Not so Great Expectations: A Model of Growth and Informational Frictions*

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ABSTRACT

I develop a model of asset markets with dispersed private information in a continuous-time, macroeconomic setting where firm managers learn from financial prices when making their investment decisions. I derive a tractable equilibrium that highlights a feedback loop between investor trading behavior and firm real investment. While the strength of real signals for the expectations of managers and investors is procyclical, financial signals are strongest during downturns and recoveries. Through this channel, contamination in price signals during financial crises can distort expectations to be more pessimistic, and lead to deeper recessions and slower recoveries. I explore the asset pricing and policy implications of my model, as well as several conceptual issues that it raises for empirical analysis.

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I. Introduction

In this paper, I introduce a tractable, dynamic framework for studying the feedback loop in learning that occurs between financial markets and firm managers when financial markets aggregate investor private information about the productivity of real investment. Through this informational channel, financial market prices are more important for learning than real activity at the trough of business cycles, and are most informative as signals about investment productivity during downturns and recoveries. My analysis establishes a link between recessions with financial origins and slow recoveries by illustrating how financial crises during downturns can delay recoveries by distorting firm manager expectations, which depresses real investment and feeds back into the incentives for financial market participants to trade on their private information.

Two observations motivate my investigation. The first is that market prices aggregate the private information of investors about macroeconomic and financial conditions, and that firms, in making their real decisions, respond to this useful information.[1] Since the mid-1980's, however, the rapid growth of the market-based financial system (Pozsar et al 2012), especially from 2002-2007 (Philippon (2008)), has increased financial opacity, as intermediaries extended credit and diversified risk through securitization and the OTC derivatives markets that arose in the wake of LTCM.[2] This heightened opacity has made it difficult for economic agents and policymakers to assess not only the depth of financial distress once a bust occurs, but also its distribution across the financial sector. This was particularly relevant in the recent recession, as regulators scrambled to map out the cross-party linkages of the unregulated financial system in late 2008 (FCIC 2011). As a result, market prices have become noisier signals about the strength of the economy, and economic actors, both real and financial, face more severe informational frictions.

That asset prices contain useful information about the macroeconomy has been well-documented in the literature.[3] Both during and in the aftermath of the financial crisis,

[1] See, for instance, Luo (2005), Chen, Goldstein, and Jiang (2007) and Bakke and Whited (2010). For evidence that firms learn from their own profit realizations, the other key signal in our model, see, for instance, Moyen and Platikanov (2013).

[2] Former FRBNY President and Treasury Secretary Timothy Geitner, in fact, made it part of his agenda before the financial crisis to move the OTC derivatives market onto exchanges to increase transparency.

[3] For stock prices, for instance, see Fama (1981), Barro (1990), and Beaudry and Portier (2006), while for credit spreads, see Gertler and Lown (1999), Gilchrist, Yankov, and Zakrasjek (2009), Gilchrist and Zakrajsek (2012), and Ng and Wright (2013), and for a wide cross-section of asset classes, see Stock and Watson (2003) and Andreu, Ghysels, and Kourtellos (2013).

many viewed the dramatic fall in asset prices as a signal that the US economy was entering a recession potentially as deep as the Great Depression.[4] When the stock market bottomed out in March 2009, in fact, the Michigan Survey of Consumers "fear of a prolonged depression" question had its lowest score since the 1991 recession.

The second observation is that recessions with financial origins appear to be deeper and have slower recoveries. A salient feature of the recent US experience, for instance, is the anemic economic recovery compared to previous cycles, especially in GDP, lending, and productivity (Haltmaier (2012), Reifschneider et al (2013)). As highlighted in a speech by former Federal Reserve Chairman, Ben Bernanke, this weak growth in productivity following the 2007 to 2009 recession represents "a puzzle whose resolution is important for shaping expectations about longer-term growth" (Bernanke (2014)). While there is growing evidence that financial crises lead to deeper recessions, however, it is less clear if, and how, they also slow recoveries.[5] My model provides a framework for addressing conceptual questions about business cycles and uncertainty that explicitly incorporates a financial sector, and can also help explain why financial shocks have asymmetric impacts over the business cycle (Aizenman et al (2012)).[6] The uncertainty I consider here that distorts investment arises from learning, and is therefore different from that in Bloom (2009), which focuses on shocks to firm fundamental volatility. It is also different from the policy uncertainty featured in Fernández-Villaverde et al (2013), which is over future corporate tax policies, and in Baker, Bloom, and Davis (2013).

Informational frictions can lead firms to voluntarily withdraw from investment because of weak expectations about the state of the economy, rather than from uncertainty itself, a phenomenon which can help explain several stylized facts. First, the FRB Senior Loan Officer Survey cites weak credit demand as a reason for the low level of C&I loans until the end of 2010. Second, since the recession, firms have been increasing the cash on their

[4] For evidence regarding the fall in the stock market, see, for instance, Robert Barro's March 2009 WSJ Article "What Are the Odds of a Depression?" that accompanies Barro and Ursúa (2009), and Gerald Dwyer's September 2009 article, "Stock Prices in the Financial Crisis" from FRB Atlanta's Notes from the Vault.

[5] While studies like Reinhart and Rogoff (2009a,b, 2011), Ng and Wright (2013), and Jorda, Schularick and Taylor (2013), for instance, argue that financial crises result in slower recoveries, others such as Haltmaier (2012) and Stock and Watson (2012) find little difference, and those such as Bloom (2009), Muir (2014), and Bordo and Haubrich (2012) predict faster upswings following financial crises.

[6] For instance, while the S&L crisis and the bursting of the housing bubble accompanied recessions that had slow recoveries, the collapse of Long-Term Capital Management (LTCM) in 1998, arguably an event that almost led to the meltdown of the whole financial system, had no significant impact on the real economy.

balance sheets and saving their income as retained earnings rather than investing (Baily and Bosworth (2013), Sanchez and Yurgadul (2013), Kliesen (2013)).[7] Third, firms appear reluctant to fill vacancies, as studies such as Daly et al (2012) and Leduc and Liu (2013) find a potential shift in the Beveridge Curve after the recent recession, which reflects a higher vacancy rate compared to the unemployment rate, while Davis, Faberman, and Haltiwanger (2013) document a fall in recruiting intensity. Though, for simplicity, my model will only involve capital, the same forces depressing real investment would also depress labor market demand in a more general framework. This evidence suggests that the slow recovery may, at least in part, be driven by firms choosing to delay investment because of a persistent poor economic outlook.

To study the implications of learning in the presence of informational frictions for financial market trading and real activity, I integrate the classic information aggregation framework of Grossman and Stiglitz (1980) and Hellwig (1980) into a standard, general equilibrium macroeconomic model in continuous-time. This setting allows me not only to examine the dynamic, real consequences of informational frictions when there is a feedback loop between real activity and financial markets, but also to depart from the CARA-normal and risk-neutral-normal frameworks, which are less desirable for addressing macroeconomic questions, and to study agents with log utility without the need for approximation. Both tasks have posed a well-known and substantial challenge in the information aggregation literature, and separate strands have developed to examine feedback in each direction. A finance literature, including Albagi (2010), Goldstein, Ozdenoren, and Yuan (2013), and Subrahmanyam and Titman (2013), examines how asset prices impact real activity through the learning channel, while a macroeconomic literature, including Angeletos, Lorenzoni, and Pavan (2012), investigates how real investment decisions are distorted by the ability to manipulate asset prices in the presence of informational frictions. I am able to make progress by appealing to the local linearity inherent in working in continuous-time, as well as to a standard assumption about the information structure of households and a convenient functional form for firm real investment.

The model presented herein features a continuum of non-overlapping generations of households that trade riskless debt and claims to the assets of firms in centralized financial mar-

[7] Pinkowitz, Stulz, and Williamson (2013) provide evidence that this increase in cash holdings is driven by perceived low investment opportunities by firms, since it is concentrated among the highly profitable firms in their sample.

kets. Households here represent the hedge funds, financial analysts, intermediaries, and other investors that participate in financial markets. Households each possess a private signal regarding the underlying strength of the economy when they trade, and are subject to preference shocks that reflect their private liquidity needs. Asset prices in my economy aggregate the private information of agents, and liquidity shocks represent a source of noise that prevents them from being fully revealing to both households and firms. To avoid both the infinite regress problem of Townsend (1983) and a time-varying correlation between the wealth of households and the persistence of their beliefs, I follow Allen, Morris, and Shin (2006), Bacchetta and van Wincoop (2008), and Straub and Ulbricht (2013) and assume that, though households in each generation pass along their wealth to their children, they do not pass along their private information. This assumption of investor myopia is necessary to maintain tractability in learning by helping me avoid these two issues.

Perfectly competitive, identical firms in my economy produce output and are run by managers who use financial prices, which aggregate private information dispersed among households, and real signals from production to form their expectations about the underlying state of the economy when making investment decisions. This introduces a channel for liquidity shocks from financial markets to feed into real activity by distorting the expectations of firm managers, since the impact of financial shocks on prices cannot be fully disentangled from fundamental trading. By affecting the returns on their securities and the informativeness of real economic signals through their investment choices, firms, in turn, impact the incentives of investors to trade on their private information to take advantage of the uncertain economic environment. This can lead to an adverse feedback loop that exacerbates real shocks to the economy during downturns that can deepen and lengthen recessions.

With these ingredients, I derive a tractable, linear noisy rational expectations equilibrium that offers several insights about learning from real and financial signals over the business cycle when there is this feedback loop. First, time-varying second moments are important for macroeconomic dynamics even without the real-options "wait-and-see" channel of Bernanke (1983) and Bloom (2009) for investment. In most environments with learning and asymmetric information, the conditional variance of beliefs is either constant or deterministically converging toward a (possibly trivial) limit. In my setting, this conditional variance varies stochastically with the level of investment, and this gives rise to countercyclical uncertainty in the economy. The second insight is that, while real signals about the macroeconomy are procyclical in their informativeness in learning, similar to the mechanism in Van Nieuwerburgh and Veldkamp (2006), financial signals are strongest during downturns and recoveries. This feature arises because households have dispersed information and trade more aggressively against each other when there is uncertainty about the state of the economy, and this increase in trading leads more of their private information to be incorporated into prices. The strength of the financial signal trades off the return to investment with the level of uncertainty in the economy, and these two quantities are negatively correlated over the business cycle. Finally, nonlinearity in investment slows recoveries since the informativeness of real and financial signals is tied to real investment. As investment falls, both real and financial signals weaken, which leads uncertainty to remain high and persistent until investment recovers. Real signals flatten because firms are less active, and financial signals flatten because the value of household private information anchors on the return to real investment.

I next offer an explanation of the slow US recovery in the context of my mechanism as stemming from confusion in financial price signals brought about by the financial crisis. This confusion led real investment to fall further during the recession and real and financial signals to flatten, which made it more difficult for agents to act on the recovery. I characterize welfare in the economy and identify a role for policy in improving the provision of public information about current economic conditions, since investors and firms do not fully internalize the benefit of the information that their activities produce.

Lastly, I turn to some of the empirical implications of my framework. I illustrate how informational frictions give rise to an informational component in risk premia. This component has predictive power for future returns and real activity, which varies with the level of uncertainty and investment in the economy. It also gives rise to business cycle varation in asset turnover based on informational trading. I then conclude by discussing how taking advantage of the business cycle behavior of financial market signals can help macroeconomic forecasting, as well as conceptual issues that informational frictions raise for identifying structural shocks originating from financial markets.

II. Related Literature

I view my amplification mechanism from feedback in learning as playing a contributing role in transmitting financial shocks to the real US economy to bring about deeper recessions and anemic recoveries, and frame it as being complementary to other channels highlighted in the macroeconomics literature linking recessions and financial crises. My paper is also part of several literatures on asymmetric information and the real consequences of asset prices. I discuss my relation to each of these literatures in turn.

Most such studies focus on the balance sheet and/or collateral channels for financial crises to amplify real shocks and depress real activity. He and Krishnamurthy (2012), for instance, explores the quantitative impact of the balance sheet channel for constrained intermediaries, while Mian and Sufi (2012) examines empirically how the deleveraging of household balance sheets can prolong recessions through debt overhang. A slow recovery explained purely by intermediary balance sheet impairment, for instance, is difficult to reconcile with the quick recapitalization of banks by early 2009 because of the TARP and SCAP programs. An explanation based purely on credit constraints confronts the empirical challenges that C&I loan terms had, on average, loosened to around 2005 levels by mid-2011, according to the FRB Senior Loan Officer Survey, and that corporate bond markets continued to function both during and after the recession.[8]

The channel I highlight is also distinct from those in other models of financial opacity, such as Gorton and Ordoñez (2012), Dang, Gorton, and Holmström (2013), and Hanson and Sunderam (2013). These studies tend to focus on the time-inconsistency in the design of informationally-insensitive securities that are deployed as collateral in lending agreements. Through a similar mechanism, Moreira and Savov (2013) attempt to explain the slow US recovery in the context of neglected risk and the fragility of the shadow banking system. A similar literature, which includes Kobayashi and Nutahara (2007), Kobayashi, Nakajima, and Inaba (2012), and Gunn and Johri (2013), explores the impact of news shocks on business cycles in the presence of financial market imperfections, such as collateral constraints or costly state verification.

My work is related to the literature on dynamic models of asymmetric information, such as Foster and Viswanathan (1996), He and Wang (1995), and Allen, Morris and Shin (2006), which do not have real sectors and feature static economic environments where the asset's fundamental is fixed. Foster and Viswanathan (1996) models strategic, dynamic trading between investors with private information and a market maker in a static informational environment, while He and Wang (1995) examines the impact on trading volume when investors trade on public signals and dynamic private information in the presence of persistent

[8] According to sifma statistics, for example, US Corporate Bond and ABS issuance, for instance, actually climbed in 2009.

noise supply shocks. Allen, Morris, and Shin (2006) and Bacchetta and van Wincoop (2006, 2008) investigate the role of higher-order expectations introduced by dispersed information in the determination of asset prices, and Nimark (2012) extends these implications to the term structure of interest rates. Albagi, Hellwig, and Tsyvinski (2013) rationalizes the credit spread puzzle with dynamic dispersed information and the nonlinear payoff profile of debt, and neither has a real sector nor long-lived incomplete information about the firm's fundamentals. My study focuses on the impact on asset prices and real activity when agents learn not only from endogenous information in prices generated by dispersed information, but also from the endogenous information in the return process governing the asset's time-varying fundamentals. To my knowledge, my work is also one of the first studies to study the long-run implications of a dynamic model of asymmetric information.

While my work exploits the local linearity of continuous-time and a non-overlapping generational informational structure for investors to help maintain tractability, the literature has developed other settings of information aggregation that deliver tractable equilibria outside of the CARA-Normal paradigm. Albagi, Hellwig, and Tsyvinski (2012), for instance, construct an equilibrium with log-concavity and an unboundedness assumption on the distribution of private signals that delivers a sufficient statistic for the market price as the private signal of the marginal trader. Goldstein, Ozdenoren, and Yuan (2013) and Albagi, Hellwig, and Tsyvinski (2012, 2014) employ risk-neutral agents with normally-distributed asset fundamentals and position limits to deliver tractable nonlinear equilibria in a static setting. Other papers like Sockin and Xiong (2014a,b) develop analytic log-linear equilibria in a static setting by exploiting Cobb-Douglas utility with fundamentals that have log-normal distributions. Straub and Ulbricht (2013) makes use of a conjugate prior framework with one period-lived, risk-neutral agents to maintain tractability in learning in a dynamic setting.

My work also contributes to the literature on informational frictions and the macroeconomy, which include Greenwood and Jovanovic (1990), Woodford (2003), Van Nieuwerburgh and Veldkamp (2006), Lorenzoni (2009), Kurlat (2013), Angeletos and La'O (2013), Blanchard, L'Huillier, and Lorenzoni (2013), Straub and Ulbricht (2013), Hassan and Mertens (2014a,b), Fajgelbaum, Schaal, and Taschereau-Dumouchel (2014), and David, Hopenhayn, and Venkateswaran (2014).[9] Only Straub and Ulbricht (2013), Hassan and Mertens (2014a,b),

[9]There is also a large literature on quantifying the impact of news shocks, which stresses the informational asymmetry between private agents and the econometrician, as well as situations in which agents have incomplete information. For a survey of this literature, see Beaudry and Portier (2013).

and David, Hopenhayn, and Venkateswaran (2014) consider the real consequences of informational frictions with centralized asset market trading to aggregate information. Informational frictions are, however, static in Hassan and Mertens (2014a,b), because of the assumption of perfect consumption insurance across agents, and in David, Hopenhayn, and Venkateswaran (2014), who focus on resource misallocation across firms from imperfect information, because firms observe their fundamentals after revenue is realized each period.[10] Straub and Ulbricht (2013) explore the feedback loop between learning and the collateral channel, which destroys information during busts when agents become financially constrained because of a decline in the value of collateral with an exogenous, but hidden fundamental.[11] My focus instead is on the adverse feedback between asset prices and real investment that arises through the persistent distortion of the beliefs that govern real investment. In contrast to models like Albagi (2010), Kurlat (2013), and Straub and Ulbricht (2013), my learning mechanism does not arise because of financial frictions, but only informational frictions, which implies, for instance, that relieving credit conditions for firms will do little in my setting to improve economic conditions.

Finally, my paper also relates to the growing literature on the real effects of asset prices, which includes Bray (1981), Subrahmanyam and Titman (2001), Albagi (2010), Tinn (2010), Goldstein, Ozdenoren, and Yuan (2011, 2013), Angeletos, Lorenzoni, and Pavan (2012), Ordoñez (2012), Albagi, Hellwig, and Tsyvinski (2014), Sockin and Xiong (2014), and Gao, Sockin, and Xiong (2014).[12] Goldstein, Ozdenoren, and Yuan (2013) explores the coordination motive among financial investors when stock prices inform real investment decisions, while Albagi (2010) examines the distortion to real investment that occurs when financial market participants face funding constraints. Angeletos, Lorenzoni, and Pavan (2012) investigates the distortion to real investment and financial prices in a sequential game when entrepreneurs make investment decisions before claims are sold to the market to rationalize the dot com bubble. Albagi, Hellwig, and Tsyvinski (2014) highlights the inefficiency that asymmetric information introduces into real investment when existing shareholders extract informational rent by making investment decisions before selling shares to imperfectly-

[10] In my setting, firms face more severe information frictions than in Hassan and Mertens (2014) and David, Hopenhayn, and Venkateswaran (2014) because they neither observe private signals nor the past history of the realized fundamental. As a result, learning occurs more slowly and uncertainty about the fundamental fluctuates endogenously over time.

[11] In a similar spirit, a working paper version of Kurlat (2013) illustrates how adverse selection in asset markets can lead to countercylical uncertainty when there is incomplete information.

[12] See Bond, Edmans, and Goldstein (2012) for a survey of this literature.

informed capital markets. Tinn (2010) features a similar setup to Angeletos, Lorenzoni, and Pavan (2012) of perfectly informed entrepreneurs selling to investors who observe a noisy public signal, where uncertainty is short-lived and again entrepreneurs have superior information to market participants. My dynamic model features feedback both from real investment to the beliefs and trading incentives of financial market participants, as in Tinn (2010), Angeletos, Lorenzoni, and Pavan (2012), Ordoñez (2012), and Albagi, Hellwig, and Tsyvinski (2014), and from financial markets back to real investment, as in Albagi (2010), Goldstein, Ozdenoren, and Yuan (2013), and Sockin and Xiong (2014) for firms, and Gao, Sockin and Xiong (2014) for home buyers. In contrast to these studies, my focus is on the dynamic consequences for real activity of learning from endogenous real and financial signals.

III. A Model of Informational Frictions

A. The Environment

I consider an infinite-horizon production economy in continuous-time on a probability triple $(\Omega, \mathbf{F}, \mathcal{P})$ equipped with a filtration $\mathcal{F}_t$. There are three fundamental shocks in the economy $\left\{Z_t^\theta, Z_t^\xi, Z_t^k\right\}$ which are standard independent Weiner processes. To focus on the impact of informational frictions in financial markets on real activity, I turn off the conventional channels for financial markets to feed back to real activity through financial frictions in borrowing and lending.

There are perfectly competitive, identical firms in the economy that manage capital K_t for households with which they produce output Y_t according to

$$Y_t = aK_t,$$

for $a > 0$. Firm managers are able to grow capital according to

$$\frac{dK_t}{K_t} = (I_t\theta_t - \delta)\, dt + \sigma_k dZ_t^k, \tag{1}$$

where I_t is investment per unit of assets, θ_t is the productivity of real investment in installing new capital, similar to the investment-specific technology shock of Greenwood, Hercowitz, and Krusell (1997, 2000), δ is depreciation, and Z_t^k is a Total Factor Productivity (TFP) shock to existing capital. Importantly, the productivity of real investment θ_t is unobservable to firm managers and all other economic agents in the economy.[13] It evolves according to an Ornstein-Uhlenbeck process

$$d\theta_t = \lambda\left(\bar{\theta} - \theta_t\right)dt + \sigma_\theta dZ_t^\theta, \tag{2}$$

which has the known solution, found by applying Itô's Lemma to $e^{\lambda t}\theta_t$ and integrating from 0 to t,

$$\theta_t = \theta_0 e^{-\lambda t} + \bar{\theta}\left(1 - e^{-\lambda t}\right) + \int_0^t \sigma_\theta e^{\lambda(s-t)} dZ_s^\theta. \tag{3}$$

The OU process is the continuous-time analogue of an AR(1) process in discrete-time and has a mean-reverting drift and iid shocks.[14]

Households consume the output from firms and invest in two assets in the economy: claims to the cash flows of the assets of firms which have price q_t and in (locally) riskless debt, which is an inside asset, with instantaneous interest rate r_t. Importantly, both assets are traded in centralized asset markets, so that prices are observable to both households and firm managers when forming their expectations about θ_t.

B. *Households*

There is a continuum $\mathcal{I} = [0, 1]$ of risk-averse households that are part of a non-overlapping generational structure with wealth $w_t(i)$ that invest in firm claims and riskless debt. Each household invests a fraction $x_t(i)$ of its wealth $w_t(i)$ in firm claims, which are perfectly divisible, and $1 - x_t(i)$ in riskless debt. I index time for households as $t, t + \Delta t, t + 2\Delta t$ and consider the continuous-time limit when Δt is of the order dt. Households have log utility over flow consumption $\log c_t(i)$ and subjective discount rate ρ over the bequest utility $v_{t+\Delta t}(i)$ they leave to future generations. I work with bequest utility instead of a preference over final wealth, as in He and Krishnamurthy (2012), to derive several asset pricing relationships relevant to the problem of firms. All prices, however, are ultimately pinned down by market clearing and not these relationships. Since households have log utility, and are therefore

[13] Kogan and Papanikolaou (2013) consider a setting where agents are trying to learn about the growth opportunities of firms and know the investment-specific technology shock.

[14] Theoretically, it is possible for θ_t to take negative values, similar to dividends in Wang (1993) and Campbell and Kyle (1993), though one can choose parameter values so that this occurs with negligible probability. Since beliefs over θ_t must be absolutely continuous with respect to the true distribution, such restrictions would apply to the posterior for θ_t as well.

That θ_t can potentially be negative may reflect that the scale of a firm can be suboptimally large during economic contractions, and that firms would strongly benefit from consolidating their businesses and shedding assets.

myopic, their optimal policies for consumption and investment, as well as the pricing kernel implied by their marginal utilities, will be the same regardless of whether they are part of a non-overlapping generational structure or long-lived.[15]

Households are subject to a random, private preference shock at each instant, which represents a liquidity shock and is the outcome of a Poisson random variable $N_t(i)$ with intensity $\pi \in (0,1)$, where $l_t(i) = \Delta N_t(i)$ is an indicator variable that the household has been hit. If hit by the preference shock, a household must take a fixed position in asset markets by divesting a fraction ξ_t of its wealth invested in firm claims and moving it into riskless bonds. Only those households hit by the shock observe its size ξ_t. The size of the shock may be correlated with investment productivity θ_t, and follows the law of motion

$$d\xi_t = \alpha\sigma_\xi dZ_t^\theta + \sqrt{1-\alpha^2}\sigma_\xi dZ_t^\xi,$$

where $\alpha \in (-1,1)$ represents this correlation. The innovation Z_t^ξ represents the pure liquidity shock to ξ_t. Later, when I consider the impact of financial crises in my economy, a financial crisis will be a large positive realization of this common liquidity shock. This allows me to focus on the informational effect of one feature of financial crises: asset firesales that depress financial prices. Other important features of financial crises, such as credit rationing and balance sheet impairment, would exacerbate the impact of financial crises through my channel.

Households are part of a continuum, and therefore exactly a fraction π will receive the liquidity shock at time t. Since those hit by the shock take a fixed position in asset markets, they do not trade on their superior information about its magnitude. Furthermore, because households are atomistic and, as such, do not view their preference shock as having any impact on the aggregate dynamics of market prices, those hit by the shock do not have an incentive to sell the private information of its magnitude to other households.

An unrealistic feature of the liquidity shock ξ_t is that it is not bounded between zero and one, and can also be negative. This implies that a household hit by the preference shock may be induced to take a positive position in the risky asset or a levered short position. Since ξ_t represents the noise in financial market prices that prevents them from being fully

[15] From Gennotte (1986), general homothetic preferences with incomplete information introduce a negative dynamic hedging term in addition to agents' myopic demand. Brown and Jennings (1989) provides a numerical analysis of the impact on investor trading that this additional hedging term introduces with dispersed information.

revealing about investment productivity θ_t, it is necessary that ξ_t have Gaussian innovations for tractability in learning, and therefore it cannot be restricted to the interval $[0,1]$. Given that the prices and investment will not depend on the wealth distribution of households in equilibrium, the redistributional consequences of the liquidity shocks are not significant for my results. In all discussions of welfare, I focus on the redistributional consequences of informational frictions by comparing welfare in my economy to one in which households and firms have perfect information.

Households in my economy have private information about its unobserved strength θ_t. At each date t, household i receives news about θ_t through a private signal $s_t(i)$

$$s_t(i) = \theta_t + \sigma_s Z_t^s(i),$$

where $Z_t^s(i)$ is a standard $N(0,1)$ random variable that represents household $i's$ idiosyncratic signal noise that is independent across (i,t) and independent from Z_t^θ and Z_t^ξ $\forall\ (t,i)$.[16] Households are part of a continuum and, as such, there is no aggregate risk from their idiosyncratic signal noise in the sense that the sum of the noise converges to zero in the $\mathcal{L}^2 - norm$.[17] Households at $t=0$ have a common Gaussian prior $\theta_0 \sim N\left(\hat{\theta}_0, \Sigma_0\right)$.

To simplify my analysis, and to focus on the feedback between the real sector and financial markets from learning, I assume that, while parents in a generation pass along their wealth to their children within a household, they do not pass along their private information, which includes their own private signal, the size of the liquidity shock if they were hit by it, and their initial wealth. As discussed in the introduction, models of information aggregation even in static settings are very difficult to solve, and I make this common, simplifying assumption so that learning by households and firm managers remains tractable. This lets me avoid both the infinite regress problem of Townsend (1983), where market prices partially reveal a moving-average representation of the investment productivity θ_t, and a time-varying correlation between the persistence of wealth of households and the persistence of their private beliefs.[18] In addition to making learning intractable, it would also render the equilibrium no longer Markovian.

[16] One can model this Gaussian process, for instance, as a time-change Wiener process.

[17] Since convergence of stochastic objects in continuous-time is in the $\mathcal{L}^2 - norm$, there is little reason to think about convegence in an $a.s.$ sense. There do, however, exist Fubini extensions of the Lebesgue measure for the index of agents such that the convergence is $a.s.$ See, for instance, Sun and Zhang (2009).

[18] Nimark (2012) instead takes the approach of having traders with long-lived private information but static wealth to break the time-varying correlation between trader wealth and private beliefs.

This assumption about the information structure, however, is not material for the main qualitative insights of my analysis. Relaxing it would introduce an additional component to the riskless rate that reflects that optimistic households tend to be wealthier during booms and poorer during recessions, similar to Detemple and Murphy (1994), Xiong and Yan (2010), and Cao (2011) for heterogeneous beliefs. This effect, however, is not likely to be significant given the nature of the equilibrium. The low uncertainty at business cycle peaks mitigates wealth inequality at peaks and during busts because households hold similar beliefs about investment productivity. This dampens the increased interest rate volatility that the interaction between wealth and beliefs would normally introduce.

In addition to their private signal $s_t(i)$, all households in a generation observe the history of firm asset growth in the economy $\log K_t$, investment I_t, the price of firm claims q_t, and the riskless rate r_t.[19] While private information is known by an individual, and would have to be remembered and passed along to progeny, historical public information is kept in public records and is readily available. Let the common knowledge, or public, filtration $\mathcal{F}_t^c$ be the minimal sigma-algebra generated by these public signals.

Households form rational expectations about the underlying state θ_t by Bayes' Rule given their information set $\mathcal{F}_t^i = \mathcal{F}_t^c \vee \{w_t(i), s_t(i)\}$, which is the public filtration $\mathcal{F}_t^c$ augmented with the household's current private wealth and signal. One can interpret the information structure of my economy as all households entering the current period with a common, time-varying prior based on the full history of public information $\mathcal{F}_t^c$, and then each updates its prior based on its private signal $s_t(i)$. Define $\hat{\theta}_t(i) = E[\theta_t \mid \mathcal{F}_t^i]$ to be the conditional expectation of θ_t of household i, where $E[\cdot \mid \mathcal{F}_t^i]$ is the conditional expectations operator with respect to the information set $\mathcal{F}_t^i$.

Households in each generation choose their consumption and investment to maximize their utility and their utility bequest to future generations $v_{t+\Delta t}(i)$, according to

$$0 = \sup_{c_t(i), x_t(i)} \left\{ \rho \Delta t \log c_t(i) + (1 - \rho \Delta t) E\left[v_{t+\Delta t}(i) \mid \mathcal{F}_t^i \right] - v_t(i) \right\}, \tag{4}$$

subject to the law of motion of their wealth $w_t(i)$ derived below. All households have the same initial wealth w_0. The optimization problem is solved under household $i's$ filtration $\mathcal{F}_t^i$ which incorporates household $i's$ private beliefs about investment productivity θ_t.

[19] Since output is related to asset growth by $y_t = aK_t$, observing asset growth is the same as observing output.

C. Firms

I keep the model of firms as simple as possible. There is a continuum of perfectly competitive, identical firms in the economy who issue claims to households. Firms issue equity claims to households and are run by managers who have two responsibilities: to oversee the firm's operations and to invest $I_t K_t$ to grow the firm's assets K_t according to equation (1). Firms must maintain a minimal level of investment $\underline{I}$ such that $I_t \geq \underline{I}$. This prevents the signals about investment productivity θ_t in the economy from fully flattening, since, if $I = 0$, then neither firms nor households care about the productivity of investment.

The choice of functional form for the capital accumulation equation (1) makes transparent the impact of firm beliefs on real investment and uncertainty in the economy, as well as shuts down any variation in the second moment of firm capital accumulation because of investment to turn off the real-options "wait-and-see" channel of Bernanke (1983) featured in Bloom (2009). While this law of motion will mechanically give rise to a stark relationship between asset growth and the signal strength of real investment, similar to the choice of the production function of firms in Van Nieuwerburgh and Veldkamp (2006), as well as between investment and the Sharpe ratio of the return on firm claims, the interaction between investment and the level of uncertainty in determining the behavior of the market price, which is the focus of my analysis, will be an equilibrium outcome. The insights about the relationships explored here will hold more generally as long as firms care about the current, hidden state of the economy when they invest, and that there is more information from real signals when real activity is high.[20]

Firm managers invest I_t to maximize the value to shareholders of its claims. Firms face frictions in adjusting their level of investment, and can only imperfectly control it by choosing effort g_t so that I_t evolves according to

$$dI_t = g_t I_t dt,$$

with $g_t \geq 0$ if $I_t = \underline{I}$. Thus $\underline{I}$ is a reflecting boundary for I_t. Managers incur a linear cost $\frac{1}{\rho} g_t$ for this adjustment per unit of current investment $I_t K_t$, which is rebated back to the firm as a subsidy τ_t. The cost is meant to slow the adjustment of real investment and captures that

[20] One may notice that learning from capital accumulation would be strong during recessions as well as expansions if real investment became largely negative, and firms, on aggregate, rapidly disinvested. Since aggregate US private nonresidential fixed investment historically has been nonnegative, I abstract from this artifact of the specification of the capital accumulation process.

real investment, in practice, is sluggish. As will be shown, if firms could choose the level of investment I_t directly, then I_t would have a well-defined solution between $\underline{I}$ and a because the value of their claims q_t is pinned down by household risk aversion, and is decreasing in I_t. With this slow adjustment, firms will have this same optimal I_t that they are slowly trying to adjust to by varying g_t, and therefore the policies with and without the technical restrictions are qualitatively similar. Since my comparisons for the dynamics of the economy will be relative to a perfect-information benchmark, relative business cycle asymmetries will not be driven by this assumption.

Households that hold firm claims receive a payment D_t of the residual cash flow from operations and investment

$$D_t = \left(a - I_t - \frac{1}{\rho} g_t I_t + \tau_t \right) K_t,$$

Firms finance their investment $I_t K_t$ from their cash flow from operations, the shortfall of which is made up by households through the sale of additional claims. Since financial markets are frictionless, they do not need to hold cash reserves.

For simplicity, managers do not have access to the private information of households and choose investment using only public information. While, in reality, firms are likely to have private information about the idiosyncratic component of their businesses or industries, they still have imperfect knowledge of general macroeconomic trends.[21] For managers to have access only to public information, they cannot observe the pricing kernels of their investors or their investors' ownership stakes in the firm. If they did, then managers would know the identity of the marginal buyer of its firm's claims, which would allow it to infer information about investment productivity θ_t. Given that managers make use of only public information, their investment strategies must be measurable with respect to the common knowledge filtration $\mathcal{F}_t^c$.

I assume that firm managers attempt to maximize shareholder value for their investors who are not hit by the preference shock ξ_t. The logic behind this choice is that households who trade because of the preference shock are trading for reasons unrelated to the return on firm claims, reasons for which they are happy to take whatever position the shock demands

[21] My mechanism is robust to managers having private information as long as they do not have superior information to households, in which case they would not need to learn from prices. See, for instance, David, Hopenhayn, and Venkateswaran (2014) for a setting in which firms also observe noisy private signals about their fundamentals.

regardless of managers' investment policies, and therefore it is unclear that maximizing shareholder value is the appropriate objective for them. Though managers must choose their investment policies from "behind the veil", since they do not know the composition of their shareholders, their policies in equilibrium will be robust to this uncertainty.

Let Λ_t be the pricing kernel of its shareholders not hit by the preference shock and E_t the value of firm claims. Firm managers then solve the optimization problem

$$E_0 = \sup_{\{g_t\}_{s\geq 0}} E\left[\int_0^\infty \frac{\Lambda_s}{\Lambda_0} D_s ds \mid \mathcal{F}_0^c\right], \tag{5}$$

subject to the transversality condition

$$\lim_{T\to\infty} E\left[\Lambda_T E_T \mid \mathcal{F}_0^c\right] = 0.$$

Since firms are perfectly competitive and atomistic, they take the pricing kernel of their shareholders as given. Though I restrict my attention to firm equity claims, it is worth mentioning that, since households have superior information compared to firms about general macroeconomic trends, firms could find it optimal to issue additional securities in this economy in order to have more signals from which to learn about the underlying state θ_t. Such a richer setting would introduce additional complexity, since instruments like risky debt are likely to have nonlinear payoffs, without adding much additional insight.

D. Market Clearing

Household i takes the net position firm claims $x_t(i)\, w_t(i) - q_t k_t(i)$, where $q_t k_t(i)$ is its initial holdings. Aggregating over all households then imposes the market clearing condition for the market for firm claims

$$\int_0^1 \left(x_t(i)\, w_t(i) - q_t k_t(i)\right) di = \int_0^1 x_t(i)\, w_t(i)\, di - q_t K_t = 0,$$

where $K_t = \int_0^1 k_t(i)\, di$ is the total assets of the firm at time t. Market clearing in the market for riskless debt additionally imposes that

$$\int_0^1 \left(1 - x_t(i)\right) w_t(i)\, di = 0.$$

Figure 1 in the Appendix illustrates the structure of the model. I search for a recursive competitive noisy rational expectations equilibrium.

E. Recursive Competitive Noisy Rational Expectations Equilibrium

Let ω be a state vector of publicly observable objects. A recursive competitive equilibrium for the economy is a list of policy functions $c\left(w(i), \hat{\theta}(i), \omega\right)$, $x\left(w(i), \hat{\theta}(i), \omega\right)$, $y(j, \omega)$, and $i(\omega)$, value functions $v\left(w(i), \hat{\theta}(i), \omega\right)$ and $E(\omega)$, and a list of prices $\{q(\omega), r(\omega)\}$ with $q(\omega) \geq 0$ such that

- Household Optimization: For every ω and i, given prices $\{q(\omega), r(\omega)\}$, $c\left(w(i), \hat{\theta}(i), l(i), \omega\right)$, and $x\left(w(i), \hat{\theta}(i), l(i), \omega\right)$ solve each household's problem (4) and deliver value $v\left(w(i), \hat{\theta}(i), \omega\right)$
- Firm Manager Optimization: For every ω, given prices $\{q(\omega), r(\omega)\}$, $g(\omega)$ solves the firm manager's problem (5) and delivers value $E(\omega)$
- Market Clearing: The markets for output, firm claims, and riskless debt clear

$$: \int_0^1 c\left(w(i), \hat{\theta}(i), l(i), \omega\right) di + I(\omega) K = aK \qquad (output) \tag{6}$$

$$: \int_0^1 x\left(w(i), \hat{\theta}(i), l(i), \omega\right) w(i)\, di = qK \qquad (firm\ claims\ market) \tag{7}$$

$$: \int_0^1 \left(1 - x\left(w(i), \hat{\theta}(i), l(i), \omega\right)\right) w(i)\, di = 0 \qquad (riskless\ debt\ market), \tag{8}$$

- Consistency: $w(i)$ follows its law of motion $\forall\, i \in [0,1]$, household i forms its expectation about θ based on its information set $\mathcal{F}^i$ and firm managers form their expectation about θ based on their information set $\mathcal{F}^c$ according to Bayes' Rule

and the transversality conditions are satisfied.

IV. The Equilibrium

I first state the main proposition of the section and then build up to this proposition in a sequence of key steps.

Proposition 1 *There exists a (locally) linear noisy rational expectations equilibrium in which the riskless return r is given by*

$$r = \frac{a}{a-I}\rho - \delta - \frac{\sigma_k^2}{1-\pi} + I\frac{\Sigma}{\Sigma + \sigma_s^2}\left(\theta - \hat{\theta}^c\right) - \frac{\pi\sigma_k^2}{1-\pi}\xi,$$

when $I > \underline{I}$, *and each household's investment in firm equity* $x(i)$ *can be decomposed into*

$$x(i) = x_c + x_i \left(\hat{\theta}(i) - \hat{\theta}^c \right),$$

where

$$\begin{aligned} x_c &= \frac{\frac{a}{a-I}\rho - r - \delta}{\sigma_k^2}, \\ x_i &= \frac{I}{\sigma_k^2}. \end{aligned}$$

When $I = \underline{I}$ *and* $g = 0$, *then* r *is instead given by*

$$r = \rho - \delta - \frac{\sigma_k^2}{1-\pi} + I\hat{\theta}^c + I\frac{\Sigma}{\Sigma + \sigma_s^2}\left(\theta - \hat{\theta}^c \right) - \frac{\pi \sigma_k^2}{1-\pi}\xi,$$

and x_c *is given by*

$$x_c = \frac{\rho + I\hat{\theta}^c - r - \delta}{\sigma_k^2}.$$

Similar to He and Wang (1995), individual households take a position in firm claims that can be decomposed into a component common to all households $x_c(\omega)$ and a term that reflects their informational advantage based on their private information $x_i(\omega)\left(\hat{\theta}(i) - \hat{\theta}^c \right)$. This informational advantage term reflects disagreement among households about the Sharpe Ratio of investing in firm claims. In contrast to He and Wang (1995), and other models of dispersed information like Foster and Viswanathan (1996) and Allen, Morris, and Shin (2006), the intensity with which households trade on their private information is influenced by real factors in the economy. Though private information is static, since the private information of households is short-lived because of the generational structure and because the signal-to-noise ratio of the private signals $s_t(i)$ is constant, the intensity with which households trade on their private information is now dynamic because the environment in which they trade is time-varying.

As is common in general equilibrium models of production, such as Cox, Ingersoll, and Ross (1985), interest rates adjust until all wealth is invested in firm assets. Focusing on the interaction between financial markets and real investment necessitates the adoption of such a setting that has this feature. In models of heterogeneous beliefs, such as Detemple and Murphy (1994) and Xiong and Yan (2010), the riskless rate r, which is the price at which relative pessimists are willing to offer leverage to relative optimists to hold all firm claims in equilibrium, reflects the disagreement among households about investment productivity θ_t. In my setting, it serves to aggregate their private information. This riskless rate falls during recessions to raise the expected excess return to firm claims, and shift down the level of optimism of the marginal buyer so that enough households purchase claims for asset markets to clear. Similarly, it rises during booms to shift up the level of optimism of the marginal buyer to curb the high demand of households for claims because of limited supply.

The market clearing condition for riskless debt effectively pins down the risk premium on firm claims required for asset markets to clear. As such, one can view market risk premium, whether it be the equity premium or a credit spread, as being the relevant market rate that aggregates information. Alternatively, one could interpret the interest rate in my stylized setting as being a composite market rate that arises from the trading of a well-diversified portfolio of securities. In the empirical discussion, I focus on the excess return to firm claims, or the spread between the return to firm claims and this riskless interest rate, to try to avoid taking a stance on which market rates have informative content.

The first step toward solving the equilibrium is to solve for the consumption and portfolio choice of household i given its information set $\mathcal{F}^i$. In what follows, I anticipate that the price of firm claims q will be a continuous, nonnegative function of finite total variation with respect to the level of investment I. Since q will have zero continuous quadratic variation, one has by a trivial application of Itô's Lemma that $\frac{dq}{q} = \frac{\partial_I q}{q} dI$.

I now derive the law of motion of the wealth of household i $w(i)$. Applying Itô's Lemma to K, the wealth of household i $w(i)$ then evolves according to

$$dw(i) = (rw(i) - c(i))\,dt + x(i)\,w(i)\left(\frac{(a-I)\,Kdt + Kdq + qdK}{qK} - rdt\right),$$

which can be expanded to yield

$$dw(i) = (rw(i) - c(i))\,dt + x(i)\,w(i)\left(\frac{a-I}{q} - r\right)dt + x(i)\,w(i)\left(\frac{dq}{q} + \frac{dK}{K}\right), \qquad (9)$$

and is irrespective of the measure. The variance term for $\frac{dK}{K}$ is irrespective of the measure because of diffusion invariance. Intuitively, it is easier to estimate variances than means of processes, so that even if two households disagreed on the drift of a process, they cannot disagree on its variance. The dividend $a - I$ reflects the dividend after the rebate for the adjustment cost.

To make progress in solving household $i's$ problem, I analyze each household's problem (4) in the limit as $\Delta t \searrow dt$. Since uncertainty over θ_t represents a compound lottery for households over the uncertainty in the change in θ_t, I can separate their filtering from their optimization problem and treat $\hat{\theta}_t(i) = E\left[\theta_t \mid \mathcal{F}_t^i\right]$ with variance $\Sigma_t(i) = E\left[\left(\theta_t - \hat{\theta}_t(i)\right) \mid \mathcal{F}_t^i\right]$ as θ_t in their optimization problem.

Given that households have log preferences over consumption, and that liquidity shocks are proportional to wealth, households will optimally consume a fixed fraction of their wealth at each date t. Furthermore, when they are unconstrained in investment, they will also choose a myopic portfolio in the sense that it maximizes the Sharpe Ratio of their investment and ignores market incompleteness. This is summarized in the following proposition.

Proposition 2 *The household's value function takes the form* $v\left(w(i), \hat{\theta}(i), l(i), h\right) = \frac{1}{\rho}\log w(i) + f\left(\hat{\theta}(i), l(i), h\right)$, *where* h_t *is a vector of general equilibrium objects. Furthermore, the household's optimal consumption and portfolio choice take the form*

$$
\begin{aligned}
c(i) &= \rho w(i), \\
x(i) &= \begin{cases} \dfrac{\frac{a-I}{q} + \frac{\partial_I q}{q} Ig + I\hat{\theta}(i) - r - \delta}{\sigma_k^2} & l(i) = 0 \\ -\xi & l(i) = 1 \end{cases}.
\end{aligned}
$$

Furthermore, define $\Lambda_t(i) = e^{-\rho t}\frac{1}{w_t(i)}$ *to be the pricing kernel of household* i *that is not hit by a liquidity shock. Then the riskless rate and risky firm equity satisfy* $\forall\ i$

$$
\begin{aligned}
r &= -\frac{1}{dt}E\left[\frac{d\Lambda(i)}{\Lambda(i)} \mid \mathcal{F}^i\right], \\
0 &= \frac{a-I}{q}dt + E\left[\frac{d(\Lambda(i) qK)}{\Lambda(i) qK} \mid \mathcal{F}^i\right].
\end{aligned}
$$

An immediate observation is that, similar to Detemple (1986), a separation principle applies in my noisy rational expectations equilibrium: the optimal consumption and investment policies are chosen independent of the learning process. Intuitively, since households are fully rational and update their beliefs with Bayesian learning, I can separate the filtering problem faced by households from their consumption choices and portfolio optimization.

Given the optimal choice of consumption $c(i) = \rho w(i)$ from the proposition, it follows that the law of motion of $w(i)$ can be written as

$$\frac{dw(i)}{w(i)} = (r - \rho) dt + x(i) \left(\frac{a - I}{q} dt + \frac{\partial_I q}{q} I g dt + \frac{dK}{K} - r dt \right), \tag{10}$$

which is also irrespective of the measure because of diffusion invariance.

From the market clearing conditions for the market for firm equity and riskless inside debt (7) and (8), the price of firm securities is given by

$$W = qK. \tag{11}$$

Equation (11) states that, in equilibrium, the total wealth in the economy W is equal to the total value of firm assets qK. Substituting $c(i) = \rho w(i)$ and equation (11) into the market clearing condition for output (6), it follows that

$$q = \frac{a - I}{\rho}, \tag{12}$$

from which follows that $\frac{a-I}{q} = \rho$, and the household, in equilibrium, receives a constant dividend yield from firm claims.

I now derive the conditional beliefs of households and firms about θ_t with respect to the common knowledge filtration $\mathcal{F}^c$ and their private information sets $\mathcal{F}^i$. The public signals that households have available for forming their expectations are $\log K$, q, I, and r. Since firm managers only have access to public information, it must be the case that firm investment $I \in \mathcal{F}^c$. Consequently, there is no additional information contained in I, or q given equation (12), once households have formed their beliefs. I can then generate the public filtration $\mathcal{F}^c$ with these two public signals $\mathcal{F}^c = \sigma\left(\{\log K_u, r_u\}_{u \leq t}\right)$.

Given the results of the main proposition, Proposition 1, let me now conjecture that the riskless rate r takes the form

$$r = r_0 + r_\theta (I, \Sigma) \left(\theta - \hat{\theta}^c\right) + r_\xi \xi, \tag{13}$$

where $r_\theta (I, \Sigma) \in \mathcal{F}^c$ since $(I, \Sigma) \in \mathcal{F}^c$. I assume that $|r_\xi|^{-1} > 0$ and that $r_\theta (I, \Sigma)$ is uniformly bounded and nonvanishing $a.s.$ Given equation (13), one can construct the public signal S

$$S = \frac{r - r_0 + r_\theta (I, \Sigma) \hat{\theta}^c}{r_\xi} = R_\theta (I, \Sigma) \theta + \xi. \tag{14}$$

Comparing equation (14) with the expression for the riskless rate r in Proposition 1, it follows that $R_\theta = -\frac{1-\pi}{\pi}\frac{I}{\sigma_k^2}\frac{\Sigma}{\Sigma+\sigma_s^2}$. Assuming that R_θ is a process of finite total variation, applying Itô's Lemma to S, S follows the law of motion

$$dS = \left(\partial_\Sigma R_\theta \frac{d\Sigma}{dt} + \partial_I R_\theta I g\right)\theta dt + R_\theta \lambda\left(\bar{\theta} - \theta\right)dt + \left(R_\theta \sigma_\theta + \alpha\sigma_\xi\right)dZ^\theta + \sqrt{1-\alpha^2}\sigma_\xi dZ^\xi.$$

Given these arguments, I can construct the vector of public signals $\zeta = \begin{bmatrix} \log K & S \end{bmatrix}'$ whose history, along with initial household wealth w_0 and firm assets K_0, generate the information set $\mathcal{F}^c$. Assuming that households using only the history of the public signals have a normal prior about θ_t, then after observing the two conditionally normal signals ζ_t their optimal updating rule for their beliefs about θ_t is linear, and their posterior belief about θ_t will also be conditionally normal. In continuous-time, these updating rules characterize the laws of motion for the conditional expectation and variance of these beliefs, $\hat{\theta}^c = E\left[\theta \mid \mathcal{F}^c\right]$ and $\Sigma = E\left[\left(\theta - \hat{\theta}^c\right)^2 \mid \mathcal{F}^c\right]$, respectively. In addition, ζ contains $\hat{\theta}^c$, Σ, and the level of investment I, which are all publicly observable, though we supress these arguments from the vector for simplicity since they do not contain new information about θ_t. Households then update these public estimates with their normally distributed private signals following another linear updating rule, and I have the following result.

Proposition 3 *The conditional belief of households using only public information is Gaussian with conditional expectation* $\hat{\theta}^c = E\left[\theta \mid \mathcal{F}^c\right]$ *and conditional variance* $\Sigma = E\left[\left(\theta - \hat{\theta}^c\right)^2 \mid \mathcal{F}^c\right] \in \left[0, \frac{\sigma_\theta^2}{2\lambda}\right]$ *that follow the laws of motion*

$$d\hat{\theta}^c = \lambda\left(\bar{\theta} - \hat{\theta}^c\right)dt + \sigma_{\hat{\theta}k}\left(I, \Sigma\right)d\tilde{Z}^k + \sigma_{\hat{\theta}r}\left(I, \hat{\theta}^c, \Sigma\right)d\tilde{Z}^r,$$

where

$$\begin{aligned}
\sigma_{\hat{\theta}k}\left(I, \Sigma\right) &= I\frac{\Sigma}{\sigma_k}, \\
\sigma_{\hat{\theta}r}\left(I, \hat{\theta}^c, \Sigma\right) &= \frac{R_\theta \sigma_\theta^2 + \alpha\sigma_\xi\sigma_\theta + R_\theta\left(\frac{\sigma_s^2}{\Sigma+\sigma_s^2}\frac{d\Sigma}{dt} + g\Sigma - \lambda\Sigma\right)}{\sqrt{\left(R_\theta\sigma_\theta + \alpha\sigma_\xi\right)^2 + \left(1-\alpha^2\right)\sigma_\xi^2}},
\end{aligned}$$

and

$$\frac{d\Sigma}{dt} = -\frac{B}{2A} \pm \frac{1}{2A}\sqrt{2B + 4A\left(\sigma_\theta^2 - 2\lambda\Sigma - I^2\frac{\Sigma^2}{\sigma_k^2}\right) - 1}$$

with

$$
\begin{aligned}
A &= \frac{\left(R_\theta \frac{\sigma_s^2}{\Sigma+\sigma_s^2}\right)^2}{\left(R_\theta\sigma_\theta + \alpha\sigma_\xi\right)^2 + \left(1-\alpha^2\right)\sigma_\xi^2}, \\
B &= 1 + 2R_\theta \frac{\sigma_s^2}{\Sigma+\sigma_s^2} \frac{R_\theta\sigma_\theta^2 + \alpha\sigma_\xi\sigma_\theta + R_\theta\left(g-\lambda\right)\Sigma}{\left(R_\theta\sigma_\theta + \alpha\sigma_\xi\right)^2 + \left(1-\alpha^2\right)\sigma_\xi^2},
\end{aligned}
$$

and

$$
\begin{aligned}
d\tilde{Z}^k &= \frac{1}{\sigma_k}\left(d\log K + \left(\frac{1}{2}\sigma_k^2 + \delta - I\hat{\theta}^c\right)dt\right), \\
d\tilde{Z}^r &= \frac{1}{\sqrt{\left(R_\theta\sigma_\theta + \alpha\sigma_\xi\right)^2 + \left(1-\alpha^2\right)\sigma_\xi^2}}\left(dS - R_\theta\left(\frac{\sigma_s^2}{\Sigma+\sigma_s^2}\frac{1}{\Sigma}\frac{d\Sigma}{dt} + g\right)\hat{\theta}^c dt - R_\theta\lambda\left(\bar{\theta} - \hat{\theta}^c\right)dt\right),
\end{aligned}
$$

is a vector of standard Wiener processes with respect to $\mathcal{F}^c$.

The conditional expectation of θ_t *of household* i *of generation* t $\hat{\theta}(i) = E\left[\theta \mid \mathcal{F}^i\right]$ *and the conditional variance* $\Sigma(i) = E\left[\left(\theta - \hat{\theta}(i)\right)^2 \mid \mathcal{F}^i\right]$ *are related to the average household estimates* $\hat{\theta}^c$ *and* Σ *by*

$$
\begin{aligned}
\hat{\theta}(i) &= \hat{\theta}^c + \frac{\Sigma}{\Sigma+\sigma_s^2}\left(s(i) - \hat{\theta}^c\right), \\
\Sigma(i) &= \frac{\sigma_s^2}{\Sigma+\sigma_s^2}\Sigma.
\end{aligned}
$$

The public or common knowledge belief $\hat{\theta}^c$ is derived from the endogenous public signals $\log K$ and r, while each household's private belief $\hat{\theta}(i)$ is a linear combination of this public belief and their private signal. This public belief $\hat{\theta}^c$ is an important state variable because it survives the aggregation of the beliefs of households, and because it is the forecast of firm managers. Similar to the Kalman Filter in discrete-time, the loadings on the normalized innovations $d\tilde{Z}^k$ and $d\tilde{Z}^r$ formed from the real investment and market signals, $\sigma_{\hat{\theta}k}$ and $\sigma_{\hat{\theta}r}$, respectively, represent the Kalman Gains of the public signals. Changes in the first moment of public beliefs $\hat{\theta}^c$ are a linear combination of a term capturing the deterministic mean-reversion of investment productivity, $\lambda\left(\bar{\theta} - \hat{\theta}^c\right)dt$, and a stochastic component related to the news from the innovations to the public signals, $\sigma_{\hat{\theta}k}(I,\Sigma)\,d\tilde{Z}^k + \sigma_{\hat{\theta}r}\left(I,\hat{\theta}^c,\Sigma\right)d\tilde{Z}^r$. The law of motion of the second moment of public beliefs Σ, in contrast, is (locally) deterministic and is the continuous-time analogue of the Ricatti equation for the Kalman filter, yet it is stochastic unconditionally.

An important feature of the optimal filter is that the conditional variance of public beliefs Σ is time-varying over the business cycle, and fluctuates endogenously according to its law of motion given in Proposition 3, which depends on its current value, the perceived investment productivity $\hat{\theta}^c$, and the level of investment by firms I. The stochastic time-variation in Σ is in contrast to dynamic models of asymmetric information like Wang (1993) that focus on the steady-state solution for the conditional variance of beliefs to which the economy tends deterministically. In this setting, Σ influences the quantity of private information households have, and how they trade on it in financial markets. As a result of shutting down the "wait and see" channel of Bloom (2009) for uncertainty to feed into firm investment behavior, firm investment decisions are indirectly influenced by Σ purely through how it affects the informativeness of the financial signal. Since Σ is time-varying, it is part of the state vector, along with I and $\hat{\theta}^c$, that summarizes the current state of the economy.

Learning from the endogenous market signal r that aggregates households' private information leads to either zero or two solutions for the (locally) deterministic change in the conditional variance $\frac{d\Sigma}{dt}$, which can result in nonexistence and multiple solutions.[22] With two solutions, households and firms can coordinate around either solution for the change in Σ, one which leads them to learn about investment productivity θ_t faster, and one in which they learn more slowly. In all the numerical applications, I follow the convention of selecting the larger root when two real solutions to $\frac{d\Sigma}{dt}$ exist, since the smaller, more negative root tends to lead households and firms to learn about θ_t extremely quickly.

In addition to their private signals, households learn about the underlying strength of the economy θ from the growth of firm assets $\log K$, whose informativeness (signal-to-noise ratio) is increasing in the level of firm investment I, and from the riskless rate, whose informativeness $R_\theta(I, \Sigma)$ is also influenced by I. This link from the investment choices of firms to the learning process of households represents one part of the feedback loop between real activity and asset markets that I wish to highlight. The ability of real investment decisions to distort investor expectations is similar to the channel explored in Angeletos,

[22]Nonexistence can occur because learning from market prices leads to the simultaneous determination of the change in the conditional variance $\frac{d\Sigma}{dt}$ and the strength of the market signal $\sigma_{\hat{\theta}r}$. There are situations when the real signal and the natural mean-reversion of θ_t are so strong that the conditional variance Σ falls too precipitously, as measured by $\frac{d\Sigma}{dt}$, for $\sigma_{\hat{\theta}r}$, which depends on $\frac{d\Sigma}{dt}$, to be sufficient to justify the fall in Σ. This result is reminiscent of the finding of Futia (1981) that price formation in a linear rational expectations framework can exhibit nonexistence pathologies.

Lorenzoni, and Pavan (2012) to rationalize the tech bubble of the early 2000's.

I now turn to the problem faced by firm managers. Given that firm managers only have access to public information, their conditional expectation of θ when making their investment decision g is $\hat{\theta}^c$. Furthermore, since the price of firm claims is pinned down by market clearing $q = \frac{a-I}{\rho}$, it must be the case that the optimal choice of g under the pricing kernel of investors confirms this price.

Proposition 4 *The value of firm claims is given by $E = qK$, and the optimal level of investment is given by*

$$g = \rho \left(q\hat{\theta}^c - 1 \right) \mathbf{1} \left\{ I > \underline{I} \, \cup \, \hat{\theta}^c \geq \frac{\rho}{a - \underline{I}} \right\}. \tag{15}$$

From the functional form of the optimal investment policy, it is apparent that $I = \underline{I}$ and $I = a$ are reflecting boundaries, since when $I = a$, then $q = 0$ and $g < 0$. As a result, the price of firm claims can never be negative. Similarly, when $I = \underline{I}$ and $\hat{\theta}^c \leq \frac{\rho}{a-\underline{I}}$, then $dI = 0$ and investment stays at $\underline{I}$ until g becomes positive. Since I has finite variation, its sample paths are continuous in time, and I will approach its two boundaries continuously.

To see how investment in my setting compares to one in which I allow firms to freely choose I, it is easy to see that the FOCs for the firm's problem would then be

$$-1 + q\hat{\theta}^c \leq 0,$$

with equality when $\hat{\theta}^c \geq \frac{\rho}{a-\underline{I}}$ since $q = \frac{a-I}{\rho}$, from which it follows that $I^{opt} = \underline{I} + \left(a - \frac{\rho}{\hat{\theta}^c} - \underline{I} \right) \mathbf{1} \left\{ \hat{\theta}^c \geq \frac{\rho}{a-\underline{I}} \right\}$. Since firms choose bang-bang policies, the price of capital q adjusts to make them indifferent to the optimal level of investment I^{opt}. Notice that when $I = a - \frac{\rho}{\hat{\theta}^c}$ when I can only be slowly adjusted, then $I = I^{opt}$ and $g = 0$. If I were above its optimal value $I > I^{opt}$, then $g > 0$, and similarly $g < 0$ when I is below its optimal value $I < I^{opt}$. Thus g tries to adjust I toward the optimal level the firm would choose if I could be chosen freely. This is the sense in which investment is sluggish.

Given the solution to the optimal investment strategy of firms, q has the interpretation of being Tobin's q. Investment by firms aims to equate the perceived productivity of real investment $\hat{\theta}^c$ to $1/q$, the book-to-market value of its assets. Thus informational frictions distort real investment by creating a misperception about the value of its assets. This highlights a key difference between my channel for firm beliefs to distort real activity and that of Straub and Ulbricht (2013). In their setting, entrepreneurs are never confused about the optimal level of production, but rather about the value of the collateral they must pledge to workers because of financial frictions. In my setting, firms optimally choose a level of production that is distorted because of their beliefs about investment productivity. Also, in contrast to models of uncertainty like Bloom (2009), investment in my economy declines because of shocks to the first moment of productivity rather than from shocks to the second moment through a "real-options" channel.

Learning by firm managers introduces a channel through which the first moment of beliefs about investment productivity $\hat{\theta}^c$ influences the second moment Σ. From Proposition 3, the change in uncertainty Σ $\frac{d\Sigma}{dt}$ depends on the change in investment g, which is a function of firm manager beliefs $\hat{\theta}^c$. Thus the filtering equations for $\hat{\theta}^c$ and Σ are coupled because there is feedback from second moments to first moments, which is a natural feature of the optimal nonlinear filter, and from first moments to second moments, because learning by firm managers determines their investment decisions, which influences the informativeness of the two public signals.

Since household trading behavior impacts the riskless rate r, from which both households and firms learn, the riskless rate acts as a channel for liquidity shocks in financial markets to feed into real investment decisions by influencing manager expectations. This mechanism for asset prices to distort firm investment is similar to Goldstein, Ordozen, and Yuan (2013). Along with the impact of investment decisions on household learning discussed above, these two forces characterize the feedback loop in learning between financial markets and real activity.

To derive the functional form for the riskless rate r, I must aggregate the wealth-weighted private expectations of all households, which will reveal the current true θ_t and the signal noise of households. Given that the private beliefs of each household are uncorrelated with their wealth share because households do not pass along their private information to later generations, the Law of Large Numbers will cause the aggregation of idiosyncratic signal noise to vanish. Let $\mathcal{D}_t$ be the set of households hit by the liquidity shock at time t. Let $W = \int_0^1 w(i)\, di$ be the total wealth of all households. Then I obtain the following result.

Proposition 5 *Aggregating the wealth-weighted deviation in the conditional expectation* θ

of household i $\hat{\theta}(i)$ from the common knowledge expectation $\hat{\theta}^c$ yields a.s.

$$\int_{\mathcal{D}^c} \frac{w(i)}{W} \left(\hat{\theta}(i) - \hat{\theta}^c\right) di - \int_{\mathcal{D}} \frac{w(i)}{W} \xi di = (1-\pi) \frac{\Sigma}{\Sigma + \sigma_s^2} \left(\theta - \hat{\theta}^c\right) - \pi\xi,$$

and the convergence $\forall\ t$ is in the $\mathcal{L}^2 - norm$.

By aggregating the beliefs of individual households, the riskless rate r will depend on $\hat{\theta}^c$ and θ through the productivity of investment θ revealed by the households' private signals. An important caveat to this result is that it relies on households being symmetrically informed. If, instead, households had different signal precisions $\sigma_s(i)$, then the wealth distribution of households would matter for prices.[23] Given this aggregation result, the noisy rational expectations equilibrium and the riskless rate r then satisfy the main theorem of the section. Thus it follows that the state vector ω for the economy is $\omega = \left[\theta, \xi, \hat{\theta}^c, I, \Sigma\right]$.

While the intensity with which households trade on their private information is procyclical, since $x_i(\omega)$ is monotonically increasing in the investment by firms I, the information content in the market price is monotonically increasing in uncertainty about θ, measured by Σ, because market prices aggregate the private information of households to partially reveal θ. These two forces interact so that asset prices will be strongest during downturns and recoveries, in the sense that the variation in $\hat{\theta}^c$ driven by the market signal is largest when I and Σ are in an intermediate range. To see this, Figure 2 in the Appendix plots, as a numerical example, the loading of the market signal $\sigma_{\hat{\theta}r}\left(I, \hat{\theta}^c, \Sigma\right)$ on beliefs for a fixed level of perceived investment productivity $\hat{\theta}^c$ for a set of parameters listed in the Appendix. The figure reveals that the variation from the market signal is increasing in the level of investment by firms, and increasing in uncertainty about investment productivity Σ, though for other parameter values it can be non-monotonic. Furthermore, since a decline in the perceived investment productivity $\hat{\theta}^c$ lowers investment, and also leads to greater uncertainty, it follows that $\sigma_{\hat{\theta}r}\left(I, \hat{\theta}^c, \Sigma\right)$ can be increasing or decreasing in $\hat{\theta}^c$ depending on I and Σ. These observations illustrate that more of the variation in the beliefs in households and firm managers is driven by the market signal when I and Σ are in an intermediate range. As $\Sigma \to 0$, the market price contains little information about θ at peaks, since $R_\theta \to 0$, and households do not react strongly to it.

[23] Asymptotically, however, one would expect households with superior information to eventually drive out the less well-informed households. This would lead to a degenerate wealth distribution in which wealth once again does not matter.

This last point merits some emphasis. While it is well-appreciated that risk premia in financial markets are countercyclical, it is less appreciated that the strength of asset prices as a signal of economic strength also exhibits business cycle asymmetries. This asymmetry arises because the incentives for investors to trade on their private information anchors on both the level of real investment and uncertainty in the economy.

V. The Impact of Feedback in Learning

To assess the impact of feedback in learning, I first derive the equilibrium in two benchmark economies, one with perfect information and one in which only households have perfect information, as helpful anchors for my analysis. The first benchmark gives us insight into how the economy behaves in the absence of any informational frictions, while the second will help to clarify the role that dispersed information among households plays in influencing the business cycle behavior of the market signal. I then explain the slow US recovery in the context of this feedback loop.

A. Two Benchmarks

Suppose that $\theta(t)$ is observable to all households and firm managers. Then all households will allocate identical fractions of their portfolios to risky projects and the riskless asset. In this benchmark setting, it is sufficient to solve the equilibrium for the aggregate state variables, since the wealth of households will only differ in their history of preference shocks. The following proposition summarizes the recursive competitive equilibrium that the recursive noisy rational expectations equilibrium tends to, in the aggregate, as informational frictions vanish for all agents.

Proposition 6 *When θ is observable to all households and firm managers, a) the price of firm equity is given by*

$$q = \frac{a - I}{\rho},$$

b) the riskless return r satisfies

$$r = \frac{a}{a - I}\rho - \delta - \frac{\sigma_k^2}{1 - \pi} - \frac{\pi \sigma_k^2}{1 - \pi}\xi,$$

when $I > \underline{I}$, c) optimal consumption and investment in firm equity by households who are *not hit by the liquidity shock satisfy*

$$\begin{aligned} c(i) &= \rho w(i), \\ x(i) &= \frac{\frac{a-I}{q} - \frac{I}{a-I}g + I\theta - r - \delta}{\sigma_k^2}, \end{aligned}$$

and d) optimal investment by managers is given by

$$g = \rho(q\theta - 1)\,\mathbf{1}\left\{I > \underline{I} \,\cup\, \theta \geq \frac{\rho}{a - \underline{I}}\right\}.$$

The equilibrium with perfect information appears similar to the one with informational frictions, except that the riskless rate no longer reflects the wedge between the beliefs of agents and the true underlying strength of the economy θ because households and firm managers are now perfectly informed. The economy is isomorphic to one with a representative agent household who owns and manages all assets in the economy, and chooses the riskless rate so that it invests all its resources in assets given its preference shock. In this setting, there is no role for noise from preference shocks ξ to transmit to real investment decisions because manager do not learn from prices. Financial market activity has no consequence for the business cycle at all.

The second benchmark provides an intermediate case between the informational frictions economy of the previous section and the perfect-information benchmark. Though households behave identically when they have perfect information, there is still feedback from financial market noise ξ to real investment decisions because managers still must learn about θ from market prices. The behavior of this economy is summarized in the next proposition.

Proposition 7 *When θ is observable to all households, a) the price of firm equity is given by*

$$q = \frac{a - I}{\rho},$$

b) the riskless return r satisfies

$$r = \frac{a}{a - I}\rho - \delta - \frac{\sigma_k^2}{1 - \pi} + I\left(\theta - \hat{\theta}^c\right) - \frac{\pi\sigma_k^2}{1 - \pi}\xi,$$

c) optimal consumption and investment in firm equity by households who are not hit by *the liquidity shock satisfy*

$$\begin{aligned} c(i) &= \rho w(i), \\ x(i) &= \frac{\frac{a-I}{q} - \frac{I}{a-I}g + I\theta - r - \delta}{\sigma_k^2}, \end{aligned}$$

and d) optimal investment by managers is given by

$$g = \rho\left(q\hat{\theta}^c - 1\right)\mathbf{1}\left\{I > \underline{I} \ \cup\ \hat{\theta}^c \geq \frac{\rho}{a - \underline{I}}\right\}.$$

Furthermore, beliefs, prices, and optimal policies in the economy with informational frictions approach their representative agent benchmark values as $\sigma_s \searrow 0$.

In this intermediate case, firm managers must still learn from both the growth of firm assets and market prices. Noise from market prices from preference shocks ξ can potentially feed back into firm manager learning, and therefore their investment decisions, yet there is an important distinction from the NREE equilibrium. Since households have perfect information, the level of uncertainty in the economy Σ does not affect their trading behavior, and consequently it has a smaller role in determining the influence and strength of the market signal. This can be seen from the difference in the loadings on the tracking error $\theta - \hat{\theta}^c$ in the expressions for r in Propositions 1 and 7. In the NREE economy, the signal-to-noise ratio $R_\theta = -\frac{1-\pi}{\pi}\frac{I}{\sigma_k^2}\frac{\Sigma}{\Sigma+\sigma_s^2}$, while in this representative agent setting $R_\theta = -\frac{1-\pi}{\pi}\frac{I_t}{\sigma_k^2}$. This implies that the market signal in the representative agent setting mimics much of the cyclical behavior of the real investment signal (though it is not redundant because the noise in the two signals are conditionally independent of each other). The market signal $S = \frac{1-\pi}{\pi\sigma_k^2}\left(r - \frac{a}{a-I}\rho + \frac{\sigma_k^2}{1-\pi} + I\hat{\theta}^c\right)$ for households and firm managers then has the law of motion

$$dS = R_\theta\left(g - \lambda\right)\theta dt + R_\theta\sigma_\theta dZ^\theta + \sigma_\xi dZ^\xi,$$

where R_θ is increasing in I and unrelated to uncertainty Σ. The market signal is, consequently, strongest during booms when uncertainty $\Sigma = E\left[\left(\theta - \hat{\theta}^c\right)^2 \mid \mathcal{F}^c\right]$ is low.

This setting consequently highlights the importance of dispersed information for the mechanism of the NREE economy: aggregation of dispersed information gives the market signal much of its countercyclical behavior because the quantity of private information Σ matters for how households trade on their private information. There is a dramatic difference, then, in the predictions of how an economy with a representative household behaves compared to an economy with households with heterogeneous information.

One could also consider a benchmark with a representative household that receives a noisy private signal instead of having perfect information. In this benchmark, the conditional variance of public beliefs Σ would be important for the information content of the market signal, and the market signal would exhibit more countercyclical behavior. Since the noise in the household's private signal would not vanish from market prices, however, it is less clear how the informativeness of the market signal would change over the cycle, since the noise in the price from the household's private signal would also increase as Σ increased.

B. *Explaining the Slow US Recovery*

My analysis highlights a potential channel by which recessions with financial origins can have deeper recessions and slower recoveries, and can help explain how the financial crisis of late 2008 may have contributed to the anemic US recovery. Economic agents rely more on price signals for helpful guidance about the state of the economy as the economy enters a downturn. Financial crises during downturns distort these price signals and, as a result of severe informational frictions, investors and firms interpret part of the collapse in asset prices as a signal of severe economic weakness. This further depresses real activity, causing both real and financial signals to flatten, which increases uncertainty and causes it to remain elevated. This makes it harder for private agents to act on signs of a recovery. Despite evidence of economic improvement, and a rebounding of financial markets, the heightened level of uncertainty makes it difficult for a recovery to gain traction and stifles growth.

To illustrate this story, Figure 3 depicts the impact of a one standard deviation negative liquidity shock to financial prices in the economy during a boom $\left(\hat{\theta}^c, I, \Sigma\right) = (.3, .1, 10^{-6})$ and during a bust $\left(\hat{\theta}^c, I, \Sigma\right) = (.2, .04, 10^{-5})$.[24] As a result of informational frictions, the recession is deeper in this numerical experiment compared to the perfect-information benchmark, and the recovery is also more gradual. In contrast, a one standard deviation negative financial shock during a boom has a much more attenuated impact on growth, which can help explain why financial events like the LTCM crisis had little effect on the real economy. Key to this result is that uncertainty is time-varying, with a law of motion given in Proposition 3, and countercyclical. When uncertainty is higher, noise in financial prices that is interpreted as

[24]Since time is continuous, we feed the quarterly negative shock to the model as one large innovation at time 0 equal to one fourth the annual variance of the financial shock.

bad news perpetuates low investment. This, in turn, perpetuates high uncertainty and allows the distortion to beliefs from the noise in financial prices to persistent.

My analysis consequently identifies a potential benefit of unconventional monetary policy in the presence of informational frictions. By buying treasury and mortgage-backed securities through Quantitative Easing (QE), the US government provided financing for investors to purchase assets from riskier asset classes, such as equities and speculative-grade debt. This injection of capital may have lessened the noise that constrained investors introduced into financial prices during the financial crisis that distorted the expectations of private agents about the strength of the US economy. In continuing QE in its various forms of QE1-QE3 until late 2014, however, the buoying of financial markets may have later added noise to financial prices that confused agents about the strength of the US recovery. The April 2011 WSJ article "Is the Market Overvalued?", for instance, discusses how market participants and economists like Robert Shiller could not disentangle signs of strong corporate profitability from the effects of QE behind the high valuations in the stock market.

VI. Welfare

I now turn to the welfare implications of my analysis. The economy with informational frictions may be constrained inefficient because households and firms do not fully internalize the benefit of the public information they produce by trading in asset markets and engaging in real investment. As emphasized in Greenwald and Stiglitz (1986), economies with incomplete markets and incomplete information are generically not constrained Pareto efficient, and there is a role for welfare-improving policies. In this spirit, I consider several thought experiments that augment the provision of public information in the economy to highlight this potential externality.

I begin this section by characterizing ex-ante welfare in the economy. I adopt a utilitarian weighting scheme to aggregate utility across the heterogeneous households, normalizing welfare to initial household consumption to remove the level effect of initial conditions. This helps me construct a measure of welfare in the economy that has a stationary distribution conducive to conducting thought experiments. Since the noise in financial prices stems from the preference shocks of households, the analysis avoids the issue of characterizing welfare in the presence of exogenous "noise traders" discussed in Wang (1994). Informational frictions impact welfare through two channels: a distortion to real investment and household trading, and a cost that comes from the inequality in household wealth that arises because of the dispersion of private beliefs. This is summarized in the following proposition.

Proposition 8 *Ex-ante utilitarian welfare in the economy with informational frictions is given by*

$$U = \underbrace{\frac{1}{\rho}E\left[\int_0^\infty e^{-\rho t}\left(\frac{\rho}{a-I_t}+\theta_t-\hat{\theta}_t^c\right)I_t dt \mid \mathcal{F}_0\right]}_{Efficiency\ of\ Real\ Investment} - \underbrace{\frac{1-\pi}{2\rho}\left(\frac{\sigma_s}{\sigma_k}\right)^2 E\left[\int_0^\infty e^{-\rho t}\left(\frac{I_t\Sigma_t}{\Sigma_t+\sigma_s^2}\right)^2 dt \mid \mathcal{F}_0\right]}_{Cross-Sectional\ Inequality}$$
$$-\frac{\delta}{\rho^2}-\frac{1}{2\rho^2}\left(\sigma_k^2+\frac{\pi\sigma_k^2}{1-\pi}\left(1+\xi_0\right)^2\right)-\frac{1}{2\rho^3}\frac{\pi\sigma_k^2}{1-\pi}\sigma_\xi^2.$$

Under this welfare criterion, there exists a representative household in the economy who holds all claims to firm assets and whose wealth w evolves according to

$$\frac{dw}{w}=\left(\frac{\rho I}{a-I}-\delta+I\left(\theta-\hat{\theta}^c\right)-\frac{1}{2}\left(\frac{\pi\sigma_k^2}{1-\pi}\left(1+\xi\right)^2+\left(1-\pi\right)\left(\frac{I}{\sigma_k}\frac{\Sigma}{\Sigma+\sigma_s^2}\sigma_s\right)^2\right)\right)dt+\sigma_k dZ^k.$$

From Proposition 8, the representative household under this welfare criterion is different from a representative household who holds all firm claims since the criterion reflects the inequality in wealth that arises because of informational frictions and liquidity shocks. This distinction is absent from representative agent models and comes from the aggregation of flow utility $\log c(i)$ rather than consumption $c(i)$ in the utilitarian welfare function. The effects of the distortion show up as a tax on the representative household, and consequently one can think of the transfer of wealth from liquidity shocks and the presence of informational frictions as imposing a tax on the economy. This tax vanishes when households have identical beliefs, which occurs in the limiting cases when $\sigma_s \searrow 0$, $\sigma_s \nearrow \infty$, or $\Sigma \equiv 0$.

Having derived ex-ante utilitarian welfare to understand the forces that impinge on household utility, I construct a measure of expected welfare using only public information once the economy has reached its stationary distribution, and initial conditions no longer matter, as a sensible measure for conducting my thought experiments. To target household and firm investing behavior, I introduce a proportional position cost τ^r on household trading and a linear subsidy on firm real investment τ^I. I construct these instruments so that the extracted revenue is returned to households as lump-sum transfers that households view as being proportional to their wealth. The position cost lets me manipulate households' trading decisions while the real investment subsidy lets me manipulate firms' investment decisions.

Solving for household's optimal investment in the presence of the position cost, it is straightforward to see from Proposition 2 that household i invests a fraction $x(i)$

$$x(i) = \frac{\frac{a-I}{q} + \frac{\partial_I q}{q} Ig + I\hat{\theta}(i) - r - \delta}{(1-\tau^r)\,\sigma_k^2},$$

of its wealth in firm claims when not hit by the liquidity shock. Households that are hit by the preference shock continue to take a fixed position $-\xi$ proportional to their wealth in the risky asset, regardless of the position cost. Then, by similar arguments to those in Section IV, one can arrive at the form for the riskless rate r when investment is unconstrained

$$r = \frac{a}{a-I}\rho - \delta + I\frac{\Sigma}{\Sigma+\sigma_s^2}\left(\theta - \hat{\theta}^c\right) - (1-\tau^r)\frac{\sigma_k^2}{1-\pi}(1+\pi\xi),$$

from which follows that

$$x(i) = \frac{1}{1-\pi} + \frac{\pi}{1-\pi}\xi + \frac{1}{1-\tau^r}\frac{I}{\sigma_k^2}\frac{\Sigma}{\Sigma+\sigma_s^2}\sigma_s Z^s(i).$$

The position cost has the counterintuitive property that it induces households to take larger positions in the risky asset based on their private information. This happens because households in continuous-time can rebalance their portfolios instantaneously to take a large enough position to offset the impact of the cost. Since the collateral is returned lump-sum, however, the cost introduces a distortion to household wealth. A higher position cost τ^r increases the amount of public information in the price by inducing households to trade more on their private information without affecting the position taken by households hit by the liquidity shock, but it also introduces more wealth inequality. There is then a tradeoff for welfare in increasing τ^r.

It is also straightforward to see from Proposition 4 that the real investment subsidy induces the firm to choose a growth rate for real investment g

$$g = \left((a-I)\hat{\theta}^c - \left(1-\tau^I\right)\rho\right)\mathbf{1}_{\left\{I>\underline{I}\cup\hat{\theta}^c\geq\frac{(1-\tau^I)\rho}{a-\underline{I}}\right\}}.$$

With these instruments in place, I now search for the probability law of the economy once it has reached its stationary distribution $p\left(\hat{\theta}^c, \Sigma, I\right)$, if it exists. I derive the Kolmogorov Forward Equation (KFE), or transport equation, which summarizes the (instantaneous) transition of the probability law of the economy $p_t\left(\hat{\theta}^c, \Sigma, I\right)$ and characterize the conditions under which $\partial_t p_t\left(\hat{\theta}^c, \Sigma, I\right) = 0$. This reduces to solving the appropriate boundary value problem for a second-order elliptic partial differential equation, summarized in the following proposition.

Proposition 9 *The stationary distribution of the economy* $p\left(\hat{\theta}^c, \Sigma, I\right)$ *satisfies the Kolmogorov Forward Equation*

$$
\begin{aligned}
0 \;=\;& -\partial_{\hat{\theta}^c}\left\{p\lambda\left(\bar{\theta}-\hat{\theta}^c\right)\right\} - \partial_I\left\{pI\left(\left(a-I\right)\hat{\theta}^c-\left(1-\tau^I\right)\rho\right)\right\}\mathbf{1}_{\left\{I>\underline{I}\cup\hat{\theta}^c\geq\frac{\left(1-\tau^I\right)\rho}{a-\underline{I}}\right\}} - \partial_\Sigma\left\{p\frac{d\Sigma}{dt}\right\} \\
& +\frac{1}{2}\partial_{\hat{\theta}^c\hat{\theta}^c}\left\{p\left(\sigma_{\hat{\theta}k}^2+\sigma_{\hat{\theta}r}^2\right)\right\},
\end{aligned}
$$

with boundary conditions given in the Appendix.

The KFE that defines the stationary distribution is a conservation of mass law that has an intuitive interpretation. It states that the sum of the flows of probability through a cube in the $\left(\hat{\theta}^c, \Sigma, I\right)$ space must be zero for the probability mass of the cube to be conserved over time. The stochastic component of $\hat{\theta}^c$ introduces a second-order term in the KFE related to its volatility since the high variability of Wiener processes has a first-order effect on the law of motion of $\hat{\theta}^c$.[25] In the case where $\sigma_s \nearrow \infty$ and $\alpha = 0$, the economy is analogous to that of Van Nieuwerburgh and Veldkamp (2006) in which only a real investment signal provides information.

Given the KFE, I now construct my welfare measure. Let U_p^c be utilitarian welfare in the economy, normalized to initial wealth, and $E^p\left[\cdot\right]$ be the expectation operator with respect

[25] To find the stationary distribution numerically, I follow the trick of rewriting the KFE in Proposition 9 as $\mathcal{D}^{g*}p = 0$, where $\mathcal{D}^{g*}$ is the adjoint of the infinitesimal generator $\mathcal{D}^g$ defined in the proof of the proposition. Discretizing the state space $\left(\hat{\theta}^c, \Sigma, I\right)$ into a $N_{\hat{\theta}} \times N_\Sigma \times N_I$ grid, one can stack the $N_{\hat{\theta}} \cdot N_\Sigma \cdot N_I$ linear equations for $\mathcal{D}^{g*}p = 0$ to construct the matrix equation

$$A'\mathbf{p} = \mathbf{0}_{N_{\hat{\theta}}\cdot N_\Sigma \cdot N_I \times 1},$$

where $\mathbf{p} = vec\left(p\right)$ and A is the $\left(N_{\hat{\theta}} \cdot N_\Sigma \cdot N_I\right)\times\left(N_{\hat{\theta}} \cdot N_\Sigma \cdot N_I\right)$ square matrix that approximates the derivative operator $\mathcal{D}^g$ constructed with the "upwind" method. Here A' denotes the transpose of A. Since the matrix equation defines the stationary distribution for a Markov chain with transition matrix A', it follows by the Frobenius-Perron Theorem for nonnegative compact operators that A' has a unique largest eigenvalue (in absolute value), called the principal eigenvalue, and an associated strictly positive eigenvector ϕ unique up to a scaling factor. Since A is singular, it is convenient to replace one row i of A' with $A_{ij} = \delta_{ij}$ and the i^{th} entry of the zero vector with 1. This allows me to update to the stationary distribution in one step after defining A.

In practice, I find it convenient to populate the matrix A imposing that $\hat{\theta}^c$ has reflecting boundaries on both sides, and then set the boundaries sufficiently far into the tails of the distribution that the choice is insensitive to my results.

to the stationary distribution. Then I have the following corollary.

Corollary 1: *Expected utilitarian welfare under the stationary distribution U_p^c with position cost and real investment subsidy τ^r and τ^I, respectively, is given by*

$$\begin{aligned} U_p^c &= \frac{1}{\rho} E^p \left[\frac{I_0}{a - I_0} \right] - \frac{1-\pi}{2\rho^2} \left(\frac{\sigma_s}{\sigma_k} \right)^2 E^p \left[\left(\frac{I_0}{1-\tau^r} \frac{\Sigma_0}{\Sigma_0 + \sigma_s^2} \right)^2 \right] - \frac{1}{2\rho^2} \frac{1-\pi}{\pi \sigma_k^2} E^p \left[\left(\frac{I_0}{1-\tau^r} \frac{\Sigma_0}{\Sigma_0 + \sigma_s^2} \right)^2 \Sigma_0 \right] \\ &\quad - \frac{\delta}{\rho^2} - \frac{1}{2\rho^2} \frac{\sigma_k^2}{1-\pi} \left(1 + \frac{1}{\rho} \pi \sigma_\xi^2 \right). \end{aligned}$$

The first two pieces again relate to the efficiency of real investment and cross-sectional inequality among households, while the third reflects uncertainty over the current size of the liquidity shock. The direct contributions to welfare from uncertainty about investment productivity Σ_0 are unambiguously negative, and it is unlikely that informational frictions can improve real investment efficiency since firms can only be distorted away from the level of investment they would choose with perfect-information. Welfare is about 1.9% lower compared to the perfect-information benchmark, and modestly about .5% higher than in the economy analogous to that of Van Nieuwerburgh and Veldkamp (2006) where households do not aggregate private information in financial markets. This modest gain reflects the tradeoff between the increased informativeness of public signals and the cross-sectional inequality induced by households trading on their heterogeneous private information.

To highlight the presence of information externalities in the economy, I conduct several illustrative thought experiments varying the position cost and real investment subsidy. I report the gain in welfare in consumption equivalent λ in the tradition of Lucas (1987).[26]

From Table 1, the position cost improves welfare in the economy with informational frictions. The intuition for this is that the gain in informational provision by having house-

[26] Formally, the consumption equivalent λ for an alternative level of the transaction tax or real investment subsidy that raises welfare from $\tilde{U}_p^c$ to U_p^c is defined as the fractional increase in the consumption of all households under the baseline level that delivers the same gain.

For log utility, λ satisfies

$$U_p^c = \frac{1}{\rho} E^p \left[\int_0^1 \log \left((1+\lambda) \tilde{c}(i) \right) di \right]$$

for $\tilde{U}_p^c = \frac{1}{\rho} E^p \left[\int_0^1 \log \tilde{c}(i)\, di \right]$, from which follows that

$$\lambda = \exp \left(\rho \left(U_p^c - \tilde{U}_p^c \right) \right) - 1.$$

τ^r	.05	.1	.15
$100 \times \lambda$	0.128	0.266	0.411

Table 1: Transaction Cost Experiment

holds take larger positions, is larger than the cost of generating more inequality by having households trade more on their heterogeneous private information.[27] Since better public information lowers the average level of uncertainty in the economy, however, this mitigates the rise in inequality.

To see if subsidizing real investment improves welfare by improving the informational content of public information, I give firms a proportional investment subsidy τ^I whenever investment is at least one standard deviation below its unconditional mean in the stationary distribution. This has the interpretation of being a countercyclical real investment subsidy. To capture the welfare impact of the subsidy through the informational channel, I modify the experiment by subtracting out expected welfare under the perfect-information benchmark U_p^{perf}, since the subsidy will mechanically impact welfare by raising the average level of investment in the economy. It is easy to derive the analogous KFE for the perfect-information benchmark economy

$$-\partial_\theta \left\{p\lambda\left(\bar{\theta}-\theta\right)\right\} - \partial_I \left\{pI\left(\left(a-I\right)\theta - \left(1-\tau^c\right)\rho\right)\right\} \mathbf{1}_{\left\{I>\underline{I}\cup\theta\geq\frac{(1-\tau^c)\rho}{a-\underline{I}}\right\}} + \frac{1}{2}\sigma_\theta^2\partial_{\theta\theta}p = 0,$$

which has similar boundary conditions. Subtracting out the expected welfare under the perfect-information benchmark captures the incremental benefit of the subsidy from mitigating informational frictions.

τ^I	.05	.10	.15
$100 \times \lambda$	1.875	3.709	5.502

Table 2: Investment Subsidy Experiment

Table 2 reveals that the real investment subsidy also improves welfare. Since the subsidy increases real investment, which increases the average position households take in asset markets, it also has a similar effect to implementing a position cost. Real investment subsidies, therefore, improve the provision of public information by increasing the informativeness of

[27] An important caveat is that the experiment understates the extent to which heterogeneous information generates wealth inequality because household private information is short-lived, and therefore there is no persistence in positions. With long-lived private information, the net benefit is likely to be more modest.

both real and financial signals, which might, in part, explain why the gains from this experiment are larger than for the position cost.

These two thought experiments are meant to illustrate that there is a role for welfare-improving policies that address an information externality that arises because of decentralization. If instead of continuums, there were only one trader or one firm in the economy, such an agent would internalize its impact on the formation of the endogenous public signals when choosing its investment policies. While also likely to be present in static settings of incomplete information, this externality has a dynamic dimension because households and firms learn from signals formed inefficiently because of decentralization in the past. Though Greenwald and Stiglitz (1986) demonstrate that there often exist welfare-improving policy for economies with incomplete information and incomplete markets, their analysis is silent as to what form these policies take, and whether there is an optimal policy. These thought experiments motivate a more systematic analysis of policy interventions to address such information externalities within an optimal policy framework, which is beyond the scope of my analysis.

VII. Empirical Implications

In this section, I explore several empirical implications of my framework that build off the observation that financial prices provide useful signals about the state of the economy, and that the strength of these signals is strongest during downturns and recoveries. I first discuss the asset pricing implications of my analysis, and then turn to conceptual issues my framework implies for empirical analysis and other empirical implications.

A. Implications for Asset Pricing

In this section, I characterize the business cycle implications of macroeconomic uncertainty in financial markets for asset risk premia and asset turnover. My analysis illustrates that, in the presence of informational frictions, there is an additional component to asset risk premia and asset turnover that reflects uncertainty about the state of the economy. This informational piece appears because households have heterogeneous private information and the degree to which they have heterogeneous beliefs increases as uncertainty rises about investment productivity. Furthermore, it gives asset returns predictive power for future returns and macroeconomic growth. The strength of this predictive power, however, varies over the business cycle, and I show that this variation is related to the behavior of asset turnover from informational trading.

A.1. Risk Premia

When the true state of the economy is known, then from Proposition 6 firms pay a risk premium on their claims

$$RP_{perf} = \rho - \frac{I}{a-I}g + I\theta - \delta - r = \underbrace{\frac{\sigma_k^2}{1-\pi} + \frac{\pi\sigma_k^2}{1-\pi}\xi}_{variance\ and\ liquidity\ risk},$$

which compensates households for variance risk and liquidity shocks. From Proposition 1, however, in the presence of informational frictions this risk premium includes an additional piece

$$RP_{NREE} = \underbrace{\frac{\sigma_k^2}{1-\pi} + \frac{\pi\sigma_k^2}{1-\pi}\xi}_{variance\ and\ liquidity\ risk} + \underbrace{\frac{I}{1+\Sigma/\sigma_s^2}\left(\theta - \hat{\theta}^c\right)}_{informational\ risk}.$$

that compensates investors for informational risk. This piece arises because households overreact to liquidity and capital quality shocks, and underreact to news about real investment productivity, driving a wedge between θ and $\hat{\theta}^c$. Similar to the risky asset demand of each household x_i from Proposition 1, the price of informational risk I is increasing in the level of investment by firms, while the quantity of informational risk $\frac{1}{1+\Sigma/\sigma_s^2}\left(\theta - \hat{\theta}^c\right)$ is increasing in the "average" pessimism of economic agents $\theta - \hat{\theta}^c$ and the level of informational frictions σ_s through the relative precision of public-to-private information Σ/σ_s^2. Consequently, investors earn risk compensation not only because of financial shocks and variance risk, but also because of distorted beliefs.

Similar to the speculative risk premium in Nimark (2012), this additional informational piece is, by construction, orthogonal to all public information, since $E\left[\frac{I}{1+\Sigma/\sigma_s^2}\left(\theta - \hat{\theta}^c\right) \mid \mathcal{F}^c\right] = 0$. Unlike the conditional mean, however, the conditional variance of this informational piece $CV = E\left[\left(\frac{I}{1+\Sigma/\sigma_s^2}\right)^2\left(\theta - \hat{\theta}^c\right)^2 \mid \mathcal{F}^c\right] = \left(\frac{I}{1+\Sigma/\sigma_s^2}\right)^2\Sigma$ is, in principle, measurable by the econometrician. This conditional variance is increasing in investment I and can be hump-shaped in the conditional variance of beliefs Σ (since $\frac{dCV}{d\Sigma} = \left(\frac{\sigma_s^2}{1+\Sigma/\sigma_s^2}\right)^2\frac{\sigma_s^2-\Sigma}{\sigma_s^2+\Sigma}$). Consequently, this informational risk premium contributes most to the time-variation in risk premia when I is sufficiently large and Σ is in an intermediate range.

To see how this informational component of risk premia affects the predictive power of asset prices for output, $Y_t = aK_t$, I integrate equation (1) from t to $s \geq t$ to find that output growth $\log \frac{Y_s}{Y_t}$ is given by

$$\log \frac{Y_s}{Y_t} = \int_t^s I_u \theta_u du + \sigma_k \left(Z_s^k - Z_t^k \right).$$

Using only public information, the covariance between output growth and expected excess returns in asset prices is

$$\begin{aligned} Cov\left[\log \frac{Y_s}{Y_t}, RP_{NREEt} \mid \mathcal{F}_t^c\right] &= \frac{I_t}{1+\Sigma_t/\sigma_s^2} Cov\left[\int_t^s I_u \theta_u du, \theta_t - \hat{\theta}_t^c \mid \mathcal{F}_t^c\right] \\ &\quad + \frac{\pi \sigma_k^2}{1-\pi} Cov\left[\int_t^s I_u \theta_u du, \xi_t \mid \mathcal{F}_t^c\right]. \end{aligned}$$

Since the riskless rate r_t is observable, $r_t \in \mathcal{F}_t^c$, I substitute for $\frac{\pi\sigma_k^2}{1-\pi}\xi_t$ with r_t from Proposition 1 to find

$$Cov\left[\log \frac{Y_s}{Y_t}, RP_{NREEt} \mid \mathcal{F}_t^c\right] = I_t Cov\left[\int_t^s I_u \theta_u du, \theta_t - \hat{\theta}_t^c \mid \mathcal{F}_t^c\right].$$

To turn off any mechanical correlation between expected excess returns and output growth, I consider the case where investment productivity shocks and liquidity shocks are uncorrelated $\alpha = 0$. In the absence of informational frictions, then, the covariance between risk premia and output growth is zero, since there is no misperception among firms or investors about θ_t, so $\hat{\theta}_t^c \equiv \theta_t$.

In the presence of informational frictions, however, this covariance is nonzero. Informational frictions introduce a short-run positive correlation between output growth and current risk premia since the true future investment productivity θ_u $u \geq t$ and investment are positively correlated at short-horizons with the true current level of investment productivity θ_t.[28] At longer horizon, the correlation weakens because of the mean-reversion in investment productivity θ_t and the potential fall in investment as it approaches its upper bound a. Since uncertainty Σ_t is countercyclical in my economy, the covariance also weakens around the peaks of business cycles, contributing to the countercyclical properties of asset price predictability for output growth. Similar insights hold for the relationship between expected returns and the growth in real investment.[29]

[28] That future investment I_u and θ_t are positively correlated when investment is not close to its upper bound follows since the growth of investment I_u is increasing $\hat{\theta}_u^c$ from Proposition 4, and $\hat{\theta}_u^c = \theta_u + \varepsilon_u$ for some ε_u such that $E\left[\varepsilon_u \mid \mathcal{F}_u^c\right] = 0$, since $\hat{\theta}_u^c$ is an unbiased estimator of θ_u.

[29] My focus in this section is on conditional covariances. It is less clear that the signs and strengths of these covariances also hold unconditionally, since for random variables X, Y, and Z, by the Law of Total

Substituting with r_t from Proposition 1, and recognizing that ξ_s and $\theta_t - \hat{\theta}_t^c$ are correlated only insofar as $\theta_t - \hat{\theta}_t^c$ is correlated with ξ_t, I also find that

$$\begin{aligned} Cov\left[\int_t^s RP_{NREEu}du, RP_{NREEt} \mid \mathcal{F}_t^c\right] &= I_t Cov\left[\int_t^s \frac{I_u}{1+\Sigma_u/\sigma_s^2}\left(\theta_u - \hat{\theta}_u^c\right)du, \theta_t - \hat{\theta}_t^c \mid \mathcal{F}_t^c\right] \\ &\quad + \frac{I_t^2\Sigma_t^2}{\Sigma_t+\sigma_s^2}(s-t), \end{aligned}$$

from which follows that $Cov\left[\int_t^s RP_{NREEu}du, RP_{NREEt} \mid \mathcal{F}_t^c\right]$ is positive. The correlation weakens at longer horizons because θ_t and $\hat{\theta}_t^c$ are mean-reverting.

Though there is this persistence in returns, households do not trade to eliminate this predictability. By the Law of Total Covariance, I can manipulate $Cov\left[\int_t^s RP_{NREEu}du, RP_{NREEt} \mid \mathcal{F}_t^c\right]$ to arrive at

$$\begin{aligned} & E\left[Cov\left[\int_t^s RP_{NREEu}du, RP_{NREEt} \mid \mathcal{F}_t^i\right] \mid \mathcal{F}_t^c\right] \\ = \; & Cov\left[\int_t^s RP_{NREEu}du, RP_{NREEt} \mid \mathcal{F}_t^c\right] \\ & - Cov\left[E\left[\int_t^s RP_{NREEu}du \mid \mathcal{F}_t^i\right], E\left[RP_{NREEt} \mid \mathcal{F}_t^i\right] \mid \mathcal{F}_t^c\right], \end{aligned}$$

from which it is apparent that the "average" perceived covariance of expected returns by household i $Cov\left[\int_t^s RP_{NREEu}du, RP_{NREEt} \mid \mathcal{F}_t^i\right]$ differs from the "average" covariance of expected returns $Cov\left[\int_t^s RP_{NREEu}du, RP_{NREEt} \mid \mathcal{F}_t^c\right]$ because of heterogeneous information. Consequently, households differ not only in their beliefs about expected returns, but also in their beliefs about the persistence of returns, which gives them incentive to trade without eliminating the predictability found with only public information.

This exercise illustrates that, in the presence of informational frictions, asset risk premia inherently contain an informational component that reflects uncertainty over current macroeconomic conditions above and beyond the correlation between real and financial shocks (since ξ may, in practice, be correlated with θ). Such a positive relationship between returns and future real activity, which arises because of the underreaction of investors to changes in the prospects of firms, is consistent, for instance, with the findings of Barro (1990), Fama (1990), and Schwert (1990). Moreover, this additional informational component exhibits

Covariance

$$Cov[X,Y] = E(Cov[X,Y \mid Z]) + Cov[E(X \mid Z), E(Y \mid Z)].$$

This implies that empirical tests would ideally focus on these conditional relationships.

countercyclical behavior, since uncertainty about investment productivity is countercyclical in the economy, and larger when financial markets are dysfunctional (larger, negative ξ shocks which depress $\hat{\theta}^c$). This may help explain why studies such as Stock and Watson (2003) and Ng and Wright (2013) find that the predictive power of asset prices for macroeconomic outcomes is somewhat episodic over business cycles, since the informational content of asset prices displays business cycle variation.

In addition to providing a measure of market liquidity ξ, which is documented, for instance, in Gilchrist, Yankov, and Zakrajsek (2009), market risk measures reflect the average expectations of market participants about the strength of the economy. This provides a strong empirical prediction that asset returns have predictive power for future returns and macroeconomic aggregates that varies with the business cycle, which is strongest during downturns and recoveries, and motivates more tests of asset pricing predictability that take this explicitly into account. Henkel, Martin, and Nardari (2011) and Dangl and Halling (2012), for instance, provide evidence of business cycle asymmetries in stock market return predictability.

Given the risk premia from the firm's perspective RP_{NREE}, one can construct the risk premium demanded by an individual household to hold firm claims

$$RP_{NREE}(i) = RP_{NREE} + \frac{I}{1+\Sigma/\sigma_s^2}\left(\hat{\theta}^c - \hat{\theta}(i)\right).$$

Since $RP_{NREE}(i)$ is increasing in the pessimism of household i, lower $\hat{\theta}(i)$ relative to the average $\hat{\theta}^c$, it follows that more pessimistic households demand higher compensation to hold firm claims, and for sufficient pessimism instead sit on their capital by investing it in the riskless asset. This pattern is consistent with the tightening of lending standards seen in the FRB Senior Loan Officer Survey during the recent recession and recovery. In support of this prediction, the survey respondents often cited a poor economic outlook, along with bank competition, as a key factor in shaping their lending standards.

A.2. Asset Turnover

Though trading volume and asset turnover have been studied extensively in the literature, relatively little attention has been given to their business cycle properties.[30] Sarolli (2013) and DeJong and Espino (2011), for instance, provide evidence of business cycle variation in

[30] See Lo and Wang (2009) for a survey of this literature.

turnover. My analysis aims to help understand how differential information influences asset turnover over the business cycle and provides new empirical predictions.

To explore these issues, I derive a measure $\mathcal{V}$ on asset turnover (trading volume / shares outstanding) from informational trading at any given instant in the economy. To do so, I recognize that households that trade because of preference shocks with take an aggregate position -$\pi\xi W$ in firm claims, and that households that trade for informational and market-making reasons each invest a fraction of their wealth

$$x(i) = \frac{1}{1-\pi} + \frac{\pi}{1-\pi}\xi + \frac{I}{\sigma_k^2}\frac{\Sigma}{\Sigma+\sigma_s^2}\sigma_s Z^s(i),$$

and take an aggregate position $(1+\pi\xi)W$. Intuitively, informational and market-making households take the offsetting position against liquidity traders plus a directional bet on the prospects of the economy based on the noise in their private signals. I thus construct a pseudo liquidity trader that takes a position $-\pi\xi W$ each period, and pseudo informational and market-making traders of mass $1-\pi$ that start with wealth W and always receive the same signal noise $Z^s(i)$.

This construction of pseudo traders is meant to mitigate the trading that arises because of preference shocks and the OLG structure of households, which mechanically leads to large changes in individual trader positions. I do not view the simplification as material for my results since I are abstracting from changes in positions that occur because of preference shocks and large changes in beliefs because of the myopic nature of households, which are both static effects over the business cycle.

The informational and market-making traders each enter the market with a position $X_I = x(i)W$ and will trade to have a position

$$\begin{aligned} dX_I &= W\frac{I}{\sigma_k^2}\frac{\Sigma\sigma_s}{\Sigma+\sigma_s^2}\left(g + \frac{\sigma_s^2}{\Sigma+\sigma_s^2}\frac{1}{\Sigma}\frac{d\Sigma}{dt}\right)Z^s(i)\,dt + x(i)W\left(I\theta - \delta - \frac{I}{a-I}g\right)dt \\ &\quad + \frac{\pi}{1-\pi}W\sigma_\xi dZ^\xi + x(i)W\sigma_k dZ^k. \end{aligned}$$

Following the insights of Xiong and Yan (2010), I aggregate the local volatility of these position changes and normalize by the price / share of firm claims q as a measure of trading volume[31]

$$\frac{1}{dt}E\left[v \mid q, W, \hat{\theta}^{c}, I, \Sigma\right] = \left(\frac{W}{q}\right)^2 \int_0^1 \left(\left(\frac{\pi}{1-\pi}\sigma_\xi\right)^2 + \left(x\left(i\right)\sigma_k\right)^2\right) di.$$

Substituting for $x\left(i\right)$ and applying the weak LLN, I arrive at

$$\frac{1}{dt}E\left[v \mid q, W, \hat{\theta}^{c}, I, \Sigma\right] = K^2\left(\frac{\pi^2}{1-\pi}\sigma_\xi^2 + \frac{\left(1+\pi\xi\right)^2}{1-\pi}\sigma_k^2 + \left(1-\pi\right)\left(\frac{I}{\sigma_k}\frac{\Sigma}{\Sigma+\sigma_s^2}\sigma_s\right)^2\right).$$

From Section IV, $W = qK$, and therefore $K = W/q$ is the total number of shares outstanding for firm claims. From Section IV, $W = qK$ is the total market capitalization of firms, and therefore $K = W/q$ is the total number of shares outstanding for firm claims.

When $\sigma_s \nearrow \infty$, and there is no private information, then this expression reduces to

$$\frac{1}{dt}E\left[v^* \mid q, W, \hat{\theta}^{c}, I, \Sigma\right] = K^2\left(\frac{\pi^2}{1-\pi}\sigma_\xi^2 + \frac{\left(1+\pi\xi\right)^2}{1-\pi}\sigma_k^2\right),$$

which represents the level of pseudo trading volume not driven by information. Thus the difference $\frac{1}{dt}E\left[v \mid W, \hat{\theta}^{c}, I, \Sigma\right] - \frac{1}{dt}E\left[v^* \mid q, W, \hat{\theta}^{c}, I, \Sigma\right]$ normalized by total shares outstanding K delivers me my measure of share turnover from informational trading

$$\mathcal{V} = \left(1-\pi\right)\left(\frac{I}{\sigma_k}\frac{\Sigma}{\Sigma+\sigma_s^2}\sigma_s\right)^2.$$

When there is no asymmetric information among households, either $\sigma_s \searrow 0$, and households all know the hidden investment productivity θ, $\sigma_s \nearrow \infty$, and all households are equally naíve, or $\Sigma \searrow 0$, and there is no uncertainty about θ, then $\mathcal{V} \searrow \prime$, and there is no informational trading. Intuitively, households trade when they have heterogeneous information on which to speculate against each other.

Asset turnover $\mathcal{V}$ from informational trading is increasing in both real investment I and the level of uncertainty Σ. Similar to Xiong and Yan (2010), this measure of turnover is increasing in the disagreement among investors, as measured by Σ, since $\Sigma\left(i\right)$ is increasing in Σ. Li and Li (2014) provide evidence that belief dispersion about macroeconomic conditions positively correlates with stock market turnover. Asset turnover from informational trading

[31] Xiong and Yan (2010) motivates this measure by recognizing that the absolute value of realized position changes over small intervals is finite and increasing, on average, in the volatility of the position change.

is, consequently, strongest when real investment and uncertainty are in an intermediate range. This pattern helps us understand why market prices are most informative about investment productivity during downturns and recoveries, which is when a negative financial shock can be particularly devastating. Market prices have their highest information content during these parts of the business cycle because they are when households are trading intensely on their private information, and asset markets have high turnover.

B. *Implications for Econometric Models*

My analysis has several conceptual implications for empirical models that I now explore in this section. Building off the discussion in the previous section of the business cycle properties of risk premia in financial markets that arises because of learning, my analysis motivates econometricians to take advantage of this behavior for macroeconomic forecasting. Since real signals are procyclical, and those of financial markets are strongest during downturns and recoveries, a weighting scheme that weighs financial market data more heavily around troughs and real data near peaks is likely to be fruitful. My analysis also stresses the importance of including measures of uncertainty as forecasting variables because of the information aggregation channel in financial markets, yet cautions that uncertainty is itself endogenous and driven by fluctuations in both the real economy and financial markets.

A second econometric issue my model highlights occurs when the econometrician tries to disentangle the channels by which financial market dysfunction propagates to the real economy in the presence of informational frictions using structural vector autoregressions (SVARs) or factor models.[32] Since a financial market shock impacts expectations about the real economy through learning from prices, it is, in part, perceived as a negative shock to real economic fundamentals. Specifically, the riskless rate in my economy is the sum of real investment productivity θ_t and the aggregate market liquidity shock ξ_t. In the presence of informational frictions, however, firms decompose θ_t and ξ_t instead into their perceived counterparts, $\hat{\theta}_t^c$ and $\hat{\xi}_t^c$, respectively. For them to react to the financial market shock, it must be the case that this decomposition results in $\hat{\theta}_t^c < \theta_t$ and $\hat{\xi}_t^c < \xi_t$, and thus the shock propagates to the real economy by depressing firm expectations about θ_t. This highlights an invertibility issue that arises when firms learn from prices when making real decisions that

[32] There are abundant similarities in recovering structural shocks from reduced-form VARs and from factor models, since factor innovations estimated by principal components are unique only up to orthonormal rotations of the $SO(n)$ group.

prevents the econometrician from finding an orthonormal rotation that can recover the true historical decomposition of structural financial market shocks from reduced-form VAR or factor model innovations.[33][34]

Finally, a third implication of learning from financial markets over the business cycle is that shocks to uncertainty are inherently entangled with shocks to financial markets. As illustrated in Section V, prices that measure financial distress, such as market risk premia and credit spreads, can contain an informational component in the presence of informational frictions that reflects uncertainty about current economic conditions. Since private agents learn from prices, adverse financial shocks will affect the conditional variance of their expectations, as can be seen in Proposition 3 in Section IV, and consequently will also propagate through the economy as uncertainty shocks back to prices. This makes it difficult to separate structural shocks stemming from financial market dislocation from innovations to uncertainty because of learning, and relates to the use of prices as external instruments in disentangling these structural shocks from reduced-form VAR and factor model innovations. Such a channel, for example, can help explain the high correlation between the recovered financial distress and uncertainty shocks found in Stock and Watson (2012).[35]

C. Other Empirical Implications

Several additional empirical predictions of the impact of feedback in learning merit mention. First, since uncertainty in my framework is countercyclical and downturns stem from real shocks to investment productivity, my model is consistent with the observations of Nakamura et al (2012) that, unconditionally, first moment shocks are negatively correlated with movements in uncertainty.

Second, while not the central focus of my analysis, another implication of asymmetric learning over the business cycle with dispersed information is that my model predicts coun-

[33] This invertibility issue is different from the one that arises because private agents and the econometrician have nested information sets, as explored, for instance, in Hansen and Sargent (1991) and Leeper, Walker, and Yang (2013). There is a large literature on dealing with news shocks when agents have superior information to the economerician. See, for instance, Beaudry and Portier (2006), Fujiwara, Hirose, and Shintani (2011), and Schmitt-Grohé and Uribe (2012).

[34] Sockin and Xiong (2014) make a similar point about trying to disentangle supply and demand shocks in commodity markets in the presence of informational frictions.

[35] Stock and Watson (2012) use innovations to the VIX and the poliicy news uncertainty index of Baker, Bloom, and Davis (2013) as instruments for uncertainty shocks. The VIX, as a measure of market volatility, has a direct analogue with prices in my economy. Innovations to the policy uncertainty index have a correlation of about 0.2 with the forecast dispersion of the Survey of Professional Forecasters, which can be viewed as a noisy analogue of uncertainty in my economy.

tercyclical dispersion in wealth across households, a feature consistent with evidence from the latest recession.[36] This arises because informational frictions are most severe at the trough, where agents have incentive to trade on their private information, whereas, at the peak, uncertainty about the underlying strength of the economy Σ is low and households coordinate around the common knowledge belief $\hat{\theta}^c$ (since Σ/σ_s^2 is small).

Finally, my model features asset prices as a coordination mechanism among firms in making their investment decisions. My model, therefore, offers an additional information channel through which learning by individual firms can give rise to the strong comovement in macroeconomic aggregates documented in Christiano and Fitzgerald (1998), and since heavily exploited through factor model analysis in the macroeconometric literature. This channel is distinct from the information externality channel informally discussed in Christiano and Fitzgerald (1998), as well as the mechanism of strategic complementarity in common information that arises because of costly sector-specific information acquisition featured in Veldkamp and Wolfers (2007).

VIII. Conclusion

In this paper, I develop a dynamic model of information aggregation in financial markets in a macroeconomic setting where both financial investors and firm managers learn about the productivity of investment from market prices. My dynamic framework features a feedback loop between investor trading behavior and firm real investment decisions by which noise in financial prices can feed into real investment through learning by firm managers, and then feed back into financial prices through the impact of learning and investment on the trading incentives of market participants. This feedback loop highlights a possible amplification mechanism through which the financial crisis of 2008 contributed to the deep recession and anemic recovery in the US by distorting firm expectations about the strength of the US economy.

While the strength of signals from real activity is procyclical, that of financial signals is strongest during downturns and recoveries. This occurs because the value of private information that financial investors have increases with uncertainty about real investment

[36] Since the noise in household private signals is unbiased, the wealth distribution is a mean-preserving spread of the wealth of an agent who has perfect-information. The wealth of this perfectly-informed pseudo-agent will, in general, not be the same as the wealth of the representative household in either benchmark because heterogeneous information impacts both investment decisions and the risk premia on firm claims.

productivity, which is countercyclical, and more information is aggregated into prices as investors start to trade against each other on their private information. As a result, financial signals are strongest when real investment and uncertainty are in an intermediate range.

I then explore the welfare and empirical implications of my model. Informational frictions introduce a role for policy to provide guidance to economic agents about the current state of the economy. As an empirical prediction of my model, informational frictions also give rise to an informational component in asset risk premia that has predictive power for future returns and real activity. This predictive power is greatest during downturns and recoveries when asset turnover from informational trading is highest. Finally, informational frictions make it difficult to disentangle the effects of financial and uncertainty shocks in the data, and confound attempts to recover historical structural shocks stemming from the financial crisis of 2008.

References

Aizenman, Joshua, Brian Pinto, and Vladyslav Sushko (2012), Financial Sector Ups and Downs and the Real Sector: Up by the Stairs and Down by the Parachute, mimeo UCSC, The World Bank, and BIS.

Albagi, Elias (2010), Amplification of Uncertainty in Illiquid Markets, mimeo University of Southern California.

Albagi, Elias, Christian Hellwig, and Aleh Tsyvinski (2012), A Theory of Asset Prices based on Heterogeneous Information, mimeo USC Marshall, Toulouse School of Economics, and Yale University.

Albagi, Elias, Christian Hellwig, and Aleh Tsyvinski (2013), Dynamic Dispersed Information and the Credit Spread Puzzle, mimeo USC Marshall, Toulouse School of Economics, and Yale University.

Albagi, Elias, Christian Hellwig, and Aleh Tsyvinski (2014), Risk-Taking, Rent-Seeking, and Investment when Financial Markets are Noisy, mimeo USC Marshall, Toulouse School of Economics, and Yale University.

Allen, Frank, Stephen Morris, and Hyun Song Shin (2006), Beauty Contests and Iterated Expectations in Asset Markets, *The Review of Financial Studies* 19, 719-752.

Angeletos, George-Marios, Guido Lorenzoni, and Alessandro Pavan (2012), Wall Street and Silicon Valley: A Delicate Interaction, mimeo MIT and Northwestern University.

Bacchetta, Philippe and Eric van Wincoop (2006), Can Information Heterogeneity Explain the Exchange Rate Determination Puzzle, *American Economic Review* 96, 552-576.

Bacchetta, Philippe and Eric van Wincoop (2008), Higher Order Expectations in Asset Pricing, *Journal of Money, Banking, and Credit* 40, 837-866.

Baily, Martin Neil and Barry Bosworth (2013), The United States Economy: Why such a Weak Recovery?, Paper prepared for the Nomura Foundation's Macro Economy Research Conference, "Prospects for Growth in the World's Four Major Economies," September 11, 2013.

Baker, Scott R., Nicholas Bloom, and Steven J. Davis (2013), Measuring Economic Policy Uncertainty, mimeo Stanford University and University of Chicago Booth School of Business.

Bakke, Tor-Erik and Toni Whited (2010), Which Firms Follow the Market? An Analysis of Corporate Investment Decisions, *The Review of Financial Studies* 23, 1941-1980.

Barro, Robert J. (1990), The Stock Market and Investment, *Review of Financial Studies* 3, 115-131.

Barro, Robert J. and José F. Ursúa (2009), Stock Market Crashes and Depressions, mimeo Harvard University.

Beaudry, Paul and Franck Portier (2006), Stock Prices, News, and Economic Fluctuations, *American Economic Review* 96, 1293-1307.

Beaudry, Paul and Franck Portier (2013), News Driven Business Cycles: Insights and Challenges, mimeo Vancouver School of Economics and Toulouse School of Economics.

Bernanke, Ben (1983), Irreversibility, Uncertainty, and Cyclical Investment, *Quarterly Journal of Economics* 98, 85-106.

Bernanke, Ben (2014), The Federal Reserve: Looking Back, Looking Forward, AEA Meeting 2014.

Blanchard, Olivier J., Jean-Paul L'Huillier, and Guido Lorenzoni (2013), News, Noise, and Fluctuations: An Empirical Exploration, *American Economic Review* 103, 3045-3070.

Bloom, Nicholas (2009), The Impact of Uncertainty Shocks, *Econometrica* 77, 623-685.

Bond, Phillip, Alex Edmans, and Itay Goldstein (2012), The Real Effects of Financial Markets, *Annual Review of Financial Economics* 4, 1-22.

Brown, David P. and Robert H. Jennings (1989), On Technical Analysis, *The Review of Financial Studies* 2, 527-551.

Cao, Daniel (2011), Collateral Shortages, Asset Price and Investment Volatility with Heterogeneous Beliefs, mimeo Georgetown University.

Cambell, John Y. and Albert S. Kyle, (1993), Smart Money, Noise Trading and Stock Price Behavior, *Review of Economic Studies* 60, 1-34.

Cambell, John Y. and Luis M. Viceira, (2002), Strategic Asset Allocation, Cambridge University Press, Cambridge, UK.

Chen, Qi, Itay Goldstein, and Wei Jiang (2007), Price Informativeness and Investment Sensitivity to Stock Price, *The Review of Financial Studies* 20, 619-650.

Christiano, Lawrence J., and Terry J. Fizgerald (1998), The Business Cycle: It's Still a Puzzle, *Federal Reserve Bank of Chicago Economic Perspectives* 22, 56-83.

Cox, John C., Jonathan E. Ingersoll, and Stephen A. Ross (1985), A Theory of the Term Structure of Interest Rates, *Econometrica* 53, 385-408.

Daly, Mary C., Bart Hobijn, Asyengül Sahin, and Robert G. Valletta (2012), A Search and Matching Approach to Labor Markets: Did the Natural Rate of Unemployment Rise?, *Journal of Economic Perspectives* 26, 3-26.

Dang, Tri Vi, Gary Gorton, and Bengt Holmström (2013), Ignorance, Debt, and Financial Crises, mimeo Columbia University, Yale University, and MIT.

Dangl, Thomas, and Michael Halling (2012), Predictive Regressions with Time-Varying Coefficients, *Journal of Financial Economics* 106, 157-181.

David, Joel, Hugo A. Hopenhayn, and Venky Venkateswaran (2014), Information, Misallocation and Aggregate Productivity, mimeo USC, UCLA, and NYU Stern.

Davis, Steven J., R. Jason Faberman, and John C. Haltiwanger (2013), The Establishment-Level Behavior of Vacancies and Hiring, *Quarterly Journal of Economics,* forthcoming.

Detemple, Jerome and Shashidhar Murphy (1994), Intertemporal Asset Pricing and Heterogeneous Beliefs, *Journal of Economic Theory* 62, 294-320.

DeJong, David N. and Emilio Espino (2011), The Cyclical Behavior of Equity Turnover, *Quantitative Economics* 2, 99-133.

Fajgelbaum, Pablo, Edouard Schaal, and Mathieu Taschereau-Dumouchel (2014), Uncertainty Traps, mimeo UCLA, NYU, and Wharton.

Fama, Eugene F (1981), Stock Returns, Real Activity, Inflation, and Money, *American Economic Review* 71, 545–565.

Fama, Eugene F. (1990), Stock Returns, Expected Returns, and Real Activity, *Journal of Finance* 45, 1089-1108.

FCIC (2011), The Financial Crisis Inquiry Report: Final Report of the National Commission of the Causes of the Financial and Economic Crisis of the United States.

Fernández-Villaverde, Jesús, Pablo Guerrón-Quintana, Keith Kuester, and Juan Rubio-Ramírez (2013), Fiscal Volatility and Economic Activity, mimeo NBER.

Fujiwara, Ippei, Yasuo Hirose, and Mototsugu Shintani (2011), Can News Be a Major Source of Aggregate Fluctuations? A Bayesian DSGE Approach, *Journal of Money, Banking, and Credit* 43, 1-29.

Futia, Carl A. (1981), Rational Expectations in Stationary Linear Models, *Econometrica* 49, 171-192.

Foster, F. Douglas and S. Viswanathan (1996), Strategic Trading When Agents Forecast the Forecast of Others, *Journal of Finance* 51, 1437-1478.

Gao, Zhenyu, Sockin, Michael, and Wei Xiong (2014), Learning about the Neighborhood: The Role of Supply Elasticity for Housing Cycles, mimeo Chinese University of Hong Kong and Princeton University.

Gertler, Mark, and Cara S. Lown (1999), The Information in the High-Yield Bond Spread for the Business Cycle: Evidence and Some Implications, *Oxford Review of Economic Policy* 15, 132–150.

Gilchrist, Simon, V. Yankov, and Egon Zakrajsek (2009), Credit Market Shocks and Economic Fluctuations: Evidence from Corporate Bond and Stock Markets, *Journal of Monetary Economics* 56, 471-493.

Gilchrist, Simon and Egon Zakrajsek (2012), Credit Spreads and Business Cycle Fluctuations, *American Economic Review* 102, 1692-1720.

Goldstein, Itay, Emre Ozdenoren, and Kathy Yuan (2013), Trading frenzies and their impact on real investment, *Journal of Financial Economics* 109, 566-582.

Gorton, Gary and Guillermo Ordoñez (2012), Collateral Crises, mimeo Yale University.

Greenwald, Bruce and Joseph E. Stiglitz (1986), Externalities in Economies with Imperfect Information and Incomplete Markets, *Quarterly Journal of Economics* 101, 229-264.

Greenwood, Jeremy, Zvi Hercowitz, and Per Krusell (1997), Long-Run Implications of Investment-Specific Technological Change, *American Economic Review* 87, 342-362.

Greenwood, Jeremy, Zvi Hercowitz, and Per Krusell (2000), The Role of Investment-Specific Technological Change in the Business Cycle, *European Economic Review* 44, 91-115.

Greenwood, Jeremy and Boyan Jovanovic (1990), Financial Development, Growth, and the Distribution of Income, *Journal of Political Economy* 98, 1076-1107.

Gunn, Christopher M. and Alok Johri (2013), An Expectations-Driven Interpretation of the "Great Recession", *Journal of Monetary Economics* 60, 391-407.

Hansen, Lars Peter and Thomas J. Sargent (1991), Two Difficulties in Interpreting Vector Autoregressions, in *Rational Expectations Econometrics* edited by Lars Peter Hansen and Thomas J. Sargent, Boulder CO: Westview Press, 77-119.

Hassan, Tarek and Thomas M. Mertens (2014), Information Aggregation in a DSGE Model, *NBER Macroeconomics Annual 2014*, forthcoming.

Hassan, Tarek and Thomas M. Mertens (2014), The Social Cost of Near-Rational Investment, mimeo University of Chicago Booth School of Business and NYU Stern School of Business.

He, Hua and Jiang Wang (1995), Differential Information and Dynamic Behavior of Stock Trading Volume, *The Review of Financial Studies* 8, 919-972.

He, Zhiguo and Arvind Krishnamurthy (2012), Intermediary Asset Pricing, *American Economic Review* 103, 732-770.

Henkel, Sam James., J. Spencer Martin, and Federico Nardari (2011), Time-Varying Short-Horizon Predictability, *Journal of Financial Economics* 99, 560–580

Judd, Kenneth (1985), The Law of Large Numbers with a Continuum of IID Random Variables, *Journal of Economic Theory* 35, 19-25.

Kitagawa, G. (1996), Monte Carlo Filter and Smoother for Non-Gaussian Nonlinear State Space Models, *Journal of Computational and Graphical Statistics* 5, 1–25.

Kobayashi, K., and K. Nutahara (2007), Collateralized Capital and News-Driven Cycles, *Economics Bulletin* 5, 1-9.

Kobayashi, K., T. Nakajima, and M. Inaba (2012), Collateral Constraint And News-Driven Cycles, *Macroeconomy Dynamics* 16, 752-776.

Kogan, Leonid and Dimitris Papanikolaou (2013), Firm Characteristics and Stock Returns: The Role of Investment-Specific Shocks, *The Review of Financial Studies,* forthcoming.

Kurlat, Pablo (2013), Lemon Markets and the Transmission of Aggregate Shocks, *American Economic Review* 103, 1463-1489.

Leduc, Sylvain, and Zheng Liu (2013), Uncertainty and the Slow Labor Market Recovery, *FRBSF Economic Letter* 2013-21.

Leeper, Eric M., Todd B. Walker, and Shu-Chun Susan Yang (2013), Fiscal Foresight and Information Flows, *Econometrica* 81, 1115-1145.

Li, Dan and Geng Li (2014), Are Household Investors Noise Traders? Evidence from Belief Dispersion and Stock Trading Volume, mimeo Federal Reserve Board.

Lo, Andrew and Jiang Wang (2009), Stock Market Trading Volume, in *Handbook of Financial Econometrics* edited by Yacine Ait-Sahalia and Lars Peter Hansen, New York: North-Holland: Elsevier.

Lorenzoni, Guido (2009), A Theory of Demand Shocks, *American Economic Review* 99, 2050-84.

Lucas, Robert. E, Jr. (1987), Models of Business Cycles, Basil Blackwell, New York.

Luo, Yuanzhi (2005), Do Insiders Learn from Outsiders? Evidence from Mergers and Acquisitions, *Journal of Finance* 60, 1951–1982.

Mian, Atif, and Amir Sufi (2012), What Explains High Unemployment? The Aggregate Demand Channel, mimeo NBER.

Moreira, Alan and Alexi Savov (2013), The Macroeconomics of Shadow Banking, mimeo Yale University and NYU Stern School of Business.

Moyen, Nathalie, and Stefan Platikanov, Corporate Investments and Learning, *Review of Finance* 17, 1437-1488.

Muir, Tyler (2014), Financial Crises and Risk Premia, mimeo Yale School of Management.

Nakamura, Emi, Dmitriy Sergeyev, and Jon Steinsson (2012), Growth-Rate and Uncertainty Shocks in Consumption: Cross-Country Evidence, mimeo NBER.

Ng, Serena and Johnathan Wright (2013), Facts and Challenges from the Great Recession for Forecasting and Macroeconomic Modeling, *Journal of Economic Literature* 51, 1120-1154.

Nimark, Kristoffer (2012), Speculative Dynamics in the Term Structure of Interest Rates, mimeo CREI.

Ordoñez, Guillermo (2012), The Asymmetric Effects of Financial Frictions, mimeo UPENN.

Pinkowitz, Lee, René M. Stulz, and Rohan Williamson (2013), Is there a U.S High Cash Holdings Puzzle after the Financial Crisis?, mimeo Georgetown University and Ohio State University.

Philippon, T (2008), The Evolution of the US Financial Industry from 1860 to 2007: Theory and Evidence, mimeo NYU Stern School of Business.

Pozsar, Zoltan, Tobias Adrian, Adam Ashcraft, and Hayley Boesky (2012), Shadow Banking, Federal Reserve Staff Report 458.

Reifschneider, Dave, William L. Wascher, David Wilcox (2013), Aggregate Supply in the United States: Recent Developments and Implications for the Conduct of Monetary Policy, mimeo Federal Reserve Board.

Reinhart, Carmen M. and Kenneth S. Rogoff (2009a), The Aftermath of Financial Crises, *American Economic Review* 99, 466-472.

Reinhart, Carmen M. and Kenneth S. Rogoff (2009b), This Time is Different: Eight Centuries of Financial Folly, Princeton and Oxford: Princeton University Press

Reinhart, Carmen M. and Kenneth S. Rogoff (2011), From Financial Crash to Debt Crisis, *American Economic Review* 101, 1676-1706.

Sarolli, Gian Domenico (2014), Cleaning the Gears: Counter-cyclical Asset Trading with Financial Transactions Taxes, forthcoming *The Quarterly Review of Economics and Finance.*

Schwert, G. William (1990), Stock Returns and Real Economic Activity: A Century of Evidence, *Journal of Finance* 45, 1237-1257.

Sockin, Michael and Wei Xiong (2014), Information Frictions and Commodity Markets, forthcoming *Journal of Finance.*

Straub, Ludwig and Robert Ulbricht (2013), Credit Crunches, Information Failures, and the Persistence of Pessimism, mimeo Northwestern University.

Stock, James and Mark Watson (2003), Forecasting Output and Inflation: The Role of Asset Prices, *Journal of Economic Literature* 41, 788-829.

Stock, James and Mark Watson (2012), Disentangling the Channels of the 2007-2009 Recession, *Brookings Papers on Economic Activity* Spring 2012, 81-156.

Subrahmanyam, Avanidhar and Sheridan Titman (2013), Financial Market Shocks and the Macroeconomy, *The Review of Financial Studies* 26, 2687-2717.

Sun, Yeneng and Yongchao Zhang (2009), Individual Risk and Lebesgue Extension Without Aggregate Uncertainty, *Journal of Economic Theory* 144, 432-443.

Tinn, Katrin (2010), Technology Adoption and Exit in Imperfectly Informed Equity Markets, *American Economic Review* 10, 925-957.

Townsend, Robert (1983), Forecasting the Forecasts of Others, *Journal of Political Economy* 91, 546-588.

Van Nieuwerburgh, Stijn and Laura Veldkamp (2006), Learning Asymmetries in Real Business Cycles, *Journal of Monetary Economics* 53, 753-772.

Veldkamp, Laura and Justin Wolfers (2007), Aggregate Shocks or Aggregate Information? Costly Information and Business Cycle Comovement, *Journal of Monetary Economics* 54, 37-55.

Wang, Jiang (1993), A Model of Intertemporal Asset Prices Under Asymmetric Information, *Review of Economic Studies* 60, 249-282.

Wang, Jiang (1994), A Model of Competitive Stock Trading Volume, *Journal of Political Economy* 102, 127-168.

Uhlig, Harald (1996), A Law of Large Numbers for Large Economies, *Economic Theory* 8, 41-50.

Woodford, Michael (2003), Imperfect Common Knowledge and the Effects of Monetary Policy, in P.Aghion, R.Frydman, J.Stiglitz and M.Woodford, eds., Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps. Princeton, NJ: Princeton University Press, 25– 58.

Xiong, Wei and Hongjun Yan (2010), Heterogeneous Expectations and Bond Markets, *The Review of Financial Studies* 23, 1433-1466.

Appendix: Proofs of Propositions

Proof of Proposition 2:

Households solve the optimization problem (4) subject to equation (9). In a recursive competitive equilibrium, all equilibrium objects are functions of the state of the economy from the household's perspective $\left(w(i), \hat{\theta}(i), l(i), h\right)$, where h is a list of general equilibrium objects including $\log K$ and r. Since the household treats prices as exogenous in the competitive equilibrium, the price of firm equity q and the riskless rate r are additional states for the household. This, however, only affects their optimal consumption and portfolio choices, in which they do not see the dependence of these prices on the Markov states. By the Martingale Representation Theorem, all these objects will be continuous Itô-semimartingales with respect to the smallest filtration on which they are measurable to the household. The Wiener processes to which they are adapted, which will be common to all households, are absolutely continuous with respect to the true processes for investment productivity θ, household liquidity shocks ξ, and the aggregate diffusion for K.

Taking the limit of problem (4) as $\Delta t \searrow dt$, assuming v is twice differentiable in its arguments, I can differentiate v and take expectations to find

$$\rho v = \sup_{\{c,x\}} \log c + \partial_w v \frac{1}{dt} E\left[dw(i) \mid \mathcal{F}^i\right] + \frac{1}{2}\partial_{ww} v \frac{1}{dt} d\left\langle w(i) \mid \mathcal{F}_t^i\right\rangle + \frac{1}{dt}\partial_t v, \tag{A-1}$$

subject to the law of motion of $w(i)$ (9), and $\langle \cdot \mid \mathcal{F}_t^i \rangle$ indicates quadratic variation under the measure $\mathcal{F}_t^i$. The $\partial_t v$ term is meant to capture the additional dependence of the drift of the household's bequest utility v on the vector of general equilibrium objects h that the household takes as given. Equation (A-1) is the usual Hamilton-Jacobi-Bellman (HJB) equation for optimal control. Necessity and sufficiency of the FOCs for the optimal controls $\{c, x\}$ follows from the concavity of their programs.

Before deriving the FOCs of the HJB equation (A-1) for households, it is useful to first recognize that all Wiener processes $\tilde{Z}_t^{\xi}(i)$ and $\tilde{Z}_t^{k}(i)$ will be uncorrelated under each household $i's$ measure since the true processes are uncorrelated and the change of measure under Girsanov's Theorem is equivalent to a change in drift. Innovations in these processes must be uncorrelated but, in general, they will be correlated unconditionally because of correlation in these drifts.

Suppressing arguments for the bequest utility v, the FOCs of the HJB equation (A-1) are given by

$$\begin{aligned}
c(i) &: \frac{1}{c(i)} - \partial_w v \leq 0 \ \ (= \ if \ c > 0), \\
x(i) &: 0 = w(i)\,\partial_w v\left(\frac{a-I}{q} + \frac{\partial_I q}{q} Ig + I\hat{\theta}(i) - r - \delta\right) + x(i)\,w(i)^2\,\partial_{ww} v \sigma_k^2 \\
&\quad + w(i)\,\partial_{wh} v d\left\langle \tilde{Z}^k(i), h \mid \mathcal{F}^i \right\rangle,
\end{aligned}$$

when household i is not hit by the liquidity shock $l(i) = 0$, from which follows that

$$x(i) = -\frac{\partial_w v\left(\frac{a-I}{q} + \frac{\partial_I q}{q} Ig + I\hat{\theta}(i) - r - \delta\right)}{w(i)\,\partial_{ww} v \sigma_k^2} - \frac{\partial_{wh} v d\left\langle \tilde{Z}^k(i), h \mid \mathcal{F}^i\right\rangle}{w(i)\,\partial_{ww} v \sigma_k^2}.$$

While objects in h like r all have Itô-semimartingale representations by the Martingale Representation Theorem, I do not expand out the quadratic covariation expressions for brevity.

Given that households have log utility, I conjecture that $v\left(w(i), \hat{\theta}(i), l(i), h\right) = A \log w(i) + f\left(\hat{\theta}(i), l(i), h\right)$. This conjecture implies that

$$\begin{aligned}
c(i) &= \frac{w(i)}{A}, \\
x(i) &= \begin{cases} \frac{\frac{a-I}{q} + \frac{\partial_I q}{q} Ig + I\hat{\theta}(i) - r - \delta}{\sigma_k^2} & l(i) = 0 \\ -\xi & l(i) = 1 \end{cases}.
\end{aligned}$$

Substituting this conjecture and the controls into the maximized HJB equation

$$\begin{aligned}
\rho v &= \log c + \partial_w v\left(x(i)\left(\frac{a-I}{q} + \frac{\partial_I q}{q} Ig + I\hat{\theta}(i) - r - \delta\right) w(i) + r w(i) - c(i)\right) \\
&\quad + \frac{1}{2}\partial_{ww} v x(i)^2\, w(i)^2\, \sigma_k^2 + \partial_t f\left(\hat{\theta}(i), l(i), h\right),
\end{aligned}$$

where $\partial_t f\left(\hat{\theta}(i), l(i), h\right)$ is shorthand for remaining terms in the HJB equation, it follows that $A = \frac{1}{\rho}$, $c(i) = \rho w(i)$, and that $f\left(\hat{\theta}(i), l(i), h\right)$ implicitly satisfies

$$\begin{aligned}
\rho f\left(\hat{\theta}(i), l(i), h\right) &= \log \rho + \frac{1}{\rho}\left(r - \rho + x(i)\left(\frac{a-I}{q} + \frac{\partial_I q}{q} Ig + I\hat{\theta}(i) - r - \delta\right) - \frac{1}{2} x(i)^2\, \sigma_k^2\right) \\
&\quad + \partial_t f\left(\hat{\theta}(i), l(i), h\right),
\end{aligned}$$

which confirms the conjecture since $x(i)$ does not depend on $w(i)$.

When the household is hit by the liquidity shock, $l(i) = 1$, then $x(i) = -\xi$. Direct verification of the value function $v\left(w(i), \hat{\theta}(i), l(i), h\right) = A \log w(i) + f\left(\hat{\theta}(i), l(i), h\right)$ in the maximized HJB equation again confirms the conjectured functional form and that $c(i) = \rho w(i)$.

Recognizing that $v\left(w(i), \hat{\theta}(i), l(i), h\right) = A \log w(i) + f\left(\hat{\theta}(i), h\right)$, the envelope condition for the maximized HJB equation (A-1) evaluated at the optimal controls takes the form

$$\begin{aligned}
\rho \partial_w v &= \partial_{ww} v \left(x(i) w(i) \left(\frac{a-I}{q} + \frac{\partial_I q}{q} I g + I \hat{\theta}(i) - r - \delta \right) + r w(i) - c(i) \right) \\
&\quad + \frac{1}{2} \partial_{www} v x(i)^2 w(i)^2 \sigma_k^2 + \partial_{ww} v x(i)^2 w(i) \sigma_k^2 \\
&\quad + \partial_w v \left(x(i) \left(\frac{a-I}{q} + \frac{\partial_I q}{q} I g + I \hat{\theta}(i) - r - \delta \right) + r \right).
\end{aligned}$$

Applying Itô's Lemma directly to $\partial_w v$, one also has that

$$\begin{aligned}
d(\partial_w v) &= \partial_{ww} v \left(x(i) w(i) \left(\frac{a-I}{q} + \frac{\partial_I q}{q} I g + I \hat{\theta}(i) - r - \delta \right) + r w(i) - c(i) \right) dt \\
&\quad + \frac{1}{2} \partial_{www} v x(i)^2 w(i)^2 \sigma_k^2 + \partial_{ww} v x(i) w(i) \sigma_k d\tilde{Z}^k.
\end{aligned}$$

Taking expectations and substituting the envelope condition, it follows that

$$\frac{1}{dt} E\left[\frac{d(\partial_w v)}{\partial_w v} \mid \mathcal{F}^i \right] = \rho - r - x(i) \left(\frac{a-I}{q} + \frac{\partial_I q}{q} I g + I \hat{\theta}(i) - r - \delta \right) - \frac{\partial_{ww} v}{\partial_w v} x(i)^2 w(i) \sigma_k^2.$$

Given $\partial_w v = \frac{1}{w}$, the solution for $x(i)$ when the household is not hit by the liquidity shock, and defining $\Lambda_t(i) = e^{-\rho t} \frac{1}{w_t(i)}$ to be the pricing kernel of household i, it follows that

$$r = -\frac{1}{dt} E\left[\frac{d\Lambda(i)}{\Lambda(i)} \mid \mathcal{F}^i \right]. \tag{A.2}$$

From $\Lambda_t(i) = e^{-\rho t} \frac{1}{w_t(i)}$, the optimal choice of $x(i)$, and equation (A.2), it follows that

$$\frac{a-I}{q} dt + E\left[\frac{d\Lambda(i)}{\Lambda(i)} + \frac{d(qK)}{qK} \mid \mathcal{F}^i \right] = x(i) \sigma_k^2 dt. = -Cov\left[\frac{d(qK)}{qK}, \frac{d\Lambda(i)}{\Lambda(i)} \mid \mathcal{F}^i \right],$$

from which one arrives at

$$\frac{a-I}{qK}Kdt+E\left[\frac{d\left(\Lambda\left(i\right)qK\right)}{\Lambda\left(i\right)qK}\mid\mathcal{F}^{i}\right]=0,$$

for household i not hit by the liquidity shock, which completes the proof.

Proof of Proposition 3:

Define $\bar{R}_{\theta}\left(\zeta_{t}\right)=R_{\theta}\left(I_{t},\Sigma_{t}\right)$, and $\bar{g}_{t}\left(\zeta_{t}\right)=g_{t}$. Given ζ_{t}, one can express the law of motion of the vector of public signals as

$$\begin{aligned} d\zeta_{t} &= A_{0}\left(\zeta_{t}\right)dt+\left[\begin{array}{c} I_{t} \\ \partial_{\Sigma}\bar{R}_{\theta}\left(\zeta_{t}\right)\frac{d\Sigma_{t}}{dt}+\partial_{I}\bar{R}_{\theta}\left(\zeta_{t}\right)I_{t}\bar{g}_{t}\left(\zeta_{t}\right)-\lambda\bar{R}_{\theta}\left(\zeta_{t}\right)\end{array}\right]\theta_{t}dt \\ &\quad +\bar{b}_{t}\left(\zeta_{t}\right)dZ_{t}^{\theta}+\bar{B}_{t}\left(\zeta_{t}\right)dZ_{t}, \end{aligned}$$

where $Z_{t}=\left[Z_{t}^{k},Z_{t}^{\xi}\right]'$ and

$$\begin{aligned} A_{0}\left(\zeta_{t}\right) &= \left[\begin{array}{c} -\delta-\frac{1}{2}\sigma_{k}^{2} \\ -\bar{R}_{\theta}\left(\zeta_{t}\right)\lambda\bar{\theta}\end{array}\right] \\ \bar{b}_{t}\left(\zeta_{t}\right) &= \left[\begin{array}{c} 0 \\ \bar{R}_{\theta}\left(\zeta_{t}\right)\sigma_{\theta}+\alpha\sigma_{\xi}\end{array}\right], \\ \bar{B}_{t}\left(\zeta_{t}\right) &= \left[\begin{array}{cc} \sigma_{k} & 0 \\ 0 & \sqrt{1-\alpha^{2}}\sigma_{\xi}\end{array}\right], \end{aligned}$$

with $\bar{R}_{\theta}\left(\zeta_{t}\right)$ uniformly bounded and $\bar{R}_{\theta}\left(\zeta_{t}\right)>0\ \forall\ \zeta_{t}$. By Theorem 7.17 of Lipster and Shiryaev (1977), then one can construct the vector of standard Wiener processes $\tilde{Z}=\left(\tilde{Z}_{t},\mathcal{F}_{t}^{c}\right)$ where $\tilde{Z}_{t}=\left[\tilde{Z}_{t}^{k},\tilde{Z}_{t}^{r}\right]'$ admits the representation

$$\begin{aligned} \tilde{Z}_{t} &= \int_{0}^{t}\left[\bar{b}_{t}\left(\zeta_{t}\right)\bar{b}_{t}\left(\zeta_{t}\right)'+\bar{B}_{s}\left(\zeta_{s}\right)\bar{B}_{s}\left(\zeta_{s}\right)'\right]^{-1/2}\times \\ &\quad \left(d\zeta_{s}-A_{0}\left(\zeta_{t}\right)dt-\left[\begin{array}{c} I_{t} \\ \partial_{\Sigma}\bar{R}_{\theta}\left(\zeta_{t}\right)\frac{d\Sigma_{t}}{dt}+\partial_{I}\bar{R}_{\theta}\left(\zeta_{t}\right)I_{t}\bar{g}_{t}\left(\zeta_{t}\right)-\lambda\bar{R}_{\theta}\left(\zeta_{t}\right)\end{array}\right]\hat{\theta}_{t}^{c}dt\right), \end{aligned}$$

where $\hat{\theta}_{t}^{c}=E\left[\theta_{t}\mid\mathcal{F}_{t}^{c}\right]$ is the conditional expectation of θ_{t} w.r.t. $\mathcal{F}_{t}^{c}$. That $\tilde{Z}$ are standard Wiener processes can be verified directly from Levy's three properties that uniquely identify Wiener processes. That $\tilde{Z}$ is a martingale generator for $\mathcal{F}_{t}^{c}$ follows since $\tilde{Z}$ generates K and

r trivially, from which the other objects of $\mathcal{F}_t^c$ can be generated, and Lemma 4.9 guarantees the existence of a representation for the driver (which possibly depends on the unobservable θ_t) in the Martingale Representation Theorem (Theorem 5.8) that is measurable w.r.t ζ_t P$-a.s.$

Given that θ_t has the representation

$$\theta_t = \int_0^t \lambda\left(\bar{\theta} - \theta_s\right) ds + \int_0^t \sigma_\theta dZ_s^\theta,$$

it follows from similar arguments that lead to the proof of Theorem 12.7 that $\hat{\theta}_t^c$ has the representation

$$\begin{aligned}\hat{\theta}_t^c \;&=\; \int_0^t \left(d\left\langle \begin{matrix} S \\ Q \end{matrix}, \sigma_\theta Z^\theta \right\rangle_s + Cov\left[\theta_s, \left[\begin{matrix} I_s \\ \partial_\Sigma \bar{R}_\theta\left(\zeta_t\right)\frac{d\Sigma_t}{dt} + \partial_I \bar{R}_\theta\left(\zeta_t\right) I_s \bar{g}_t\left(\zeta_t\right) - \lambda \bar{R}_\theta\left(\zeta_t\right) \end{matrix}\right] \theta_s \mid \mathcal{F}_s^c\right]\right)' \times \\ &\left[\bar{b}_t\left(\zeta_t\right)\bar{b}_t\left(\zeta_t\right)' + \bar{B}_s\left(\zeta_s\right)\bar{B}_s\left(\zeta_s\right)'\right]^{-1/2} d\tilde{Z}_s + \int_0^t \lambda\left(\bar{\theta} - \hat{\theta}_s^c\right) ds, \end{aligned} \tag{A.3}$$

where $d\left\langle \xi, Z^\theta \right\rangle_t$ is the quadratic covariation of ξ_t and Z_t^θ. It is easy to see that $Cov\left[\theta_s, \theta_s \mid \mathcal{F}_s^c\right] = Var\left[\theta_s \mid \mathcal{F}_s^c\right] = \Sigma_s$. The covariance matrix in equation (A.3) is given by

$$\bar{b}_t\left(\zeta_t\right)\bar{b}_t\left(\zeta_t\right)' + \bar{B}_s\left(\zeta_s\right)\bar{B}_s\left(\zeta_s\right)' = \left[\begin{matrix} \sigma_k^2 & 0 \\ 0 & \left(\bar{R}_\theta\left(\zeta_t\right)\sigma_\theta + \alpha\sigma_\xi\right)^2 + \left(1-\alpha^2\right)\sigma_\xi^2 \end{matrix}\right],$$

from which follows that

$$\left[\bar{b}_t\left(\zeta_t\right)\bar{b}_t\left(\zeta_t\right)' + \bar{B}_s\left(\zeta_s\right)\bar{B}_s\left(\zeta_s\right)'\right]^{-1/2} = \left[\begin{matrix} \frac{1}{\sigma_k} & 0 \\ 0 & \frac{1}{\sqrt{\left(\bar{R}_\theta(\zeta_t)\sigma_\theta + \alpha\sigma_\xi\right)^2 + (1-\alpha^2)\sigma_\xi^2}} \end{matrix}\right]$$

Thus it follows that $\hat{\theta}_t^c$ follows the law of motion

$$d\hat{\theta}_t^c = \lambda\left(\bar{\theta} - \hat{\theta}_t^c\right) dt + I_t \frac{\Sigma_t}{\sigma_k} d\tilde{Z}_t^k + \frac{\bar{R}_\theta\left(\zeta_t\right)\sigma_\theta^2 + \alpha\sigma_\xi\sigma_\theta + \left(\partial_\Sigma \bar{R}_\theta\left(\zeta_t\right)\frac{d\Sigma_t}{dt} + \partial_I \bar{R}_\theta\left(\zeta_t\right) I_t \bar{g}_t\left(\zeta_t\right) - \lambda \bar{R}_\theta\left(\zeta_t\right)\right)\Sigma_t}{\sqrt{\left(\bar{R}_\theta\left(\zeta_t\right)\sigma_\theta + \alpha\sigma_\xi\right)^2 + \left(1-\alpha^2\right)\sigma_\xi^2}} d\tilde{Z}_t^r.$$

Given the common Gaussian prior of households $N\left(\hat{\theta}_0^c, \Sigma_0\right)$, establishing the conditional Gaussianity of the posterior $\theta_t \mid \mathcal{F}_t^c$ can be done through similar arguments to those made in Chapter 11 of Lipster and Shiryaev (1977) with the appropriate regularity conditions. Similar to the arguments of Theorem 12.7, one can the also establish that the conditional variance of beliefs $\Sigma_t = Var\left[\theta_t \mid \mathcal{F}_t^c\right]$ follows the deterministic law of motion

$$\frac{d\Sigma_t}{dt} = \sigma_\theta^2 - 2\lambda\Sigma_t - I_t^2\frac{\Sigma_t^2}{\sigma_k^2} - \frac{\left(\bar{R}_\theta\left(\zeta_t\right)\sigma_\theta^2 + \alpha\sigma_\xi\sigma_\theta + \left(\partial_\Sigma\bar{R}_\theta\left(\zeta_t\right)\frac{d\Sigma_t}{dt} + \partial_I\bar{R}_\theta\left(\zeta_t\right)I_t\bar{g}_t\left(\zeta_t\right) - \lambda\bar{R}_\theta\left(\zeta_t\right)\right)\Sigma_t\right)^2}{\left(\bar{R}_\theta\left(\zeta_t\right)\sigma_\theta + \alpha\sigma_\xi\right)^2 + \left(1-\alpha^2\right)\sigma_\xi^2}, \tag{A.4}$$

which is a second-order polynomial in $\frac{d\Sigma_t}{dt}$, from which follows from equation (A.4) that

$$\frac{d\Sigma_t}{dt} = -\frac{B\left(\zeta_t\right)}{2A\left(\zeta_t\right)} \pm \frac{1}{2A\left(\zeta_t\right)}\sqrt{2B\left(\zeta_t\right) - 4A\left(\zeta_t\right)\left(2\lambda\Sigma_t - \sigma_\theta^2 + I_t^2\frac{\Sigma_t^2}{\sigma_k^2}\right) - 1},$$

where

$$\begin{aligned} A\left(\zeta_t\right) &= \frac{\left(\partial_\Sigma\bar{R}_\theta\left(\zeta_t\right)\Sigma_t\right)^2}{\left(\bar{R}_\theta\left(\zeta_t\right)\sigma_\theta + \alpha\sigma_\xi\right)^2 + \left(1-\alpha^2\right)\sigma_\xi^2}, \\ B\left(\zeta_t\right) &= 1 + 2\partial_\Sigma\bar{R}_\theta\left(\zeta_t\right)\Sigma_t\frac{\bar{R}_\theta\left(\zeta_t\right)\sigma_\theta^2 + \alpha\sigma_\xi\sigma_\theta + \left(\partial_I\bar{R}_\theta\left(\zeta_t\right)I_t\bar{g}_t\left(\zeta_t\right) - \lambda\bar{R}_\theta\left(\zeta_t\right)\right)\Sigma_t}{\left(\bar{R}_\theta\left(\zeta_t\right)\sigma_\theta + \alpha\sigma_\xi\right)^2 + \left(1-\alpha^2\right)\sigma_\xi^2}. \end{aligned}$$

Substituting $R_\theta = -\frac{1-\pi}{\pi}\frac{I}{\sigma_k^2}\frac{\Sigma}{\Sigma+\sigma_s^2}$ into the above expressions delivers the laws of motion stated in the proposition.

The conditional variance of beliefs Σ is trivially bounded from below by 0. To find the upper bound, consider the case when all public signals are completely uninformative $\forall\ t$, then Σ follows the law of motion

$$\frac{d\Sigma}{dt} = \sigma_\theta^2 - 2\lambda\Sigma_t,$$

which has the steady-state solution $\Sigma_t = \frac{\sigma_\theta^2}{2\lambda}$. Since any informativeness of the public signals reduces the conditional variance of beliefs, $\Sigma_t \leq \frac{\sigma_\theta^2}{2\lambda}$.

To find the relationship between $\hat{\theta}_t^c$ and $\hat{\theta}_t^c(i)$ for households, I make use of the Law of Iterated Expectations to write

$$\hat{\theta}_t^c(i) = E\left[\theta_t \mid \mathcal{F}_t^i\right] = E\left[\theta_t^c \mid s_t(i)\right],$$

where $\theta_t^c = \theta_t \mid \mathcal{F}_t^c$. Consider the common knowledge estimate $\hat{\theta}_t^c$, one I can arrive at the estimate of household i $\hat{\theta}_t(i)$ by updating $\mathcal{F}_t^c$ with household i's private signal $s_t(i)$. Since both the average household estimate $\hat{\theta}_t^c$ and the signal $s_t(i)$ are jointly Gaussian, which is apparent from the linearity of the Kalman Filter in the data $\{\zeta_s, \theta_s\}_{s\leq t}$, the process of updating the conditional mean is an exercise in the updating of two sets of Gaussian random variables. It then follows that

$$\hat{\theta}_t(i) = \hat{\theta}_t^c + Cov\left[\theta_t, s_t(i) \mid \mathcal{F}_t^c\right] Var\left[s_t(i) \mid \mathcal{F}_t^c\right]^{-1}\left(s_t(i) - E\left[s_t(i) \mid \mathcal{F}_t^c\right]\right) = \hat{\theta}_t^c + \frac{\Sigma_t}{\Sigma_t + \sigma_s^2}\left(s_t(i) - \hat{\theta}_t^c\right).$$

Similarly, the conditional variance of household $i's$ estimate of θ is

$$\Sigma_t(i) = \Sigma_t - Cov\left[\theta_t, s_t(i) \mid \mathcal{F}_t^c\right] Var\left[s_t(i) \mid \mathcal{F}_t^c\right]^{-1} Cov\left[\theta_t, s_t(i) \mid \mathcal{F}_t^c\right] = \Sigma_t - \frac{\Sigma_t^2}{\Sigma_t + \sigma_s^2} = \frac{\sigma_s^2}{\Sigma_t + \sigma_s^2}\Sigma_t.$$

Proof of Proposition 4:

To find the optimal level of investment I, let me conjecture that $E = E(t, K, I)$. Then, by the Feyman-Kac Theorem and $\frac{\Lambda_t}{\Lambda_0}, E_t > 0$, the function E that solves each manager's problem (5) must solve the necessary condition

$$0 \geq \sup_{g_t} \frac{\left(a - I_t - \frac{1}{\rho}g_t I_t + \tau_t\right) K_t}{E_t} dt + E\left[\frac{d\left(\Lambda_t E_t\right)}{E\left[\Lambda_t \mid \mathcal{F}_t^c\right] E_t} \mid \mathcal{F}_t^c\right],$$

which can be rewritten as

$$0 \geq \sup_{g_t} \frac{\left(a - I_t - \frac{1}{\rho}g_t I_t + \tau_t\right) K_t}{E_t} dt + E\left[\frac{dE_t}{E_t} \mid \mathcal{F}_t^c\right] + \frac{E\left[d\Lambda_t \mid \mathcal{F}_t^c\right]}{E\left[\Lambda_t \mid \mathcal{F}_t^c\right]} + \frac{d\left\langle \Lambda_t, E_t \mid \mathcal{F}_t^c\right\rangle}{E\left[\Lambda_t \mid \mathcal{F}_t^c\right] E_t}. \quad \text{(A.6)}$$

By Proposition 2, the pricing kernel of investor j $\Lambda_t(j)$ satisfies $\frac{1}{dt}E\left[\frac{d\Lambda_t(j)}{\Lambda_t(j)} \mid \mathcal{F}^j\right] = -r_t$. Thus, by the Law of Iterated Expectations, $E\left[\frac{d\Lambda_t}{\Lambda_t} \mid \mathcal{F}^c\right] = -r_t$, regardless of the distribution of ownership among households. Then, applying Itô's Lemma to E, equation (A.6) becomes

$$0 \geq \sup_{g_t} \frac{a - I_t}{E_t} K_t - \frac{I_t}{\rho} g_t K_t + \tau_t K_t + \frac{\partial_K E_t}{E_t}\left(I_t \hat{\theta}_t^c - \delta\right) K_t + \frac{1}{2}\frac{\partial_{KK} E_t}{E_t}\sigma_k^2 K_t^2 + \partial_t E_t - r_t + \frac{1}{dt}\frac{d\left\langle \Lambda_t, E_t \mid \mathcal{F}_t^c\right\rangle}{E\left[\Lambda_t \mid \mathcal{F}_t^c\right] E_t}, \quad \text{(A.7)}$$

where $\frac{1}{dt}\frac{d\langle \Lambda_t, E_t \mid \mathcal{F}_t^c\rangle}{\Lambda_t E_t}$ is the risk premium on firm claims. Since firms are perfectly competitive, they do not recognize, in equilibrium, that their actions affect the riskless rate r_t or the pricing kernel of shareholders Λ_t.

Firm effort g_t is chosen by the firm to achieve its optimal level of investment. Since equation (A.7) is (locally) riskless and linear in investment I_t, firm managers are effective risk-neutral and it follows that it must be the case that g_t satisfies

$$-1 + \partial_K E_t \hat{\theta}_t^c - \frac{1}{\rho} g_t = 0, \tag{A.8}$$

or else there is a riskless gain to changing g if the marginal return to investment for firm value is positive or negative. By market clearing, the value of firm claims must be such that $E_t = q_t K_t$, where $q_t = \frac{a - I_t}{\rho}$. To see that $E_t = q_t K_t$ satisfies the maximized form of equation (A.6), recall from Proposition 2 that $E_t = q_t K_t$ satisfies at the optimal I_t

$$\Lambda_t(i) \frac{a - I_t}{E_t} K_t dt + E\left[\frac{d(\Lambda_t(i) E_t)}{E_t} \mid \mathcal{F}_t^i\right] = 0.$$

Let $u_t(i)$ be the share of the firm owned by household i that has not experienced a preference shock, such that $\Lambda_t = \int u_t(i) \Lambda_t(i) di$. Assuming that the firm equal weights the pricing kernels of investing households $\Lambda_t = \int u_t(i) \Lambda_t(i) di = \int e^{-\rho t} u_t(i) \frac{1}{w_t(i)} di$, then it follows, by linearity and the finiteness of Λ_t, that

$$\frac{1}{dt} \frac{d\langle \Lambda_t, E_t \mid \mathcal{F}_t^c \rangle}{E[\Lambda_t \mid \mathcal{F}_t^c] E_t} = \int u_t(i) \frac{1}{dt} \frac{d\left\langle \frac{1}{w_t(i)}, K_t \mid \mathcal{F}_t^c \right\rangle}{K_t E\left[\int u_t(i) \frac{1}{w_t(j)} dj \mid \mathcal{F}_t^c\right]} di = -\sigma_k^2 \frac{\int u_t(i) E\left[\frac{x_t(i)}{w_t(i)} \mid \mathcal{F}_t^c\right] di}{E\left[\int u_t(i) \frac{1}{w_t(j)} dj \mid \mathcal{F}_t^c\right]}.$$

Given the optimal position of investing households from Proposition 2, and that $w_t(i)$ is independent of $\hat{\theta}(i)$ because of the generational structure of the economy, it follows that

$$-\frac{1}{dt} \frac{d\langle \Lambda_t, E_t \mid \mathcal{F}_t^c \rangle}{E[\Lambda_t \mid \mathcal{F}_t^c] E_t} = \frac{a_t - I_t}{q_t} - \frac{I_t}{a - I_t} g_t + I_t \hat{\theta}_t^c - r_t - \delta.$$

Thus by direct integration, the linearity of the expectation and covariance operators, and the Law of Iterated Expectations, it follows that

$$\frac{a - I_t}{E_t} K_t + \frac{1}{dt} E\left[\frac{dE_t}{E_t} \mid \mathcal{F}_t^c\right] + \frac{1}{dt} \frac{E[d\Lambda_t \mid \mathcal{F}_t^c]}{E[\Lambda_t \mid \mathcal{F}_t^c]} + \frac{1}{dt} \frac{d\langle \Lambda_t, E_t \mid \mathcal{F}_t^c \rangle}{E[\Lambda_t \mid \mathcal{F}_t^c] E_t} = 0.$$

Therefore, if E_t satisfies each household's Euler equation, then $E_t = q_t K_t$ solves each manager's problem.

Thus from equation (A.8), it follows that

$$g = \rho\left(q\hat{\theta}^c - 1\right) \mathbf{1}\left\{I > \underline{I} \cup \hat{\theta}^c \geq \frac{\rho}{a - \underline{I}}\right\}.$$

Proof of Proposition 5:

By the second part of Proposition 3

$$\int_{\mathcal{D}_t^c} \frac{w_t(i)}{W_t}\left(\hat{\theta}_t(i)-\hat{\theta}_t^c\right) di = \frac{\Sigma_t}{\Sigma_t+\sigma_s^2}\left(\theta_t-\hat{\theta}_t^c\right)\int_{\mathcal{D}_t^c} \frac{w_t(i)}{W_t} di + \frac{\Sigma_t}{\Sigma_t+\sigma_s^2}\int_{\mathcal{D}_t^c} \frac{w_t(i)}{W_t} Z_t^s(i)\, di. \tag{A.5}$$

Let me define the integral X_t

$$X_t = \int_0^1 \psi_t(i)\, dZ_t^s(i)\, di.$$

where $\psi_t(i) = \frac{w_t(i)}{W_t} > 0$ is now a weight function, with $\psi_t(i) \in (0,1)$ on a set of full measure, whose integral is bounded on any set of positive measure and is 1 over the set $i \in [0,1]$.

Importantly, since the law of motion of the price of firm equity q and the riskless rate r by conjecture do not depend on the wealth share or signal noise of any one household, the only difference in the wealth shares of households at time t are the histories of the fraction of wealth invested in firm equity $\{x_u(i)\}_{u\leq t}$, which differ across households only because of differences in signal noise. Therefore, conditional on the initial wealth share of households and the history of the fundamentals $\mathcal{G}_t = \sigma\left(\{\theta_u, K_u, \xi_u\}_{u\leq t} \vee w_0\right)$, the weights $\psi_i(t)$ are independent across households.

First, I establish that X_t converges to its cross-sectional expectation $E[X_t \mid \mathcal{G}_t]$ in the $\mathcal{L}^2 - norm$. As an aside, I do not require convergence $a.s.$ and rely on a weaker notion of convergence because of the issues discussed in Judd (1985).

Similar to Uhlig (1996), one can discretize the integral across i into a Riemann sum $\Sigma(t,\varphi)$ with a partition φ with $0 = i_0 < ...i_j < ...i_m = 1$ and midpoints $\phi_j \in [i_{j-1}, i_j]$, $j \in \{1,...,m\}$

$$\Sigma(t,\varphi) = \sum_{j=1}^{m} \psi_t(\phi_j)\, Z_t^s(\phi_j)\,(i_j - i_{j-1}).$$

Conditional on $\mathcal{G}_t$, $E[X_t \mid \mathcal{G}_t]$ is a constant, and one recognizes by Chebychev's Inequality that

$$\begin{aligned}
E\left[\left(\Sigma\left(t,\varphi\right)-E\left[X_{t}\mid\mathcal{G}_{t}\right]\right)^{2}\ \mid\mathcal{G}_{t}\right] &= E\left[\left(\sum_{j=1}^{m}\left(\psi_{t}\left(\phi_{j}\right)Z_{t}^{s}\left(\phi_{j}\right)-E\left[X_{t}\mid\mathcal{G}_{t}\right]\right)\left(i_{j}-i_{j-1}\right)\right)^{2}\ \mid\mathcal{G}_{t}\right] \\
&= E\left\{\sum_{j=1}^{m}E\left[\left(\psi_{t}\left(\phi_{j}\right)Z_{t}^{s}\left(\phi_{j}\right)-E\left[X_{t}\mid\mathcal{G}_{t}\right]\right)^{2}\ \mid\mathcal{G}_{t}\right]\left(i_{j}-i_{j-1}\right)^{2}\right\} \\
&\leq \sum_{j=1}^{m}\left(i_{j}-i_{j-1}\right)^{2} \\
&\leq \varepsilon\left(\varphi\right),
\end{aligned}$$

where $\varepsilon\left(\varphi\right)=\max_{j}\left(i_{j}-i_{j-1}\right)$. As $\varepsilon\left(\varphi\right)\searrow 0$, the above integral converges to the $\mathcal{L}^{2}$ distance between $\Sigma\left(t,\varphi\right)$ and $E\left[X_{t}\mid\mathcal{G}_{t}\right]$ on the LHS and 0 on the RHS.

Therefore

$$\lim_{\varepsilon(\varphi)\searrow 0}E\left[\left(\Sigma\left(t,\varphi\right)-E\left[X_{t}\mid\mathcal{G}_{t}\right]\right)^{2}\ \mid\mathcal{G}_{t}\right]=0.$$

By Dominated Convergence and Slusky's Theorem

$$\lim_{\varepsilon(\varphi)\searrow 0}E\left[\left(\Sigma\left(t,\varphi\right)-E\left[X_{t}\mid\mathcal{G}_{t}\right]\right)^{2}\ \mid\mathcal{G}_{t}\right]=E\left[\left(X_{t}-E\left[X_{t}\mid\mathcal{G}_{t}\right]\right)^{2}\ \mid\mathcal{G}_{t}\right].$$

Therefore

$$E\left[\left(X_{t}-E\left[X_{t}\mid\mathcal{G}_{t}\right]\right)^{2}\ \mid\mathcal{G}_{t}\right]=0,$$

which does not depend on the wealth share or signal noise of any individual household because $E\left[X_{t}\mid\mathcal{G}_{t}\right]=g\left(\tilde{\omega}_{t}\right)$ for some $\tilde{\omega}_{t}\in\mathcal{G}_{t}$.

Since the choice of partition φ was arbitrary, the convergence result did not depend on my choice of partition, and therefore X_{t} and its convergence to $g\left(\tilde{\omega}_{t}\right)$ in $\mathcal{L}^{2}$ are well-defined. Furthermore, since convergence is in $\mathcal{L}^{2}$, the integral is $g\left(\tilde{\omega}_{t}\right)$ *a.s.* and I can choose a modification of the process, if need be, under which it is always 0.[37] Given that this convergence is ex-post the realized sample path of the aggregate state variables $\mathcal{G}_{t}$, this convergence also holds unconditionally.

Recognizing that $E\left[\bar{Z}\left(i\right)\ \mid\mathcal{G}_{t}\right]=0$, it follows that

$$g\left(\tilde{\omega}_{t}\right)=\frac{\Sigma_{t}}{\Sigma_{t}+\sigma_{s}^{2}}E\left[\psi_{t}\left(i\right)Z_{t}^{s}\left(i\right)\ \mid\mathcal{G}_{t}\right]=\frac{\Sigma_{t}}{\Sigma_{t}+\sigma_{s}^{2}}E\left[\psi_{t}\left(i\right)\ \mid\mathcal{G}_{t}\right]E\left[Z_{t}^{s}\left(i\right)\ \mid\mathcal{G}_{t}\right]=0,$$

[37] Though the convergence implies that the variance of X_{t} is zero over time, X_{t} can deviate from its expected value on a negligible subset of times.

since $\psi_t(i)$ is independent of $Z_t^s(i)\ \forall\, i$ and $E[Z_t^s(i) \mid \mathcal{G}_t] = 0$. Similarly, I can apply a weak LLN to $\int_0^1 \frac{w_t(i)}{W_t} di$, which holds on subintervals of $[0,1]$ $a.s.$, to arrive at

$$\begin{aligned}
W_t &= E[w_t(i) \mid \mathcal{G}_t], \\
\int_{\mathcal{D}_t^c} \frac{w_t(i)}{W_t} di &= 1-\pi, \\
\int_{\mathcal{D}_t} \frac{w_t(i)}{W_t} di &= \pi.
\end{aligned}$$

Thus equation (A.5) becomes

$$\int_{\mathcal{D}_t^c} \frac{w_t(i)}{W_t} \left(\hat{\theta}_t(i) - \hat{\theta}_t^c\right) di - \int_{\mathcal{D}_t} \frac{w_t(i)}{W_t} \xi_t di = (1-\pi) \frac{\Sigma_t}{\Sigma_t + \sigma_s^2} \left(\theta_t - \hat{\theta}_t^c\right) - \pi \xi_t.$$

Proof of Proposition 1:

Substituting $q = \frac{a-I}{\rho}$, optimal household demand for firm claims $x(i)$ from Proposition 2, and optimal firm investment g from Proposition 4 into the market clearing condition for the market for riskless debt (8), and imposing $W > 0$ and Proposition (5), one has, when $I > \underline{I}$, that

$$r = \frac{a}{a-I}\rho - \delta + I \frac{\Sigma}{\Sigma + \sigma_s^2} \left(\theta - \hat{\theta}^c\right) - \frac{1+\pi\xi}{1-\pi}\sigma_k^2,$$

and therefore, matching this with the conjectured representation equation (13), it follows that

$$\begin{aligned}
r_0 &= \frac{a}{a-I}\rho - \delta - I \frac{\Sigma}{\Sigma + \sigma_s^2}\hat{\theta}^c - \frac{1}{1-\pi}\sigma_k^2, \\
r_\theta &= I \frac{\Sigma}{\Sigma + \sigma_s^2}, \\
r_\xi &= -\frac{\pi}{1-\pi}\sigma_k^2,
\end{aligned}$$

which confirms the conjecture. Given optimal firm equity demand $x(i)$ from Proposition 2, it follows that $x(i)$ can be decomposed as

$$x(i) = x_c + x_i \left(\hat{\theta}(i) - \hat{\theta}^c\right)$$

where

$$\begin{aligned}
x_c &= \frac{\frac{a}{a-I}\rho - r - \delta}{\sigma_k^2}, \\
x_i &= \frac{I}{\sigma_k^2}.
\end{aligned}$$

When $I = \underline{I}$ and $g = 0$, then r is instead given by

$$r = \rho - \delta + I\hat{\theta} + I\frac{\Sigma}{\Sigma + \sigma_s^2}\left(\theta - \hat{\theta}^c\right) - \frac{1+\pi\xi}{1-\pi}\sigma_k^2,$$

and x_c is insteady

$$x_c = \frac{\rho + I\hat{\theta} - r}{\sigma_k^2}.$$

Proof of Proposition 6:

When θ, then optimal investment I and the firm equity price q are given by equations (12) and (15)

$$q = \frac{a-I}{\rho}$$

and

$$g = \rho\left(q\theta - 1\right).$$

Since all households are now perfectly informed, it follows that the only heterogeneity among them is whether they are hit by liquidity shocks. Following the arguments of Proposition 2, their optimal policies are

$$\begin{aligned}
c(i) &= \rho w(i), \\
x(i) &= \frac{\frac{a-I}{q} - \frac{I}{a-I}g + I\theta - r - \delta}{\sigma_k^2}.
\end{aligned}$$

By the market clearing condition for riskless debt (8), it follows that

$$r = \frac{a}{a-I}\rho - \delta - \frac{1+\pi\xi}{1-\pi}\sigma_k^2.$$

Proof of Proposition 7:

When households are perfectly informed about θ, they consume a fixed fraction of their wealth and follow identical investment strategies

$$\begin{aligned} c(i) &= \rho w(i), \\ x(i) &= \frac{\frac{a-I}{q} - \frac{I}{a-I} g + I\theta - r - \delta}{\sigma_k^2}, \end{aligned}$$

when not hit by the preference shock. Since managers still learn from prices, it follows that the optimal g still satisfies

$$g = \rho \left(q\hat{\theta}^c - 1 \right).$$

It follows by market clearing condition for riskless debt (8) that the riskless rate satisfies

$$r = \frac{a}{a-I}\rho - \delta + I\left(\theta - \hat{\theta}^c\right) - \frac{1+\pi\xi}{1-\pi}\sigma_k^2.$$

As $\sigma_s \searrow 0$, from the law of motion of $\hat{\theta}^c$ and $\hat{\theta}(i)$ from Proposition 3, it follows that $\Sigma(i) \searrow 0$ while

$$\frac{d\Sigma_t}{dt} \to \sigma_\theta^2 - 2\lambda\Sigma - I^2\frac{\Sigma^2}{\sigma_k^2} - \frac{\left(\alpha\sigma_\xi\sigma_\theta + R_\theta\sigma_\theta^2 + R_\theta(g-\lambda)\Sigma_t\right)^2}{\left(R_\theta\sigma_\theta + \alpha\sigma_\xi\right)^2 + (1-\alpha^2)\sigma_\xi^2}$$

where R_θ, the loading of the riskless rate r on the household expectational error $\theta - \hat{\theta}^c$ converges to

$$R_\theta \to -\frac{1-\pi}{\pi}\frac{I}{\sigma_k^2}.$$

Thus it follows that Σ does not converge to 0 as $\sigma_s \searrow 0$, reflecting the uncertainty that firm managers still face about θ from observing only $\log K$ and r, and g does not converge to its perfect-information benchmark value.

Since $\hat{\theta}^c \to \theta$, $\hat{\theta}(i) \to \theta$, it follows that the investment strategy of households $x(i)$ converges to

$$x(i) = \frac{\frac{a-I}{q} - \frac{I}{a-I} g + I\theta - r - \delta}{\sigma_k^2},$$

from which it follows that the riskless rate r approaches its representative agent benchmark value. Thus beliefs, prices, and optimal policies in the economy with informational frictions approach their representative agent benchmark values as $\sigma_s \searrow 0$.

Proof of Proposition 8:

From Proposition 1, it follows that each household's demand for the risky asset when not hit by the liquidity shock can be rewritten as

$$x(i)=\frac{1+\pi\xi}{1-\pi}+\frac{I}{\sigma_k^2}\frac{\Sigma_t}{\Sigma_t+\sigma_s^2}\sigma_s Z^s(i). \tag{A.11}$$

Substituting my expressions for q and g into the law of motion of household wealth $w_t(i)$ equation (9), it follows by Itô's Lemma that

$$d\log w(i)=(1-x(i))(r-\rho)\,dt+x(i)\left(\left(\frac{\rho I}{a-I}-\delta+I\left(\theta-\hat{\theta}^c\right)\right)dt+\sigma_k dZ^k\right)-\frac{1}{2}x(i)^2\sigma_k^2 dt,$$

Substituting for $x(i)$ with equation (A.11) and aggregate across households, one then has that

$$\begin{aligned}\int_0^1 d\log w(i)\,di &= \left(\frac{\rho I}{a-I}-\delta+I\left(\theta-\hat{\theta}^c\right)\right)dt+\sigma_k dZ^k\\ &\quad -\frac{1}{2}\left(\pi\sigma_k^2\xi^2+\sigma_k^2\frac{(1+\pi\xi)^2}{1-\pi}+(1-\pi)\left(\frac{I}{\sigma_k}\frac{\Sigma}{\Sigma+\sigma_s^2}\sigma_s\right)^2\right)dt \quad \text{(A.12)}\end{aligned}$$

With equation (A.12), one can then express aggregate flow utility $\int_0^1 \log c_s(i)\,di=\log\rho+\log w_0+\int_0^1\int_0^s d\log w_u(i)\,du\,di$ as

$$\begin{aligned}\int_0^1 \log c_t(i)\,di &= \int_0^t\left(\frac{\rho I_s}{a-I_s}-\delta+I_s\left(\theta_s-\hat{\theta}_s^c\right)\right)ds+\sigma_k Z_t^k+\log\rho+\log w_0\\ &\quad -\frac{1}{2}\int_0^t\left(\sigma_k^2+\frac{\pi\sigma_k^2}{1-\pi}(1+\xi_s)^2+(1-\pi)\left(\frac{I_s}{\sigma_k}\frac{\Sigma_s}{\Sigma_s+\sigma_s^2}\sigma_s\right)^2\right)ds.\end{aligned}$$

It follows then that utilitarian welfare at time 0 $U=E\left[\int_0^\infty e^{-\rho t}\int_0^1\log\frac{c_t(i)}{c_0(i)}di\,dt\mid\mathcal{F}_0\right]$ in the economy under the physical measure $\mathcal{P}$ defined on $\mathcal{F}_0$ is given by

$$\begin{aligned}U &= E\left[\int_0^\infty e^{-\rho t}\left[\int_0^t\left(\frac{\rho I_s}{a-I_s}-\delta+I_s\left(\theta_s-\hat{\theta}_s^c\right)\right)ds\right]dt\mid\mathcal{F}_0\right]+E\left[\int_0^\infty e^{-\rho t}\sigma_k Z_t^k dt\mid\mathcal{F}_0\right]\\ &\quad -\frac{1}{2}E\left[\int_0^\infty e^{-\rho t}\left[\int_0^t\left(\sigma_k^2+\frac{\pi\sigma_k^2}{1-\pi}(1+\xi_s)^2+(1-\pi)\left(\frac{I_s}{\sigma_k}\frac{\Sigma_s}{\Sigma_s+\sigma_s^2}\sigma_s\right)^2\right)ds\right]dt\mid\mathcal{F}_0\right].\end{aligned}$$

Taking expectations under $\mathcal{P}$, it follows that

$$
\begin{aligned}
U &= E\left[\int_0^\infty e^{-\rho t}\left[\int_0^t \left(\frac{\rho I_s}{a-I_s} - \delta + I_s\left(\theta_s - \hat{\theta}_s^c\right) - \frac{1-\pi}{2}\left(\frac{I_s}{\sigma_k}\frac{\Sigma_s}{\Sigma_s+\sigma_s^2}\sigma_s\right)^2\right) ds\right] dt \mid \mathcal{F}_0\right] \\
&\quad -\frac{1}{2}\left(\sigma_k^2 + \frac{\pi\sigma_k^2}{1-\pi}\left(1+\xi_0\right)^2\right) E\left[\int_0^\infty e^{-\rho(s-t)} t dt \mid \mathcal{F}_0\right] - \frac{1}{4}\frac{\pi\sigma_k^2}{1-\pi}\sigma_\xi^2 E\left[\int_0^\infty e^{-\rho(s-t)} t^2 dt \mid \mathcal{F}_0\right].
\end{aligned}
$$

Recognizing that $\int_0^\infty e^{-\rho\tau}\tau d\tau = \frac{1}{\rho^2}$ and $\int_0^\infty e^{-\rho\tau}\tau^2 d\tau = \frac{2}{\rho^3}$, one arrives at

$$
\begin{aligned}
U &= E\left[\int_0^\infty e^{-\rho t}\int_0^t \left(\frac{\rho}{a-I_s} + \theta_s - \hat{\theta}_s^c\right) I_s ds dt \mid \mathcal{F}_0\right] - \frac{1}{2\rho^2}\left(\sigma_k^2 + \frac{\pi\sigma_k^2}{1-\pi}\left(1+\xi_0\right)^2\right) \\
&\quad -\frac{1-\pi}{2}\left(\frac{\sigma_s}{\sigma_k}\right)^2 E\left[\int_0^\infty e^{-\rho t}\int_0^t \left(\frac{I_s\Sigma_s}{\Sigma_s+\sigma_s^2}\right)^2 ds dt \mid \mathcal{F}_0\right] \\
&\quad -\frac{\delta}{\rho^2} - \frac{1}{2\rho^3}\frac{\pi\sigma_k^2}{1-\pi}\sigma_\xi^2.
\end{aligned} \tag{A.13}
$$

By stacking the terms in the two double integrals in equation (A.13), I can rewrite them to arrive at

$$
\begin{aligned}
U &= \frac{1}{\rho}E\left[\int_0^\infty e^{-\rho t}\left(\frac{\rho}{a-I_t} + \theta_t - \hat{\theta}_t^c\right) I_t dt \mid \mathcal{F}_0\right] - \frac{1-\pi}{2\rho}\left(\frac{\sigma_s}{\sigma_k}\right)^2 E\left[\int_0^\infty e^{-\rho t}\left(\frac{I_t\Sigma_t}{\Sigma_t+\sigma_s^2}\right)^2 dt \mid \mathcal{F}_0\right] \\
&\quad -\frac{\delta}{\rho^2} - \frac{1}{2\rho^2}\left(\sigma_k^2 + \frac{\pi\sigma_k^2}{1-\pi}\left(1+\xi_0\right)^2\right) - \frac{1}{2\rho^3}\frac{\pi\sigma_k^2}{1-\pi}\sigma_\xi^2.
\end{aligned}
$$

Defining w such that $d\log w = \int_0^1 d\log w(i)\, di$, from equation (A.12) and Itô's Lemma it follows that w has the law of motion

$$
\frac{dw}{w} = \left(\frac{\rho I}{a-I} - \delta + I\left(\theta - \hat{\theta}^c\right) - \frac{1}{2}\left(\frac{\pi\sigma_k^2}{1-\pi}\left(1+\xi\right)^2 + \left(1-\pi\right)\left(\frac{I}{\sigma_k}\frac{\Sigma}{\Sigma+\sigma_s^2}\sigma_s\right)^2\right)\right) dt + \sigma_k dZ^k. \tag{A.14}
$$

Thus I can think of the economy as having a representative household who holds all firm claims in the economy and whose wealth evolves according to the law of motion (A.14).

Proof of Proposition 9:

To find the law of motion of the probability law of the economy $p_t\left(\hat{\theta}^c, \Sigma, I\right)$, I find the probability law implied by households and firms whose optimization is consistent with their HJB equations. This is commonly referred to as the Kolmogorov Forward Equation. To find this, I recognize that, under the optimal control for the change in investment

$g\left(\hat{\theta}^c_s, \Sigma_s, I_s\right)_{s\geq 0}$, $\mathcal{D}^g f = 0$ where $\mathcal{D}^g$ is the infinitesimal generator that satisfies

$$\mathcal{D}^g f = \partial_{\hat{\theta}^c} f \lambda \left(\bar{\theta} - \hat{\theta}^c\right) + \partial_\Sigma f \frac{d\Sigma}{dt} + \partial_I f I \left((a - I)\hat{\theta}^c - \left(1 - \tau^I\right)\rho\right) \mathbf{1}_{\left\{I > \underline{I} \cup \hat{\theta}^c \geq \frac{(1-\tau^I)\rho}{a - \underline{I}}\right\}} + \frac{1}{2}\partial_{\hat{\theta}^c \hat{\theta}^c} f \left(\sigma^2_{\hat{\theta}k} + \sigma^2_{\hat{\theta}r}\right),$$

where $\sigma_{\hat{\theta}k}$ and $\sigma_{\hat{\theta}r}$ are given in Proposition 3 appropriately modified for the transaction cost τ^r. In the above expression, the variance of $\hat{\theta}^c$ is unchanged under the physical measure because of diffusion invariance.

Let $z\left(\hat{\theta}^c, \Sigma, I\right) \in \mathcal{C}^\infty_0\left(\mathbb{R} \times \left[0, \frac{\sigma^2_\theta}{2\lambda}\right] \times [\underline{I}, a]\right)$ be an arbitrarily, infinitely differentiable test function with compact support. Then $E\left[z\left(\hat{\theta}^c_t, \Sigma_t, I_t\right)\right] = \int z\left(\hat{\theta}^c, \Sigma, I\right) p_t\left(\hat{\theta}^c, \Sigma, I\right) d\hat{\theta}^c d\Sigma dI$ can be written as

$$\begin{aligned} E\left[z\left(\hat{\theta}^c_t, \Sigma_t, I_t\right)\right] &= E\left[\int_0^t dz\left(\hat{\theta}^c_s, \Sigma_s, I_s\right)\right] \\ &= E\left[\int_0^t \mathcal{D}^g z\left(\hat{\theta}^c_s, \Sigma_s, I_s\right) ds\right] \\ &= \int \int_0^t \mathcal{D}^g z\left(\hat{\theta}^c, \Sigma, I\right) p_t\left(\hat{\theta}^c, \Sigma, I\right) d\hat{\theta} d\Sigma dI. \end{aligned}$$

Differentiating w.r.t t, one finds that

$$\int z\left(\hat{\theta}^c, \Sigma, I\right) \partial_t p_t\left(\hat{\theta}^c, \Sigma, I\right) d\hat{\theta} d\Sigma dI = \int \mathcal{D}^g z\left(\hat{\theta}^c, \Sigma, I\right) p_t\left(\hat{\theta}^c, \Sigma, I\right) d\hat{\theta}^c d\Sigma dI.$$

Since z has compact support, I can perform integration by parts to arrive at

$$\int z\left(\hat{\theta}^c, \Sigma, I\right) \partial_t p_t\left(\hat{\theta}^c, \Sigma, I\right) d\hat{\theta}^c d\Sigma dI = \int z\left(\hat{\theta}^c, \Sigma, I\right) \mathcal{D}^{g*} p_t\left(\hat{\theta}^c, \Sigma, I\right) d\hat{\theta}^c d\Sigma dI,$$

where $\mathcal{D}^{g*}$ is the adjoint of $\mathcal{D}^g$ and is the time-homogeneous infinitesimal generator associated with the Koopman operator. Assuming $\partial_t p_t\left(\hat{\theta}^c, \Sigma, I\right) - \mathcal{D}^{g*} p_t\left(\hat{\theta}^c, \Sigma, I\right)$ is continuous, it follows, since z is arbitrary, that

$$\partial_t p_t\left(\hat{\theta}^c, \Sigma, I\right) = \mathcal{D}^{g*} p_t\left(\hat{\theta}^c, \Sigma, I\right), \tag{A.9}$$

Importantly, $\mathcal{D}^{g*}$ is a (uniformly) elliptic operator that has divergence form. When p_t has reached its stationary distribution p, where $p = \lim_{t \nearrow \infty} p_t$, it follows that $\partial_t p_t = 0$. Thus equation (A.9) is a second-order parabolic equation and can can be rewritten when p_t has reached its stationary distribution, suppressing arguments, as

$$
\begin{aligned}
0 &= -\partial_{\hat{\theta}^c}\left\{p\lambda\left(\bar{\theta}-\hat{\theta}^c\right)\right\}-\partial_I\left\{pI\left((a-I)\,\hat{\theta}^c-\left(1-\tau^I\right)\rho\right)\right\}\mathbf{1}_{\left\{I>\underline{I}\cup\hat{\theta}^c\geq\frac{(1-\tau^I)\rho}{a-\underline{I}}\right\}}-\partial_\Sigma\left\{p\frac{d\Sigma}{dt}\right\} \\
&\quad +\frac{1}{2}\partial_{\hat{\theta}^c\hat{\theta}^c}\left\{p\left(\sigma_{\hat{\theta}k}^2+\sigma_{\hat{\theta}r}^2\right)\right\}, \qquad \text{(A.10)}
\end{aligned}
$$

which is the expression given in the proposition.

That $p_t\left(\hat{\theta}^c,\Sigma,I\right)$ will satisfy the conservation of mass law $\int p_t\left(\hat{\theta}^c,\Sigma,I\right)d\hat{\theta}^c d\Sigma dI=1$, where the integral is understood to be taken over the entire space $\mathbb{R}\times\left[0,\frac{\sigma_\theta^2}{2\lambda}\right]\times\left[\underline{I},a\right]$, gives rise to my spatial boundary conditions. Notice that I can rewrite equation (A.10) as

$$
\nabla\cdot S\left(B,g,A\right)=0,
$$

where

$$
S\left(\hat{\theta}^c,\Sigma,I\right)=\begin{bmatrix} S^{\hat{\theta}^c}\left(\hat{\theta}^c,\Sigma,I\right) \\ S^{\Sigma}\left(\hat{\theta}^c,\Sigma,I\right) \\ S^{I}\left(\hat{\theta}^c,\Sigma,I\right)\end{bmatrix}=\begin{bmatrix}\lambda\left(\bar{\theta}-\hat{\theta}^c\right)p\left(\hat{\theta}^c,\Sigma,I\right)-\frac{1}{2}\partial_{\hat{\theta}^c}\left\{\left(\sigma_{\hat{\theta}k}^2+\sigma_{\hat{\theta}r}^2\right)p\left(\hat{\theta}^c,\Sigma,I\right)\right\} \\ \frac{d\Sigma}{dt}p\left(\hat{\theta}^c,\Sigma,I\right) \\ \left((a-I)\,\hat{\theta}^c-\left(1-\tau^I\right)\rho\right)Ip\left(\hat{\theta}^c,\Sigma,I\right)\mathbf{1}_{\left\{I>\underline{I}\cup\hat{\theta}^c\geq\frac{(1-\tau^I)\rho}{a-\underline{I}}\right\}}\end{bmatrix}.
$$

Here $S\left(\hat{\theta}^c,\Sigma,I\right)$ is the "probability flux" representing the flow or flux of particles through the point $\left(\hat{\theta}^c,\Sigma,I\right)$. Consequently, a reflecting boundary condition will ultimately impose that the flux through boundary points must be zero.

For $\hat{\theta}^c$, which have unbounded support, one has that for $\varepsilon>0$ arbitrary that

$$
\lim_{\hat{\theta}^c\nearrow\infty}\left(\hat{\theta}^c\right)^{2(1+\varepsilon)}p\left(\hat{\theta}^c,\Sigma,I\right)=0\;\forall\;I,
$$

while for $\Sigma=0$, one has that $\partial_\Sigma p\left(\hat{\theta}^c,\frac{\sigma_\theta^2}{2\lambda},I\right)=0$, since $\frac{\sigma_\theta^2}{2\lambda}$ is a reflecting boundary, and $\lim_{\Sigma\searrow 0}p\left(\hat{\theta}^c,\Sigma,I\right)=0$, since arbitrarily small precision becomes arbitrarily unlikely given that new unobservable innovations to θ_t occur at each instant.

Integrating this expression over the entire space, imposing that $\int\partial_t p_t\left(\hat{\theta}^c,\Sigma,I\right)d\hat{\theta}^c d\Sigma dI=\partial_t\int p_t\left(\hat{\theta}^c,\Sigma,I\right)d\hat{\theta}^c d\Sigma dI=0$, applying the Divergence Theorem, it follows that the appropriate "reflecting" boundary condition for I is $\hat{n}_{I=\underline{I}}\cdot S\left(\hat{\theta}^c,\Sigma,\underline{I}\right)=\hat{n}_{I=a}\cdot S\left(\hat{\theta}^c,\Sigma,a\right)=0\;\forall\;\left(\hat{\theta}^c,\Sigma\right)$, where $\hat{n}_{I=i}$ is the unit (outward) normal vector perpendicular to the $I=i$

boundary. The intuition for these two boundary conditions is that the probability flux, or flow, through the two walls $I = \underline{I}$ and $I = a$ must be zero for probability mass not to leak out through them.

Proof of Corollary 1:

Let U^c be ex-ante utilitarian welfare under the common knowledge filtration. Then U^c satisfies

$$U^c = E\left[\int_0^\infty e^{-\rho t} \int_0^1 \log c_t(i)\, didt \mid \mathcal{F}_0^c\right] = E\left[E\left[\int_0^\infty e^{-\rho t} \int_0^1 \log c_t(i)\, didt \mid \mathcal{F}_0\right] \mid \mathcal{F}_0^c\right] = E\left[U \mid \mathcal{F}_0^c\right],$$

from which follows from the expression for U from 8, and since $\hat{\theta}_t^c \sim \mathcal{N}(0, \Sigma)$, that the above reduces by the LIE to

$$\begin{aligned} U^c \;=\; & E\left[\int_0^\infty e^{-\rho t} \frac{I_t}{a - I_t} dt \mid \mathcal{F}_0^c\right] - \frac{1-\pi}{2\rho}\left(\frac{\sigma_s}{\sigma_k}\right)^2 E\left[\int_0^\infty e^{-\rho t}\left(\frac{I_t}{1-\tau^r}\frac{\Sigma_t}{\Sigma_t + \sigma_s^2}\right)^2 dt \mid \mathcal{F}_0^c\right] \\ & -\frac{1}{2\rho^2}\frac{1-\pi}{\pi\sigma_k^2}\left(\frac{I_0}{1-\tau^r}\frac{\Sigma_0}{\Sigma_0+\sigma_s^2}\right)^2 \Sigma_0 - \frac{1}{2\rho^2}\left(\sigma_k^2 + \frac{\pi\sigma_k^2}{1-\pi}\left(1 + E\left[\xi_0 \mid \mathcal{F}_0^c\right]\right)^2\right) \\ & -\frac{\delta}{\rho^2} - \frac{1}{2\rho^3}\frac{\pi\sigma_k^2}{1-\pi}\sigma_\xi^2, \end{aligned} \tag{A.12}$$

since $\left(r, \hat{\theta}^c, \hat{\xi}^c\right) \in \mathcal{F}^c \subseteq \mathcal{F}$, from the expression for the riskless rate r in Proposition 1, and it follows

$$\xi - \hat{\xi}^c = \frac{1-\pi}{\pi\sigma_k^2}\frac{I}{1-\tau^r}\frac{\Sigma}{\Sigma+\sigma_s^2}\left(\theta - \hat{\theta}^c\right),$$

and therefore

$$E\left[\left(\xi_0 - E\left[\xi_0 \mid \mathcal{F}_0^c\right]\right)^2 \mid \mathcal{F}_0^c\right] = \left(\frac{1-\pi}{\pi\sigma_k^2}\frac{I_0}{1-\tau^r}\frac{\Sigma_0}{\Sigma_0+\sigma_s^2}\right)^2 \Sigma_0,$$

Assume now that the economy is initialized from the stationary distribution $p\left(\hat{\theta}^c, I, \Sigma\right)$ and that the stationary distribution is bounded $p\left(\hat{\theta}^c, I, \Sigma\right) \in \mathcal{L}^\infty\left(\mathbb{R}, \left[0, \frac{\sigma_\theta^2}{2\lambda}\right], [\underline{I}, a]\right)$. Let U_p^c be the expected welfare in economy under the stationary distribution, and $E^p[\cdot]$ be the expectation operator w.r.t. the stationary distribution. Then the first expectation, when taken w.r.t. the stationary distribution, can be rewritten as

$$\begin{aligned} E^p\left[\int_0^\infty e^{-\rho t}\frac{I_t}{a-I_t}dt\right] &= \int_0^\infty e^{-\rho t}\int P_t\frac{I_0}{a-I_0}p\left(\hat{\theta}_0^c,\Sigma_0,I_0\right)d\hat{\theta}d\Sigma dIdt \\ &= \int_0^\infty e^{-\rho t}\int \frac{I_0}{a-I_0}P_t^*p\left(\hat{\theta}_0^c,\Sigma_0,I_0\right)d\hat{\theta}d\Sigma dIdt, \quad \text{(A.13)} \end{aligned}$$

where $P_t = e^{t\mathcal{D}^g}$ is the Ruelle-Frobenius-Perron operator and $P_t^* = e^{t\mathcal{D}^{g*}}$ is its adjoint, often called the Koopman operator. P_t^* is defined such that, for a bounded, Borel measurable function f and measure ν $\left\langle\ P_t f,\quad \nu\ \right\rangle = \int_{\mathbb{R}\times\mathbb{R}_+\times\mathbb{R}_+} P_t f d\nu = \left\langle\ f,\quad P_t^*\nu\ \right\rangle$. Probabilistically, P_t^* corresponds to time-reversal and acts on measures, whereas P_t acts on functions. By construction, since $\mathcal{D}^{g*}p = 0$,

$$e^{t\mathcal{D}^{g*}}p\left(\hat{\theta}_0^c,\Sigma_0,I_0\right) = p\left(\hat{\theta}_0^c,\Sigma_0,I_0\right),$$

and therefore equation (A.13) simplies to

$$E^p\left[\int_0^\infty e^{-\rho t}\frac{\rho}{a-I_t}I_t dt\right] = \frac{1}{\rho}E^p\left[\frac{I_0}{a-I_0}\right].$$

A similar result obtains for the second expectation, under the assumption that Σ is essentially bounded. Since $\Sigma \leq \frac{\sigma_\theta^2}{2\lambda}$ from Proposition 3, this assumption is justified for σ_θ finite and $\lambda > 0$. It follows from these results, $E^p[\xi_0] = 0$, and equation (A.12), that U_p^c takes the form

$$\begin{aligned} U_p^c &= \frac{1}{\rho}E^p\left[\frac{I_0}{a-I_0}\right] - \frac{1-\pi}{2\rho^2}\left(\frac{\sigma_s}{\sigma_k}\right)^2 E^p\left[\left(\frac{I_0}{1-\tau^r}\frac{\Sigma_0}{\Sigma_0+\sigma_s^2}\right)^2\right] - \frac{1}{2\rho^2}\frac{1-\pi}{\pi\sigma_k^2}E^p\left[\left(\frac{I_0}{1-\tau^r}\frac{\Sigma_0}{\Sigma_0+\sigma_s^2}\right)^2\Sigma_0\right] \\ &\quad -\frac{\delta}{\rho^2} - \frac{1}{2\rho^2}\frac{\sigma_k^2}{1-\pi}\left(1+\frac{1}{\rho}\pi\sigma_\xi^2\right). \end{aligned}$$

Appendix: Figures

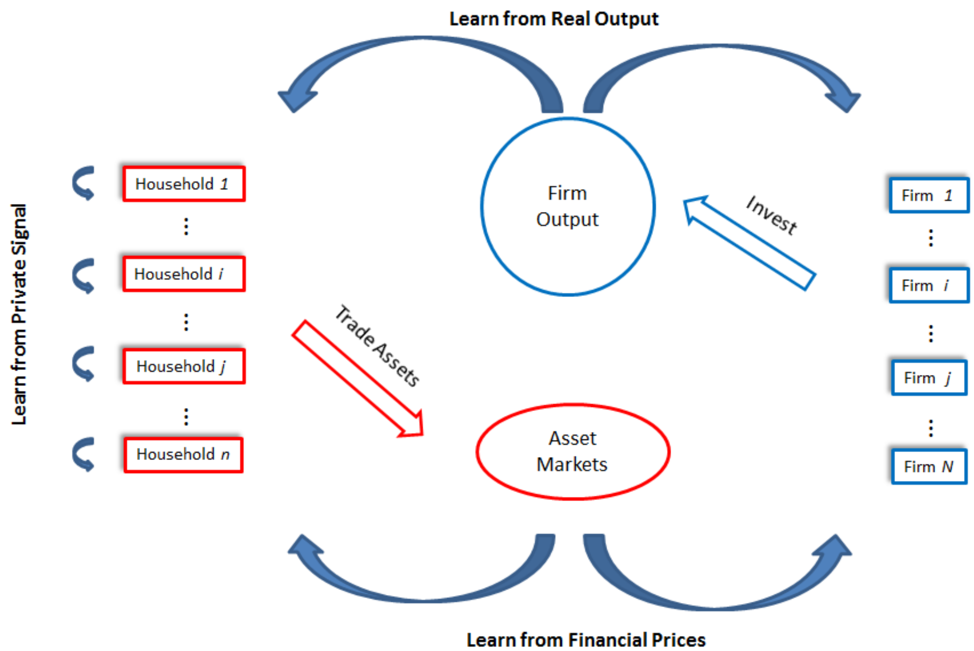

Figure 1: Structure of the Model

In the numerical experiments that follow, I treat one time unit (t.u) as a year. I set the subjective discount rate ρ to be .02 and depreciation δ to be .10 following the literature. I choose a to be .2 so that the maximum level of investment I in my model is three standard deviations above its mean of the ratio of US private nonresidential fixed investment to real GDP since 1973. Given the stylized structure of my model, I choose reasonable values for the remaining parameters.

I set the mean-reversion and standard deviation of investment productivity shocks, λ and σ_θ, respectively, to both be .02. I set the long-run mean $\bar{\theta}$ to be .3. I set the standard deviations of capital and financial shocks to be the same $\sigma_k = \sigma_\xi = .05$ so that the exogenous noise in both the real and financial signals are the same. I set the standard deviation of private information σ_s to .03 and the fraction of households hit by the preference shock π to .4. Finally, to shut off any mechanical learning from market prices, I set the correlation between investment productivity and financial shocks α to zero.

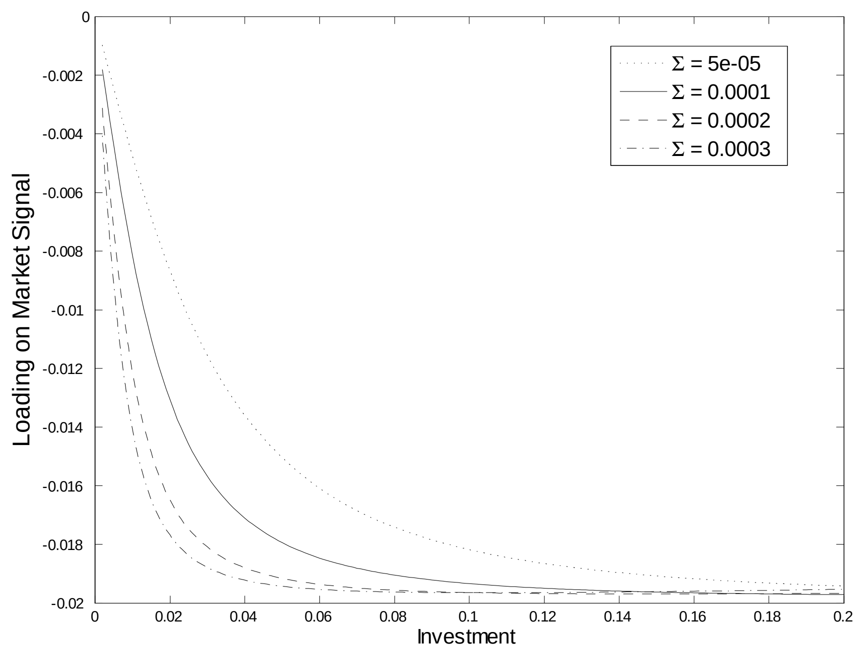

Figure 2: Loading on Market Signal for Fixed Perceived Investment Productivity $\hat{\theta}^c = 0.3$

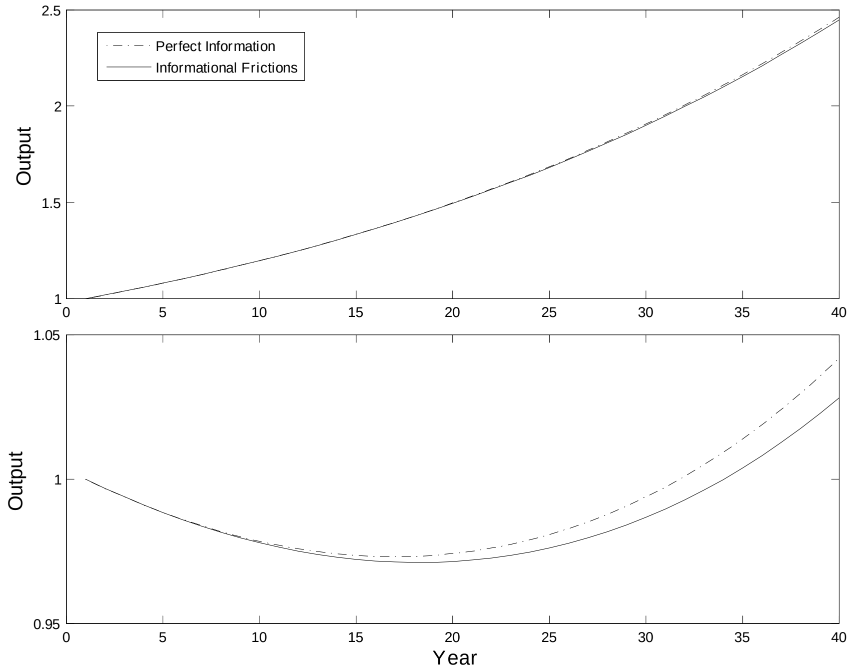

Figure 3: Impulse Response of the Economy to a One Standard Deviation Negative Financial Shock in a Boom (Panel 1) and a Bust (Panel 2) (Output is normalized to 1 at time 0).