha_{z,n,j} \theta_n + \epsilon_{z,n,j}, \quad j \in \{1, \ldots, J_n\} \tag{20}$$

where individual control variables, including parental education and wealth, initial education level, and lagged schooling are omitted from the measurement equations. The measurement errors $\epsilon_{z,c,j}, \epsilon_{z,n,j}$ are assumed to be independently distributed. The unconditional distribution of $(\theta_c, \theta_n)$ is assumed to be jointly normal. To incorporate both continuous and binary measurements, we assume that the following relationship holds for each measurement at every point of time:

$$Z_{i,j} = \begin{cases} Z_{i,j}^* & \text{if } Z_{i,j} \text{ is continuous} \\ 1(Z_{i,j}^* > 0) & \text{if } Z_{i,j} \text{ is binary} \end{cases}, \quad i \in \{c, n\}. \tag{21}$$

3 Data and Regression Analysis

We use data from the National Longitudinal Survey of Youth 1997 (NLSY97). The NLSY97 is a nationally representative sample of approximately 9,000 youths born during the years 1980 through 1984. Over the sample period 1997 to 2013, NLSY97 provides extensive information every year on the respondents' schooling, employment, earnings, and monetary transfers from parents and government. It also provides individuals' information on cognitive skills measures, earlier-life adverse behaviors, and parental education and wealth.

We restrict our sample to white males, so the estimation results on inequality are isolated
from discrimination by race or gender. We use the unweighted data.\textsuperscript{10} Our final sample contains 2,103 individuals, with 25,641 individual-year observations. Table A1 in the Web Appendix reports the number of observations dropped in each of our sample selection step.

3.1 Variable Description

Measures of Cognitive Ability and Noncognitive Ability

We use the Armed Services Vocational Aptitude Battery (ASVAB) scores as measures of cognitive ability.\textsuperscript{11} Specifically, we consider the scores from Mathematical Knowledge (MK), Arithmetic Reasoning (AR), Word Knowledge (WK), and Paragraph Comprehension (PC). These four scores have been used by NLSY staff to create the Armed Forces Qualification Test (AFQT) score, which has been used commonly in the literature as a measure of IQ or cognitive ability. These ASVAB scores are only asked in year 1999.

Our measures of noncognitive ability include three variables that indicate respondents’ adverse behaviors at very early ages. Specifically, we use: violent behavior in 1997 (ever attack anyone with the intention of hurting or fighting), theft behavior in 1997 (ever steal something worth $50 or more), and any sexual intercourse before age 15. Individuals with high noncognitive ability are less likely to display adverse behaviors. (See Heckman and Kautz, 2014 and Kautz and Zanoni, 2015 for discussions of these measures.)

Education and Labor Market Outcomes

Education is measured by the highest grade completed. We manually recode this variable by cross-checking the highest grade completed with data on enrollment and the highest degree received, in order to correct for missing data, data coding errors, and GEDs. In particular, a high school dropout with a GED is recoded to his highest grade of school actually completed.

\textsuperscript{10}See Johnson (2013) for the same procedure.

\textsuperscript{11}The CAT-ASVAB is an automated computerized test developed by the United States Military which measures overall aptitude. The test is composed of 12 subsections and has been well-researched for its ability to accurately capture a test-takers aptitude.
The NLSY97 records the number of hours worked in each week, number of weeks worked in a year, and total income earned in a year. We define full-time working to be working no less than 30 hours a week, and part-time working to be working less than 30 hours a week but more than or equal to 10 hours a week. Frequency distributions of weeks and hours worked are provided in the Web Appendix Figure A4. For employed workers, the hourly wage rate is the ratio between total earned income and total actual hours worked (in 2004 dollars).

The NLSY97 collects detailed information on assets and debts of respondents at ages 20, 25, and 30. Because we are primarily interested in the effects of borrowing constraints on youth schooling decisions, we focus on financial assets and unsecured borrowing, and we measure youth net worth as all financial assets and vehicles minus financial debts and money owed with respect to a vehicle owned. Financial assets include business, pension and retirement accounts, savings accounts, checking accounts, stocks, and bonds. We do not directly observe consumptions. Instead, we use changes in net worth and income to infer total consumption expenditure.

Parental Education, Net Worth, and Transfers

NLSY97 asks each respondent about their parents’ schooling and net worth information only in round 1 (1997). We define parents’ education as the average years of schooling of father and mother if both the father’s and mother’s schooling are available. For single-parent families where only one parent’s schooling level is available, we define the parents’ schooling
only using the single parent’s schooling level. Parents’ net worth is defined as all assets (including housing assets and all financial assets) minus all debt (including mortgages and all other debts). Parental transfer data is constructed as total monetary transfers received from parents in each year, including allowance, non-allowance income, college financial aid gift, and inheritance. \(^{16}\)

### 3.2 Summary Statistics

Table 2 reports the statistics of key variables over age groups.\(^ {17}\) At age 17, 87% of the youth are enrolled in school and the fraction of the youth in school decreases to 10% at age 25. The fraction of the youth who work full time steadily increases from 43% at age 20 to 76% at age 30; the fraction of part-time employment decreases from 29% at age 20 to 6% at age 30. Average years of schooling increase from 10.3 at age 17 to 13.78 at age 30. The average net worth increases from -$95 at age 20 to $13,645 at age 30. Average hourly wages (both part-time job and full-time job) increase between age 17 and age 30. Average full-time hourly wage rate is $18 at age 30. All the variables are measured in 2004 dollars. Table A3 reports average years of work experience, wages, and net worth by 4 education groups at age 25. Measures of cognitive and noncognitive skills at age 17 are presented in Table 3.

\(^{16}\)College financial aid gift includes any financial aid respondents received from relatives and friends that is not expected to be paid back for each college and term attended in each school year.

\(^{17}\)The summary statistics for the entire sample over year 1997 to 2011 is reported in Table A2.
Table 2: Key Variables over Age

<table>
<thead>
<tr>
<th></th>
<th>Age 17</th>
<th>Age 20</th>
<th>Age 25</th>
<th>Age 30</th>
</tr>
</thead>
<tbody>
<tr>
<td>In School</td>
<td>0.87</td>
<td>0.37</td>
<td>0.10</td>
<td>0.03</td>
</tr>
<tr>
<td>Full-Time Working</td>
<td>0.04</td>
<td>0.43</td>
<td>0.71</td>
<td>0.76</td>
</tr>
<tr>
<td>Part-Time Working</td>
<td>0.48</td>
<td>0.29</td>
<td>0.12</td>
<td>0.06</td>
</tr>
<tr>
<td>Part-Time Working while in School</td>
<td>0.46</td>
<td>0.23</td>
<td>0.07</td>
<td>0.03</td>
</tr>
<tr>
<td>Education</td>
<td>10.30</td>
<td>12.23</td>
<td>13.41</td>
<td>13.78</td>
</tr>
<tr>
<td>Years Worked</td>
<td>0.00</td>
<td>1.25</td>
<td>4.58</td>
<td>8.56</td>
</tr>
<tr>
<td>Net Worth</td>
<td>0.00</td>
<td>-95.02</td>
<td>2122.72</td>
<td>13645.17</td>
</tr>
<tr>
<td>Full-Time Hourly Wage</td>
<td>6.10</td>
<td>9.55</td>
<td>14.71</td>
<td>18.25</td>
</tr>
<tr>
<td>Part-Time Hourly Wage</td>
<td>6.16</td>
<td>8.46</td>
<td>15.28</td>
<td>15.77</td>
</tr>
<tr>
<td>Receive Parental Transfers</td>
<td>0.36</td>
<td>0.46</td>
<td>0.18</td>
<td>0.06</td>
</tr>
<tr>
<td>Total Parental Transfers</td>
<td>428.32</td>
<td>1766.64</td>
<td>315.89</td>
<td>83.51</td>
</tr>
</tbody>
</table>

Table 3: Measures of Initial Health, Cognitive and Noncognitive Ability (Year 1997)

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>sd</th>
<th>min</th>
<th>max</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASVAB: Arithmetic Reasoning (1997)</td>
<td>-0.08</td>
<td>0.95</td>
<td>-3.14</td>
<td>2.37</td>
<td>1,787</td>
</tr>
<tr>
<td>ASVAB: Mathematics Knowledge (1997)</td>
<td>0.06</td>
<td>0.99</td>
<td>-2.80</td>
<td>2.68</td>
<td>1,782</td>
</tr>
<tr>
<td>ASVAB: Paragraph Comprehension (1997)</td>
<td>-0.16</td>
<td>0.93</td>
<td>-2.36</td>
<td>1.83</td>
<td>1,785</td>
</tr>
<tr>
<td>ASVAB: Word Knowledge (1997)</td>
<td>-0.28</td>
<td>0.89</td>
<td>-3.15</td>
<td>2.35</td>
<td>1,786</td>
</tr>
<tr>
<td>Noncognitive: Violent behavior (1997)</td>
<td>0.22</td>
<td>0.42</td>
<td>0.00</td>
<td>1.00</td>
<td>2,098</td>
</tr>
<tr>
<td>Noncognitive: Had sex before Age 15</td>
<td>0.18</td>
<td>0.38</td>
<td>0.00</td>
<td>1.00</td>
<td>2,101</td>
</tr>
<tr>
<td>Noncognitive: Theft behavior (1997)</td>
<td>0.10</td>
<td>0.29</td>
<td>0.00</td>
<td>1.00</td>
<td>2,099</td>
</tr>
</tbody>
</table>

21
Figure 1: Parental Monetary Transfers By Parental Characteristics

(a) By Parents’ Net Worth & Education

(b) By Youth’s Age

Source: NLSY97. Parental transfer is the total monetary transfers received from parents in each year, including allowance, non-allowance income, college financial aid gift, and inheritance.

The distribution of parental transfers is skewed. The amount of parental transfers to children is either positive or zero. On average, 29% of the youths receive zero monetary transfers from their parents. Among those who receive positive parental transfers, the average amount of transfers received is $3,116, and the median amount is $907. As shown in Figure 1, on average, the amount of parental transfers depends crucially on parental education and net worth and varies over the youth’s life cycle.

There is a positive impact of parental education and wealth on educational decisions. As seen in Figure 2, even after controlling for measures of the youths’ own cognitive ability, there is still a strong positive correlation between parents’ education and net worth and an individual’s college attendance and 4-year college completion. Table 4 reports the OLS regression results of years of schooling at age 30 on ASVAB, number of early adverse behavior, parental education, and parental net worth. After controlling the ASVAB score, individuals’ years of schooling are still positively correlated with both parental education and parental net worth. Furthermore, college attendance decisions are negatively correlated with the number

---

18Conditional on parental transfers being positive, the top 1 percentile of the parental transfers amount is about $24,639. We top-code the maximum amount of positive parental transfers to be $30,000 per year.
of early adverse behavior, which suggests a positive associate between years of schooling and noncognitive ability.\footnote{Table A5 reports the OLS estimation results for logarithm of hourly wages among individuals who always work after leaving school upon completing the highest degree.}

Figure 2: Relationships Between Early Endowments and Environments and College Choices

Source: NLSY97 white males. 4-Year college graduate rate is calculate as the fraction of individual whose years of schooling are more than or equal to 16 at age 25.
Table 4: OLS Regression of Adult Educational Outcomes on Early Endowment and Family Influence

<table>
<thead>
<tr>
<th></th>
<th>Education</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASVAB</td>
<td>1.03*** (0.03)</td>
</tr>
<tr>
<td>Num of Adverse Behaviors</td>
<td>-0.60*** (0.04)</td>
</tr>
<tr>
<td>Parents’ Education</td>
<td>0.27*** (0.02)</td>
</tr>
<tr>
<td>Parents’ Net Worth 2nd</td>
<td>0.67*** (0.07)</td>
</tr>
<tr>
<td>3rd Tercile</td>
<td>1.12*** (0.07)</td>
</tr>
<tr>
<td>Age</td>
<td>0.09*** (0.02)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.46</td>
</tr>
<tr>
<td>Observations</td>
<td>5354</td>
</tr>
</tbody>
</table>

Standard errors in parentheses.
Source: NLSY97 white males aged 25 to 30.
* \(p < 0.10\), ** \(p < 0.05\), *** \(p < 0.01\)

4 Empirical Strategy

Our model is fully parameterized. The specifications used are reported in Section 4.1. In Section 4.2, we discuss external calibration for parameters that can be identified using externally supplied data. After that, we turn to a description of our model identification (Section 4.3) and estimation (Section 4.4).

4.1 Model Parameterization

We use a semi-separable utility function:

\[
U(c_{t}, d_{e,t}, d_{k,t}; \Omega_{t}) = \frac{(c_{t}/es_{t,e})^{1-\gamma} - 1}{1 - \gamma} + u_{e}(\Omega_{t})d_{e,t} + u_{k}(d_{k,t}, \Omega_{t}) + \phi_{k,e}d_{e,t}1(d_{k,t} = 0.5)
\]  

(22)

where \(es_{t,e}\) is the equivalence scales of family size, \(u_{e}(\Omega_{t})\) and \(u_{k}(d_{k,t}, \Omega_{t})\) are flow utility (or disutility if negative) associated with the individual’s choices of schooling and working,

---

\(^{20}\)Web Appendix Section A.2 describes the parameterization of other components of the model.

\(^{21}\)Household equivalence scales measure the change in consumption expenditures needed to keep the welfare of a family constant when its size varies. We calculate the equivalence scales of different household sizes following Fernández-Villaverde and Krueger (2007). For example, this scale implies that a household of two needs 1.34 times the consumption expenditure of a single household. We do not model endogenous changes in family size. Instead we allow family size to vary exogenously depending on education level \(e\) and age \(t\). The average family size for each education group at every age is obtained from CPS data 1997 to 2012.
respectively:

\[
\begin{align*}
   u_e(\Omega_t) &= \phi_{e,0} \mathbf{1}(d_{e,t} + e_t \leq 12) + (\phi_{e,1} + \phi_{e,a} \mathbf{1}(t > 22)) \cdot \mathbf{1}(d_{e,t} + e_t > 12 & d_{e,t} + e_t \leq 16) \\
   &+ \phi_{e,2} \mathbf{1}(d_{e,t} + e_t > 16) + \alpha_{e,c}\theta_c + \alpha_{e,n}\theta_n + \phi_{e,p} \mathbf{1}(e_p \geq 16) - \phi_{e,e}(1 - d_{e,t-1}) + \sigma_{e}\epsilon_{e,t} \\
   u_k(d_{k,t}, \Omega_t) &= [\mathbf{1}(d_{k,t} = 0.5) \cdot \phi_{k,0} + \mathbf{1}(d_{k,t} = 1) \cdot (\phi_{k,1} + \phi_{k,2} \mathbf{1}(t \geq 23))] \\
   &\cdot (1 + \alpha_{k,c}\theta_c + \alpha_{k,n}\theta_n + \phi_{k,3} \mathbf{1}(age < 18))
\end{align*}
\]

(23) where the schooling preference shock \(\epsilon_{e,t}\) is i.i.d. standard normal distributed.

We allow for the psychic costs of schooling to depend on an individual’s cognitive and noncognitive abilities. \(\phi_{e,0}, \phi_{e,1},\) and \(\phi_{e,2}\) controls the level of psychic costs for attending high school, college, and graduate school respectively, \(\phi_{e,e}\) is the psychic cost of re-entering school.

We also allow for preference heterogeneity in schooling depending on parental education level \(e_p\) to allow for direct impact of parental education on schooling. We allow for preference shocks to the utility of schooling. Based on the individual’s previous schooling status, \(d_{e,t-1}\), he may face a different cost of returning to school in the current period.

Following De Nardi (2004), we assume that the terminal value function at age \(T+1\) takes the following functional form:

\[
V_{T+1}(\Omega_{T+1}) = \phi_s \frac{(s_{T+1}/es_{T,e} + b_s)^{1-\gamma} - 1}{1 - \gamma},
\]

(25) where \(\phi_s\) controls for the mean net worth at age \(T + 1\) and \(b_s \geq 0\) is maximum debt amount allowed at age \(T + 1\). Equation (25) approximates an individual’s value function at age \(T + 1\). It does not imply that individuals die at age \(T + 1\) or that other state variables in \(\Omega_{T+1}\) do not matter. It just implies that the marginal effects of other state variables (such as accumulation of education and experience) on the individual’s value function at age \(T + 1\) is small. As noted in Section 2.2, we set the terminal age to be \(T + 1 = 51\).

We allow the subjective discount rate \(\rho(\theta_c, \theta_n)\) to depend on an individuals’ cognitive
ability and noncognitive ability,

$$\rho(\theta_c, \theta_n) = \rho_0(1 - \rho_c\theta_c - \rho_n\theta_n) \quad (26)$$

Therefore the associated subjective discount factor is $$\exp(-\rho_0(1 - \rho_c\theta_c - \rho_n\theta_n))$$.

An individual’s wage function and human capital function are given by:

$$\log w_t = \log \psi_t + 1(d_{k,t} = 0.5)\left(\beta_{w,0} + \beta_{w,1}d_{e,t}\right) \quad (27)$$

$$\psi_t = \exp(\beta_{\psi,0} + \beta_{\psi,k}k_t + \beta_{\psi,kk}k_t^2 + \beta_{\psi,e,0}(e_t - 12) + (\beta_{w,e,1} + \alpha_{\psi,e,1}\theta_c + \alpha_{\psi,n,1}\theta_n) \cdot 1(e_t \geq 12 \& e_t < 16) + (\beta_{w,e,2} + \alpha_{\psi,e,2}\theta_c + \alpha_{\psi,n,2}\theta_n) \cdot 1(e_t \geq 16)) \cdot \epsilon_{w,t} \quad (28)$$

Following Chatterjee, Corbae, Nakajima, and Ros-Rull (2007), we assume the following one-parameter functional form for the distribution function of $$\epsilon_{w,t} \subset [\epsilon_w, \tau_w] \subset \mathbb{R}_{++}$$:

$$\text{Prob}(\epsilon_w \leq z) = \left(\frac{z - \epsilon_w}{\tau_w - \epsilon_w}\right)^{\phi_w} \quad (29)$$

where $$\phi_w$$ controls the shape of the shock distribution and $$\frac{\tau_w - \epsilon_w}{\epsilon_w} > 0$$ determines the bounds of the shock distribution. Without loss of generality, we normalize the lowest possible value of $$\epsilon_w$$ to be 1: $$\epsilon_w = 1$$.

### 4.2 External Calibration

For parameters that can be easily identified without the structural model, such as the monetary cost of schooling and government transfers, we rely on external data sources. Table 5 summarizes all the parameters that are externally specified in our structural model. We now discuss these choices in detail.

---

$$^{22}\mathbb{E}(\epsilon_{w,t}) = \tau_w - \frac{(\tau_w - \epsilon_w)^{\phi_w}}{\phi_w + 1}.$$ When $$\phi_w = 1$$, $$\epsilon_{w,t}$$ is uniformly distributed over the subset $$[\epsilon_w, \tau_w]$$ and $$\mathbb{E}(\epsilon_{w,t}) = (\tau_w + \epsilon_w)/2$$. When $$\phi_w < 1$$, $$\epsilon_{w,t}$$ has a long right tail.
<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>College Tuition &amp; Fees</td>
<td>$tc(e = 13, 14)$</td>
<td>$5073$</td>
<td>IPEDS data on average tuition and fees 1999-2006.</td>
</tr>
<tr>
<td></td>
<td>$tc(e ≥ 15)$</td>
<td>$10653$</td>
<td></td>
</tr>
<tr>
<td>College Grants and Scholarship</td>
<td>$gr(e = 13, 14, s_p = T1)$</td>
<td>$2581$</td>
<td>NLSY97 data on average grants and scholarship by years of schooling and parental wealth terciles.</td>
</tr>
<tr>
<td></td>
<td>$gr(e = 13, 14, s_p = T2)$</td>
<td>$2287$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$gr(e = 13, 14, s_p = T3)$</td>
<td>$2476$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$gr(e ≥ 15, s_p = T1)$</td>
<td>$3604$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$gr(e ≥ 15, s_p = T2)$</td>
<td>$2569$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$gr(e ≥ 15, s_p = T3)$</td>
<td>$2607$</td>
<td></td>
</tr>
<tr>
<td>College Room and Board</td>
<td>$tr(e = 13, 14)$</td>
<td>$4539$</td>
<td>Johnson (2013) room and board for 2-year college and 4-year college</td>
</tr>
<tr>
<td></td>
<td>$tr(e ≥ 15)$</td>
<td>$6532$</td>
<td></td>
</tr>
<tr>
<td>GSL Borrowing Flow</td>
<td>$\bar{\ell}(e = 13)$</td>
<td>$2625$</td>
<td>Annual Stafford loan limits 1993 to 2007</td>
</tr>
<tr>
<td>Annual</td>
<td>$\bar{\ell}(e = 14)$</td>
<td>$3500$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\bar{\ell}(e = 15, 16)$</td>
<td>$5500$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\bar{\ell}(e &gt; 16)$</td>
<td>$8500$</td>
<td></td>
</tr>
<tr>
<td>GSL Borrowing Aggregate Limit</td>
<td>$\bar{T}^g(e_t ≥ 13 &amp; e_t ≤ 16)$</td>
<td>$23000$</td>
<td>Undergraduate</td>
</tr>
<tr>
<td></td>
<td>$\bar{T}^g(e_t ≥ 16)$</td>
<td>$65500$</td>
<td>Graduate + Undergraduate</td>
</tr>
<tr>
<td>Borrowing Interest Rate</td>
<td>$r_b$</td>
<td>5%</td>
<td>Federal Student Aid</td>
</tr>
<tr>
<td>Lending Interest Rate</td>
<td>$r_l$</td>
<td>1%</td>
<td>Average real interest rate on 1-year U.S. government bonds from 2001 to 2007</td>
</tr>
<tr>
<td>Parental Transfer Function</td>
<td>$tr_p(e_p, s_p, d_{c,t}, d_{k,t}, e_t, t)$</td>
<td>Table A6</td>
<td>NLSY97 sample</td>
</tr>
<tr>
<td>Parents Consumption Subsidy</td>
<td>$tr_{c,t} = \chi \cdot 1(t &lt; 18)$</td>
<td>$7800$</td>
<td>Kaplan (2012) &amp; Johnson (2013)</td>
</tr>
<tr>
<td>Full-time Annual Hours of Work</td>
<td>$h$</td>
<td>2080</td>
<td>NLSY97 sample (see FigureA2)</td>
</tr>
<tr>
<td>Unemployment Benefit</td>
<td>$b_g(e ≤ 12)$</td>
<td>$540 \times 3$</td>
<td>NLSY97 UI benefits</td>
</tr>
<tr>
<td></td>
<td>$b_g(e ≥ 13 &amp; e ≤ 16)$</td>
<td>$600 \times 3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$b_g(e &gt; 16)$</td>
<td>$740 \times 3$</td>
<td></td>
</tr>
<tr>
<td>Minimum Consumption Floor</td>
<td>$c_{min}$</td>
<td>$2800.0000$</td>
<td>NLSY sample average means-tested transfers among recipients</td>
</tr>
<tr>
<td>Risk Aversion Coefficient</td>
<td>$\gamma$</td>
<td>2.0000</td>
<td>Lochner and Monge-Naranjo (2012) and Johnson (2013)</td>
</tr>
<tr>
<td>Terminal Value function</td>
<td>$\phi_s$</td>
<td>1.6900</td>
<td>PSID 1999-2011: Median($s_{51}/c_{50}$)=1.30</td>
</tr>
</tbody>
</table>

IPEDS = Integrated Postsecondary Education Data System. Average tuition and fees are weighted by full-time enrollment and are deflated in 2004 dollars. Because expenditures are higher at four-year institutions than at two-year institutions, there is a noticeable jump in cost between two and three years of college. Within our sample period, the aggregate subsidized Stafford Loan Limits is $23000 for undergraduate and $65500 for graduate and undergraduate in total. The Interest rate ranges from 3.34 to 8.25% for Stafford Loans over the time period 1997 to 2011. Parental consumption subsidy is given by $tr_{c,t} = \chi \cdot 1(t < 18)$, where $\chi$ is the value of direct consumption subsidy provided by the parents such as shared housing and meals when the youth attends high school.
We calculate the cost of college tuition and fees and grants and scholarships from the following two sources: (i) Total direct expenditures (including tuition and fees) of higher education level $e_t$ are calculated as the average expenditures per student using data from The Integrated Postsecondary Education Data System (IPEDS); (ii) We also calculate the average amount of the grant for each education level associated with every parental net worth tercile using the NLSY97 sample. We also obtain the average cost of college room and board from IPEDS for two year college and 4 year college, respectively. We set the borrowing interest rate equal to 5 percent annually. We set the lending interest rate $r_l$ to be 1 percent annually, which is the average real interest rate on 1-year U.S. government bonds from 2001 to 2007.

We estimate the logarithm of parental monetary transfers, $\log(tr_{p,t}+1)$ using our NLSY97 sample (see Section A.2 in the Web Appendix for parameterization); the parameter estimates are reported in Web Appendix Table A6. In the sample, 94% of youth who are attending high school live with their parents. Following Kaplan (2012) and Johnson (2013), we set the consumption subsidy provided by parents for those who are living with their parents, $\chi$, to be $650$ monthly ($7800$ annually); $\chi$ includes both the direct and indirect costs of housing as well as shared meals.

We set the annual hours of working of a full-time employed worker to be 2080 hours. We set the monthly unemployment benefits to be $540$ for unemployed workers without a college degree, $600$ for some college or 4-year college workers, and $740$ for workers with a graduate degree. Conditional on receiving unemployment benefits, the mean monthly unemployment insurance benefits are $800$ for workers with at most a high school degree, $900$ for workers with some college or 4-year college, and $1100$ for workers with a graduate degree. In the model, we assume individuals who are not working or in school receive unemployment benefits, which are substantially more generous than the actual unemployment benefits; we thus reduce the predicted unemployment benefits amount by one-third following Kaplan (2012).
the government means-tested minimum consumption floor $c_{\text{min}}$ to be $2800,000.

We set the relative risk aversion parameter to be $\gamma = 2.0000$ following Lochner and Monge-Naranjo (2011) and Johnson (2013). A majority of existing microstudies on consumption and savings estimates the value of $\gamma$ between one and three.\footnote{See Browning, Hansen, and Heckman (1999) for a summary of the early literature.}

Because we assume that individuals cannot leave positive debt in the private debt market at the end of age $T$, i.e., $s_{T+1} \geq 0$, we set $b_s = 0$ in the terminal value function (Equation (25)). The first-order optimal condition at age $T$ can be written as $c_T^{-\gamma} = \phi_s s_{T+1}^{-\gamma}$. From the Panel Study of Income Dynamics (PSID) 1999 to 2011 the median value of $\frac{s_{T+1}}{c_T}$ is 1.30 among households whose head aged $T + 1 = 51$, therefore we set $\phi_s = \left(\frac{s_{T+1}}{c_T}\right)^{\gamma} = (1.30)^{\gamma} = 1.6900$.

4.3 Identification

This section discusses identification of key features of the model.

4.3.1 Factor Model and Measurement System

The identification of factor models requires normalizations that set the location and scale of the factors (see Anderson and Rubin (1956)). For each factor $(\theta_c, \theta_n)$, we normalize its unconditional mean to be zero, i.e., $E_{e_p,s_p}(\mu_c(e_p, s_p)) = E_{e_p,s_p}(\mu_n(e_p, s_p)) = 0$, and standard deviation to be one, i.e., $\sigma_c = \sigma_n = 1$. We allow intercepts in all measurement equations.

4.3.2 Dynamic Model and Structural Parameters

This section provides an overview of identification. The parameters on the subjective discount rate are identified by using consumption data formed from the asset data. To illustrate, consider the Euler equation under a CRRA utility specification for those who are far away
from borrowing constraints (abstracting from uncertainty):\footnote{For illustrative purposes, here we assume $u_c(c_t; \Omega_t) = c_t^{1-\gamma}/1-\gamma$ and $r$ is the borrowing/lending interest rate.}

$$
\gamma \cdot (\log c_{t+1} - \log c_t) = -\rho(\theta_c, \theta_n) + \log(1 + r),
$$

using the fact that $\gamma$ is set externally. The identification of the parameters of the subjective discount rate relies on variations in consumption growth and thus savings. The level of average net worth identifies the constant term of the subjective discount rate, $\rho_0$.\footnote{Alternatively, if we fix $\rho$ externally, we can identify $\gamma$.}

### 4.3.3 Identification

Identification of the remaining parameters of the model follows from an extension of the reasoning in Heckman and Navarro (2007). Using a version of a large support condition, we can find an unconstrained subset of agents with access to Arrow-Debreu insurance contracts. Assuming access to continuous instruments, from this group of agents we can identify the remaining model parameters. With $\gamma$ in hand, we can monetize the consumption value of future income flows and use the analysis of Heckman and Navarro (2007). The identification of the human capital production functions follows Cunha and Heckman (2008) and Cunha, Heckman, and Schennach (2010).

### 4.4 Estimation Method

We use a two-step estimation procedure. In the first step, we estimate the parameters of the measurement system and the joint distribution of cognitive ability and noncognitive ability at age 17. The initial conditions for cognitive ability and noncognitive ability in the second step are obtained by simulation using the parameter estimates from the first step.

In the second step, we use the method of simulated moments to estimate parameters of individuals’ preferences (14 parameters), human capital production function and wage
equation (14 parameters), and discount factors (3 parameters).\textsuperscript{30} In total, we estimate 31 parameters in the second step and the total number of moments is 232. Table 6 lists targeted moments, which includes choices probabilities and outcome variables over age and by education categories as well as conditional moments between outcome variables and measures of cognitive and noncognitive abilities. We use diagonal moments of the data following Altonji and Segal (1996).

5 Estimation Results

This section discusses our estimates. Sections 5.1 and 5.2 discusses the parameter estimates and the goodness of model fit respectively. Section 5.3 presents the estimated natural borrowing limits and the fraction of youths who are credit constrained under estimated model parameters. Section 5.5 discuss the sorting pattern into education based on unobserved cognitive ability and noncognitive ability. Section 5.6 discuss the inequality in human capital, education, and consumption in the estimated model (baseline case).

5.1 Parameter Estimates

5.1.1 Measurement System Parameters

The initial distribution of \((\theta_c, \theta_n)\) is reported in Web Appendix Table A7. The parameter estimates of the measurement equations are reported in Table A8. These three initial endowments are positively correlated with each other.\textsuperscript{31} The correlation between cognitive ability and noncognitive ability is moderate (0.280).\textsuperscript{32}

\textsuperscript{30}The choice variables in the model include not only discrete controls such as schooling and working decisions but also continuous controls such as asset level. As a result, we use Simulated Method of Moments (SMM) to estimate the model.

\textsuperscript{31}The variance of each factor is normalized to one for identification.

\textsuperscript{32}Heckman, Humphries, and Veramendi (2016) report an estimate of .40 for a related model.
Table 6: Targeted Moments for SMM Estimation

<table>
<thead>
<tr>
<th>Targeted Moments</th>
<th># Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Choice probabilities, state variables, and outcome variables over the life-cycle</strong></td>
<td></td>
</tr>
<tr>
<td>Probabilities of schooling for each age 17 to 30</td>
<td>14</td>
</tr>
<tr>
<td>Probabilities of working part-time for each age 17 to 30</td>
<td>14</td>
</tr>
<tr>
<td>Probabilities of working full-time for each age 17 to 30</td>
<td>14</td>
</tr>
<tr>
<td>Average hourly full-time wage rate for each age 18 to 30</td>
<td>13</td>
</tr>
<tr>
<td>Average hourly part-time wage rate for each age 18 to 30</td>
<td>13</td>
</tr>
<tr>
<td>Average net worth at ages 20, 25, and 30</td>
<td>3</td>
</tr>
<tr>
<td>Percent of negative net worth at ages 20, 25, and 30</td>
<td>3</td>
</tr>
<tr>
<td>Average negative net worth at ages 20, 25, and 30</td>
<td>3</td>
</tr>
<tr>
<td>Probability of enrolling in college at age 21</td>
<td>1</td>
</tr>
<tr>
<td>Probability of graduating from 4-year college at age 25</td>
<td>1</td>
</tr>
<tr>
<td>Probabilities of high school graduation, some college, and 4-year college at ages 25 and 30</td>
<td>3 × 2</td>
</tr>
<tr>
<td>Average years of schooling × parents’ education at ages 21 and 25</td>
<td>2</td>
</tr>
<tr>
<td>Average years of schooling × parents’ net worth terciles = 3 at ages 21 and 25</td>
<td>2</td>
</tr>
<tr>
<td>Probability of working part-time while attending school over ages 18 to 22</td>
<td>1</td>
</tr>
<tr>
<td>Mean and Variance of years of schooling at age 30</td>
<td>2</td>
</tr>
<tr>
<td>Mean and Variance of years of work experience at age 30</td>
<td>2</td>
</tr>
<tr>
<td><strong>Covariance terms from auxiliary models (Indirect Inference)</strong></td>
<td></td>
</tr>
<tr>
<td>Regression coefficients of log wages on work experience, work experience squared, years of schooling, HSG, SCL , CLG, ASVAB AR × HSD, num of adverse behaviors × HSD, ASVAB AR × HSG, num of adverse behaviors × HSG, ASVAB AR × SCL, num of adverse behaviors × SCL, ASVAB AR × CLG, num of adverse behaviors × CLG, working part-time, working part-time while in school, previously not working</td>
<td>17</td>
</tr>
<tr>
<td>Regression coefficients of log net worth on ASVAB AR, num of adverse behaviors, and log wage, at ages 20, 25, and 30</td>
<td>3</td>
</tr>
<tr>
<td>Regression coefficients of school enrollment on parents’ education, ASVAB AR, num of adverse behaviors, previous period’s enrollment status</td>
<td>4</td>
</tr>
<tr>
<td>Regression coefficients of full-time working on ASVAB AR, num of adverse behaviors</td>
<td>2</td>
</tr>
<tr>
<td><strong>Conditional moments for each of the 4 education categories</strong></td>
<td></td>
</tr>
<tr>
<td>Probability of working part-time by 4 education categories at ages 25 and 30</td>
<td>4 × 2</td>
</tr>
<tr>
<td>Probability of working full-time by 4 education categories at ages 25 and 30</td>
<td>4 × 2</td>
</tr>
<tr>
<td>Average years of work experience by 4 education categories at ages 25 and 30</td>
<td>4 × 2</td>
</tr>
<tr>
<td>Average hourly full-time wage by 4 education categories at ages 25 and 30</td>
<td>4 × 2</td>
</tr>
<tr>
<td>Average hourly part-time wage by 4 education categories at ages 25 and 30</td>
<td>4 × 2</td>
</tr>
<tr>
<td>Median log hourly wages by 4 education categories at ages 25 and 30</td>
<td>4 × 2</td>
</tr>
<tr>
<td>Average log hourly wages by 4 education categories at ages 25 and 30</td>
<td>4 × 2</td>
</tr>
<tr>
<td>Standard deviation of log hourly wages by 4 education categories at ages 25 and 30</td>
<td>4 × 2</td>
</tr>
<tr>
<td>Bottom 5 percentile of log hourly wages by 4 education categories at ages 25 and 30</td>
<td>4 × 2</td>
</tr>
<tr>
<td>Top 5 percentile of log hourly wages by 4 education categories at ages 25 and 30</td>
<td>4 × 2</td>
</tr>
<tr>
<td>Median net worth by 4 education categories at ages 25 and 30</td>
<td>4 × 2</td>
</tr>
<tr>
<td>Mean net worth by 4 education categories at ages 25 and 30</td>
<td>4 × 2</td>
</tr>
<tr>
<td>Percent of negative net worth by 4 education categories at ages 25 and 30</td>
<td>4 × 2</td>
</tr>
<tr>
<td>Average negative net worth by 4 education categories at ages 25 and 30</td>
<td>4 × 2</td>
</tr>
</tbody>
</table>
5.1.2 Structural Model Parameters

Web Appendix Table A9 reports preference parameter estimates for the schooling utility functions (Panel A) and work utility function (Panel B). The psychic benefit of schooling is higher for individuals with higher cognitive and noncognitive abilities. Individuals whose parents have higher education also have higher flow utility of schooling.

Parameter estimates of the discount rate function are reported in Table A10. Figure 3 plots the density of estimated discount factors in the benchmark model.

![Kernel density estimate](image)

Figure 3: Density of Estimated Discount Factors: \( \exp(-\rho(\theta_c, \theta_n)) \)

5.1.3 Labor Market Skill Production and Wages

Table A11 in the Web Appendix reports parameter estimates for the human capital production function and the wage equation. A one standard deviation increase in cognitive ability increases an individual’s human capital level as well as offered wages by \( \alpha_{\psi,c} = 0.1054 \) log points. The effects of noncognitive ability on human capital level and wages is small and not statistically different from zero.
5.2 Goodness of Model Fit

Our model closely fits the lifecycle patterns of school enrollment (Figure 4(a)) and employment (see Figures 4(b) and 4(c)). Figures 4(d) shows that our estimated model fits the observed accepted wage patterns over time. Figures 4(e) and 4(f) plot our model fits of average net worth and average negative net worth at ages 20, 25, and 30. The only bad fit is for negative net worth by age.

As seen in Figure 5, our model can replicate the patterns of number of years worked for each of the four education categories at age 30; Our model can also replicate the average wages for each education categories at age 30.

Tables A12 and A13 in the Web Appendix reports model fits of the auxiliary model for enrollment and employment, respectively. Table A14 in the Web Appendix reports model fits

33 Figures A5 in the Web Appendix plots the model fit on years of schooling over age, over parent’s education categories, and over parents’ net worth terciles. The fit is generally good.
of the auxiliary model for log hourly wage. Table A15 shows the model fits of the auxiliary model for log net worth. Our model generally fits the data well.

5.3 Natural Borrowing Limit $\bar{L}_t^s$

Figure 6(a) plots both the average amount of natural borrowing limit for all individuals from age 17 to age 50. On average, the natural borrowing limit first increases as an individual accumulates schooling and work experience, and then gradually decreases as individuals age and the remaining lifetime earnings declines. Such a pattern is in sharp contrast with that of a life cycle model without human capital accumulation in the absence of life cycle wage growth in which the natural borrowing limit decreases monotonically with age.

Next, we explore the relationship between an individual’s natural borrowing limit in the private lending market $\bar{L}_t^s(e_{t+1}, k_{t+1}, \theta)$ and his human capital $\psi_t = F^\psi(e_t, k_t, \theta_c, \theta_n, \epsilon_t)$ at age $t$. Figure 6(b) plots the model implied average natural borrowing limit over human capital levels at age 30. The average natural borrowing limit is relatively flat with respect to an individual’s human capital level when the human capital level is low. When the human capital level is high, the natural borrowing limit increases with the human capital level. Similarly, conditional on age, the natural borrowing limit generally increases with education, cognitive ability, and noncognitive ability (see Web Appendix Figure A6).

34The decline to zero at age 50 is an artifact of our assumed horizon of 51 years. The figure is qualitatively correct for later terminal ages but shifts the hump rightward.
We plot the evolution of natural borrowing limit with age for individuals with different cognitive and noncognitive abilities in Figure 7. The natural borrowing limit generally increases with age during the period age 17 to 30. However, the growth rate of the natural borrowing limit is much higher for individuals with higher initial ability endowments.

5.4 Borrowing Constrained Youths

An important contribution of our model is the estimation of life cycle credit constraints governing behavior. Using our model, we can calculate the fraction of individuals who are bor-
rowing constrained by evaluating the Lagrangian multiplier associated with an individual’s optimal saving’s decision. In particular, using the first order conditions from an individual’s optimization problem (Equations (17) and (18)), we can calculate the Lagrangian multiplier associated with individuals’ next period asset decisions as follows:

$$\lambda_{s,t}(c_t, s_{t+1}; \Omega_t) = \frac{\partial u_c(c_t; \Omega_t)}{\partial c_t} - \exp(-\rho(\theta_c, \theta_n)) \left( \frac{\partial E V_{t+1}}{\partial s_{t+1}} \right)$$

An individual is borrowing constrained at period $t$ if $\lambda_{s,t} > 0$. Figures 8, 9, and 10 graph—at ages 21, 30, and 40, respectively—the estimated multipliers ($\lambda_{s,t}$) as functions of various arguments and the fraction of the sample constrained as a function of the same arguments. The most interesting figures are those in the “d” and “h” sequences across age corresponding to cognitive ability and human capital stocks, respectively. Figures 8(d), 9(d), and 10(d) report results by levels of cognitive ability. Figures 8(h), 9(h), and 10(h) report results by levels of human capital stocks.

At age 21, these graphs exhibit a bimodality that sharpens by age 30. Two groups of people are constrained—those with low ability/low human capital endowment who have relatively flat life cycle wage growth and those who are high ability/high human capital people with high wage growth who, because of borrowing restrictions, cannot fully access their future income. By age 40 (Figure 10), when life cycle earnings growth is more fully realized, the second hump associated with the agent’s high ability and high initial endowments disappears. The first hump remains throughout the life cycle.

The other graphs in this sequence are also informative. There is little evidence of bimodality in constraints with respect to non-cognitive ability. There is a fairly steady monotonic decline in binding constraints as non-cognitive ability increases, except possibly at age 21. The patterns with respect to net worth show a sharp cliff. Beyond a certain level, agents are not constrained.

Figure 11 shows the life cycle pattern of the proportion borrowing constrained. The

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$^{35}$Human capital and cognitive ability are strongly correlated.
Figure 8: Lagrangian Multiplier \( \lambda_{s,t} \) and Borrowing Constrained Youths \( \lambda_{s,t} > 0 \) at Age 21
Figure 9: Lagrangian Multiplier $\lambda_{s,t}$ and Borrowing Constrained Youths $\lambda_{s,t} > 0$ at Age 30
Figure 10: Lagrangian Multiplier $\lambda_{s,t}$ and Borrowing Constrained Youths $\lambda_{s,t} > 0$ at Age 40
U shape is a consequence of initial borrowing constraints and the approach of the terminal horizon. The constrained in college are 5% more likely to work than other college students.

Figure 11: % Borrowing Constrained over Age

5.5 Sorting into Education

Using simulated data based on estimated model parameters, we illustrate the magnitude of sorting into education at age 30 based on unobserved abilities. Figure 12 is the density plot of three unobservables by education groups in the simulated data. Based on cognitive and noncognitive ability, people sort into education.

Figure 12: Density of Initial Factors Conditional on Age-30 Education

The age 50 horizon artificially accelerates the borrowing constraint into middle age.
5.6 Inequality in Education, Earnings, and Consumption

Both employment decisions and borrowing constraints have important implications for earnings and consumption inequality. The first row of Table 7 shows the age-30 inequality in education, earnings, and consumption in our benchmark model. The cross-sectional inequality is measured by the variance of logs. Earnings inequality is higher than education inequality because of the existence of uninsured human capital risk, differences in employment choices (part-time vs full-time), and large variation in individuals’ work experience, cognitive ability, and noncognitive ability. Consumption inequality is lower than earnings inequality because individuals use savings to smooth consumption fluctuations. We discuss the other rows of this table in the next section.

Table 7: Inequality in Education, Earnings, and Consumption (Age 30)

<table>
<thead>
<tr>
<th>Inequality (Var of log)</th>
<th>Changes in Inequality (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Educ</td>
</tr>
<tr>
<td>Benchmark</td>
<td>0.0457</td>
</tr>
<tr>
<td>Equalizing Cognitive Ability</td>
<td>0.0284</td>
</tr>
<tr>
<td>Equalizing Noncognitive Ability</td>
<td>0.0277</td>
</tr>
<tr>
<td>Equalizing Parental Factors</td>
<td>0.0363</td>
</tr>
<tr>
<td>Subsidizing College Tuition</td>
<td>0.0462</td>
</tr>
<tr>
<td>Increasing Student Loan Limits</td>
<td>0.0464</td>
</tr>
</tbody>
</table>

Note: Inequality in Education (Educ), earnings, and consumption (C) are measured using variance of log years of schooling, log earnings, and log consumption at age 30, respectively. Changes in inequality is calculated as the percentage changes in inequality compared to the benchmark model. When equalizing cognitive ability, we set every individual’s cognitive ability equal to the population mean, i.e., zero. Similarly, we set every individual’s noncognitive ability equal to zero when equalizing noncognitive ability. When equalizing parental factors, we set parents’ education equal to 12 years of schooling and parent’s net worth to be in the second tercile.
6 Counterfactual Exercises

We next discuss counterfactual simulations based on the model.

6.1 Equalizing Initial Endowments

In this section, we use the model to analyze the determinants of the inequality in education, earnings, and consumption. In particular, we are interested in the following question: To what extent do differences in cognitive ability, noncognitive ability, and parental factors account for the observed cross-section inequality in years of schooling, earnings, and consumption at age 30? To address this question, we perform three counterfactual experiments that equalize initial heterogeneity in (i) cognitive ability, (ii) noncognitive ability, and (iii) parental factors (including both schooling and net worth). Equalization is made by putting all observations at the mean of the selected variable, keeping other variables at their sample values.

As shown in Table 7, equalizing cognitive ability reduces age-30 inequality in education by 38%. Equalizing noncognitive ability reduces age-30 education inequality by 39%. Finally, equalizing parental factors (including parent’s education and net worth) reduces inequality by 21%.

Equalizing cognitive endowments reduces earnings inequality by 11% and consumption inequality by 26%. Equalizing noncognitive endowments reduces earnings inequality by 2.5% and consumption inequality by 6.5%. Equalizing parental factors reduces earnings inequality by 2% and consumption inequality by 1.6%.

6.2 Subsidizing College Tuition

This section reports the effects of tuition subsidy on schooling choices. We give each agent a college tuition subsidy of $1500.\footnote{In the simulation, we do not allow the amount of student loan that an individual can borrow if he decides to attend college to be directly affected by such college tuition subsidy.} As shown in Table 8, compared to the benchmark model,
the college attendance rate at age 21 increases by 2.87 percentage points and the 4-year college graduation rate at age 25 increases by 3.34 percentage points. The implied elasticity of college going with respect to tuition is $-0.21$, which is within the range of estimates reported in the literature.

Table 8: Years of Schooling and College Attendance and Graduation under Different Experiments

<table>
<thead>
<tr>
<th></th>
<th>Years of Schooling at age 30</th>
<th>College Attendance at age 21</th>
<th>4-Year College Graduation at age 25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>13.60</td>
<td>49.05</td>
<td>26.19</td>
</tr>
<tr>
<td>Subsidizing College Tuition</td>
<td>13.77</td>
<td>51.92</td>
<td>29.53</td>
</tr>
<tr>
<td>Increasing Student Loan Limits</td>
<td>13.64</td>
<td>49.05</td>
<td>27.46</td>
</tr>
</tbody>
</table>

Note: Education is measured by the average years of schooling at age 30. College attendance is measured as the percentage of individuals whose years of education is more than or equal to 13 at age 21. The rate of 4-year college graduation is measured as the percentage of individuals whose years of schooling is more than or equal to 16 at age 25.

Figure 13 plots the college attendance and the 4-year college graduation rates for the benchmark and reduced tuition policy over the full distribution of cognitive ability and noncognitive ability. Our college tuition subsidy increases the college attendance at age 21 among the individuals whose cognitive ability is within the second and third quartiles. The rate of 4-year college graduation increases much more among high ability individuals compared to low ability individuals.

This policy promotes increasing inequality in education. However the consumption inequality at age 30 is slightly reduced by 1.05% as more individuals stay in school for longer period of time (see Table 7).

6.2.1 Increasing Student Loan Limits

We next increase student loan limits. In this counterfactual experiment, we increase the borrowing limit from the student program also by $1,500 for each academic year. As shown
in Table 8, compared to the benchmark model, the college attendance rate at age 21 changes little and the 4-year college graduation rate at age 25 increases by 1.27 percentage points. Figure 14 plots the college attendance and the 4-year college graduation rates for the benchmark and enhanced student loan limit policy over the full distribution of cognitive ability and noncognitive ability. The differences are small.\(^\text{38}\)

Finally, as shown in Table 7, this counterfactual policy experiment leads to an increase in inequality in education. However, the consumption inequality at age 30 is slightly reduced (3.12\%) as more individuals stay in school for longer period of time and as more individuals need to pay back student loans.

\(^\text{38}\)These results are comparable to those reported in Johnson (2013).
Figure 14: Effects of Increasing Student Loan Limits

7 Summary and Conclusion

This paper estimates a life cycle model of human capital and work experience with parental transfers in the presence of endogenous borrowing limit and precautionary savings motives. In our model, individuals are subject to uninsured human capital risks and choose to invest in education, accumulate work experience and assets, and smooth consumption. Borrowing is permitted up to an endogenous limit, which is an explicitly derived natural borrowing limit accounting for the private lending market and government student loan programs. We use the model to investigate the determinants of human capital inequality and to examine the relationship between educational attainment, cognitive and noncognitive abilities, and
parental education and wealth. We analyze the effects of tuition subsidies and enhanced student loan limits on educational attainment and human capital inequality.

We find substantial evidence of life cycle credit constraints that affect human capital accumulation and inequality. The constrained fall into two groups: (a) the chronically poor with low initial endowments and abilities and low levels of acquired skills over the lifetime, and (b) the initially well-endowed persons with high levels of acquired skills. The first group has flat life cycle wage profiles. They remained constrained over most of their lifetimes. The second group has rising life cycle wage profiles. They are constrained only early on in life because they cannot immediately access their future earnings. As they age, their constraints are relaxed as they access their future earnings.

Equalizing cognitive ability has dramatic effects on reducing inequality in education (Table 7). Equalizing non-cognitive ability has a similar strong impact. Earnings and consumptions, including family background, has much less dramatic effects after controlling for the other first order effects of cognitive ability. Reducing tuition substantially promotes schooling, but has only minor effects on our measures of inequality. Enhancing student loan limits has minor effects on all outcomes studied. There are dramatic effects of equalizing cognition but equalizing other factors.
References


Fernández-Villaverde, Jesus, and Dirk Krueger, 2007, Consumption over the Life Cycle:


