Unrestricted Information Acquisition

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Introduction

- A theory of information acquisition in games
  - Endogenize assumptions on players' information
  - Common extra layer of strategic interaction
- Flexible learning about state and what others know
  - $\neq$ Bergemann & Valimaki 2002, Hellwig & Veldkamp 2009, Myatt & Wallace 2012, Yang 2015,...
  - Expose primitive incentives to acquire information
- Broad assumptions on cost of information
  - Costly to learn state and what others know
  - Example: Shannon mutual information
- Applications
  - Investment games: risk-dominance selection
  - Games on networks: Bonacich centrality
  - Large games: endogenous informational smallness

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  - Investment games: risk-dominance selection
  - Games on networks: Bonacich centrality
  - Large games: endogenous informational smallness

Today

Investment games: bunk runs, currency crises,...

Exogenous information

- Common knowledge: multiplicity
  - Diamond & Dybvig 1983, Obstfeld 1996,...
- Global games: risk-dominance selection
  - Carlsson & van Damme 1993, Morris & Shin 1998,...
- Any perturbation: any selection
  - Weinstein & Yildiz 2007

Endogenous information w/ mutual information

- Flexible info acquisition about state: multiplicity
  - Yang 2015
- Unrestricted info acquisition: risk-dominance selection
  - Extend to potential games

Endogenous information w/out mutual information: next talk

Outline

- Investment game with incomplete information
  - Basic game: actions, states, utilities
  - Exogenous Information structure
- Recap: common knowledge, global games,...
- Investment game with information acquisition
  - Basic game: actions, states, utilities
  - Information acquisition technology
- Flexible info acquisition about state: multiplicity
- Unrestricted info acquisition: risk-dominance selection

Basic Game

- $N = \{1, \ldots, n\}$: finite set of *players*
- $A_i = \{invest, not\ invest\}$: set of i's *actions*
- $\Theta \subseteq \mathbb{R}$: closed set of *states*
- $P_\Theta \in \Delta(\Theta)$: state distribution
- $\rho(\bar{a}_{-i}, \theta) \in \mathbb{R}$: i's non-decreasing *return*
  - $\bar{a}_{-i}$: share of opponents who invest
  - ρ integrable in θ w.r.t. P_Θ
  - $P_\Theta(\{\theta : \rho(1, \theta) < 0\}) > 0$: dominance region
  - $P_\Theta(\{\theta : \rho(0, \theta) > 0\}) > 0$: dominance region
- $u_i(a, \theta) = \mathbf{1}_{\{invest\}}(a_i)\rho(\bar{a}_{-i}, \theta)$: i's *utility*

Today:

	invest	*not invest*
invest	θ, θ	$\theta - 1, 0$
not invest	$0, \theta - 1$	$0, 0$

Exogenous Information Structure

- $(\Omega, \mathcal{F}, P)$: underlying probability space
- $\boldsymbol{\theta} : \Omega \to \Theta$: random variable with $\boldsymbol{\theta} \sim P_\Theta$
- X_i: Polish space of i's *messages*
- $\boldsymbol{x}_i : \Omega \to X_i$: i's *signal*, random variable

Recap

Common knowledge: multiplicity

- $\boldsymbol{x}_i = \boldsymbol{\theta}$ for all i
- $\boldsymbol{\theta} \in [0, 1]$: equilibrium indeterminacy

Global games: risk-dominance selection

- $\boldsymbol{x}_i = \boldsymbol{\theta} + \lambda \boldsymbol{\epsilon}_i$ for all i
  - $\lambda > 0$: scale factor
  - $\boldsymbol{\epsilon}_i$: idiosyncratic noise
- $\lambda \to 0$: perturbation of complete information
- $1/2$: risk-dominance threshold

Any perturbation: any selection

- Weinstein & Yildiz 2007

Information Acquisition Technology

- $(\Omega, \mathcal{F}, P)$: underlying probability space
- $\boldsymbol{\theta} : \Omega \to \Theta$: random variable with $\boldsymbol{\theta} \sim P_\Theta$
- X_i: Polish space of i's *messages*
- $\boldsymbol{X}_i$: nonempty set of i's *signals* $\mathbf{x}_i : \Omega \to X_i$
- $C_i : \Delta(X \times \Theta) \to [0, \infty]$: *$i$'s cost of information*

Game with Information Acquisition

Basic game + info technology = strategic form game:

- Set of players: N
- i's strategy: signal $\boldsymbol{x}_i \in \boldsymbol{X}_i$, *contingency plan* $s_i \in S_i$
  - S_i: set of all measurable $s_i : X_i \to A_i$
- i's payoff: $E[u_i(s(\boldsymbol{x}), \boldsymbol{\theta})] - \lambda C_i(P_{(\boldsymbol{x},\boldsymbol{\theta})})$
  - $\lambda > 0$: scale factor
  - $P_{(\boldsymbol{x},\boldsymbol{\theta})} \in \Delta(X \times \Theta)$: joint distribution of $\boldsymbol{x}$ and $\boldsymbol{\theta}$

Solution concept: pure-strategy Nash equilibrium

To ease notation: $C_i(\boldsymbol{x}, \boldsymbol{\theta}) = C_i(P_{(\boldsymbol{x},\boldsymbol{\theta})})$

$\lambda \to 0$: multiplicity/selection of equilibria?

Flexible Info Acquisition about State

Yang 2015: for all players i

Assumption 0. $|X_i| \geq |A_i|$. Moreover, if random variable $\boldsymbol{x}_i' : \Omega \to X_i$ is measurable w.r.t. some $\boldsymbol{x}_i \in \boldsymbol{X}_i$, then $\boldsymbol{x}_i' \in \boldsymbol{X}_i$.

Assumption 1. Take any $\boldsymbol{x} \in \boldsymbol{X}$. Then $(\boldsymbol{x}_i \perp \boldsymbol{x}_{-i})|\boldsymbol{\theta}$.

Assumption 2. Take any $P_{X_i \times \Theta} \in \Delta(X_i \times \Theta)$.
If $\boldsymbol{\theta} \sim \text{marg}_\Theta(P_{X_i \times \Theta})$, then $(\boldsymbol{x}_i, \boldsymbol{\theta}) \sim P_{X_i \times \Theta}$ for some $\boldsymbol{x}_i \in \boldsymbol{X}_i$.

Assumption 3. For all $\boldsymbol{x} \in \boldsymbol{X}$, $C_i(\boldsymbol{x}, \boldsymbol{\theta}) = I(\boldsymbol{x}_i; \boldsymbol{x}_{-i}, \boldsymbol{\theta})$.

Mutual information: for X_i finite, p p.m.f. of $\boldsymbol{x}_i$,

$$I(\boldsymbol{x}_i; \boldsymbol{x}_{-i}, \boldsymbol{\theta}) = E\left[\log \frac{p(\boldsymbol{x}_i | \boldsymbol{x}_{-i}, \boldsymbol{\theta})}{p(\boldsymbol{x}_i)}\right].$$

Flexible Info Acquisition about State

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Revelation Principle

Direct technology: $X_i = A_i$ for all i

Basic game + direct technology = strategic form game:

- Set of players: N
- i's strategy: *direct signal* $\boldsymbol{x}_i \in \boldsymbol{X}_i$
- i's payoff: $E[u_i(\boldsymbol{x}, \boldsymbol{\theta})] - \lambda C_i(\boldsymbol{x}, \boldsymbol{\theta})$

Solution concept: pure-strategy Nash equilibrium

Revelation principle (Yang 2015):
"A0-A3 $\Rightarrow$ w.l.o.g. technology and signals are direct."

Multiplicity

Theorem (Yang 2015)

Assume A0-A3.

Let P_Θ be abs. continuous w.r.t. Lebesgue measure.

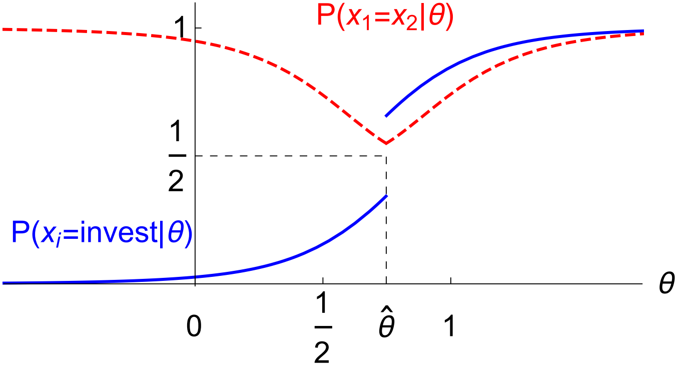
Then $\forall \hat{\theta} \in [0,1]$, $\exists$*equilibria* $(\boldsymbol{x}_\lambda : \lambda > 0)$*:* $\forall i \in N$

$$P(\boldsymbol{x}_{i,\lambda} = invest | \boldsymbol{\theta}) \xrightarrow{a.s.} \begin{cases} 1 & \text{if } \boldsymbol{\theta} \geq \hat{\theta}, \\ 0 & \text{if } \boldsymbol{\theta} < \hat{\theta}. \end{cases}$$

Remark. Also non-monotone equilibria

Monotone Equilibria

$\lambda > 0$:



Monotone Equilibria

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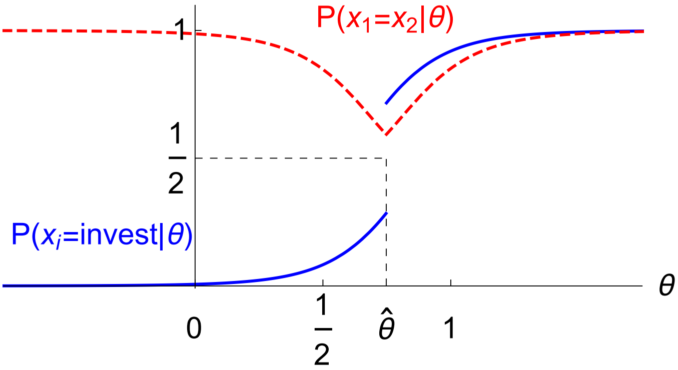

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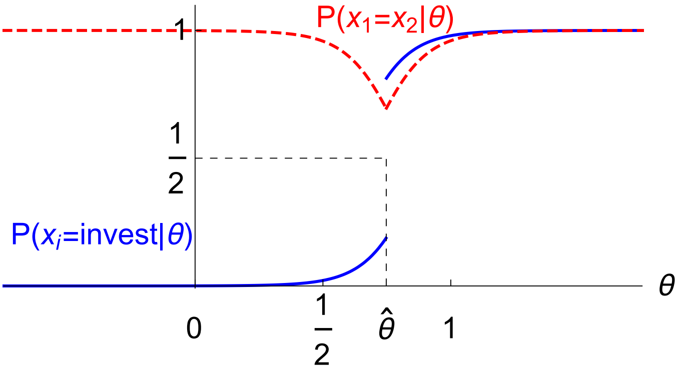

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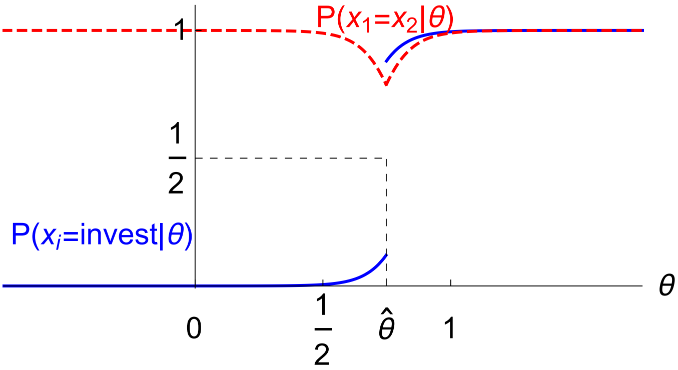
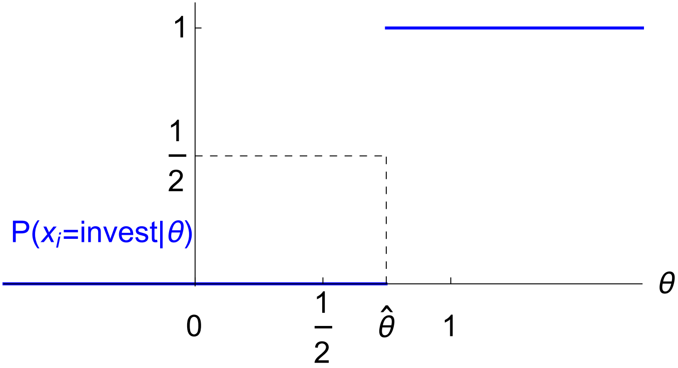


Monotone Equilibria

$\lambda \to 0$:



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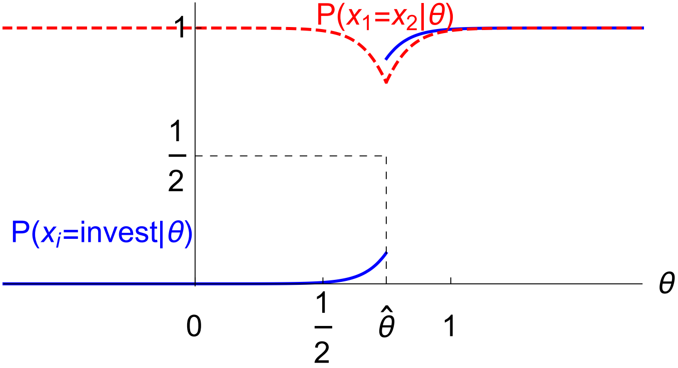

Monotone Equilibria

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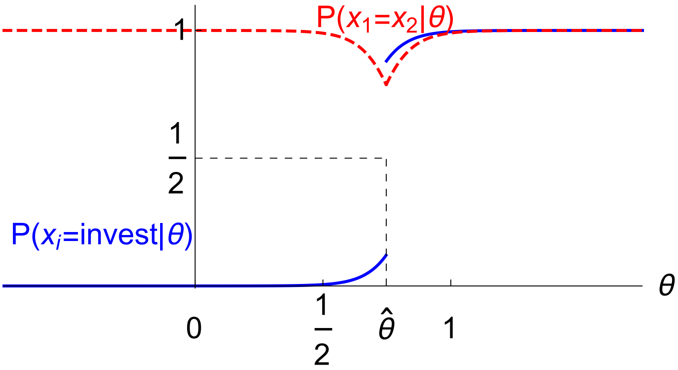


- i's primitive incentive: learn $\{\rho(\bar{\mathbf{x}}_{-i}, \boldsymbol{\theta}) \geq 0\}$
- $\{\rho(\bar{\mathbf{x}}_{-i}, \boldsymbol{\theta}) \geq 0\} \neq \{\boldsymbol{\theta} \geq \hat{\theta}\}$ since $Var(\bar{\mathbf{x}}_{-i}|\boldsymbol{\theta}) \neq 0$

Unrestricted Info Acquisition

For all players i:

Assumption 0. $|X_i| \geq |A_i|$. Moreover, if random variable $x_i' : \Omega \to X_i$ is measurable w.r.t. some $x_i \in \boldsymbol{X}_i$, then $x_i' \in \boldsymbol{X}_i$.

Assumption 3. For all $\boldsymbol{x} \in \boldsymbol{X}$, $C_i(\boldsymbol{x}, \boldsymbol{\theta}) = I(\boldsymbol{x}_i; \boldsymbol{x}_{-i}, \boldsymbol{\theta})$.

Assumption 4. Take any $\boldsymbol{x}_{-i} \in \boldsymbol{X}_{-i}$ and $P_{X \times \Theta} \in \Delta(X \times \Theta)$. If $(\boldsymbol{x}_{-i}, \boldsymbol{\theta}) \sim \text{marg}_{X_{-i} \times \Theta}(P_{X \times \Theta})$, then $(\boldsymbol{x}, \boldsymbol{\theta}) \sim P_{X \times \Theta}$ for some $\boldsymbol{x}_i \in \boldsymbol{X}_i$.

▸ Finite-game Construction ▸ General Construction

Unrestricted Info Acquisition

For all players i:

Assumption 0. $|X_i| \geq |A_i|$. Moreover, if random variable $x'_i : \Omega \to X_i$ is measurable w.r.t. some $x_i \in \boldsymbol{X}_i$, then $x'_i \in \boldsymbol{X}_i$.

Assumption 3. For all $x \in \boldsymbol{X}$, $C_i(\boldsymbol{x}, \boldsymbol{\theta}) = I(\boldsymbol{x}_i; \underline{\boldsymbol{x}_{-i}, \boldsymbol{\theta}})$.

Assumption 4. Take any $\boldsymbol{x}_{-i} \in \boldsymbol{X}_{-i}$ and $P_{X \times \Theta} \in \Delta(X \times \Theta)$. If $(\boldsymbol{x}_{-i}, \boldsymbol{\theta}) \sim \text{marg}_{X_{-i} \times \Theta}(P_{X \times \Theta})$, then $(\boldsymbol{x}, \boldsymbol{\theta}) \sim P_{X \times \Theta}$ for some $\boldsymbol{x}_i \in \boldsymbol{X}_i$.

Finite-game Construction | General Construction

Unrestricted Info Acquisition

For all players i:

Assumption 0. $|X_i| \geq |A_i|$. Moreover, if random variable $\boldsymbol{x}'_i : \Omega \to X_i$ is measurable w.r.t. some $\boldsymbol{x}_i \in \boldsymbol{X}_i$, then $\boldsymbol{x}'_i \in \boldsymbol{X}_i$.

Assumption 3. For all $\boldsymbol{x} \in \boldsymbol{X}$, $C_i(\boldsymbol{x}, \boldsymbol{\theta}) = I(\boldsymbol{x}_i; \underline{\boldsymbol{x}_{-i}, \boldsymbol{\theta}})$.

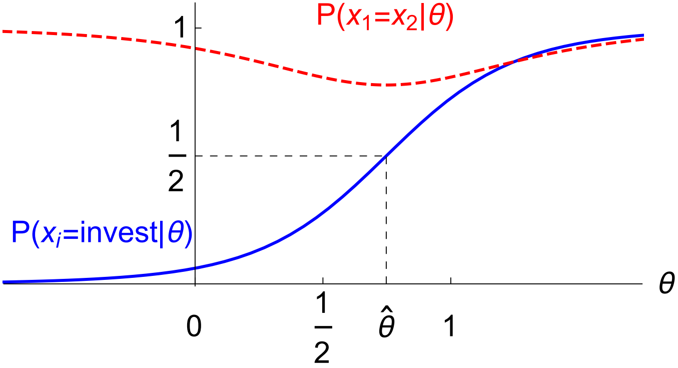
Assumption 4. Take any $\boldsymbol{x}_{-i} \in \boldsymbol{X}_{-i}$ and $P_{X \times \Theta} \in \Delta(X \times \Theta)$. If $(\boldsymbol{x}_{-i}, \boldsymbol{\theta}) \sim \text{marg}_{X_{-i} \times \Theta}(P_{X \times \Theta})$, then $(\boldsymbol{x}, \boldsymbol{\theta}) \sim P_{X \times \Theta}$ for some $\boldsymbol{x}_i \in \boldsymbol{X}_i$.

▸ Finite-game Construction ▸ General Construction

Revelation principle:
"A0, A3, A4 $\Rightarrow$ w.l.o.g. technology and signals are direct."

All Equilibria

$\lambda > 0$:



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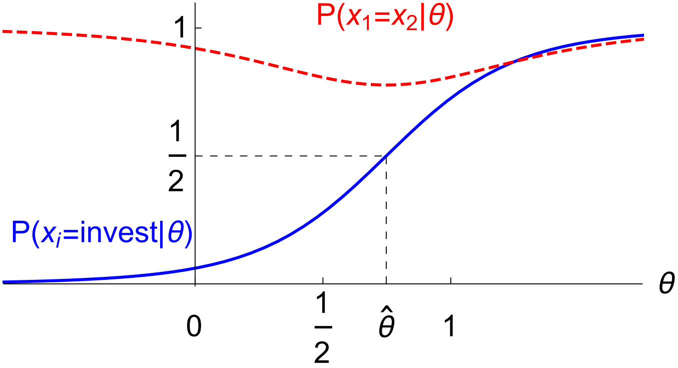

$$\hat{\theta} = \frac{1}{2} - \lambda \log \frac{P(\boldsymbol{x}_i = \mathit{invest})}{P(\boldsymbol{x}_i = \mathit{not\ invest})}$$

All Equilibria

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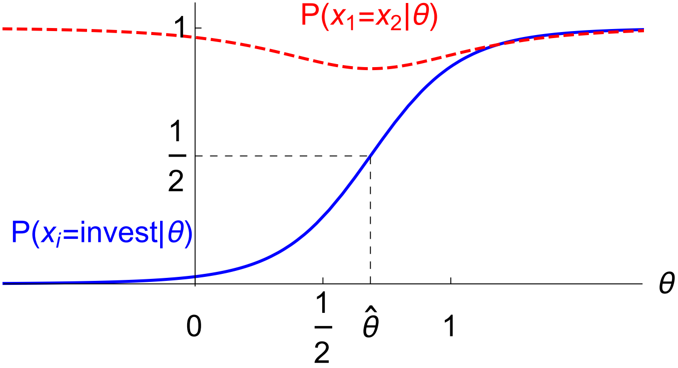

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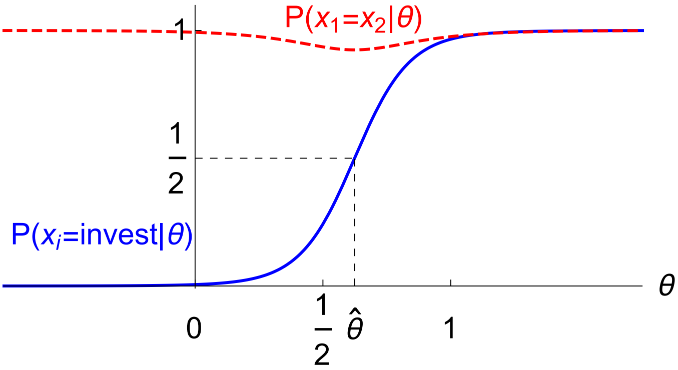

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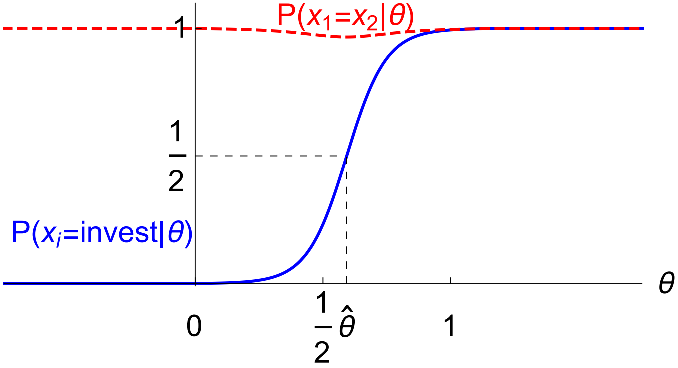
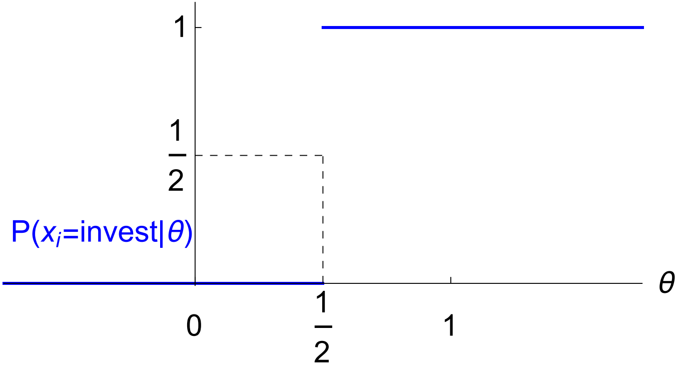


$$\hat{\theta} = \frac{1}{2} - \lambda \log \frac{P(\boldsymbol{x}_i = invest)}{P(\boldsymbol{x}_i = not\ invest)}$$

All Equilibria

$\lambda \to 0$:



Risk-dominance Selection

Theorem

Assume A0, A3, and A4.

Then $\forall$*equilibria* $(\boldsymbol{x}_\lambda : \lambda > 0)$*:* $\forall i \in N$

$$P(\boldsymbol{x}_{i,\lambda} = invest|\boldsymbol{\theta}) \xrightarrow{a.s.} \begin{cases} 1 & if\ \boldsymbol{\theta} > \frac{1}{2}, \\ 0 & if\ \boldsymbol{\theta} < \frac{1}{2}. \end{cases}$$

Theorem extends to potential games.

Proof

- $(\boldsymbol{x}_\lambda)$: family of equilibria
- $v : A \times \Theta \to \mathbb{R}$: potential s.t. for all i, a_i, and a_i'

$$u_i(a_i, \cdot) - u_i(a_i', \cdot) = v(a_i, \cdot) - v(a_i', \cdot)$$

- Investment games: for all a and θ

$$v(a, \theta) = \sum_{m=0}^{|a|-1} \rho\left(\frac{m}{n-1}, \theta\right)$$

  with $|a|$ number of players who invest
- a^* risk-dominant at θ: $v(a^*, \theta) > v(a, \theta)$ for all $a \neq a^*$
- Info acquisition w/ mutual information, $\lambda > 0$:
  - Static 1-player: Csiszar 1974, Matejka & McKay 2016
  - Dynamic 1-player: previous talk
  - Static n-player: next slide

Key Lemma

$P_{(\boldsymbol{x}_\lambda, \boldsymbol{\theta})}$: *joint distribution of* $\boldsymbol{x}_\lambda$ *and* $\boldsymbol{\theta}$

1. *Quality.* *Almost surely,*

$$\frac{dP_{(\boldsymbol{x}_\lambda, \boldsymbol{\theta})}}{d(P_{\boldsymbol{\theta}} \times_{i \in N} P_{\boldsymbol{x}_{i,\lambda}})}(a, \theta) = \frac{e^{\frac{v(a,\theta)}{\lambda}}}{\int_A e^{\frac{v(a',\theta)}{\lambda}} (\times_{i \in N} P_{\boldsymbol{x}_{i,\lambda}})(da')}.$$

2. *Quantity.* $P_{\boldsymbol{x}_{1,\lambda}}, \ldots, P_{\boldsymbol{x}_{n,\lambda}}$ *eq. of potential game* V:

$$V(P_{A_1}, \ldots, P_{A_n}) = \int_\Theta \log \left(\int_A e^{\frac{v(a,\theta)}{\lambda}} (\times_{i \in N} P_{A_i})(da) \right) P_\Theta(d\theta).$$

$P_{A_1}, \ldots, P_{A_n}$: *equilibrium of* V ▸ Existence

- *There is equilibrium* $\boldsymbol{x}_\lambda$ *s.t.* $\boldsymbol{x}_{i,\lambda} \sim P_{A_i}$ *for all* $i \in N$.

Key lemma: for all θ and $a \neq a^*$

$$\frac{dP_{(\boldsymbol{x}_\lambda,\boldsymbol{\theta})}}{d(P_{\boldsymbol{\theta}} \times_{i\in N} P_{\boldsymbol{x}_{i,\lambda}})}(a,\theta) = \frac{e^{\frac{v(a,\theta)}{\lambda}}}{\int_A e^{\frac{v(a',\theta)}{\lambda}} (\times_{i\in N} P_{\boldsymbol{x}_{i,\lambda}})(da')}$$

$$\leq \frac{e^{\frac{v(a,\theta)}{\lambda}}}{e^{\frac{v(a^*,\theta)}{\lambda}} (\times_{i\in N} P_{\boldsymbol{x}_{i,\lambda}})(\{a^*\})}$$

$$= \frac{1}{e^{\frac{v(a^*,\theta)-v(a,\theta)}{\lambda}} (\times_{i\in N} P_{\boldsymbol{x}_{i,\lambda}})(\{a^*\})} \to 0.$$

- Recall: $v(a^*,\theta) > v(a,\theta)$ for all $a \neq a^*$
- Dominance regions: $\liminf_{\lambda\to 0} P_{\boldsymbol{x}_{i,\lambda}}(\{a_i^*\}) > 0$ for all i

$\Rightarrow \; P(\boldsymbol{x}_\lambda = a^* | \boldsymbol{\theta} = \theta) \to 1.$ □ Extension Infinite N

Conclusion

Today: multiplicity/selection of eq. in coordination games

- Exogenous: info players have about others' info
- Endogenous: info players want about others' info

Unrestricted info acquisition

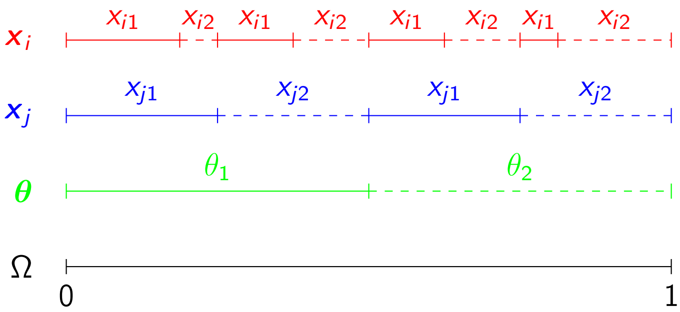
- Broad assumptions on cost of information: A5, A6
- Games on networks, Bonacich centrality
- Large games, endogenous informational smallness

$\Rightarrow$ Rich yet tractable language for info acquisition in games

Appendix

Finite-game Construction

- Θ and X finite
- $(\Omega, \mathcal{F}, P)$: nonatomic probability space
- $\boldsymbol{X}_i$: all measurable functions $\boldsymbol{x}_i : \Omega \to X_i$ Back



Finite-game Construction

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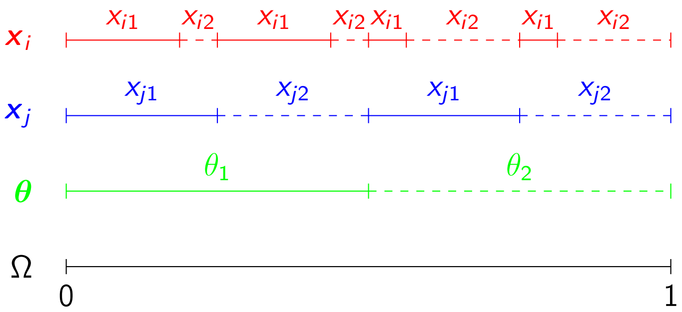

Finite-game Construction

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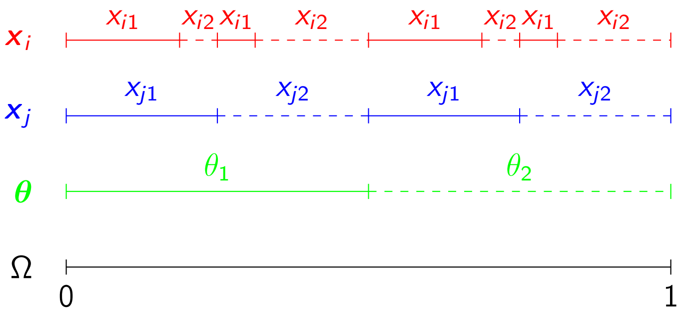

General Construction

T: uncountable set.

$(\Omega, \mathcal{F}, P)$:

- $\exists \boldsymbol{\theta} : \Omega \to \Theta$: $\boldsymbol{\theta} \sim P_\Theta$.
- $\exists \boldsymbol{z}_t : \Omega \to [0, 1]$, $t \in T$:
  - $\boldsymbol{\theta}$ and $\boldsymbol{z}_t$, $t \in T$, independent
  - $\boldsymbol{z}_t$, $t \in T$, uniformly distributed

$\forall i \in N$, $\boldsymbol{x}_i \in \boldsymbol{X}_i$ iff $\exists$countable $Q \subset T$:

- $\boldsymbol{x}_i$ measurable w.r.t. $\boldsymbol{\theta}$ and $\boldsymbol{z}_t$, $t \in Q$. Back

Existence

- A_i, Θ: Polish spaces
- $v : A \times \Theta \to \mathbb{R}$: measurable function s.t.

$$-\infty < \int_\Theta \inf_{a \in A} v(a,\theta) P_\Theta(d\theta) < \int_\Theta \sup_{a \in A} v(a,\theta) P_\Theta(d\theta) < \infty$$

- $V : \Delta(A_1) \times \cdots \times \Delta(A_n) \to \mathbb{R}$:

$$V(P_{A_1}, \ldots, P_{A_n}) = \int_\Theta \log \left(\int_A e^{v(a,\theta)} (\times_{i \in N} P_{A_i})(da) \right) P_\Theta(d\theta)$$

Fact

Function V has a maximum if:

- *A_i is compact for all $i \in N$.*
- *v is upper semi-continuous in a.* Back

Unique Selection for Potential Games

- $N = \{1, \dots, n\}$: finite set of *players*
- A_i: Polish space of i's *actions*
- Θ: Polish space of *states*
- $P_\Theta \in \Delta(\Theta)$: state distribution
- $v : A \times \Theta \to \mathbb{R}$: *potential*

Integrability: $\int_\Theta \sup_{a \in A} v(a, \theta) P_\Theta(d\theta) < \infty$

Theorem

Assume A0, A3, and A4.

Take any $a \in A$ *s.t.* $\forall i \in N \; \exists \Theta_{a_i} \subseteq \Theta$:

- $P_\Theta(\Theta_{a_i}) > 0$
- $\inf_{\theta \in \Theta_{a_i}} \inf_{a_i' \neq a_i} \inf_{a_{-i} \in A_{-i}} v(a, \theta) - v(a_i', a_{-i}, \theta) > 0$

Then $\forall$*equilibria* $(\boldsymbol{x}_\lambda : \lambda > 0)$*: almost surely*

$$v(a, \theta) > \sup_{a' \neq a} v(a', \theta) \quad \Rightarrow \quad \lim_{\lambda \to 0} P(\boldsymbol{x}_\lambda = a | \boldsymbol{\theta} = \theta) = 1.$$

Back

Infinite N: Multiplicity

Theorem

Assume A0, A3, and A4.

Let $N = \{1, 2, \ldots\}$.

Let P_Θ be abs. continuous w.r.t. Lebesgue measure.

Then $\forall \hat{\theta} \in [0, r]$, $\exists$equilibria $(\boldsymbol{x}_\lambda : \lambda > 0)$: $\forall i \in N$

- $(\boldsymbol{x}_i \perp \boldsymbol{x}_{-i}) | \boldsymbol{\theta}$
- $P(\boldsymbol{x}_{i,\lambda} = invest | \boldsymbol{\theta}) \xrightarrow{a.s.} \begin{cases} 1 & \text{if } \boldsymbol{\theta} \geq \hat{\theta} \\ 0 & \text{if } \boldsymbol{\theta} < \hat{\theta} \end{cases}$

Proof

- Assume $(\boldsymbol{x}_i \perp \boldsymbol{x}_{-i})|\boldsymbol{\theta}$ for all $i \in N$.
- Law of large numbers: $Var(\bar{\boldsymbol{x}}|\boldsymbol{\theta}) = 0$.
- Hence, $Var(\rho(\bar{\boldsymbol{x}}, \boldsymbol{\theta})|\boldsymbol{\theta}) = 0$.
- Independent information acquisition is optimal.
- Multiplicity as in Yang 2015. Back

Assumption 5

Take any $i \in N$, $\boldsymbol{x}_{-i} \in \boldsymbol{X}_{-i}$, and $\boldsymbol{x}_i, \boldsymbol{x}'_i \in \boldsymbol{X}_i$.

Assume that $(\boldsymbol{x}'_i \perp (\boldsymbol{x}_{-i}, \boldsymbol{\theta}))|\boldsymbol{x}_i$.

Then $C_i(\boldsymbol{x}_i, \boldsymbol{x}_{-i}, \boldsymbol{\theta}) \geq C_i(\boldsymbol{x}'_i, \boldsymbol{x}_{-i}, \boldsymbol{\theta})$.

Equality holds only if $(\boldsymbol{x}_i \perp (\boldsymbol{x}_{-i}, \boldsymbol{\theta}))|\boldsymbol{x}'_i$. Back

Assumption 6

Take any $i \in N$, $\boldsymbol{x}_{-i} \in \boldsymbol{X}_{-i}$, and $\boldsymbol{x}_i, \boldsymbol{x}_i' \in \boldsymbol{X}_i$.

Assume there is measurable $f : X_{-i} \times \Theta \to Z$:

- $(\boldsymbol{x}_i, f(\boldsymbol{x}_{-i}, \boldsymbol{\theta})) \sim (\boldsymbol{x}_i', f(\boldsymbol{x}_{-i}, \boldsymbol{\theta}))$
- $(\boldsymbol{x}_i' \perp (\boldsymbol{x}_{-i}, \boldsymbol{\theta}))|f(\boldsymbol{x}_{-i}, \boldsymbol{\theta})$

Then $C_i(\boldsymbol{x}_i, \boldsymbol{x}_{-i}, \boldsymbol{\theta}) \geq C_i(\boldsymbol{x}_i', \boldsymbol{x}_{-i}, \boldsymbol{\theta})$.

Equality holds only if $(\boldsymbol{x}_i \perp (\boldsymbol{x}_{-i}, \boldsymbol{\theta}))|f(\boldsymbol{x}_{-i}, \boldsymbol{\theta})$. Back

Proof

Set $\boldsymbol{z} = f(\boldsymbol{x}_{-i}, \boldsymbol{\theta})$:

$$I(\boldsymbol{x}_i'; \boldsymbol{x}_{-i}, \boldsymbol{\theta}) = I(\boldsymbol{x}_i'; \boldsymbol{z}) = I(\boldsymbol{x}_i; \boldsymbol{z}) \leq I(\boldsymbol{x}_i; \boldsymbol{x}_{-i}, \boldsymbol{\theta}). \quad \square$$