Efficiently Inefficient Markets for Assets and Asset Management*

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Preliminary – Do Not Distribute

Abstract

We consider a model where investors can invest directly or search for an asset manager, information about assets is costly, and managers charge an endogenous fee. In equilibrium, the efficiency of asset prices is linked to the efficiency of the asset management market: (1) if investors can find managers more easily then more money is allocated to active management, fees are lower, and asset prices are more efficient; (2) as search cost diminish, asset prices become efficient in the limit, even if information-collection costs remain large; (3) managers of complex assets earn larger fees and are fewer, and such assets are less efficiently priced; (4) good managers outperform after fees, bad managers underperform after fees, and the net performance of the average manager depends on the number of "noise allocators."

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The efficiency of market prices is one of the central questions in financial economics. The key benchmark is that security markets are perfectly efficient (Fama (1970)), but this leads to two paradoxes: First, no one has an incentive to collect information in an efficient market, so how does the market become efficient (Grossman and Stiglitz (1980))? Second, if asset markets are efficient, then positive fees to active managers implies inefficient markets for asset management (Pedersen (2015)).

When one has collected information about securities, one can invest on this information on behalf of others, so professional asset managers arise naturally as a result of the returns to scale in collecting and trading on information (Admati and Pfleiderer (1988), Ross (2005), Garcia and Vanden (2009)). Therefore, professional asset managers are central to understanding market efficiency. Indeed, we must understand the efficiency of asset markets jointly with the efficiency of the markets for asset management.

One benchmark for the efficiency of asset management is provided by Berk and Green (2004), which considers the implications of perfectly efficient asset-management markets (in the context of exogenous and inefficient asset prices). However, fire sales resulting from asset managers' outflows contribute to the limits of arbitrage (Shleifer and Vishny (1997)), the contractual features that arise in equilibrium can distort asset prices (Stein (2005), Cuoco and Kaniel (2011), Buffa, Vayanos, and Woolley (2014)),[1] and empirical evidence suggests that investors face search frictions associated with finding asset managers (Sirri and Tufano (1998), Jain and Wu (2000), and Hortaçsu and Syverson (2004)).

We seek to shed further light on the efficiency and interactions of the markets for assets and asset management by considering a general equilibrium with two levels of frictions: (i) investors' search frictions for finding and vetting asset managers and (ii) asset managers' cost of collecting information about assets. We show that there exists an efficient level of inefficiency of both asset markets and the markets for asset management, and the efficiencies of these markets are closely linked in equilibrium. Our model thus accommodates the paradoxes

[1] While these papers focus on effects that arise as a result a manager's distorted incentives or forced sale, we assume that managers invest in their clients' interest, thus focusing on managers' incentives to acquire information about assets and investors' incentive to search for good managers in a general equilibrium.

mentioned above and produces a number of new results with clear empirical predictions.

The equilibrium works as follows. A number of asset managers decide to enter the market and acquire information, while investors must decide whether to expend search costs to find an asset manager. In an interior equilibrium, investors are indifferent between passive investing (i.e., investing that does not rely on information collection) and searching for an asset manager. Investors do not collect information on their own, since the costs of doing so are higher than the benefits to an individual due to the relatively high equilibrium efficiency of the asset markets. This high equilibrium efficiency arises from investors' ability to essentially "share" information collection costs by investing through an asset manager.[2] When an investor meets an asset manager, they negotiate a fee, and asset prices are set in a competitive noisy rational expectations market. We solve for the equilibrium number of investors who invest directly, respectively through managers, the equilibrium number of asset managers, the equilibrium management fee, and the equilibrium asset prices.

We derive several results that link the efficiency of asset markets to that of the asset management market. First, if search costs are lower such that investors more easily can identify good managers, then more money is allocated to active management, fees are lower, and security markets are more efficient.

If investors search costs go to zero and the pool of potential investors is large, then the asset markets become efficient in the limit. Indeed, as search costs diminish, fewer and fewer asset managers with more and more asset under management collect smaller and smaller fees (both per investor and in total), and this evolution makes asset prices more and more efficient even though information-collection costs remain constant (and potentially large). It may appear surprising (and counter to the result of Grossman and Stiglitz (1980)) that markets can become close to efficient despite large information collection costs, but this result is driven by the fact that the costs are shared by investors through an increasingly efficient and consolidated group of asset managers. We discuss how these model-implied effects of

[2] In our benchmark model with symmetric investors, no one collects information on their own; one could consider an extension with investors with different abilities, in which case some investors may collect information on their own.

changing search costs can help explain cross-sector, cross-country, and time-series evidence on the efficiency, fees, and asset management industry for mutual funds, hedge funds, and private equity and gives rise to new tests.

We also consider the effect of the magnitude of information-collection costs. Higher information-collection costs leads to fewer active investors, fewer asset managers, higher fees, and lower asset market efficiency. One can interpret a high information-collection cost as a "complex" asset and, hence, the result can be stated as saying that complex assets have fewer asset managers, higher asset management fees, and lower efficiency, predictions that we relate to the empirical literature.

In our basic model, all investors and managers are identical, but several interesting results arise when we relax these assumptions and consider asset managers that vary in their skill. Good asset managers can acquire information about assets while bad ones try in vain. Some investors try to invest only with good managers, but they must spend resources to determine a manager's quality. The model therefore features search frictions and two layers of asymmetric information, namely about managers and assets, a "double-decker" information model.[3] In equilibrium, good asset managers attract more investors, consistent with the evidence of Berk and Binsbergen (2012). However, fund flows are not fully revealing of manager quality due to what we call "noise allocators." Noise allocators invest with whichever asset manager they find first (due to behavioral or liquidity reasons) and, thus, their noisy asset allocations play a similar role in the asset-management market to the role played by noise traders in the asset market.

Good managers outperform passive investment even after fees, thus rewarding investors for their due diligence in finding the good manager. In equilibrium, some optimizing investors may decide to save on their due diligence expenses and, instead, rely on the noisy information contained in fund flows. Their returns reflect a weighted average of good and bad managers, but the overall net performance should also be positive. Lastly, the performance of noise allocators is also a weighted average of good and bad managers, but noise

[3]Diamond (1984) and Holmstrom and Tirole (1997) consider double-decker models of moral hazard and credit frictions.

allocators are weighted more towards bad managers and, thus, their performance after fees may be worse than that of passive investment. The asset-weighted average net performance of all investors (i.e., the average manager's net return) can be positive or negative, depending on the importance of noise allocators.

The model's predictions about the performance of asset managers is related to a large empirical literature. First, the prediction that managers should be able to outperform passive investing before fees is consistent with evidence of mutual fund returns (Grinblatt and Titman (1989), Wermers (2000), Kacperczyk, Sialm, and Zheng (2008), Kosowski, Timmermann, Wermers, and White (2006)). Second, the evidence suggests that active U.S. equity mutual funds underperform after fees (e.g., Carhart (1997), but see Berk and Binsbergen (2012) for a critique), but hedge funds may outperform after fees (Kosowski, Naik, and Teo (2007), Fung, Hsieh, Naik, and Ramadorai (2008), Jagannathan, Malakhov, and Novikov (2010)), which could be consistent with the idea that noise allocators are more prominent in the mutual fund industry, although this prediction is yet to be tested more directly.

Third, our model predicts that investors (who are not noise allocators) should be able to collect sufficient information to find an asset manager that delivers a positive expected net return. Given that fund of funds are in the business of generating such information about asset managers, our model can help explain their value added documented by Ang, Rhodes-Kropf, and Zhao (2008).

The paper is related to a large body of research in addition to that cited above. The empirical literature is discussed in detail in relation to our empirical predictions in Section 5. The related theoretical literature includes, in addition to the papers cited above, noisy rational expectations models (Grossman (1976), Hellwig (1980), Diamond and Verrecchia (1981), Admati (1985)) and other models of informed trading (Glosten and Milgrom (1985), Kyle (1985)) and noise trading (Black (1986)); search models in finance (Duffie, Gârleanu, and Pedersen (2005), Lagos (2010)); and models of asset management (Pastor and Stambaugh (2012), Stambaugh (2014)) and the role of trust in this context (Gennaioli, Shleifer, and Vishny (2015)) .

The next section lays out the basic model, Section 2 provides the solution, and Section 3 derives the key properties of the equilibrium. Section 4 consider the extension of the model with noise allocators and good vs. bad asset managers. Section 5 lays out the empirical predictions of the model and their relation to the existing evidence. Section 6 concludes.

1 Model of Assets and Asset Managers

1.1 Investors and Asset Managers

The economy has two types of agents trading in a financial market: investors and asset managers. Both investors and managers can obtain a signal about the asset value by paying a fixed cost k, but while investors can only trade on their own behalf, managers have the ability to manage funds on behalf of a group of investors.

More specifically, there exist N investors and each investor can either (i) invest directly in asset markets without the signal, (ii) invest directly in asset markets after having acquired the signal, or (iii) invest through an asset manager. Due to economies of scale, a natural equilibrium outcome is that investors do not acquire the signal, but, rather, invest as uninformed or through a manager. We highlight below some weak conditions under which our equilibria take this form, and assume that these conditions are met. Consequently in our equilibria we focus on the number of investors who make informed investments through a manager I, inferring the number of uninformed investors as the residual, $U = N - I$.

There exist an unlimited number of potential asset managers, of whom M elect to pay a cost k to acquire the signal and thereby become active managers. More precisely, there is a continuum of active managers of mass M and, similarly, we think of the group of investors as a continuum.[4] All agents act competitively, taking as given the actions of others.

To invest with an asset manager, investors must search for, and vet, managers, which is a costly activity. Specifically, the cost of finding a manager and confirming that he has the signal (i.e., performing due diligence) is $c(M, I)$, which depends on both the number of

[4]Treating agents as a continuum keeps the exposition as simple as possible, but the model's properties also obtain in a limit of a finite-investor model.

active asset managers M and the number of investors I in these asset management firms. We make the natural assumption that finding a manager is easier when there are more managers (e.g., because more managers means that the geographical distance between investors and managers is smaller)[5] and fewer investors, that is, the cost c decreases weakly with M and increases weakly with I.

The search cost c captures the realistic feature that most investors spend significant resources finding an asset manager they trust with their money. In fact, real-world institutional investors often travel the world to meet potential managers in person, spend time meeting potential managers visiting them, perform due diligence on the premises, try to understand the investment strategy at a high level, and examine the trading infrastructure, risk management, back office, valuation practices, the custody of the assets, and so on. In fact, a large industry of investment consulting firms help institutional investors perform these tasks. Likewise, individual investors search for asset managers, some via local branches of financial institutions, others with the aid of investment advisors, and yet others via the internet or otherwise.

We assume that all agents have constant absolute risk aversion (CARA) utility over end-of-period consumption with risk-aversion parameter γ (following Grossman and Stiglitz (1980)). Hence, with end of period wealth $\tilde{W}$, an agent's utility is $E(-e^{-\gamma\tilde{W}})$. Each investor is endowed with an initial wealth W while managers have a zero initial wealth (without loss of generality).

When an investor finds an asset manager, he confirms that the manager has the signal and then they negotiate the asset management fee f. The fee is set through Nash bargaining and, at this bargaining stage, all costs are sunk, both the manager's information acquisition cost and the investor's search cost.

A manager who does not pay the cost k has no investors and receives a utility of $-e^{\gamma\cdot 0} = 1$

[5]Sialm, Sun, and Zheng (2014) find that "funds of hedge funds overweight their investments in hedge funds located in the same geographical areas" and have an information advantage in doing so, consistent with the similar results for individual investors' stock investments due to Coval and Moskowitz (1999).

given his zero initial wealth. The utility of an active manager is given by

$$\mathrm{E}\left[-e^{-\gamma(fI/M-k)}\right] \tag{1}$$

where I/M is the number of investors per manager, relying informally on the law of large numbers. Hence, $f\,I/M$ is the manager's total fee revenue; k is his cost of operation.

1.2 Assets and Information

We adopt the asset-market structure of Grossman and Stiglitz (1980), aiming to focus on the consequences of introducing asset managers into this framework. Specifically, there exists a risk-free asset normalized to deliver a zero net return, and a risky asset with payoff v distributed normally with mean m and standard deviation σ_v.

Agents can obtain a signal s of the payoff, where

$$s = v + \varepsilon. \tag{2}$$

The noise ε has mean zero and standard deviation σ_ε, is independent of v, and is normally distributed.

The risky asset is available in a stochastic supply given by q, which is jointly normally distributed with, and independent of, the other exogenous random variables. The mean supply is Q and the standard deviation of the supply is σ_q. We can think of the noisy supply as the number of shares outstanding plus the demand from "noise traders" who are not motivated by informational issues but, rather, are trading due to a liquidity need, hedging, or behavioral reasons. We can also think of the noisy supply more broadly as a modeling device that captures that it can be difficult to infer fundamentals from prices.

Given this asset market, uninformed investors buy a number of shares x_u to maximize their utility U_u, taking into account that the price p may reflect information about the value:

$$U_u(W) = \max_{x_u} \mathrm{E}\left[-e^{-\gamma(W+x_u(v-p))}|p\right] = e^{-\gamma W}U_u(0) \equiv e^{-\gamma W}U_u \tag{3}$$

We see that, because of the CARA utility function, an investor's wealth level simply scales his utility function and does not affect his optimal behavior. Therefore, we define the scalar U_u as the wealth-independent part of the utility function (a scalar that naturally depends on the asset-market equilibrium, in particular the price efficiency).

Asset managers observe the signal and invest in the best interest of their investors. This informed investing gives rise to the utility U_i of an active investor (not taking into account his search cost and the asset management fee — we study those later):

$$U_i(W) = \max_{x_i} \mathrm{E}\left[-e^{-\gamma(W+x_i(v-p))}|p, s\right] = e^{-\gamma W} U_i(0) \equiv e^{-\gamma W} U_i \tag{4}$$

As above, we define the scalar U_i as the wealth-independent part of the utility function. We see that the utility of an active investor differs from that of an uninformed via conditioning on the signal s.

1.3 Equilibrium Concept

We first consider the (partial) equilibrium in the asset market given the number of active investors I: An *asset-market equilibrium* is an asset price p such that the asset market clears

$$q = (N - I)x_u + I x_i \tag{5}$$

for the uninformed investors' demand x_u that maximizes their utility (3) given p and the demand from investors using asset managers x_i that maximizes their utility (4) given p and the signal s. The market clearing condition equates the noisy supply q with the total demand from the $N - I$ uninformed investors and the I informed investors.

Second, we define a *general equilibrium for assets and asset management* as a number of asset managers in operation M, a number of active investors I, an asset price p, and an asset management fee f such that (i) no manager would like to change his decision to enter or exit the asset management industry, (ii) no investor would like to switch status from active to passive or vice-versa, (iii) the price is an asset-market equilibrium, and (iv) the asset management fee is the outcome of Nash bargaining.

2 Solving the Model

2.1 Asset-Market Equilibrium

We first derive the asset-market equilibrium. The price p of the risky asset is determined as in a market in which I investors have the signal and the $N-I$ other investors are uninformed, i.e., the asset-market equilibrium is as in Grossman and Stiglitz (1980). We consider only their linear asset-market equilibrium and, for completeness, we record the main results in this section.[6]

In the linear equilibrium, an informed agent's demand for the asset is a linear function of prices and signals and the price is a linear function of the signal and the noisy supply:

$$p = \theta_0 + \theta_s \left((s - m) + \theta_q q\right), \tag{6}$$

where, as we show in the appendix, the coefficients are given by

$$\theta_0 = m - \frac{\gamma Q \operatorname{var}(v|s)}{I + (N - I)\frac{\operatorname{var}(v|s)}{\operatorname{var}(v|p)}} \tag{7}$$

$$\theta_s = \frac{I\frac{\sigma_v^2}{\sigma_v^2+\sigma_\varepsilon^2} + (N-I)\frac{\operatorname{var}(v|s)}{\operatorname{var}(v|p)}\frac{\sigma_v^2}{\sigma_v^2+\sigma_\varepsilon^2+\theta_q^2\sigma_q^2}}{I + (N - I)\frac{\operatorname{var}(v|s)}{\operatorname{var}(v|p)}} \tag{8}$$

$$\theta_q = \gamma \frac{\sigma_\varepsilon^2}{I}. \tag{9}$$

As we see, the equilibrium price depends on the ratio $\frac{\operatorname{var}(v|s)}{\operatorname{var}(v|p)}$, which is given explicitly in Proposition 1 and has an important interpretation. Indeed, following Grossman and Stiglitz (1980), we define the *efficiency* (or informativeness) of asset prices as this ratio or,

[6] Our results in this section differ slightly from those of Grossman and Stiglitz (1980) because of differences in notation (not just in the naming of variables, but also in the modeling of the information structure), but there exists a mapping from our results to theirs. Palvolgyi and Venter (2014) derive interesting non-linear equilibria in the Grossman and Stiglitz (1980) model.

equivalently, its square root, $\sqrt{\frac{\text{var}(v|s)}{\text{var}(v|p)}}$. This ratio is the uncertainty about the asset value given that one knows the signal s relative to the uncertainty given that one knows only the price p. If the price fully reflected the signal, then this measure of informativeness would equal one, and, the noisier the price, the lower the measure of informativeness.

The relative utility of investing as uninformed versus benefitting from the manager's information also plays a central role in the remainder of the paper. It is defined as

$$\Lambda \equiv \frac{U_i}{U_u} \tag{10}$$

We note that the relative utility of active versus uninformed investors does not depend on the investors' wealth level as seen in Equations (3)–(4) that define these utilities. Further, Λ is positive and less than 1 because the CARA utilities U_i and U_u are negative numbers. We can also think of Λ as a measure of the outperformance of informed investors relative to uninformed ones, where a Λ further away from 1 corresponds to greater outperformance.

As we shall see, the relative utility Λ is central for our analysis for several reasons: It affects investors' incentive to search for managers, the equilibrium asset management fee, managers' incentive to acquire information, and the informativeness of asset prices. Importantly, in equilibrium, investors' relative utility is linked to the asset price efficiency and depends on the number of active investors as described in the following proposition.

Proposition 1 *There exists a unique linear asset-market equilibrium given by* (6)*. In the linear asset-market equilibrium, the ratio of the utilities of informed investors to uninformed investors equals the efficiency of the price*

$$\Lambda = \frac{U_i}{U_u} = \sqrt{\frac{\text{var}(v|s)}{\text{var}(v|p)}}. \tag{11}$$

Further, Λ is increasing in the number of active investors I and can be written as

$$\Lambda = \left(1 - \frac{\gamma^2\sigma_q^2\sigma_\varepsilon^2}{I^2 + \gamma^2\sigma_q^2\sigma_\varepsilon^2}\frac{\sigma_v^2}{\sigma_\varepsilon^2 + \sigma_v^2}\right)^{\frac{1}{2}} \quad \in (0,1). \tag{12}$$

Naturally, informed and uninformed investors receive more similar utility (i.e., Λ is closer to one) when there are more active investors (larger I), because having more active investors makes the equilibrium price more informative, diminishing the benefit of information.

We note that the asset price efficiency does not depend directly on the the number of asset managers M. What determines the asset price efficiency is the risk-bearing capacity of agents investing based on the signal, and this risk-bearing capacity is ultimately determined by the number of active investors (not the number of managers they invest through). The number of asset managers does affect asset price efficiency *indirectly*, however, since the number of active investors and asset manages are determined jointly in equilibrium as we shall see.

2.2 Investors' Decision to Search for Asset Managers

An investor optimally decides to look for a manager as long as

$$U_u \leq U_i e^{-\gamma(-c-f)} = U_i e^{\gamma(c+f)}. \tag{13}$$

Here, the left-hand side is simply the utility of investing directly as an uninformed investor while the right-hand side is the utility of incurring a search cost c, an asset management fee f, and then benefitting from the manager's information.

Recalling the notation $\Lambda = U_i/U_u$ for the relative benefit of information, we can equivalently say that an investor looks for a manager if

$$\Lambda \leq e^{-\gamma(c+f)}. \tag{14}$$

This relation must hold with equality in an "interior" equilibrium (i.e., an equilibrium in which strictly positive amounts of investors decide to invest as uninformed and through asset managers — as opposed to all investors making the same decision).

Using similar arguments, we see that an investor would prefer being uninformed to acquiring the signal himself if $e^{-\gamma k} \leq \Lambda$. More importantly, the investor prefers using an asset manager to acquiring the signal singlehandedly provided $e^{-\gamma k} \leq e^{-\gamma(c+f)}$. Using the equilibrium asset management fee derived below in Equation (17), the condition that asset management is preferred to buying the signal can be written as $k \geq 2c$. In other words, finding an asset manager should be at most half the cost of actually being one, which seems like an assumption that is clearly satisfied in the real world. We can also make use of (15) to express this condition equivalently as $I \geq 2M$, i.e., there must be at least two investors for every manager, another realistic implication.

2.3 Entry of Asset Managers

A prospective asset manager must pay the cost k to acquire information and then, in equilibrium, manages the capital of I/M investors. Therefore, she chooses to enter and become an active manager provided that the total fee revenue covers the cost of operations:

$$f\, I/M \geq k. \tag{15}$$

This manager condition must hold with equality for an interior equilibrium.

2.4 Asset Management Fee

The asset-management fee is set through Nash bargaining between an investor and a manager. If no agreement is reached, the investor's utility (i.e., outside option) equals $e^{-\gamma(W-c)}U_u$ as the cost c is already sunk. The utility he obtains in an agreement is $e^{-\gamma(W-c-f)}U_i$. Similarly, if V is the outside option of the manager, then $e^{-\gamma f}V$ is the utility achieved following an agreement (the cost k is sunk and there is no marginal cost to taking on the investor). The bargaining outcome maximizes the product of the utility gains from agreement:

$$\left(e^{-\gamma(W-c-f)}U_i - e^{-\gamma(W-c)}U_u\right)\left(e^{-\gamma f}V - V\right) \tag{16}$$

The objective (16) is maximized by the asset management fee f given by

$$e^{-\gamma f} = \Lambda^{\frac{1}{2}}. \qquad \text{[equilibrium asset management fee]} \tag{17}$$

Expressed differently, the equilibrium fee is $f = -\frac{\log(\Lambda)}{2\gamma}$, which would naturally be zero if asset markets were perfectly efficient, so that no benefit of information existed ($\Lambda = 1$), and the fee is higher when the benefits of information are greater (Λ further away from one).

2.5 General Equilibrium for Assets and Asset Management

Given the outcome for the asset market and the equilibrium asset management fee, we can characterize the general equilibrium by the managers' entry decision and the investors' search decision. Inserting the asset management fee (17) into investor's search condition (14) and into the manager's entry condition (15), we arrive at following two indifference conditions:

$$\Lambda^{\frac{1}{2}} = e^{-\gamma c(M,I)} \qquad \text{[investors' indifference condition]} \tag{18}$$

$$\Lambda^{\frac{1}{2}} \frac{I}{M} = e^{-\gamma k} \qquad \text{[asset managers' indifference condition]} \tag{19}$$

where Λ is a function of I given explicitly by (12). Hence, solving the general equilibrium comes down to solving these two explicit equations of two unknowns (I, M).

The derivation of equilibrium is illustrated in Figure 1, which shows various possible combinations of the number of investors I and of managers M. The solid blue line indicates investors' indifference condition (18). When (I, M) is to the North-West of the solid blue line, investors prefer to search for asset managers because managers are easy to find and attractive to find due to the limited efficiency of the asset market. In contrast, when (I, M) is South-East of the blue line, investors prefer to be passive as the costs of finding a manager is not outweighed by the benefits. The indifference condition is naturally increasing as investors are more willing to be active when there are more asset managers.

Similar, the dashed red line shows the managers' indifference condition (19). When (I, M) is above the red line, managers prefer not to incur the information cost k since too many

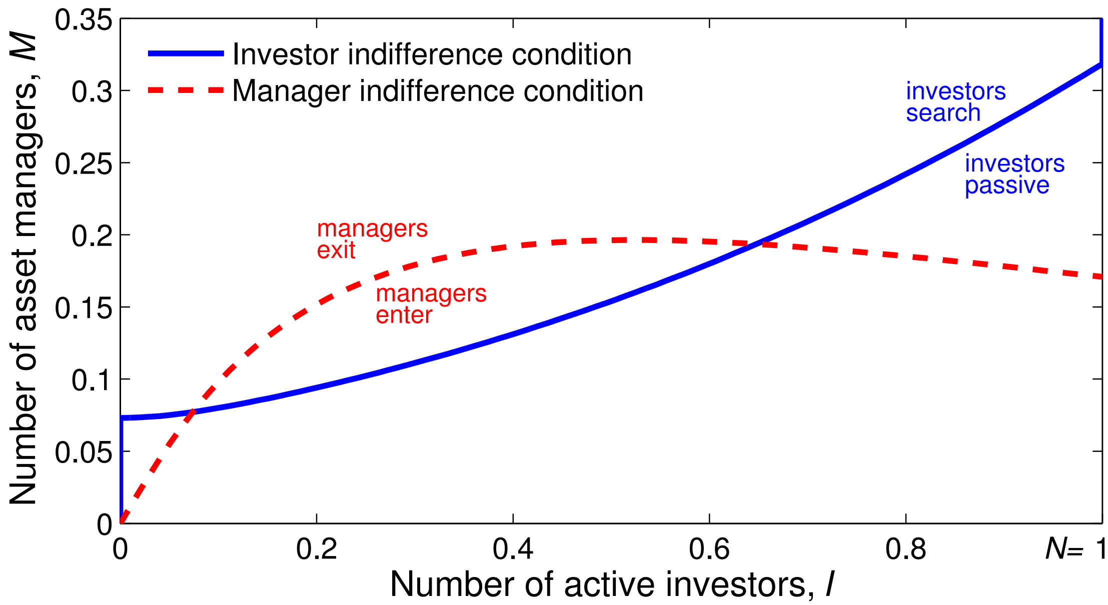

Figure 1: **Equilibrium for assets and asset management.** Illustration of the equilibrium determination of the number of active investors I (among all investors N) and the number of asset managers M. Each investor decides whether to search for an asset manager or be passive depending on the actions (I, M) of everyone else, and, similarly, managers decide whether or not to pay the information cost to enter the asset management industry. The right-most crossing of the indifference conditions is a stable equilibrium.

managers are seeking to service the investors. Below the red line, managers want to enter the asset management industry. Interestingly, the manager indifference condition can be hump shaped for the following reason: When the number of active investors increases from zero, the number of entering managers also increases from zero, since they are encouraged to earn the fees paid by investors. However, the total fee revenue is the product of the number of active investors I and the fee f. The equilibrium asset management fee decreases with number of active investors because active investment increases the asset-market efficiency, thus reducing the value of the asset management service. Hence, when so many investors have become active that this fee-reduction dominates, additional active investment decreases the number of entering managers.

The economy in Figure 1 has three equilibria. One equilibrium is that there is no asset management: $(I, M) = (0, 0)$. In this equilibrium, no investor searches for asset managers as there is no one to be found, and no asset manager sets up operation because there are no investors. This type of equilibrium is the only stable[7] equilibrium if the parameters are such that the blue line stays above the red line for all values of $I \in (0, N]$.

Figure 1 also shows two interior equilibria where the red and blue lines cross each other. The equilibrium on the left is unstable while the equilibrium on the right is stable. Consider first the left equilibrium. If slightly more managers enter, then a lot more investors will search for managers. Given a higher number of active investors, more managers will enter, and so on until we arrive at the stable equilibrium at the right. For arbitrary specifications of the search cost function c, the investor indifference condition can in principle "wiggle" enough to create additional crossings of the two lines, i.e., additional interior equilibria.

Lastly, if we change the parameters in Figure 1 such that the blue line ends below the red line for $I = N$, we have a "corner solution" in which all investors search for asset managers and the equilibrium number of asset managers is given by their indifference condition evaluated at the right side of the graph, $I = N$. We state these results more formally in the following proposition.

Proposition 2 *There always exists a general equilibrium of masses* (I, M) *of active investors and asset managers, a linear asset-market equilibrium* p*, and fee* f*, which can be characterized as follows. (i) If frictions* k *and* c *are sufficiently large, the unique equilibrium features zero asset managers. (ii) If frictions are sufficiently low, all investors search for asset managers and the number of managers is given by* (19) *with* $I = N$*. (iii) For intermediate levels of frictions, there exist (generically) an even number of interior equilibria. The masses* I *and* M *are positively related across equilibria.*

In the interest of being specific, in particular in the comparative statics that follow, we focus on the *largest equilibrium*, that is, the equilibrium with the highest levels of I and M. As stated formally in Proposition 1, this is the equilibrium in which the asset market is most efficient, and it is stable. The concept of largest equilibrium is well defined due to the results

[7] As is standard, we denote an equilibrium as stable (unstable) if a deviation in I or M from the equilibrium amounts results in incentives for agents to change their behavior towards (away from) the behavior required by the equilibrium.

in Proposition 2.

3 Equilibrium Properties

We now turn to our central results on how the frictions in the market for money management interact with the efficiency of the asset market. Our results use the fact that the asset-market efficiency is determined by the number of active investors I in equilibrium, as shown in Proposition 1. We say that the asset price is *fully efficient* if $\Lambda = 1$, meaning that the price fully reflects the signal (which is never the case in equilibrium, but it can happen in the limit). We say that the asset price is *constrained efficient* if Λ is given by (12) with $I = N$, meaning that the price reflects as much information as it can when all investors are active. Finally, *efficiently inefficient* simply refers to the equilibrium efficiency given the frictions.

We start by considering some basic properties of performance in efficiently inefficient markets in the benchmark model. We use the term outperformance to mean that an informed investor's performance yields a higher expected utility than that of an uninformed, and vice versa for underperformance.

Proposition 3 *In an equilibrium, asset managers outperform passive investing before and after fees. In an interior equilibrium, active investors' outperformance net of fees just compensates their search costs.*

These results follow in straightforward way from the fact that investors must have an incentive to incur search costs to find an asset manager and pay the asset-management fees. We note that, when we introduce asymmetric information about asset managers' quality in Section 4, the performance results will be more nuanced, as seen in Propositions 9–10.

Next, we consider the effect of investors' cost c of searching for asset managers.

Proposition 4

(i) Consider two search cost functions, c_1 and c_2, with $c_1 > c_2$ and the corresponding largest equilibria. In the equilibrium with the lower search costs c_2, the number of

active investors I is larger, the number of managers M may be higher or lower, the asset price is more efficient, the asset management fee f is lower, and the total fee revenue fI may be either higher or lower.

(ii) If $\{c_j\}_{j=1,2,3,...}$ is a decreasing series of cost functions that converges to zero at every point, then $I = N$ when the cost is sufficiently low, that is, the asset price becomes constrained efficient. If the number of investors $\{N_j\}$ increases towards infinity as j goes to infinity, then Λ goes to 1 (full price efficiency in the limit), the asset management fee f goes to 0, the number of asset managers M goes to 0, the number of investors per manager goes to infinity, and the total fee revenue of all asset managers fI goes to zero.

This proposition provides several intuitive results, which we illustrate in Figure 2. As seen in the figure, a lower search costs means that the investor indifference curve moves down, leading to a larger number of active investors in equilibrium. This result is natural, since investors have stronger incentives to enter when their cost of doing so is lower.

The number of asset managers can increase or decrease (as in the figure), depending on the location of the hump in the manager indifference curve. This ambiguous change in M is due to two countervailing effects. On the one hand, a larger number of active investors increases the total management revenue that can be earned given the fee. On the other hand, more active investors means more efficient asset markets, leading to lower asset management fees. When the search cost is low enough, the latter effect dominates and the number of managers starts falling as seen in part (ii) of Proposition 4.

As search costs continue to fall, the asset-management industry becomes increasingly concentrated, with fewer and fewer asset managers managing the money of more and more investors. This leads to an increasingly efficient asset market and market for asset management. Specifically, the asset-management fee and the total fee revenue decrease toward zero, and increasingly fewer resources are spent on information collection as only a few managers incur the cost k, but invest on behalf of an increasing number of investors.

We next consider the effect of changing the cost of acquiring information.

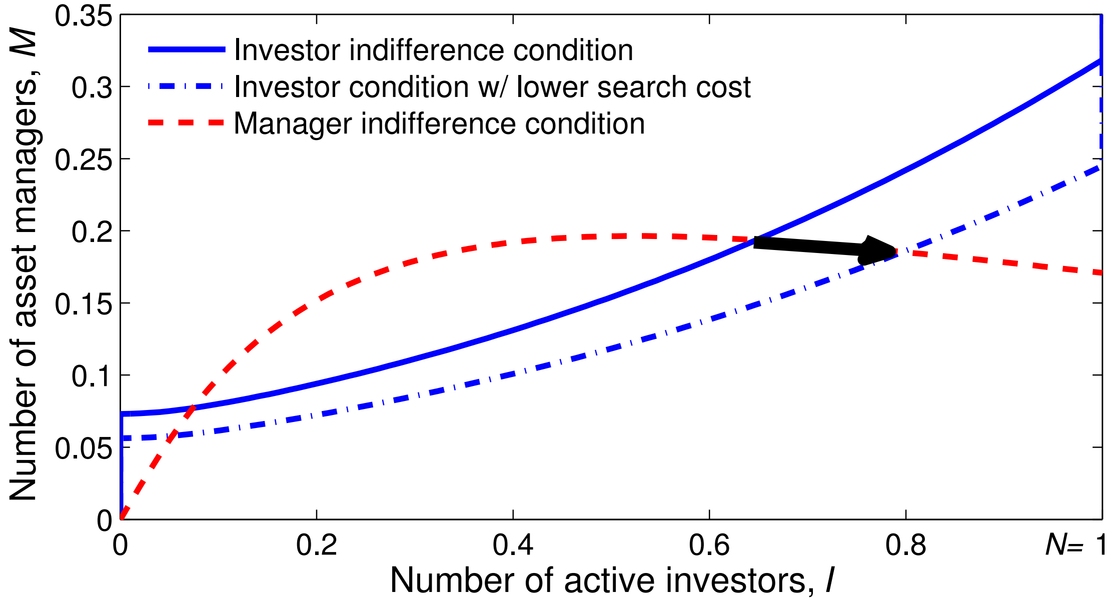

Figure 2: **Equilibrium effect of lower investor search costs.** The figure illustrates that lower costs of finding asset managers implies more active investors in equilibrium and, hence, increased asset-market efficiency.

Proposition 5 *As the cost of information k decreases, the largest equilibrium changes as follows: The number of active investors I increases, the number of asset managers M increases, the asset-price efficiency increases, the asset-management fee f goes down, while the total fee revenue fI increases for large values of k and decreases for the other ones. If k is sufficiently small, all investors are active and the asset price is constrained efficient.*

The results of this proposition are illustrated in Figure 3. As seen in the figure, a lower information cost for asset managers moves their indifference curve out. This leads to a higher number of asset managers and active investors in equilibrium, which increases the asset-price efficiency. Naturally, less "complex" assets — assets with lower k — are priced more efficiently than more complex ones, and the more complex ones have fewer managers, higher fees, and fewer investors.

The cost of information is closely related to the precision of the information that is acquired, as we consider next.

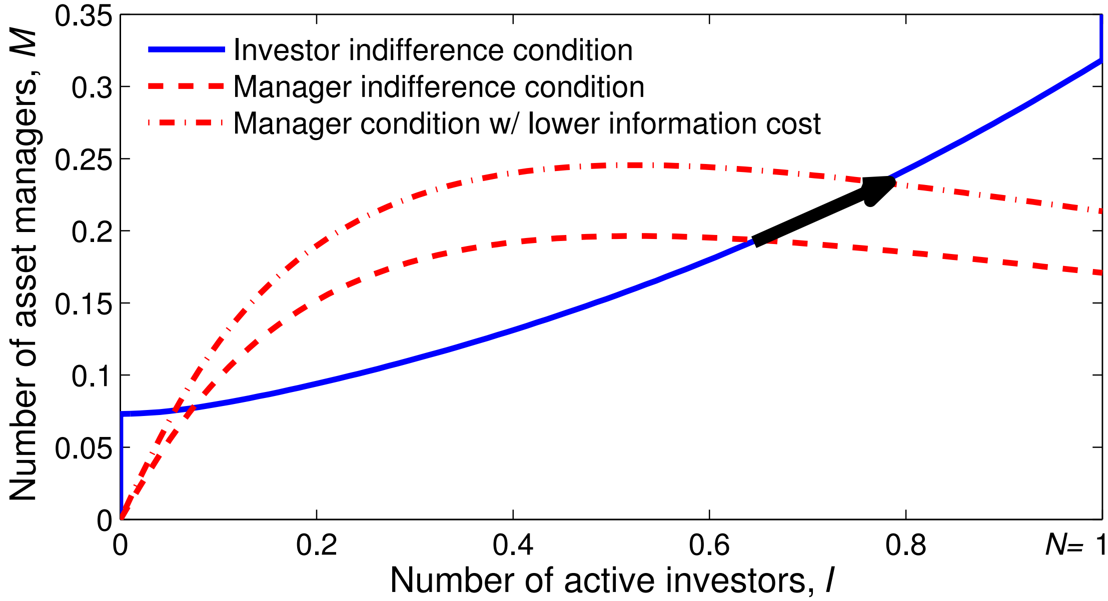

Figure 3: **Equilibrium effect of lower information acquisition costs.** The figure illustrates that lower costs of getting information about assets implies more active investors and more asset managers in equilibrium and, hence, increased asset-market efficiency.

Proposition 6 *As the amount of noise in the signal, σ_ε, decreases from a high enough value, the number of active investors I increases, the number of asset managers M increases, the asset-price efficiency decreases, and the asset management fee f goes up. All effects are reversed if σ_ε is low.*

We see that the effect of a more precise signal is similar to that of a cheaper signal as long as the noise in the signal is sufficiently large. However, as the noise in the signal approaches zero, the informed investors face a vanishing risk and trade increasingly aggressively and, eventually, these results are reversed as fewer and fewer investors enter.

As a final comparative static, we consider the importance of fundamental asset risk and noise trader risk.

Proposition 7 *An increase in the fundamental volatility σ_v or in the noise-trading volatility σ_q leads to more active investors I, more asset managers M, and higher total fee revenue fI. The effect on the efficiency of asset prices and the asset-management fee f is ambiguous.*

The same results obtain with a proportional increase in $(\sigma_v, \sigma_\varepsilon)$ *or in all risks* $(\sigma_v, \sigma_\varepsilon, \sigma_q)$.

An increase in risk increases the disadvantage of investing uninformed, which attracts more investors and more managers to service them. Interestingly, the asset-market efficiency may increase or decrease. For instance, if the search cost depends only on the number of investors searching, then new investor entry will mitigate the disadvantage of being uninformed only partially — so as to justify the higher search cost. On the other hand, if it depends only on the number of managers, then the higher number of managers decreases the search cost and investors enter until the market efficiency exceeds the original level.

4 Good Managers, Bad Managers, and Noise Allocators

In the benchmark model we have considered so far, investors incur a cost c to find an asset manager and ascertain her quality (i.e., the search and due diligence processes are modeled as a single combined effort). We now consider a model in which investors' efforts to search and perform due diligence are separated, and consider managers that differ in their quality. This leads us to consider a richer "double-decker" model in which there are two levels of asymmetric information, regarding both assets and asset managers, and to consider the role of noise allocators.

4.1 Model: Information about Assets and Asset Managers

The structure of the asset market is the same as in the basic model. Also as before, an endogenous number of investors I search asset managers M at a cost c. New to the extended model, asset managers can be of two types: a fraction π_g are good (g), while the rest $1 - \pi_g$ are bad (b). Good managers acquire information about the asset (as in the basic model), while bad ones fail to do so, providing investors with an expected utility equal to that of an uninformed investor U_u before fees.

Investors, who have already paid a search cost c, can elect to pay a due-diligence cost C to learn whether the manager is good or bad. The number of investors who perform due diligence is denoted by $I^i \leq I$. The $I - I^i$ remaining investors rely instead on observing the manager's assets under management, but these assets are not fully revealing due to the presence of noise allocators.

Finally, noise allocators invest with all managers they meet as long as they receive the "normal" equilibrium fee. The number of noise allocators who approach a given manager is given by the cumulative distribution function F^M given by $F^M(x) = F(xM)$. In other words, we consider a general noise-trader distribution F, and the total number of noise allocators is independent of the number of asset managers M. In order to simplify some technical arguments, we require that F has a corresponding log-concave probability density function ϕ (a relatively standard regularity condition).

When an asset manager decides whether or not to pay the cost k to set up her operation and create an "investment process" she does not know whether she is good or bad. Said differently, she knows that it is costly to set up her investment process, that is, her method of collecting information, but she does not know whether her process will be successful in generating information about the asset. Further, she does not know the number of noise allocators that will approach her. To simplify computations, we assume that the managers are risk neutral. For the purposes of bargaining, we wish to give all agents the same utilities over certain payoffs. We consequently specify the manager's utility over final wealth $\tilde{W}$ as $-e^{-\gamma E[\tilde{W}]}$.

The asset management fee f is determined through bargaining between each investor and manager. To avoid the complications of bargaining under asymmetric information, we assume that the fee is determined before the investor decides whether to do due diligence and before the manager finds out whether her information collection was successful.

Subsequently, the investors decide whether to pay the fee and stay with the manager (rather than to invest on their own as uninformed) conditioning on the outcome of their due diligence if they engaged in any. Investors who didn't engage in due diligence can base their decision on the realized total number of investors in the fund, which we denote as the "assets under management" (AUM).

The timing of the model is therefore: (i) investors and managers decide whether to enter; (ii) investor meets manager; (iii) fee is determined; (iv) investor decides whether to do due diligence about the manager; (v) investor decides whether to invest in the fund, conditional on due diligence or equilibrium AUM; (vi) asset markets clear.

The definition of the equilibrium is standard. The one remark we would like to make concerns the investors' ability to condition on equilibrium AUM when deciding whether to invest; differently phrased, their ability to make investment plans contingent on the AUM. This assumption is the equivalent of conditioning on price in rational-expectation models.

4.2 Equilibrium and its Properties

To solve the model, we proceed via backward induction, starting with the investor's decision of whether to stay with the manager, taking the asset-market outcome as given. We relegate the details of the derivation of the equations characterizing the equilibrium to the appendix, and instead here describe its nature and discuss the results.

In equilibrium, the probability of a manager's type being good conditional on the AUM increases in the AUM. There consequently exists a cutoff level for the AUM above which investors benefit from staying in the fund, while the opposite holds for AUM below the cutoff. The cutoff, naturally, depends on the fee already bargained and the utility gain a good manager can provide, which is unaffected by the outcome for one manager. It also depends on the number of agents who are informed about the manager, introducing a fixed-point element. Given the cutoff and fee, the expected utility of an investor who does not do due diligence is computed, and can be compared to that of an investor who does do due diligence, thus determining the equilibrium number of investors who do it.

At the bargaining stage, both parties can estimate their utility gains from agreeing on a particular fee, taking into account its effect on due diligence, participation in the fund, and inference, as appropriate. They then settle on an outcome that is beneficial for both of them.

The asset-market equilibrium, and in particular Λ, for a given I, is determined by the level of I^i and the cutoff AUM level, which give the number of agents investing with good managers and the number investing with bad managers. The price informativeness Λ is the result of a fixed point, taking I and M as given.

Finally, both prospective managers and investors take the expected utilities obtained at the bargaining stage as given, and choose to enter if they compare favorably with their respective entry costs. In fact, given the combination of bargaining solution and asset-market equilibrium, similar loci to the ones depicted in Figure 1 obtain, the intersection of which represent general equilibria.

Proposition 8 *There exists an equilibrium in which the investors who perform due diligence invest only with good managers, and a number $\bar{A}$ such that other investors invest if and only if the total number of investors in the fund is above $\bar{A}$. As a result, good managers service more investors than bad managers, on average.*

The implications for performance are more nuanced than those in the basic model, as shown in the following proposition.

Proposition 9 *The following holds in equilibrium.*

(i) Good managers outperform passive investing both before and after fees. Bad managers underperform passive investing after fees.

(ii) Investors who invest based on due diligence expect to outperform before and after the asset-management fee. Investors who don't do due diligence, but invest based on an optimal use of public information also expect to outperform before and after the fee, but by less.

(iii) Noise allocators can outperform or underperform after fees, depending on the parameters. The same holds for the average of all investors or, equivalently, the average manager.

The following proposition characterizes situations when we expect the average manger to, respectively, outperform or underperform.

Proposition 10

(i) If the number of noise allocators is large enough and π_g and C are small enough, then the average manager underperforms after fees.

(ii) If the number of noise allocators is small enough, then the average manager outperforms after fees.

The proposition shows that a large number of noise allocators combined with sufficiently many bad managers (low π_g) leads to poor performance for the average manager (or, equivalently, the average investor). The result in part (i) also relies on C being small, which ensures that most active investors find it optimal to perform due diligence.

In contrast, a low number of noise traders leads to outperformance by the asset-weighted average manager. This holds even when there are many bad managers (low π_g) because rational investors can avoid these to an optimal extent through their due diligence. Naturally, given that the basic model features no noise allocators, part (i) of Proposition 10 contradicts Proposition 3, while part (ii) is in line with Proposition 3.

5 Empirical Implications

In this section, we lay out some of our model's testable empirical implications for asset markets, asset management, and their interaction. The model has implications both for the cross-section of assets and asset managers — e.g., in cross-country comparisons or across different asset classes or market segments — and the time series, e.g., studying secular trends as access to information changes. We consider both empirical predictions that correspond to existing evidence as well as new predictions that are yet to be tested.

A. Search frictions and asset managers. Search frictions in the asset management fund industry are documented by Sirri and Tufano (1998), Jain and Wu (2000), and

Hortaçsu and Syverson (2004) and, consistent with our model, proxies for lower search costs are associated with more investors. In a cross-country study, Khorana, Servaes, and Tufano (2008) find that mutual fund "fees are lower in wealthier countries with more educated populations," which may be related to lower search frictions for well educated investors. If search costs have gone down over time due to improvements in information technology (e.g., the internet), our model predicts an increasing allocation to managers and a downward trend in fees, somewhat consistent with the result of French (2008), although the fees may not have come down as much as our model might predict.

B. Average performance of asset managers: mutual funds and hedge funds. Consistent with the evidence cited in the introduction, our model implies that some managers can outperform passive investing before and after fees while other managers underperform after fees. The performance of the average manager depends on the role of noise allocators.

C. Predicting manager performance. In our model, investors who collect information about asset managers can help predict their performance. Even without such information about managers, investors can get a noisier prediction about manager performance based on fund flows. Indeed, good managers receive more funds than bad managers, so fund size (or flows) should predict performance, as found empirically by Berk and Binsbergen (2012). However, as emphasized by Berk and Green (2004), this effect may by mitigated by decreasing returns to scale in asset management, e.g., due to larger transaction costs for large managers.

D. Asset management fees. We predict that asset-management fees should be larger for managers of more inefficient assets and in more inefficient asset management markets. For instance, if search costs for managers are large, this leads to less active investing and higher management fees. Note that the higher management fee in this example is not driven by higher information costs for managers, but, rather, by the equilibrium dynamics between the markets for the asset and asset management. This may help explain why hedge funds have historically charged higher fees than mutual funds. Also, markets for more complex assets that are costly to study should be more inefficient and have higher management fees. This can help explain why equity funds tend to have higher fees than bond funds and why global equity funds have higher fees than domestic ones.

E. Efficiently inefficient markets. While the efficient market hypothesis is a powerful theory, it can nevertheless be difficult to test because of the so-called "joint hypothesis" problem. However, the existence of deviations from the Law of One Price (securities with the same cash flows that trade at different prices) is a clear rejection of fully efficient asset markets. The theory of efficiently inefficient markets is *not* the entire complement to fully efficient markets, but, rather, it should be viewed as an equally well-defined null hypothesis. Efficiently inefficient markets means that investors should be indifferent between passive investing and searching for asset managers, where the latter should deliver an expected outperformance balanced by asset management fees and search costs, consistent with the findings of Gerakos, Linnainmaa, and Morse (2014) for professional asset managers. The average manager might not deliver this outperformance due to noise allocators, but investors should be able to collect sufficient information to achieve an outperformance that compensates their costs in an efficiently inefficient market.

F. Anomalies. In an efficiently inefficient market, anomalies are more likely to arise the more resources a manager needs to trade against them (higher k) and the more difficult it is for investors to build trust in such managers (higher c).

G. Fraction of active investors: the size of the asset management industry. Our model also has several implications for the size of the asset management industry. The asset management industry grows when investors search cost diminish or when asset managers' information costs go down, leading to more efficient asset markets, consistent with the evidence of Pastor, Stambaugh, and Taylor (2014). Other important models that speak to the size of asset management industry include Berk and Green (2004), Garcia and Vanden (2009), and Pastor and Stambaugh (2012).

H. Number and size of asset managers. When investors' search costs go down, our model predicts that the number of managers will fall, but the remaining managers will be larger (in fact so much larger that the total size of the asset management industry grows as mentioned above). Such consolidation of the asset management industry is discussed in the press, but we are not aware of a direct test of this model prediction.

I. Private equity and venture capital. We can also think our model as a description of the markets for private equity and venture capital, where investors search for asset managers who in turn examine private (and public) companies. Our model's predictions may help explain the puzzling performance persistence documented by Kaplan and Schoar (2005), given the complications of finding a good manager (Korteweg and Sorensen (2014)).

Further, consistent with our double-decker equilibrium in Proposition 8, private equity funds often raise money from investors *contingent* on reaching a total amount of at least, say, 1 billion dollars. According to our model, this contractual feature could help some investors gain confidence that they only invest if the total assets are sufficient to signal the manager's quality; alternatively, this feature might serve the role of ensuring that the manager only needs to perform her duties if she has sufficient capital – testing the relative merits of these explanations would be interesting.

J. Investment consulting firms, investment advisors, and funds of funds. Lastly, investors' search frictions in our model are consistent with the demand for investment consulting services and funds of funds who may essentially help lower these frictions. Hence, these industries could be analyzed in light of our model.

6 Conclusion

This paper describes the joint dynamics of the markets for assets and asset management. We show that asset managers can increase asset price efficiency by letting investors essentially share information costs, but their ability to do so is limited by the frictions in the asset-management industry. Therefore, the efficiency of asset markets is fundamentally connected to the efficiency of the asset management market.

Our model illuminates the determinants of asset price efficiency, the fees in the asset management industry, the performance of asset managers before and after fees, the number of asset managers and their size, the drivers of asset management consolidation, the fraction of assets allocated to good managers, the effects of noise allocators, and the overall costs of intermediation to all investors. These model predictions help explain a number of existing empirical facts and lay the ground for further tests.

A Proofs

Proof of Proposition 1. This result is effectively provided, and proved, in Grossman and Stiglitz (1980). ■

The following proofs make use of the following result.

Lemma 1 *The positive function of I given by $-I\log(\Lambda)$ increases up to a point $\bar{I}$ and then decreases, converging to zero.*

Proof of Lemma 1. The function of interest is a constant multiple of

$$h(x) := x\log\left(\frac{a+x^2}{b+x^2}\right), \tag{A.1}$$

with $a > b > 0$. Its derivative equals

$$\begin{aligned} h'(x) &= \log\left(\frac{a+x^2}{b+x^2}\right) + x\frac{b+x^2}{a+x^2}\frac{2x(b+x^2)-2x(a+x^2)}{(b+x^2)^2} \\ &= \log\left(\frac{a+x^2}{b+x^2}\right) - \frac{2(a-b)x^2}{(a+x^2)\,(b+x^2)}. \end{aligned}$$

For $x = 0$, the first term is clearly higher: $h'(0) > 0$. For $x \to \infty$, the second is larger, so that $\lim h'(x) < 0$. Finally, letting $y = x^2$ and differentiating $h'(y)$ with respect to y one sees that $h''(y) = 0$ when y satisfies the quadratic

$$y^2 - (a+b)y - 3ab = 0, \tag{A.2}$$

which clearly has a root of each sign. Thus, since $y = x^2$ is always positive, $h''(x)$ changes sign only once. Given that $h'(x)$ starts positive and ends negative and its derivative changes sign only once, we see that h' itself must change sign exactly once. This result means that h is hump-shaped. Finally, we can apply L'Hôpital's rule to $h(x) = \log\left(\frac{a+x^2}{b+x^2}\right)/(1/x)$ to conclude that $\lim_{x\to\infty} h(x) = 0$. ■

Proof of Proposition 2. Let $\mathcal{M}$ denote the function that assigns to a value of I the corresponding value M at which managers are indifferent between entering and being passive — the red line in Figure 1. Likewise, let $\mathcal{I}$ give the equilibrium number of investors as a function of M; note that the blue line in the figure graphs the function $\mathcal{I}^{-1} : I \mapsto M$, which is clearly increasing as seen from (18).

By virtue of the lemma, we know that the function $\mathcal{M}$ increases in I and then decreases to approach zero as I increases without bound. It follows that the graph of $\mathcal{M}$ crosses that of $\mathcal{I}^{-1}$ the last time from above, for large enough I.

As I increases from zero, generically, $\mathcal{M}(I)$ crosses $\mathcal{I}^{-1}(I)$ an even number of times at points $I > 0$. If the maximal value N occurs after an even number of such values, then the largest among them is an equilibrium, and it is stable, since the crossing by $\mathcal{M}(I)$ is from above. The value $I = N$ is not an equilibrium. On the other hand, if $\mathcal{M}(N) > I^{-1}(N)$, then $I = N$ and $M = \mathcal{M}(N)$ is an equilibrium, and it is stable. ■

Proof of Proposition 3. This proposition follows from the derivations in the text. ■

Proof of Proposition 4. (i) One can offer a graphical argument. As c decreases, the blue line $\mathcal{M}(I)$ shifts to the right. Given that the blue line crosses the red one from below, the investment I unambiguously increases, while the number of managers M increases if and only if the red curve, thus $\mathcal{I}^{-1}(I)$, increases at the equilibrium point. The increase in I translates into a higher Λ because of (12) and a lower f because of (17). Further, from (15) we see that $fI = Mk$ so that the total fee revenue behaves is proportional to M, thus unimodal.

(ii) Remember first that c_j increases in M and decreases in I. For any value $\hat{I}$, $c_j(0, \hat{I})$ becomes arbitrarily small with j, so that in equilibrium I exceeds $\hat{I}$ for j high enough — otherwise, Λ would be too low given the upper bound on c_j. Thus I tends to infinity, Λ tends to one, and f decreases to zero. In addition, from Lemma 1 we know that $M = \frac{-I\log(\Lambda)}{2\gamma k} \to 0$ and so $fI = Mk \to 0$. ■

Proof of Proposition 5. A decrease in k causes the red line $\mathcal{M}(I)$ to shift upwards — i.e., M increases for every level of I. ■

Proof of Proposition 6. We start by rewriting (18)–(19) abstractly as

$$0 = \Lambda^{\frac{1}{2}} - e^{-\gamma c(M,I)} \equiv g^I(I, M) = g^I(\mathcal{I}(M), M) \tag{A.3}$$

$$0 = \Lambda^{\frac{1}{2}} - e^{-\gamma k \frac{M}{I}} \equiv g^M(I, M) = g^M(I, \mathcal{M}(I)), \tag{A.4}$$

and note that the fact that $\mathcal{M}$ crosses $\mathcal{I}^{-1}$ from above means that $\mathcal{M}'(I) < (\mathcal{I}^{-1})'(I)$, which, using subscripts to indicate partial derivatives, translates into

$$-\frac{g_I^M}{g_M^M} < -\frac{g_I^I}{g_M^I}, \tag{A.5}$$

which is equivalent to

$$g_M^I g_I^M < g_I^I g_M^M \tag{A.6}$$

given $g_M^I < 0$ and $g_M^M > 0$.

We continue by noting that $\frac{\partial \Lambda}{\partial \sigma_\varepsilon^2} < 0$ if and only if σ_ε^2 is low enough. The dependence of I and M on σ_ε^2 is given as a solution to

$$\begin{pmatrix} g_I^M & g_M^M \\ g_I^I & g_M^I \end{pmatrix} \begin{pmatrix} I_{\sigma_\varepsilon^2} \\ M_{\sigma_\varepsilon^2} \end{pmatrix} = \left(-\frac{\partial \Lambda^{\frac{1}{2}}}{\partial \sigma_\varepsilon^2} \right) \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \tag{A.7}$$

which is given by

$$\begin{pmatrix} I_{\sigma_\varepsilon^2} \\ M_{\sigma_\varepsilon^2} \end{pmatrix} = \frac{1}{g_M^I g_I^M - g_I^I g_M^M} \begin{pmatrix} g_M^I - g_M^M \\ g_I^M - g_I^I \end{pmatrix} \left(-\frac{\partial \Lambda^{\frac{1}{2}}}{\partial \sigma_\varepsilon^2} \right). \tag{A.8}$$

We note that $g_M^I - g_M^M < 0$ and $g_I^M - g_I^I < 0$, while the determinant $g_M^I g_I^M - g_I^I g_M^M$ is negative because the function $\mathcal{M}(I)$ crosses $\mathcal{I}^{-1}(I)$ from above. Thus, at an interior stable equilibrium both I and M increase as σ_ε^2 increases if σ_ε^2 is small enough, and decrease otherwise. At a corner equilibrium, given by $I = N < \mathcal{M}^{-1}(M)$, when σ_ε^2 is small enough M increases with σ_ε^2 — the curve $\mathcal{M}$ shifts up — and vice-versa. (I is constant.) ■

Proof of Proposition 7. Letting x denote either σ_v^2 or σ_q^2, we note that $\frac{\partial \Lambda}{\partial x} < 0$ — graphically, this translates in an upward shift in the red curve. The rest of the argument mimics the one in Proposition 6 to obtain that M and I increase. The total amount of fees fI increases with M.

The effect on the efficiency of the asset market, on the other hand, is not determined. To see this clearly, differentiate (18) to get

$$\frac{d\Lambda^{\frac{1}{2}}}{dx} = -\gamma \Lambda^{\frac{1}{2}} \left(c_M M_x + c_I I_x \right) \tag{A.9}$$

and remember that $c_M \leq 0$ and $c_I \geq 0$. Since $M_x > 0$ and $I_x > 0$, by setting one of the partial derivatives c_M and c_I to zero and keeping the other non-zero, the sign of $\frac{d\Lambda^{\frac{1}{2}}}{dx}$ can be made either positive or negative. Consequently the efficiency may increase as well as decrease, a conclusion that translates to the fee f.

Exactly the same argument works when increasing $(\sigma_v, \sigma_\varepsilon)$ or $(\sigma_v, \sigma_\varepsilon, \sigma_q)$ proportionally. ■

Proof of Proposition 8. To be added. ■

Proof of Proposition 9. To be added. ■

Proof of Proposition 10. To be added. ■

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