Abstract

We show that (i) different collateral requirements across countries in securitized markets lead to net international capital flows, and (ii) the ability to tranche asset-backed securities in one country leads to offsetting portfolio flows, creating gross financial flows. We use a general equilibrium model with two countries, collateralized borrowing, and securitized markets. Financial systems differ in the collateral requirement for loans and the ability to tranche asset-backed securities. Net financial flows increase the greater the difference between collateral requirements in each country. Tranching leads to gross flows, and the gross flows are larger the greater the difference between collateral requirements in each country. Exogenous fluctuations in collateral requirements and the introduction of tranching have important implications for both net and gross international flows.

Keywords: asset-backed securities, current account deficit, global imbalances, gross international asset positions, idiosyncratic risk.

JEL classification: D52, F32, F34, F36, G15.

1 Introduction

In the previous three decades, the current account deficit in the United States increased persistently and then dramatically collapsed following the 2007–2009 financial crisis. At the same time, there has been incredible growth
and proliferation of securitized financial markets. Not only has there been a large increase in the creation of asset-backed securities, until the peak of the financial crisis securitized markets in the U.S. became increasingly defined by lower margin requirements for the underlying loan, with margins spiking tremendously after the financial crisis. In this paper we provide a mechanism that links collateral levels in securitized markets and international capital flows. We show that net international capital flows arise when countries have different collateral requirements in securitized markets. Furthermore, we show that the ability to tranche asset-backed securities in one country leads to offsetting portfolio flows so that, even when net flows are roughly unchanged, gross flows increase tremendously.

We illustrate this mechanism in a general equilibrium model with two countries, collateralized borrowing, and securitized markets. The two countries, Home and Foreign, each are populated by a continuum of ex ante identical entrepreneurs and have financial sectors that create asset-backed securities from collateralized loans. Entrepreneurs have investment projects that are subject to uninsurable idiosyncratic risk, but they effectively share some of the risk by borrowing against their projects and using the proceeds to invest in asset-backed securities. The financial system in the Home country differs from the Foreign system in two ways. First, the collateral requirement is lower for Home investors, implying not only higher leverage but also a greater degree of risk-sharing. Second, the Home financial system can divide asset-backed securities into “tranches” that pay different state-

\[\text{See Figure 1 of Fostel and Geanakoplos (2012) for down payments on mortgages in the U.S.}\]
contingent dividends. International trade in assets allows Foreign investors to buy Home asset-backed securities, which offer attractive returns and state-contingent payoffs, and as a result capital flows from Foreign to Home. The lower is the Home collateral requirement relative to Foreign, the greater are these flows.

There are two key intuitions for our results. First, when collateral levels are low, agents can better self-insure against idiosyncratic shocks by sharing risk through securitized markets. When collateral levels differ across countries, countries have different levels of precautionary savings, different demand for safe assets, and different autarkic interest rates. As has been shown in other environments, trade leads to net flows as higher precautionary savings from the country with more uninsured idiosyncratic risk flows to the country with a higher interest rate.

The second intuition is that when collateral levels differ across countries, investors naturally hold different portfolios, and so the risk exposures of agents in each country differ by aggregate state. Tranching allows agents more flexibility in choosing their assets, holding portfolios that pay differently in different states of the world, thus better hedging existing risks. Additional trade flows arise as agents hold combinations of tranches that match the residual risk in their portfolios better than pass-through securities do.

We are not the first paper to investigate the possibility that differences in financial systems can drive global imbalances. [Caballero et al. (2008)] emphasize the role of heterogeneous domestic financial systems in explaining global imbalances in which financial imperfections are captured by a country’s ability to supply assets in a world without uncertainty. Other pa-
pers have emphasized different abilities of the financial systems to insure idiosyncratic risk. Willen (2004) shows in a constant absolute risk aversion (CARA)-normal framework that incomplete markets (as opposed to complete markets) lead to trade imbalances, which is related to our result that the differences in the degree of market incompleteness leads to imbalances. Mendoza et al. (2009) and Angeletos and Panousi (2011) argue that different levels of financial development can lead to sustained global imbalances. Mendoza et al. (2009) consider the extent to which a country’s legal system can enforce financial contracts among its residents, and so financial development refers to the development of the legal system. Angeletos and Panousi (2011) take a somewhat “reduced form” approach and assume that financial development is equivalent to decreasing the level of idiosyncratic risk. Our paper is closely related to this literature on risk-sharing as a mechanism driving international capital flows. A key difference is that we model the ability of countries to insure idiosyncratic risks by the collateral level required for securitized loans. Countries that require a high margin cannot insure risks as well as countries with low margins. This is because since collateralized loans are non-recourse, the higher the margin is, there is less room for default in bad idiosyncratic states, which leads to less risk sharing.

In contrast to Mendoza et al. (2009) and Angeletos and Panousi (2011), our mechanism does not require the interpretation that the U.S. is “more developed” in the sense that other countries ought to follow suit. Although low margins and high leverage could reflect a high degree of sophistication, they could also reflect undesirable lending practices. We do not take a stand on why margins are what they are. Instead, as in many papers in economics
that take technical progress as exogenous, our approach is to treat margins as an exogenous process. As well, because we emphasize leverage rates and not the development of the legal system, our model can account for sudden changes in the financial system. Whereas the level of development of a financial system is likely to be persistent and improving, leverage in securitized markets will almost certainly fluctuate; recent events suggest that these fluctuations can be large and sudden. As margins for securitized loans increased in the U.S., a decline in the current account deficit would naturally follow as a prediction of our model.

An important contribution of our analysis is highlighting the role of tranching in driving capital flows. We show that modeling the underlying assets traded is important for capturing essential features of the data. In our model, tranching leads to offsetting gross capital flows that are of the same order of magnitude as the net flows. Economists have begun to recognize that the rapid growth of gross global financial flows poses serious risks for macroeconomic and financial stability (Obstfeld 2012). Gross capital flows are an essential feature of international finance, but few models studying net capital flows also produce gross flows. Because we explicitly model trade in underlying financial securities (specifically tranched ABS) rather than just supposing countries can trade in risk-free bonds, risk-sharing differentials in our model produce both net and gross flows. Mendoza et al. (2009) model gross flows by allowing agents to invest managerial capital across countries.

2Doubtless there are many other reasons for the current account collapse (see for example Eaton et al. 2011). We are simply highlighting one channel by which changes in leverage in financial markets would affect net financial flows.

3Fostel and Geanakoplos (2012) illustrate how tranching affects asset prices in a closed economy; our model illustrates how tranching affects international portfolios.
Literally interpreted, capital flows arise in our model when countries have different levels of leverage in securitized markets. However, one can more broadly interpret securitized markets to represent the more complex mechanisms by which financial systems hedge and insure idiosyncratic risks, in the process producing less risky liabilities from risky assets. In this case, low margins for securitized loans generally represent lending standards by financial institutions in that country and how easily borrowers can convert asset risk into liability risk. Relatedly, Caballero and Krishnamurty (2009) argue that a structural factor behind the financial crisis was the large demand for riskless assets from the rest of the world, emphasizing the role of securitized markets in driving capital flows, and differential abilities of countries to convert risky investments into risk-free assets, and Acharya and Schnabl (2010) emphasize the importance of securitized markets (specifically asset-backed commercial paper) and global banking flows for understanding the financial crisis.

1.1 Related literature

The basic framework of our model is the collateral equilibrium introduced by Geanakoplos (1997, 2003) and Geanakoplos and Zame (2014). Fostel and Geanakoplos (2008) study how collateralized borrowing can lead to spillovers across different, seemingly unrelated, international markets. Toda (2013) shows that due to the competition among financial intermediaries, securitization in a general equilibrium model with idiosyncratic risk leads to high leverage and the endogenous emergence of “sub-prime” loans.
Gourinchas and Jeanne (2006) show that in a neoclassical model the welfare gains to financial integration for capital-poor countries are small. In our model, it is the financial undeveloped country that benefits the most because of gains in risk-sharing; thus our mechanism provides an explanation for why capital-poor countries, which are likely to have less financial development, would benefit from financial integration. As well, Gourinchas and Jeanne (2013) document that capital does not flow from capital rich to capital poor countries and emphasize the role of low domestic financial development as an explanation; thus, our paper provides a mechanism that could potentially explain a portion of the capital allocation puzzle. Our paper also relates to the literature on financial adjustment and capital allocation. Gourinchas and Rey (2007) document the importance of financial adjustment for understanding capital flows and Gourinchas and Obstfeld (2012) document the importance of increases in leverage for explaining crises in both developed and emerging countries.

Kalemli-Ozcan et al. (2012) document differential levels of leverage among financial intermediaries across countries, with more leverage in the U.S. Altunbas et al. (2009) study the role of securitization on the effectiveness of monetary policy through the bank lending channel. Gorton and Metrick (2012) study the role of securitized markets in the financial crisis. DeMarzo (2005) and Farhi and Tirole (2012) study tranching and asymmetric information. These papers focus on the strategic interaction between the seller and buyer of ABS but abstract from the default decision of borrowers; our paper is complementary.
2 Benchmark One Country Model

In this section we present a basic one country, two-period general equilibrium model with securitized lending and asset-backed securities. In Section 3 we extend the model to two countries to show how different collateral requirements drive capital flows. Section 4 introduces aggregate risk and tranching.

2.1 Description of the economy

Consider a two period model of collateral equilibrium with idiosyncratic shocks as in Toda (2013). Time is indexed by $t = 0, 1$. There is a unit continuum of investors indexed by $i \in I = [0, 1]$. There is also a unit continuum of risk-neutral, perfectly competitive profit maximizing financial intermediaries who service loans and issue asset-backed securities.

2.1.1 Entrepreneurs

Entrepreneurs$^4$ have identical Kreps-Porteus-Epstein-Zin-Weil constant relative risk aversion, constant elasticity of intertemporal substitution (CRRA/CEIS) utility function defined by

$$U(C_0, C_1) = \left( \frac{1}{1 + \beta} C_0^{1-1/\varepsilon} + \frac{\beta}{1 + \beta} \mathbb{E}[C_1^{1-\gamma}]^{1-1/\gamma} \right)^{1-1/\varepsilon}, \quad (2.1)$$

$^4$Below, we use “entrepreneurs”, “investors”, and “agents” interchangeably.
where $\beta > 0$ is the time preference rate, $\gamma > 0$ is the coefficient of relative risk aversion, and $\varepsilon > 0$ is the elasticity of intertemporal substitution. Agent $i$ is endowed with $W^i$ units of initial capital at $t = 0$, but there is no endowment at $t = 1$. The entrepreneurs have access to a constant-returns-to-scale technology with stochastic, idiosyncratic productivity. Investor $i$’s investments yield $A^i$ (gross return on investment), which realizes at the beginning of time 1. So, if agent $i$ invests $k^i$ he gets $A^i k^i$ at the beginning of time 1. Idiosyncratic returns $A^i$ are independently distributed across investors; there are no aggregate risks for now.

### 2.1.2 Financial Structure

Markets are incomplete and investors cannot directly insure against the idiosyncratic risk, possibly due to moral hazard, costly state verification, or other reasons. However, investors can borrow from financial intermediaries by putting up their investments as collateral. Intermediaries can offer arbitrary amount of loans from a finite menu of loan types indexed by $l \in \{1, \ldots, L\}$. Loan $l$ is characterized by an exogenous collateral requirements $c^l \geq 1$ and a gross borrowing rate $R_{b,l} \in [0, \infty]$, to be determined in equilibrium. For each dollar investor $i$ takes from loan $l$, he must invest $c^l$ dollars in the investment technology and put up its return (product) $A^i c^l$ as collateral. Following Geanakoplos and Zame (2014), loans are non-recourse, that is, the sole penalty of default is the confiscation of collateral: for each unit taken from

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5If $\gamma = 1$, $E[C_1^{1-\gamma}]^{1-\gamma}$ should be replaced by $\exp(E[\log C_1])$. If $\varepsilon = 1$, (2.1) becomes

$$U(C_0, C_1) = \exp \left( \frac{1}{1 + \beta} \log C_0 + \frac{\beta}{1 + \beta} \log \left( E[C_1^{1-\gamma}]^{1-\gamma} \right) \right).$$

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loan \( l \) at \( t = 0 \), the investor has the option of either paying back the promised interest rate \( R_{b,l} \) or surrendering the collateral \( A^i c_l \) at \( t = 1 \). Therefore, at \( t = 1 \) the investor chooses the better option and optimally delivers

\[
\min \{ A^i c_l, R_{b,l} \}
\]  

(2.2)

to the financial intermediary.

We can equivalently think of non-recourse collateralized loans as convertible bonds. Notice that the payoff of 1 unit of loan with collateral level \( c_l \) and interest rate \( R_{b,l} \) is

\[
\min \{ A^i c_l, R_{b,l} \} = c_l \min \{ A^i, \frac{R_{b,l}}{c_l} \},
\]

so this is equal to the payoff of \( c_l \) units of convertible bonds with strike \( \frac{R_{b,l}}{c_l} \).

Therefore, we can interchangeably think of default—delivering the project investment instead of the promised payment—as a flexible capital structure that shares investment risk in low-payoff states.

In order to raise capital for lending, each financial intermediary pools all debt contracts of the same type \( l \in \{1, \ldots, L\} \) and issues asset-backed securities (ABS), or in this case collateralized debt obligations (CDO). By risk neutrality and perfect competition, the intermediary’s profit must be zero. Therefore the total dividend to ABS \( l \) is the cross-sectional sum of individual deliveries (2.2),

\[
\int_I s_l \min \{ A^i c_l, R_{b,l} \} \, di,
\]
where \( s^i_l \geq 0 \) is the size of loan \( l \) investor \( i \) takes from the financial intermediary. Since the amount of collateral is proportional to the loan size, \( s^i_l \) does not affect the default decision. Since the productivities are independent across investors, by the law of large numbers we can write the gross return on ABS \( l \) as

\[
R_{\text{ABS},l} = \frac{\int_I s^i_l \min \{ A^i c_l, R_{b,l} \} \text{di}}{\int_I s^i_l \text{di}} = E \left[ \min \{ A^i c_l, R_{b,l} \} \right], \quad (2.3)
\]

which is simply the cross-sectional average of individual deliveries \((2.2)\). The following diagram shows the flow of capital at each point in time.

In reality, securitization follows a more complex procedure.\(^6\) First, the bank originating loans sells individual loans to a trust called special purpose vehicle (SPV). Second, the SPV pools debt contracts and create tranches\(^7\) of claims to the pool according to seniority to be repaid (the more senior tranches gets repaid first in case of default), and sells them to investors. Typically the bank keeps servicing the loans (collect monthly payments from

\(^6\)See Gorton and Metrick (2013) for more institutional details about securitization.

\(^7\)“Tranche” is the French term for “slice”.

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the borrower) and pass through the proceeds to the SPV after deducting a small service fee. Hence the “financial intermediary” in our model can be regarded as the combination of the bank and SPV, with no transaction costs. It is certainly interesting and important to model how to set the collateral requirement or how to split the pool into tranches, but in this paper we abstract from these institutional details.

2.1.3 Budget and Portfolio Constraints

Let $\theta \geq 0$ be the fraction of capital a typical entrepreneur invests in the technology. Let $\phi_l \geq 0$ be the fraction invested in the asset-backed security $l$, and let $\psi_l \geq 0$ be the fraction borrowed from loan $l$. By accounting,

$$\theta + \sum_{l=1}^{L} \phi_l - \sum_{l=1}^{L} \psi_l = 1.$$  

The collateral requirement (collateral constraint) is

$$\theta \geq \sum_{l=1}^{L} c_l \psi_l,$$

that is, the total investment in the technology must exceed the total collateral required. Note that since loans are collateralized, once some part of the investment is used as collateral, it cannot be used again. This is why the fraction of investment $\theta$ must be at least as much as the total collateral requirement $\sum_{l=1}^{L} c_l \psi_l$. 

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The total return on portfolio \( \pi = (\theta, \phi, \psi) \) is

\[
R^i(\pi) = A^i \theta + \sum_{l=1}^{L} R_{\text{ABS},l} \phi_l - \sum_{l=1}^{L} \min \{ A^i c_l, R_{b,l} \} \psi_l, \tag{2.4}
\]

where \( R_{\text{ABS},l} \) is the gross return on ABS \( l \) defined by (2.3) and \( \min \{ A^i c_l, R_{b,l} \} \) is the delivery of investor \( i \) to the financial intermediary for each unit of loan \( l \) taken as in (2.2).

### 2.1.4 Entrepreneur’s problem

The objective of each investor is to maximize the utility subject to the budget and portfolio constraints:

\[
\text{maximize} \quad \left( \frac{1}{1+\beta} C_0^{1-1/\varepsilon} + \frac{\beta}{1+\beta} \mathbb{E}[C_1^{1-\gamma}]^{1-1/\varepsilon} \right)^{1/(1-\gamma)}
\]

subject to

\[
C_1 = R^i(\pi)(W^i - C_0), \tag{2.5a}
\]

\[
\theta + \sum_{l=1}^{L} \phi_l - \sum_{l=1}^{L} \psi_l = 1, \tag{2.5b}
\]

\[
\theta \geq \sum_{l=1}^{L} c_l \psi_l. \tag{2.5c}
\]

(2.5a) is the intertemporal budget constraint, where \( R^i(\pi) \) is the total return on portfolio \( \pi = (\theta, \phi, \psi) \) defined by (2.4). (2.5b) is the intratemporal budget constraint (accounting). (2.5c) is the collateral constraint.

Once the agent chooses consumption at \( t = 0 \), the remaining decision is to choose the portfolio, which (by the monotonicity of the utility function)
is equivalent to maximizing

$$E[R^t(\pi)^{1-\gamma}]^{\frac{1}{1-\gamma}} \quad (2.6)$$

subject to the portfolio constraints (2.5b) and (2.5c). Since the productivities $A^t$ are i.i.d. across agents, so are portfolio returns $R^t(\pi)$ for a given portfolio $\pi$. Since the objective function of the optimal portfolio problem (2.6) is the same across agents, the maximum of (2.6) takes a common value. Let

$$\rho = \max_{\pi} E[R^t(\pi)^{1-\gamma}]^{\frac{1}{1-\gamma}}$$

be the maximum value.

Substituting the budget constraint (2.5a) into the objective function (2.1) and using the definition of $\rho$, we can easily solve for the optimal consumption rule by calculus. The solution is

$$(C^t_0, C^t_1) = \left( \frac{1}{1 + \beta^t \rho'^{-1}} W^t, \frac{\beta^t \rho'^{-1} R^t(\pi)}{1 + \beta^t \rho'^{-1}} W^t \right), \quad (2.7)$$

where $\pi$ is the optimal portfolio. Although this expression was derived under the condition $\varepsilon \neq 1$, it is also valid for $\varepsilon = 1$. The growth rate of consumption is

$$\frac{C^t_1}{C^t_0} = \beta^t \rho'^{-1} R^t(\pi). \quad (2.8)$$

Substituting (2.7) into (2.1), the maximum utility (in units of consumption equivalent per unit of wealth, that is, the maximum of (2.1) when $W^t = 1$)
is

\[ V(\beta, \varepsilon, \rho) = \begin{cases} 
(1 + \beta)^{\frac{\rho}{1+\beta}} (1 + \beta^\varepsilon \rho^{\varepsilon-1})^{\frac{1}{1+\beta}}, & (\varepsilon \neq 1) \\
\frac{1}{1+\beta}(\beta \rho)^{\frac{\beta}{1+\beta}}, & (\varepsilon = 1)
\end{cases} \] (2.9)

Note that \( V(\beta, \varepsilon, \rho) \) is continuous at \( \varepsilon = 1 \).

### 2.1.5 Equilibrium and Properties

A collateral equilibrium with ABS is defined by borrowing rates, consumption choices, and portfolio choices such that (i) agents optimize and (ii) markets clear. Since there are no aggregate shocks, all ABS pools are risk-free (idiosyncratic risks are diversified away).

**Definition 2.1 (Collateral Equilibrium).** Given initial wealth \((W^i)_i \in I\) and the collateral requirement \((c_l)_l \in L\), the individual consumption and portfolio choice \((C^i_0, C^i_1, \theta^i, \phi^i, \psi^i)_i \in I\) and borrowing rates \((R^i_l)_l \in L\) constitute a collateral equilibrium if

1. Each investor \(i \in I\) solves the optimal consumption-portfolio problem \[(2.5): \; \pi^i = (\theta^i, \phi^i, \psi^i) \text{ maximizes } (2.6) \text{ and the optimal consumption rule is } (2.7).\]

2. For each loan type \(l \in L\), lending and borrowing are matched:

\[ \int_I (W^i - C^i_0) \phi^i_i \, di = \int_I (W^i - C^i_0) \psi^i_i \, di. \]

3. Capital markets clear:

\[ \int_I (W^i - C^i_0) \theta^i \, di = \int_I (W^i - C^i_0) \psi^i \, di. \]
Note that the profit maximization by intermediaries is implicit in the definition of the ABS return in (2.3).

As proved in Toda (2013), an equilibrium exists. Furthermore the single loan traded in equilibrium is the loan with the lowest collateral requirement. Relabeling loans so that the collateral requirements are

\[ 1 \leq c_1 < c_2 < \cdots < c_L, \]

loan \( l = 1 \) has the lowest collateral requirement and is thus the equilibrium contract traded. In other words, investors will borrow only from this loan and exhaust their collateral (borrow to the maximum) for any menu of loans. Furthermore, in this economy assets in zero net supply or splitting the payoffs of the assets (such as tranching) are irrelevant for equilibrium.

Equilibrium looks as follows. Since agents are homogeneous, investment levels and portfolios are given by

\[ \theta = 1, \quad \phi_1 = \psi_1 = \frac{1}{c_1}, \quad \text{and} \quad (\forall l > 1) \phi_l = \psi_l = 0. \]

Thus, interest rates are determined such that agents invest their capital in their projects, borrow against the projects and reinvest the proceeds in ABS in a matched portfolio.

Because investors leverage to the max and are constrained by their collateral, it is useful to consider the downpayment associated with borrowing in collateralized contract \( c_l \). If an agent invests 1 in the project, she can borrow \( \frac{1}{c_l} \) against the project in loan \( c_l \). As a result, an investor needs to
put up $1 - \frac{1}{c_l}$ as a downpayment in order to invest 1 unit. Thus, collateral levels define the percent downpayment required on a loan, and in equilibrium agents will use the loans with the lowest required downpayment.

How do securitized markets affect economic growth and welfare? Collateralized lending and asset-backed securities allow risk sharing because default—or loan convertibility—makes the delivery of individual loans different from that of ABS, even though the two positions cancel out in the aggregate. This is clear to see from the portfolios that agents choose. In the absence of securitized lending, the objective function of the optimal portfolio problem \[ (2.6) \] becomes a constant

$$\rho_{\text{no sec}} = \mathbb{E}[(A^i)^{1-\gamma}]^{\frac{1}{1-\gamma}}$$

because investors have no portfolio choice: $\theta = 1$ and $\phi = \psi = 0$ necessarily. With securitized markets, investors borrow against their investment projects, but to invest in ABS. Because investors strategically default on their loans, their overall position becomes less risky. Asset-backed securities have no idiosyncratic risk—in this setting they are risk-free—and by defaulting investors can thus share idiosyncratic risk and improve welfare. To see this formally, since the portfolio without securitization $(\theta, \phi, \psi) = (1, 0, 0)$ is still feasible with securitization, the maximum value of the optimal portfolio problem will be larger:

$$\rho_{\text{sec}} = \max_{\pi} \mathbb{E}[R^{i}(\pi)^{1-\gamma}]^{\frac{1}{1-\gamma}} > \rho_{\text{no sec}}.$$
Since the maximum utility (2.9) is a monotonic function of $\rho$, securitization improves welfare.

Consider when the collateral level is $c_1 = 1$, corresponding to infinite leverage and zero margins on collateralized loans. In this case, investors always default on their loans and turn over the proceeds of their investment projects to the financial intermediaries. Asset-backed securities, in this case, are exactly the expected project payoff, and thus investors receive a risk-free payoff—the expected investment return without any idiosyncratic risk. This is the perfect risk-sharing case. The interesting case is when the collateral level is $c_1 > 1$. As $c_1$ decreases and approaches 1, agents share more and more idiosyncratic risk through default.

Turning to growth, since individual consumption growth is given by (2.8), taking the cross-sectional average, by the law of large numbers the aggregate consumption growth is

$$g_C = E[C_i^t/C_0^t] = \beta^\varepsilon \rho^{\varepsilon - 1} E[R^t(\pi)] = \beta^\varepsilon \rho^{\varepsilon - 1} E[A^t], \quad (2.10)$$

where the last equality uses the market clearing condition $\theta = 1$ and $\phi = \psi$. Note that the growth rate $g_C$ is increasing or decreasing in $\rho$ according as $\varepsilon > 1$ or $\varepsilon < 1$. Since more securitization increases $\rho$, it follows that with securitization the economy grows faster if the elasticity of intertemporal substitution (EIS) $\varepsilon$ is greater than 1 and slower if $\varepsilon < 1$. This dependency is not surprising. In fact, Schmidt and Toda (2015) show in a general setting that there is a precautionary saving motive if and only if EIS $< 1$. Since securitization offers risk sharing, it reduces the precautionary saving motive.
if EIS < 1.

3 Two Country Model

In this section we consider two countries that are identical except for the size (aggregate wealth) and the collateral level required for securitized loans. Capital flows arise between countries because the low-margin country can better insure idiosyncratic risk than the high-margin country. Trade arises in ABS as a result. Throughout the rest of the paper, a subscript refers to time or an asset type; a superscript refers to a country or an agent.

3.1 Description of the economy

Consider two countries—Home and Foreign—denoted by $H$ and $F$. In each country there is a unit continuum of investors with identical preferences given by (2.1) and investment projects with idiosyncratic risks. Each country has a continuum of risk-neutral, perfectly competitive financial intermediaries who offer securitized loans. Since by homotheticity the inequality among agents within a country does not affect aggregate outcomes, for simplicity assume that the initial wealth of an agent in country $j$ is $W^j$, where $j = H, F$.

Investors can only borrow from intermediaries in their country of residence and intermediaries can only make loans in their domestic country. Since agents will always choose the lowest collateral loan in equilibrium, we will simplify, without loss of generality, by considering that countries each offer a single collateral level, $c_H$ and $c_F$.8 The Home financial sector offers contracts

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8Equivalently, $c_H$ and $c_F$ could be the lowest collateral loan offered from the menu of
with lower collateral requirements, reflected by \( c_H < c_F \). Home investors borrow at rate \( R_{b,H} \) subject to collateral level \( c_H \), and Foreign investors borrow at rate \( R_{b,F} \) subject to \( c_F \).

Countries can trade ABS, and thus investors can hold asset-backed securities from either country. In this way, capital can flow between countries as investors buy and sell ABS through international markets. Since there are no aggregate risks, ABS are risk-free and therefore the ABS in both countries must offer the same return, i.e., \( R_{ABS,H} = R_{ABS,F} \). However, the borrowing rates \( R_{b,H} \) and \( R_{b,F} \) will differ because the collateral rates differ.

The portfolio of an investor in country \( j \) consists of the fraction of capital invested in the technology \( \theta^j \), the fraction borrowed in (the domestic) collateralized loans \( \psi^j \), and the fraction invested in ABS issued in each country \( \phi_{H}^j \) and \( \phi_{F}^j \). Summing up, the intratemporal budget constraint (accounting) satisfies

\[
\theta^j + \phi_{H}^j + \phi_{F}^j - \psi^j = 1
\]

and the collateral constraint is as in (2.5c), subject to the respective collateral level in the investor’s country of residence.

Equilibrium is as before—borrowing rates and portfolios such that markets clear and agents optimize—with the modification that market clearing in capital and asset markets reflects international trade. Let \( C_{0,j}^j \) be the aggregate consumption in country \( j \) at \( t = 0 \), given by (2.7). Since country \( j \) has remaining capital \( W^j - C_{0,j}^j \) after consumption, the market clearing condition
for the asset-backed securities are

\[(W^H - C^H_0)\phi^H_j + (W^F - C^F_0)\phi^F_j = (W^j - C^j_0)\psi^j\]  

(3.1)

for \(j = H, F\), where \(\phi^H_j, \phi^F_j\) denote the Home and Foreign portfolio share of ABS issued by country \(j\). The left-hand side is the world demand of the ABS issued by country \(j\). The right-hand side is the world supply of that ABS, which is supplied only by country \(j\). The market clearing condition for capital is

\[(W^H - C^H_0)\theta^H + (W^F - C^F_0)\theta^F = (W^H - C^H_0) + (W^F - C^F_0),\]  

(3.2)

where \(\theta^H, \theta^F\) denote the Home and Foreign portfolio share of the investment in the technology. The left-hand side is the total capital invested. The right-hand side is the total capital available, which must end up in the technology because lending and borrowing through securitization cancel out.

As in autarky, investors continue to leverage to the maximum, but the difference now is that \(\theta \neq 1\). Letting \(\psi^j\) be the aggregate borrowing by country \(j\), by the maximum leverage property we have

\[\psi^j = \frac{\theta^j}{c^j} \quad (j = H, F)\]

and investments in ABS \((\phi^H_j + \phi^F_j)\) are determined from the budget constraint, given \(\theta^j\). Importantly, it is no longer the case that investors will hold “matched” portfolios, borrowing against their project to invest in the domestic ABS: we will have \(\phi^H_j + \phi^F_j \neq \psi^j\) in general. Agents in the low-margin
country will borrow to increase investment in their project.

3.2 Numerical Example

The two country model admits no closed-form solutions and therefore we must resort to a numerical solution. To solve for equilibrium, let the coefficient of relative risk aversion be $\gamma = 2$, the EIS be $\varepsilon = .9$, $\beta = .95$, and the initial wealths be equal with $W^H = W^F = 1$. We vary the collateral level in Home from $c^H = 1.03$ to $1.25$, and keep the Foreign collateral level fixed at $c^F = 1.25$. We then solve for investment levels, borrowing rates, and risk free rates in autarky and with international capital flows.

Figure 1 plots investment levels, $\theta$ for each country as the Home collateral level $c^H$ varies. In autarky both countries would have investment levels equal to one. As the Home collateral level falls, investment in Home increases. Trivially, Home investors can borrow more against their projects when the collateral rate falls—but this was true in autarky, too. In autarky, as collateral rates drop, investors borrow more against their projects but increase their purchases of ABS, because market clearing requires that investment in the project is constant at $\theta = 1$. With international capital flows, however, Home investors choose to borrow more to invest in their project. On the other hand, Foreign investors invest less of their own capital in their projects, choosing instead to invest in ABS.

Why does Home investment increase when collateral decreases and coun-

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9 The distribution for returns is a mixture of log-normals with $\mu = .1$ and $\sigma = .1$ and $\sigma = .4$, with equal weight on the mixtures. We choose this mixture to match the aggregate distributions we will use later, but the results would be qualitatively identical if we used a single log-normal distribution.
tries can trade? Remember that investors hold a portfolio of ABS and borrow using collateralized loans as a way of insuring idiosyncratic risk. When the collateral rate drops, investors can hold a larger portfolio of ABS, which are risk-free, and thus insure more risk. However, the Foreign collateral rate is fixed, but Foreign investors can buy Home ABS in order to insure their risk. Thus, Foreign investors buy Home ABS as a way of insuring more risk than they can using the Foreign financial sector. Trade allows Foreign investors to “get access” to risk-sharing in the Home financial sector. As result, capital flows toward Home.

Figure 2 shows how borrowing rates change as a result of international capital flows and changing collateral levels. Home and Foreign borrowing rates get closer as a result of trade. Figure 3 shows the effect of trade and collateral levels on risk-free rates. Notably, as a result of trade the risk-free rate drops in Home and increases in Foreign. Thus, our model delivers the “savings glut” hypothesis that financial integration explains low risk-free
Collateral levels and trade have potentially important consequences for consumption levels and growth. Figure 4 plots consumption and consumption growth as functions of $c_H$, with trade and in autarky. With EIS<1, higher interest rates lead to higher consumption in period 0. As a result of trade,
Home interest rates fall and Foreign interest rates rise, leading to a slight decrease in Home initial consumption, $C^H_0$, and a substantial increase in Foreign consumption $C^F_0$.

Figure 4: Effects of Collateral Levels and Trade on Consumption and Consumption Growth.

(a) Period-0 Consumption. (b) Consumption Growth.

Lower collateral levels lead to lower consumption growth in autarky as risk-sharing improves. Interestingly, the effect of trade on consumption growth can be quite substantial. Consumption growth falls for Foreign agents, perhaps not surprisingly since initial consumption increases by so much. Home consumption growth increases as a result of trade, and this effect is larger for lower collateral levels. This result is in contrast to Toda (2013), where consumption growth decreases after securitization. The reason for this difference is that in an open economy the country with low margins can borrow from the other country to invest in the production technology.
4 Aggregate Shocks, Risk, and Tranching

One of the most important recent financial innovations has been the “tranching” of assets or collateral. In tranched securitizations the collateral dividend payments are divided among a number of bonds which are sold off to separate buyers. The essential feature of tranching is that different bonds have different state-contingent payoffs. For example, mortgage pools are often tranched into “floaters” and “inverse floaters,” in which the payoff of the bond is tied to current interest rates. The more a floater pays, the less an inverse floater pays and so on. In this case, tranche payments are contingent on an aggregate state (interest rates), even if there is no risk in the underlying collateral pool, which is typically the case for Prime mortgages.

Another important example is tranching a pool of collateral into bonds that differ in seniority/subordination of when payments are made. In the simplest case, a pool of assets can be tranched into a “debt” bond, with a near-constant payoff, and an “equity” bond, with a payoff that varies with the aggregate payoff of the pool. Even when the tranches of subprime mortgages appear to have the form of debt for the higher tranches, and equity for the lower tranches, the presence of various triggers which move cash flows from one tranche to another can make the payoffs go in opposite directions. For these tranches of a pool of collateral to be meaningful, there must be risk in what the pool will pay.

Thus far in our analysis, there is no aggregate risk and thus collateral pools have a risk-free payoff; there is no room for meaningful tranching. To analyze the effect of tranching on international flows, we will introduce
multiple aggregate states and allow for the cleanest form of tranching. To simplify, we will consider two aggregate states; thus, a pool of collateral will have two possible values. We will consider tranching the Home pool of collateral into Arrow securities that pay in each state: the first tranche pays the value of the Home collateral in the first state, and zero otherwise; and the second tranche pays the value of the Home collateral in the second state, and zero otherwise. Investors in each country can hold different combinations of the two tranches to isolate payoffs in each aggregate state.

It is worth understanding why tranching would be meaningful in this situation, and why it would change results. The result of our paper so far has been that because collateral rates differ across countries, agents in each country have different exposures to risk, and international trade transpires precisely because agents face different levels of risk. Trade allows agents to hold portfolios that share risk more effectively than the autarkic portfolios. With ex ante identical agents, Toda (2013) has shown that tranching has no effect on the autarkic equilibrium because investors hold identical portfolios and thus have identical exposure to aggregate risks. However, when agents in each country are subject to different collateral rates, investors in different countries hold different portfolios and are thus subject to different aggregate risk exposure. Because collateral levels differ across countries, the payoffs to ABS are linearly independent. However, investors cannot sell short ABS, and thus investors cannot span the aggregate states. Tranching, by design, allows agents to isolate aggregate risks, and thus tranching will lead to international flows and greater risk sharing.

With this in mind, we will consider aggregate risk in the form of second-
moment shocks to idiosyncratic risk. In one state idiosyncratic risk is low; in the other idiosyncratic risk is high. In this case, the marginal utilities of agents in each countries will differ in each aggregate state because countries differ in how well securitize markets allow agents to share idiosyncratic risk.

First we introduce aggregate risk in the form of second-moment shocks, which does not change our earlier results without aggregate risk, and then we introduce tranching.

4.1 Aggregate Shocks

There are two aggregate states, \( s = 1, 2 \), occurring with probabilities \( p \) and \( 1 - p \), which index the distribution of payoffs to investors’ projects. The return to agent \( i \) is distributed according to the cumulative distribution function \( F_s(\cdot) \) in state \( s \). The expected payoff in each state is the same but the variance of returns in higher in state 2. Thus,

\[
E[A^i \mid s = 1] = E[A^i \mid s = 2], \\
\text{Var}[A^i \mid s = 1] < \text{Var}[A^i \mid s = 2].
\]

Agents’ problems are as before, with the exception that they include aggregate risk when taking expectations. The payoffs to ABS are now state-contingent, so that ABS pay \( R_{\text{ABS},j}(s) \) in state \( s \) in country \( j = H, F \) defined

\footnote{As a result, our paper relates to the literature on “uncertainty shocks,” for example Bloom et al. (2007), Bloom (2009), and Bloom et al. (2012). Alessandria et al. (2015) study second-moment shocks and trade.}

\footnote{We have also investigated the effects of first-moment shocks, but the effects are insignificant. This is because when only expected payoffs differ across states, marginal utilities in each country are quite similar.}
as before but the distribution corresponds to the aggregate state.

We solve for equilibrium with trade (and in autarky) for $\gamma = 2$, log-normally distributed returns with $\mu = .1, \sigma_1 = .1, \sigma_2 = .4, p = .5$. Figure 5 plots investment levels $\theta^H, \theta^F$ and borrowing rates $R_{b,H}, R_{b,F}$ as a function of $c_H$. Notice that results are nearly identical to the results without aggregate risk.

![Graph of Investment Levels](image1)

![Graph of Borrowing Interest Rates](image2)

(a) Investment levels. (b) Borrowing interest rates.

Figure 5: Effect of collateral levels and capital flows.

Figure 6 plots portfolio holdings for Home and Foreign investors. The left frame plots holdings of Home ABS $\phi_H$, and right frame plots holdings of Foreign ABS, $\phi_F$. When Home collateral levels are low, capital flows from Foreign to Home through the purchase of Home ABS by Foreign Investors; Home investors in contrast invest exclusively in Home ABS.

Consumption and Welfare

Collateral levels and trade similarly effects on consumption levels and growth as we saw in Figure 4 without aggregate risk. Figure 7 plots consumption and consumption growth as functions of $c_H$, with trade and in autarky, illustrating
the same features as already documented.

Figure 6: Home and Foreign ABS Portfolios $\phi_H$ and $\phi_F$.

Figure 7: Effects of Collateral Levels and Trade on Consumption and Consumption Growth.

(a) Period-0 Consumption.  
(b) Consumption Growth.

The effects on welfare are equally interesting. Figure 8 plots welfare as a function of $c_H$ with trade and in autarky, and plots the changes in welfare as a result of trade. In autarky, lower collateral levels lead to higher welfare as risk-sharing improves through securitization. Capital flows from Foreign to Home lower the Home borrowing rate, which decreases the fraction of
downside risk that entrepreneurs can pledge to lenders; therefore there is
less risk sharing, which hurts Home welfare. For this reason, consumption
growth increases with trade because low risk sharing increases precautionary
saving (if EIS < 1). In contrast, Foreign investors benefit from trade, both
because they can access higher savings rates and because higher borrowing
rates improve risk-sharing.\footnote{We see similar welfare effects without aggregate risk as well.}

\begin{figure}[h]
\centering
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{welfare.png}
\caption{Welfare.}
\end{subfigure}
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{welfare_trade.png}
\caption{Welfare and Trade.}
\end{subfigure}
\caption{Effects of Collateral Levels and Trade on Welfare.}
\end{figure}

\section{4.2 Tranching}

We model tranching as the ability of financial intermediaries to divide the
payments from a pool of collateral into different bonds that pay in different
states. Since there are two states, without loss of generality complete
tranching transpires when intermediaries issue “Arrow Securities” that pass
through the collateral values in only one state.\footnote{While considering Arrow securities is an extreme case, the equilibrium asset holdings
are not so far from “balanced” so that we could instead consider two tranches, one safer}

\footnotetext[12]{We see similar welfare effects without aggregate risk as well.}
\footnotetext[13]{While considering Arrow securities is an extreme case, the equilibrium asset holdings
are not so far from “balanced” so that we could instead consider two tranches, one safer}
that only the Home financial sector has the ability to tranche assets. We call these tranches “Home-1”, denoted by “H1,” and “Home-2,” denoted by “H2.” The tranche payoffs are

\[ R_{\text{ABS},H1} = (R_{\text{ABS},H(1)}, 0), \]
\[ R_{\text{ABS},H2} = (0, R_{\text{ABS},H(2)}). \]

Denote the price of H1 by \( q \), implying the price of H2 is \( 1 - q \), where the prices sum to 1 precisely because the sum of both tranches is the per unit value of the underlying collateral. The budget constraint is modified as follows. Let \( \phi_{H1}, \phi_{H2}, \phi_F \geq 0 \) be the fraction invested in the ABS tranches of each country, and let \( \psi \geq 0 \) be the fraction borrowed. By accounting the budget constraint with tranching becomes,

\[ \theta^i + q\phi_{H1}^i + (1 - q)\phi_{H2}^i + \phi_F^i - \psi^i = 1. \]

Equilibrium is modified to include market clearing in tranches. Financial intermediaries create as many tranches as are backed by collateral. Thus, lending in each Home tranche must equal total Home borrowing:

\[ \phi_{H1}^H + \phi_{H1}^F = \psi^H \]
\[ \phi_{H2}^H + \phi_{H2}^F = \psi^H \]

As before, borrowing rates and tranche prices are determined such that agents and one slightly riskier, and still get similar quantitative results.
optimize and asset markets clear.

Equilibrium borrowing rates are hardly changed by the introduction of tranching, but there are important changes in allocations. The first result is that tranching leads to non-monotonic, but second-order, capital flows. Figure 9 plots the difference between investment with tranching and without. Denoting investment with tranching by $\theta_T^i$, Figure 9 plots $\theta_T^H - \theta^H$ and $\theta_T^F - \theta^F$.

Figure 9: Change in Investment as a Result of Tranching.

When home collateral levels are very low, tranching increases capital flows to Home, but the results are slightly negative for intermediate values of Home collateral levels. These results are highly sensitive to parameter values, so we do not emphasize either the qualitative or quantitative predictions. However, the results illustrate that tranching can be important for capital flows.

Figure 10 plots $\phi_{H1}^H - \phi_{H2}^H$, which measures how much the portfolios of Home agents differ from just holding ABS bonds. In other words, this value reflects the extent to which tranching affects portfolio holdings. The tranche difference is not uniformly positive for different parameters, as we discuss
below.

Figure 10: Difference in Home Tranche Portfolios $\phi_{H1}^H - \phi_{H2}^H$ as Collateral Levels Change.

One reason that agents hold different tranches is precisely because the payoffs between these two tranches differ, in addition to their expected marginal utilities in different states. Figure 11 plots the spread between the payoffs to tranche 1 and 2 when they pay in their respective states, $R_{ABS,H}(1) - R_{ABS,H}(2)$. The spread is non-monotonic in collateral levels. Lower collateral levels make the H2 tranche less attractive since default is increasingly likely, and the H2 tranche is disproportionately affected because idiosyncratic risk is high, but portfolio holdings also respond in equilibrium, with increasing demand for H1 tranches as collateral levels drop. These two forces combine to yield hump-shaped tranche spreads.

Finally, and most importantly, tranching leads to gross capital flows. With no tranching, Home agents did not buy any Foreign ABS; no capital flowed from Home to Foreign. With tranching, net flows are of the same order of magnitude as without tranching, but Home investors purchase Foreign
ABS. Figure 12 plots Home and Foreign holdings of Foreign ABS, $\phi^H_F$ and $\phi^F_F$. Notice that the Home portfolios are of the same order of magnitude as the net capital flows ($\theta^H - 1$).

Without tranching, Home investors would like to short Foreign ABS to hedge their exposure to aggregate risk. By holding different combinations
of Home tranches together with a positive amount of Foreign ABS, Home
investors are able to better span the aggregate risk in their portfolios.

The non-linear relation between collateral and the effect of tranching is
quite sensitive to parameters. Figure 13a plots the same results for \( \gamma = 
2, 3, 4, 5 \). The non-linearities become more pronounced, but also potentially
of opposite direction. This is because the risk-sharing properties of tranch-
ing. Agents prefer different tranches depending on when default is more
likely. Agents default whenever \( A^i c_i < R_{b,i} \), which means that the likelihood
of default increases as collateral rates drop both mechanically and because
equilibrium borrowing rates increase. This means that the attractiveness
of the safe and risky tranches (states 1 and 2) depend heavily on the equi-
librium borrowing rates. When risk-aversion is higher, borrowing rates are
lower, implying different cutoffs for when agents default.

![Graph](image)

(a) Change in Investment as a Result of (b) Effect of Collateral Levels and Cap-
Tranching.

Similarly, Figure 13b plots the difference in tranche holdings for agents
for different levels of risk-aversion. Here we see the mechanism driving the
non-linear effects on investment levels. While the composition of tranche
holdings are not monotonic in collateral levels, the composition is monoton-
ically affected by risk-aversion. When the tranche balance switches from
overweighting state 1 to state 2, the risk-sharing properties change and thus
capital flows respond. This, in part, explains the non-monotonicity in capital
flows resulting from tranching.

**Tranching and Welfare**

Whereas we saw that trade improves welfare for Foreign agents but decreases
welfare for Home agents, tranching improves the welfare of both agents rel-
ative to trade alone. Figure 14 plots the change in welfare with tranching
compared to with only trade. Since tranching allows agents to better hedge
residual portfolio risk, risk-sharing improves for both agents, and Foreign
investors benefit the most.  

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Figure 14: Welfare Gains from Tranching.

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14The effects on consumption growth, relative to trade, are non-monotonic in a similar
way to the effects of tranching on investment relative to trade. As with investment, the
effects on consumption growth depend on the coefficient of relative risk-aversion.
5 Conclusion

This paper shows that financial integration can lead to large international financial flows when countries have different collateral requirements for loans that are securitized into asset-backed securities. Capital flows from countries with low financial development (represented by high margins), and thus low risk-sharing and high demand for risk-free assets, to countries with high financial development (represented by low margins), with high interest rates and high returns to investment. Gross financial flows arise when one country can tranche collateral into securities with different state-contingent payoffs. Financial integration improves welfare for the country with high collateral requirements and hurts welfare for the country with low collateral requirements, but the effects are not symmetric. Tranching improves welfare in both countries, relative to trade, as agents can better isolate residual portfolio risks.

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