The Gains From Input Trade in Firm-Based Models of Importing*

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Abstract

Trade in intermediate inputs allows firms to lower their costs of production by using better, cheaper, or novel inputs from abroad. Quantifying the aggregate impact of trade in inputs, however, is challenging. As importing firms differ markedly in how much they buy in foreign input markets, results based on aggregate trade models do not apply. We develop a methodology to quantify the aggregate gains from input trade for a class of firm-based models of importing. We derive a sufficiency result: as long as firms' demand system between domestic and foreign inputs is CES, the change in consumer prices induced by input trade is fully determined from the observable joint distribution of firms' value added and domestic expenditure shares in material spending. We provide a simple formula that can be readily evaluated given the micro-data. Three features of our methodology are that: (i) it does not require information on prices and qualities of the different sourcing countries, (ii) it does not impose any restrictions on how foreign inputs are combined for production, (iii) it is consistent with any model of the extensive margin, i.e. of how firms find their foreign input suppliers. In an application, we consider a multi-sector, general-equilibrium trade model with a rich input-output structure. Using data for French importers, we find that input trade leads to a 27% reduction in consumer prices in the manufacturing sector.

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1 Introduction

A large fraction of world trade is accounted for by firms sourcing intermediate inputs from abroad. Trade theory highlights one particular margin of how domestic consumers benefit from producers engaging in international sourcing. By providing access to novel, cheaper or higher quality inputs, input trade reduces firms' unit costs and lowers domestic prices, therefore increasing consumers' purchasing power. In this paper, we develop a methodology to quantify this channel and provide an application to France.

Quantifying the welfare consequences of input trade is not straightforward. Recent quantitative trade models that allow for trade in inputs feature the convenient property that welfare can be measured with aggregate data only - e.g. Eaton et al. (2011), Caliendo and Parro (2015) and Costinot and Rodriguez-Clare (2014). This property, however, relies on the assumption that firms' import intensities are equalized - a feature that is at odds with the data. In particular, importing firms differ substantially in the share of material spending they allocate to foreign inputs. In this paper, we show that accounting for this heterogeneity in import exposure, which requires resorting to firm-based models of importing, significantly affects the measurement of the welfare gains from input trade.

We provide a sufficiency result that applies to a class of firm-based models of importing where firms' demand system between domestic and foreign inputs is CES.[1] In particular, we show that firm-level data on domestic shares of intermediate spending and value added is sufficient to compute the *consumer price gains from input trade*, i.e. the change in consumer prices relative to a situation of "input autarky" where firms can use only domestic inputs. Not only is such data sufficient but, more importantly, it implies the exact same consumer price gains for any model in this class. Hence, this result does not rely on any particular mechanism of how firms determine their trading partners, e.g. whether importing is limited by the presence of fixed costs, a process of network formation or costly search. Conveniently, we provide a closed-form expression that makes calculating the consumer price gains straightforward.

Our result builds on a simple insight. By inverting the demand system for intermediates, we can link the firm's unit cost to its spending pattern on domestic inputs. When such demand system is CES, the unit cost reduction from importing, which we refer to as the *producer gains*, is fully determined by the domestic expenditure share and two structural parameters.[2] In particular, the producer gains are high when the domestic share is low. In a second step, we then show how these producer gains can be aggregated to compute the consumer price gains of input trade taking general equilibrium effects into account. In a multi-sector trade model with intersectoral linkages and monopolistic competition, such consumer price gains are akin to a value-added weighted average of the producer gains. In this way, the joint distribution of domestic shares and value added is sufficient to characterize the effect of input trade on consumer prices. Importantly, a key aspect of the data

[1] This class nests several frameworks used in the literature, e.g. Halpern et al. (2011), Gopinath and Neiman (2014), Antràs et al. (2014) and Goldberg et al. (2010).

[2] These are the elasticity of firm output to intermediate inputs and the elasticity of substitution between domestic and international varieties.

is how firm size and domestic shares correlate; if bigger firms feature higher trade shares, then the consumer gains will turn out to be large.

Our procedure places no restrictions on several components of the theory related to firms' import environment. First, we do not require information on the prices and qualities of the foreign inputs, nor on how these are combined for production.[3] While in principle required to compute the firm's unit cost, these elements are fully summarized by the domestic expenditure share. Consider next the extensive margin of trade. Because our sufficiency result is derived purely from the cost minimization problem taking the set of trading partners as given, specifying a mechanism of how the firm finds its suppliers is not necessary. In this way, our approach bypasses important data requirements as well as functional form and behavioral assumptions and therefore holds in a variety of settings.

An important parameter in our methodology is the elasticity of substitution between domestically sourced and imported inputs. Because our economy does not generate a standard gravity equation, this parameter is not identified from aggregate data. We therefore devise a strategy to identify it from firm-level variation. Specifically, we exploit the fact that the sensitivity of firm revenue to import spending depends on this elasticity of substitution. To address the endogeneity concern that unobserved productivity shocks might lead to both higher import spending and higher revenue, we use changes in the world supply of particular varieties as an instrument for firms' import spending.

We apply our methodology to the population of manufacturing firms in France. We estimate the distribution of trade-induced changes in unit costs across firms. We find substantial cross-sectional dispersion in these producer gains, which is induced by the observed variation in domestic expenditure shares. While the median unit cost reduction is 11%, the average is 22%. Moreover, bigger firms benefit more from input trade. We then aggregate the producer gains to compute the consumer gains by relying on the joint distribution of domestic shares and value added. We find that input trade reduces consumer prices in the manufacturing sector by 27%.[4] There are three reasons why the consumer gains exceed the median producer gains, which go back to the above-mentioned patterns. First, the dispersion in producer gains is valued by consumers given their elastic demand. Second, the positive relation between the producer gains and firm size is beneficial because the endogenous productivity gains from importing and firm efficiency are complements. Finally, there are important linkages between firms whereby non-importers buy intermediates from importing firms. This structure of round-about production amplifies the gains from input trade in general equilibrium.

We then consider the effect of input trade on a broader notion of welfare. While the consumer price gains are an important component of the welfare gains from input trade, they do not take into account any resources spent by firms to attain their equilibrium sourcing strategies. Because such resource loss cannot be read-off the data, we need to commit to a particular model of the extensive

[3]We consider a production structure where foreign inputs are aggregated into an import bundle. We require that such import bundle is combined with a bundle of domestic inputs in a CES fashion, but place no restrictions on the foreign input aggregator.

[4]When we include the non-manufacturing sector, the consumer price gains amount to only 9%. This is due to the fact that manufacturing accounts for a relatively small share in aggregate consumer spending and that production links between the manufacturing and the non-manufacturing sector, which we assume to be closed to international trade, are limited.

margin of trade and fully calibrate it. We consider a model where participation in international markets is limited by fixed costs. We parametrize the distributions of qualities, prices and fixed costs, and discipline the model with moments of the French data. We target the joint distribution of domestic expenditure shares and value added, which as argued above contains important information about the gains from input trade. The main result of this exercise is that the full welfare gains are about half as large as the consumer price gains.

Because our methodology stresses the importance of micro-data, a natural question is: how do our estimates change when only aggregate data is used? Relying on aggregate data affects the estimates of the gains from input trade in two distinct ways. First, there is a bias that arises from ignoring the heterogeneity in firms' import shares for given parameters. While this bias can be positive or negative, we show that the sign depends only on parameters and not on the micro-data. A second type of bias is related to the estimation of the elasticity of substitution. Approaches that rely on a standard gravity equation to estimate this parameter may lead to different results than an analysis based on micro-data. In our application to the French data, the first bias leads to overestimating the consumer gains by about 10%, while the second one leads to underestimating them by 50%.[5] Thus, the magnitude of the different errors from using aggregate data can be substantial.

Our paper contributes to a recent literature on quantitative models of input trade. On the one hand, there are aggregative trade models as Eaton et al. (2011), Caliendo and Parro (2015) and Costinot and Rodriguez-Clare (2014). These models have the convenient implication that the welfare consequences of input trade are fully determined from readily available aggregate data.[6] This property, however, crucially relies on a theoretical structure where firms' import shares are equalized - an implication which is strongly at odds with the data. On the other hand, there is a small literature on firm-based models of importing - see Halpern et al. (2011), Gopinath and Neiman (2014) or Ramanarayanan (2014). Our approach is different in two aspects. First, the existing contributions do not rely on firms' domestic expenditure shares to directly measure the unit cost reductions from importing at the firm level. Instead, they measure these producer gains indirectly by estimating the entire structural model. Because the firm's extensive margin problem is tractable only under particular assumptions and the structure of output markets needs to be fully specified, their results rely on these restrictions. Secondly, the existing papers do not target the joint distribution of value added and domestic shares in their estimations, nor exploit the fact that such data is sufficient to characterize the consumer price gains from trade. Ramanarayanan (2014) and Gopinath and Neiman (2014) for example consider a model that generates a perfect, and hence counterfactual, correlation between firm-size and domestic shares. We explicitly show that the consumer price gains in such type of model are too high.[7]

[5] Using the micro-data to estimate the elasticity of substitution turns out to be important as we obtain a value close to two. Estimation approaches that rely on aggregate data typically find values closer to four.

[6] Specifically, the welfare gains are summarized by the change in the aggregate domestic expenditure share and a trade elasticity, which can be estimated from aggregate trade flows.

[7] Specifically, models where physical efficiency is the single source of firm heterogeneity generate a perfect assignment between efficiency and the domestic share. As more efficient firms experience larger reductions in their unit cost, the aggregate gains from input trade turn out to be too large.

On a more technical level, our paper builds on a recent literature which stresses that complementarities across inputs of production make the import problem different from the better known export problem. In particular, firms' extensive margin of trade is in general harder to characterize - see Blaum et al. (2013) and Antràs et al. (2014). On the export side, recent work has been able to quantitatively account for firms' entry behavior into different markets.[8] In contrast, theories that can account for the pattern of entry into import markets are far less developed. A notable exception is the recent contribution by Antràs et al. (2014), who study a firm-based model of importing and adapt the estimation procedure by Jia (2008) to match positive aspects of import behavior. In contrast, our paper focuses on normative aspects of input trade. Our main result stresses that, conditional on the micro-data, the consumer price gains from trade do not depend on the mechanics of the extensive margin or other aspects of firms' import environment.

At a conceptual level, our paper is related to Arkolakis et al. (2012) in that our sufficient statistic for the firm's unit cost is related to their sufficient statistic for aggregate welfare. In particular, we show that, conditional on the micro-data on firms' domestic shares and a "trade elasticity", which in our setup corresponds to the elasticity of substitution of the firm's import demand system, a wide class of models will imply the exact same distribution of producer gains across firms.

Finally, a number of empirically oriented papers study trade liberalization episodes to provide reduced-form evidence on the link between imported inputs and firm productivity - see e.g. Kasahara and Rodrigue (2008), Amiti and Konings (2007), Goldberg et al. (2010) or Khandelwal and Topalova (2011).[9] Our results are complementary to this literature as we provide a structural interpretation of this reduced-form empirical evidence. In particular, from the point of view of applied researchers, our sufficiency result provides a way to analyze episodes of trade liberalization, or other changes in firms' import environment, without having to fully specify and solve a structural model of importing. The observable change in the domestic expenditure shares correctly measures the effect of the policy on firms' unit costs, taking all adjustments into account. If micro-data on value added is also available, our formula for the consumer gains can be used to gauge the full effect of the policy on consumer prices in general equilibrium.

The remainder of the paper is structured as follows. In Section 2, we present direct evidence from the population of French firms for why firm-based models of importing are necessary to study the normative properties of input trade. Section 3 lays out the class of models we consider and derives our sufficiency results for the producer and consumer gains from input trade. The empirical application to France is contained in Sections 4.1 and 4.2. In Section 5, we calibrate a version of our model with a fully-specified extensive margin of importing to provide a full measure of welfare. Section 6 concludes.

[8] See in particular Eaton et al. (2011), Arkolakis (2010), Arkolakis and Muendler (2011) and Bernard et al. (2011). We also refer the reader to Bernard et al. (2007) and Bernard et al. (2012) for two recent surveys of the literature.

[9] See also De Loecker and Goldberg (2013) for a recent survey about firms in international markets.

2 Why Firm-Based Models of Importing?

In this section, we present data on firms' heterogeneous import behavior that is informative about the consequences of input trade. We rely on data from the population of manufacturing firms in France.[10] In Figure 1, we display the cross-sectional distribution of importers' domestic shares, i.e. the share of material spending allocated to domestic inputs. These differ markedly. While the majority of importers spend less than 10% of their material spending on foreign inputs, some firms are heavy importers with import shares exceeding 50%. This heterogeneity in import intensities is at odds with aggregative models which presume that import shares are equalized across importers. To rationalize the data of Figure 1, we therefore have to resort to firm-based models of importing.

[Figure 1 here]

In this paper, we show that the dispersion in firms' import exposure documented in Figure 1 has aggregate implications. The intuition is simple. As a firm's domestic share measures the extent to which it benefits from foreign input sourcing, Figure 1 shows that the gains from input trade are heterogeneous at the micro-level. To correctly aggregate these producer gains, we have to know firms' relative importance in the economy. In particular, the consumer gains from input trade will be high whenever intense importers, i.e. firms with low domestic shares, are large. Figure 2 displays the extent to which this is the case in France.In the left panel, we depict the distribution of value added by import status. While importers are significantly larger than non-importers, there is ample overlap in their distribution of value added. In the right panel, we show the conditional distribution of domestic shares for different value added quantiles. The relationship between firms' import intensity and size is mildly negative. While larger importers have lower domestic expenditure shares on average, there is substantial dispersion in home-shares conditional on size.

[Figure 2 here]

These patterns are important for our understanding of input trade. Holding the marginal distribution of domestic spending displayed in Figure 1 fixed, the gains from input trade would be higher if import intensity and firm size were more tightly linked. The joint distribution of domestic shares and value added therefore contains important information about the normative implications of input trade. In the next section, we make these statements precise and derive a simple formula to quantify the effect of input trade on consumer prices that only relies on the data displayed in Figures 1 and 2.

3 Theory

In this section, we lay out the general theoretical framework of importing firms that we use to quantify the gains from input trade. In Section 3.1, we study the firm's import problem and formally

[10]We describe the dataset in more detail in Section 4.1 below.

show our unit cost sufficiency result. In Section 3.2, we embed the firm problem into an general equilibrium trade trade model with input-output linkages to quantify the effect of input trade on consumer prices.

3.1 The Producer Gains from Input Trade

Consider the problem of a firm, which we label as i, that uses local and foreign inputs according to the following production structure:

$$y_i = \varphi_i f(l, x) = \varphi_i l^{1-\gamma} x^{\gamma} \tag{1}$$

$$x = \left(\beta_i (q_D z_D)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\beta_i) x_I^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2}$$

$$x_I = h_i \left([q_{ci} z_c]_{c \in \Sigma_i} \right). \tag{3}$$

The firm combines intermediate inputs x with primary factors l, which for we for simplicity refer to as labor, in a Cobb-Douglas fashion with efficiency φ_i.[11] Intermediate inputs are a CES composite of a domestic variety z_D, with quality q_D, and a foreign import bundle x_I with relative efficiency for imported inputs given by β_i. The firm has access to foreign inputs from multiple countries $[z_c]$ which may differ in their quality $[q_{ci}]$, where c is a country index.[12] Foreign inputs are aggregated according a constant returns to scale production function $h_i(\cdot)$.[13] An important endogenous object in the production structure is the set of foreign countries the firm sources from, which we denote by Σ_i and henceforth refer to as the sourcing strategy. Because our approach relies entirely on optimality conditions from the intensive margin problem, we do not impose any restrictions on how Σ_i is determined.

As far as the market structure is concerned, we assume that the firm is a price-taker in input markets and faces prices $(p_D, [p_{ci}])$ for domestic and foreign inputs respectively. Note that prices include trade costs. Similarly, we assume that labor can be hired frictionlessly at a given wage w. On the output side, we do not impose any restrictions, i.e. we do not specify whether firms produce a homogeneous or differentiated final good and how they compete.

The setup above describes a class of firm-based models that have been used in the literature. In particular, it nests the contributions by Gopinath and Neiman (2014), Halpern et al. (2011), Antràs et al. (2014), Kasahara and Rodrigue (2008), Amiti et al. (2012) and Goldberg et al. (2010), among others.[14] While these contributions differ in the specifics of their import environment, they all share

[11] For notational simplicity we consider a single primary factor. It will be clear below that that all our results apply to $l = g(l_1, l_2, ..., l_T)$, where $g(.)$ is a CRS production function and l_j are primary factor of different types.

[12] We do not distinguish for now between products and varieties, i.e. imports of a given product stemming from different countries. As will be clear below, we do not need to take stand on this distinction for the results of this section.

[13] We note that this setup allows for an interaction between quality flows and the firm's efficiency, i.e. a form of non-homothetic import demand that consistent with the findings in Kugler and Verhoogen (2011) and Blaum et al. (2013).

[14] While Antràs et al. (2014) consider a model of importing in the spirit of Eaton and Kortum (2002) instead of a variety-type model, our results apply to their framework once the Frechet parameter θ is substituted for the CES parameter $\varepsilon - 1$.

a common prediction for the relation between the firm's unit cost and its domestic expenditure share. To see this, consider the firm's import demand problem conditional on the sourcing strategy, i.e. the intensive margin of trade problem. The firm's unit cost is characterized by

$$UC\left(\Sigma_i;\varphi_i,\beta_i,[q_{ci}],[p_{ci}],h_i\right) \equiv \min_{z,l}\left\{wl+p_D z_D+\sum_{c\in\Sigma_i}p_{ci}z_c \text{ s.t. } \varphi_i l^{1-\gamma}x^{\gamma}\geq 1\right\}, \tag{4}$$

subject to (2)-(3). For notational simplicity we refer to the unit cost as UC_i. Standard calculations imply that there is an *import price index* given by

$$A\left(\Sigma_i,[q_{ci}],[p_{ci}],h_i\right)\equiv\frac{m_I}{x_I}, \tag{5}$$

where m_I denotes import spending and x_I is the foreign import bundle defined in (3). Importantly, conditional on Σ_i, this price-index is exogenous from the point of view of the firm and we henceforth denote it by $A_i\left(\Sigma_i\right)$. Next, given the CES production structure between domestic and foreign inputs, the price index for intermediate inputs is given by

$$Q_i\left(\Sigma_i\right)=\left(\beta_i^{\varepsilon}\left(p_D/q_D\right)^{1-\varepsilon}+\left(1-\beta_i\right)^{\varepsilon}A_i\left(\Sigma_i\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}, \tag{6}$$

so that $x=m/Q_i(\Sigma_i)$ where m denotes total spending in materials. It follows that the firm's unit cost is given by[15]

$$UC_i=\frac{1}{\varphi_i}w^{1-\gamma}Q_i\left(\Sigma_i\right)^{\gamma}. \tag{7}$$

We see that input trade affects the unit cost through the price index for intermediate inputs.[16] This price index, however, depends on a number of unobserved parameters related to the trading environment, e.g. the prices and qualities of the foreign inputs. We use the fact that the unobserved price index $Q_i\left(\Sigma_i\right)$ is related to the observed expenditure share on domestic inputs via

$$s_{Di}\left(\Sigma_i\right) = \frac{p_D z_{Di}}{Q_i\left(\Sigma_i\right)x_i}=Q_i\left(\Sigma_i\right)^{\varepsilon-1}\beta_i^{\varepsilon}\left(\frac{q_D}{p_D}\right)^{\varepsilon-1}. \tag{8}$$

Substituting (8) into (7) yields

$$UC_i=\frac{1}{\tilde{\varphi}_i}\times\left(s_{Di}\right)^{\frac{\gamma}{\varepsilon-1}}\times\left(\frac{p_D}{q_D}\right)^{\gamma}w^{1-\gamma}, \tag{9}$$

where $\tilde{\varphi}_i\equiv\varphi_i\beta_i^{\frac{\varepsilon\gamma}{\varepsilon-1}}$. (9) is a sufficiency result: conditional on the firm's domestic expenditure share s_{Di}, no aspects of the import environment, including the sourcing strategy Σ_i, the prices p_{ci}, the qualities q_{ci} or the technology h_i, affect the firm's unit cost. With (9) at hand, we can derive what we call the *producer gains from input trade*.

[15]With a slight abuse of notation we suppress the constant $\left(\frac{1}{1-\gamma}\right)^{1-\gamma}\left(\frac{1}{\gamma}\right)^{\gamma}$ in the definition of (7).

[16]In particular, the firm has an incentive to enlarge Σ_i to lower $Q_i(\Sigma_i)$ and thus lower UC_i.

Proposition 1. *Consider the model above. We define the producer gains from input trade as the reduction in unit cost relative to autarky holding prices fixed, i.e.* $G_i \equiv ln\left(\frac{UC_i^{Aut}}{UC_i}\right)\Big|_{p_D,w}$. *Then*

$$G_i = \frac{\gamma}{1-\varepsilon} ln\left(s_{Di}\right). \tag{10}$$

Proof. Follows directly from (9) and the fact that the domestic share in autarky is by construction equal to unity. □

Proposition 1 shows that the effect of participating in international input markets on the firm's unit cost is observable given values of the elasticities γ and ε.[17] More precisely, the increase in production costs that firm i would experience if it (and only it) was excluded from international markets can be recovered from the firm's domestic expenditure share. Intuitively, input trade benefits the firm by reducing the price index of intermediate inputs Q_i. Conditional on an import demand system, we can invert the change in this price index from the change in the domestic expenditure share - see (8). Because in general p_D and w may change when the economy moves to input autarky, Proposition 1 is a partial equilibrium result. While we explicitly allow for general equilibrium effects in Section 3.2 below, we note that (10) identifies the *distribution* of the producer gains from input trade across firms.

The sufficiency result in Proposition 1 allows us to measure the change in the unit cost without specifying several components of the theory. As equations (5)-(7) show, the firm's unit cost depends on the import environment parameters $[p_{ci}, q_{ci}, h_i, \beta_i]$. The domestic expenditure share conveniently encapsulates all the information from these parameters that is relevant for the unit cost - see (9). Instead, the standard approach in the literature consists of estimating these parameters in the context of a fully-specified model of importing. This approach requires researchers to specify the entire import environment, including the structure of output markets, and to solve for firms' optimal sourcing strategies which, as discussed below, can be a non-trivial problem. Hence Proposition 1 is useful because it allows us to bypass the challenges of firm-based models of importing and quantify the producer gains in a wide class of models. In this sense, Proposition 1 is akin to a firm-level analogue of Arkolakis et al. (2012). In the same vain as consumers gain purchasing power by sourcing cheaper or complementary products abroad, firms can lower the effective price of material services by tapping into foreign input markets. The extent to which such prices are lower in the observed trade equilibrium is directly given by firms' domestic share, which is observable in the firm-level data.

Finally, we note that (10) can be used for a structural evaluation of observed changes in trade policy, as long as data on firms' domestic shares before and after the change is available. In particular,

[17]In Section 9.1 in an Online Appendix, we consider two additional generalizations. We derive a local version of (9) for the case where domestic and foreign inputs are not necessarily combined in a CES fashion, and we derive Proposition 1 for the case where the output elasticity of material inputs is not constant. There we discuss what additional information is required to perform counterfactual analysis in the latter case.

the distribution of changes in firms' unit costs is simply given by

$$ln\left(\frac{UC_i^{post}}{UC_i^{pre}}\right) = \frac{\gamma}{1-\varepsilon} ln\left(\frac{s_{Di}^{pre}}{s_{Di}^{post}}\right). \tag{11}$$

Consider for concreteness the productivity effects of an episode of trade liberalization (e.g. Chile in 1980s (Pavcnik, 2002), Indonesia in the late 1980s and early 1990s (Amiti and Konings, 2007) or India in the 1990s (De Loecker et al., 2012)). One can use (11) to estimate the direct effect of improved access to international inputs on firms' unit costs.[18] In particular, (11) contains both the exogenous change in foreign prices due to lower trade barriers and tariffs as well as the endogenous change resulting from adjustments in the sourcing pattern.[19]

An Example with Fixed Costs To compare our approach with the existing literature, consider the following example of a static economy where international sourcing is limited by the presence of fixed costs. In particular, suppose that sourcing an input from country c entails paying a fixed cost f_{ci}. We denominate fixed costs in units of labor. The profit maximization problem is then given by

$$\pi_i \equiv \max_{\Sigma, y}\left\{(p(y) - UC_i)\, y - w \sum_{c\in\Sigma} f_{ci}\right\}, \tag{12}$$

where the unit cost is

$$UC_i = \frac{1}{\varphi_i} w^{1-\gamma}\left[\beta^{\varepsilon}\left(p_D/q_D\right)^{1-\varepsilon} + (1-\beta)^{\varepsilon} A\left(\Sigma_i\right)^{1-\varepsilon}\right]^{\frac{\gamma}{1-\varepsilon}}, \tag{13}$$

and $p(y)$ denotes the demand function. Firms choose their size y and set of imported varieties Σ to maximize profits.

Albeit conceptually easy, solving the firm's profit maximization problem presents us with two practical challenges. First, one has to specify the entire set of structural primitives of the model, including the distribution of prices, qualities and fixed costs across countries, the local output demand function and the full market structure on output markets. Second, even after making such assumptions, the choice of the optimal sourcing strategy can be computationally difficult (see Blaum et al. (2013) and Antràs et al. (2014) for a detailed discussion).[20] The reason is the interdependence between entry decisions in different import markets. A particular variety is imported whenever the

[18] This methodology is of course subject to the caveat that the domestic shares may have changed for reasons unrelated to the policy under study. This concern, however, is equally relevant for any empirical analysis trying to infer the causal effect of trade liberalization.

[19] Of course, opening up to trade might induce firms' to engage in other productivity enhancing activities like R&D, in which case innate efficiency φ would also increase. Such increases in complementary investments are not encapsulated in (11), which only measures the static gains from trade holding efficiency fixed. To disentangle the dynamic from the static gains from trade, more structure and data is required - see for example Eslava et al. (2014).

[20] Note that the extensive margin problem cannot by sidestepped even in cases where the researcher is interested in computing unit cost changes between two states where the sourcing sets are known - e.g. the current trade equilibrium and autarky. The reason is that, to evaluate (13), one needs to know the full set of structural parameters. While these parameters can be in principle estimated, such estimation would typically entail solving for the optimal sourcing set in (12).

reduction in the average production cost outweighs the incurred fixed costs. When imported varieties are imperfect substitutes, the cost reduction associated with entering a particular foreign market depends on the quantities sourced from all other markets - see (5). If foreign inputs differ in both quality and fixed costs, the profit maximization problem in (12) is in general non-convex and the choice of the optimal sourcing set requires evaluating all possible sourcing strategies, entailing substantial computational burden. This interdependence of entry decisions makes the extensive margin of imports different from that of exports, where the sourcing strategy can typically be solved market by market, and has the implication that more productive firms need not source their inputs from more countries, unless more restrictions are imposed.[21]

The benefit of Proposition 1 is that we can bypass both of these challenges. With micro-data on domestic expenditure shares and the two structural parameters γ and ε, we can directly measure the endogenous reduction in unit cost arising from input trade at the firm-level. Not only is the calculation straight-forward but it does not rely on any assumptions made to make the solution to (12) feasible.

3.2 The Consumer Gains from Input Trade

In this section, we embed the model of firm behavior of Section 3.1 in a macroeconomic environment and study the aggregate effects of input trade. We focus on the change in consumer prices, i.e. how much more would domestic consumers pay for the locally produced goods if firms were not allowed to source their inputs from abroad. To isolate the effect of input trade, we abstract from trade in final goods. That is, we consider an environment where domestic consumers solely benefit from trade openness indirectly through firms' cost-reductions. The micro result in Proposition 1 above is crucial as it allows us to measure such firm-level unit cost reductions in the data. To aggregate these producer gains, the macroeconomic environment needs to take a stand on two aspects: (i) the nature of input-output linkages across firms and (ii) the degree of pass-through, which depends on consumers' demand system and the market structure on output across producers. While the former determines the effect of trade on the price of domestic inputs p_D, the latter determines how much of the trade-induced cost reductions actually benefit consumers.

We consider the following multi-sector CES monopolistic competition environment, which is for example also used in Caliendo and Parro (2015).[22] There are S sectors of production, each comprised of a measure N_s of firms which we treat as fixed. There is a unit measure of consumers who supply

[21] Note, however, that this does not imply that general results concerning the extensive margin cannot be derived. If for example fixed costs are not firm-specific, more productive firms import and more productive firms adopt a sourcing strategy that leads to lower unit costs. However, this does *not* imply that more productive firms source *more* varieties or products. Antràs et al. (2014) show in a related model that sourcing can be shown to be hierarchical as long as the profit function function has increasing differences.

[22] While this framework shuts down any issues of imperfect pass-through, we note that data on firm-specific prices would be necessary to discipline the extent of pass-through in a more general framework. See Dhingra and Morrow (2012) and Fabinger and Weyl (2012) for recent contributions on incomplete pass-through in international trade.

L units of labor inelastically and whose preferences are given by

$$U = \prod_{s=1}^{S} C_s^{\alpha_s} \tag{14}$$

$$C_s = \left(\int_0^{N_s} c_{is}^{\frac{\sigma_s - 1}{\sigma_s}} di \right)^{\frac{\sigma_s}{\sigma_s - 1}}. \tag{15}$$

Firm i in sector $s = 1, ..., S-1$ produces according to the production technology given by (1)-(3) in Section 3.1 above, where the structural parameters ε and γ are allowed to be sector-specific. As before, we do not assume any particular mechanism of how the extensive margin of trade is determined nor impose any restrictions on $[p_{ci}, q_{ci}, h_i, \beta_i]$. That is, the distribution of prices and qualities across countries and the aggregator of foreign inputs can take any form. Additionally, these parameters can vary across firms in any way. We assume sector S to be comprised of firms that do not trade inputs and refer to it as the non-manufacturing sector.[23]

We assume the following structure of roundabout production. Firms use a sector-specific domestic input that is produced using the output of all other firms in the economy according to

$$z_{Ds} = \prod_{j=1}^{S} Y_j^{\zeta_j^s} \text{ where } Y_j = \left(\int_0^{N_j} y_{\nu js}^{\frac{\sigma_j - 1}{\sigma_j}} d\nu \right)^{\frac{\sigma_j}{\sigma_j - 1}}, \tag{16}$$

where z_{Ds} denotes the bundle of domestic inputs, ζ_j^s is a matrix of input-output linkages with $\zeta_j^s \in [0,1]$ for all s and j and $\sum_{j=1}^{S} \zeta_j^s = 1$ for all s, and $y_{\nu js}$ is the output of firm ν in sector j demanded by a firm in sector s. In this setting, the price of the domestic input p_{Ds} is endogenous and affected by trade policy. Hence, domestic firms are affected by trade policy via their purchases of intermediate inputs from importers.

Building on our result from Section 3.1, we now show that the consumer price index associated with (14)-(15) can be expressed in terms of observables. Given the CES demand and monopolistic competition structure, the consumer price index for sector s is given by

$$P_s = \mu_s \left(\int_{i=0}^{N_s} UC_i^{1-\sigma_s} di \right)^{\frac{1}{1-\sigma_s}} = \mu_s \left(\frac{p_{Ds}}{q_{Ds}} \right)^{\gamma_s} \times \left(\int_0^{N_s} \left(\frac{1}{\tilde{\varphi}_i} \left(s_{Di} \right)^{\gamma_s / (\varepsilon_s - 1)} \right)^{1-\sigma_s} di \right)^{\frac{1}{1-\sigma_s}}, \tag{17}$$

where $\mu_s \equiv \sigma_s / (\sigma_s - 1)$ is the mark-up in sector s and we treat labor as the numeraire in this economy. The second equality follows from (9) above which allows us to express firms' unit costs in terms of their domestic expenditure shares (s_{Di}) and efficiency ($\tilde{\varphi}_i$). (17) shows that, holding domestic input prices fixed, the effect of input trade on consumers' purchasing power is an efficiency-

[23] We introduce this sector for empirical reasons. In the next section we consider an application to France where we do not have data on firm-level imports outside of the manufacturing sector. To make aggregate statements about input trade, we take the non-manufacturing sector into account. See Section 4 for details.

weighted average of the firm-level gains. While firm efficiency $\tilde{\varphi}_i$ is not observed, it can be recovered (up so scale) from data on value added and domestic spending as

$$va_i \propto \left(\tilde{\varphi}_i \left(s_{Di}\right)^{\gamma_s/(1-\varepsilon_s)}\right)^{\sigma_s - 1}. \tag{18}$$

Next, we have to take into account (a) that consumers consume a basket of S goods so that $P \propto \prod_s P_s^{\alpha_s}$ and (b) that domestic input prices are affected via input-output linkages so that $p_{Ds} \propto \prod_{j=1}^{S} P_j^{\zeta_j^s}$. We thus arrive at a simple expression for the aggregate gains from input trade in terms of observables.

Proposition 2. *Let P be the domestic price index in the trade equilibrium and P^{Aut} the price index in autarky. We define the consumer price gains from input trade as the reduction in the domestic price index relative to autarky, i.e. $G \equiv ln\left(P^{Aut}/P\right)$. Then,*

$$G = \alpha' \left(\Gamma \left(\mathcal{I} - \Xi \times \Gamma\right)^{-1} \Xi + \mathcal{I}\right) \times \Lambda, \tag{19}$$

where

$$\Lambda_s = \frac{1}{1-\sigma_s} ln \left(\int_{i=0}^{N_s} \omega_i s_{Di}^{\frac{\gamma_s}{1-\varepsilon_s}(1-\sigma_s)} di\right) \tag{20}$$

is observable from the micro-data as $\omega_i = \frac{va_i}{\int_i^{N_s} va_i di}$ denotes firm i's share in value added, $\Lambda = [\Lambda_1, \Lambda_2, ..., \Lambda_S]'$, $\Xi = \left[\zeta_j^s\right]$ is the $S \times S$ matrix of production interlinkages, α is the $S \times 1$ vector of demand coefficients, $\mathcal{I}$ is an identity matrix and $\Gamma = diag\left(\gamma\right)$, where γ is the $S \times 1$ vector of input intensities.

Proof. See Section 8.2 in the Appendix. □

Proposition 2 is the main result of the paper. It shows that the information contained in the micro-data on domestic spending and value added is sufficient to characterize the consumer price gains from input trade relative to autarky in the class of models considered in this section. Thus, the consumer price gains can essentially be "read off" the micro-data given the parameters for consumer demand and production. Information about firms' import environment or firms' endogenous choice of their extensive margin of importing is not required.

To understand Proposition 2, it is instructive to consider the case of a single sector economy. Expression (19) then becomes

$$G = \gamma\Psi + \Lambda = \frac{\Lambda}{1-\gamma}, \tag{21}$$

that is, the consumer price gains are simply given by the aggregate unit cost reduction Λ, inflated by $1/\left(1-\gamma\right)$ to capture the presence of roundabout production.

Proposition 2 is a useful organizing tool for the existing models of importing. It shows that in terms of their normative implications, existing models differ only in their implied distribution of domestic shares and value added which translate into different unit costs reductions Λ. Consider first the aggregative models of importing where firms' domestic expenditure shares are equalized -

see Eaton et al. (2011), Caliendo and Parro (2015) and Costinot and Rodriguez-Clare (2014). In these models the aggregate unit cost reductions are given by

$$\Lambda_s^{Agg} = \frac{\gamma_s}{1-\varepsilon_s} ln\left(s_{Ds}^{Agg}\right) = \frac{\gamma_s}{1-\varepsilon_s} ln\left(\int_{i=0}^{N_s} \omega_i s_{Di} di\right), \tag{22}$$

where s_{Ds}^{Agg} is the aggregate domestic expenditure share in sector s.[24] While these frameworks have the benefit of only requiring aggregate data, Figure 1 in Section 2 shows that their implication of equalized domestic shares is rejected in the micro-data and Proposition 2 shows that such deviation has aggregate consequences. In particular, (20) and (22) show that the bias from measuring the aggregate unit cost reductions in sector s through the lens of an aggregative model is given by

$$Bias_s \equiv \Lambda_s^{Agg} - \Lambda_s = \frac{\gamma_s}{\varepsilon_s - 1} \times ln\left[\frac{\left(\int_{i=0}^{N_s} \omega_i s_{Di}^{\chi_s} di\right)^{1/\chi_s}}{\int_{i=0}^{N_s} \omega_i s_{Di} di}\right], \tag{23}$$

where $\chi_s = \frac{\gamma_s(\sigma_s-1)}{\varepsilon_s-1}$. Heterogeneity in import shares induces a bias in the estimates of the gains from trade of aggregative models, as long as $\chi_s \neq 1$. The *magnitude* of the bias depends on the underlying dispersion in domestic shares and their correlation with firm size - we quantify it in our empirical application below. The *sign* of the bias, however, depends only on parameters and not on the underlying micro-data. In particular the generalized mean inequality directly implies that

$$Bias_s > 0 \text{ if and only if } \chi_s = \frac{\gamma_s(\sigma_s - 1)}{\varepsilon_s - 1} > 1. \tag{24}$$

Hence, aggregative trade models are upward biased, i.e. predict larger gains from trade, whenever demand is elastic (σ is high), there are strong complementarities between foreign and domestic inputs (ε is low) or materials are important (γ is high). The key intuition why this is the case is the following. Because the current trade equilibrium is observed in the data, quantifying the gains from trade boils down to predicting consumer prices in the counterfactual autarky allocation. Such prices are fully determined by producers' physical productivity $\tilde{\varphi}_i^{\sigma-1}$. As the latter is unobserved, they are inferred from (18). Crucially, given the data on value added, (18) shows that physical productivity is exactly proportional to $s_{D,i}^{\chi}$. In the same vain as dispersion in unit costs is valued by consumers whenever demand is elastic, dispersion in the inferred productivity dispersion is valued as long as $\chi \geq 1$. This is exactly condition (24). Hence, the counterfactual autarky allocation implied by using the aggregative deflator Λ_s^{Agg} is *worse* than in actuality, making the gains from trade *upward* biased.

On the other side of the spectrum are firm-based models of importing. These models generate heterogeneity in firms' import shares via sorting into different import markets and also induce a joint distribution between import intensity and firm size. Gopinath and Neiman (2014), Amiti et al. (2012) and Ramanarayanan (2014) for example assume that firms differ only in their efficiency and

[24] Note that because of Cobb-Douglas production firm value added is proportional to material spending, so that s_D^{Agg} is indeed equal to the aggregate share of material spending allocated towards domestic producers.

thus generate a perfect negative correlation between domestic shares and value added, conditional on importing and imply that all importers are larger than domestic firms. By assigning the largest unit cost reductions to the largest firms, this tends to magnify the aggregate gains from trade. Figure 2 in Section 2, however, shows that the correlation between firm-size and domestic spending is negative but far from perfect and that many importers are small. Antràs et al. (2014) and Halpern et al. (2011) allow for firm heterogeneity in productivity and fixed costs and thus generate a non-trivial distribution of value added and domestic spending. Whether or not the model-implied distribution is *quantitatively* consistent with the micro-data and hence informative for normative questions is then a question about the particular calibration.

Finally, we note that Proposition 2 can be generalized to study counterfactuals beyond input autarky. In particular, consider a policy that changes firms' domestic expenditure shares from $[s_{Di}]$ to $[s'_{Di}]$. The effect of such policy on consumer prices is given by the same expression as in Proposition 2, except that Λ_s is now given by

$$\Lambda_s^* \quad = \quad \frac{1}{1-\sigma_s} ln \left(\int_{i=0}^{N_s} \omega_i \left(s_{Di}/s_{Di}^{'} \right)^{\frac{\gamma_s}{\varepsilon_s - 1}(\sigma_s - 1)} di \right). \tag{25}$$

We see that the consumer gains from *observed* policy experiments can be easily computed as long as data on domestic shares before and after the change is available. Thus, the main takeaway from this section is that the observable joint distribution of value added and domestic shares contains crucial information about the aggregate consequences of changes in the import environment. Finally, we note that (25) is also useful for unobserved counterfactuals. In particular, it stresses that all models (within our class) will have the exact same normative counterfactual implications as long as they generate the same counterfactual distribution of domestic shares. Hence, the microstructure of the underlying import environment determines whether or not different models are succesful in predicting firms' domestic shares - conditional on such predictions, the implied consumer gains are same.

4 Quantifying the Producer and Consumer Gains

We now take the framework laid out above to data on French firms to quantify the gains from input trade both at the firm and aggregate level. Implementing Propositions 1 and 2 empirically requires a set of parameters. We deal with their estimation in Section 4.1 and compute the producer and consumer gains in Section 4.2.

4.1 Estimation of Parameters

Our approach relies on both micro and aggregate data. We use the micro-data to estimate the production function parameters, i.e. the material elasticities $[\gamma_s]$ and the elasticities of substitution $[\varepsilon_s]$, as well as the sector-specific demand elasticities $[\sigma_s]$. We identify the input-output structure on the production side $\left[\zeta_j^s\right]$ and the aggregate demand parameters $[\alpha_s]$ from the input-output tables. This allows us also to account for the non-manufacturing sector and doing so is quantitatively important.

Data. The main source of information we use is a firm-level dataset from France. A detailed description of how the data is constructed is contained in Section 8.3 of the Appendix. Because we are interested in trade in inputs, we restrict the analysis to manufacturing firms. We observe import flows for every manufacturing firm in France from the official custom files. Manufacturing firms account for 30% of the population of French importing firms and 53% of total import value in 2004. Import flows are classified at the country-product level, where products are measured at the 8-digit (NC8) level of aggregation. Using unique firm identifiers we can match this dataset to fiscal files, which contain detailed information on firm characteristics. The final sample consists of an unbalanced panel of roughly 170,000 firms which are active between 2002 and 2006, 38,000 of which are importers. Table 9 in the Appendix contains some basic descriptive statistics. We augment this data with two additional data sources. First, we employ data on input-output linkages in France from the STAN database of the OECD. Second, we use global trade flows from the UN Comtrade Database to measure aggregate export supplies which we use to construct an instrument to estimate the elasticity of substitution ε below.

Identification of α, ζ and σ. We compute the demand parameters α_s and the matrix of input-output linkages $\left[\zeta_j^s\right]$ using data from the French input-output tables on the distribution of firms' intermediate spending and consumers' expenditure by sector.[25] Sectors are classified at the 2-digit level. Letting Z_j^s denote total spending on intermediate goods from sector j by firms in sector s and E_s total consumption spending in sector s, our theory implies

$$\zeta_j^s = \frac{Z_j^s}{\sum_{j=1}^S Z_j^s} \text{ and } \alpha_s = \frac{E_s}{\sum_{j=1}^S E_j}. \tag{26}$$

We aggregate all non-manufacturing sectors into one residual sector and construct its consumption share α_{NM} and input-output matrix ζ_j^{NM} directly from the Input-Output Tables.

Our dataset does not have information on firm-specific prices but only revenues. We therefore use industry-specific average mark-ups to get the demand elasticities $[\sigma_s]$. In the model, mark-ups in sector s are equal to $\sigma_s/(\sigma_s - 1)$. As in Oberfield and Raval (2014), we identify mark-ups from firms' ratios of revenues to total costs. We include the costs of capital and calculate total costs as the sum of material spending, payments to labor and the costs of capital, which we measure as Rk_i, where k_i denotes firm i's capital stock. We take $R = 0.20$ as our benchmark and compute averages at the sector level to identify σ_s.

Table 1 below contains the results. Column three reports the consumption share α_s for each of the 20 sectors in France. The non-manufacturing sectors are important as they account for a large share of the budget of consumers. Column five reports the demand elasticities σ_s which, consistent

[25] See Section 9.2 in the Online Appendix for a detailed description of how we construct the input-output matrix.

with the literature, are estimated at around 3. For brevity, we report the input-output matrix ζ_j^s in Section 9.2 of the Online Appendix.

[Table 1 here]

Estimation of ε and γ. Of particular importance are the elasticities of substitution ε_s and the intermediate input shares γ_s, as they directly affect the producer gains. To understand our identification strategy, note that firm output can be written as[26]

$$y_{is} = \varphi_i s_{Di}^{-\frac{\gamma_s}{\varepsilon_s - 1}} k_i^{\phi_{ks}} l_i^{\phi_{ls}} m_i^{\gamma_s} \times B \tag{27}$$

where m_i is total material spending by firm i and B collects all general equilibrium variables, which are constant across firms. By expressing output in terms of *spending* in materials instead of quantities, (27) shows that we can estimate ε_s by treating the domestic share as an additional input in a production function estimation exercise.[27] We also see that the domestic share is akin to a productivity shifter.

Because we do not observe firm-specific prices, we rely on the demand structure assumed in Section 3.2 and express (27) in terms of firm revenue

$$ln\left(Rev_{is}\right) = \delta + \tilde{\phi}_{ks} ln\left(k_i\right) + \tilde{\phi}_{ls} ln\left(l_i\right) + \tilde{\gamma}_s ln\left(m_i\right) + ln\left(\omega_i\right), \tag{28}$$

where the productivity residual ω_i is given by

$$ln\left(\omega_i\right) = \frac{1}{1-\varepsilon_s}\tilde{\gamma}_s ln\left(s_{Di}\right) + \frac{\sigma_s - 1}{\sigma_s} ln\left(\varphi_i\right) \tag{29}$$

and $\tilde{\gamma_s} = \frac{\sigma_s - 1}{\sigma_s}\gamma_s$ and $\tilde{\phi_{ks}}$ and $\tilde{\phi_{ls}}$ are defined accordingly.

We use equations (28) and (29) to estimate ε_s and γ_s following three complementary approaches. The first two methods estimate (28) and (29) separately. They only differ in the way in which the output elasticities $[\phi_{ks}, \phi_{ls}, \gamma_s]$ are obtained from (28). We consider both a factor shares approach and a proxy method. We then use such elasticities to construct productivity residuals $ln\left(\omega_i\right)$ and use (29) together with data on domestic shares to estimate ε_s. To increase the power of the estimation, we pool firms from all sectors together and estimate a single ε. The third approach treats the domestic share as an additional input and estimates all parameters in (28)-(29) simultaneously. In this approach we allow for sector-specific ε_s.

Consider first the approach based on observed factor shares, which is a simple and easy-to-

[26]In this section, we augment the production function considered in Section 3 to include capital, i.e. $y_{is} = \varphi_i k^{\phi_{ks}} l^{\phi_{ls}} x^{\gamma_s}$, where ϕ_k and ϕ_l denote the capital and labor output elasticities.

[27]Note that it is common in the literature to rely on material spending as a measure of input use as quantities are rarely observed. (27) shows that in this case the domestic expenditure share turns out to be the appropriate deflator for material spending. Not controlling for the domestic shares therefore results in biased estimates (De Loecker and Goldberg, 2013).

implement benchmark. The Cobb-Douglas production structure implies that

$$\tilde{\gamma_s} = \frac{m_i}{p_i y_i}, \tag{30}$$

so that we can measure $\tilde{\gamma_s}$ as the average share of material spending across firms. We can similarly measure $\tilde{\phi}_{ks}$ and $\tilde{\phi}_{ls}$, and hence construct the productivity residuals $ln\left(\omega_i\right)$ from (28) up to an inconsequential constant. In a second step, we then use the estimated $\tilde{\gamma_s}$, the productivity residuals and data on domestic shares to estimate equation (29).

Clearly, we cannot estimate (29) via OLS as the required orthogonality restriction fails: s_D is not orthogonal to innate productivity φ under most reasonable models of import behavior. In particular, more productive firms are likely to sort into more and different sourcing countries and this variation in the extensive margin of trade will induce variation in firm-specific price indices and hence domestic shares. Hence, we estimate ε from (29) using an instrumental variable strategy. In particular, we follow Hummels et al. (2011) and instrument s_D with shocks to world export supplies, which we construct from the Comtrade data. More precisely, we construct the instrument

$$z_{it} = \Delta ln\left(\sum_{ck} WES_{ckt} \times s_{cki}^{pre}\right), \tag{31}$$

where WES_{ckt} denotes the total exports for product k of county c in year t to the entire world excluding France, s_{cki}^{pre} is firm i's import share on product k of county c *prior* to our sample, and Δ denotes the change between year $t-1$ and year t. Hence, z_{it} can be viewed as a firm-specific index of shocks to the supply of the firm's input bundle. Movements in this index should induce variation in firms' domestic shares that are plausibly orthogonal to firm productivity. Intuitively, if we see China's exports in product k increasing in year t, French importers that used to source product k from China will be relatively more affected by this positive supply shock and should increase their import activities. Using this source of variation in import prices at the firm-level, we can identify the elasticity of substitution ε. We estimate (29) in first differences using (31) to instrument the domestic share according to the following specification

$$\Delta ln\left(\hat{\omega}_{ist}\right) = \delta_s + \delta_t + \frac{1}{1-\varepsilon} \times \Delta\tilde{\gamma}_s ln\left(s_{ist}^{D}\right) + x_{ist}'\zeta + u_{ist}, \tag{32}$$

where δ are sector and year fixed effects, x_{ist} are firm-level controls and $\Delta ln\left(\hat{\omega}_{ist}\right)$ and $\Delta\tilde{\gamma}_s ln\left(s_{Dist}\right)$ are the changes in firm residual productivity and domestic shares respectively, which are instrumented by (31). For our baseline results, we define products at the 6-digit level and consider all importing firms in our sample, taking their respective first year as an importer to calculate the pre-sample expenditure shares s_{cki}^{pre}. As stated above, to increase statistical power we estimate a unique ε from (32) by pooling firms from all sectors together.

As an alternative to the factor shares approach, we employ a proxy method from the production function estimation literature, akin to Levinsohn and Petrin (2012), to obtain the output coefficients in equation (28). We assume labor to be a dynamic input, which seems plausible for the French labor market, and estimate the obtained equation using GMM as in Wooldridge (2009) to arrive at estimates of the vector of coefficients $[\phi_{ks}, \phi_{ls}, \gamma_s]$. We experiment with the standard Cobb-Douglas specification, as well as a more flexible translog specification where we continue to assume a constant output elasticity for intermediate inputs but allow for second-order terms in capital and labor. The second step is as in our previous approach: we construct productivity residuals $ln\left(\omega_i\right)$ for each firm and estimate ε from (32) using the instrumental approach described above. Hence, if the production function estimation were to give us the same $[\phi_{ks}, \phi_{ls}, \gamma_s]$ as the factor shares approach, the implied estimate for ε would be numerically identical.

Our third method consists of estimating firms' production function with an integrated GMM approach. Instead of treating (28) and (29) as separate estimation equations, we estimate the firms' production function in a single step with four inputs and again follow Wooldridge (2009) to estimate the four parameters via GMM. We follow the literature in using lagged values of capital, labor and materials to proxy for φ, and two-years lagged values of intermediate inputs as an instrument for current intermediate inputs (the only static input). We use the trade instrument discussed above to account for the endogeneity of firms' domestic shares.

The results of the three estimation approaches for ε are reported in Table 2 and Figure 3 below. For brevity, we report the results concerning the other production function parameters in Section 9.3 in the Online Appendix. Table 2 contains the estimation results for ε using the factor shares approach and the proxy method based on Levinsohn and Petrin (2012) and Wooldridge (2009). For the latter procedure we report both the results based on the Cobb Douglas assumption and the more general translog specification. In the respective first column, we show the first stage relationship between changes in aggregate export supply z_{it} and firms' changes in domestic spending. Reassuringly, there is a negative relationship that is statistically significant, i.e. firms whose trading partners see an increase in their aggregate exports also reduce their domestic spending.[28] Turning to the results for ε, we see that the different procedures yield relatively similar results as the estimates of ε lie between 1.7 and 2.4.[29] In particular, the point estimate is essentially unchanged when we estimate the second stage equation only for importing firms. Note however that the standard error increases substantially as we lose a large amount of data by conditioning on import status.

[Table 2 here]

[Figure 3 here]

Figure 3 summarizes the results of our integrated GMM approach. Because we estimate firms' production function for each industry, this procedure gives us sector-specific estimates for ε. We depict both the point estimate and confidence intervals based on two standard deviations. While we lack precision in some industries, the point estimates are mostly in the same ballpark as the

[28] The reason why the first stage results for the 2-step GMM procedure are not numerically equivalent is that the estimated material elasticity is different. Recall that the independent variable is $\Delta\tilde{\gamma}_s ln\left(s^D_{ist}\right)$.

[29] In a related approach, Kasahara and Rodrigue (2008) find estimates in the range of 3.1 to 4.4 for Chilean data. However, they do not use an external instrument for firms' domestic shares. Our estimates are close to the ones in Antràs et al. (2014) who rely on cross-country variation.

pooled results from above.[30] Note that the source of variation is slightly different. While the results of Table 2 are based on (32), which is estimated in first differences, the one-step GMM approach treats firms' domestic shares as an explicit input and estimates the production function in levels. It is comforting to see that all these approaches yield consistent results. Conceptually, we prefer the identification strategy in first differences as we find the underlying exogeneity assumptions more plausible. Hence, for the quantitative analysis that follows we take the estimate stemming from the factor shares approach, i.e. $\varepsilon = 2.378$, as the benchmark.[31] While we lock in to this benchmark value for the remainder of the paper, we report confidence intervals for all quantitative results which take into account the sampling variation in this benchmark estimate. Note additionally that our choice of benchmark ε is conservative as far as the magnitude of the gains from trade is concerned, since the unit cost reductions are decreasing in ε. In Section 8.4 in the Appendix we also provide further robustness checks to our estimates of ε, which lead to similar conclusions.

4.2 Results

With the structural parameters at hand, we now quantify the gains from input trade in France. We proceed as in the theory. We follow Proposition 1 and use data on domestic expenditure shares to measure the producer gains from input trade, i.e. distribution of unit cost reduction across firms. We then augment this data with information on firm size and use Proposition 2 to measure the consumer price gains from input trade. There we also exploit our decomposition of the gains from trade in expression (23) to quantify the importance of using the micro-data by comparing our results to an analysis based on aggregate data.

Input Trade and Producer Gains. Given our estimates of ε and γ_s and the micro-data on firms' domestic shares, Proposition 1 states that the unit cost reductions from input trade are given by $\frac{\gamma}{1-\varepsilon} ln\left(s_D\right)$. We depict these producer gains in Figure 4 and summarize them in Table 3. We see that there is substantial dispersion in the gains from trade. While the median firm would see its unit cost increase by 10.6% if moved to autarky, firms above the 90th percentile of the distribution would experience losses of 62% or more. According to Proposition 1, any model within the class covered in Section 3.1 will arrive at exactly the same conclusions about the distribution of the gains from trade at the micro-level, as long as it matches Figure 4 and utilizes the same values for γ_s and ε.

[Figure 4 here]

[30] Halpern et al. (2011) use a related framework and estimate it on Hungarian micro-data. They derive a production function equation analog to (28)-(29), as well as an import demand equation. The main difference with our approach is that they obtain the parameters of their structural model, namely the trade elasticity (analog to ε) and the quality of foreign varieties, by *simultaneously* estimating the production function and import demand equations. Because both of these estimating equations are derived after solving for the extensive margin of trade, they only hold under rather restrictive assumption. In contrast, we identify ε not within the structural model but by using exogenous variation in input supplies. This allows to estimate ε without having to impose additional assumptions and without having to take a stance how the extensive margin of trade is determined. Halpern et al. (2011) find a much bigger elasticity of substitution between the domestic and foreign variety of 7.3.

[31] Another reason why the factor shares or the two-step GMM approaches may be preferable to the one-step GMM method is that in the latter there are sectors for which we cannot reject $\varepsilon_s < 1$, a feature that leads to the prediction that all firms in such sector ought to be importers.

[Table 3 here]

We can also use the micro-data to learn about firm characteristics that are correlated with the producer gains. In particular, consider the following diagnostic regressions:

$$\frac{\gamma_s}{1-\varepsilon} ln\left(s_{Dist}\right) = \delta_s + \delta_t + \mu x_{ist} + u_{ist}, \tag{33}$$

where δ_s and δ_t are industry and time fixed effects and x_{ist} are different firm characteristics. To interpret μ, recall from (8) that the observed domestic shares can reflect firm-variation in exogenous "import capabilities" (such as lower prices $[q_{ci}/p_{ci}]$ or an import bias β_i) and firms' endogenous sourcing strategies Σ_i. The results are contained in Table 4 and are intuitive. Bigger firms, as measured by either value added or employment, see higher gains. Being an exporter or a member of an international group is associated with a reduction in the unit cost of 8.5% and 14.8%, respectively. When we restrict the analysis to the sample of importers, the positive relation between firm size and the producer gains becomes substantially weaker. This is consistent with the pattern documented in Section 2 above which showed a mild correlation between import intensity and value added for importers. Next, we consider the role of the firm's sourcing strategy, which we measure by the average number of countries that the firm sources its products from. According to the theory of Section 3, firms source their inputs internationally to reduce their unit cost. Consistent with the theory, we find a strong positive relation between firms' extensive margin of importing and the producer gains. Note that the importance of other firm characteristics is diminished once the number of varieties is controlled for.[32]

[Table 4 here]

Input Trade and the Consumer Price Gains. We now have all the ingredients required by Proposition 2 to quantify the effect of input trade on consumer prices. Using firm-level data on domestic expenditure shares and value added, together with the elasticities γ_s, σ_s and ε estimated above, we compute the sector-specific aggregate unit cost reductions Λ_s given in (20). With Λ_s and the demand and input-output parameters α_s, ζ_j^s estimated above, we calculate the consumer price gains by solving the linear system in (19). Table 5 contains the results. We find that French consumer prices in the manufacturing sector would be 27.5% higher if French producers were forced to source their inputs domestically. When the non-manufacturing sector is taken into account, the consumer price gains amount to 9%. The reason why the economy-wide gains are smaller than in the manufacturing sector lies in the non-manufacturing sector, which experiences only a 3% price reduction and accounts for 70% of consumers' budget - see Table 1. Table 5 also reports the consumer price gains predicted by a representative-firm approach, which requires data on domestic spending aggregated at the sector level only. This aggregative approach implies gains of 31.4% and 9.9% in

[32]In particular, firm size is substantially negatively correlated with import spending holding the number of imported varieties fixed. This is intuitive. If a small firm decides to source from a large number of sourcing countries and not be an exporter or a member of a foreign group, it is likely that this firm is a proficient importer, which manifests itself in a low share of domestic spending.

the manufacturing sector and the entire economy, respectively. Thus, ignoring the heterogeneity in firms' import behavior within sectors results in an over-estimation of the consumer price gains by 3.4 percentage points for the manufacturing sector and 1 percentage point for the entire economy.

[Table 5 here]

With the micro-data at hand, we can also quantify our confidence in these results. There are two sources of uncertainty. First, we base our analysis on a large but finite sample. Second, the structural parameters ε, γ_s and σ_s are estimated with error. To quantify the magnitude of this uncertainty, Table 5 also reports the 90-10 confidence intervals of the bootstrap distribution of the respective statistics in italics.[33] It is clearly seen that the uncertainty about the parameters and the sample itself introduces quite a bit of variation in the objects of interest. The consumer price gains in the manufacturing sector lie between 21% and 36% with 80% probability, and the gains for the entire economy lie between 7% and 12%.[34] It is also seen that an aggregative approach will almost surely lead to an over-estimation of the gains. A graphical depiction of this sampling uncertainty is contained in Figure 5. This figure also shows that the distribution of the bias has the majority of its mass on positive numbers.

[Table 6 here]

Table 6 reports the gains by sector and provides a decomposition to isolate the importance of production linkages across sectors. We first report the sectoral consumer price gains, P_s^{Aut}/P_s, which measure the change in the price of the output bundle of sector s. We then decompose these gains into the direct unit cost reduction from firms in sector s sourcing internationally, Λ_s, and the indirect gains stemming from firms in sector s buying inputs from local firms in other sectors who in turn engage in trade, p_{Ds}^{Aut}/p_{Ds}.[35] Note that these indirect gains vary across sectors due to heterogeneity in input-output linkages - sectors that rely on relatively open sectors more intensively benefit more from input trade as their upstream suppliers experience larger unit cost reductions. Table 6 shows a large degree of heterogeneity in these different measures across sectors. While e.g. the textile sector experiences a direct unit cost reduction of 31%, these are only 8% for producers in the wood industry. This heterogeneity stems mostly from the observed import behavior in that textile producers source a larger share of their inputs abroad.[36] This is also true for the total changes in the sector prices,

[33] We explain the details of the bootstrap procedure in Section 9.4 of the Appendix. A sketch of the procedure is as follows. For each bootstrap iteration, we construct a new sample of the French manufacturing sector by drawing firms from the empirical distribution with replacement. We then redo the analysis of Section 4.1 and obtain new estimates for the structural parameters. Finally, for each iteration, we recalculate the consumer price gains and the other statistics of interest.

[34] Given the large sample size, most of the uncertainty stems from the variation in the structural parameters and not from the re-sampling of firms. We display the bootstrap distribution of both the underlying parameters and the resulting sectoral productivity gains in Figure 8 in the Appendix.

[35] Note that p_{Ds} is the price-index of a unit of the bundle of domestic inputs z_{Ds} which is an aggregator of all the goods produced locally, see (16).

[36] Note that the share of intermediate inputs is almost the same for textile and wood producers (see Table 1) and that they - by construction - share the same elasticity of substitution ε.

P_s^{Aut}/P_s. While prices for textile products would be 56% higher if textile producers were not allowed to source their inputs from abroad, this effect is only 18% for metal products.

Table 6 also contains the sectoral consumer price gains that arise from a representative firm model. In line with the results of Table 5, in 12 of the 18 manufacturing sectors the gains based on aggregate data are higher than those based on micro-data. The reason for this sectoral asymmetry goes back to the condition in (24) which characterizes the sign of the bias as a function of parameters. It turns out that for most sectors the estimated σ_s and γ_s satisfy the condition for the bias from using aggregate data to be positive. Note also that the bias can be quite substantial. Consider for example the office and computing machinery sector, where an analysis based on an aggregative model would imply price changes of 37%, while the exact firm-based formula tells us that the direct unit cost reduction due to trade amounts to only 20%.

Importantly, there is a second source of bias that arises when using an aggregative approach which pertains to the "correct" elasticity of substitution ε. While we treat ε as a production function parameter and estimate it from micro-data, aggregative models often estimate ε from a gravity equation using aggregate trade flows. While there is a large literature concerning this particular parameter[37], most aggregative approaches find estimates that are larger than our preferred estimate of 2.37.[38] Costinot and Rodriguez-Clare (2014) for example use a trade elasticity of four as their benchmark value. As the implied gains from trade are decreasing in the elasticity of substitution, such choice would lead to substantially smaller gains from trade. In Section 9.5 of the Online Appendix, we redo the analysis of Tables 5 and 6 for a range of values of ε spanning the estimates from the aggregative literature. Moving to $\varepsilon = 4$, for example, tends to reduce the consumer price gains from trade of the aggregative approach by 50%. The bias in the estimates of the gains from trade arising from the use of an inappropriate elasticity of substitution can therefore be substantial.

Table 6 is also helpful to understand why the economy-wide gains are only a third of the gains in the manufacturing sector. The last row in Table 6 shows that aggregate non-manufacturing prices fall only by 3%. This relatively modest cost reduction in the non-manufacturing sector stems from two sources. Not only does this sector lack any *direct* cost reduction, as it does not engage in input trade by assumption, but it also relies little on the manufacturing sectors as its share of intermediate inputs γ^{NM} is only 40% (see Table 1). Hence, it sees its input prices decrease by only 7.5%.

Finally, to assess the importance of interconnections between sectors, we consider the case with no cross-industry input-output linkages where each sector uses only its own products as inputs.[39] In this case, we find a point estimate for the consumer prices gains from trade of

$$G = \sum_{s=1}^{S} \alpha_s \frac{\Lambda_s}{1-\gamma_s} = 12\%.$$

That is, shutting down input trade would reduce consumer prices by 12%. Compared to the actual gains of 9%, the economy without interlinkages over-estimates the aggregate gains by about a third.

[37] See e.g. Simonovska and Waugh (2013, 2014).

[38] Recall that our benchmark was chosen conservatively, as all other estimates of ε in Table 2 are smaller.

[39] In this case, the matrix of input-output linkages is given by $\zeta_j^s = 0$ for $j \neq s$ and $\zeta_j^j = 1$.

The reason is that the non-manufacturing sector is not only important for final consumers but also as a provider of inputs to other manufacturing firms.[40] And as the non-manufacturing sector is not a direct beneficiary of input trade, such linkages actually dampen the aggregate effect of input trade.

5 Input Trade and Welfare

In the previous sections, we considered the aggregate impact of input trade via its effect on consumer prices. By exploiting the sufficiency results in Propositions 1 and 2, we were able to measure such reduction in consumer prices without specifying a particular extensive margin mechanism. These consumer price gains, however, do not take into account the resources spent by firms to attain their equilibrium sourcing strategies. Measuring the full effect of input trade on welfare therefore requires taking a stand on how the extensive margin of trade is determined. To this end, we consider a model where foreign sourcing is limited by the presence of fixed costs and calibrate it to French micro-data. We use the calibrated model to measure the full welfare gains from input trade, as well as to assess the importance of the micro-data. For expositional simplicity, we consider a one-sector version of the model and leave the analysis with multiple sectors to the Appendix.[41]

5.1 A Model of Fixed Costs

We maintain the theoretical structure of Section 3 and assume additionally that engaging in international trade requires payment of fixed costs. As discussed above, computing firms' optimal sourcing strategies can be challenging when prices, qualities and fixed costs vary by country in an arbitrary way. To ensure tractability, we assume that the fixed cost of sourcing is constant across countries, i.e. $f_c = f$ for all c. In this case, the firm selects its sourcing countries based purely on price-adjusted quality and the sourcing strategy reduces from a set Σ to a scalar, a price-adjusted quality cutoff $\overline{q}$.[42] Furthermore, we assume there is a continuum of countries so that this cutoff can be characterized by a first order condition. Finally, there is also a fixed cost f_I to start importing. We impose the following functional form assumptions that ensure a parsimonious characterization of the firm's problem.

Assumption 1. *Consider the environment above and assume the following:*

1. *Foreign prices p_c and qualities q_c are related by $p_c = \alpha q_c^{\nu}$.*

2. *Country quality is Pareto distributed:*

$$G(q) = \Pr(q_c \leq q) = 1 - (q_{min}/q)^{\theta}.$$

[40] See Table 1 where we report the average intensity with which sectors are used as a factor of production in the rest of the economy.

[41] See Section 9.6 in the Online Appendix.

[42] More precisely, if country c with price-adjusted quality q_c/p_c is an element of Σ so are all countries c' with $q_{c'}/p_{c'} > q_c/p_c$. See Antràs et al. (2014) for a solution method that does not require this assumption.

3. Imported inputs are combined according to:

$$x_I = \left(\int_{c \in \Sigma} (q_c z_c)^{\frac{\rho-1}{\rho}} \, dc \right)^{\frac{\rho}{\rho-1}}. \tag{34}$$

4. The following parametric condition is satisfied: $(\rho - 1)(1 - \nu) < \theta$.

The isoelastic relation between prices and qualities allows us to work with qualities instead of price-adjusted qualities.[43] The Pareto distribution for country quality, together with the CES production function for the foreign bundle, imply that the firm's import price index, given by equation (5) above, takes a convenient power form:

$$A(\Sigma) = \left(\int_{c \in \Sigma} (p_c/q_c)^{1-\rho} \, dc \right)^{\frac{1}{1-\rho}} = z n^{-\eta} \equiv A(n), \tag{35}$$

where n is the share of countries the firm sources foreign inputs from[44] and z and η are "auxiliary" parameters which depend on the underlying parameters governing import prices (α, ν), the distribution of quality (q_{min}, θ) and the elasticity of substitution of foreign varieties ρ.[45] Thus, the underlying structure of the import environment matters for the firm's problem *only* through (z, η). In this way, knowledge of the deep parameters $(\alpha, q_{min}, \theta, \rho, \nu)$ is irrelevant for all aggregate outcomes as long as (z, η) are known.[46]

Under the above assumptions, the firm's profit maximization problem is given by:

$$\pi = \max_n \left\{ UC(n)^{1-\sigma} \times B - w \left(nf + f_I I (n > 0) \right) \right\}, \tag{36}$$

where the unit cost function is given by

$$UC(n) \equiv \frac{1}{\varphi} w^{1-\gamma} \left[(p_D/q_D)^{1-\varepsilon} + z^{1-\varepsilon} n^{-\eta(1-\varepsilon)} \right]^{\frac{\gamma}{1-\varepsilon}}, \tag{37}$$

and $B \equiv \left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} P^{\sigma-1} S$, with P and S denoting the consumer price index and aggregate spending, which is determined in general equilibrium. Conditional on importing, the optimal sourcing strategy

[43] When $\nu < 1$, we have that $q/p = \frac{1}{\alpha} q^{1-\nu}$ is increasing in quality and hence the firm selects countries based on their quality. Otherwise, one can reorder countries so that low-quality countries have the highest price-adjusted quality.

[44] Given the distribution of country quality, the firm's sourcing strategy can be equivalently described by the mass of countries sourced n or the quality cutoff $\overline{q}$.

[45] See Section 9.7 in the Appendix for the derivation of (35) and precise expressions for z and η. Note that the parametric condition in item 4 of Assumption 1 is required to compute the integral in (35).

[46] This however, does not mean that the degree of quality heterogeneity (θ) or the substitutability of inputs (ρ) do not have a role in shaping firms' import demand. For example, it can be easily shown that diversity and substitutability increase import productivity z (i.e. $\frac{\partial z}{\partial \theta} < 0$ and $\frac{\partial z}{\partial \rho} > 0$) and that diversity and substitutability are complements (i.e. $\frac{\partial^2 z(E[q], \theta, \rho)}{\partial \theta \partial \rho} < 0$) if and only if $(\rho - 1)(1 - \nu) > 1$. When $(\rho - 1)(1 - \nu) > 1$, the firm is effectively *risk loving* and values diversity. As only the best countries are selected, more variance in the unconditional distribution of quality increases the benefit of importing. Such gains from diversity however are only available if inputs are sufficiently substitutable, and so when ρ is higher firms can leverage such quality differences more. Thus, quality heterogeneity and technological substitutability do affect import demand. However, for given estimates of η and z, they do not change the researchers' conclusion on firms' import demand or the aggregate gains from trade.

n is given by the solution to a first order condition - see Appendix for details.[47] Intuitively, firms choose n by trading off the import-induced reduction in unit costs vs the payment of fixed costs.

To generate a rich distribution of firm size and import intensity as shown in Figure 2 above, we allow for two sources of firm heterogeneity: efficiency φ_i and fixed costs f_i.[48] As φ_i and the endogenous unit costs reduction through input trade are complements, there is a fixed-cost-specific efficiency cutoff $\overline{\varphi}(f)$ above which firms select into importing.[49] Conditional on importing, the optimal number of sourcing countries $n(\varphi_i, f_i)$ is increasing in φ_i and decreasing in f_i.[50]

We now impose equilibrium in the labor market and balanced trade between the domestic economy and the rest of the world. We assume that the supply of foreign inputs from country c is perfectly elastic at price p_c. We also assume that foreign firms demand the output of local firms with a CES demand structure given by (14)-(15) above, i.e. similar to that of domestic consumers and producers.[51][52] Letting z_i^{ROW} be the foreign demand for firm i's production, balanced trade requires that

$$\int_i p_i z_i^{ROW} = \int_i (1 - s_{D,i})\, m_i di, \tag{38}$$

where m_i denotes material spending of firm i, so that $(1 - s_{D,i})\, m_i$ is firm i's spending on imported varieties, and p_i is firm i's price.

Definition 1. *An equilibrium is a set of prices $w, [p_i]_i$, labor demands for production and fixed costs $\left[l_i, l_i^F\right]_i$, differentiated product quantities, consumption levels and foreign demands $\left[y_i, c_i, z_i^{ROW}\right]_i$, domestic and international input demands $[y_{vi}]_{vi}, [z_{ci}]_{ci}$ and sourcing strategies $[n_i]_i$ such that:*

1. *Firms maximize profits given by* (36)*,*

2. *Consumers maximize utility given by* (14) *and* (15) *subject to their budget constraint*

$$\int_i p_i c_i di = wL + \int_i \pi_i di, \tag{39}$$

3. *Trade is balanced* (38)*,*

[47] Proposition 5 in Section 9.7 of the Online Appendix gives the full characterization of the extensive margin.

[48] With a single source of heterogeneity, the model would generate a one-to-one assignment between firm size and import intensity - a feature that is counterfactual in the French data.

[49] We note that, given our functional form assumptions, the first order condition that characterizes n features an interior solution for any value of φ or f. To guarantee the existence of non-importers, it is necessary to have a fixed cost to start importing f_I. For simplicity, this fixed cost does not vary by firm.

[50] See Proposition 5 in Section 9.7 of the Online Appendix for the details.

[51] This simplifies the problem of local producers, who treat their different customers - consumers, local firms and foreign firms - alike. Hence, their profit maximization problem is given by (36)-(37).

[52] We view this situation as reflecting the local economy exporting its domestic inputs. Of course, in this single sector economy with roundabout production, domestic inputs and final goods are equivalent. This is no longer the case with multiple sectors, see Section 9.6 in the Online Appendix.

4. *Labor and good markets clear*

$$
\begin{aligned}
L &= \int_i \left(l_i + l_i^F\right) di \\
y_i &= c_i + z_i^{ROW} + \int y_{vi} dv.
\end{aligned}
$$

The following proposition characterizes full gain in consumer welfare relative to autarky.

Proposition 3. *Consider the above setup and let W and W^{Aut} denote the total welfare in the trade equilibrium and autarky respectively. Then*

$$
\frac{W}{W^{Aut}} = \frac{P^{Aut}}{P} \times \left(\frac{L - \int_i^N l_i^F di}{L}\right). \tag{40}
$$

Proof. See Section 9.6 in the Online Appendix for a proof for the multi-sector economy. □

Proposition 3 provides an intuitive decomposition of the welfare gains from input trade into two components. First, there is the reduction in consumer prices associated with input trade, which is captured by the term $P^{Aut}/P > 1$. This was the focus of Sections 3-4.2 above. Second, there is the resource loss associated with attaining the equilibrium sourcing strategies, captured by the second term in expression (40). When foreign sourcing is costly, it is as if the actual labor force was smaller.[53]

5.2 Calibration And Welfare Gains

We now have all the ingredients in place to estimate the full welfare effects of input trade along the lines of Proposition 3. A natural requirement for the structural model is to match the consumer price gains from trade which can be read off directly from the micro-data. It follows from Proposition 2 that this is ensured by targeting the joint distribution of value added and domestic expenditure shares. Our strategy is as follows. We use the estimates of ε, γ and σ from Section 4.1 above[54] and estimate η directly from the micro-data. Recall that η determines the price index of the import bundle - see (35)- and hence the demand for foreign varieties.[55] We identify this parameter from the cross-sectional relationship between firms' extensive margin of trade and their domestic shares. Next, we parametrize the distribution of firm efficiency and fixed costs as a joint log-normal distribution

$$
\begin{pmatrix} ln(\varphi) \\ ln(f) \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_\varphi \\ \mu_f \end{pmatrix}, \begin{pmatrix} \sigma_\varphi^2 & \rho\sigma_\varphi\sigma_f \\ \rho\sigma_\varphi\sigma_f & \sigma_f^2 \end{pmatrix}\right), \tag{41}
$$

[53]While we calculate $\int_i^N l_i^F di$ within a model of fixed costs, note also that (40) did not use any information on the extensive margin yet. Hence, if one thought that importers found their trading partners through a process of costly search, (40) would still hold. However, the environment to calculate $\int_i^N l_i^F di$ would naturally be different.

[54]Section 4.1 provides estimates of σ and γ by sector. In this Section, we use value-added weighted averages of these sectoral estimates.

[55]While the import-price index (35) also depends on z, it turns out that this parameter is not required for the calibration of the model.

where ρ controls the correlation between efficiency and fixed costs. We normalize μ_φ and calibrate the rest of the parameters in (41) to match salient features of the joint distribution of value added and domestic expenditure shares. Finally, we choose the fixed cost of being an importer f_I to match the share of importers in the French population.

Estimation of η. As seen from (35), η parametrizes the sensitivity of the import bundle price-index with respect to firms' extensive margin of trade (n). Because import prices map into import spending, as seen in (8) above, we rely on the cross-sectional relation between firms' domestic expenditure shares and the extensive margin to identify η. More specifically, (35) implies that the domestic expenditure share is given by

$$s_D(n) \quad = \quad \frac{1}{1+\left(\frac{p_D}{q_D}\frac{1}{z}n^{\eta}\right)^{\varepsilon-1}}. \tag{42}$$

(42) predicts a log-linear relation between n and the term $(1-s_D)/s_D$, with a slope given by η. At this point, we need to take a stand on what the counterpart of n is in the data. We focus on the number of countries the firm sources their products from, i.e. the number of foreign varieties.[56] We run the following regression:

$$ln\left(\frac{1-s_{Dist}}{s_{Dist}}\right) = \delta_s + \delta_t + \delta_{NK} + \eta\,(\varepsilon-1)\,ln\,(n_{ist}) + u_{ist}, \tag{43}$$

where n_{ist} denotes firm i's average number of countries per product sourced, δ_{NK} contains a set of fixed effects for the number of products sourced and δ_s and δ_t are year and sector fixed effect. Hence, we identify η from firms sourcing the same number of products from a different number of supplier countries. We measure products at the 8-digit level. Section 9.8 in the Online Appendix contains the results for a variety of specifications and also provides evidence that there is ample variation in the number of varieties French importers source. Our preferred specification yields a value of η of 0.382 that is precisely estimated.

Calibration. To calibrate the five remaining structural parameters $(\mu_f, \sigma_f, \sigma_\varphi, \rho, f_I)$ we target the following five moments: (i) the aggregate domestic share of the French manufacturing sector, (ii) the share of importing firms, (iii) the standard deviation of log domestic shares, (iv) the standard deviation of log value added and (v) the correlation between log value added and log domestic shares. While all parameters are calibrated jointly, the average level of fixed costs (μ_f) controls mostly the aggregate domestic share, the fixed cost of importing (f_I) is mostly identified from the share of importers and the dispersion in fixed costs (σ_f) and efficiency (σ_φ) from the dispersion in domestic shares and value added, respectively. Finally, the correlation between efficiency and fixed costs (ρ) is

[56] This notion of "varieties" is widely used in the literature - see e.g. Broda and Weinstein (2006) and Goldberg et al. (2010). Moreover, the choice of the number of products sourced may be determined to a large degree by technological considerations, while the demand for multiple supplier countries within a given product category may plausibly stem from love-for-variety effects, which are at the heart of the mechanism stressed by our theory. However, we note that the analysis that follows can be done under alternative interpretations of n.

disciplined by the correlation between value added and domestic spending.[57] Note that, by explicitly targeting the economy's aggregate domestic share, we can again compare our results to those of an aggregative approach where the moments from the micro-data are not used.

Table 7 contains the results of the calibration. As can be seen, the model can be calibrated to match the data accurately. Note in particular that the correlation between firm efficiency and fixed costs turns out to be positive. This is necessary to match the far from perfectly negative correlation between value added and domestic shares in the data. We show in Section 8.5 of the Appendix that the calibrated model is also able to match a number of non-targeted moments relatively well.[58]

[Table 7 here]

Welfare Gains from Input Trade. With the calibrated model at hand, we now compute the full welfare gains from input trade. Table 8 contains the results. The first column shows that the model-predicted consumer price gains are very close to those measured in the data.[59] This should not come as a surprise since such gains are a function of the joint distribution of value added and domestic shares which is a direct calibration target. Column two contains the main result of this section: the full welfare gains from input trade between the current trade equilibrium and autarky are predicted to be 17.54%. Thus, we see that only about half of the consumer price gains translate into welfare gains once the resources spent in fixed costs are taken into account. The reason is that, as seen in column three, a move to autarky would free up about 15% of the labor force, which counteracts the increase in prices.

[Table 8 here]

5.3 The Importance of Domestic Shares

In this subsection, we assess the value of the micro-data on domestic shares for estimating the gains from input trade. Note that the analysis of Section 4 to measure the producer and consumer price gains, as well as the calibration exercise of Section 5.2 to quantify the welfare gains, both

[57] We describe the details of our calibration strategy in Section 9.9 in the Online Appendix. Solving the model in principle entails finding a fixed point for firms' optimal sourcing strategies, as the general equilibrium variables depend on the domestic shares of all other firms, which in turn depend on the general equilibrium variables. The structure of the economy, however, suggests a calibration approach which bypasses this computation. Intuitively, we can calibrate a normalized version of both fixed costs and efficiency, where these are scaled by an appropriate transformation of the general equilibrium variables. Because the general equilibrium variables depend on firms' import behavior only via domestic shares, which are itself a calibration target, we can compute all prices *after* the calibration and thus back out the underlying true fixed costs. This not only reduces the computational burden substantially (as we do not have to solve for a fixed point), but also implies that the parameter z is not required for the calibration.

[58] Table 11 reports the average domestic expenditure share, both for importers and the full population, as well as features of the joint distribution of value added and domestic shares for importers. Figure 6 reports the marginal distributions of domestic shares and log value added for importers both in the model and in the French data. Figure 7 reports average domestic shares by value added quintile for the sample of importers.

[59] The number reported in Table 8 for the French data comes from applying the one-sector consumer price gains formula (21) to the Manufacturing sector pooling all sub-sectors together. Additionally, we leave the non-manufacturing sectors out of the analysis. For these reasons, the price-index gains reported in Table 8 do not coincide with those reported for the Manufacturing sector in Table 5 above.

relied directly on the domestic share data. When such data is not available, quantifying any type of gain from input trade - e.g. changes in producer unit costs, consumer prices or full consumer welfare - requires calibrating a structural model of importing. The role of the model is to generate a distribution of domestic expenditure shares. A natural question is: how well does a standard model of fixed costs do in predicting such shares?

To answer this question, we consider the following simple exercise. We calibrate the model of Section 5.1 without targeting the moments associated with domestic expenditure shares, i.e. the dispersion of domestic shares and their correlation with value added. Accordingly, we set the dispersion in fixed costs and their correlation with efficiency both to zero, i.e. $\sigma_f = \rho = 0$. Note that firm efficiency is the single source of heterogeneity in this model.

We report the results in Table 12 in the Appendix, where the baseline calibration is also displayed for comparison. The calibrated parameters in the model without data on domestic shares - henceforth NSD - imply aggregate gains from trade that are upward biased relative to those of the baseline. In particular, the NSD model over-predicts both the consumer price and the welfare gains from trade relative to the baseline. This is intuitive. By relying on efficiency as the single source of heterogeneity, the NSD model generates a perfectly negative correlation between firm efficiency and the domestic share. This means that firms with higher efficiency experience larger reductions in their unit costs, a feature that tends to make input trade more attractive. This manifests itself as a counterfactually strong negative correlation between value added and domestic shares. The resulting biases in the estimates of the gains from trade can be quantitatively meaningful, of about 14% for the consumer price gains and 24% for welfare.

6 Conclusion

Firms around the world routinely engage in input trade. By accessing cheaper, better or novel inputs from abroad, they reduce their costs of production. Quantifying the aggregate consequences of input trade, however, has been limited by an inherent difficulty. On the one hand, firms differ vastly in the intensity with which they participate in international markets. This feature of the data makes aggregative trade models inapplicable to measure the gains from input trade. On the other hand, fully-specified firm-based models of import behavior are challenging, at least as long as they are sufficiently rich to match salient features of the micro-data. Not only is firms' extensive margin of input trade non-trivial to characterize but, more importantly, such an approach requires researchers to fully specify (and estimate) the entire environment, including production technologies, the heterogeneity across firms and sourcing countries and the structure of output markets.

In this paper, we developed a methodology that bypasses these concerns. In particular, we show how one can use readily available micro-data to easily quantify the gains from input trade at both the firm and aggregate level for a wide class of models. Our first main result showed that the firm's domestic expenditure share in material spending is a sufficient statistic for its unit costs, as long as domestic and foreign inputs are combined with a constant elasticity of substitution. Importantly, this result does not rely any restrictions on firms' import environment. We can allow for arbitrary distributions of qualities and prices across potential sourcing countries, and for an arbitrary production function for imported inputs which can vary by firm. Crucially, we do not have to take a stand on how firms end up with their set of trading partners. Hence, regardless of the microstructure of firms' trading environment, all models within the CES class will imply exactly the same unit-cost reductions as long as they are consistent with the micro-data and share the same estimate of the elasticity of substitution. Our second main result concerns the effect of input trade on consumer prices. Within the context of a multi-sector, general-equilibrium trade model with a rich input-output structure we show that the observable joint distribution of value added and domestic expenditure shares fully determines the change in consumer prices due to input trade. Hence, despite the fact that the economy is non-aggregative, the micro-data on value added and domestic shares contains sufficient information to perform the correct aggregation.

We apply our methodology to the French economy. Using micro-data on the population of French manufacturing firms, we first propose a methodology to estimate the elasticity of substitution between domestic and imported varieties. Using the co-movement between changes in domestic spending and changes in revenue productivity at the firm-level, and exploiting changes in aggregate trade flows as an instrument, we estimate an elasticity of substitution of about two and hence substantially smaller than estimates from aggregate data. We then focus on the normative implications of input trade. We first show that there is substantial variation in the benefits from international sourcing across French producers. While some firms manage to reduce their unit costs by 50% holding aggregate prices fixed, the median importer would see its costs increase by only 10% if the French economy moved into input autarky. At the aggregate level, manufacturing prices would be 27% higher and the economy-wide consumer price level would increase by 9% under autarky. We also demonstrate that aggregative trade models lead to biased conclusions and that such biases can be substantial depending on the choice of the elasticity. By calibrating a model with fixed costs, we finally show that taking into account the resource loss of engaging in international reduces the consumer price gains by about 50%.

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7 Tables and Figures

Industry	ISIC	α	$\overline{\zeta}$	σ	γ	VA share	s_D^{Agg}
Mining	10-14	0.02%	1.50%	2.58	0.33	1.28%	0.90
Food, tobacco, beverages	15-16	9.90%	1.66%	3.85	0.73	15.24%	0.80
Textiles and leather	17-19	3.20%	3.25%	3.35	0.63	3.96%	0.54
Wood and wood products	20	0.13%	2.90%	4.65	0.60	1.67%	0.81
Paper, printing, publishing	21-22	1.37%	3.91%	2.77	0.50	7.96%	0.75
Chemicals	24	2.04%	6.69%	3.29	0.67	12.91%	0.60
Rubber and plastics products	25	0.44%	3.48%	4.05	0.59	5.88%	0.63
Non-metallic mineral products	26	0.24%	2.41%	3.48	0.53	4.54%	0.72
Basic metals	27	0.01%	6.52%	5.95	0.67	2.07%	0.60
Metal products (ex machinery and equipment)	28	0.26%	6.16%	3.27	0.48	9.27%	0.81
Machinery and equipment	29	0.66%	4.15%	3.52	0.62	7.00%	0.69
Office and computing machinery	30	0.43%	2.29%	7.39	0.81	0.35%	0.59
Electrical machinery	31	0.47%	2.46%	4.49	0.60	3.99%	0.64
Radio and communication	32	0.63%	4.27%	3.46	0.62	1.92%	0.64
Medical and optical instruments	33	0.35%	2.58%	2.95	0.49	3.83%	0.66
Motor vehicles, trailers	34	4.31%	2.13%	6.86	0.76	9.99%	0.82
Transport equipment	35	0.37%	2.62%	1.87	0.35	4.72%	0.64
Manufacturing, recycling	36-37	1.79%	0.92%	3.94	0.63	3.42%	0.75
Non-manufacturing		73.39%	40.11%	na	0.41		1

Notes: σ denotes the demand elasticity, which we measure with industry-specific average markups. Markups are constructed as the ratio of firm revenues to total costs. α is the sectoral share in consumer expenditure, which is taken from the Input-Output Tables. γ is the sectoral share of material spending in total costs, which is measured in the French micro-data. $\overline{\zeta}_s = \sum_{j=1}^{S} \zeta_s^j$ is the average intensity with which sector s is used as an input to production. "VA share" is the sectoral share of value added in manufacturing as observed in the micro-data. s_D^{Agg} is the aggregate domestic shares, i.e. $s_D^{Agg} = \sum_{i=1}^{n} s_{Di} \times va_i$, which in the model is equal to $\frac{M-IM}{M}$, where M is aggregate intermediary spending and IM is total import spending. See Appendix for the details.

Table 1: Structural Parameters by Industry

	Factor shares			2-step GMM					
				Cobb-Douglas			Translog		
	First Stage	ε	N	First Stage	ε	N	First Stage	ε	N
Full sample	-0.019***	2.378***	526,687	-0.017***	1.776***	331,412	-0.016***	1.727***	331,412
	(0.003)	(0.523)		(0.003)	(0.288)		(0.003)	(0.235)	
Importers	-0.010***	2.322**	65,799	-0.008**	1.896**	53,349	-0.008**	1.802**	53,349
	(0.004)	(1.014)		(0.003)	(0.850)		(0.003)	(0.735)	

Notes: Robust standard errors in parentheses with ***, **, and * respectively denoting significance at the 1%, 5% and 10% levels. The first stage column refers to the estimation of (32) with the instrument given in (31). We estimate γ_s based on factor shares, as per (30), or on the proxy method used in Levinsohn and Petrin (2012) and Wooldridge (2009). For the latter we report results based on Cobb-Douglas technology (28) and or Translog (**??**). For the factor share specification, we use data for the years 2002-2006. For the proxy method we use data for the years 2004-2006, as two lagged values are required to build the appropriate instruments for the estimation of the production function. For the 2-step GMM procedure we construct standard errors via bootstrap to take the sampling variation in the generated regressor $\gamma_s \Delta ln\left(s_D\right)$ into account.

Table 2: Estimating the Elasticity of Substitution ε

Mean	Quantile				
	10	25	50	70	90
22.21%	0.64%	2.75%	10.6%	29.07%	61.91%

Notes: The Table reports moments of the empirical distribution of $\frac{\gamma}{1-\varepsilon} ln\left(s_{D,i}\right)$, which is a consistent estimator of the distribution of the gains from input trade holding aggregate prices fixed (see Proposition 1). The data is taken to be the cross-section of importing firms in 2004 and ε and γ_s are estimated as in Section 4.1.

Table 3: Inequality in the Producer Gains from Input Trade in France

	Dependent variable: Producer gains $\frac{\gamma}{1-\varepsilon} ln\left(s_{Di}\right)$							
ln(Value Added)	0.028***		0.013***	0.005***		-0.008***		-0.029***
	(0.000)		(0.000)	(0.001)		(0.001)		(0.001)
ln(Employment)		0.028***			-0.000			
		(0.000)			(0.001)			
Exporter			0.085***			0.040***		0.024***
			(0.001)			(0.002)		(0.002)
Intl. Group			0.148***			0.138***		0.113***
			(0.003)			(0.003)		(0.003)
ln (Num. Varieties)							0.128***	0.144***
							(0.002)	(0.002)
Sample	Full sample			Importers Only				
Observations	633,240	640,610	633,240	118,799	120,344	118,799	120,344	118,799

Notes: Robust standard errors in parentheses with ***, ** and * respectively denoting significance at the 1%, 5% and 10% levels. All regressions include year fixed effects and 3-digit industry fixed effects. The number of varieties is the number of countries firms source from. A firm is member of an international group if at least one affiliate or the headquarter is located outside of France.

Table 4: Cross-Sectional Variation in Producer Gains

	Manufacturing Sector		Entire Economy	
Consumer Price Gains	27.5	*[21.2,35.9]*	9	*[7.1,11.6]*
Aggregate Data	30.9	*[21.5,45.3]*	9.9	*[7.1,14]*
Bias	3.4	*[0.2,10]*	0.9	*[0,2.6]*

Notes: The table reports the reduction in consumer prices for the manufacturing sector (left panel) and the entire economy (right panel) associated with input autarky. The measure in the first row is based on Proposition 2 and takes firm heterogeneity into account. The associated Λ_s are reported in Table 6 and the structural parameters Ξ, γ_s, σ_s and α_s given in Table 1. The second row contains results based on an aggregative model with identical input-output structure and parameters. Specifically, they are based on Proposition 2 where the sectoral gains are measured by Λ_s^{Agg} as per (22) instead of Λ_s. The third row reports the bias, defined as the difference between the first two rows - see (23). We report 90-10 confidence intervals for all measures in brackets. These are calculated via a bootstrap procedure described in Section 9.4 of the Appendix. We estimate the empirical distribution of all statistics using 200 bootstrap iterations.

Table 5: Consumer Price Gains From Input Trade in France

Estimated				
Parameter	Value	Identified from		
σ	3.83	Revenue/Cost Data, Section 4.2		
ε	2.38	Prod. Function Estimation, Section 4.3		
γ	0.61	Prod. Function Estimation, Section 4.3		
η	0.38	Dom Share and Ext. Margin, Section 6.3		
Calibrated				
		Moment	Data	Model
μ_f	2.43	Aggregate Dom Share	0.72	0.72
f_I	0.12	Share of Importer	0.20	0.20
σ_φ	0.51	Dispersion in log Value Added	1.52	1.52
σ_f	2.41	Dispersion in log Dom Shares	0.36	0.36
ρ	0.73	Correlation log Value Added - log Dom Shares	-0.31	-0.31

Notes: This table contains all the estimates and calibrated structural parameters. While $(\sigma, \varepsilon, \gamma, \eta)$ are estimated directly from the micro-data, $\left(\mu_f, f_I, \sigma_\varphi, \sigma_f, \rho\right)$ are calibrated to match the 5 moments listed in column 3. All moments are calibrated simultaneously to match the set of moments.

Table 7: Structural Parameters

Industry	ISIC	UC Reductions		Domestic Inputs		Sectoral Price Gains		Aggregate Data	
Mining	10-14	3.0	*[1.8,4.2]*	14.9	*[11.1,19.2]*	7.8	*[5.2,10.3]*	2.5	*[1.6,3.6]*
Food, tobacco, beverages	15-16	11.1	*[7.5,14.6]*	8.4	*[6.2,10.6]*	17.8	*[12.4,23.4]*	12.6	*[7.8,18.2]*
Textiles and leather	17-19	31.1	*[24.2,39.9]*	31.4	*[24.3,40.3]*	55.6	*[42.4,74]*	31.9	*[22.4,46.9]*
Wood and wood products	20	8.2	*[6.4,10.5]*	9.6	*[7.4,12.1]*	14.4	*[11.1,18.2]*	9.6	*[6.7,13.7]*
Paper, printing, publishing	21-22	12.2	*[9,16]*	14.5	*[10.9,18.7]*	20.1	*[14.7,26.5]*	11.0	*[7.7,15.4]*
Chemicals	24	27.2	*[20.1,36.4]*	21.6	*[16.1,28.2]*	45.1	*[32.7,60.7]*	28.1	*[18.7,41.8]*
Rubber and plastics products	25	20.1	*[14.3,26.5]*	27.3	*[20.2,36]*	38.4	*[27.5,50.9]*	21.5	*[13.9,31]*
Non-metallic mineral products	26	13.4	*[9.6,17.9]*	12.7	*[9.7,16.3]*	20.8	*[15.3,27.4]*	13.3	*[9,19]*
Basic metals	27	21.8	*[16.3,27.7]*	21.5	*[16.4,27.3]*	38.9	*[28.2,50.2]*	28.8	*[19,41.6]*
Metal products (ex machinery and equipment)	28	8.2	*[6.2,10.5]*	20.5	*[15.5,26.2]*	18.3	*[13.8,23.5]*	7.7	*[5.5,10.8]*
Machinery and equipment	29	17.6	*[12.8,23.2]*	20.0	*[15,25.7]*	31.7	*[23,41.6]*	18.2	*[12.2,26.2]*
Office and computing machinery	30	20.4	*[15.4,25.5]*	25.2	*[18.3,32.1]*	44.6	*[31.9,57]*	37.0	*[22.4,60.3]*
Electrical machinery	31	19.8	*[14.6,25.6]*	23.9	*[17.7,30.6]*	36.1	*[26.4,46.6]*	21.6	*[14.8,30.7]*
Radio and communication	32	21.5	*[13.1,31.1]*	23.3	*[16.6,30.5]*	38.5	*[23.5,54.8]*	22.1	*[12.5,36.1]*
Medical and optical instruments	33	17.9	*[12.8,23.4]*	20.4	*[15.1,26.2]*	29.2	*[21.1,38.3]*	15.9	*[10.7,22.5]*
Motor vehicles, trailers	34	6.2	*[3.2,16.4]*	21.7	*[17,29.3]*	23.3	*[17.4,39]*	11.2	*[6.1,24.3]*
Transport equipment	35	15.3	*[10.5,22]*	19.9	*[14.5,27.2]*	22.9	*[16,33.2]*	11.8	*[7.9,18.2]*
Manufacturing, recycling	36-37	12.9	*[9.7,16.3]*	19.0	*[14.5,24]*	26.0	*[19.2,33.4]*	14.1	*[9.5,20.4]*
Non-manufacturing		0.0	*[0,0]*	7.5	*[5.7,9.4]*	3.0	*[2.3,3.8]*	0.0	*[0,0]*

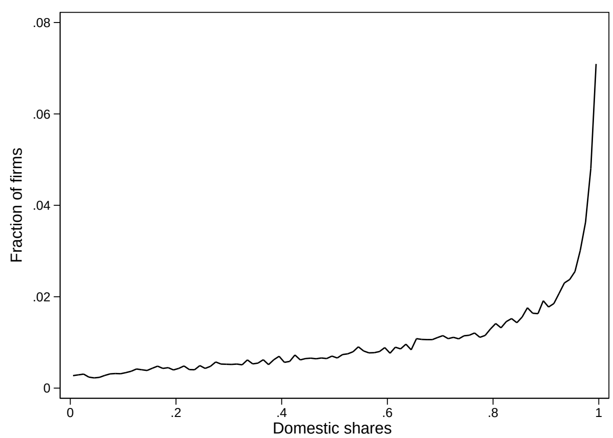
Notes: The first column reports the direct unit costs reductions from international sourcing relative to autarky, Λ_s, which are calculated according to (20). The second column reports the reductions in the price of domestically sourced intermediate inputs, p_{Ds}^{Aut}/p_{Ds}. The third column contains the full change in sector prices relative to autarky, P_s^{Aut}/P_s. Column 4 reports the full change in sector prices predicted by an aggregative approach whereby the direct unit costs reductions from international sourcing relative to autarky are given by Λ_s^{Agg}, as per (22).

Table 6: The Consumer Price Gains across Sectors

	Consumer Price Gains (in %)	Welfare Gains (in %)	% of Labor in Fixed Cost Production
Model	38.07	17.53	14.87
French Data	41.53	-	-

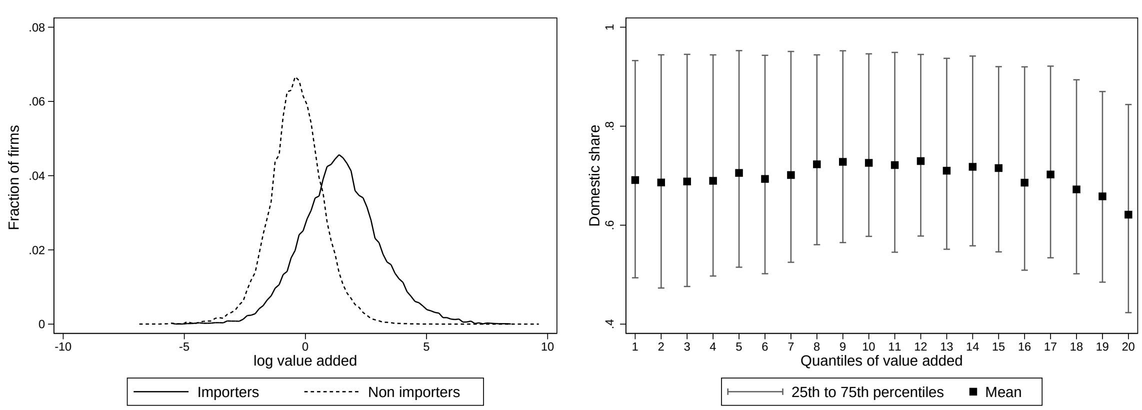
Notes: The consumer price gains are given by $\left(P^{AUT}/P-1\right)\times 100$. The full welfare gains are given by $\left(W/W^{AUT}-1\right)\times 100$. The results correspond to the calibration of Section 5.2. See the main text for details.

Table 8: Welfare Gains from Input Trade



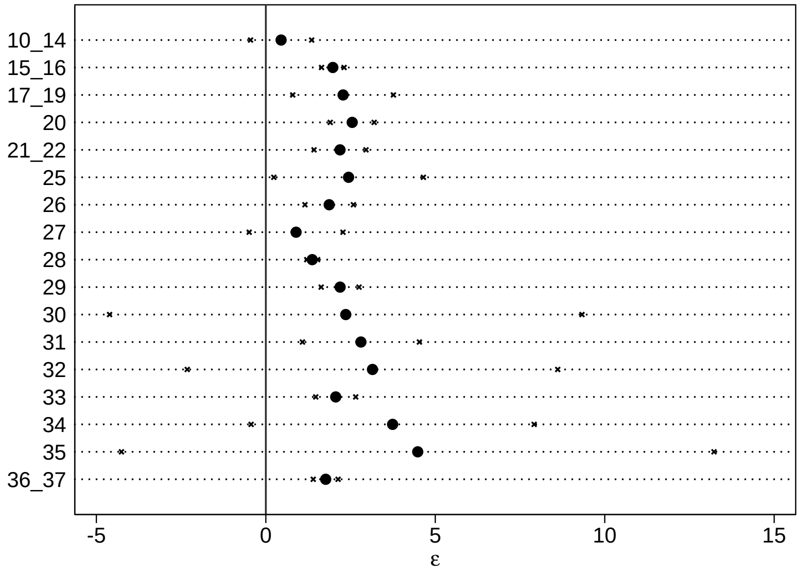
Notes: The figure shows the cross-sectional distribution of domestic expenditure shares, i.e. the share of material spending allocated to domestic inputs, for the population of importing manufacturing firms in France in 2004.

Figure 1: The Dispersion in Import Intensity



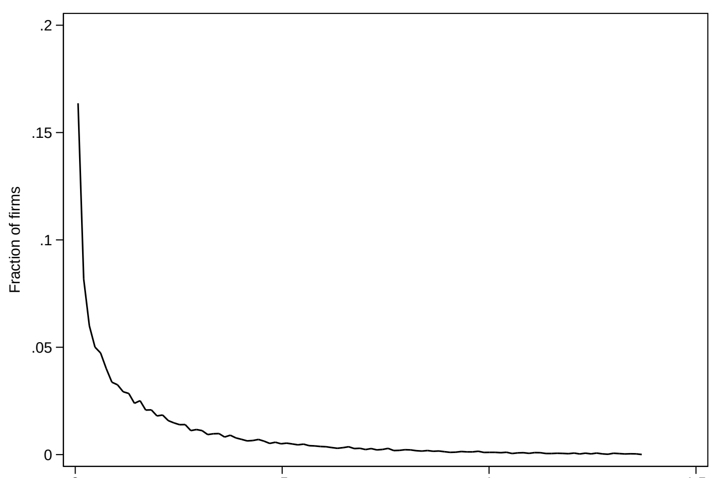
Notes: The left panel shows the distribution of log value added by import status. The right panel shows both the average and the interquartile-range of domestic shares by value added quantile.

Figure 2: Domestic Shares and Size of French Importers



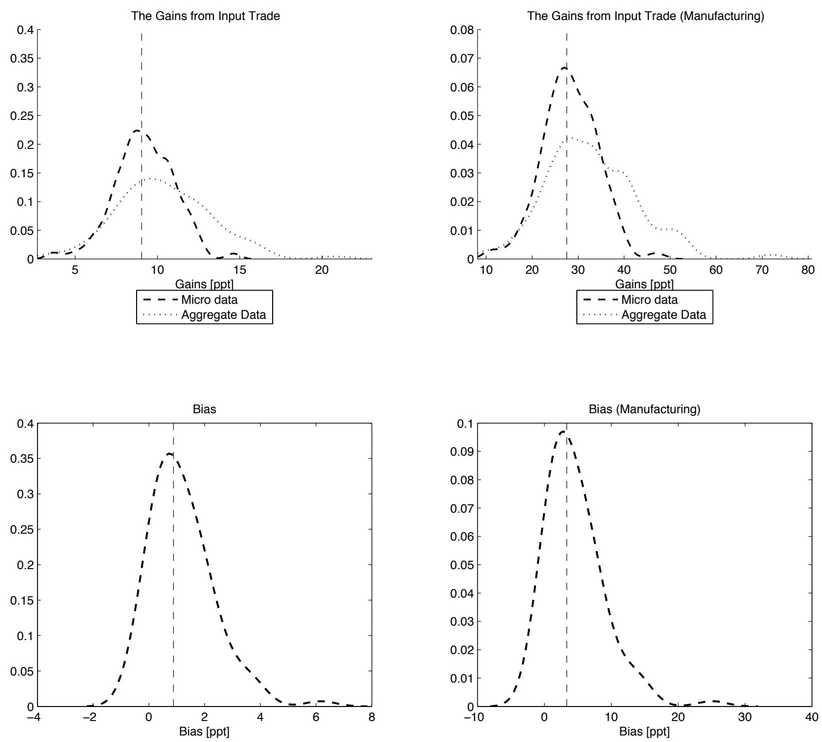
Notes: The figures displays the sector-specific point estimates for ε from the integrated GMM approach based on (**??**). We also display two standard deviations confidence intervals. The full results are contained in Table 18 in Section 9.3 in the Online Appendix.

Figure 3: Sector-specific substitution elasticities ε_s



Notes: The figure displays the distribution of $\frac{\gamma_s}{1-\varepsilon} ln\left(s_{Di}\right)$ which measures the change in firm unit cost when moving to autarky, keeping prices fixed - see Proposition 1. The data for domestic expenditure shares corresponds the cross-section of importing firms in 2004 and ε and γ_s are estimated as in Section 4.1.

Figure 4: The Producer Gains from Input Trade in France



Notes: The figure depicts the bootstrap distribution of the consumer price gains from input trade and the bias from using aggregate data for the entire economy (left panels) and the manufacturing sector (right panels). The results for "micro-data" are based on Λ_s, the results for "aggregate data" are based on Λ_s^{Agg}. See Proposition 2 and equations (22) and (23) for details. The gains are reported in percetange points.

Figure 5: Bootstrap Distribution of Consumer Price Gains and Bias

8 Appendix

8.1 Proof of Proposition 1

Consider firm i with efficiency φ_i and an extensive margin Σ_i, facing prices $[p_{ci}]_{c\in\Sigma_i}$. The profit-maximizing demand for imported varieties and domestic inputs also has to solve the dual problem, i.e. the cost-minimization problem (for simplicity we drop the subscript i from now on)

$$\Gamma\left(y,\varphi,[p_c],\Sigma\right) \equiv \min_z \left\{ p_D z_D + \sum_{c\in\Sigma} p_c z_c \text{ s.t. } x \geq \left(\frac{y}{\varphi l^{\gamma}}\right)^{\frac{1}{1-\gamma}} \right\},$$

where

$$x = \left(\beta_i \left(q_D z_D\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\beta_i)\left(h_i\left([\eta\left(q_c,\varphi\right) z_c]_{c\in\Sigma}\right)\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

We can first solve the import problem

$$\lambda\left(x_I\right) = \min_z \left\{ \sum_{c\in\Sigma} p_c z_c \text{ s.t. } h_i\left([\eta\left(q_c,\varphi\right) z_c]_{c\in\Sigma}\right) \geq x_I \right\}.$$

Because $h\left(.\right)$ has constant-returns-to scale, we get that

$$\lambda\left(x_I\right) = \min_{\tilde{z}} \left\{ x_I \sum_{c\in\Sigma} p_c \tilde{z}_c \text{ s.t. } h_i\left([\eta\left(q_c,\varphi\right) \tilde{z}_c]_{c\in\Sigma}\right) \geq 1 \right\} = x_I \times \lambda\left(1\right) = x_I \lambda. \tag{44}$$

Hence, $\lambda = \lambda\left(\Sigma,\varphi,[p_{ci}],G_q\right)$ is the price index for firm i, which depends on the underlying heterogeneity in quality G_q, firm productivity φ, the set of prices and the production functions. Crucially: from the point of view of the firm, it is constant for given Σ. The cost-minimization problem is hence given by

$$\min_z \left\{ p_D z_D + \lambda\left(\Sigma,\theta\right) x_I \text{ s.t. } x \geq \left(\frac{y}{\varphi l^{\gamma}}\right)^{\frac{1}{1-\gamma}} \right\}, \tag{45}$$

where

$$x = \left(\beta_i \left(q_D z_D\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\beta_i)\, x_I^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

This implies that

$$p_D z_D + \lambda\left(\Sigma,\theta\right) x_I \quad = \quad Q x^{1/\varepsilon}\left(\beta q_D^{\frac{\varepsilon-1}{\varepsilon}} z_D^{\frac{\varepsilon-1}{\varepsilon}} + (1-\beta)\, x_I^{\frac{\varepsilon-1}{\varepsilon}}\right) = Qx,$$

where

$$Q \quad = \quad \left((1-\beta)^{\varepsilon}\left(\lambda\left(\Sigma,\theta\right)\right)^{1-\varepsilon} + \beta^{\varepsilon}\left(\frac{p_D}{q_D}\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}.$$

Note also that

$$\beta\left(q_D z_D\right)^{\frac{\varepsilon-1}{\varepsilon}} \quad = \quad Q^{\varepsilon-1} x^{(\varepsilon-1)/\varepsilon} \beta^{\varepsilon}\left(\frac{q_D}{p_D}\right)^{\varepsilon-1}, \tag{46}$$

so that

$$Q = s_D^{\frac{1}{\varepsilon-1}}\left(\frac{1}{\beta}\right)^{\frac{\varepsilon}{\varepsilon-1}}\left(\frac{p_D}{q_D}\right). \tag{47}$$

The unit costs are then

$$UC \quad = \quad \frac{1}{\varphi} w^{1-\gamma} Q^{\gamma} = \frac{1}{\varphi}\left(\frac{1}{\beta}\right)^{\frac{\varepsilon\gamma}{\varepsilon-1}} s_D^{\frac{\gamma}{\varepsilon-1}} w^{1-\gamma}\left(\frac{p_D}{q_D}\right)^{\gamma},$$

which is the required expression given in (9).

8.2 Proof of Proposition 2

Consider sector s. Let $[s_{D,i}]$ be the distribution of domestic shares. The unit costs of firm i in sector s are hence given by (see (9))

$$UC_{i,s} = \frac{1}{\tilde{\varphi}_i} \times \left(s_{D,i}\right)^{\gamma_s/(\varepsilon_s-1)} \times \left(\frac{p_{D,s}}{q_{D,s}}\right)^{\gamma_s}. \tag{48}$$

Note that we normalized $w = 1$. Monopolistic competition implies that $p_{i,s} = \mu_s UC_{i,s}$, where $\mu_s = \frac{\sigma_s}{\sigma_s-1}$ is the equilibrium mark-up. Let $[p_{D,s}]$ be given. The ideal price index for sector $s$output, P_s, is given by

$$P_s \quad = \quad \mu_s\left(\int_{i=0}^{N_s} UC_i^{1-\sigma_s} di\right)^{\frac{1}{1-\sigma_s}} = \mu_s\left(\frac{p_{D,s}}{q_{D,s}}\right)^{\gamma_s}\left(\int_{i=0}^{N_s}\left(\frac{1}{\tilde{\varphi}_i}\left(s_{D,i}\right)^{\gamma_s/(\varepsilon_s-1)}\right)^{1-\sigma_s} di\right)^{\frac{1}{1-\sigma_s}} \tag{49}$$

$$\equiv \quad \mu_s\left(\frac{p_{D,s}}{q_{D,s}}\right)^{\gamma_s} \Upsilon_s, \tag{50}$$

where

$$\Upsilon_s = \left(\int_{i=0}^{N_j}\left(\frac{1}{\tilde{\varphi}_i}\left(s_{D,i}\right)^{\gamma_j/(\varepsilon_j-1)}\right)^{1-\sigma_j} di\right)^{\frac{1}{1-\sigma_j}}. \tag{51}$$

The costs of domestic intermediaries for firms in sector s are therefore given by

$$p_{D,s} = \zeta_s^* \prod_{j=1}^{S} P_j^{\zeta_j^s}, \tag{52}$$

where $\zeta_s^* = \prod_{j=1}^{S}\left(\zeta_j^s\right)^{-\zeta_j^s}$. Using (49) we get

$$\begin{aligned} p_{D,s} &= \zeta_s^*\left(\prod_{j=1}^{S}\mu_j^{\zeta_j^s}\right)\left(\prod_{j=1}^{S}\left(\frac{1}{q_{D,j}}\right)^{\gamma_j\zeta_j^s}\right)\left(\prod_{j=1}^{S}p_{D,j}^{\gamma_j\zeta_j^s}\right)\left(\prod_{j=1}^{S}\Upsilon_j^{\zeta_j^s}\right) \\ &= \left[\zeta_s^*\mu_s^*q_s^*\left(\prod_{j\neq s}p_{D,j}^{\gamma_j\zeta_j^s}\right)\left(\prod_{j=1}^{S}\Upsilon_j^{\zeta_j^s}\right)\right]^{1/(1-\gamma_s\zeta_s^s)}, \end{aligned} \tag{53}$$

where $\mu_s^* = \prod_{j=1}^{S}\mu_j^{\zeta_j^s}$ and $\frac{1}{q_s^*} = \left(\prod_{j=1}^{S}\left(1/q_{D,j}\right)^{\gamma_j\zeta_j^s}\right)$. Given the technological constants ζ_s^*, μ_s^* and q_s^* and the endogenous trade-induces productivity terms $\{\Upsilon_s\}_s$, (53) is a system of S equation in the S unknowns $[p_{D,s}]_{s=1}^{S}$, which can be easily solved.

Given $[p_{D,s}]_{s=1}^{S}$, the consumer price index is then given by

$$P = \prod_{s=1}^{S}\left(\frac{P_s}{\alpha_s}\right)^{\alpha_s} = \alpha^*\mu^*\frac{1}{q^*}\prod_{s=1}^{S}p_{D,s}^{\gamma_s\alpha_s}\prod_{s=1}^{S}\Upsilon_s^{\alpha_s}, \tag{54}$$

where P_s follows from (49) and $\alpha^* = \prod_{s=1}^{S}\alpha_s^{-\alpha_s}$, $\mu^* = \prod_{s=1}^{S}\mu_s^{\alpha_s}$, $\frac{1}{q^*} = \prod_{s=1}^{S}q_{D,s}^{-\gamma_s\alpha_s}$.

The Consumer Gains from Trade are then defined by

$$G = ln\left(\frac{P^{Aut}}{P}\right),$$

where P is recursively defined by (54) and (53) and P^{Aut} accordingly as

$$\begin{aligned} P^{Aut} &= \alpha^*\mu^*\frac{1}{q^*}\prod_{s=1}^{S}\left(p_{D,s}^{Aut}\right)^{\gamma_s\alpha_s}\prod_{s=1}^{S}\Psi_s^{\alpha_s} \\ p_{D,s}^{Aut} &= \left[\zeta_s^*\mu_s^*q_s^*\left(\prod_{j\neq s}\left(p_{D,j}^{Aut}\right)^{\gamma_j\zeta_j^s}\right)\left(\prod_{j=1}^{S}\Psi_j^{\zeta_j^s}\right)\right]^{1/(1-\gamma_s\zeta_s^s)}, \end{aligned}$$

where Ψ is defined akin to (51) with $s_{D,i} = 1$.

Hence

$$exp\left(G\right) = \frac{\alpha^*\mu^*\frac{1}{q^*}\prod_{s=1}^{S}\left(p_{D,s}^{Aut}\right)^{\gamma_s\alpha_s}\prod_{s=1}^{S}\Psi_s^{\alpha_s}}{\alpha^*\mu^*\frac{1}{q^*}\prod_{s=1}^{S}p_{D,s}^{\gamma_s\alpha_s}\prod_{s=1}^{S}\Upsilon_s^{\alpha_s}} = \prod_{s=1}^{S}\left(\frac{p_{D,s}^{Aut}}{p_{D,s}}\right)^{\gamma_s\alpha_s}\prod_{s=1}^{S}\left(\frac{\Psi_s}{\Upsilon_s}\right)^{\alpha_s}, \tag{55}$$

where

$$\frac{p_{D,s}^{Aut}}{p_{D,s}} = \left(\prod_{j\neq s}\left(\frac{p_{D,j}^{Aut}}{p_{D,j}}\right)^{\gamma_j\zeta_j^s}\prod_{j=1}^{S}\left(\frac{\Psi_j}{\Upsilon_j}\right)^{\zeta_j^s}\right)^{1/(1-\gamma_s\zeta_s^s)}. \tag{56}$$

Now note that

$$\frac{\Psi_s}{\Upsilon_s} = \left(\frac{\int_{i=0}^{N_s} (1/\tilde{\varphi}_i)^{1-\sigma_s} di}{\int_{i=0}^{N_s} \left(\frac{1}{\tilde{\varphi}_i} (s_{D,i})^{\gamma_s/(\varepsilon_s - 1)} \right)^{1-\sigma_s} di} \right)^{\frac{1}{1-\sigma_s}}.$$

In the trade equilibrium, firm i's value added is given by

$$va_i = \kappa \times \left(\tilde{\varphi}_i (s_{D,i})^{\gamma_s/(1-\varepsilon_s)} \right)^{\sigma - 1}.$$

Hence

$$\begin{aligned}
\Lambda_s \equiv \frac{\Psi_s}{\Upsilon_s} &= \left(\frac{\int_{i=0}^{N_s} (\tilde{\varphi}_i)^{\sigma_s - 1} di}{\int_{i=0}^{N_s} \left(\tilde{\varphi}_i (s_{D,i})^{\gamma_s/(1-\varepsilon_s)} \right)^{\sigma_s - 1} di} \right)^{\frac{1}{1-\sigma_s}} \\
&= \left(\frac{\int_{i=0}^{N_s} va_i (s_{D,i})^{\gamma_s(\sigma_s - 1)/(\varepsilon_s - 1)} di}{\int_{i=0}^{N_s} va_i di} \right)^{\frac{1}{1-\sigma_s}} \\
&= \left(\int_{i=0}^{N_s} \frac{va_i}{\int_{i=0}^{N_s} va_i di} (s_{D,i})^{\gamma_s(\sigma_s - 1)/(\varepsilon_s - 1)} di \right)^{\frac{1}{1-\sigma_s}}.
\end{aligned}$$

(56) implies that

$$\frac{p_{D,s}^{Aut}}{p_{D,s}} = \left(\prod_{j=1}^{S} \Lambda_j^{\zeta_j^s} \right) \prod_j \left(\frac{p_{D,j}^{Aut}}{p_{D,j}} \right)^{\gamma_j \zeta_j^s}. \tag{57}$$

Hence, (55) and (57) yield

$$G = \sum_{s=1}^{S} \gamma_s \alpha_s \pi_s + \sum_{s=1}^{S} \alpha_s \Lambda_s, \tag{58}$$

$$\pi_s = \sum_{j=1}^{S} \zeta_j^s \Lambda_j + \sum_{j=1}^{S} \gamma_j \zeta_j^s \pi_j, \tag{59}$$

where $\pi_s \equiv ln \left(\frac{p_{D,s}^{Aut}}{p_{D,s}} \right)$. To express (58) and (59) in matrix notation, note that (59) can be written as

$$\begin{bmatrix} \pi_1 \\ \pi_2 \\ ... \\ \pi_S \end{bmatrix} = \begin{bmatrix} \zeta_1^1 & \zeta_2^1 & .. & \zeta_S^1 \\ \zeta_1^2 & \zeta_2^2 & ... & ... \\ ... & & & \\ \zeta_1^S & \zeta_2^S & & \zeta_S^S \end{bmatrix} \begin{bmatrix} \Lambda_1 \\ \Lambda_2 \\ ... \\ \Lambda_S \end{bmatrix} + \begin{bmatrix} \zeta_1^1 \gamma_1 & \zeta_2^1 \gamma_2 & .. & \zeta_S^1 \gamma_S \\ \zeta_1^2 \gamma_1 & \zeta_2^2 \gamma_2 & ... & ... \\ ... & & & \\ \zeta_1^S \gamma_1 & \zeta_2^S \gamma_2 & & \zeta_S^S \gamma_S \end{bmatrix} \begin{bmatrix} \pi_1 \\ \pi_2 \\ ... \\ \pi_S \end{bmatrix}$$

or in matrix form

$$\pi = \Xi \times \Lambda + \Xi \times \Gamma \times \pi,$$

where

$$\Gamma = \begin{bmatrix} \gamma_1 & 0 & & 0 \\ 0 & \gamma_2 & & \\ & & ... & \\ 0 & & & \gamma_S \end{bmatrix}.$$

Hence,

$$\pi = (\mathcal{I} - \Xi \times \Gamma)^{-1} \Xi \times \Lambda. \tag{60}$$

Given (60), we can then solve for the aggregate gains from (58) as

$$ln(G) = \alpha'\Gamma\pi + \alpha'\Lambda = \alpha'\Gamma(\mathcal{I} - \Xi \times \Gamma)^{-1} \Xi\Lambda + \alpha'\Lambda.$$

For counterfactuals other than autary, suppose that the counterfactual domestic share is $s^{'}_{D,i}$. Then

$$\Lambda_s^* = \left(\frac{\int_{i=0}^{N_s} \left(\tilde{\varphi}_i \left(s^{'}_{D,i}\right)^{\gamma_s/(1-\varepsilon_s)}\right)^{\sigma_s - 1} di}{\int_{i=0}^{N_s} \left(\tilde{\varphi}_i \left(s_{D,i}\right)^{\gamma_s/(1-\varepsilon_s)}\right)^{\sigma_s - 1} di} \right)^{\frac{1}{1-\sigma_s}} = \left(\frac{\int_{i=0}^{N_s} va_i \left(\left(\frac{s^{'}_{D,i}}{s_{D,i}}\right)^{\gamma_s/(1-\varepsilon_s)}\right)^{\sigma_s - 1} di}{\int_{i=0}^{N_s} va_i di} \right)^{\frac{1}{1-\sigma_s}}$$

$$= \left(\int_{i=0}^{N_s} \frac{va_i}{\int_{i=0}^{N_s} va_i di} \left(\frac{s_{D,i}}{s^{'}_{D,i}}\right)^{\frac{\gamma_s(\sigma_s-1)}{\varepsilon_s - 1}} di \right)^{\frac{1}{1-\sigma_s}}.$$

8.3 Data Description

Our main data set stems from the information system of the French custom administration (DGDDI) and contains the universe of import and export flows by French manufacturing firms.[60] The data is collected at the 8-digit (NC8) level and a firm located within the French metropolitan territory must report this detailed information as long as the following criteria are met. For imports from outside the EU, reporting is required from each firm and flow if the imported value to exceeds 1,000 Euros. For within EU imports, import flows had to be reported (between 2001 and 2006)[61] as long as the firm's annual trade value exceeds 100,000 Euros. However, some firms (ca. 10,000 firm year observations out of ca. 130,000) also report while they are below the threshold. Clearly, the existence of this administrative threshold induces a censoring for small EU importers. While it does not affect aggregate, value weighted statistics, it will generate some attenuation bias in our econometric analyses, which would bias our results against finding any result.

In spite of this limitation, the attractive feature of the French data is the presence of unique firm identifiers (the SIREN code), which is available in all French administrative files. Hence, various other datasets can be matched to the trade data at the firm level. To learn about the characteristics

[60] For imports from outside the EU, all shipments must be reported to the custom administration. The conditions are more stringent for exports since all shipments (even within EU) must be reported to the custom administration.

[61] Between 1993 and 2001, the threshold was ca. 40,000 euros. After 2006, it was raised to 150,000 euros and to 460,000 euros after 2011.

	Full sample	Importers	Non importers	Exporters	Non exporters
Employment	25	92	8	81	9
Sales	5,455	21,752	1,379	19,171	1,468
Sales per worker	126	208	105	196	105
Value added	1,515	5,972	400	5,294	416
Value added per worker	45	55	43	55	43
Capital	2,217	8,728	588	7,661	634
Capital per worker	44	64	40	61	40
Inputs	2,600	10,225	693	8,943	756
Domestic share	0.943	0.698	1	0.790	0.986
Share of importers	0.200	1	0	0.677	0.061
Share of exporters	0.225	0.762	0.091	1	0
Share of firms that are part of an international group	0.029	0.131	0.004	0.113	0.005
Productivity (factor shares)	39.173	65.450	32.989	63.858	32.359
Number of observations (firm * year)	650,401	130,135	520,266	146,496	503,905
Number of firms	172,244	38,240	148,619	44,648	146,423

Notes: Sales, wages, expenditures on imports or exports are all expressed in 2005 prices using a 3-digit industry level price deflator. Our capital measure is the book value reported in firms' balance sheets ("historical cost"). We measure employees by occupation. Skilled workers are engineers, technicians and managers, workers of intermediate skills are skilled blue and white collars and low skilled workers are members of unskilled occupations. A firm is member of an international group if at least one affiliate or the headquarter is located outside of France.

Table 9: Characteristics of importers, exporters and domestic firms

of the firms in our sample we employ fiscal files.[62] Sales are deflated using price indices of value added at the 3 digit level obtained from the French national accounts. To measure the expenditure on domestic inputs, we subtract the total import value from the total expenditure on wares and inputs reported in the fiscal files. Capital, used for the TFP estimation, is measured at book value (historical cost).

Finally we incorporate information on the ownership structure from the LIFI/DIANE (BvDEP) files. These files are constructed at INSEE using a yearly survey (LIFI) describing the structure of ownership of all of the French firms in the private sector whose financial investments in other firms (participation) are higher than 1.2 million Euros or having sales above 60 million Euros or more than 500 employees. This survey is complemented with the information about ownership structure available in the DIANE (BvDEP) files, which are constructed using the annual mandatory reports to commercial courts, and with the register of firms that are controlled by the State.

Using these datasets, we construct a non-balanced panel dataset spanning the period from 2001 to 2006. Some basic characteristics of importing and non-importing firms are contained in Table 9. For comparison, we also report the results for exporting firms. Expectedly, importers outperform

[62]The firm level accounting information is retrieved from two different files: the BRN ("Bénéfices Réels Normaux") and the RSI ("Régime Simplifié d'Imposition"). The BRN contains the balance sheet of all firms in the traded sectors with sales above 730,000 Euros. The RSI is the counterpart of the BRN for firms with sales below 730,000 Euros. Although the details of the reporting differs, for our purpose these two data sets contain essentially the same information. Their union covers nearly the entire universe of all French firms.

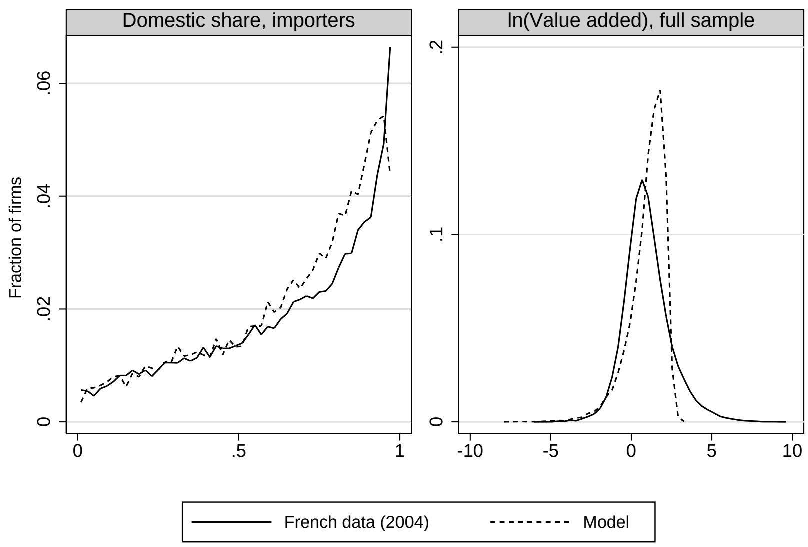
domestic firm in essentially all dimensions we look at (see also Bernard et al. (2012)). Furthermore, import and export status are highly correlated.

8.4 Estimates of the Elasticity of Substitution ε

Table 10 below contains various robustness checks for our estimates for the elasticity of substitution using both factor shares and proxy methods to arrive at an estimate for γ.

8.5 Non-Targeted Moments and Model Comparison

Table 11 and Figures 6 and 7 report the fit between the model and additional non-targeted moments. Table 11 compares the model with the data at various dimensions. The model performs relatively well. That fact that it under predicts the dispersion of value added is related to the fact that the log-normal distribution of efficiency has too thin tails. This also explains why the model under predicts the share of value added by importers - there are simply too few very large firms in our model.



Notes: The left panel (right panel) shows the distribution of domestic shares (log value added) in the data (solid line) and in the model (dashed line). The firms are grouped in 50 bins of equal length. The distributions of log value added have been normalized to have means of unity.

Figure 6: Marginal Distributions: Model and Data

Figure 6 reports the marginal distributions of domestic shares and log value added both in the model and in the French data. The model captures the marginal distribution of domestic shares quite well. It under predicts the density for very small importers, which is natural in a model of fixed costs - it not worth it pay the fixed costs to then import tiny amounts. When we compare the distribution of value added between model and the data, we again see that the model generates too little dispersion.

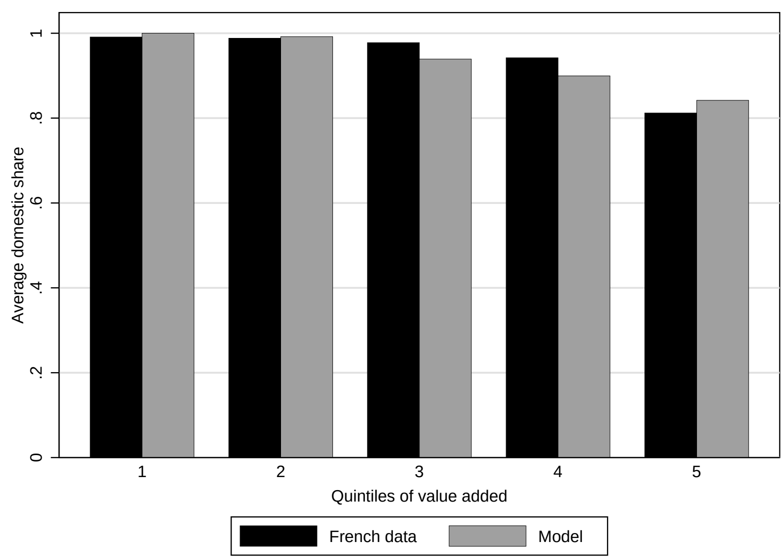
			$\Delta \ln(WES)$	$\tilde{\gamma}_s \times \Delta \ln(s_D)$	ε	N
Factor shares	Full sample	First stage	-0.019***	-	-	526,687
Sample: 2002-2006			(0.003)			
	Importers only	First stage	-0.010***	-	-	65,799
			(0.004)			
Factor shares	Full sample	All weights	-	-0.726***	2.378***	526,687
(bootstrapped SE)				(0.197)	(0.523)	
Sample: 2002-2006		Pre-sample (2001) weights	-	-1.407***	1.711***	443,954
				(0.356)	(0.166)	
	Importers only	All weights	-	-0.756	2.322**	65,799
	(2 digit dummies)			(0.537)	(1.014)	
		Pre-sample (2001) weights	-	-1.121*	1.892***	54,604
				(0.632)	(0.541)	
2-step GMM	Full sample	First stage	-0.017***	-	-	331,421
Sample: 2004-2006			(0.003)			
	Importers only	First stage	-0.008**	-	-	53,349
			(0.003)			
2-step GMM	Full sample	All weights	-	-1.288***	1.776***	331,421
(bootstrapped SE)				(0.395)	(0.288)	
Sample: 2004-2006		Pre-sample (2001) weights	-	-2.152***	1.465***	258,957
				(0.652)	(0.148)	
	Importers only	All weights	-	-1.116	1.896**	53,349
	(2 digit dummies)			(1.203)	(0.850)	
		Pre-sample (2001) weights	-	-1.968	1.508***	43,393
				(1.910)	(0.402)	
2-step GMM, translog	Full sample	First stage	-0.016***	-	-	331,421
Sample: 2004-2006			(0.003)			
	Importers only	First stage	-0.008**	-	-	53,349
			(0.003)			
2-step GMM, translog	Full sample	All weights	-	-1.376***	1.727***	331,421
(bootstrapped SE)				(0.402)	(0.235)	
Sample: 2004-2006		Pre-sample (2001) weights	-	-2.371***	1.422***	258,957
				(0.693)	(0.119)	
	Importers only	All weights	-	-1.246	1.802**	53,349
	(2 digit dummies)			(1.071)	(0.735)	
		Pre-sample (2001) weights	-	-2.171	1.461***	43,393
				(1.835)	(0.332)	

Table 10: Estimating the Elasticity of Substitution ε

Non-Targeted Moments	French Data	Baseline
Avg Domestic Share (Importers)	0.70	0.67
Avg Domestic Share (Population)	0.94	0.93
Agg Domestic Share (Importers)	0.63	0.52
Dispersion log Value Added (Importers)	1.62	1.02
Dispersion log Dom Shares (Importers)	0.69	0.65
Correlation log Value Added - log Dom Shares (Importers)	-0.01	-0.06
Share of Value Added by Importers	0.79	0.59

Notes: This table reports some non-targeted moments in the micro-data (column 2) and in our baseline calibration (column 3). The calibrated parameters of the benchmark model are contained in Table 7.

Table 11: Non-Targeted Moments



Notes: The graph depicts the average domestic expenditure share for different size groups in the French economy. We depict both the micro-data and the data from the calibrated model.

Figure 7: Correlation Structure: Model and Data

Figure 7 reports the average domestic share by value added quintile. The model fits that moment relatively well. In particular, the model captures that sales and import behavior are not perfectly aligned.

Table 12 finally reports the calibrated parameters and model fit of our alternative calibration strategy if no information domestic share was available.

	Baseline		No s_D Data	
	Model	Parameter	Model	Parameter
Aggregate Domestic Share	0.72	$\mu_f = 2.43$	0.72	$\mu_f = 2.88$
Share of Importers	0.20	$f_I = 0.12$	0.20	$f_I = 0.16$
Dispersion in log Value Added	1.52	$\sigma_\varphi = 0.51$	1.52	$\sigma_\varphi = 0.51$
Dispersion in Domestic Shares	0.36	$\sigma_f = 2.41$	0.14	$\sigma_f = 0$
Correlation log Value Added - Dom Shares	-0.31	$\rho = 0.73$	-0.72	$\rho = 0$
Consumer Price Gains	38.07 %		43.26%	Bias:13.65 %
Welfare Gains	17.53%		21.69%	Bias:23.73 %

Notes: The table contains an alternative calibration which does not use information on domestic expenditure shares. This model is only calibrated to the first three moments. For completeness we also report our baseline calibration.

Table 12: Calibration Without Domestic Shares

While the model is calibrated to match the first three moments, the results in Table 12 show that it is unsuccessful to predict a distribution of import shares that is consistent with the data. Not only is there too strong a negative correlation with firm value added, but the model also under-predicts the cross-sectional dispersion in domestic shares.

9 Online Appendix

This Appendix contains additional results and material for our paper "The Gains from Input Trade in Firm-Based Models of Importing".

1. Section 9.1 considers two generalizations of Proposition 1,
2. Section 9.2 contains details about the identification of the Input-Output Matrix $[\Xi_{ij}]$,
3. Section 9.3 contains the parameter estimates of the production function coefficients,
4. Section 9.4 explains our bootstrap procedure,
5. Section 9.5 reports the consumer gains for a range of values of ε,
6. Section 9.6 extends Proposition 3 to a multi-sector environment,
7. Section 9.7 characterizes the firm's problem with fixed costs contained in Section 5,
8. Section 9.8 contains the estimation of η,
9. Section 9.9 contains details about the algorithm used to calibrate the model of Section 5.

9.1 Generalizations of Proposition 1

In this section, we consider two generalizations of our main result that firm's unit costs are given by (see (9))

$$UC_i = \frac{1}{\tilde{\varphi}_i} \times (s_{D,i})^{\frac{\gamma}{\varepsilon-1}} \times \left(\frac{p_D}{q_D}\right)^{\gamma} w^{1-\gamma}. \tag{61}$$

(61) was derived under two important restrictions: (i) the production function has a constant elasticity of materials γ and (ii) domestic and foreign inputs are combined in a CES fashion with elasticity of substitution ε. Here we generalize both of these assumptions. We allow for firms' production function to be CES between materials and primary factors and we consider the case of domestic and foreign inputs to be combined in a general, smooth, production function with constant returns.

Firms with CES production function For our analysis we restrict ourselves to the case of Cobb-Douglas production functions. More precisely: we require that the elasticity with respect to material inputs is constant. While this assumption is not conceptually crucial, it is very useful for the counterfactual analysis. To see this, suppose for example we had a CES production function

$$y = \varphi\left((1-\gamma)\, l^{\frac{\zeta-1}{\zeta}} + \gamma x^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}}.$$

Let again Q denote the price of x and w denote the price of l. Hence, the price of the bundle is

$$\nu = \left(\gamma^{\zeta} Q^{1-\zeta} + (1-\gamma)^{\zeta} w^{1-\zeta}\right)^{\frac{1}{1-\zeta}},$$

so that

$$UC = \frac{1}{\varphi}\nu.$$

Note that by using (47), we can again write

$$\nu \quad = \quad s_M^{\frac{1}{\zeta-1}} \left(\frac{1}{\gamma}\right)^{\frac{\zeta}{\zeta-1}} Q = s_M^{\frac{1}{\zeta-1}} \left(\frac{1}{\gamma}\right)^{\frac{\zeta}{\zeta-1}} s_D^{\frac{1}{\varepsilon-1}} \left(\frac{1}{\beta}\right)^{\frac{\varepsilon}{\varepsilon-1}} \left(\frac{p_D}{q_D}\right), \tag{62}$$

where s_M is the expenditure share on materials. Hence,

$$UC \quad = \quad \frac{1}{\varphi}\left(\frac{1}{\gamma}\right)^{\frac{\zeta}{\zeta-1}} \left(\frac{1}{\beta}\right)^{\frac{\varepsilon}{\varepsilon-1}} \left(\frac{p_D}{q_D}\right) \times s_M^{\frac{1}{\zeta-1}} s_D^{\frac{1}{\varepsilon-1}} \propto s_M^{\frac{1}{\zeta-1}} s_D^{\frac{1}{\varepsilon-1}}. \tag{63}$$

With non-Cobb-Douglas preferences, the firm-specific part in firms' UC is given by

$$s_M^{\frac{1}{\zeta-1}} s_D^{\frac{1}{\varepsilon-1}}, \tag{64}$$

both of which are in principle observable. If we had an estimate of ζ, we could again use (64), the micro-data on $[s_M, s_D]$ and estimates for (ε, ζ) to simply measure firms' unit costs up to scale. However, (64) is less useful for the counterfactual analysis. The reason is that a move to say autarky would not only change firms' domestic expenditure shares from s_D to unity, but it would also endogenously change their material shares s_M. As inputs are cheaper (relative to labor) in the trade equilibrium, firms will substitute away from materials towards primary inputs when we take the economy to autarky. How strong the scope for such substitution is, depends on the underlying parameters and the quality and prices of domestic inputs.[63] To see this, note that the optimal material share is given

$$s_M \quad = \quad \frac{\gamma^\zeta Q^{1-\zeta}}{\nu^{1-\zeta}} = \frac{\gamma^\zeta Q^{1-\zeta}}{\gamma^\zeta Q^{1-\zeta} + (1-\gamma)^\zeta w^{1-\zeta}}. \tag{65}$$

Firms' unit costs in (63) can therefore be written as

$$UC \propto s_D^{\frac{1}{\varepsilon-1}} \left(\frac{\gamma^\zeta Q^{1-\zeta}}{\gamma^\zeta Q^{1-\zeta} + (1-\gamma)^\zeta w^{1-\zeta}}\right)^{\frac{1}{\zeta-1}} = s_D^{\frac{1}{\varepsilon-1}} \left(\frac{\gamma^\zeta s_D^{\frac{1-\zeta}{\varepsilon-1}} \left(\frac{1}{\beta}\right)^{\frac{\varepsilon(1-\zeta)}{\varepsilon-1}} \left(\frac{p_D}{q_D}\right)^{1-\zeta}}{\gamma^\zeta s_D^{\frac{1-\zeta}{\varepsilon-1}} \left(\frac{1}{\beta}\right)^{\frac{\varepsilon(1-\zeta)}{\varepsilon-1}} \left(\frac{p_D}{q_D}\right)^{1-\zeta} + (1-\gamma)^\zeta w^{1-\zeta}}\right)^{\frac{1}{\zeta-1}}.$$

[63] Recall that we are free to take labor as the numeraire.

Hence, the partial equilibrium firm-level gains gains from input trade are given

$$\frac{UC^{Aut}}{UC^{Trade}} = \left(\frac{\gamma^{\zeta} s_D^{\frac{1-\zeta}{\varepsilon-1}} \left(\frac{1}{\beta}\right)^{\frac{\varepsilon(1-\zeta)}{\varepsilon-1}} \left(\frac{p_D}{q_D}\right)^{1-\zeta} + (1-\gamma)^{\zeta} w^{1-\zeta}}{\gamma^{\zeta} \left(\frac{1}{\beta}\right)^{\frac{\varepsilon(1-\zeta)}{\varepsilon-1}} \left(\frac{p_D}{q_D}\right)^{1-\zeta} + (1-\gamma)^{\zeta} w^{1-\zeta}}\right)^{\frac{1}{\zeta-1}}$$

$$= \left(\frac{1 + \left(\frac{\gamma}{1-\gamma}\right)^{\zeta} s_D^{\frac{1-\zeta}{\varepsilon-1}} \left(\frac{1}{\beta}\right)^{\frac{\varepsilon(1-\zeta)}{\varepsilon-1}} \left(\frac{p_D/q_D}{w}\right)^{1-\zeta}}{1 + \left(\frac{\gamma}{1-\gamma}\right)^{\zeta} \left(\frac{1}{\beta}\right)^{\frac{\varepsilon(1-\zeta)}{\varepsilon-1}} \left(\frac{p_D/q_D}{w}\right)^{1-\zeta}}\right)^{\frac{1}{\zeta-1}} \tag{66}$$

In principle one can still determine the gains from input trade *without* any knowledge about the details of the trading environment (as Q does not feature in (66)). However, to evaluate (66), one requires many more parameters. We want to note, however, that the micro-data can again be useful to identify these directly if one is willing to make stringent assumptions. Using (65), we know that for a domestic firm we have

$$s_M^D = \frac{\left(\frac{\gamma}{1-\gamma}\right)^{\zeta} \left(\frac{1}{\beta}\right)^{\frac{\varepsilon(1-\zeta)}{\varepsilon-1}} \left(\frac{p_D/q_D}{w}\right)^{1-\zeta}}{\left(\frac{\gamma}{1-\gamma}\right)^{\zeta} \left(\frac{1}{\beta}\right)^{\frac{\varepsilon(1-\zeta)}{\varepsilon-1}} \left(\frac{p_D/q_D}{w}\right)^{1-\zeta} + 1}. \tag{67}$$

If the whole variation in firms' material shares was indeed given by some firms facing different prices through input trade (i.e. there are no firm-specific biases or input prices (i.e. β and p_D was equalized across firms), one could use (66) to arrive at

$$\frac{UC^{Aut}}{UC^{Trade}} = \left(\frac{1 + s_D^{\frac{1-\zeta}{\varepsilon-1}} \times \frac{s_M^D}{1-s_M^D}}{1 + \frac{s_M^D}{1-s_M^D}}\right)^{\frac{1}{\zeta-1}} = \left(1 - s_M^D + s_D^{\frac{1-\zeta}{\varepsilon-1}} \times s_M^D\right)^{\frac{1}{\zeta-1}}.$$

This expression only requires the micro-data on domestic and material shares and *two* elasticities of substitution, ε and ζ. However, it obviously requires much stronger assumption on the underlying heterogeneity to identify the structural parameters from the observed material shares of domestic firms. Note also that

$$\lim_{\zeta\to 1} ln\left(\frac{UC^{Aut}}{UC^{Trade}}\right) = \lim_{\zeta\to 1}\left[\frac{ln\left(1 - s_M^D + s_D^{\frac{1-\zeta}{\varepsilon-1}} \times s_M^D\right)}{\zeta - 1}\right] = \lim_{\zeta\to 1}\left[\frac{\partial}{\partial\zeta} ln\left(1 - s_M^D + s_D^{\frac{1-\zeta}{\varepsilon-1}} \times s_M^D\right)\right]$$

$$= \lim_{\zeta\to 1}\left[\frac{1}{1 - s_M^D + s_D^{\frac{1-\zeta}{\varepsilon-1}} \times s_M^D} s_M^D s_D^{\frac{1-\zeta}{\varepsilon-1}} \frac{1}{1-\varepsilon} ln\left(s_D\right)\right]$$

$$= \frac{s_M^D}{1-\varepsilon} ln\left(s_D\right).$$

For $\zeta \to 1$ we have that (see (67))

$$\lim_{\zeta \to 1} s_M^D = \frac{\left(\frac{\gamma}{1-\gamma}\right)}{\left(\frac{\gamma}{1-\gamma}\right)+1} = \gamma,$$

so that

$$ln\left(\frac{UC^{Aut}}{UC^{Trade}}\right) = \frac{\gamma}{1-\varepsilon} ln\left(s_D\right),$$

which is exactly our main expression given in (9).

A General production function for material services Let us now suppose that the aggregator for material services is given by

$$x = g\left(q_D z_D, x_I\right), \tag{68}$$

where x_I denotes the service flow of imported inputs. In the main version of the paper, we assumed that $g(.)$ is a CES with elasticity of substitution ε. Under (68) we can derive a "local" version of our main result. In that case the cost-minimization problem (45) takes the form

$$\min_z \left\{p_D z_D + \lambda\left(\Sigma, \theta\right) x_I \text{ s.t. } g\left(q_D z_D, x_I\right) \geq g_0\right\},$$

where again $\lambda(\Sigma, \theta)$ is the endogenous implicit price index given in (44). The optimality conditions are

$$\begin{aligned} p_D &= Q \frac{\partial g\left(q_D z_D, x_I\right)}{\partial x_D} q_D \\ \lambda\left(\Sigma, \theta\right) &= Q \frac{\partial g\left(q_D z_D, x_I\right)}{\partial x_I}. \end{aligned}$$

Now consider any shock to the trading environment, which affects $\lambda(\Sigma, \theta)$. Then $dUC = z_I d\lambda$,which yields

$$dln\left(UC\right) = \frac{dUC}{UC} = \frac{z_I \lambda}{UC} \frac{d\lambda}{\lambda} = \left(1 - s_D\right) dln\left(\lambda\right). \tag{69}$$

Using the optimality conditions we get

$$p_D = \lambda\left(\Sigma, \theta\right) q_D \frac{\partial g\left(q_D z_D, x_I\right)/\partial x_D}{\partial g\left(q_D z_D, x_I\right)/\partial x_I} = \lambda\left(\Sigma, \theta\right) q_D H\left(\frac{q_D z_D}{x_I}\right),$$

where

$$H\left(\frac{q_D z_D}{x_I}\right) \equiv \frac{\partial g\left(q_D z_D, x_I\right)/\partial x_D}{\partial g\left(q_D z_D, x_I\right)/\partial x_I} = \frac{\partial g\left(\frac{q_D z_D}{x_I}, 1\right)/\partial x_D}{\partial g\left(\frac{q_D z_D}{x_I}, 1\right)/\partial x_I} \equiv \frac{g_1'\left(\frac{q_D z_D}{x_I}, 1\right)}{g_2'\left(\frac{q_D z_D}{x_I}, 1\right)}.$$

Hence,

$$0 = dln\left(\lambda\right) + \frac{\partial ln\left(H\left(\frac{q_D z_D}{x_I}\right)\right)}{\partial ln\left(\frac{q_D z_D}{x_I}\right)} dln\left(\frac{q_D z_D}{x_I}\right). \tag{70}$$

Also

$$\frac{p_D z_D}{\lambda(\Sigma,\theta)\, x_I} = \frac{q_D z_D}{x_I} H\left(\frac{q_D z_D}{x_I}\right),$$

so that

$$dln\left(\frac{p_D z_D}{\lambda(\Sigma,\theta)\, x_I}\right) = \left(1 + \frac{\partial ln\left(H\left(\frac{q_D z_D}{x_I}\right)\right)}{\partial ln\left(\frac{q_D z_D}{x_I}\right)}\right) dln\left(\frac{q_D z_D}{x_I}\right). \tag{71}$$

As

$$dln\left(\frac{p_D z_D}{\lambda(\Sigma,\theta)\, x_I}\right) = dln\left(\frac{s_D}{1-s_D}\right) = \frac{1-s_D}{s_D} \times \frac{(1-s_D)+s_D}{(1-s_D)^2} ds_D = \frac{1}{1-s_D} dln(s_D), \tag{72}$$

we get from (70), (71) and (72) that

$$\begin{aligned} dln(\lambda) &= -\frac{\partial ln\left(H\left(\frac{q_D z_D}{x_I}\right)\right)}{\partial ln\left(\frac{q_D z_D}{x_I}\right)} dln\left(\frac{q_D z_D}{x_I}\right) \\ \frac{1}{1-s_D} dln(s_D) &= \left(1 + \frac{\partial ln\left(H\left(\frac{q_D z_D}{x_I}\right)\right)}{\partial ln\left(\frac{q_D z_D}{x_I}\right)}\right) dln\left(\frac{q_D z_D}{x_I}\right), \end{aligned}$$

which yields

$$dln(\lambda) = -\frac{\left(-\frac{1}{\varepsilon_L}\right)}{1-\frac{1}{\varepsilon_L}} \frac{1}{1-s_D} dln(s_D),$$

where

$$-\frac{1}{\varepsilon_L} \equiv \frac{\partial ln\left(H\left(\frac{q_D z_D}{x_I}\right)\right)}{\partial ln\left(\frac{q_D z_D}{x_I}\right)}$$

is the local elasticity of substitution. Substituting this into (69) yields

$$\begin{aligned} dln(UC) &= (1-s_D)\, dln(\lambda) = \frac{\frac{1}{\varepsilon_L}}{1-\frac{1}{\varepsilon_L}} dln(s_D) \\ &= -\frac{1}{1-\varepsilon_L} dln(s_D). \end{aligned} \tag{73}$$

Hence, the unit cost reductions of any small trade shocks are given by, which is simply the local version of our main result. In case the elasticity of substitution is constant, i.e. $\varepsilon_L = \varepsilon$, (73) can be integrated to yield (9).

9.2 Identification of Input-Output and Demand Structure

We use the French input-output tables from the OECD to discipline the demand parameters $[\alpha_s]_s$ and the matrix of input-output linkages Ξ. To determine Ξ, we focus on the distribution of the value added components, which determine the intermediate supply from each industry j for each industry s. As Ξ is the expenditure share by sourcing sector, we abstract from any taxes and subsidies so that:

$$\zeta_j^s = \frac{\text{Intermediate supply from industry } j \text{ for industry } s}{\text{Intermediate consumption at final prices from industry } s},$$

i.e. ζ_j^s measures the importance of sector j in the production process of sector s. By construction, this ensures that $\sum_{j=1}^{S} \zeta_j^s = 1$ for all s. We arrange the input-output matrix so that the columns contain the distribution of expenditure for the different sectors:

$$\Xi = \begin{bmatrix} \zeta_1^1 & \zeta_1^2 & & \zeta_1^S \\ \zeta_2^1 & & & \\ & & & \\ \zeta_S^1 & & & \zeta_S^S \end{bmatrix}.$$

To determine the distribution of final demand α, we also use the input-output tables as they contain information on the composition of final demand. Since there is no trade in final goods in the theory, we exclude any exports and imports in final goods in the data. More specifically, in the input-output tables we observe $HHFC_j$ as final consumption expenditure by households on sector j and hence set

$$\alpha^s = \frac{HHFC^s}{\sum_{j=1}^{S} HHFC^j}.$$

The OECD input-output tables report their data at the ISIC Rev. 3 level. This gives us 37 industries. Some of them fall outside the manufacturing industry so we cannot link them to the firm-level data, as prescribed by Proposition . To match this to our theory, we adopt the following procedure: we let sector S be the "residual sector" of the economy, which is not itself engaged in input trade but might still benefit from input trade if it is uses output of the manufacturing industry. To incorporate this sector in the analysis we first set

$$\alpha_S = 1 - \sum_{j \in M} \alpha_j, \tag{74}$$

where M is the set of manufacturing sectors. Similarly, we think of this sector as not being in engaged in input trade and hence set

$$\Lambda_S = 0.$$

The input-output structure from sector S can be recovered from the input-output tables. In particular we set

$$\zeta_j^S \equiv \frac{\sum_{n=1}^{NM} \text{Intermediate supply from industry } j \text{ for industry } n}{\sum_{n=1}^{NM} \text{Intermediate consumption at final prices from industry } n}.$$

where NM is the set of non-manufacturing sectors. Similarly, we have to measure the importance of materials in the production of this non-manufacturing sector, γ^S. While we estimate γ from the micro-data for manufacturing sectors, we measure it from the Input-Output Matrix for the non-manufacturing sector. As we observe value added and intermediary spending for each sector s, we set

$$\gamma^S = \frac{\sum_{n=1}^{NM} X_n}{\sum_{n=1}^{NM} (X_n + VA_n)}, \tag{75}$$

where X_n denotes total intermediary spending by sector n. Table (9.2) summarizes how the $\alpha' s$ and $\gamma' s$ are computed. As for the calculation of Ξ, we simply aggregate the data before we calculate $\left[\zeta_j^s\right]_{j,s}$ from the data on expenditure. In particular, let m_j^s be total spending of firms *in* sector s on intermediate goods produced by firms *from* sector j. The construction of this data is contained in Table (14). Given the aggregated data, the input-output matrix used in our empirical analysis is then contained in Table 15 below.

9.3 Estimating the Parameters of the Production Function

In the tables below we report the results of estimating the production function parameters using our different approaches. In Table 16 we report the results using the factor share approach. Note that this method imposes the assumption of constant returns, so that $\phi_{ks} + \phi_{ls} + \gamma_s = 1$. Table 17 reports the results based on proxy methods akin to Levinsohn and Petrin (2012) and Wooldridge (2009). In particular, we assume that labor is a dynamic input, which seems plausible given that the French economy is characterized by stringent regulations regarding hiring and firing. Note that we do not include firms' domestic share in material spending in the production function as we estimate ε in the second stage. Finally, Table 18 contains the full specification where we treat firms' domestic expenditure shares as a distinct input and estimate the parameter vector $(\phi_{ks}, \phi_{ls}, \gamma_s, \varepsilon_s)$ in one step.

9.4 Bootstrap

As explained in the paper, there are two reasons why our quantitative results are uncertain: there is sampling variation in the estimated structural parameters and our statistics Λ and s_D^{Agg} are random variables given our finite sample. To gauge how important these sources of uncertainty are, we depict the empirical distribution of ε, γ, Λ and λ in Figure 8. To estimate these distributions, we sample firms from our micro-data with replacement to construct 200 replicates of our micro-data. We then redo our analysis on these 200 samples, i.e. we reestimate ε and $[\gamma_s]_s$ and recalculate $[\Lambda_s]_s$ and $\left[s_{Ds}^{Agg}\right]_s$. Figure 8 depicts the distribution of these variables. For the three sector-level variables we simply report the distribution of the sectoral averages. We also depict our point estimates from our main analysis by the red vertical lines. While the variation in γ and s_D^{Agg} is relatively modest, there is a quite a bit of uncertainty regarding ε. This is consistent with the non-negligible standard errors reported in Table 2. It is this variation in ε, which induces most of the variation in Λ.

9.5 The Bias of Aggregate Data

Tables 19 and 20 report the gains from input trade for different values of the elasticity of substitution ε. Columns three and four replicate the results for our baseline estimate $\varepsilon = 2.37$. While column three reports the results based on the micro-data, column four reports the aggregate gains $\frac{\gamma}{1-\varepsilon} ln\left(s_D^{Agg}\right)$,

Direct Data						Aggregation		
ISIC	α	Value Added	Intermediate Purchases	γ	Λ	Coarse Classification	α	γ
1	α_1	VA_1	X_1	$\frac{X_1}{X_1+VA_1}$	0	Non-Manufacturing	α_S from (74)	γ_S from (75)
2	α_2	VA_2	X_2	$\frac{X_2}{X_2+VA_2}$	0			
3					0			
...					0			
10	α_{10}	VA_{10}	X_{10}	Estimate from micro-data	"Read off" from micro-data	Manufacturing	α_{10}	γ_{10}
16								
...								
39	α_{39}	VA_{39}	X_{39}				α_{39}	γ_{39}
40	α_{40}	VA_{40}	X_{40}	$\frac{X_{40}}{X_{40}+VA_{40}}$	0	Non-Manufacturing	α_S from (74)	γ_S from (75)
...					0			
99	α_{99}	VA_{99}	X_{99}	$\frac{X_{99}}{X_{99}+VA_{99}}$	0			

Notes: We calculated α_s from the distribution of final consumer demand in France. Also: Gross output is sometimes not exactly equal to the sum of value added and intermediate purchases. This is due to taxes and subsidies. As these are not in our theory, we abstract from them.

Table 13: Details of Empirical Implementation

Original data									
ISIC	*1*	*2*	..	*10*	*39*	*40*	...	*99*	
1	m_1^1	m_2^1						m_{99}^1	*NM*
2	m_2^1								*NM*
...									*NM*
10	m_{10}^1								*M*
...	..								*M*
39	m_{39}^1								*M*
40	m_{40}^1								*NM*
...	...								*NM*
99	m_{99}^1								*NM*
Interm. Cons.	$m^1 = \sum_{j=1}^{99} m_j^1$	...	..					$m^{99} = \sum_{j=1}^{99} m_j^{99}$	

$\Longrightarrow$

Aggregated Data				
ISIC	*10*	...	*39*	*S*
10	m_{10}^{10}			$m_{10}^S = \sum_{n \in NM} m_{10}^n$
...	...			...
39	m_{39}^{10}			
S	$m_S^{10} = \sum_{n \in NM} m_n^{10}$			$m_S^S = \sum_{j,n \in NM} m_n^j$
Interm. Cons.	m^1		m^{39}	m^S

Table 14: Details of Empirical Implementation: Input-Output-Linkages

Sector	10-14	15-16	17-19	20	21-22	23	24	25	26	27	28	29	30	31	32	33	34	35	36-37	NM
10-14	8.69	0.12	0.00	0.02	0.41	59.62	1.80	0.26	9.83	4.85	0.36	0.12	0.05	0.16	0.07	0.12	0.00	0.00	0.82	0.87
15-16	0.48	21.33	2.27	0.10	0.51	0.09	1.97	0.24	0.06	0.17	0.17	0.12	0.10	0.12	0.18	0.18	0.04	0.04	0.48	2.99
17-19	0.26	0.11	46.79	0.08	0.65	0.02	0.80	1.39	0.44	0.27	0.17	0.34	0.28	0.42	0.63	0.87	1.40	0.48	5.95	0.39
20	1.33	0.38	0.13	30.47	1.07	0.01	0.17	0.38	2.06	0.33	0.39	0.29	0.09	0.23	0.32	0.32	0.29	0.41	15.90	0.60
21-22	1.44	2.29	1.73	1.45	44.73	0.19	3.02	2.27	2.82	0.60	0.63	1.01	0.82	1.20	1.37	2.11	0.46	0.52	2.74	3.07
23	4.08	0.72	0.69	0.77	0.68	14.06	4.41	0.68	2.18	2.73	0.49	0.51	0.23	0.27	0.26	0.44	0.34	0.19	1.02	2.66
24	4.53	1.25	8.25	1.69	5.03	3.08	39.28	40.03	3.31	2.92	3.02	1.78	0.57	4.42	0.84	1.17	1.93	1.03	4.17	1.96
25	1.68	2.46	2.36	0.65	1.67	1.49	3.03	15.72	1.44	0.66	2.49	3.65	1.29	6.58	5.07	3.23	6.57	1.64	4.93	0.97
26	8.53	0.79	0.19	0.81	0.21	0.05	0.81	0.66	21.53	0.88	0.90	0.67	0.11	0.81	1.50	2.08	1.23	0.36	1.86	1.82
27	0.62	0.09	0.40	0.64	0.80	0.28	0.76	1.67	3.24	41.61	24.72	9.00	0.48	11.46	2.50	5.66	8.44	2.28	9.25	0.35
28	5.95	1.39	1.16	3.51	0.98	1.30	2.13	1.78	2.26	6.92	28.04	15.87	0.75	13.11	5.97	8.31	8.02	4.84	4.72	1.40
29	20.33	0.79	1.66	3.31	0.87	0.58	0.78	2.79	2.60	2.00	4.04	19.27	0.64	2.28	1.99	3.37	4.21	3.33	3.01	1.54
30	0.00	0.02	0.04	0.36	0.08	0.00	0.02	0.08	0.36	0.04	0.24	0.48	37.61	0.35	1.27	2.02	0.00	0.07	0.17	0.35
21	0.51	0.14	0.08	0.28	0.35	0.31	0.34	0.21	0.79	1.07	2.24	4.43	3.93	16.03	6.83	2.84	3.07	1.05	1.39	1.14
32	0.90	0.06	0.07	0.09	0.15	0.28	0.13	0.39	0.37	0.34	0.59	3.81	12.59	11.00	30.30	11.86	1.98	4.26	1.50	0.71
33	0.07	0.01	0.08	0.03	0.12	0.12	0.12	0.22	0.57	0.39	0.61	2.29	2.59	2.72	8.55	19.31	1.52	8.74	0.42	0.61
34	0.68	0.09	0.12	0.22	0.12	0.17	0.07	0.07	0.47	0.21	0.25	0.69	0.13	0.27	0.22	0.24	35.30	0.14	0.42	0.83
35	0.15	0.01	0.04	0.04	0.05	0.01	0.04	0.05	0.06	0.05	0.11	1.31	0.02	0.03	0.03	0.05	0.11	46.37	0.05	1.14
36-37	0.20	0.17	0.59	0.42	0.80	0.06	0.18	0.23	0.43	1.34	0.98	0.76	0.31	0.61	0.34	0.52	1.94	0.37	6.85	0.46
NM	39.56	67.77	33.34	55.05	40.72	18.28	40.12	30.88	45.19	32.61	29.55	33.61	37.40	27.94	31.76	35.29	23.12	23.90	34.35	76.16

Table 15: Input-Output-Linkages: $\Xi = \left[\zeta_j^s\right]$

Industry	ISIC	ϕ_k		ϕ_l		γ	
Mining	10-14	0.374***	(0.039)	0.293***	(0.017)	0.333***	(0.043)
Food, tobacco, beverages	15-16	0.098***	(0.004)	0.177***	(0.003)	0.725***	(0.006)
Textiles and leather	17-19	0.081***	(0.003)	0.293***	(0.009)	0.626***	(0.012)
Wood and wood products	20	0.113***	(0.004)	0.285***	(0.006)	0.602***	(0.006)
Paper, printing, publishing	21-22	0.134***	(0.007)	0.362***	(0.011)	0.504***	(0.011)
Chemicals	24	0.124***	(0.008)	0.204***	(0.01)	0.671***	(0.014)
Rubber and plastics products	25	0.124***	(0.005)	0.289***	(0.007)	0.587***	(0.011)
Non-metallic mineral products	26	0.178***	(0.01)	0.294***	(0.012)	0.529***	(0.015)
Basic metals	27	0.124***	(0.01)	0.202***	(0.015)	0.674***	(0.021)
Metal products (ex machinery and equipment)	28	0.108***	(0.002)	0.412***	(0.008)	0.479***	(0.009)
Machinery and equipment	29	0.071***	(0.003)	0.313***	(0.015)	0.616***	(0.018)
Office and computing machinery	30	0.037***	(0.012)	0.150***	(0.032)	0.813***	(0.04)
Electrical machinery	31	0.096***	(0.008)	0.306***	(0.011)	0.598***	(0.014)
Radio and communication	32	0.055***	(0.006)	0.322***	(0.048)	0.624***	(0.052)
Medical and optical instruments	33	0.071***	(0.004)	0.435***	(0.026)	0.494***	(0.029)
Motor vehicles, trailers	34	0.106***	(0.009)	0.135***	(0.016)	0.759***	(0.014)
Transport equipment	35	0.152***	(0.019)	0.499***	(0.03)	0.349***	(0.044)
Manufacturing, recycling	36-37	0.084***	(0.003)	0.283***	(0.009)	0.633***	(0.012)

Table 16: Production Function Coefficient Estimates, by 2-digit Sector: Factor Shares

Industry	ISIC	ϕ_k		ϕ_l		γ	
Mining	10-14	0.647***	(0.101)	0.626***	(0.087)	0.295**	(0.139)
Food, tobacco, beverages	15-16	0.174***	(0.010)	0.274***	(0.009)	0.538***	(0.060)
Textiles and leather	17-19	0.216***	(0.026)	0.513***	(0.037)	0.481***	(0.096)
Wood and wood products	20	0.138***	(0.023)	0.414***	(0.024)	0.521***	(0.058)
Paper, printing, publishing	21-22	0.061***	(0.022)	0.717***	(0.033)	0.600***	(0.099)
Chemicals	24	0.027	(0.081)	0.142	(0.134)	1.304***	(0.336)
Rubber and plastics products	25	0.148***	(0.031)	0.536***	(0.050)	0.357***	(0.115)
Non-metallic mineral products	26	0.221***	(0.037)	0.539***	(0.037)	0.357***	(0.119)
Basic metals	27	0.104	(0.096)	0.381***	(0.087)	0.481*	(0.263)
Metal products (ex machinery and equipment)	28	0.252***	(0.013)	0.655***	(0.016)	0.231***	(0.036)
Machinery and equipment	29	0.186***	(0.018)	0.563***	(0.025)	0.393***	(0.066)
Office and computing machinery	30	0.170**	(0.081)	0.574***	(0.110)	0.104	(0.210)
Electrical machinery	31	0.144***	(0.031)	0.448***	(0.044)	0.449***	(0.123)
Radio and communication	32	0.123**	(0.057)	0.565***	(0.114)	0.568***	(0.207)
Medical and optical instruments	33	0.231***	(0.021)	0.501***	(0.020)	0.421***	(0.073)
Motor vehicles, trailers	34	0.082	(0.091)	0.316**	(0.127)	0.765**	(0.355)
Transport equipment	35	0.194***	(0.073)	0.686***	(0.089)	0.997***	(0.256)
Manufacturing, recycling	36-37	0.242***	(0.018)	0.472***	(0.016)	0.430***	(0.055)

Table 17: Production Function Coefficient Estimates, by 2-digit Sector: 2 Step GMM

Industry	ISIC	ϕ_k		ϕ_l		γ		ε	
Mining	10-14	0.679***	(0.100)	0.617***	(0.088)	0.257*	(0.148)	0.450	(0.451)
Food, tobacco, beverages	15-16	0.173***	(0.010)	0.278***	(0.009)	0.512***	(0.063)	1.976***	(0.166)
Textiles and leather	17-19	0.247***	(0.029)	0.555***	(0.035)	0.354***	(0.101)	2.279***	(0.743)
Wood and wood products	20	0.172***	(0.025)	0.437***	(0.025)	0.441***	(0.061)	2.548***	(0.324)
Paper, printing, publishing	21-22	0.089***	(0.022)	0.726***	(0.032)	0.465***	(0.099)	2.189***	(0.383)
Chemicals	24	0.072	(0.056)	0.250***	(0.084)	1.064***	(0.229)	-6.586	(6.818)
Rubber and plastics products	25	0.163***	(0.032)	0.584***	(0.055)	0.253**	(0.128)	2.442**	(1.103)
Non-metallic mineral products	26	0.221***	(0.036)	0.526***	(0.036)	0.354***	(0.121)	1.869***	(0.358)
Basic metals	27	0.151	(0.120)	0.540***	(0.177)	-0.084	(0.583)	0.892	(0.692)
Metal products (ex machinery and equipment)	28	0.254***	(0.014)	0.667***	(0.016)	0.191***	(0.036)	1.368***	(0.0796)
Machinery and equipment	29	0.183***	(0.018)	0.553***	(0.024)	0.381***	(0.065)	2.191***	(0.279)
Office and computing machinery	30	0.132	(0.087)	0.451***	(0.110)	0.126	(0.222)	2.358	(3.485)
Electrical machinery	31	0.138***	(0.028)	0.460***	(0.037)	0.428***	(0.110)	2.806***	(0.863)
Radio and communication	32	0.132*	(0.069)	0.614***	(0.144)	0.442*	(0.264)	3.147	(2.733)
Medical and optical instruments	33	0.237***	(0.021)	0.498***	(0.020)	0.372***	(0.073)	2.062***	(0.295)
Motor vehicles, trailers	34	0.089	(0.087)	0.354***	(0.120)	0.706**	(0.344)	3.741*	(2.088)
Transport equipment	35	0.170**	(0.081)	0.664***	(0.094)	0.955***	(0.282)	4.482	(4.371)
Manufacturing, recycling	36-37	0.263***	(0.019)	0.468***	(0.017)	0.361***	(0.059)	1.765***	(0.183)

Table 18: Production Function Coefficient Estimates, by 2-digit Sector: 1 Step GMM

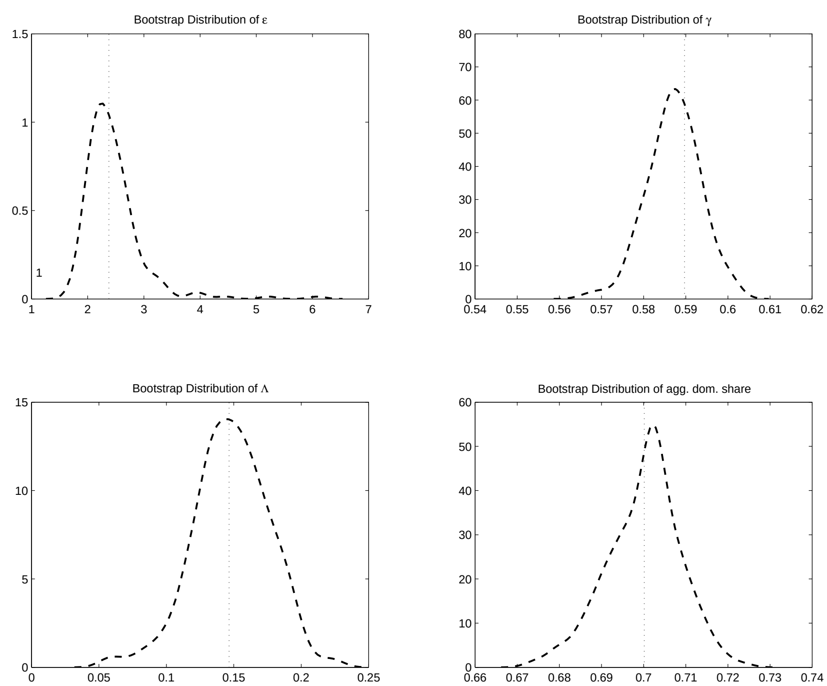


Figure 8: Bootstrap Distribution of structural parameters and the sectoral gains from trade.

	Micro-data	Aggregate Data				
			ε			
	2.378	2.378	3	4	5	6
Entire Economy	9.04	9.9	6.72	4.43	3.31	2.64
Manufacturing Sector	27.52	30.8	20.32	13.12	9.69	7.68

Notes: This table contains the consumer price gains from input trade using aggregate data (i.e. s_D^{Agg}) for different values of the elasticity of substitution ε. In the first two columns we report the baseline results for comparison. The remaining columns report the respective gains as a function of ε. Row 1 contains the results for the entire economy, row two focuses on the manufacturing sector.

Table 19: The Consumer Price Gains for different values of ε

there s_D^{Agg} denotes the aggregate domestic share. These results correspond to the ones reported in Tables 5 and 6. In the remaining columns we report the aggregate gains for different values of ε, which are consistent with studies using aggregate data to estimate the trade elasticity. In particular, Costinot and Rodriguez-Clare (2014) take $\varepsilon = 4$ as a baseline. The tables show clearly that the results are very sensitive to the value of ε. Table 20 shows that the economy-wide gains predicted by an aggregative approach under $\varepsilon = 4$ are about 50% lower than the gains predicted by the approach that relies on micro-data. Thus, analyses that ignore the micro-data can result in substantial biases in the estimates of the gains from trade through the use of an inappropriate trade elasticity.

9.6 Welfare in the Multi-Sector Model

We now characterize welfare in the multi-sector economy of Section 3.2. We will show that (40) is indeed the special case for a single sector economy. We assume that trade is balanced and that the value of exports in sector s is given by $\alpha_s^{ROW} \times IM$, where IM denotes the value of total spending on imported inputs.

Proposition 4. *Consider the setup above and let $[l_{\Sigma i}]_i$ denote the resource loss associated with firms' equilibrium sourcing strategies $[\Sigma_i]_i$. Let W and W^{Aut} denote the total welfare in the trade equilibrium and autarky respectively. Then*

$$\frac{W}{W^{Aut}} = \frac{I/P}{I^{Aut}/P^{Aut}} = \frac{I}{I^{Aut}} \times \frac{P^{Aut}}{P},$$

where I is total consumer income. In particular

$$I = L + \sum_{s=1}^{S} \frac{1}{\sigma_s} S_s - \sum_{s=1}^{S} \left(\int_i^{N_s} l_{\Sigma_i} di \right) \quad (76)$$

$$I^{Aut} = L + \sum_{s=1}^{S} \frac{1}{\sigma_s} S_s^{Aut}, \quad (77)$$

Industry	ISIC	Micro-data	Aggregate Data				
			ε				
		2.378	2.378	3	4	5	6
Mining	10-14	2.96	2.5	1.72	1.14	0.86	0.68
Food, tobacco, beverages	15-16	11.06	12.62	8.53	5.61	4.18	3.33
Textiles and leather	17-19	31.14	31.87	20.99	13.55	10	7.92
Wood and wood products	20	8.23	9.58	6.51	4.29	3.2	2.55
Paper, printing, publishing	21-22	12.15	10.96	7.43	4.89	3.65	2.91
Chemicals	24	27.23	28.14	18.62	12.06	8.91	7.07
Rubber and plastics products	25	20.12	21.53	14.37	9.37	6.95	5.52
Non-metallic mineral products	26	13.42	13.29	8.98	5.9	4.39	3.5
Basic metals	27	21.8	28.83	19.07	12.34	9.12	7.23
Metal products (ex machinery and equipment)	28	8.17	7.7	5.24	3.47	2.59	2.07
Machinery and equipment	29	17.64	18.23	12.23	7.99	5.94	4.72
Office and computing machinery	30	20.42	37	24.22	15.56	11.45	9.06
Electrical machinery	31	19.77	21.64	14.45	9.41	6.98	5.55
Radio and communication	32	21.55	22.15	14.78	9.62	7.13	5.67
Medical and optical instruments	33	17.9	15.9	10.7	7.01	5.21	4.15
Motor vehicles, trailers	34	6.24	11.23	7.61	5.01	3.73	2.98
Transport equipment	35	15.32	11.83	8.01	5.27	3.93	3.13
Manufacturing, recycling	36-37	12.87	14.06	9.48	6.23	4.63	3.69
Non-manufacturing		0	0	0	0	0	0

Notes: This table contains the consumer price gains from input trade using aggregate data (i.e. s_D^{Agg}) for different values of the elasticity of substitution ε. In the first two columns we report the baseline results for comparison. The remaining columns report the respective gains as a function of ε.

Table 20: The Sectoral Consumer Price Gains for Different Values of ε

where $[S_j]$ *and* $\left[S_j^{Aut}\right]$ *solve*

$$S_s = \alpha_s \left(L - \sum_{j=1}^{S} \left(\int_i^{N_j} l_{\Sigma_i} di \right) + \sum_{j=1}^{S} \frac{1 + \frac{\zeta_s^j}{\alpha_s} \gamma_j (\sigma_j - 1)}{\sigma_j} S_j \right) + \sum_{j=1}^{S} \left[\alpha_s^{ROW} - \zeta_s^j \right] \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_{i=0}^{N_j} (1 - s_{D,i}) \omega_i di, \tag{78}$$

where $\omega_i = \frac{va_i}{\int_i^{N_s} va_i di}$ *and*

$$S_s^{Aut} = \alpha_s \left(L + \sum_{j=1}^{S} \frac{1 + \frac{\zeta_s^j}{\alpha_s} \gamma_j (\sigma_j - 1)}{\sigma_j} S_j^{Aut} \right), \tag{79}$$

respectively. Furthermore, $\frac{P^{Aut}}{P} = G$ is given in Proposition 2 above.

Proof. As labor is the only factor of production, the consumer's budget constraint (39) implies that consumer welfare is given by real income

$$W = \frac{I}{P},$$

where P is the domestic price level and I denotes consumer income, which is given by

$$I = L + \sum_{s=1}^{S} \left(\int_{i=0}^{N_s} \pi_i di \right).$$

Here L denotes total labor income and π_i denotes firm i's profits. To derive π_i, recall that firms in sector s have a mark-up of $\frac{\sigma_s}{\sigma_s - 1}$ so that variable profits gross of any extensive margin resource loss are given by

$$\pi_i^V = (p_i - UC_i) y_i = \frac{1}{\sigma_s} p_i y_i. \tag{80}$$

Total revenue for firm i is given by

$$p_i y_i = \left(\frac{p_i}{P_s} \right)^{1-\sigma_s} S_s, \tag{81}$$

where P_s is the CES price index for differentiated products in sector s and S_s denotes total spending for sector s goods. Hence,

$$\pi_i = \frac{1}{\sigma_s} p_i y_i - l_{\Sigma_i} = \frac{1}{\sigma_s} \left(\frac{p_i}{P_s} \right)^{1-\sigma_s} S_s - l_{\Sigma_i},$$

so that

$$\begin{aligned} I &= L+\sum_{s=1}^{S}\left(\int_{i}^{N_s}\frac{1}{\sigma_s}\left(\frac{p_i}{P_s}\right)^{1-\sigma_s}S_s di\right)-\sum_{s=1}^{S}\left(\int_{i}^{N_s}l_{\Sigma_i}di\right) \\ &= L+\sum_{s=1}^{S}\frac{1}{\sigma_s}S_s\left(\int_{i}^{N_s}\left(\frac{p_i}{P_s}\right)^{1-\sigma_s}di\right)-\sum_{s=1}^{S}\left(\int_{i}^{N_s}l_{\Sigma_i}di\right) \\ &= L+\sum_{s=1}^{S}\frac{1}{\sigma_s}S_s-\sum_{s=1}^{S}\left(\int_{i}^{N_s}l_{\Sigma_i}di\right). \end{aligned} \tag{82}$$

Hence, given $[S_s]_s$ and $[l_{\Sigma i}]_i$, total income I is fully determined. Now consider $[S_s]_s$. Note that

$$S_s = S_s^C + S_s^X + S_s^{ROW}, \tag{83}$$

i.e. total spending stems from consumers, intermediary producers and the rest of the world. For our economy we have that $S_c^C = \alpha_s I$ and $S_s^{ROW} = \alpha_s^{ROW} Im$ as consumers spend a fraction α_s of their income I on sector s products and balanced trade requires that total spending by the rest of the world is equal to the value of imports Im, a fraction α_s^{ROW} of which is spent on sector s products. To derive S_s^X, let total domestic intermediary purchases in sector j be given by X_j. Then

$$S_s^X = \sum_{j=1}^{S}\zeta_s^j X_j. \tag{84}$$

Letting m_i be total material spending by firm i and s_i be total spending by firm i, we know that

$$\begin{aligned} X_j &= \int_{i=0}^{N_j} s_{D,i}m_i di = \int_{i=0}^{N_j} s_{D,i}\gamma_j s_i di = \int_{i=0}^{N_j} s_{D,i}\gamma_j\frac{\sigma_j-1}{\sigma_j}p_i y_i di \\ &= \gamma_j\frac{\sigma_j-1}{\sigma_j}S_j\int_{i=0}^{N_j} s_{D,i}\left(\frac{p_i}{P_j}\right)^{1-\sigma_j}di, \end{aligned} \tag{85}$$

where we used that firms in sector j spend a fraction γ_j of their total input spending s_i on materials and that total spending s_i accounts for a fraction $\frac{\sigma_j-1}{\sigma_j}$ of revenue (with the remaining $\frac{1}{\sigma_j}$ accounting for profits (see (80))). Hence, (84) and (85) imply that

$$S_s^X = \sum_{j=1}^{S}\zeta_s^j\gamma_j\frac{\sigma_j-1}{\sigma_j}S_j\int_{i=0}^{N_j} s_{D,i}\left(\frac{p_i}{P_s}\right)^{1-\sigma_s}di. \tag{86}$$

Similarly, total import spending is equal to

$$\begin{aligned} Im &= \sum_{j=1}^{S} Im_j = \sum_{j=1}^{S}\int_{i=0}^{N_j}\left(1-s_{D,i}\right)m_i di \\ &= \sum_{j=1}^{S}\gamma_j\frac{\sigma_j-1}{\sigma_j}S_j\int_{i=0}^{N_j}\left(1-s_{D,i}\right)\left(\frac{p_i}{P_s}\right)^{1-\sigma_s}di. \end{aligned} \tag{87}$$

Hence (86) and (87) imply that

$$
\begin{aligned}
S_s &= \alpha_s I + \alpha_s^{ROW} \left(\sum_{j=1}^{S} \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_{i=0}^{N_j} (1 - s_{D,i}) \left(\frac{p_i}{P_j} \right)^{1-\sigma_j} di \right) + \sum_{j=1}^{S} \zeta_s^j \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_{i=0}^{N_j} s_{D,i} \left(\frac{p_i}{P_j} \right)^{1-\sigma_j} di \\
&= \alpha_s I + \sum_{j=1}^{S} \zeta_s^j \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j + \alpha_s^{ROW} \left(\sum_{j=1}^{S} \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_{i=0}^{N_j} (1 - s_{D,i}) \left(\frac{p_i}{P_j} \right)^{1-\sigma_j} di \right) - \sum_{j=1}^{S} \zeta_s^j \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_{i=0}^{N_j} (1 \\
&= \alpha_s I + \sum_{j=1}^{S} \zeta_s^j \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j + \sum_{j=1}^{S} \left[\alpha_s^{ROW} - \zeta_s^j \right] \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_{i=0}^{N_j} (1 - s_{D,i}) \left(\frac{p_i}{P_j} \right)^{1-\sigma_j} di.
\end{aligned}
$$

Using 82, we get that

$$
\begin{aligned}
S_s &= \alpha_s \left(L + \sum_{j=1}^{S} \frac{1}{\sigma_j} S_j - \sum_{j=1}^{S} \left(\int_i^{N_j} l_{\Sigma_i} di \right) \right) + \sum_{j=1}^{S} \zeta_s^j \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j + \sum_{j=1}^{S} \left[\alpha_s^{ROW} - \zeta_s^j \right] \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_{i=0}^{N_j} (1 - s_{D,i}) \left(\right. \\
&= \alpha_s \left(L - \sum_{j=1}^{S} \left(\int_i^{N_j} l_{\Sigma_i} di \right) + \sum_{j=1}^{S} \frac{1 + \frac{\zeta_s^j}{\alpha_s} \gamma_j (\sigma_j - 1)}{\sigma_j} S_j \right) + \sum_{j=1}^{S} \left[\alpha_s^{ROW} - \zeta_s^j \right] \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_{i=0}^{N_j} (1 - s_{D,i}) \left(\frac{p_i}{P_j} \right)
\end{aligned}
$$

Now note that

$$
va_i = p_i y_i - m_i = p_i y_i - \gamma_s s_i = p_i y_i - \gamma_s \frac{\sigma_s - 1}{\sigma_s} p_i y_i = \left(1 - \gamma_s \frac{\sigma_s - 1}{\sigma_s} \right) p_i y_i
$$

so that

$$
\frac{va_i}{\int_i^{N_s} va_i di} = \frac{p_i y_i}{\int_i^{N_s} p_i y_i di} = \frac{\left(\frac{p_i}{P_s} \right)^{1-\sigma_s} S_s}{\int_i^{N_s} \left(\frac{p_i}{P_s} \right)^{1-\sigma_s} S_s di} = \left(\frac{p_i}{P_s} \right)^{1-\sigma_s}.
$$

Hence,

$$
S_s = \alpha_s \left(L - \sum_{j=1}^{S} \left(\int_i^{N_j} l_{\Sigma_i} di \right) + \sum_{j=1}^{S} \frac{1 + \frac{\zeta_s^j}{\alpha_s} \gamma_j (\sigma_j - 1)}{\sigma_j} S_j \right) + \sum_{j=1}^{S} \left[\alpha_s^{ROW} - \zeta_s^j \right] \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_{i=0}^{N_j} (1 - s_{D,i}) \omega_i di \qquad (88)
$$

where $\omega_i = \frac{va_i}{\int_i^{N_s} va_i di}$. Hence, given $L^{NET} = L - \sum_{j=1}^{S} \left(\int_i^{N_j} l_{\Sigma_i} di \right)$, (88) are S equations in S unknowns S_s, which we can easily solve. Now consider the case of autarky. There we have $l_{\Sigma_i} = 0$ and $s_{D,i} = 1$. Hence, (88) yields

$$
S_s^{Aut} = \alpha_s \left(L + \sum_{j=1}^{S} \frac{1 + \frac{\zeta_s^j}{\alpha_s} \gamma_j (\sigma_j - 1)}{\sigma_j} S_j^{Aut} \right).
$$

In the case of a single sector (i.e. $S = 1$) it has to be the case that

$$
\alpha_S = \alpha_S^{ROW} = \zeta_S^S = 1.
$$

Hence,

$$S^{Aut} = L + \frac{1+\gamma\left(\sigma-1\right)}{\sigma} S^{Aut} = \frac{\sigma}{\left(1-\gamma\right)\left(\sigma-1\right)} L.$$

Substituting this in (82) yields

$$I^{Aut} = L + \frac{1}{\sigma} S = \frac{1+\left(1-\gamma\right)\left(\sigma-1\right)}{\left(1-\gamma\right)\left(\sigma-1\right)} L.$$

Similarly, we get from (88) that

$$\sum_{j=1}^{S} \left[\alpha_s^{ROW} - \zeta_s^j\right] \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_{i=0}^{N_j} \left(1 - s_{D,i}\right) \omega_i di = 0$$

so that

$$\begin{aligned} S &= \frac{\sigma}{\left(1-\gamma\right)\left(\sigma-1\right)} \left(L - \left(\int_i^N l_{\Sigma_i} di\right)\right) \\ I &= \frac{1+\left(1-\gamma\right)\left(\sigma-1\right)}{\left(1-\gamma\right)\left(\sigma-1\right)} \left(L - \left(\int_i^N l_{\Sigma_i} di\right)\right). \end{aligned}$$

This implies directly (40). This concludes the proof of Propositions 3 and 4. □

9.7 Characterizing the Model with Fixed Costs

Consider the setup in the paper, in particular suppose Assumption 1 holds true. First of all we are going to derive (35). Under Assumption 1, the price index of the imported bundle (89) is given by

$$A\left(\overline{q}, \varphi\right) = \left(\int_{\overline{q}}^{\infty} \left(p\left(q\right)/q\right)^{1-\rho} dG\left(q\right)\right)^{\frac{1}{1-\rho}} = \left(\int_{\overline{q}}^{\infty} \left(q/p\left(q\right)\right)^{\rho-1} dG\left(q\right)\right)^{\frac{1}{1-\rho}} \tag{89}$$

$$= \alpha \left(\int_{\overline{q}}^{\infty} q^{(1-\nu)(\rho-1)} dG\left(q\right)\right)^{-\frac{1}{\rho-1}} \tag{90}$$

As quality follows a Pareto distribution, (89) implies

$$A\left(\overline{q}, \varphi\right) = A\left(\overline{q}\right) = \alpha \left(q_{\min}^{\theta} \frac{\theta}{\theta - \left(1-\nu\right)\left(\rho-1\right)} \overline{q}^{(1-\nu)(\rho-1)-\theta}\right)^{-\frac{1}{\rho-1}}. \tag{91}$$

The optimal number of varieties n is related to the chosen cutoff $\overline{q}$ via

$$n = P\left(q \geq \overline{q}\right) = \left(\frac{q_{\min}}{\overline{q}}\right)^{\theta}. \tag{92}$$

Substituting into (91) yields

$$\begin{aligned} A(n) &= \alpha\left(q_{\min}^{(1-\nu)(\rho-1)}\frac{\theta}{\theta-(1-\nu)(\rho-1)}n^{\frac{\theta-(1-\nu)(\rho-1)}{\theta}}\right)^{-\frac{1}{\rho-1}} \\ &= \alpha\left(E\left[q\right]\right)^{-(1-\nu)}\left(\frac{\theta-1}{\theta}\right)^{-(1-\nu)}\left(\frac{\theta}{\theta-(1-\nu)(\rho-1)}\right)^{\frac{1}{1-\rho}}n^{-\left(\frac{1}{\rho-1}-\frac{1-\nu}{\theta}\right)}, \end{aligned}$$

which is the required expression. In terms of the "auxiliary" parameters used in the main text, we have

$$z = \alpha\left(E\left[q\right]\right)^{-(1-\nu)}\left(\frac{\theta-1}{\theta}\right)^{-(1-\nu)}\left(\frac{\theta}{\theta-(1-\nu)(\rho-1)}\right)^{\frac{1}{1-\rho}} \tag{93}$$

$$\eta = \frac{1}{\rho-1}-\frac{1-\nu}{\theta}. \tag{94}$$

Finally, note that $\eta > 0$ follows from Assumption 1.

Given this functional form for $A(n)$, we can characterize firms' extensive margin of importing. This is the content of the following Proposition.

Proposition 5. *Consider the setup above and suppose that*

$$\eta(\varepsilon-1) < 1 \text{ and } \eta\gamma(\sigma-1) < 1. \tag{95}$$

Then, the optimal share of countries firms import from is given by a function $n(\varphi, f)$ and a productivity cutoff $\overline{\varphi}(f)$, such that $n(\varphi, f) = 0$ for $\varphi \leq \overline{\varphi}(f)$ with $\overline{\varphi}(\cdot)$ increasing. Furthermore, $n(\varphi, f)$ is increasing in productivity (φ), decreasing in the fixed costs of sourcing (f) and increasing in average import quality (z). Furthermore, it is increasing in the price-adjusted domestic quality (q_D/p_D) if and only if $\gamma(\sigma-1) > (\varepsilon-1)$.

Proof. Suppose first that the firm's profit function conditional on importing is indeed concave. The necessary and sufficient condition for the optimal number of sourcing countries is then given by

$$\Gamma\frac{\gamma(\sigma-1)}{\varepsilon-1}\left((q_D/p_D)^{\varepsilon-1}+\left(\frac{1}{z}\right)^{\varepsilon-1}n^{\eta(\varepsilon-1)}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}-1}\eta(\varepsilon-1)\left(\frac{1}{z}\right)^{\varepsilon-1}n^{\eta(\varepsilon-1)-1}\varphi^{\sigma-1} = wf, \tag{96}$$

where Γ contains aggregate variables, which are determined in general equilibrium but which firms take as given. Rearranging terms yields the implicit definition of $n(\varphi, f)$ as

$$\left((q_D/p_D)^{\varepsilon-1}+\left(\frac{1}{z}\right)^{\varepsilon-1}n(\varphi,f)^{\eta(\varepsilon-1)}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}-1}\eta\left(\frac{1}{z}\right)^{\varepsilon-1}n(\varphi,f)^{\eta(\varepsilon-1)-1} = \frac{1}{\gamma(\sigma-1)}\frac{w}{\Gamma}\frac{f}{\varphi^{\sigma-1}}.$$

Given this optimal sourcing strategy $n(\varphi, f)$, total profits are

$$\pi^{IM}(\varphi,f) = \Gamma\left((q_D/p_D)^{\varepsilon-1}+\left(\frac{1}{z}\right)^{\varepsilon-1}n(\varphi,f)^{\eta(\varepsilon-1)}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}\varphi^{\sigma-1}-w\left(fn(\varphi,f)+f^I\right).$$

If the firms was not importing, its profits were

$$\pi^D(\varphi, f) = \Gamma(q_D/p_D)^{\gamma(\sigma-1)}\varphi^{\sigma-1}.$$

Hence, the firm is an importer as long as $\pi^{IM}(\varphi, f) \geq \pi^D(\varphi, f)$, i.e.

$$\left[\left(1+\left(\frac{p_D}{q_D}\left(\frac{1}{z}\right)n(\varphi,f)^\eta\right)^{\varepsilon-1}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}-1\right](q_D/p_D)^{\gamma(\sigma-1)}\frac{\Gamma}{w}\varphi^{\sigma-1} > fn(\varphi,f)+f^I. \tag{97}$$

Let us now prove that n is indeed unique. The marginal product per sourcing country is given in (96) as

$$\pi'(n) = \Psi\left((q_D/p_D)^{\varepsilon-1}+\left(\frac{1}{z}\right)^{\varepsilon-1}n^{\eta(\varepsilon-1)}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}-1}\left(\frac{1}{z}\right)^{\varepsilon-1}n^{\eta(\varepsilon-1)-1}-wf, \tag{98}$$

where $\Psi = \Gamma\gamma(\sigma-1)\eta\varphi^{\sigma-1} > 0$. Hence,

$$\begin{aligned}
\pi''(n) &= \Psi\left(\frac{1}{z}\right)^{\varepsilon-1}\frac{\partial}{\partial n}\left\{\left((q_D/p_D)^{\varepsilon-1}+z^{\varepsilon-1}n^{\eta(\varepsilon-1)}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}-1}n^{\eta(\varepsilon-1)-1}\right\} \\
&= \Psi\left(\frac{1}{z}\right)^{\varepsilon-1}\left((q_D/p_D)^{\varepsilon-1}+z^{\varepsilon-1}n^{\eta(\varepsilon-1)}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}-1}\frac{n^{\eta(\varepsilon-1)-2}}{\eta(\varepsilon-1)-1}\times \\
&\quad \left\{1+\frac{(\gamma(\sigma-1)-\varepsilon+1)\eta}{(\eta(\varepsilon-1)-1)}\frac{\left(\frac{1}{z}\right)^{\varepsilon-1}n^{\eta(\varepsilon-1)}}{(q_D/p_D)^{\varepsilon-1}+\left(\frac{1}{z}\right)^{\varepsilon-1}n^{\eta(\varepsilon-1)}}\right\}.
\end{aligned}$$

The optimal choice of n is unique if π is concave, i.e. if $\pi''(n) < 0$. Note that we can write $\pi''(n)$ as

$$\pi''(n) = \Psi(n)\times\frac{1}{(\eta(\varepsilon-1)-1)^2}\left\{(\eta(\varepsilon-1)-1)+(\gamma(\sigma-1)-\varepsilon+1)\eta\sigma(n)\right\}.$$

where $\Psi(n) > 0$ and $\sigma(n) = \frac{\left(\frac{1}{z}\right)^{\varepsilon-1}n^{\eta(\varepsilon-1)}}{(q_D/p_D)^{\varepsilon-1}+\left(\frac{1}{z}\right)^{\varepsilon-1}n^{\eta(\varepsilon-1)}} \in [0,1]$. For π to be concave in n we therefore need that

$$(\eta(\varepsilon-1)-1)+(\gamma(\sigma-1)-\varepsilon+1)\eta\sigma(n) < 0. \tag{99}$$

A necessary and sufficient condition for this to be the case is $\eta(\varepsilon-1) < 1$ and $\eta\gamma(\sigma-1) < 1$ given in the Proposition. Sufficiency is trivial. To see that it is also necessary, suppose one the conditions was not satisfied. Then we could always find some γ, ε and $\sigma(n)$ such that (99) was not satisfied. The comparative static results on $n(\varphi, f)$ and $\overline{\varphi}(f)$ follow directly from (97) and (96). This proves Proposition 5. □

Note that our calibrated and estimated parameters satisfy these restrictions (see Table 7).

9.8 Estimation of η

For our baseline results, we estimate (43) on the subsample of firm-product pairs which source their respective products from at least two supplier countries. There is a sizable number of firm-product pairs in our data which are sourced from a single country and we are concerned that such single-variety interactions may not credibly identify the extensive margin of varieties but rather pick-up other variation across firms. The results are contained in Table 21.

Dep. Variable: $ln(\frac{1-s_D}{s_D})$						
	All Importers				> 1 variety	> 2 varieties
ln (Number of Varieties)	1.308***	0.707***	0.733***	0.739***	0.526***	0.463***
	(0.009)	(0.010)	(0.010)	(0.010)	(0.011)	(0.019)
ln (Capital / Employment)				-0.070***		
				(0.006)		
Exporter Dummy			-0.395***	-0.388***	-0.254***	-0.198***
			(0.013)	(0.013)	(0.017)	(0.029)
International Group			0.150***	0.174***	0.216***	0.223***
			(0.016)	(0.016)	(0.016)	(0.019)
Control for Num of products	No	Yes	Yes	Yes	Yes	Yes
Implied Eta	0.950***	0.513***	0.532***	0.536***	0.382***	0.336***
	(0.260)	(0.142)	(0.147)	(0.148)	(0.106)	(0.096)
Observations	120,344	120,344	120,344	120,344	73,651	35,751

Table 21: Estimating η

Notes: To back out a value for η we use our benchmark $\varepsilon = 2.378$ from Section 4.1 above.

Column 5 contains our baseline results of (43), where we add additional firm level controls that can affect firms' import behavior conditional on the number of varieties sourced. The implied value of η is 0.382 and it is precisely estimated. The remaining columns contain robustness checks. When we include firm-product pairs that are sourced from a single trading partner the estimated η increases, as single-variety importers have very high domestic shares in the data. The first column shows that it is important to control for the number of products sourced as import-intensive firms source both more varieties per-product and more products on international markets - without the product fixed effects, the estimated η increases substantially.[64] Columns three and four show that the estimate of η is virtually unaffected by additional firm-level controls. For our quantitative analysis we take column 5 as a benchmark but in principle the analysis that follows can be done for other values of η.

[64]Recall that the parameter η is a combinations of different structural parameters of the economy. While η is indeed sufficient to characterize the aggregate gains from trade, one might be interested to decompose the returns to international sourcing into the the elasticity of substitution across varieties ρ, the dispersion in input quality θ, and the elasticity of input prices with respect to quality ν. Using our estimate of η, we need two additional moments for identification. It turns out that this can be done in a very tractable way using reduced form methods. The two crucial additional pieces of information required are import prices (to identify ν) and data on firms' expenditure shares across trading partners (to identify θ).

9.9 Calibrating the Model of Section 5

Our calibration strategy is as follows: In Proposition 4 above, we derived all allocation and market-clearing prices as a function of parameters, the joint distribution of productivity and domestic shares $G(\varphi, s_D)$ and a given level of fixed cost labor $[l_{\Sigma,i}]_i$. To calibrate the model, we will now only have to make sure that (a) the distribution of domestic shares is consistent with firms' profit maximization problem, (b) that it is consistent with moments in the data and (c) that it satisfies all equilibrium conditions. We are going to process in two steps. First we will show that that firms' policy functions for their domestic expenditure shares depends on general equilibrium prices only through two aggregate statistics. Then we will show that we can calibrate the model in "normalized" parameters and then use the general equilibrium prices to back out the underlying structural parameters. This will ensure that all equilibrium conditions are satisfied even though we never have to solve for the equilibrium as a fixed point of firms' optimal behavior. For notational simplicity we will consider the single sector model used in Section 5. The multi-sector case is analogous.

Firms' optimal import behavior is fully determined by (96) and (97). From the price index $A = zn^{-\eta}$, we get that n and s^D are related via

$$n^{\eta(\varepsilon-1)} = \left(\frac{1-s_D}{s_D}\right) z^{\varepsilon-1} \left(\frac{q_D}{P_D}\right)^{\varepsilon-1}.$$

Substituting this in (96) and (97) yields

$$\Gamma\frac{\gamma(\sigma-1)}{\varepsilon-1}\left(\frac{q_D}{P_D}\right)^{\gamma(\sigma-1)-(\varepsilon-1)}\left(\frac{1}{s_D}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}-1}\eta(\varepsilon-1)\left(\frac{1}{z}\right)^{\varepsilon-1}\left(\left(\frac{1-s_D}{s_D}\right)^{\frac{1}{\eta(\varepsilon-1)}} z^{1/\eta}\left(\frac{q_D}{P_D}\right)^{1/\eta}\right)^{\eta(\varepsilon-1)-1}\varphi^{\sigma-1} = wf. \tag{100}$$

Rearranging terms yields

$$(s_D)^{\frac{1-\gamma(\sigma-1)\eta}{\varepsilon-1}}(1-s_D)^{1-\frac{1}{\eta(\varepsilon-1)}} = \frac{\tilde{f}}{\varphi^{\sigma-1}}, \tag{101}$$

where

$$\tilde{f} = z^{1/\eta}\left(\frac{P_D}{q_D}\right)^{\gamma(\sigma-1)-1/\eta}\frac{w}{\Gamma\gamma(\sigma-1)\eta}\times f \tag{102}$$

Similarly, (97) implies that the firm is an importer as long as

$$\left[s_D^{-\frac{\gamma(\sigma-1)}{\varepsilon-1}}-1\right]\varphi^{\sigma-1} > \left(\frac{1-s_D}{s_D}\right)^{\frac{1}{\eta(\varepsilon-1)}}\gamma(\sigma-1)\eta\tilde{f}+\tilde{f}^I, \tag{103}$$

where

$$\tilde{f}^I = \frac{w}{\Gamma}\frac{1}{(q_D/p_D)^{\gamma(\sigma-1)}}\times f^I. \tag{104}$$

From (101) and (103) it is clear that we can solve for firms' optimal domestic share (and import participation) as a function of $\left(\varphi^{\sigma-1}, \tilde{f}, \tilde{f}^I\right)$. Hence, we can directly calibrate $\tilde{f}^I$ and the parameters of the joint distribution of $\left(\varphi, \tilde{f}\right)$. This fully determines p_D. Note also that

$$l_{\Sigma,i} = fn_i + f^I = \left(\frac{q_D}{P_D}\right)^{\gamma(\sigma-1)} \frac{\Gamma}{w} \left[\left(\frac{1-s_D}{s_D}\right)^{\frac{1}{\eta(\varepsilon-1)}} \gamma\left(\sigma-1\right)\eta\tilde{f} + \tilde{f}^I\right].$$

Given the calibration, the term in brackets is know. As q_D can be normalized, w can be taken as the numeraire and Γ depends on total spending, we can use the results of Proposition 4 to solve for aggregate output, total spending and hence $\int l_{\Sigma,i}di$. We can then use (102) and (104) to solve for the underlying structural parameters. Note in particular, that z is not required to solve for any allocations, given the domestic shares. Hence, the calibration only identifies $fz^{1/\eta}$.