# Phasing out the GSEs *

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**Abstract**

We develop a new model of the mortgage market where both borrowers and lenders can default. Risk tolerant savers (risk takers) act as intermediaries between risk averse depositors and impatient borrowers. The government plays a crucial role by providing both mortgage guarantees and deposit insurance. Underpriced government mortgage guarantees lead to risky mortgage origination and excessive financial sector leverage. Mortgage crises frequently turn into financial crises and government bailouts due to the fragility of the intermediaries' balance sheets. Increasing the price of the mortgage guarantee crowds in the private sector, reduces financial fragility, leads to less and safer mortgage lending, lowers house prices, and raises mortgage and risk-free interest rates. Due to a more robust financial sector, consumption smoothing improves and aggregate welfare increases. While borrowers only incur a small welfare loss, both types of savers are substantially better off, with depositors benefiting the most.

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# 1 Introduction

Government and quasi-government entities dominate mortgage finance in the U.S. Over the past five years, the government-sponsored enterprises, Fannie Mae and Freddie Mac, and the Federal Housing Administration have stood behind 80% of the newly originated mortgages.[1] Ever since the collapse of the GSEs in September of 2008 and the conservatorship which socialized housing finance, there have been many proposals to bring back private capital into this market.[2] The main idea of these policy proposals is to dramatically reduce the size and scope of the government guarantee on standard (conforming) mortgages. Because this reform would turn a largely public into a largely private housing finance market, there is both uncertainty and concern about its impact on house prices, the availability of mortgage finance, financial sector stability, and ultimately welfare.[3]

Understanding the economic impact of wholesale mortgage finance reform requires a general equilibrium model. Such a model must recognize the important role that residential real estate and mortgage markets have come to play in the financial systems and the macro-economy of rich countries (Jorda, Schularick, and Taylor (2014)). They also must recognize the large footprint of the government in this space. This paper proposes such a general equilibrium model of the housing market where the interaction of the financial sector and the government is central.

In the benchmark calibration of the model, the government enjoys a dominant position in the provision of mortgage default insurance. The financial sector issues mortgages to borrowers and decides for how many of those mortgages to buy the government guarantee. The model ascribes the dominance of the guaranteed mortgage market to the low insurance premium

[1]Currently, of the $9.85 trillion stock of residential mortgages, 57% are Agency Mortgage-backed Securities guaranteed by Fannie Mae, Freddie Mac, and Ginnie Mae. Private-label mortgage backed securities make up less than 8% of the stock. The rest is unsecuritized first liens held by the GSEs and the banking sector (28%) and second liens (7%). Acharya, Richardson, Van Nieuwerburgh, and White (2011) provide an in-depth discussion of the history of the GSEs, their growth, and collapse.

[2]The Obama Administration released a first report along these lines in February 2011. The bills proposed -but not passed- by Corker-Warner in 2013 and Johnson-Crapo in 2014 provide the most recent attempts at legislative reform.

[3]The financial and real estate industries, the Mortgage Bankers Association, and consumer advocate groups have all vehemently argued to keep a form of government guarantee in place, in the form of a public mortgage guarantor that would succeed Fannie and Freddie, for fear that there may "not be enough private capital" in the mortgage market in a fully private system, jeopardizing the stable provision of mortgage credit for a broad cross-section of households. On the other side of the spectrum, the Congressional Budget Office recently argued that housing finance reform would have minimal impact (CBO 2014).

(guarantee fee or g-fee) that the government charges banks for that insurance, as well as to the minuscule amount of regulatory capital that banks must hold against guaranteed mortgage bonds. As in the real world, mortgages are long-term, prepayable, and non-recourse. A novel model ingredient is that the financial sector enjoys a government bailout guarantee, which is equivalent to deposit insurance in our setting. Deposit insurance is an important feature of any mortgage finance system, public or private, that the literature on mortgage finance has not considered hitherto.[4]

One of the interesting features of the model is the interaction between mortgage guarantees, which reduce the risk of banks' assets, and deposit insurance, which reduces the risk of their liabilities. As relatively risk tolerant agents, bankers in the model desire a high return-high risk portfolio. But taking advantage of the underpriced mortgage guarantee lowers the risk of banks' assets. This prompts banks to increase leverage in order to attain their desired risk-reward ratio. The favorable regulatory capital treatment of guaranteed mortgage bonds enables such high leverage. Deposit insurance further propels leverage, since it makes banks' lenders, the depositors, less sensitive to the risk of a banking collapse. A second way in which banks increase risk is through their mortgage origination decisions. They grow the size of the mortgage portfolio and increase its riskiness, as witnessed by higher mortgage debt-to-income and loan-to-value ratios. Because of the dual guarantees, equilibrium mortgage rates are low making borrowers willing to take on higher mortgage debt. In sum, the government's underpriced mortgage guarantee distorts financial sector leverage and leads to a larger financial sector and lower underwriting standards as banks pursue their desired risk-reward ratio. The financial sector bailout guarantee, or deposit insurance, amplifies these effects because it immunizes depositors from the elevated risk-taking by the bankers and removes their incentives to discipline bankers.

Ex-post, a riskier mortgage portfolio produces higher mortgage default rates and losses. These losses produce deadweight costs of foreclosures, a first source of welfare losses. Given banks' low net worth cushion and high leverage, housing crises often turn into financial crises, defined as bank insolvencies. Thus, the economy with underpriced mortgage guarantees results

[4]Deposit insurance can be thought of more broadly as encompassing implicit government guarantees for short-term financial sector liabilities such as money market funds, asset-backed commercial paper, repurchase agreements, etc. Indeed, the government stepped in to rescue those markets in the Fall of 2008 and in the spring of 2009.

in financial sector fragility. They also result in higher house price volatility. However, the government absorbs the losses from housing and financial crises by issuing debt to pay for the payoffs on mortgage default insurance and the financial sector bailouts. The government's ability to issue debt in bad times allows society to spread out the fiscal costs of mortgage defaults over time and to limit the negative effects of the crisis on consumption smoothing. It also partially protects banks' balance sheets from the mortgage losses, enabling them to continue to issue new mortgages. A key finding is that the benefit of having the government smooth out default shocks over time is smaller than the cost of increased risk taking by the private sector. The equilibrium consumption smoothing in the economy with GSEs is poor, a second source of welfare loss. Intuitively, equilibrium risk sharing is poor because a fragile intermediation sector hampers the optimal flow of funds between borrowers and savers.

Capturing the spirit of the proposed mortgage finance reforms, our main policy experiment is to start from the "status quo" and increase the price of government mortgage insurance, thereby "crowding in" the private sector. Naturally, we find that a higher guarantee fee shifts the financing of mortgages from guaranteed to private mortgage bonds. This portfolio shift increases the riskiness of bank assets, making it unnecessary for banks to go to their maximum allowed leverage ratio or to increase the size and riskiness of their mortgage portfolio. Intermediary net worth is higher on average so that banks have more "skin in the game" and the overall bank balance sheet is smaller. Ex-post, mortgage default and loss rates are lower in the private sector economy. Fewer housing crises turn into financial crises because of the sturdier bank balance sheets. Because of sufficient intermediary capital, banks are able to continue lending even during housing crises so that the provision of mortgage credit is about equally stable in the model with and without government guarantees. This result dispels the notion that the GSEs are needed to guarantee stable access to mortgage finance. The key insight is that abolition of mortgage guarantees leads banks to take less risk, moving them farther from their leverage constraints. That improves the economy's ability to allocate resources to the highest marginal utility user and thus consumption smoothing.

At g-fees that are high enough to crowd out the government completely, we find lower house prices by 9%, a smaller mortgage market by 16%, and a less levered banking sector by 9.5%. Mortgages are safer: debt-to-income ratios are 9% lower. The upshot in the "private market solution" is that the financial system is less fragile: the incidence of mortgage defaults and realized mortgage losses are only half as large and bank defaults (financial sector bailouts) are completely eliminated. The overall effect of phasing out the GSEs is that social welfare would increase the population-weighted average value function by 4.9%. Borrowers welfare is slightly lower (by 0.6%), while both types of saver households gain. Borrowers benefit from the improved risk sharing in the economy without guarantees, but they loose their mortgage subsidy, face higher mortgage rates, and tighter lending standards. House prices lower the wealth of existing homeowners but make it easier for new homeowners to buy a house. Depositors gain the most (10%), and risk-takers (bankers) also benefit substantially (4.4%). While abolishing the GSEs improves overall welfare, it increases wealth inequality.

At intermediate levels for the g-fee, we observe that the government guarantee is only taken up in bad times. This dovetails with the "mortgage insurer of last resort" option in the Obama Administration proposal which envisions crowding-in the government only in crises.

We study the effect of raising the regulatory capital requirement on guaranteed bonds to the same level as that on non-guaranteed bonds. This exercise help us understand how much of the financial sector fragility can be undone with higher capital requirements (macroprudential policy). We find that higher capital requirements eliminate most of the excessive risk taking by banks, even in the continued presence of underpriced mortgage guarantees.

We plan to explore several extensions in future drafts. First, we plan to use the model to quantitatively evaluate the 2014 Johnson-Crapo bill which proposes to put 10% private capital in front of a catastrophic government guarantee. This means that the first 10% of losses in the event of a mortgage default would be born by the private sector. The government would step in only when losses exceed that threshold. Since the financial sector encompasses the private mortgage insurance industry in our model, this policy amounts to changing the capital requirements for guaranteed mortgages to 10%, and redefining the payoff of the guaranteed mortgage bond to reflect the catastrophic nature of the insurance. Second, since we model mortgage bonds as long-term assets, we can explore the effect of the typical length or duration of a mortgage. Many have argued that the 30-year fixed rate mortgage, a staple of American housing finance, would not be feasible absent a government guarantee. To assess this claim, we compare the welfare effects of the guarantee for different levels of the duration of the mortgage.

The presence of guarantees allows for banks to lay off the credit risk and only hold duration risk. The government absorbs -or at least temporally transforms- mortgage credit risk. Hence, the pure term premium for long-term fixed rate mortgages may be substantially lower, with guarantees, affecting mortgage rates and house price levels. This has implications for the design of mortgage finance systems: adjustable rate or other short-duration mortgages may provide better outcomes in economies without government guarantees.

**Related Literature** Our paper contributes to several strands of the literature on housing, finance, and macro-economics. Unlike recent quantitative work that explores the causes and consequences of the housing boom,[5] this paper focuses on the current and future state of the housing finance system and the role the government plays in this system. It shares with these models a focus on quantitative implications and on general equilibrium considerations. In particular, house prices and interest rates are determined in equilibrium rather than exogenously specified. We simplify by working in an endowment economy with a constant housing stock.[6]

Like another strand of the literature, our model features borrowers defaulting optimally on their mortgages.[7] Unlike most of that literature, our lenders are not risk-neutral but risk averse. A default risk premium is priced into the mortgage contract which is time-varying and depends on the covariance of the risk taker's intertemporal marginal rate of substitution with the payoff on the mortgage loan. We assume that lenders impose maximum LTV ratios on borrowers, chosen to match borrower mortgage debt/income ratios and default rates in normal and housing crisis times. Unlike much of the literature, our mortgage contract is a long-term contract. This is important because of the centrality of the 30-year fixed-rate mortgage in the debate on U.S. housing finance reform. We calibrate our mortgage contract to exhibit the same amount of interest rate risk as the outstanding pool of agency mortgage-backed securities.

The biggest difference between our housing finance model and the literature is our focus on

[5]Examples include Ortalo-Magné and Rady (2006), Kiyotaki, Michaelides, and Nikolov (2011), Favilukis, Ludvigson, and Van Nieuwerburgh (2013), Landvoigt, Piazzesi, and Schneider (2015), Landvoigt (2012), Chu (2014). See Davis and Van Nieuwerburgh (2015) for a recent review of the literature.

[6]The role of housing supply and construction are studied in Favilukis, Ludvigson, and Van Nieuwerburgh (2013), Chatterjee and Eyigungor (2009), Hedlund (2014), and Boldrin, Garriga, Peralta-Alva, and Sanchez (2013).

[7]Recent examples of equilibrium models with default are Corbae and Quintin (2014), Garriga and Schlagenhauf (2009), Chatterjee and Eyigungor (2009), Jeske, Krueger, and Mitman (2013), Landvoigt (2012), Arslan, Guler, and Taskin (2013), and Hedlund (2014).

the financial sector and the role the government plays. A recent literature in asset pricing has emphasized the central role of financial intermediaries in the crisis.[8] Usually, intermediaries have access to a different technology from other agents. In our model, as in Drechsler, Savov, and Schnabl (2014), intermediaries arise endogenously from differences in risk aversion instead. The least risk averse savers choose to issue short-term debt ("deposits") and purchases risky long-term mortgage loans. Like in the part of the literature that emphasizes debt constraints, our intermediaries face borrowing or margin constraints which link the amount of short-term liabilities they can issue to the collateral value of their assets. The net worth of the financial sector is the key state variable which governs risk sharing and asset prices in those models. In our model, intermediary wealth is also an important state variable, but it is not the only one. The wealth of the depositors, the wealth of the borrowers, and the outstanding amount of government debt all affect the equilibrium allocation and prices.

Unlike most of this literature, we explicitly model the intermediary's decision to default. When intermediary net worth threatens to go negative, intermediaries can choose to offload all assets and liabilities onto the government. The government bailout option is equivalent to deposit insurance in the model. As emphasized above, the interaction of the mortgage guarantee and this bailout guarantee is one of the most interesting and novel aspects of our analysis. By studying the role of the financial sector in the provision of mortgages we capture the stylized fact pointed out by Jorda, Schularick, and Taylor (2014) that over a 5-year window run ups in mortgage lending and run ups in house prices raise the likelihood of a subsequent financial crises. Mortgage and house price booms are predictive of future financial crises, and this effect has also become much more dramatic since WW2. We ask how government intervention in the form of asset (mortgage) guarantees or liability (deposit) guarantees affects this nexus.

Our result that the economy without the government guarantee for mortgages features a better capitalized financial sector, a more stable financial system, and higher welfare echoes the arguments made in the literature on capital regulation of financial institutions.[9] Our work contributes a quantitative general equilibrium model to that discussion, with an emphasis on

[8]Recent examples include Brunnermeier and Sannikov (2012), He and Krishnamurty (2013), Gârleanu and Pedersen (2011), Adrian and Boyarchenko (2012), and Maggiori (2013). Brunnermeier, Eisenbach, and Sannikov (2013) provides a review of this literature.

[9]See for example, Kashyap, R., and Stein (2008), Hart and Zingales (2011), Admati and Hellwig (2013), and Admati, DeMarzo, Hellwig, and Pfleiderer (2013).

the role of mortgages and mortgage guarantees. The framework should be useful to investigate the quantitative implications of deposit insurance and higher capital requirements as well. See Begenau (2015) for a similar approach in this literature.

Finally, our paper contributes to the literature that quantifies the effect of government policies in the housing market. Most work focuses on studying the effect of abolishing the mortgage interest rate tax deductibility and the tax exemption of imputed rental income of owner-occupied housing.[10] When house and rental prices are determined endogenously, the policy changes lower house prices and price-rent ratios and cause an increase in home ownership rates. These policy changes redistribute consumption from the rich to the poor and increase welfare. Studying the GSE subsidies, Jeske, Krueger, and Mitman (2013) reach a similar conclusion regarding welfare. Our work differs from Jeske, Krueger, and Mitman (2013) in its emphasis on the role of the financial sector and the interaction between the mortgage guarantee and the bailout guarantee. We identify an effect that leads to an increase in inequality after the abolition of the mortgage subsidy, while they emphasize the regressive nature of the subsidy and hence a reduction in inequality from the abolition. In terms of model setup, there are several more differences. Jeske et al. model mortgage guarantees as a tax-financed mortgage interest rate subsidy. In our model, the government sells mortgage insurance to the private sector. Second, our mortgages are long-term in nature while theirs are one-period mortgages. Third, our lenders are risk-averse, while theirs are risk-neutral. Fourth, our model features aggregate risk but no idiosyncratic income risk, while their model has no aggregate risk and emphasizes idiosyncratic risk. Fifth, their model has a simple construction sector and constant house prices, while our model has a fixed housing supply and endogenous house prices. Sixth, their model has home ownership choice while ours does not. Seventh, our government can issue debt while theirs has to balance the budget every period. We view the robustness in conclusions as reassuring news for policymakers.

[10] For example, Gervais (2002), Chambers, Garriga, and Schlagenhauf (2009), Floetotto, Kirker, and Stroebel (2012), and Sommer and Sullivan (2013).

# 2 The Model

## 2.1 Endowments, Preferences, Technology, Timing

**Endowments** The model is a two-good endowment economy with a non-housing and a housing Lucas tree. The fruit of the non-housing tree, *output* $Y_t$, grows and its growth rate is subject to aggregate shocks. The different households are endowed with a fixed and non-tradeable share of this tree. This endowment can be interpreted as labor income. The size of the housing tree (housing stock) grows at the same stochastic trend as output. The total quantity of housing shares is fixed and normalized to 1. The housing stock yields fruit (housing services) proportional to the stock.

**Preferences** The model features a government and three groups of households. Impatient households are borrowers (denoted by superscript B), while patient households are savers. There are two type of savers, differentiated by their risk aversion coefficient; we refer to the less risk averse savers as "risk takers" (denoted by superscript R) and the more risk averse as "depositors" (denoted by D). Thus, for the rate of impatience we assume that $\beta_R = \beta_D > \beta_B$, and for the coefficient of relative risk aversion we assume that $\sigma_R < \sigma_B \leq \sigma_D$. All agents have Epstein-Zin preferences over the joint consumption bundle which is a Cobb-Douglas aggregate of housing and non-housing consumption with aggregation parameter $\theta$.

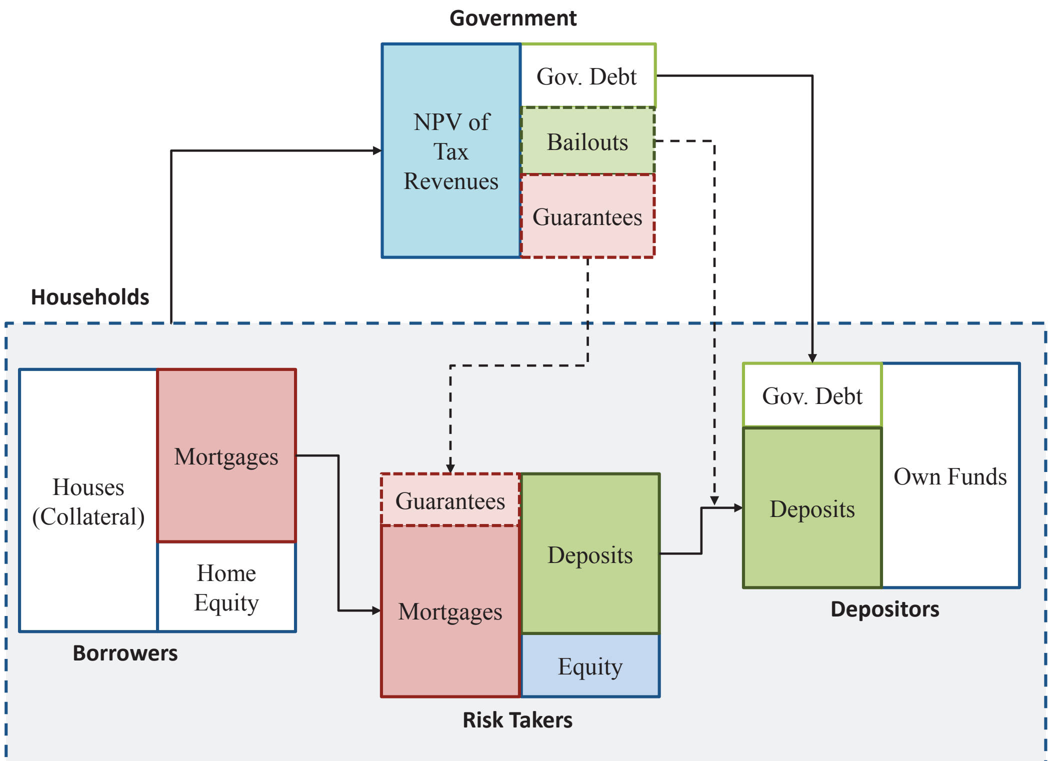
$$U_t^j = \left\{ (1-\beta_j) \left(u_t^j\right)^{1-1/\nu} + \beta_j \left( \mathrm{E}_t \left[ (U_{t+1}^j)^{1-\sigma_j} \right] \right)^{\frac{1-1/\nu}{1-\sigma_j}} \right\}^{\frac{1}{1-1/\nu}} \tag{1}$$

$$u_t^j = \left(C_t^j\right)^{1-\theta} \left(A_K K_{t-1}^j\right)^{\theta} \tag{2}$$

$C_j^t$ is numeraire non-housing consumption and the constant $A_K$ specifies the housing services from owning the housing stock, expressed in units of the numeraire. All agents share the same elasticity of intertemporal substitution $\nu$.

Figure 1 depicts the balance sheets of the different agents in the economy and the flows of funds between them.

Figure 1: Balance sheets of agents in model economy




**Technology** There are three assets in the economy. The first is a one-period short-term bond. The second is a mortgage bond, which aggregates the mortgage loans made to all borrower households. The third is mortgage insurance which the government sells to the private market. The guarantee turns a defaultable long-term mortgage bond into a default-free government-guaranteed mortgage bond.

Borrowers experience housing depreciation shocks and may choose to default on their mortgage. There is no recourse; savers and possibly the government (ultimately the tax payers) bear the loss depending on whether mortgage loans are held in the form of private or government-guaranteed mortgage bonds, respectively. A novel model ingredient is that risk takers may also choose to default and declare bankruptcy. Default wipes clean their negative wealth position with no further consequences; the losses are absorbed by the government in a "financial sector bailout."

**Timing** The timing of agents' decisions at the beginning of period $t$ is as follows:

1. Income shocks for all types of agents and housing depreciation shocks for borrower households are realized.

2. Risk takers (financial intermediaries) decide on a bankruptcy policy. In case of a bankruptcy, their financial wealth is set to zero and they incur a utility penalty. At the time of the decision, the magnitude of the penalty is unknown.[11] Savers know its probability distribution and maximize expected utility by specifying a binding decision rule for each possible realization of the penalty.[12]

3. Borrower households decide what fraction of mortgage debt to default on.

4. Risk takers' utility penalty shock is realized and they follow their bankruptcy decision rule from step 2. In case of bankruptcy, the government picks up the shortfall in repayments to debt holders (depositors).

5. Borrowers choose how much of the remaining mortgage balance to prepay (refinance). All agents solve their consumption and portfolio choice problems. Markets clear. All agents consume.

Each agent's problem depends on the wealth of others; the entire wealth distribution is a state variable. Each agent must forecast how that state variable evolves, including the bankruptcy decisions of borrowers and risk takers. We now describe each of the three types of household problems and the government problem in detail.

## 2.2 Borrower's Problem

There is a representative family of borrowers, consisting of a measure one of members. Each member receives the same stochastic labor income $Y_t^B$, chooses the same quantity of housing $k_{t-1}^B$ s.t. $\int_0^1 k_{t-1}^B di = K_{t-1}^B$, and the same quantity of outstanding mortgage bonds $a_t^B$ s.t. $\int_0^1 a_t^B di = A_t^B$. The mortgage is a long-term contract, modeled as a perpetuity. Bond coupon

[11]Introducing a random utility penalty is a technical assumption we make for tractability. It makes the value function differentiable and allows us to use our numerical methods which rely on this differentiability. This randomization assumption is common in labor market models (Hansen (1985)).

[12]The assumption of making a binding default decision is necessitated in the presence of Epstein-Zin preferences.

(mortgage) payments decline geometrically, $\{1, \delta, \delta^2, \ldots\}$, where $\delta$ captures the duration of the mortgage. Labor income is subject to a tax rate, $\tau_t^B$, and mortgage payments are tax deductible at tax rate $\tau_t^M$.[13]

After having received income and having chosen house and mortgage size, each family member draws an idiosyncratic housing depreciation shock $\omega_{i,t} \sim F_\omega(\cdot)$ which proportionally lowers the value of the house by $(1-\omega_{i,t})p_t k_{t-1}^B$. The value of the house after stochastic depreciation is $\omega_{i,t} p_t k_{t-1}^B$. We denote the cross-sectional mean and standard deviation by $\mu_\omega = \mathrm{E}_i[\omega_{i,t}]$ and $\sigma_{t,\omega} = (\mathrm{Var}_i[\omega_{i,t}])^{0.5}$, where the latter varies over time.

Each family member then optimally decides whether or not to default on the mortgages. The houses that the borrower family defaults on are turned over to (foreclosed by) the lender. Let the function $\iota(\omega) : [0, \infty) \to \{0, 1\}$ indicate the borrower's decision to default on a house of quality $\omega$. We conjecture and later verify that the optimal default decision is characterized by a threshold level $\omega_t^*$, such that borrowers default on all houses with $\omega_{i,t} \le \omega_t^*$ and repay the debt for all other houses. Using the threshold level $\omega_t^*$, we define $Z_A(\omega_t^*)$ to be the fraction of debt *repaid* to lenders and $Z_K(\omega_t^*)p_t K_{t-1}^B$ to be the value of the housing stock to the borrowers after default decisions have been made, where:

$$Z_A(\omega_t^*) = \int_0^\infty (1-\iota(\omega))\, f_\omega(\omega) d\omega = \Pr[\omega_{i,t} \ge \omega_t^*], \tag{3}$$

$$Z_K(\omega_t^*) = \int_0^\infty (1-\iota(\omega))\omega\, f_\omega(\omega) d\omega = \Pr[\omega_{i,t} \ge \omega_t^*]\ \mathrm{E}[\omega_{i,t} \,|\, \omega_{i,t} \ge \omega_t^*] \tag{4}$$

After making a coupon payment of 1 per unit of remaining outstanding mortgage, the amount of outstanding mortgages declines to $\delta Z_A(\omega_t^*) A_t^B$.

Next, the households can choose to prepay (call) a quantity of the outstanding mortgages $R_t^B$ by paying the "face value" $F = \frac{\alpha}{1-\delta}$ per unit to the lender. We denote by $Z_t^R = R_t^B / A_t^B$ the ratio of prepaid mortgages to beginning-of-period mortgages. Prepayment incurs a monetary cost. We use an adjustment cost function $\Psi(R_t^B, A_t^B)$ that is convex in the fraction prepaid $Z_t^R$, capturing bottlenecks in the mortgage refinance infrastructure when too large a share of

[13] In reality, mortgage interest payments are deductible at the marginal income tax rate. Because a mortgage in the model is a perpetuity, there is no clean separation of principal and interest payments. Having a different tax rate at which mortgage payments are deducted allows us to calibrate the model to a realistic level of deductibility. Appendix A discusses the mapping between our geometric mortgage bonds and mortgages in the real world in great detail.

mortgages are prepaid at once.

The borrower family's problem is to choose consumption $C_t^B$, housing $K_t^B$, default threshold $\omega_t^*$, prepayment quantity $R_t^B$, and new mortgage debt $B_t^B$ to maximize life-time utility $U_t^B$ in (1), subject to the budget constraint:

$$C_t^B+(1-\tau_t^m)Z_A(\omega_t^*)A_t^B+p_tK_t^B+FR_t^B+\Psi(R_t^B,A_t^B)\leq(1-\tau_t^B)Y_t^B+Z_K(\omega_t^*)p_tK_{t-1}^B+q_t^mB_t^B+G_t^{T,B}, \tag{5}$$

an evolution equation for outstanding mortgage debt:

$$A_{t+1}^B=\delta Z_A\left(\omega_t^*\right)A_t^B-R_t^B+B_t^B, \tag{6}$$

a maximum loan-to-value constraint:

$$FA_{t+1}^B\leq\phi p_tK_t^B. \tag{7}$$

and a double constraint on the amount of mortgages that can be refinanced:

$$0\leq R_t^B\leq\delta Z_A\left(\omega_t^*\right)A_t^B. \tag{8}$$

Outstanding mortgage debt at the end of the period (equation 6) is the sum of the remaining mortgage debt after default and new borrowing $B_t^B$ minus prepayments. The borrower household uses after-tax labor income, net transfer income from the government ($G_t^{T,B}$), housing wealth, and new mortgage debt raised to pay for consumption, mortgage debt service net of mortgage interest deductibility, new home purchases, prepayments $FR_t^B$ and associated prepayment costs $\Psi(R_t^B,A_t^B)$ . New mortgage debt raised is $q_t^mB_t^B$, where $q_t^m$ is the price of one unit of mortgage bonds in terms of the numeraire good.

The borrowing constraint in (7) caps the face value of mortgage debt at the end of the period, $FA_{t+1}^B$, to a fraction of the market value of the underlying housing, $p_tK_t^B$, where $\phi$ is the maximum loan-to-value ratio. With such a constraint, declines in house prices (in bad times) tighten borrowing constraints.

The refinancing constraints in equation (8) ensure that the amount prepaid is between 0 and the outstanding balance after the default decision was made. Equivalently, the share prepaid, $Z_t^R$, must be between 0 and $\delta Z_A(\omega_t^*)$.

## 2.3 Depositors

The first type of savers, depositors, receive labor income, $Y_t^D \propto Y_t$, own a fixed share of the housing stock $K_t^D$, and can invest in three assets: short-term risk-free bonds, long-term private mortgage bonds, and long-term government-guaranteed mortgage bonds.

A private mortgage bond is a simple pass-through vehicle, aggregating the mortgages of the borrowers. The coupon payment on performing mortgages in the current period is $A_t^B Z_A(\omega_t^*)$. For mortgages that go in foreclosure, the saver repossesses the homes. These homes are worth $(1-\zeta)\left(\mu_\omega - Z_K(\omega_t^*)\right) p_t K_{t-1}^B$, where $\zeta$ is the fraction of home value destroyed in a foreclosure, a deadweight loss. Thus, the total payoff per unit of private mortgage bond is:

$$M_{t,P} = Z_A(\omega_t^*) + \frac{(1-\zeta)(\mu_\omega - Z_K(\omega_t^*))p_t K_{t-1}^B}{A_t^B}.$$

The price of the bond is $q_t^m$.

A government-guaranteed bond is a security with the same duration (maturity and cash-flow structure) as a private mortgage bond. The only difference is that it carries no mortgage default risk because of the government guarantee. To prevent having to keep track of an additional state variable, we model guarantees as one-period default insurance.[14] Combining one unit of a private mortgage bond with one unit of default insurance creates a mortgage bond that is government-guaranteed for one period. One unit of a government-guaranteed mortgage bond has the following payoff:

$$M_{t,G} = 1 + (1 - Z_A(\omega_t^*))\delta F$$

The first term is the coupon of 1 on all loans in the pool. The second term is compensation for the loss in principal of defaulted loans. Owners of guaranteed loans receive a "principal repayment" $F = \frac{\alpha}{1-\delta}$, a constant parameter that does not depend on the value of the collateral

[14] Rolling over default insurance every period for the life of the loan is the equivalent to the real-world guarantees provided by Fannie Mae and Freddie Mac. Having the choice of renewal each period makes our guarantees more flexible, and hence more valuable, than those in the real world.

or any state variable of the economy. We explain the choice of $F$ below. In exchange for the guarantee, savers pay a fee $\gamma_t$ per unit of bond to the government. The government sets the price of insurance $\gamma_t$.

Entering with wealth $W_t^D$, the depositor's problem is to choose consumption $C_t^D$, holdings of private mortgage bonds $A_{t+1,P}^D$, holdings of government-guaranteed mortgage bonds $A_{t+1,G}^D$, and short-term bonds $B_t^D$ to maximize life-time utility $U_t^D$ in (1), subject to the budget constraint:

$$C_t^D + q_t^m A_{t+1,P}^D + (q_t^m + \gamma_t) A_{t+1,G}^D + q_t^f B_t^D + (1-\mu_\omega) p_t K_{t-1}^D \leq (1-\tau_t^S) Y_t^D + G_t^{T,D} + W_t^D \quad (9)$$

short-sales constraints on all bond holdings:

$$A_{t+1,P}^D \geq 0, \quad (10)$$

$$A_{t+1,G}^D \geq 0, \quad (11)$$

$$B_t^D \geq 0,. \quad (12)$$

The budget constraint (9) shows that the depositor uses after-tax labor income, net transfer income, and beginning-of-period wealth to pay for consumption, purchases of private and government-guaranteed mortgage bonds and of short-term bonds, and for housing repairs. Housing repairs undo the effects of depreciation. Since the mortgage guarantee is valid for only one period, both private and government-guaranteed bonds bought last period trade for the same price $q_t^m$. We do not allow for negative positions in either long-term mortgage bond (equations 10 and 11). We also do not allow depositor's to take a negative position in the short-term bond (12), consistent with our assumption that the depositor must not declare bankruptcy.

### 2.4 Risk Takers

After shocks to income and housing depreciation have been realized, the risk taker (financial intermediary) chooses whether or not to declare bankruptcy. Risk takers who declare bankruptcy have all their assets and liabilities liquidated. They also incur a stochastic utility penalty $\rho_t$, with $\rho_t \sim F_\rho$, i.i.d. over time and independent of all other shocks. At the time of the bankruptcy decision, risk takers do not yet know the realization of the bankruptcy penalty. Rather, they have to commit to a bankruptcy decision rule $D(\rho) : \mathbb{R} \rightarrow \{0, 1\}$, that specifies the optimal decision for every possible realization of $\rho_t$. Risk takers choose $D(\rho)$ to maximize expected utility at the beginning of the period. We conjecture and later verify that the optimal default decision is characterized by a threshold level $\rho_t^*$, such that risk takers default for all realizations for which the utility cost exceeds the threshold.

After the realization of the penalty, risk takers execute their bankruptcy choice according to the decision rule. They then face a consumption and portfolio choice problem identical to that of the depositor with two exceptions. First, while intertemporal preferences are still specified by equation (1), intraperiod utility $u_t^j$ depends on the bankruptcy decision and penalty:

$$u_t^R = \frac{\left(C_t^R\right)^{1-\theta}\left(A_K K_{t-1}^R\right)^{\theta}}{\exp\left(D(\rho_t)\rho_t\right)}.$$

Second, we replace equation (12) with the following borrowing constraint:

$$-B_t^R \leq q_t^m(\xi_P A_{t+1,P}^R + \xi_G A_{t+1,G}^R). \tag{13}$$

A negative position in the short-term bond is akin to the risk taker issuing short-term bonds, or equivalently deposits. The negative position in the short-term bond must be collateralized by the market value of the risk taker's holdings of long-term mortgage bonds. The parameters $\xi_P$ and $\xi_G$ determine how useful private and government-guaranteed mortgage bonds are as collateral. In the calibration, we will assume that guaranteed mortgages are better collateral: $\xi_G > \xi_P$.[15]

Denote risk-taker wealth by:

$$\begin{aligned} W_t^R = &(M_{t,P} + \delta Z_A(\omega_t^*)q_t^m - Z_t^R[q_t^m - F])A_{t,P}^R \\ &+ (M_{t,G} + \delta Z_A(\omega_t^*)q_t^m - Z_t^R[q_t^m - F])A_{t,G}^R + B_{t-1}^R. \end{aligned}$$

[15] In the language of finance, this short-term borrowing is exactly like a repo contract. It allows the saver to buy a mortgage bond by borrowing a fraction $\xi$ of the purchase price while only using a fraction $1 - \xi$ of the purchase price, the margin requirement, of her own capital. One can think of the guaranteed bond as a private mortgage bond plus a government guarantee (a credit default swap or mortgage insurance). Implicit in constraint (13) is the assumption that the government guarantee itself is an off-balance sheet item that cannot be collateralized.

## 2.5 Government

We model the government as set of exogenously specified tax, spending, bailout, and debt issuance policies.[16] Government tax revenues, $T_t$, are labor income tax receipts minus mortgage interest deduction tax expenditures plus guarantee fee receipts:

$$T_t = \tau_t^B Y_t^B + \tau_t^S (Y_t^R + Y_t^D) - \tau_t^m Z_A(\omega_t^*) A_t^B + \gamma_t (A_{t,G}^R + A_{t,G}^D)$$

Government expenditures, $G_t$ are the sum of payoffs on mortgage guarantees, financial sector bailouts, other exogenous government spending, $G_t^o$, and transfer spending $G_t^T$:

$$G_t = (M_{t,G} - M_{t,P})(A_{t,G}^R + A_{t,G}^D) - D(\rho_t) W_t^R + G_t^o + G_t^T$$

The bailout to the financial sector equals the negative of the financial wealth of the risk taker, $W_t^R$, in the event of a bankruptcy.

The government issues one-period risk-free debt. Debt repayments and government expenditures are financed by new debt issuance and tax revenues, resulting in the budget constraint:

$$B_{t-1}^G + G_t \leq q_t^f B_t^G + T_t \tag{14}$$

We impose a transversality condition on government debt:

$$\lim_{u \to \infty} \mathrm{E}_t \left[ \tilde{\mathcal{M}}_{t,t+u}^D B_{t+u}^G \right] = 0$$

where $\tilde{\mathcal{M}}^D$ is the SDF of the depositor.[17]

Because of its unique ability to tax and repay its debt, the government can spread out the cost of mortgage default waves and financial sector rescue operations over time.

Government policy parameters are $\Theta_t = \left(\tau_t^B, \tau_t^S, \tau_t^m, \gamma_t, G_t^o, \phi, \xi_G, \xi_P\right)$. The parameters $\phi$ in

[16] We consolidate the role of the GSEs and that of the Treasury department into one government, reflecting the reality as of September 2008.

[17] We show below that the risk averse saver is the marginal agent for short-term risk-free debt. In the numerical work below, we keep the ratio of government debt to GDP contained between $\underline{b^G}$ and $\overline{b^G}$ by increasing other government expenditure $G_t^o$ exponentially when debt-to-GDP threatens to fall below $\underline{b^G}$ and lowering $G_t^o$ exponentially (until it hits zero) when debt-to-GDP threatens to exceed $\overline{b^G}$.

equation (7) and $(\xi_G, \xi_P)$ in equation (13) can be thought of as macro-prudential policy tools. One could add the parameters that govern the utility cost of bankruptcy of risk takers to the set of policy levers, since the government may have some ability to control the fortunes of the financial sector in the event of a bankruptcy.

## 2.6 Equilibrium

Given a sequence of income shocks $\{Y_t\}$, housing depreciation shocks $\{\omega_{t,i}\}_{i\in B}$, and utility costs of default shocks $\rho_t$, and given a government policy $\Theta_t$, a competitive equilibrium is an allocation $\{C_t^B, K_t^B, B_t^B\}$ for borrowers and $\{C_t^S, A_{t,P}^S, A_{t,G}^S, B_t^S\}$ for savers $S \in \{R, D\}$, default policies $\iota(\omega_{it})$ and $D(\rho_t)$, and a price vector $\{p_t, q_t^m, q_t^f\}$, such that given the prices, borrowers, depositors, and risk-takers maximize life-time utility subject to their constraints, the government satisfies its budget constraint, and markets clear.

The market clearing conditions are:

1. Risk-free bonds: $B_t^G = B_t^D + B_t^R$
2. Mortgages: $A_t^B = A_{t,G}^R + A_{t,P}^R + A_{t,G}^D + A_{t,P}^D$
3. Housing tree shares: $K_t^B + K_t^R + K_t^D = 1$
4. Consumption: $Y_t^B + Y_t^R + Y_t^D = C_t^B + C_t^R + C_t^D + (1-\mu_{t,\omega})p_t + G_t^o + \zeta(\mu_{t,\omega} - Z_K(\omega_t^*))\frac{p_t K_{t-1}^B}{A_t^B} + \Psi(R_t^B, A_t^B)$

The last equation states that total non-housing resources equal the sum of non-housing consumption expenditures and home renovations by the households, (wasteful) spending by the government, and lost resources due to the deadweight costs of foreclosure and mortgage refinancing.

## 2.7 Welfare

In order to compare economies that differ in the policy parameter vector $\Theta_t$, we must take a stance on how to weigh the different agents. We propose a utilitarian social welfare function summing value functions of the agents according to their population weights $\ell$:

$$\mathcal{W}_t(\cdot;\Theta_t) = \ell^B V_t^B + \ell^D V_t^D + \ell^R V_t^R,$$

where the $V^i(\cdot)$ functions are the value functions defined in the appendix.

# 3 Model Solution and Calibration

## 3.1 First Order Conditions

Appendix A presents the Bellman equations for each of the three household types and derives first-order conditions for optimality. We highlight some key features of the solution here. First, since borrowers are the only households freely choosing their housing position, their choice pins down the price of housing in the economy. Let $\tilde{\mathcal{M}}^i_{t,t+1}$ be the intertemporal marginal rate of substitution (or stochastic discount factor) for agent $i \in \{B, D, R\}$, with expressions provided in the Appendix. At the optimum, house prices satisfy the recursion:

$$p_t\left[1-\tilde{\lambda}_t^B\phi\right] = \mathrm{E}_t\left[\tilde{\mathcal{M}}^B_{t,t+1}e^{g_{t+1}}\left\{p_{t+1}Z_K(\omega^*_{t+1}) + \frac{\theta C^B_{t+1}}{(1-\theta)K^B_t}\right\}\right] \tag{15}$$

The marginal cost of housing on the left-hand side consists of the house price $p_t$ minus a term which reflects the collateral benefit of housing; an extra unit of housing relaxes the maximum LTV constraint (7). The right hand side captures the expected discounted future marginal benefits which depends on the resale value of the non-defaulted stock and the dividend from housing, which is the intratemporal marginal rate of substitution between housing and non-housing goods.

Second, we analyze the borrower's optimal foreclosure decision. In the appendix, we show that the optimal default threshold is given by:

$$\omega^*_t = \frac{(1-\tau^m_t+\delta q^m_t-\delta\tilde{\lambda}^{RB}_t)A^B_t}{p_tK^B_{t-1}}.$$

At the threshold level $\omega^*_t$, the cost from foreclosure (and mortgage debt relief), which is the loss of a house valued at $\omega_t^* p_t K_{t-1}^B$, exactly equals the expected cost from continuing the service the mortgage (including the option to default in the future which is encoded in $q_t^m$) and keeping the house. The cutoff has an intuitive interpretation. It is the aggregate loan-to-value ratio of the borrowers, with both mortgage debt and housing valued at market prices. When the *market* leverage of the borrower increases, the house value threshold rises and default becomes more likely. Note that when the borrower exercises her prepayment option to its maximum extent, $\tilde{\lambda}_t^{RB} > 0$ and default becomes less likely. Hence the default option and the prepayment option interact. A valuable refinancing option gives the borrower incentives to postpone a default decision.

Third, the optimal share of outstanding mortgages that the borrower chooses to prepay, $Z_t^R = R_t^B / A_t^B$ equals:

$$Z_t^R = \frac{1}{\psi}\left(q_t^m - F + \tilde{\mu}_t^{RB} - \tilde{\lambda}_t^{RB}\right) \tag{16}$$

This balances the marginal cost of refinancing (governed by $\psi$) with the marginal benefit which is to increase the value of mortgage debt raised by $q_t^m - F$. Intuitively, when current mortgage rates are lower than when the mortgage was originated, the mortgage is a premium bond and trades at a price $q^m$ above par value $F$. By refinancing a marginal unit of debt, the borrower gains $q^m - F$. If $q^m - F$ is large enough, the borrower will want to refinance all outstanding debt ($Z_t^R = \delta Z_A(\omega^*)$). The multiplier on the refinancing upper bound activates ($\tilde{\lambda}_t^{RB} > 0$). Conversely, when $q_t^m < F$, refinancing is not useful and the multiplier on the lower refinancing bound, $\tilde{\mu}_t^{RB} > 0$, turns positive to keep $Z_t^R = 0$.

Fourth, from the borrower's first order condition for $A^B$, we can read off the demand for mortgage debt.

$$q_t^m = \tilde{\lambda}_t^B F + \mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^B Z_A(\omega_{t+1}^*)\left(1 - \tau^m - \frac{\psi\left(Z_{t+1}^R\right)^2}{2Z_A(\omega_{t+1}^*)} - \delta\tilde{\lambda}_{t+1}^{RB} + \delta q_{t+1}^m\right)\right]. \tag{17}$$

A unit of mortgage debt obtained generates an amount $q_t^m$ today but uses up some borrowing capacity, which is costly when the borrower's loan-to-value constraint binds ($\tilde{\lambda}_t^B > 0$). The non-defaulted part of the debt must be serviced in future periods, modulo a mortgage interest tax deduction, as long as it is not prepaid.

In this version of the paper, we restrict the depositor to only invest in deposits, i.e., short-term debt issued by the risk taker. This debt is equivalent to government debt by virtue of the deposit insurance.[18] The depositor's first-order condition for the short-term bond is:

$$q_t^f \;=\; \mathrm{E}_t\left[\tilde{\mathcal{M}}^D_{t,t+1}\right]$$

Next, we turn to the risk taker's default decision. The risk taker will optimally default whenever the utility costs of doing so is sufficiently small: $\rho_t < \rho_t^*$. The threshold depends on her wealth $W_t^R$ and the state variables $\mathcal{S}_t^R$ that are exogenous to the risk taker, including the wealth of the borrower and of the depositor, and the outstanding amount of government debt. At the threshold, she is indifferent between defaulting and offloading her (negative) wealth onto the government or carrying on:

$$V^R(0,\rho_t^*,\mathcal{S}_t^R) = V^R(W_t^S,0,\mathcal{S}_t^R).$$

Finally, the risk taker can invest in both government guaranteed and private MBS. The respective first-order conditions are:

$$\begin{aligned}
q_t^m(1-\xi_G\tilde{\lambda}_t^R)+\gamma_t \;&=\; \mathrm{E}_t\left[\tilde{\mathcal{M}}^R_{t,t+1}\left(M_{G,t+1}+\delta Z_A(\omega^*_{t+1})q^m_{t+1}-Z^R_{t+1}[q^m_{t+1}-F]\right)\right]\\
q_t^m(1-\xi_P\tilde{\lambda}_t^R) \;&=\; \mathrm{E}_t\left[\tilde{\mathcal{M}}^R_{t,t+1}\left(M_{P,t+1}+\delta Z_A(\omega^*_{t+1})q^m_{t+1}-Z^R_{t+1}[q^m_{t+1}-F]\right)\right].
\end{aligned}$$

The marginal cost of a guaranteed mortgage bond is the price $q_t^m$ plus the guarantee fee $\gamma_t$ (expressed as a price) while the benefit is the expected discounted value of the bond tomorrow, which consists of the coupon payment and the repayment of principal in case of default (both are in $M_G$) plus the resale value of the non-defaulted portion of the mortgage bond. When there are prepayments, the market value of the bond is adjusted for the difference between the market value and the face value, on the share of mortgages that gets prepaid. The cost is lowered by the relaxation of the margin constraints, and depends on the haircut $\xi_G$ for guaranteed mortgages. The first-order condition for private mortgages is similar, without the guarantee fee term, with a different collateral requirement term ($\xi_P$), and a different mortgage payoff $M_P$. One way

[18] We plan to relax this assumption of $A^D_{t,G}=0$ in later versions of the paper.

of restating the risk taker's choice is in terms of how many units of the mortgage to lend to borrowers, and for how much of these holdings to buy default insurance from the government. The optimal amount of default insurance to buy solves:

$$\gamma_t = \mathrm{E}_t\left[\tilde{\mathcal{M}}^R_{t,t+1}\left(M_{G,t+1} - M_{P,t+1}\right)\right] + \tilde{\lambda}^R_t q^m_t\left(\xi_G - \xi_P\right)$$

Risk takers will buy insurance until the marginal cost of insurance on the left equals the marginal benefit. An extra unit of default insurance increases the payoff of the mortgage and it increases the collateralizability of a mortgage, a benefit which only matters when the borrowing constraint binds.

In equilibrium, the risk-taker's short-sales constraint for government-guaranteed bonds may be binding, depending on the level of the guarantee fee. In that case, her position in these bonds would be zero. In contrast, she is always the marginal agent for private mortgage bonds. She finances these bonds with her own wealth but also by issuing short-term debt (deposits). The bailout guarantee that the government provides to the risk takers makes these deposits risk-free and equivalent to short-term government debt. That is, the bailout guarantee is deposit insurance. The risk-taker may or may not borrow up to the limit in (13). If she does, the Lagrange multiplier on that constraint $\tilde{\lambda}^R_t > 0$. Since mortgage bonds enable the issuance of more short-term debt, they increase risk taker demand for mortgage bonds when the constraint binds (lower the marginal cost by $q^m_t \xi_P \tilde{\lambda}^R_t$).

The above equations describe the supply of mortgage credit from the risk taker, or banks, perspectively. We find the equilibrium mortgage price and the quantity borrowed at the intersection of the supply and demand for mortgage debt.

## 3.2 Calibration

The parameters of the model and their targets are summarized in Table 1.

Table 1: Calibration

This table reports the parameter values of our model.

| | Parameter | Description | Value | Target |
|---|---|---|---|---|
| | | Exogenous Shocks | | |
| 1 | $\bar{g}$ | mean income growth | 1.9% | Mean rpc GDP gr 1929-2013 |
| 2 | $\sigma_g$ | volatility income growth | 3.9% | Vol rpc GDP gr 1929-2013 |
| 3 | $\rho_g$ | persistence income growth | 0.41 | AC(1) rpc GDP gr 1929-2013 |
| 4 | $\mu_\omega$ | mean idiosync. house value shock | 2.5% | Housing depreciation Census |
| 5 | $\sigma_\omega$ | volatility idiosync. house value shock | {0.10,0.22} | Mortgage loss rates (Appendix B.2) |
| 6 | $p^\omega_{LL}$ | transition prob | 0.2 | frequency of mortgage crises of 10% |
| 7 | $p^\omega_{HH}$ | transition prob | 0.99 | duration of mortgage crises of 2y |
| | | Population, Income, and Housing Shares | | |
| 8 | $\ell^i$, $i \in \{B, D, R\}$ | population shares | {47,51,2}% | Population shares SCF 1995-2013 |
| 9 | $Y^i$, $i \in \{B, D, R\}$ | income shares | {38,52,10}% | Income shares SCF 1995-2013 |
| 10 | $K^i$, $i \in \{B, D, R\}$ | housing shares | {39,49,12}% | Housing wealth shares SCF 1995-2013 |
| | | Mortgages | | |
| 11 | $\delta$ | average life of mortgage pool | 0.95 | Duration Fcn. (Appendix B.3) |
| 12 | $\alpha$ | guarantee payout fraction | 0.52 | Duration Fcn. (Appendix B.3) |
| | | Preferences | | |
| 13 | $\sigma^B$ | risk aversion borrower | 8 | Vol household mortgage debt to GDP 1985-2014 |
| 14 | $\beta^B$ | time discount factor borrower | 0.88 | Mean housing wealth to GDP 1985-2014 |
| 15 | $\theta^B$ | housing expenditure share | 0.20 | Housing expenditure share NIPA |
| 16 | $\sigma^D$ | risk aversion depositor | 20 | volatility risk-free interest rate 1998-2014 |
| 17 | $\beta^D = \beta^R$ | time discount factor savers | 0.975 | Mean risk-free interest rate 1998-2014 |
| 18 | $\sigma^R$ | risk aversion risk taker | 4 | Financial sector leverage Flow of Funds 1985-2014 |
| 19 | $\nu$ | intertemp. elasticity of subst. | 1 | |
| | | Government Policy | | |
| 20 | $\tau^S = \tau^B$ | income tax rate | 19.83% | BEA govmt revenues to trend GDP 1985-2013 |
| 21 | $G^o$ | exogenous govmt spending | 15.8% | BEA govmt spending to trend GDP 1985-2013 |
| 22 | $G^T$ | govmt transfers to agents | 3.41% | BEA govmt net transfers to trend GDP 1985-2013 |
| 23 | $\tau^m$ | mortgage interest rate deductibility | 0.48 $\tau^B$ | See text |
| 24 | $\phi$ | collateral constr | 0.65 | Mean borrowers' mortgage debt-to-income SCF 1995-2013 |
| 25 | $\xi_G$ | margin guaranteed MBS | 1.6% | Basel 2/3 regulatory capital charge agency MBS |
| 26 | $\xi_P$ | margin private MBS | 8% | Basel 2/3 regulatory capital charge non-agency mortgages |

**Aggregate Income** The model is calibrated at annual frequency. Aggregate endowment or labor income $Y_t$ follows:

$$
\begin{aligned}
Y_t &= Y_{t-1}\exp(g_t) \\
g_t &= \rho_g g_{t-1} + (1-\rho_g)\bar{g} + \epsilon_t, \qquad \epsilon_t \sim iid\, \mathcal{N}(0,\sigma_g)
\end{aligned}
$$

We scale all variables by permanent income in order render the problem stationary. Given the persistence of income growth, $g_t$, becomes a state variable. We discretize the $g_t$ process into a 5-state Markov chain using the method of Rouwenhorst. The procedure matches the mean, volatility, and persistence of GDP growth by choosing both the grid points and the transition probabilities between them. We use annual data on real per capita GDP growth from the BEA NIPA tables from 1929-2014 and exclude the war years 1940-1945. The resulting mean is 1.9%, the standard deviation is 3.9%, and the persistence is 0.42. The states, the transition probability matrix, and the stationary distribution are listed in Appendix B.1.

**Foreclosure crises** The stochastic depreciation shocks or idiosyncratic house value shocks, $\omega_{i,t}$, are drawn from a Gamma distribution characterized by shape and a scale parameters $(\chi_{t,0}, \chi_{t,1})$. $F_\omega(\cdot; \chi_{t,0}, \chi_{t,1})$ is the corresponding CDF. We choose $\{\chi_{t,0}, \chi_{t,1}\}$ to keep the mean $\mu_\omega$ constant at 0.975, implying annual depreciation of housing of 2.5%, and to let the cross-sectional standard deviation $\sigma_{t,\omega}$ take on one of two values, a high and a low value. Together with the maximum loan-to-value parameter $\phi$, $(\sigma_{H,\omega}, \sigma_{L,\omega})$ determine the expected losses from mortgage default. We set these three parameters to fit three targets: the mortgage loss rate during normal times, the mortgage loss rate during foreclosure crises, and the average mortgage debt-to-income ratio for borrowers. These targets are 0.3%, 3.4%, and 130%, respectively. Appendix B.2 explains the rationale behind these targets and the data sources. The resulting parameter values are $\phi = 0.65$, $\sigma_{H,\omega} = 0.22$, and $\sigma_{L,\omega} = 0.10$. The calibration produces a loss rates on banks' overall mortgage portfolio of 1.7% during normal times and 4.6% during crisis times, and the mortgage debt-to-income ratio among borrowers is 140%.

To pin down the transition probabilities of the 2-state Markov chain for $\sigma_{t,\omega}$, we assume that when the aggregate income growth rate in the current period is high ($g$ is in one of the top three income states), there is a zero chance of transitioning from the $\sigma_{L,\omega}$ to the $\sigma_{H,\omega}$ state and a 100% chance of transitioning from the $\sigma_{H,\omega}$ to the $\sigma_{L,\omega}$ state. Conditional on low growth ($g$ is in one of the bottom two income states) we calibrate the two transition probability parameters (rows have to sum to 1), $p^{\omega}_{LL}$ and $p^{\omega}_{HH}$, to match the frequency and length of mortgage crises. Based on the argument by Jorda et al. (2014) that most financial crises are related to the mortgage market and based on the historical frequency of financial crises in Reinhart and Rogoff, we target a 10% probability of a foreclosure crisis. Conditional on a crisis, we set the expected length to 2 years, based on evidence in Jorda et al. and Reinhart and Rogoff. As such, the model implies that not all recessions are mortgage crises, but all mortgage crises are recessions. In a long simulation, 33% of recessions are also crises. This compares to a fraction of 6/22 (≈27%) in Jorda et al. (2014). The correlation between $\sigma_{t,\omega}$ and $g_t$ is -0.42. The model generates persistence in the mortgage default rate of .036 in the low g-fee economy and 0.35 in the high g-fee economy. The persistence depends on, among other things, the persistence of $\sigma_{t,\omega}$.

**Population and wealth shares** To pin down the labor income and housing shares for borrowers, depositors, and risk takers, we calculate a net fixed-income position for each household in the Survey of Consumer Finance (SCF).[19] Net fixed income equals total bond and bond-equivalent holdings minus total debt. If this position is positive, we consider a household to be a saver, otherwise it is a borrower. For savers, we calculate the amount of risky assets, defined as their holdings of stocks, business wealth, and real estate wealth, as well as the share of these risky assets in total wealth. We define risk takes as households that are within the top 5% of risky asset holdings and have a risky asset share of at least 75%. This delivers population shares of $\ell^B = 47\%$, $\ell^D = 51\%$, and $\ell^R = 2\%$. Based on this classification and the same SCF data, borrowers receive 38% of aggregate income and own 39% of residential real estate. Depositors receive 52% of income and 49% of housing wealth. Finally, risk takers receive 10% of income and 12% of housing wealth. By virtue of the calibration, the model thus matches basic aspects of the observed income and wealth inequality.

**Mortgages** In our model, a government-guaranteed MBS is a geometric bond. The issuer of one bond at time $t$ promises to pay the holder 1 at time $t+1$, $\delta$ at time $t+2$, $\delta^2$ at time

[19]We use all survey waves from 1995 until 2013 and average across them.

$t+3$, and so on. If the borrower defaults on the mortgage, the government guarantee entitles the holder to receive a "principal repayment" $F = \frac{\alpha}{1-\delta}$, a constant parameter that does not depend on the value of the collateral or any state variable of the economy. We estimate values for $\delta$ and $F$ such that the duration of the geometric mortgage matches the duration of the portfolio of outstanding mortgage-backed securities, as measured by the Barclays MBS Index, across a history of observed mortgage rates. This novel procedure recognizes that the mortgage in the model represents the pool of all outstanding mortgages of all vintages. Appendix B.3 provides the details. The repayment $F$ reflects the fact that prepayments happen when rates are low and reinvestment opportunities are poor. Thus, there are some losses associated with (default-induced or other) prepayments on guaranteed bonds. We find that values of $\delta = 0.95$ and $\alpha = 0.52$ imply a relationship between price and mortgage rate for the geometric mortgage that closely matches the price-rate relationship for a real-life MBS pool consisting of fixed-rate mortgages issues across a range of vintages. The average duration in model and data of the mortgage (pool) is about 4 years. Like the real-life MBS pool, the geometric mortgage price is convex in rates when rates are high (the prepayment option is out-of-the-money) and concave when rates are low ("negative convexity" when the prepayment option is in-the-money). Thus, the geometric mortgage has the same interest rate risk (duration) of real-life mortgages for different interest rate scenarios. Despite its simplicity, the perpetual mortgage with prepayment captures the key features of real-life guaranteed MBS pools.

**Government parameters** Government policy consists of mortgage guarantee policy and general taxation and spending policy. In our desire to have a quantitatively meaningful model, we believe it is important to also capture the latter. After the conservatorship of Fannie Mae and Freddie Mac in September 2008, the merger of the GSEs and the Treasury Department became a reality.

Starting with the guarantee policy, our parameter $\gamma$ specifies the cost of a guarantee, expressed in the same units as the price of the mortgage. Real-world guarantee fees are expressed as a surcharge to the interest rate. We will consider several values for $\gamma$ with implied g-fees ranging from 0 basis points to 70 basis point. Freddie Mac's management and g-fee rate was stable at around 20bps from 2000 to 2012 and has increased gradually from 20bps at the start of 2012 to 32 bps at the end of 2014. Fannie Mae's single-family effective g-fee was also right around 20bps between 2000 and 2009, and has been increasing gradually from 20bps at the start of 2009 to 41 bps at the end of 2014 (Urban Institute Housing Finance Policy Center, December 2014 update). Fannie's g-fees on new single-family originations currently average 63bps.

We set the proportional income tax rate equal to $\tau^S = \tau^B = 19.83\%$ in order to match average discretionary tax revenue to trend GDP in U.S. data. The discretionary tax revenue in the 1985-2013 data of 19.4% is after mortgage interest deductions, which is about 0.43% of trend GDP. Hence, we set a tax rate before MID of 19.83%. We allow for mortgage interest rate deductibility. Because our geometric mortgages do not distinguish between interest and principal payments, we assume that the entire mortgage payment is deductible but at a lower rate, $(1 - \alpha) \times \tau$.[20] We set exogenous government spending equal to $G^o = 0.158$ (times trend GDP of 1) in order to match average exogenous government spending to trend GDP in the 1985-2013 U.S. data of 15.8%.[21] This exogenous spending is wasted. We also allow for transfer spending of 3.41% of GDP, which equals the net transfer spending in the 1985-2013 data. This spending is distributed lump-sum to the agents in proportion to their population. As a fraction of realized GDP, expenditures fluctuate, mimicking their counter-cyclicality in the data. Tax revenues are pro-cyclical, as in the data. Every dollar of income is taxed at the same tax rate. Risk takers are only 2% of the population but pay 11% of the income taxes since they earn 11% of the income.

We can interpret the risk-taker borrowing constraint parameters, $\xi_G$ and $\xi_P$ as regulatory capital constraints set by the government. Under Basel II and III, "first liens on a single-family home that are prudently underwritten and performing" enjoy a 50% risk weight and all others a 100% risk weight. Agency MBS receive a 20% risk weight. Given that we think of the non-guaranteed mortgage market as the subprime and Alt-A market, a capital charge of 8% (100%

[20]As discussed in Appendix B.3, the sum of all mortgage payments is $1/(1 - \delta)$ and $F = \alpha/(1 - \delta)$ is the payment of "principal." Hence, the fraction of "interest payments" is the fraction $(1 - \alpha)$. In the equilibrium with low g-fees, the mortgage interest deductibility expense is 0.46% of trend GDP, very close to the target.

[21]The data are from Table 3.1 from the BEA. Exogenous government spending is defined as consumption expenditures (line 18) plus subsidies (line 27) minus the surplus of government enterprises (line 16). It excludes interest service on the debt and net spending on social security and other entitlement programs. Government revenues are defined as current receipts (line 1), which excludes social security tax receipts. Trend GDP is calculated with the Hodrick Prescott Filter.

risk weight) seems most appropriate for $\xi_P$. Given that the government guaranteed mortgages are the counterpart to agency MBS, we set a capital charge of 1.6% (20% risk weight) for $\xi_G$.

**Preference parameters** We set the elasticity of inter-temporal substitution equal to 1 for all agents, a common value in the literature. The coefficients of risk aversion are $\sigma_R = 4$, $\sigma_B = 8$, and $\sigma_D = 20$. The annual subjective time discount factors are $\beta_R = \beta_D = 0.975$ and $\beta_B = 0.88$. Risk aversion and the time discount factor of the depositor target the mean one-year real risk-free interest rate of 0.15% and its volatility of 2.0%.[22] The borrower's discount factor is set to match the value of housing wealth to GDP over 1985-2014, which is 1.51 (Flow of Funds). Borrower risk aversion is set to target the volatility of the annual change in household mortgage debt to GDP (Flow of Funds and NIPA), which is 4.2% in the 1985-2014 data. Our low g-fee economy produces a close fit with a volatility of 4.6%. The risk taker subject discount factor is set equal to that of the other savers, the depositors. Their risk aversion is chosen to match average leverage ratios of the financial sector. Since mortgage assets are predominantly held by leveraged financial institutions, we calculate leverage for those kinds of institutions. The average ratio of total debt to total assets for 1985-2014 is 95.6%.[23] Our low g-fee economy produces a close fit with a value of 93.1%.

**Utility cost of risk-taker bankruptcy** The model features a random utility penalty that risk takers suffer when they default. Because random default is mostly a technical assumption, it is sufficient to have a small penalty. We assume $\rho_t$ is normally distributed with a mean of

[22]To calculate the real rate, we take the nominal one year constant maturity Treasury yield (FRED) and subtract expected inflation over the next 12 months. To form expected inflation, we take either realized inflation over the past 12 months, the realized inflation over the next 12 months, or the average of the two. We consider the period 1998-2014, the period which corresponds to the low g-fee regime. The preceding period still contains the remnants of the Volcker disinflation, which our model is silent on. All three concepts of expected inflation give rise to an average real rate between 0.10 and 0.15% with a volatility between 1.77% and 2.00%.

[23]Specifically, we include U.S. Chartered Commercial Banks and Savings Institutions, Foreign Banking offices in U.S., Bank Holding Companies, Banks in U.S. Affiliated Areas, Credit Unions, Finance Companies, Security Brokers and Dealers, Funding Corporations (Fed Bailout entities e.g. Maiden Lanes), GSEs, Agency- and GSE-backed Mortgage pools (before consolidation), Issuers of ABS, REITs, and Life and Property-Casualty Insurance Companies. Krisnamurthy and Vissing-Jorgensen (2011) identify a group of financial institutions as net suppliers of safe, liquid assets. This group is the same as ours except that we add insurance companies and take our money market mutual funds, since we are interested in leveraged financial firms. For comparison, leverage for the Krisnamurthy and Vissing-Jorgensen institutions is 90.7% for the 1985-2014 sample. The group of excluded, non-levered financial institutions are Money Market Mutual Funds, other Mutual Funds, Closed-end funds and ETFs, and State, Local, Federal, and Private Pension Funds. Total financial sector leverage, including these non-levered institutions, is 60.6%.

$\mu_\rho$ =1, i.e., a zero utility penalty on average, and a small standard deviation of $\sigma_\rho = 0.05$. The mean size of the penalty affects the frequency of financial sector defaults (and government bailouts). The lower $\mu_\rho$, the lower the resistance to declare bankruptcy, and the higher the frequency of bank defaults. The standard deviation affects the correlation between negative financial intermediary wealth and bank defaults. Given those parameters, the frequency of financial crises (government bailouts of the risk-taker) depends on the frequency of foreclosure crises, and the endogenous choices (asset composition and liability choice) of the risk taker.

**Deadweight cost of foreclosure** We assume a $\zeta = 50\%$ deadweight cost of foreclosure, in line with values used in the literature. This parameter affects mortgage loss rates, which were discussed above.

**Prepayment cost** We consider a quadratic prepayment cost, governed by the parameter $\phi$. A value of 0.15 produces an average conditional prepayment rate of 7.8% in the model, which is close to the historical average.

# 4 Main Results: Phasing out the GSEs

The main experiment in the paper is to compare an economy with and without government-guaranteed mortgages. More precisely, we compute a sequence of three economies that differ by the guarantee fee $\gamma_t$ that the government charges for providing the default insurance. All economies feature a government bailout guarantee to the financial sector (risk takers), or equivalently, deposit insurance. Higher g-fees "crowd-in" the private sector, a key objective of any reform proposal. We evaluate how equilibrium prices and quantities, and ultimately welfare are affected from an increase in g-fees. We do so based on a long simulation of the model. Tables 2 and 3 summarize the results.

## 4.1 Status Quo

We start with a model where the government provides the mortgage guarantee relatively cheaply. We set $\gamma_t$ to a value that implies an annual rate spread of 20bps[24] We think of this "low g-fee" case as representing actual g-fees observed between the late 1990s and the late 2000s. In this economy, a dominant share of mortgages are held in the form of government-guaranteed bonds. We refer to this calibration as the "status quo" or "20bps g-fee" economy. The unconditional mean and standard deviation of the key objects of interest in this economy are in the first two columns of Table 2.

Risk takers hold 63% of their assets on average in the form of government-guaranteed bonds. In unreported results, we find that the guaranteed MBS portfolio share is 61% in normal times (low $\sigma_\omega$ states) and 99% in housing crises (high $\sigma_\omega$ states). Risk takers finance these MBS holdings in equilibrium by issuing deposits and combining them with their own equity capital contributions. By lending in long-term guaranteed MBS and borrowing short-term, banks bear interest rate risk. They perform the traditional role of liquidity transformation.[25] Because banks have a modest risk aversion, compared with the much higher risk aversion of the depositors, banks are willing to provide safe assets to the depositors. They can do so paying essentially zero interest on deposits. Bearing little default risk on their assets, banks use substantial leverage in order to achieve their desired risk-return combination. The average bank leverage (book value of debt to market value of assets) ratio is 95.5%. Given that the margin requirement on guaranteed mortgage bonds is 1.6% and that on private bonds is 8%, banks are close to their leverage limit. Specifically, the leverage constraint binds in 92% of the periods. Average risk taker wealth is modest, at 2.9% of trend GDP. Banks have little "skin in the game." Due to low risk taker wealth and high leverage, the banking system is fragile. When adverse income or house value shocks hit, risk taker net worth threatens to fall below zero. When that happens,

[24]The interest rate on private bonds can be calculated as $r_{P,t} = \log\left(\frac{1}{q_t^m} + \delta\right)$, and the rate on guaranteed bonds is $r_{G,t} = \log\left(\frac{1}{q^m + \gamma_t} + \delta\right)$. The effective g-fee, quoted as a difference in rates, is therefore given by $r_{P,t} - r_{G,t}$.

[25]Guaranteed bonds in the model do suffer some losses when there is a default-induced or rate-induced prepayment, mimicking what happens in reality to agency MBS. Since agency bonds usually trade above par prior to the prepayment, and prepayments come in at par, the prepayment constitutes a loss for the holder or agency MBS. Put differently, prepayments happen when interest rates are low and reinvestment opportunities are poor. Mortgage REITS investing in agency MBS play a similar role to banks in reality. Some hybrid mortgage REITS combine agency and non-agency (private-label) MBS.

the government steps in to bail out the financial sector. Such financial crises happen in 18.6% of the periods. In housing crises (high $\sigma_\omega$ periods), the frequency of financial crises is even 26.5%. Thus housing crises are likely to trigger financial crises, as documented in the empirical work of Jorda et al. (2014). The average equilibrium return on the risk taker's wealth, the ROE of the banking sector, is high when excluding bankruptcy episodes: 13.1%. However, because bankers get frequently wiped out, the ROE is volatile and much lower once those episodes are considered. The volatility is large because wealth frequently comes close to zero as a result of high leverage.

Turning to the borrowers, we find that mortgage rates are low in this economy. The average (real) mortgage rate is 3.5%, not unlike the real mortgage rates observed in the 2000s. Borrowers enjoy low rates in part due to the subsidized mortgage guarantees. Simultaneously house prices are high. Faced with high house prices and low mortgage rates, borrowers take out a lot of mortgage debt in equilibrium. The steady state stock of mortgages outstanding is high (0.60 market value or 0.051 units $A^B$). The average borrower LTV ratio is 63.9% and borrowers' mortgage debt-to-income is 1.41 on average, both are close to the averages in the recent SCF data. The average default rate on mortgages is 4.6%, and the loss rate from mortgage default is 2.3%, with losses of 9.3% in housing crises and 1.4% in normal times. The volatility of mortgage debt to income growth, our measure of the stability of the provision of mortgage credit, is 4.8%. Rate-induced prepayments average to 8.7% per year, an average prepayment rate that is consistent with average conditional prepayment rates observed in the data. Bonds that prepay result in an average mark-to-market loss of 11%, reflecting the fact that these prepayments occur in low interest rate environments when reinvestment opportunities are weak.

Depositors have a strong precautionary savings demand for risk-free assets, given their high risk aversion. They are happy to lend to the banks at low interest rates because of their risk aversion and because the deposit insurance makes their claims on the banks risk free. In equilibrium, most of the supply of safe assets is provided by the banks because government debt averages only 19.5% of trend GDP. The debt is a result of frequent financial sector bailouts (deposit insurance fund payouts) and payouts on mortgage guarantees. The reasons that government debt is not higher are that the government raises more revenue from taxes than it spends on exogenous expenditures (by about 0.6% of trend GDP each year), and that debt service is low due to the low interest rates. During housing crises, the depositor's precautionary savings incentives strengthen substantially and she desires to hold more risk-free assets. In those times, the government faces more payouts on mortgage guarantees and more frequent financial sector bailouts, which cause a deficit and increase its debt to 27.6% of trend GDP. The risk taker accommodates the net increase in demand for risk-free debt by taking in more deposits. Risk taker leverage increases to 97.8% in these periods. The risk free interest rate falls to -0.6%. The rise in bond prices in crisis times can be thought of as a flight-to-quality effect.

In summary, the economy with low g-fees features high house prices, high levels of mortgage debt, high mortgage debt to income ratios for borrowers, high rates of mortgage defaults and severities, high levels of bank debt, low levels of intermediary capital, frequent financial crises, higher government debt, and low interest rates.

## 4.2 Higher g-fees

The second and third economies we compute are cases where the g-fee is 25 basis points and 65 basis points on average, respectively. The 25 bps value reflects roughly what Fannie Mae and Freddie Mac charged the financial sector in the years from 2010 to 2012, while the 65 bps g-fee is just above what Fannie currently charges on new originations. It is meant to be above the observable range and to imply a cost of protection at which the private sector is crowded in.[26] Since the results are monotone in g-fees, we describe these two economies in parallel. The results are in column 3-6 of Table 2.

For the intermediate cost of mortgage default insurance, we find that risk-takers portfolio holdings more strongly depend on the state of the economy, with the average share of guaranteed bonds being 10.2%. During crisis times (high $\sigma_\omega$ states), banks hold only 2.5% of private mortgages, similar to the 20bps g-fee economy. However, during normal times, they hold 99% of private MBS. Put differently, banks mostly buy the government insurance in bad times but not in good times. The state uncontingent g-fee is too cheap in bad times but too expensive in good times. This intermediate g-fee economy is reminiscent of Option B of the Obama

[26] Back-of-the-envelope calculations by J.P. Morgan suggest that at g-fees of above 50bps, private execution dominates GSE execution for many loans (J.P. Morgan Securitized Products update 2014).

Table 2: Main Results: Unconditional Moments

| | **20 bp g-fee** | | **25 bp g-fee** | | **65 bp g-fee** | |
|---|---|---|---|---|---|---|
| | mean | stdev | mean | stdev | mean | stdev |
| | | | **Prices** | | | |
| Risk free rate | -0.001 | 0.018 | 0.023 | 0.031 | 0.032 | 0.035 |
| Mortgage rate | 0.035 | 0.003 | 0.039 | 0.003 | 0.042 | 0.003 |
| House price | 2.121 | 0.138 | 2.041 | 0.100 | 1.934 | 0.100 |
| | | | **Risk-Taker** | | | |
| Market value of bank assets | 0.603 | 0.027 | 0.550 | 0.020 | 0.507 | 0.021 |
| Market value of private bonds | 0.213 | 0.240 | 0.494 | 0.166 | 0.507 | 0.021 |
| Market value of guaranteed bonds | 0.390 | 0.243 | 0.056 | 0.162 | 0.000 | 0.000 |
| Risk taker leverage | 0.955 | 0.034 | 0.866 | 0.048 | 0.849 | 0.055 |
| Risk taker wealth | 0.029 | 0.026 | 0.086 | 0.027 | 0.089 | 0.027 |
| Fraction $\lambda^R > 0$ | 0.922 | 0.269 | 0.205 | 0.404 | 0.154 | 0.361 |
| Bankruptcy frequency | 0.186 | 0.389 | 0.002 | 0.039 | 0.000 | 0.014 |
| Return on RT wealth[a] | -0.053 | 0.839 | 0.040 | 0.230 | 0.033 | 0.217 |
| Return on RT wealth (excld. bankr.)[b] | 0.131 | 0.730 | 0.041 | 0.226 | 0.033 | 0.217 |
| Excess return on guaranteed bond | -0.003 | 0.024 | -0.017 | 0.029 | -0.066 | 0.032 |
| Excess return on private bond | 0.001 | 0.062 | 0.001 | 0.035 | 0.000 | 0.037 |
| | | | **Borrower** | | | |
| Mortgage debt | 0.051 | 0.002 | 0.049 | 0.001 | 0.046 | 0.001 |
| Borrower LTV | 0.639 | 0.051 | 0.637 | 0.031 | 0.637 | 0.033 |
| Market value of debt LTV | 0.760 | 0.073 | 0.724 | 0.057 | 0.706 | 0.058 |
| Borrower debt to income | 1.410 | 0.049 | 1.356 | 0.016 | 1.284 | 0.021 |
| Debt/income growth | 0.001 | 0.048 | 0.001 | 0.041 | 0.001 | 0.044 |
| Default rate | 0.046 | 0.105 | 0.022 | 0.051 | 0.018 | 0.046 |
| Rate-induced prepayment rate | 0.087 | 0.025 | 0.048 | 0.026 | 0.029 | 0.026 |
| Loss Given Default | 0.484 | 0.015 | 0.479 | 0.015 | 0.477 | 0.015 |
| MTM Loss Given Prepayment | 0.110 | 0.029 | 0.062 | 0.034 | 0.036 | 0.037 |
| Loss rate private | 0.023 | 0.054 | 0.011 | 0.027 | 0.009 | 0.024 |
| Loss rate guaranteed | 0.006 | 0.012 | 0.002 | 0.004 | 0.001 | 0.003 |
| Loss rate portfolio | 0.010 | 0.028 | 0.008 | 0.022 | 0.009 | 0.024 |
| | | | **Government** | | | |
| Government debt / GDP | 0.195 | 0.179 | 0.051 | 0.059 | 0.021 | 0.005 |

The table reports unconditional means and standard deviations of the main outcome variables from a 10,000 period simulation of three different models. The model in the first 2 columns has a mortgage guarantee fee of 0 (0 bp g-fee), the model in columns 3 and 4 has an average g-fee of 0.15% (expressed as a rate), and the model in the last two columns has an average g-fee of 0.70%.
[a]: Return on wealth is the return on the risk takers total portfolio i.e. their positive position in mortgages and negative position in deposits. It is set to -100% in periods, when risk takers declare bankruptcy.
[b]: Return on wealth computed by excluding simulation periods when risk takers declare bankruptcy.

Administration housing reform plan, which envisions g-fees high enough that they are only attractive in bad times.

In the high g-fee economy, in the third column of Table 2, risk takers shift exclusively towards holding private MBS. They do not buy default insurance from the government, neither in good nor in bad times. The g-fee is high enough to "crowd-in" the private sector at all times. The high g-fee economy evaluates Option A in the Obama plan which envisions an entirely private mortgage market.

Our key result is that increasing the g-fee lowers the riskiness of the financial sector. Risk taker leverage in the intermediate g-fee economy averages 86.8% compared to 95.2% in the status quo economy. It falls further to 84.9% in the high g-fee economy. This results is not just driven by the fact that private mortgages carry higher regulatory capital requirements ($\xi_P > \xi_G$). Indeed, the banks' leverage constraint binds in only about 15% of the periods in the high g-fee economy, suggesting that banks choose to stay away from their leverage constraint in most periods. Since banks' portfolios now bear a substantial amount of mortgage default risk, there is less need to lever up in order to achieve the desired risk-return relationship for intermediary wealth. The return on risk-taker wealth, the banks' ROE, *increases* as g-fees rise from 20 bps to 65bps, from $-5.3\%$ to 3.3%. This happens even as the amount of risk-taker wealth (intermediary capital) triples from 2.9% of GDP to 8.9%. Despite being better capitalized and having lower leverage, they earn higher returns on equity on average. The reason is that risk takers now avoid bankruptcy episodes due to their safer balance sheet.

Borrowers face higher interest rates in the economies with higher g-fees. The mortgage rate increases from 3.5% in the 20bps g-fee economy to 3.9% in the intermediate case and 4.2% in the high g-fee economy. The government guarantee props up house price levels. House prices are 3.9% lower in the intermediate g-fee and 9.8% lower in the high g-fee economy than in the status quo. In equilibrium, because of lower house prices and higher mortgage rates, there is less mortgage debt outstanding: 10.8% less in book value and 18.9% less in market value term in the high g-fee economy compared to the status quo. Borrowers have less mortgage debt relative to their income. Borrower debt to income ratios fall from 141% in the 20 bps g-fee economy to 128% in the high g-fee economy. Because of lower house prices, loan-to-value ratios are almost identical across economies in book debt terms and 5% points lower for market-debt based LTV.

The lower indebtedness of the borrowers results in fewer defaults and hence in smaller loss rates for banks. The portfolio loss rate falls from 1.0% to 0.8% as we raise the g-fee. This decline is largely driven by smaller default rates during good times and lower severities. While crisis loss rates on private mortgages fall with higher g-fees, banks now have a larger share of these private bonds in portfolio so that the portfolio loss rate in crises actually rises. Nevertheless, the overall default rate and loss rate fall by half and almost half as we raise g-fees. In other words, absent government guarantees, banks not only reduce their leverage, they also take less risk on their mortgage portfolio. The safer intermediary balance sheet (higher equity, lower leverage) reduces the frequency of financial crises, or government bailouts dramatically from 18.6% in the status quo to 0.2% in the intermediate economy and to 0 (in ten thousand simulations) in the high g-fee economy. Even though housing crises are just as frequent in all three economies by design (low $\sigma_\omega$ states happen with the same probability), they are not as severe in terms of losses the banking sector suffers, and combined with lower leverage and larger bak equity cushions, they do not result in financial crises. In the 20 bps g-fee economy, 26.5% of housing crises lead to financial crises while in the intermediate economy only 0.8% do, and in the high g-fee economy none do. Thus, we conclude that it is the presence of the GSEs and the incentives the mortgage guarantees create for banks to lever up and make riskier mortgages that cause housing crises to result in systemic financial crises.

The increased stability of the financial system allows banks to expand mortgage credit in bad times (housing crises) in the high g-fee economy more than in the low g-fee economies. The overall volatility in mortgage originations, as measured by the standard deviation of the growth rate of mortgage debt/income, is higher in the low g-fee economy (4.8%) than in the low g-fee economy (4.4%). The popular fear that a private mortgage system would lead to large swings in the availability of mortgage credit, especially in bad times, is unwarranted in our model.

Turning to depositors, we find that they enjoy higher average deposit interest rates in the lower g-fee economies. The equilibrium risk-free rate goes up from 0% to 2.3% to 3.2%. This result is surprising at first blush. The supply of risk-free securities is lower as g-fees rise because the intermediary balance sheet is smaller and leverage is smaller. Furthermore, because bank bailouts are less frequent and there is little equilibrium mortgage default insurance provision, the government spends less which further reduces the supply of risk-free assets. Indeed, government debt is substantially lower in the high g-fee economy. All else equal, a lower supply of risk-free debt should lead to higher prices and lower interest rates. However, the depositor's demand for risk-free assets falls in the high g-fee economies, more than offsetting the reduced supply. The reason is a weaker precautionary savings motive. Indeed, depositors' consumption is less volatile in the high g-fee economies. The intuition for this result is that the optimal allocation of risk between risk takers and depositors is distorted in the low g-fee economy. When the g-fee is high enough, risk takers are well enough capitalized and never constrained in their intermediation capacity. They bear (and hence internalize) all mortgage default risk. In contrast, in the status quo economy, mortgage crisis episodes lead to frequent bankruptcies of risk takers, impeding their intermediation function. During these crises the risk free rate drops sharply, effectively making depositors bear a greater part of the mortgage default risk. A more stable financial system reduces precautionary motives and leads to higher real rates.

Finally, in terms of the government sector, the models with higher g-fees have lower government debt and less volatile government debt-to-GDP ratios: 19.5% with 17.9% volatility in the status quo compared to 5.1% with 5.9% volatility in the intermediate economy and 2.1% with 0.5% volatility in the high g-fee world. The latter two models avoid an increase in government debt during housing crises because there are much fewer or even no payouts on mortgage guarantees and much less frequent/no financial sector bailouts. In the low g-fee economy, the government can mop up the fallout from the housing crisis without incurring prohibitive increases in the cost of debt exactly because the financial fragility encourages savings and hampers the proper allocation of risk.

## 4.3 Welfare

Aggregate welfare, measured as the population-weighted average of the value functions of the three types of agents increase with g-fees. It is 4.7% higher in the high g-fee economy than in the status quo. In the presence of deadweight costs of foreclosures, the lower losses on mortgages lead to an efficiency gain for the entire economy. The deadweight loss are 1.4% of GDP in the 20bps economy, respectively, but only 0.6% and 0.4% of GDP in the 25bps and high g-fee economies, respectively. The lower deadweight costs leave more resources for private

Table 3: Welfare and Risk Sharing

| | **20 bp g-fee** | | **25 bp g-fee** | | **65 bp g-fee** | |
|---|---|---|---|---|---|---|
| | mean | stdev | mean | stdev | mean | stdev |
| Aggregate Welfare | 0.309 | 0.008 | 0.313 | 0.008 | 0.324 | 0.009 |
| Deadweight Loss from Foreclosure | 0.014 | 0.031 | 0.006 | 0.014 | 0.004 | 0.011 |
| Value function borrower | 0.322 | 0.010 | 0.321 | 0.010 | 0.320 | 0.010 |
| Value Function depositor | 0.302 | 0.007 | 0.311 | 0.007 | 0.332 | 0.008 |
| Value function risk taker | 0.178 | 0.005 | 0.178 | 0.005 | 0.187 | 0.005 |
| MU ratio borrower/risk taker | -0.766 | 0.226 | -0.774 | 0.139 | -0.852 | 0.116 |
| MU ratio risk taker/depositor | 1.109 | 0.154 | 1.146 | 0.082 | 1.174 | 0.051 |
| Consumption borrower | 0.300 | 0.040 | 0.296 | 0.031 | 0.295 | 0.030 |
| Consumption depositor | 0.393 | 0.018 | 0.413 | 0.012 | 0.420 | 0.009 |
| Consumption risk taker | 0.077 | 0.011 | 0.075 | 0.005 | 0.073 | 0.004 |

The table reports unconditional means and standard deviations from a 10,000 period simulation of three different models. The model in the first 2 columns has a mortgage guarantee fee of 10 (10 bps g-fee), the model in columns 3 and 4 has an average g-fee of 0.15% (expressed as a rate), and the model in the last two columns has an average g-fee of 0.65%.
[a]: Marginal utility ratios are calculated as the difference of the logarithm of marginal utilities.
[b]: With unit EIS the value functions are in units of composite consumption $C^{1-\rho}K^{\rho}$. Therefore differences in values have a direct interpretation as consumption-equivalent welfare differences.

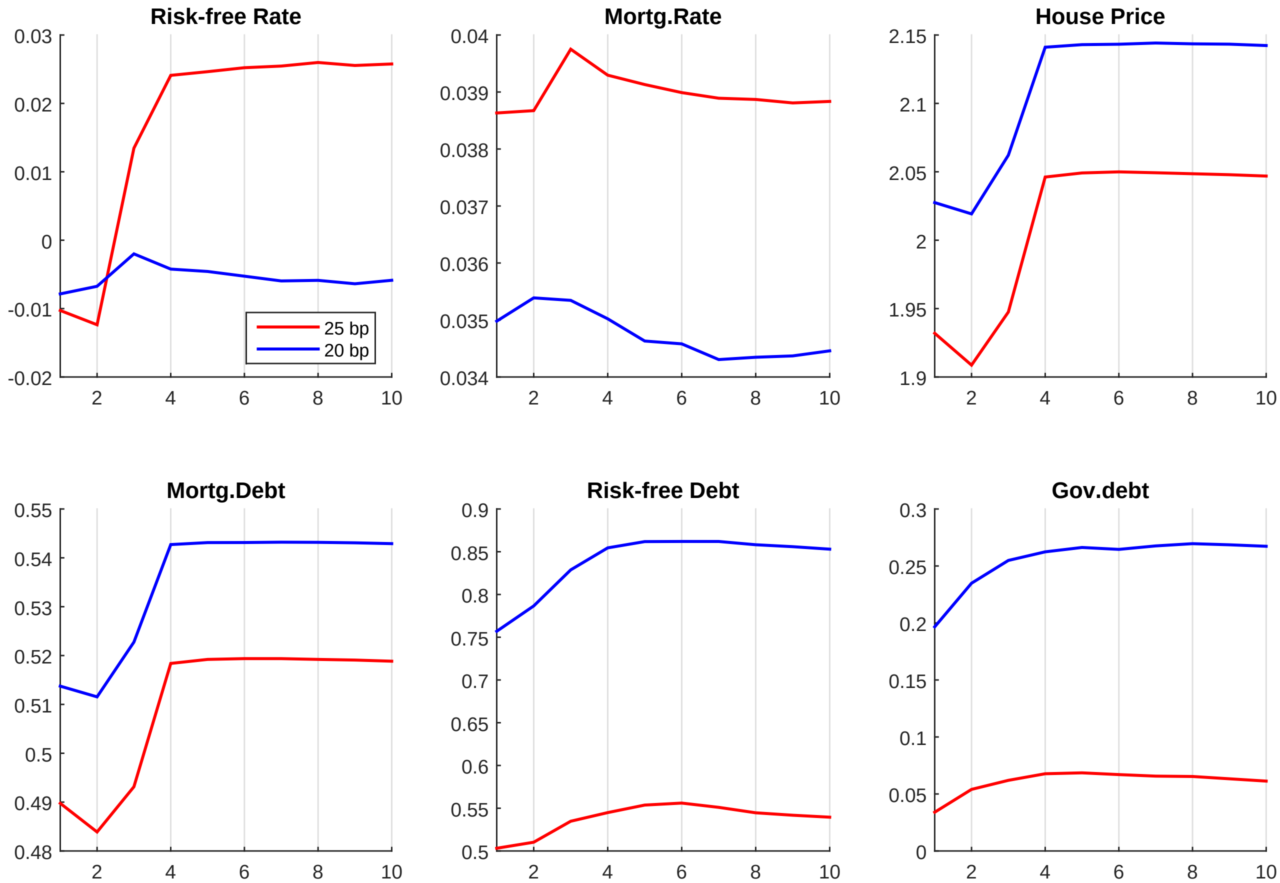
consumption each period.

A second, more important, source of welfare gains is that risk sharing between the different types of agents improves as g-fees increase. To measure the extent of the improvement, we compute the ratios of (log) marginal utilities between the different types. If markets were complete, agents would be able to achieve perfect risk sharing by forming portfolios that keep these ratios constant. Hence larger volatilities of these marginal utility (MU) ratios indicate worse risk sharing between the different types of agents. Table 3 lists the average MU ratios and their volatilities for borrowers/risk takers, and risk takers/depositors, as these are the pairs of agents that directly trade with each other. The volatilities of both ratios decrease substantially as g-fees rise, indicating an improvement in risk sharing. The presence of cheap mortgage guarantees effectively leads to more fragile financial intermediaries (risk takers). This in turn means that both the flow of funds from patient savers to impatient borrowers, and the risk allocation among savers of different risk tolerance break down more frequently.

Table 3 show the increase in welfare due to improved risk sharing and fewer deadweight losses. The borrower's welfare only declines slightly in the full public-to-private transition. The absence of a non-trivial loss in welfare for borrowers is surprising since providing subsidized mortgage guarantees obviously directly benefits borrowers since it lowers mortgage rates and increases property values. Taking away these subsidies hurts them. The important offset comes from the improved risk sharing in the private economy that the borrowers also enjoy. While their consumption is a bit lower, that consumption is also less volatile. Depositor's welfare increases by 9.8% while risk taker welfare increases by 4.7%. The reform redistributes wealth to savers, raising inequality. But it makes no type of agent significantly worse off, and thus the economy as a whole, better off.

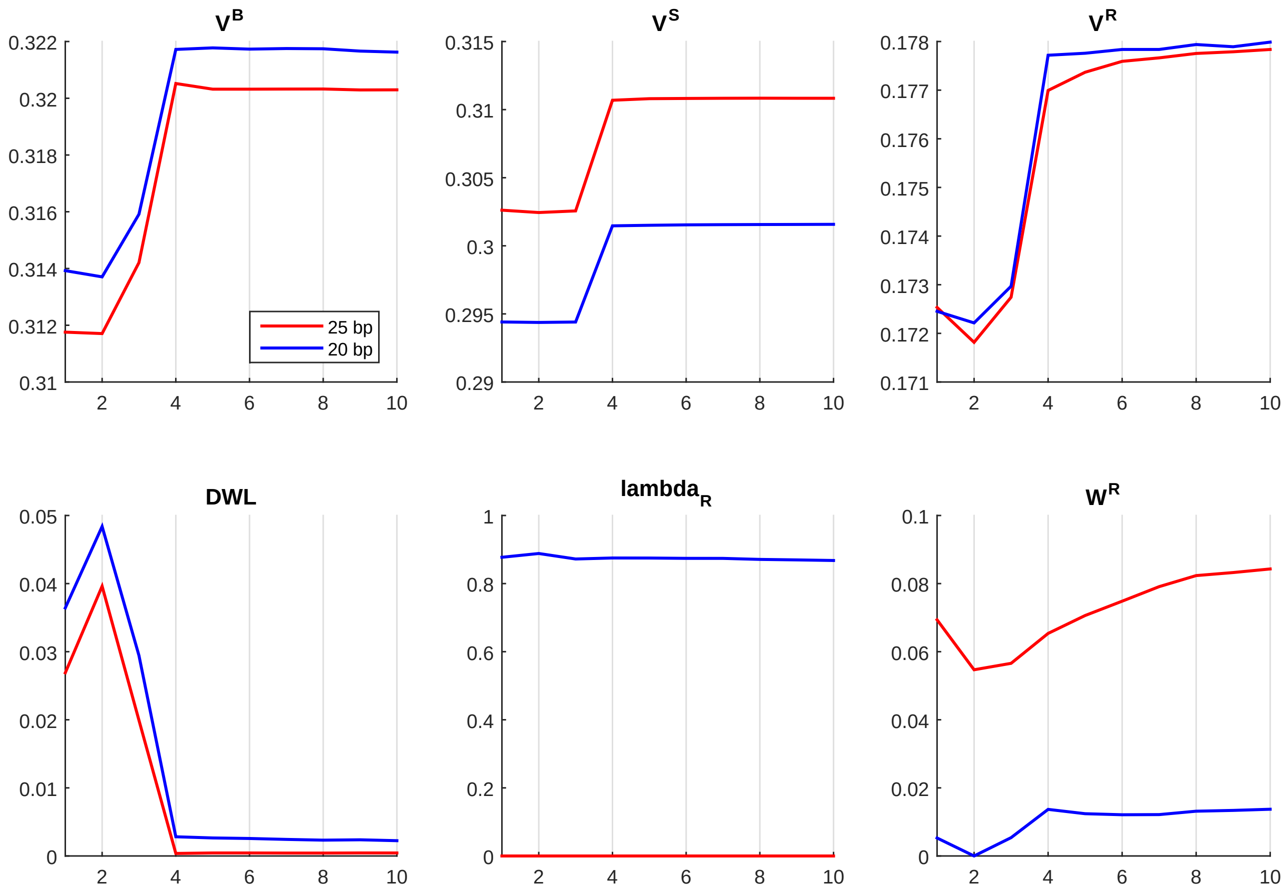
**Impulse-Responses** Figures 2 and 3 illustrate the sources of the welfare differences. The graphs show for several model variables the median path of the economy through a mortgage crisis, defined as an episode with high mortgage risk $\sigma_\omega$. The crisis episodes are based on the same 10,000 period simulation of the model that was used to calculate the moments in tables 2 and 3.

Figure 2: Mortgage crisis episode: interest rates, house price, and debt quantities




The graphs show the median path of the economy through a mortgage crisis episode starting at time 2 (defined as a sequence of periods with high $\sigma_\omega$). The crisis episodes are taken from the same 10,000 period simulation used to compute the moments in tables 2 and 3. **Blue line:** 20 bp economy **red line:** 25 bp economy.

Figure 3: Mortgage crisis episode: welfare




The graphs show the median path of the economy through a mortgage crisis episode starting at time 2 (defined as a sequence of periods with high $\sigma_\omega$). The crisis episodes are taken from the same 10,000 period simulation used to compute the moments in tables 2 and 3. **Blue line:** 20 bp economy **red line:** 25 bp economy.

Figure 2 shows that both the risk free rate and the mortgage rate are permanently low in the 20 bp economy, with the onset of the mortgage crisis only having a small effect. In the 25 bp economy, however, the risk free initially drops sharply before returning to its long run average over the course of several periods. House prices, mortgage debt, and risk-free debt (liabilities of risk takers) also drop sharply and only recover over the course of several periods. The dynamic behavior of the 20 and 25 bps economies is similar, with the recovery of the quantity variables being somewhat slower in the 20 bp case. Guarantee payouts cause government debt to accumulate to an elevated level and only slowly return to its long run average.

Figure 3 displays the value functions of the three types of households in the top row, and the sources of the welfare differences between the two economies in the bottom row. The bottom left graph shows the difference in deadweight losses from foreclosures. The middle and right-hand graphs in the bottom row illustrate that risk takers are more severely constrained in their intermediation capacity in the 20 bp economy. The value of the Lagrange multiplier on their leverage constraint spikes sharply as their wealth drops to zero for the 20 bp case. In the 25 bp economy, on the contrary, risk takers choose to stay away from the leverage constraint during crisis times.

All three types suffer large welfare losses during mortgage crises in both economies. Borrowers' welfare is slightly lower in the 25 bp case. The positive effect of lower mortgage rates is almost perfectly offset by higher deadweight losses and a more severely constrained risk taker in the 20 bp economy. Depositors are much better off in the 25 bp economy as they benefit from better risk sharing and higher deposit rates. Risk takers welfare is almost unchanged. Higher deposit rates in the 25 bp mean that their average level consumption is lower compared to the 20 bp case. However, their consumption is also less volatile and they are less constrained in their intermediation role. These effects almost offset each other.

One rationale for mortgage guarantees is the stability in the provision of mortgages due to the government backstop. Taken as given a highly levered balance sheet of banks and the riskiness of the mortgages they originate, a shock to output (for example) could decimate the banks in the absence of guarantees and lead them to sharply reduce new mortgage originations. With guarantees, the government would step in and use its ability to issue debt to smooth out the negative shock over time. Banks would not be hurt nearly as much because of the mortgage default insurance they enjoy. This intertemporal smoothing technology of the government is operational in our model and does mitigate the effects of the shock, as can be seem from the bottom right panel in figure 2. But the logic ignores the endogenous response of the banks' leverage and mortgage origination choice. Absent guarantees, banks become more prudent which benefits the stability of mortgage origination and of the financial system. In our model economy, the private sector turns out to be able to deliver superior consumption smoothing and risk sharing outcomes.

While absent in the model, if income taxation were distortionary and some of the higher spending in the low g-fee economies due to the dual guarantee had to be raised from taxes, such distortions would further amplify the welfare costs of low g-fees. Similarly, if financial crises had negative effects on other unmodeled sectors of the economy, that would further amplify the welfare effects as well.

## 4.4 Limited Liability and Capital Requirements

To highlight the interaction of mortgage guarantees on the one hand, and limited liability and low capital requirements of intermediaries on the other hand, we solve the status quo 20bps economy with two parameter variations. In both cases, we vary only a single parameter: (i) we set a much higher mean of the utility penalty for risk taker bankruptcy ($\mu_\rho = 2$), and (ii) we set the capital requirement for guaranteed bonds to 8%, i.e., we set $\xi_G = \xi_P = 0.92$.

Table 4 shows the results in the right two columns. The left column contains the unaltered 20bps "status quo" economy as a benchmark. Both a high cost of bankruptcy and a higher capital charge for guaranteed bonds lead to an equilibrium that is much closer to the high g-fee (65bps) economy of Table 2. In spite of identical g-fees of 20 bps in all three economies in Table 4, risk takers choose portfolios mainly consisting of private bonds in the economies with high $\mu_\rho$ and low $\xi_G$. At the same time, risk free rates and mortgage rates are higher and borrowers' debt-to-income ratios are lower in these economies, leading to lower mortgage default rates and loss rates. As a consequence mortgages are less risky, and it is less attractive for risk takers to purchase mortgage guarantees, which in turn explains their portfolio composition.

Taken together, these results show that the low interest rates, high debt, and greater financial fragility of the benchmark "status quo" economy only occur if high leverage is both feasible *and* optimal for intermediaries. In the economy with a high cost of intermediary bankruptcy, the allocation of the benchmark economy with risk taker leverage of 95.5% is generally feasible. However, due to the much larger penalty associated with bankruptcy, risk takers are effectively more risk averse when they get close to their leverage constraint, and they choose to avoid a binding constraint most of the time. Risk taker leverage is only 86.5% and borrowing constraints bind only in 21.6% of the periods. In the economy with $\xi_G = 0.92$, guaranteed bonds have the same equity capital requirement as private bonds, capping the feasible leverage of risk takers at 92% independent of portfolio shares. Yet again, risk takers choose to stay away from the

Table 4: The Role of Limited Liability and Capital Requirements

| | **Benchmark** | | **high $\mu_\rho$** | | $\boldsymbol{\xi_G = 92\%}$ | |
|---|---|---|---|---|---|---|
| | mean | stdev | mean | stdev | mean | stdev |
| | | | **Prices** | | | |
| Risk free rate | -0.001 | 0.018 | 0.020 | 0.032 | 0.017 | 0.029 |
| Mortgage rate | 0.035 | 0.003 | 0.039 | 0.003 | 0.038 | 0.003 |
| House price | 2.121 | 0.138 | 2.099 | 0.101 | 2.063 | 0.106 |
| | | | **Risk Taker** | | | |
| Market value of bank assets | 0.603 | 0.027 | 0.572 | 0.020 | 0.563 | 0.019 |
| Market value of private bonds | 0.213 | 0.240 | 0.512 | 0.175 | 0.504 | 0.171 |
| Market value of guaranteed bonds | 0.390 | 0.243 | 0.060 | 0.172 | 0.059 | 0.169 |
| Risk taker leverage | 0.955 | 0.034 | 0.865 | 0.048 | 0.869 | 0.044 |
| Risk taker wealth | 0.029 | 0.026 | 0.089 | 0.027 | 0.084 | 0.026 |
| Fraction $\lambda^R > 0$ | 0.922 | 0.269 | 0.216 | 0.411 | 0.243 | 0.429 |
| Bankruptcy frequency | 0.186 | 0.389 | 0.000 | 0.000 | 0.001 | 0.033 |
| Return on RT wealth[a] | -0.053 | 0.839 | 0.040 | 0.225 | 0.054 | 0.242 |
| Return on RT wealth (excld. bankr.)[b] | 0.131 | 0.730 | 0.040 | 0.225 | 0.055 | 0.239 |
| Excess return on guaranteed bond | -0.003 | 0.024 | -0.013 | 0.028 | -0.011 | 0.026 |
| Excess return on private bond | 0.001 | 0.062 | 0.001 | 0.035 | 0.003 | 0.036 |
| | | | **Borrower** | | | |
| Mortgage debt | 0.051 | 0.002 | 0.050 | 0.001 | 0.049 | 0.001 |
| Borrower LTV | 0.639 | 0.051 | 0.638 | 0.030 | 0.637 | 0.034 |
| Market value of debt LTV | 0.760 | 0.073 | 0.731 | 0.058 | 0.732 | 0.059 |
| Borrower debt to income | 1.410 | 0.049 | 1.397 | 0.016 | 1.370 | 0.018 |
| Debt/income growth | 0.001 | 0.048 | 0.001 | 0.040 | 0.001 | 0.038 |
| Default rate | 0.046 | 0.105 | 0.024 | 0.054 | 0.025 | 0.057 |
| Rate-induced prepayment rate | 0.087 | 0.025 | 0.054 | 0.028 | 0.056 | 0.025 |
| Loss Given Default | 0.484 | 0.015 | 0.480 | 0.015 | 0.480 | 0.015 |
| MTM Loss Given Prepayment | 0.110 | 0.029 | 0.071 | 0.034 | 0.074 | 0.031 |
| Loss rate private | 0.023 | 0.054 | 0.012 | 0.028 | 0.013 | 0.030 |
| Loss rate guaranteed | 0.006 | 0.012 | 0.002 | 0.005 | 0.002 | 0.006 |
| Loss rate portfolio | 0.010 | 0.028 | 0.009 | 0.023 | 0.009 | 0.024 |
| | | | **Government** | | | |
| Government debt / GDP | 0.195 | 0.179 | 0.082 | 0.092 | 0.064 | 0.072 |
| | | | **Welfare** | | | |
| Aggregate Welfare | 0.309 | 0.008 | 0.318 | 0.009 | 0.320 | 0.009 |

The table reports unconditional means and standard deviations of the main outcome variables from a 10,000 period simulation of three different models. All models have the same g-fee of 20 bp. The model in the first 2 columns is the benchmark model from table 2. The middle economy has a higher utility penalty of risk taker bankruptcy. The last two columns display results for an economy with a capital charge for guaranteed bonds set to 8% (same as for private bonds).

[a]: Return on wealth is the return on the risk takers total portfolio i.e. their positive position in mortgages and negative position in deposits. It is set to -100% in periods, when risk takers declare bankruptcy.

[b]: Return on wealth computed by excluding simulation periods when risk takers declare bankruptcy.

constraints most of the time (in 75.8% of the periods) and bank leverage is 86.9%. In both economies, banks' equity buffers are much stronger and financial crises are entirely eliminated.

Both results show that risk takers only shift their holdings towards guaranteed bonds if they can simultaneously increase their return through higher leverage. Macro-prudential policy (higher capital requirements) and policies that make bank insolvencies more costly ex-ante are effective at reducing financial fragility, reducing deadweight losses from foreclosures, and improving the allocation of risk in society. Aggregate welfare is 3.5% higher in both experiments than in the status quo, capturing 73% of the benefits of moving to high g-fees of 65bps.

# 5 Conclusion

Our main findings are that underpriced, government-provided mortgage default insurance distorts the incentives of the financial sector in a way that leads it to take more risk in the mortgages it originates and in the leverage it takes on. While the policy leads to higher house prices, low mortgage rates, and low interest rates, it also leads to more frequent mortgage defaults, and financial crises when banks become insolvent. Our model thus highlights the connection between housing market risk and financial sector risk, a prominent feature of financial systems in developed economies.

While the government can mitigate the fallout from such crises by spreading the costs out over time via the issuance of government debt, the ultimate allocation of risk remains suboptimal. We document large welfare gains from transitioning to a private mortgage system, a transition which can be effectuated by raising the cost of the government mortgage guarantees. A safer financial sector emerges which is better able to intermediate between borrowers and savers so as to implement the optimal allocation of risk in the economy. Since the improvement in aggregate welfare benefits savers and bankers, with borrowers' welfare remaining essentially unchanged, the policy increases wealth inequality.

More broadly, the paper brings together the literatures of financial intermediary-based asset pricing and housing finance reform. It brings in the role of the government into the former and the importance of the financial sector into the latter. New is the possibility of default for both borrowers and banks and the government's provision of bailout guarantees to the creditors' of the banking sector. An important element is the interaction of such guarantees on banks' liabilities with the mortgage guarantee on banks' assets.

Our model is a natural laboratory to explore the effects of government purchases of whole mortgage loans, guaranteed mortgage bonds, and private mortgage bonds. The GSEs were a large buyer in all categories accumulating a combined portfolio of $1.7 trillion dollars by 2007. Equally interesting would be to study the impact of the subsequent 50% reduction of that balance sheet to $850 billion at the end of 2014. Similarly, the Federal Reserve was a large buyer of guaranteed mortgage bonds, accumulating $1.8 trillion as part of its QE1 and QE3 programs. While current policy is to keep the size of this portfolio constant, studying the effects of a change to a no-reinvestment policy would be interesting. Over the next several years, we are likely to see a change from governmental to private sector ownership of at least 25% of one of the largest fixed income markets in the world. A complete understanding of the impact of these large purchases or sales on the mortgage market, house prices, the macro-economy, and the financial sector remains an important challenge for future research.

There are several other promising avenues for further exploration. The model currently abstracts from the choice between owning and renting. Abolishing the mortgage guarantees may well affect the home ownership rate. If house price-to-rent ratios fall in the aftermath of the policy reform, as they do in recent models that study the abolition of mortgage interest rate deductibility, the policy may well boost the home ownership rate. A second ingredient our work abstracts from is the feedback effect from the mortgage lending complex to the rest of the financial sector and to the real economy. In a world with subsidized mortgage lending, lending to non-financial businesses with good ideas gets crowded out. Future work could add a group of capital-constrained entrepreneurs with productive investment opportunities, thereby endogenizing the exogenous endowment process considered here. Such a model would add a further cost of government mortgage guarantees and financial sector bailout guarantees.

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# A Model Appendix

We reformulate the problem of risk taker, depositor, and borrower to ensure stationarity of the problem. We do so by scaling all variables by permanent income.

## A.1 Borrower

### A.1.1 Preliminaries

We start by defining some preliminaries.

$$
\begin{aligned}
Z_A(\omega_t^*) &= [1 - F_\omega(\omega_t^*; \chi)] \\
Z_K(\omega_t^*) &= [1 - F_\omega(\omega_t^*; \chi)] \mathrm{E}\left[w_{i,t} \mid \omega_{i,t} \geq \omega_t^*; \chi\right]
\end{aligned}
$$

and $F_\omega(\cdot; \chi)$ is the CDF of $\omega_{i,t}$ with parameters $\chi$. Assume $\omega_{i,t}$ are drawn from a Gamma distribution with shape and scale parameters $\chi = (\chi_0, \chi_1)$ such that

$$
\begin{aligned}
\mu_\omega &= \mathrm{E}_i[\omega_{i,t}; \chi_0, \chi_1] = \chi_0 \chi_1 \\
\sigma_{t,\omega}^2 &= \mathrm{Var}_i[\omega_{i,t}; \chi_0, \chi_1] = \chi_0 \chi_1^2
\end{aligned}
$$

From Landsman and Valdez (2004, equation 22), we know that

$$
\mathrm{E}[\omega | \omega \geq \bar{\omega}] = \mu_\omega \frac{1 - F_\omega(\bar{\omega}; \chi_0 + 1, \chi_1)}{1 - F_\omega(\bar{\omega}; \chi_0, \chi_1)}
$$

so the closed form expression for $Z_K$ is

$$
Z_K(\omega_t^*) = \mu_\omega \left[1 - F_\omega(\omega_t^*; \chi_0 + 1, \chi_1)\right]
$$

It is useful to compute the derivatives of $Z_K(\cdot)$ and $Z_A(\cdot)$:

$$
\begin{aligned}
\frac{\partial Z_K(\omega_t^*)}{\partial \omega_t^*} &= \frac{\partial}{\partial \omega_t^*} \int_{\omega_t^*}^{\infty} \omega f_\omega(\omega) d\omega = -\omega_t^* f_\omega(\omega_t^*), \\
\frac{\partial Z_A(\omega_t^*)}{\partial \omega_t^*} &= \frac{\partial}{\partial \omega_t^*} \int_{\omega_t^*}^{\infty} f_\omega(\omega) d\omega = -f_\omega(\omega_t^*),
\end{aligned}
$$

where $f_\omega(\cdot)$ is the p.d.f. of a Gamma distribution with parameters $(\chi_0, \chi_1)$.

**Prepayment Cost** Let

$$
\Psi(R_t^B, A_t^B) = \frac{\psi}{2} \left(\frac{R_t^B}{A_t^B}\right)^2 A_t^B
$$

Then partial derivatives are

$$
\Psi_R(R_t^B, A_t^B) = \psi \frac{R_t^B}{A_t^B} \tag{18}
$$

$$
\Psi_A(R_t^B, A_t^B) = -\frac{\psi}{2} \left(\frac{R_t^B}{A_t^B}\right)^2 \tag{19}
$$

### A.1.2 Statement of stationary problem

Let $\mathcal{S}_t^B = \left(g_t, \sigma_{\omega,t}, W_t^R, W_t^D, B_{t-1}^G\right)$ represent state variables exogenous to the borrower's decision. We consider the borrower's problem in the current period after income and house depreciation shocks have been realized, after the risk taker has chosen a default policy, and after the random utility penalty is realized. Then the borrower's value function, transformed to ensure stationarity, is:

$$V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^B) = \max_{\{C_t^B, K_t^B, \omega_t^*, R_t^B, B_t^B\}} \Bigg\{ (1-\beta_B) \left[ \left(C_t^B\right)^{1-\theta} \left(A_K K_{t-1}^B\right)^{\theta} \right]^{1-1/\nu} + \\ + \beta_B \mathrm{E}_t \left[ \left( e^{g_{t+1}} \tilde{V}^B(K_t^B, A_{t+1}^B, \mathcal{S}_{t+1}^B) \right)^{1-\sigma_B} \right]^{\frac{1-1/\nu}{1-\sigma_B}} \Bigg\}^{\frac{1}{1-1/\nu}}$$

subject to

$$C_t^B = (1-\tau_t^B) Y_t^B + Z_K(\omega_t^*) p_t K_{t-1}^B + q_t^m B_t^B - (1-\tau_t^m) Z_A(\omega_t^*) A_t^B - p_t K_t^B - F R_t^B - \Psi(R_t^B, A_t^B) \tag{20}$$

$$A_{t+1}^B = e^{-g_{t+1}} \left[ \delta Z_A(\omega_t^*) A_t^B - R_t^B + B_t^B \right] \tag{21}$$

$$\phi p_t K_t^B \geq F \left[ \delta Z_A(\omega_t^*) A_t^B - R_t^B + B_t^B \right] \tag{22}$$

$$0 \leq R_t^B \leq \delta Z_A(\omega_t^*) A_t^B \tag{23}$$

$$\mathcal{S}_{t+1}^B = h(\mathcal{S}_t^B) \tag{24}$$

where the functions $Z_K$ and $Z_A$ are defined in the preliminaries above.

The continuation value $\tilde{V}^B(\cdot)$ must take into account the default decision of the risk taker at the beginning of next period. We anticipate here and show below that the default decision takes the form of a cutoff rule:

$$\begin{aligned} \tilde{V}^B\left(K_{t-1}^B, A_t^B, \mathcal{S}_t^B\right) =& F_\rho(\rho_t^*) \mathrm{E}_\rho \left[ V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^B) \,|\, \rho < \rho_t^* \right] + (1 - F_\rho(\rho_t^*)) \, \mathrm{E}_\rho \left[ V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^B) \,|\, \rho > \rho_t^* \right] \\ =& F_\rho(\rho_t^*) V^S(K_{t-1}^B, A_t^B, \mathcal{S}_t^S(\rho_t < \rho_t^*)) + (1 - F_\rho(\rho_t^*)) \, V^S(K_{t-1}^B, A_t^B, \mathcal{S}_t^S(\rho_t > \rho_t^*)), \end{aligned} \tag{25}$$

where (25) obtains because the expectation terms conditional on realizations of $\rho_t$ and $\rho_t^*$ only differ in the values of the aggregate state variables.

Denote the value function and the partial derivatives of the value function as:

$$\begin{aligned} V_t^B &\equiv V(K_{t-1}^B, A_t^B, \mathcal{S}_t^B), \\ V_{A,t}^B &\equiv \frac{\partial V(K_{t-1}^B, A_t^B, \mathcal{S}_t^B)}{\partial A_t^B}, \\ V_{K,t}^B &\equiv \frac{\partial V(K_{t-1}^B, A_t^B, \mathcal{S}_t^B)}{\partial K_{t-1}^B}. \end{aligned}$$

Therefore the marginal values of borrowing and of housing of $\tilde{V}^B(\cdot)$ are:

$$\begin{aligned} \tilde{V}_{A,t}^B &= F_\rho(\rho_t^*) \frac{\partial V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^B(\rho_t < \rho_t^*))}{\partial A_t^B} + (1 - F_\rho(\rho_t^*)) \frac{\partial V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^B(\rho_t > \rho_t^*)}{\partial A_t^B} \\ \tilde{V}_{K,t}^B &= F_\rho(\rho_t^*) \frac{\partial V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^B(\rho_t < \rho_t^*))}{\partial K_{t-1}^B} + (1 - F_\rho(\rho_t^*)) \frac{\partial V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^B(\rho_t > \rho_t^*)}{\partial K_{t-1}^B} \end{aligned}$$

Denote the certainty equivalent of future utility as:

$$CE_t^B = \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^B(K_t^B, A_{t+1}^B, \mathcal{S}_{t+1}^B)\right)^{1-\sigma_B}\right]^{\frac{1}{1-\sigma_B}}$$

Recall that

$$u_t^B = \left(C_t^B\right)^{1-\theta}\left(A_K K_{t-1}^B\right)^{\theta}$$

### A.1.3 First-order conditions

**New mortgages** The FOC for new mortgage loans $B_t^B$ is:

$$\begin{aligned}
0 = \frac{1}{1-1/\nu}&\Bigg\{(1-\beta_B)\left[\left(C_t^B\right)^{1-\theta}\left(A_K K_{t-1}^B\right)^{\theta}\right]^{1-1/\nu} + \\
&+ \beta_B \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^B(K_t^B, A_{t+1}^B, \mathcal{S}_{t+1}^B)\right)^{1-\sigma_B}\right]^{\frac{1-1/\nu}{1-\sigma_B}}\Bigg\}^{\frac{1}{1-1/\nu}-1} \times \\
&\times\Bigg\{(1-1/\nu)(1-\beta_B)\left[\left(C_t^B\right)^{1-\theta}\left(A_K K_{t-1}^B\right)^{\theta}\right]^{-1/\nu}(1-\theta)(A_K K_{t-1}^B)^{\theta}(C_t^B)^{-\theta}q_t^m + \\
&+ \beta_B\frac{1-1/\nu}{1-\sigma_B}\mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^B(K_t^B, A_{t+1}^B, \mathcal{S}_{t+1}^B)\right)^{1-\sigma_B}\right]^{\frac{1-1/\nu}{1-\sigma_B}-1} \times \\
&\times E_t\left[(1-\sigma_B)\left(e^{g_{t+1}}\tilde{V}^B(K_t^B, A_{t+1}^B, \mathcal{S}_{t+1}^B)\right)^{-\sigma_B} e^{g_{t+1}}\tilde{V}_{A,t+1}^B e^{-g_{t+1}}\right]\Bigg\} - \lambda_t^B F
\end{aligned}$$

where $\lambda_t^B$ is the Lagrange multiplier on the borrowing constraint.

Simplifying, we get:

$$\begin{aligned}
&q_t^m\frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu} = \\
&\lambda_t^B F - \beta_B \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B}\tilde{V}_{A,t+1}^B](CE_t^B)^{\sigma_B-1/\nu}(V_t^B)^{1/\nu}
\end{aligned} \tag{26}$$

Observe that we can rewrite equation (26) as:

$$q_t^m = \frac{C_t^B}{(1-\theta)(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}}\left\{\lambda_t^B F - \beta_B \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B}\tilde{V}_{A,t+1}^B](CE_t^B)^{\sigma_B-1/\nu}(V_t^B)^{1/\nu}\right\}.$$

We define the rescaled Lagrange multiplier of the borrower as the original multiplier divided by marginal utility of current consumption:

$$\tilde{\lambda}_t^B = \lambda_t^B\frac{C_t^B}{(1-\theta)(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}}.$$

Then we can solve for the mortgage price as:

$$q_t^m = \tilde{\lambda}_t^B F - \beta_B\frac{C_t^B\left\{\mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B}\tilde{V}_{A,t+1}^B](CE_t^B)^{\sigma_B-1/\nu}(V_t^B)^{1/\nu}\right\}}{(1-\theta)(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}}. \tag{27}$$

**Houses** The FOC for new purchases of houses $K_t^B$ is:

$$0 = \frac{1}{1-1/\nu}(V_t^B)^{1/\nu} \times \Big\{-(1-1/\nu)(1-\beta_B)(u_t^B)^{-1/\nu}(1-\theta)(A_K K_{t-1}^B)^{\theta}(C_t^B)^{-\theta}p_t+$$
$$+\frac{1-1/\nu}{1-\sigma_B}\beta_B(CE_t^B)^{\sigma_B-1/\nu}\mathrm{E}_t[(1-\sigma_B)(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B}e^{g_{t+1}}\tilde{V}_{K,t+1}^B]\Big\} + \lambda_t^B\phi p_t.$$

Simplifying, we get:

$$p_t\frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu} =$$
$$\lambda_t^B\phi p_t + \beta_B\mathrm{E}_t[e^{(1-\sigma_B)g_{t+1}}(\tilde{V}_{t+1}^B)^{-\sigma_B}\tilde{V}_{K,t+1}^B](CE_t^B)^{\sigma_B-1/\nu}(V_t^B)^{1/\nu} \tag{28}$$

**Default Threshold** Taking the first-order condition with respect to $\omega_t^\star$ and using the expressions for the derivatives of $Z_K(\cdot)$ and $Z_A(\cdot)$ in the preliminaries above yields:

$$f_\omega(\omega_t^*)\left[\omega_t^* p_t K_{t-1}^B - (1-\tau_t^m)A_t^B\right]\frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu} =$$
$$\delta A_t^B f_\omega(\omega_t^*)\left\{\lambda_t^B F - \lambda_t^{RB} - \beta_B\mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}_{t+1}^B\right)^{-\sigma_B}\tilde{V}_{A,t+1}^B\right]\times(CE_t^B)^{\sigma_B-1/\nu}(V_t^B)^{1/\nu}\right\}.$$

This can be simplified by replacing the term in braces on the right-hand side using the FOC for new loans (27) and solving for $\omega_t^*$ to give:

$$\omega_t^* = \frac{A_t^B(1-\tau_t^m+\delta q_t^m - \delta\tilde{\lambda}_t^{RB})}{p_t K_{t-1}^B}, \tag{29}$$

where the rescaled Lagrange multiplier on the upper refinancing bound is:

$$\tilde{\lambda}_t^{RB} = \lambda_t^{RB}\frac{C_t^B}{(1-\theta)(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}}.$$

**Prepayment** The FOC for repayments $R_t^B$ is:

$$[F+\Psi_R(R_t^B,A_t^B)]\frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu} =$$
$$\mu_t^{RB} - \lambda_t^{RB} + \lambda_t^B F - \beta_B\mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B}\tilde{V}_{A,t+1}^B](CE_t^B)^{\sigma_B-1/\nu}(V_t^B)^{1/\nu}, \tag{30}$$

where $\lambda_t^{RB}$ is the Lagrange multiplier on the upper bound on $R_t^B$ and $\mu_t^{RB}$ is the Lagrange multiplier on the lower bound. Combining with (27), we obtain:

$$\Psi_R(R_t^B,A_t^B) = q_t^m - F + \tilde{\mu}_t^{RB} - \tilde{\lambda}_t^{RB},$$

where we defined the lower bound Lagrange multiplier on refinancing as the original multiplier divided by marginal utility of consumption:

$$\tilde{\mu}_t^{RB} = \mu_t^B\frac{C_t^B}{(1-\theta)(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}}.$$

Recall the definition $Z_t^R = R_t^B/A_t^B$. Using the functional form of $\Psi_R$ from (18), the optimal prepayment fraction is:

$$Z_t^R = \frac{1}{\psi}\left(q_t^m - F + \tilde{\mu}_t^{RB} - \tilde{\lambda}_t^{RB}\right) \tag{31}$$

### A.1.4 Marginal Values of State Variables and SDF

**Mortgages** Taking the derivative of the value function with respect to $A_t^B$ gives:

$$V_{A,t}^B = -\left(1 - \tau_t^m + \frac{\Psi_A(R_t^B, A_t^B)}{Z_A(\omega_t^*)}\right) Z_A(\omega_t^*) \frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}$$
$$- \delta Z_A(\omega_t^*)\{\lambda_t^B F - \lambda_t^{RB} - \beta_B \mathrm{E}_t[e^{g_{t+1}}(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B}\tilde{V}_{A,t+1}^B] \times (CE_t^B)^{\sigma_B - 1/\nu}(V_t^B)^{1/\nu}\}.$$

Note that we can substitute for the term in braces using equation (26) and for $\Psi_A$ using (19):

$$V_{A,t}^B = -Z_A(\omega_t^*)\left(1 - \tau_t^m - \frac{\psi\left(Z_t^R\right)^2}{2Z_A(\omega_t^*)} + \delta q_t^m - \delta\tilde{\lambda}_t^{RB}\right)\frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}. \tag{32}$$

**Houses** Taking the derivative of the value function with respect to $K_{t-1}^B$ gives:

$$V_{K,t}^B = \left[p_t Z_K(\omega_t^*) + \frac{\theta C_t^B}{(1-\theta)K_{t-1}^B}\right]\frac{1-\theta}{C_t^B}(V_t^B)^{1/\nu}(1-\beta_B)(u_t^B)^{1-1/\nu}. \tag{33}$$

**SDF** Define the borrower's intertemporal marginal rate of substitution between $t$ and $t+1$, conditional on a particular realization of $\rho_{t+1}$ as:

$$\begin{aligned}
\mathcal{M}_{t,t+1}^B(\rho_{t+1}) &= \frac{\partial V_t^B/\partial C_{t+1}^B}{\partial V_t^B/\partial C_t^B} = \frac{\partial V_t^B}{\partial V_{t+1}^B} e^{-g_{t+1}} \frac{\partial V_{t+1}^B/\partial C_{t+1}^B}{\partial V_t^B/\partial C_t^B} \\
&= (V_t^B)^{1/\nu}\beta_B(CE_t^B)^{\sigma_B - 1/\nu}(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B}\frac{\frac{1-\theta}{C_{t+1}^B}(1-\beta_B)(V_{t+1}^B)^{1/\nu}(u_{t+1}^B)^{1-1/\nu}}{\frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}} \\
&= \beta_B e^{-\sigma_B g_{t+1}}\left(\frac{C_{t+1}^B}{C_t^B}\right)^{-1}\left(\frac{u_{t+1}^B}{u_t^B}\right)^{1-1/\nu}\left(\frac{V_{t+1}^B}{CE_t^B}\right)^{-(\sigma_B - 1/\nu)}
\end{aligned}$$

We can then define the stochastic discount factor (SDF) of borrowers as:

$$\tilde{\mathcal{M}}_{t,t+1}^B = F_\rho(\rho_{t+1}^*)\mathcal{M}_{t,t+1}^B(\rho_{t+1} < \rho_{t+1}^*) + (1 - F_\rho(\rho_{t+1}^*))\mathcal{M}_{t,t+1}^B(\rho_{t+1} > \rho_{t+1}^*),$$

where $\mathcal{M}_{t,t+1}^B(\rho_{t+1} < \rho_{t+1}^*)$ and $\mathcal{M}_{t,t+1}^B(\rho_{t+1} > \rho_{t+1}^*)$ are the IMRSs, conditional on the two possible realizations of state variables.

### A.1.5 Euler Equations

**Mortgages** Recall that $\tilde{V}_{A,t+1}^B$ is a linear combination of $V_{A,t+1}^B$ conditional on $\rho_t$ being below and above the threshold, and with each $V_{A,t+1}^B$ given by equation (32). Substituting in for $\tilde{V}_{A,t+1}^B$ in (27) and using the SDF expression, we get the recursion:

$$q_t^m = \tilde{\lambda}_t^B F + \mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^B Z_A(\omega_{t+1}^*)\left(1 - \tau^m - \frac{\psi\left(Z_{t+1}^R\right)^2}{2Z_A(\omega_{t+1}^*)} - \delta\tilde{\lambda}_{t+1}^{RB} + \delta q_{t+1}^m\right)\right]. \tag{34}$$

**Houses** Likewise, observe that we can write (28) as:

$$p_t\left[1 - \tilde{\lambda}_t^B\phi\right] = \frac{\beta_B \mathrm{E}_t[e^{g_{t+1}}(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B}\tilde{V}_{K,t+1}^B](CE_t^B)^{\sigma_B - 1/\nu}(V_t^B)^{1/\nu}}{\frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}}$$

Recall that $\tilde{V}^B_{K,t+1}$ is a linear combination of $V^B_{K,t+1}$ conditional on $\rho_t$ being below and above the threshold, and with each $V^B_{K,t+1}$ given by equation (33). Substituting in for $\tilde{V}^B_{K,t+1}$ and using the SDF expression, we get the recursion:

$$p_t\left[1-\tilde{\lambda}^B_t\phi\right] = \mathrm{E}_t\left[\tilde{\mathcal{M}}^B_{t,t+1}e^{g_{t+1}}\left\{p_{t+1}Z_K(\omega^*_{t+1}) + \frac{\theta C^B_{t+1}}{(1-\theta)K^B_t}\right\}\right] \tag{35}$$

## A.2 Depositor

We solve a restricted problem where we assume that the depositor does not buy private long-term mortgage bonds. After solving the model under this assumption, we go back and verify that the depositor indeed does not want to deviate from a zero position in equilibrium.

### A.2.1 Statement of stationary problem

Let $\mathcal{S}^D_t = \left(g_t, \sigma_{\omega,t}, W^R_t, A^B_t, B^G_{t-1}\right)$ be the depositor's state vector capturing all exogenous state variables. Scaling by permanent income, the stationary problem of the depositor -after the risk taker has made default her decision and the utility cost of default is realized- is:

$$\begin{aligned} V^D(W^D_t, \mathcal{S}^D_t) = \max_{\{C^D_t, B^D_t, A^D_{t+1,G}\}} & \left\{(1-\beta_D)\left[\left(C^D_t\right)^{1-\theta}\left(A_K K^D_{t-1}\right)^{\theta}\right]^{1-1/\nu} + \right. \\ & \left. + \beta_D \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^D(W^D_{t+1}, \mathcal{S}^D_{t+1})\right)^{1-\sigma_D}\right]^{\frac{1-1/\nu}{1-\sigma_D}}\right\}^{\frac{1}{1-1/\nu}} \end{aligned}$$

subject to

$$\begin{aligned} C^D_t &= (1-\tau^S_t)Y^D_t + W^D_t - (q^m_t + \gamma_t)A^D_{t+1,G} - q^f_t B^D_t - (1-\mu_{t,\omega})p_t K^D_{t-1} && (36) \\ W^D_{t+1} &= e^{-g_{t+1}}\left[(M_{t+1,G} + \delta Z_A(\omega^*_{t+1})q^m_{t+1} - Z^R_{t+1}[q^m_{t+1} - F])A^D_{t+1,G} + B^D_t\right] && (37) \\ B^D_t &\geq 0 && (38) \\ A^D_{t+1,G} &\geq 0 && (39) \\ \mathcal{S}^D_{t+1} &= h(\mathcal{S}^D_t) && (40) \end{aligned}$$

As before, we will drop the arguments of the value function and denote marginal values of wealth and mortgages as:

$$\begin{aligned} V^D_t &\equiv V^D_t(W^D_t, \mathcal{S}^D_t), \\ V^D_{W,t} &\equiv \frac{\partial V^D_t(W^D_t, \mathcal{S}^D_t)}{\partial W^D_t}, \end{aligned}$$

Denote the certainty equivalent of future utility as:

$$CE^D_t = \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^D(W^D_t, \mathcal{S}^D_t)\right)^{1-\sigma_D}\right],$$

and the composite within-period utility as:

$$u^D_t = (C^D_t)^{1-\theta}(A_K K^D_{t-1})^{\theta}.$$

Like the borrower, the depositor must take into account the risk-taker's default decisions and the realization of the utility penalty of default. Therefore the marginal value of wealth is:

$$\tilde{V}^D_{W,t} = F_\rho(\rho^*_t)\frac{\partial V^D(W^D_t, \mathcal{S}^D_t(\rho_t < \rho^*_t))}{\partial W^D_t} + (1 - F_\rho(\rho^*_t))\,\frac{\partial V^D(W^D_t, \mathcal{S}^D_t(\rho_t > \rho^*_t)}{\partial W^D_t}.$$

### A.2.2 First-order conditions

The first-order condition for the short-term bond position is:

$$\begin{aligned} &q^f_t \frac{1-\theta}{C^D_t}(1-\beta_D)(V^D_t)^{1/\nu}(u^D_t)^{1-1/\nu} = \\ &\lambda^D_t + \beta_D \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}^D_{t+1})^{-\sigma_D}\tilde{V}^D_{W,t+1}](CE^D_t)^{\sigma_D - 1/\nu}(V^D_t)^{1/\nu} \end{aligned} \tag{41}$$

where $\lambda^D_t$ is the Lagrange multiplier on the no-borrowing constraint (38).

The first order condition for the government-guaranteed mortgage bond position is:

$$\begin{aligned} &(q^m_t + \gamma_t)\frac{1-\theta}{C^D_t}(1-\beta_D)(V^D_t)^{1/\nu}(u^D_t)^{1-1/\nu} = \\ &\mu^D_{G,t} + \beta_D \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}^D_{t+1})^{-\sigma_D}\tilde{V}^D_{W,t+1}\left(M_{G,t+1} + \delta Z_A(\omega^*_{t+1})q^m_{t+1} - Z^R_{t+1}[q^m_{t+1} - F]\right)](CE^D_t)^{\sigma_D - 1/\nu}(V^D_t)^{1/\nu}, \end{aligned} \tag{42}$$

where $\mu^D_{t,G}$ is the Lagrange multiplier on the no-shorting constraint for guaranteed loans (39).

### A.2.3 Marginal Values of State Variables and SDF

Marginal value of wealth is:

$$V^D_{W,t} = \frac{1-\theta}{C^D_t}(1-\beta_D)(V^D_t)^{1/\nu}(u^D_t)^{1-1/\nu}, \tag{43}$$

and for the continuation value function:

$$\tilde{V}^D_{W,t} = F_\rho(\rho^*_t)\frac{\partial V^D(W^D_t, \mathcal{S}^D_t(\rho_t < \rho^*_t))}{\partial W^D_t} + (1 - F_\rho(\rho^*_t))\,\frac{\partial V^D(W^D_t, \mathcal{S}^D_t(\rho_t > \rho^*_t)}{\partial W^D_t}.$$

Defining the SDF in the same fashion as we did for the borrower, we get:

$$\mathcal{M}^D_{t,t+1}(\rho_t) = \beta_D e^{-\sigma_D g_{t+1}}\left(\frac{V^D_{t+1}}{CE^D_t}\right)^{-(\sigma_D - 1/\nu)}\left(\frac{C^D_{t+1}}{C^D_t}\right)^{-1}\left(\frac{u^D_{t+1}}{u^D_t}\right)^{1-1/\nu},$$

and

$$\tilde{\mathcal{M}}^D_{t,t+1} = F_\rho(\rho^*_{t+1})\mathcal{M}^D_{t,t+1}(\rho_{t+1} < \rho^*_{t+1}) + (1 - F_\rho(\rho^*_{t+1}))\mathcal{M}^D_{t,t+1}(\rho_{t+1} > \rho^*_{t+1}).$$

### A.2.4 Euler Equations

Combining the first-order condition for short-term bonds (41) with the marginal value of wealth, and the SDF, we get the Euler equation for the short-term bond:

$$q^f_t = \tilde{\lambda}^D_t + \mathrm{E}_t\left[\tilde{\mathcal{M}}^D_{t,t+1}\right] \tag{44}$$

where $\tilde{\lambda}^D_t$ is the original multiplier $\lambda^D_t$ divided by the marginal value of wealth.

Similarly, from (42) we get the Euler Equation for guaranteed mortgages:

$$q^m_t + \gamma_t = \tilde{\mu}^D_{G,t} + \mathrm{E}_t\left[\tilde{\mathcal{M}}^D_{t,t+1}\left(M_{G,t+1} + \delta Z_A(\omega^*_{t+1})q^m_{t+1} - Z^R_{t+1}[q^m_{t+1} - F]\right)\right] \tag{45}$$

## A.3 Risk Takers

### A.3.1 Statement of stationary problem

Denote by $W_t^R$ risk taker wealth at the beginning of the period, before their bankruptcy decision. Then wealth after realization of the penalty $\rho_t$ is:

$$\tilde{W}_t^R = (1 - D(\rho_t))W_t^R,$$

and the effective utility penalty is:

$$\tilde{\rho}_t = D(\rho_t)\rho_t.$$

Let $\mathcal{S}_t^R = \left(g_t, \sigma_{\omega,t}, W_t^D, A_t^B, B_{t-1}^G\right)$ denote all other aggregate state variables exogenous to risk takers.

After the default decision, risk takers face the following optimization problem over consumption and portfolio composition, formulated to ensure stationarity:

$$\begin{aligned} V^R(\tilde{W}_t^R, \tilde{\rho}_t, \mathcal{S}_t^R) = \max_{C_t^R, A_{t+1,P}^R, A_{t+1,P}^R, B_t^R} & \left\{ (1-\beta_R)\left[\frac{(C_t^R)^{1-\theta}(K_{t-1}^R)^{\theta}}{e^{\tilde{\rho}_t}}\right]^{1-1/\nu} \right. \\ & \left. + \beta_R \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^R\left(W_{t+1}^R, \mathcal{S}_{t+1}^R\right)\right)^{1-\sigma_R}\right]^{\frac{1-1/\nu}{1-\sigma_R}} \right\}^{\frac{1}{1-1/\nu}} \end{aligned} \tag{46}$$

subject to:

$$(1-\tau^S)Y_t^R + \tilde{W}_t^R = C_t^R + (1-\mu_\omega)p_t K_{t-1}^R + q_t^m A_{t+1,P}^R + (q_t^m + \gamma_t)A_{t+1,G}^R + q_t^f B_t^R, \tag{47}$$

$$\begin{aligned} W_{t+1}^R = e^{-g_{t+1}} & \left[(M_{t+1,P} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m - Z_{t+1}^R[q_{t+1}^m - F])A_{t+1,P}^R \right. \\ & \left. + (M_{t+1,G} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m - Z_{t+1}^R[q_{t+1}^m - F])A_{t+1,G}^R + B_t^R\right], \end{aligned} \tag{48}$$

$$B_t^R \geq -q_t^m(\xi_P A_{t+1,P}^R + \xi_G A_{t+1,G}^R), \tag{49}$$

$$A_{t+1,G}^R \geq 0, \tag{50}$$

$$A_{t+1,P}^R \geq 0, \tag{51}$$

$$\mathcal{S}_{t+1}^R = h(\mathcal{S}_t^R). \tag{52}$$

The continuation value $\tilde{V}^R\left(W_{t+1}^R, \mathcal{S}_{t+1}^R\right)$ is the outcome of the optimization problem risk takers face at the beginning of the following period, i.e., before the decision over the optimal bankruptcy rule. This continuation value function is given by:

$$\tilde{V}^R(W_t^R, \mathcal{S}_t^R) = \max_{D(\rho)} \mathrm{E}_\rho\left[D(\rho)V^R(0, \rho, \mathcal{S}_t^R) + (1 - D(\rho))V^R(W_t^R, 0, \mathcal{S}_t^R)\right] \tag{53}$$

Define the certainty equivalent of future utility as:

$$CE_t^R = \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^R\left(W_{t+1}^R, \mathcal{S}_{t+1}^R\right)\right)^{1-\sigma_R}\right]^{\frac{1}{1-\sigma_R}}. \tag{54}$$

and the composite within-period utility (evaluated at $\rho = 0$) as:

$$u_t^R = (C_t^R)^{1-\theta}(A_K K_{t-1}^R)^{\theta}.$$

### A.3.2 First-order conditions

**Optimal Default Decision** The optimization consists of choosing a function $D(\rho) : \mathbb{R} \to \{0, 1\}$ that specifies for each possible realization of the penalty $\rho$ whether or not to default.

Since the value function $V^R(W, \rho, \mathcal{S}_t^R)$ defined in (46) is increasing in wealth $W$ and decreasing in the penalty $\rho$, there will generally exist an optimal threshold penalty $\rho^*$ such that for a given $W_t^R$, risk-takers optimally default for all realizations $\rho < \rho^*$. Hence we can equivalently write the optimization problem in (53) as

$$\begin{aligned}\tilde{V}^R(W_t^R, \mathcal{S}_t^R) =& \max_{\rho^*} \mathrm{E}_\rho \left[\mathbb{1}[\rho < \rho^*]\; V^R(0, \rho, \mathcal{S}_t^R) + (1 - \mathbb{1}[\rho < \rho^*])\, V^R(W_t^R, 0, \mathcal{S}_t^R)\right] \\ =& \max_{\rho^*} F_\rho(\rho^*)\; \mathrm{E}_\rho \left[V^R(0, \rho, \mathcal{S}_t^R) \,|\, \rho < \rho^*\right] + (1 - F_\rho(\rho^*))\, V^R(W_t^R, 0, \mathcal{S}_t^R).\end{aligned}$$

The solution $\rho_t^*$ is characterized by the first-order condition:

$$V^R(0, \rho_t^*, \mathcal{S}_t^R) = V^R(W_t^S, 0, \mathcal{S}_t^R).$$

By defining the partial inverse $\mathcal{F} : (0, \infty) \to (-\infty, \infty)$ of $V^S(\cdot)$ in its second argument as

$$\left\{(x, y) : y = \mathcal{F}(x) \Leftrightarrow x = V^R(0, y)\right\},$$

we get that

$$\rho_t^* = \mathcal{F}(V^R(W_t^R, 0, \mathcal{S}_t^R)), \tag{55}$$

and by substituting the solution into (53), we obtain

$$\tilde{V}^R(W_t^R, \mathcal{S}_t^R) = F_\rho(\rho_t^*)\mathrm{E}_\rho \left[V^R(0, \rho, \mathcal{S}_t^R) \,|\, \rho < \rho_t^*\right] + (1 - F_\rho(\rho_t^*))\, V^R(W_t^R, 0, \mathcal{S}_t^R). \tag{56}$$

Equations (46), (55), and (56) completely characterize the optimization problem of risk-takers.

To compute the optimal bankruptcy threshold $\rho_t^*$, note that the inverse value function defined in equation (55) is given by:

$$\mathcal{F}(x) = \begin{cases} \log((1 - \beta_R)u_t^R) - \frac{1}{1-1/\nu}\log\left(x^{1-1/\nu} - \beta_R(CE_t^R)^{1-1/\nu}\right) & \text{for } \nu > 1 \\ (1 - \beta_R)\log(u_t^R) + \beta_R \log(CE_t^R) - \log(x) - (1 - \beta_R) & \text{if } \nu = 1. \end{cases}$$

**Optimal Portfolio Choice** The first-order condition for the short-term bond position is:

$$\begin{aligned} & q_t^f \frac{1-\theta}{C_t^R}(1 - \beta_R)(V_t^R)^{1/\nu}(u_t^R)^{1-1/\nu} = \\ & \lambda_t^R + \beta_R \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^R)^{-\sigma_R}\tilde{V}_{W,t+1}^R](CE_t^R)^{\sigma_R - 1/\nu}(V_t^R)^{1/\nu} \end{aligned} \tag{57}$$

where $\lambda_t^R$ is the Lagrange multiplier on the borrowing constraint (49).

The first order condition for the government-guaranteed mortgage bond position is:

$$\begin{aligned} & (q_t^m + \gamma_t)\frac{1-\theta}{C_t^R}(1 - \beta_R)(V_t^R)^{1/\nu}(u_t^R)^{1-1/\nu} = \lambda_t^R \xi_G q_t^m + \mu_{G,t}^R \\ & + \beta_R \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^R)^{-\sigma_R}\tilde{V}_{W,t+1}^R \left(M_{G,t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m - Z_{t+1}^R[q_{t+1}^m - F]\right)](CE_t^R)^{\sigma_R - 1/\nu}(V_t^R)^{1/\nu}, \end{aligned} \tag{58}$$

where $\mu_{t,G}^R$ is the Lagrange multiplier on the no-shorting constraint for guaranteed loans (50).

The first order condition for the private mortgage bond position is:

$$\begin{aligned} & q_t^m \frac{1-\theta}{C_t^R}(1 - \beta_R)(V_t^R)^{1/\nu}(u_t^R)^{1-1/\nu} = \lambda_t^R \xi_P q_t^m + \mu_{P,t}^R \\ & + \beta_R \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^R)^{-\sigma_R}\tilde{V}_{W,t+1}^R \left(M_{P,t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m - Z_{t+1}^R[q_{t+1}^m - F]\right)](CE_t^R)^{\sigma_R - 1/\nu}(V_t^R)^{1/\nu}, \end{aligned} \tag{59}$$

where $\mu_{t,P}^R$ is the Lagrange multiplier on the no-shorting constraint for guaranteed loans (51).

### A.3.3 Marginal value of wealth and SDF

Differentiating (56) gives the marginal value of wealth

$$\tilde{V}^R_{W,t} = (1 - F_\rho(\rho^*_t))\,\frac{\partial V^R(W^R_t, 0, \mathcal{S}^R_t)}{\partial W^R_t},$$

where

$$\frac{\partial V^R(W^R_t, 0, \mathcal{S}^R_t)}{\partial W^R_t} = \frac{1-\theta}{C^R_t}(1-\beta_R)(V^R(W^R_t, 0, \mathcal{S}^R_t))^{1/\nu}(u^R_t)^{1-1/\nu},$$

The stochastic discount factor of risk-takers is therefore

$$\mathcal{M}^R_{t,t+1} = \beta_R e^{-\sigma_R g_{t+1}} \left(\frac{V^R(W^R_{t+1}, 0, \mathcal{S}^R_{t+1})}{CE^R_t}\right)^{-(\sigma_R - 1/\nu)} \left(\frac{C^R_{t+1}}{C^R_t}\right)^{-1} \left(\frac{u^R_{t+1}}{u^R_t}\right)^{1-1/\nu},$$

and

$$\tilde{\mathcal{M}}^R_{t,t+1} = (1 - F_\rho(\rho^*_{t+1}))\mathcal{M}^R_{t,t+1}$$

### A.3.4 Euler Equations

It is then possible to show that the FOC with respect to $B^R_t$, $A^R_{t+1,G}$, and $A^R_{t+1,P}$ respectively, are:

$$q_t = \tilde{\lambda}^R_t + \mathrm{E}_t\left[\tilde{\mathcal{M}}^R_{t,t+1}\right], \tag{60}$$

$$q^m_t(1 - \xi_G\tilde{\lambda}^R_t) + \gamma_t = \tilde{\mu}_{t,G} + \mathrm{E}_t\left[\tilde{\mathcal{M}}^R_{t,t+1}\left(M_{G,t+1} + \delta Z_A(\omega^*_{t+1})q^m_{t+1} - Z^R_{t+1}[q^m_{t+1} - F]\right)\right], \tag{61}$$

$$q^m_t(1 - \xi_P\tilde{\lambda}^R_t) = \tilde{\mu}_{t,P} + \mathrm{E}_t\left[\tilde{\mathcal{M}}^R_{t,t+1}\left(M_{P,t+1} + \delta Z_A(\omega^*_{t+1})q^m_{t+1} - Z^R_{t+1}[q^m_{t+1} - F]\right)\right]. \tag{62}$$

## A.4 Equilibrium

The optimality conditions describing the problem are (20), (29), (31), (34) and (35) for borrowers, (36), (44), and (45) for depositors, and (47), (60), (61), and (62) for risk takers. We add complementary slackness conditions for the constraints (22) and (23) for borrowers, (38) and (39) for depositors, and (49), (50), and (51) for risk-takers. Together with the market clearing conditions, these equations fully characterize the economy.

# B Calibration Appendix

## B.1 States and transition Probabilities

After discretizing the aggregate real per capita income growth process as a Markov chain using the Rouwenhorst method, we obtain the following five states for $g$:

$$[0.943, 0.980, 1.018, 1.058, 1.101]$$

with $5 \times 5$ transition probability matrix:

$$\begin{bmatrix} 0.254 & 0.415 & 0.254 & 0.069 & 0.007 \\ 0.103 & 0.381 & 0.363 & 0.134 & 0.017 \\ 0.042 & 0.242 & 0.430 & 0.242 & 0.042 \\ 0.017 & 0.134 & 0.363 & 0.381 & 0.103 \\ 0.007 & 0.069 & 0.254 & 0.415 & 0.254. \end{bmatrix}$$

We discretize the process for $\sigma_\omega^2$ into a two-state Markov chain that is correlated with income growth $g$. The two states are:

$$[.078, .203]$$

The transition probability matrix, conditional on being in one of the bottom two $g$ states is:

$$\begin{bmatrix} 0.80 & 0.20 \\ 0.01 & 0.99 \end{bmatrix}$$

The transition probability matrix, conditional on being in one of the top three $g$ states is:

$$\begin{bmatrix} 1.0 & 0.0 \\ 1.0 & 0.0 \end{bmatrix}$$

The stationary distribution for the joint Markov chain of $g$ and $\sigma_\omega^2$ is

| State | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $g$ | 0.943 | 0.943 | 0.980 | 0.980 | 1.018 | 1.058 | 1.101 |
| $\sigma_\omega^2$ | 0.078 | 0.203 | 0.078 | 0.203 | 0.078 | 0.078 | 0.078 |
| Prob. | 0.039 | 0.023 | 0.167 | 0.081 | 0.372 | 0.255 | 0.063 |

From a long simulation, we obtain the following mean, standard deviation, and persistence for $g$: 1.019, .039, and .42, respectively. We obtain the following mean, standard deviation, and persistence for $\sigma_\omega^2$: .092, .039, and .46, respectively. We obtain a correlation between $g$ and $\sigma_\omega$ of -0.42.

## B.2 Evidence on default rates and mortgage severities

Since not all mortgage delinquencies result in foreclosures (loans can cure or get modified), we use the fraction of loans that 90-day or more delinquent or in foreclosure as the real world counterpart to our model's default rate. Some loans that were 90-day delinquent or more received a loan modification, but many of these modifications resulted in a redefault 12 to24 months later. Given that our model abstracts from modifications, using a somewhat broader criterion of delinquency than foreclosures-only seems warranted.

The observed 90-day plus (including foreclosures) default rate rose from 2% at the start of 2007 to just under 10% in 2010.Q1. Since then, the default rate has been gradually falling back, to 4.7% by 2014.Q3 (Mortgage Bankers Association and Urban Institute). The slow decline in foreclosure rates in the data is partly due to legal delays in the foreclosure process, especially in judicial states like New York and Florida where the average foreclosure process takes up to 1000 days. In other part it is due to re-defaults on modified loans. Since, neither is a feature of the model, it seems reasonable to interpret the abnormally high default rates of the post-2013 period as due to such delays, and to reassign them to the 2010-2012 period. If we assume that the foreclosure rate will return to its normal 2% level by the end of 2016, then such reassignment delivers an average foreclosure rate of 8.5% during the 2007-2012 foreclosure crisis. Absent reassignment, the average default rate would be 5.9% over the 2007-2016 period. Jeske et al. (2014) target only a 0.5% foreclosure rate, but their calibration is to the pre-2006 sample. The evidence from the post-2006 period dramatically raises the long-term mean default rate.

Fannie Mae's 10K filings for 2007 to 2013 show that severities, or losses-given-default, on conventional single-family loans were 4% in 2006, 11% in 2007, 26% in 2008, 37% in 2009, 34% in 2010, 35% in 2011, 31% in 2012, and 24% in 2013. Severities on Fannie's non-conforming (mostly Alt-A and subprime) portfolio holdings exceed 60% in all these years. If anything, the severity rate on Fannie's *non-conforming* holdings is lower than that of the overall non-conforming market due to advantageous selection (Adelino et al 2014). Given that the non-conforming market accounted for half of all mortgage originations in 2004-2007, the severities on conventional loans are too low to accurately reflect the market-wide severities. To take account of this composition effect, we target a market-wide severity rate of 40% in the crisis (2007-2012). We target a severity rate of 15% in non-crisis years (pre-2007 and post-2012), based on Fannie's experience in that period and the much smaller size of the non-conforming mortgage market in those years.

Combining a default rate of 2% in normal times with a severity of 15%, we obtain a loss rate of 0.3% in normal times. Combining the default rate of 8.5% during a foreclosure crisis with the severity of 40% in crises, we obtain a 3.4% loss rate.

To obtain mortgage debt to GDP in normal times and in crisis times, we calculate a time series of household mortgage debt (including debt on multi-family real estate owned by the household sector) and divide by GDP. Since mortgage debt-GDP saw a gradual decrease for reasons related to new technology, such as automated underwriting and securitization, we focus attention on the post-1985 period. Mortgage debt-GDP averages to 54% in the 1985-1999 period. We target this for our normal times value. Mortgage debt-GDP averages to 78% in the 2000-2014 period. We target that number for our crisis number.

## B.3 Long-term mortgages

Our model's mortgages are geometrically declining perpetuities, and as such have no principal. The issuer of one unit of the bond at time $t$ promises to pay the holder 1 at time $t+1$, $\delta$ at time $t+2$, $\delta^2$ at time $t+3$, and so on. If the borrower defaults on the mortgage, the government guarantee entitles the holder to receive a "principal repayment" $F = \frac{\alpha}{1-\delta}$, a constant parameter that does not depend on the value of the collateral or any state variable of the economy. Real life mortgages have a finite maturity (usually 30 years) and a principal payment. They also have a vintage (year of origination), whereas our mortgages combine all vintages in one variable. This appendix explains how to map the geometric mortgages in our model into real-world mortgages.

Our model's mortgage refers to the entire pool of all outstanding mortgages. In reality, this pool not only consists of newly issued 30-year fixed-rate mortgages (FRMs), but also of newly issued 15-year mortgages, other mortgage types such as hybrid adjustable-rate mortgages (ARMs), as well as all prior vintages of all mortgage types. This includes, for example, 30 year FRMs issued 29 years ago. The Barclays U.S. Mortgage Backed Securities (MBS) Index is the best available measure of the overall pool of outstanding government-guaranteed mortgages. It tracks agency mortgage backed pass-through securities (both fixed-rate and hybrid ARM) guaranteed by Ginnie Mae (GNMA), Fannie Mae (FNMA), and Freddie Mac (FHLMC). The index is constructed by grouping individual TBA-deliverable MBS pools into aggregates or generics based on program, coupon and vintage. For this MBS index we obtain a time series of monthly price, duration (the sensitivity of prices to interest rates), weighted-average life (WAL), and weighted-average coupon (WAC) for January 1989 until December 2014.

Our calibration strategy is to choose values for $\delta$ and $F$ so that the relationship between price and interest rate (duration) is the same for the observed Barclays MBS Index and for the model's geometric bond. We proceed in two steps. In the first step, we construct a simple model to price a pool of MBS bonds and calibrate it to match the observed time series of MBS durations. With this auxiliary model in hand, we then choose the two parameters to match the price-rate curve in the auxiliary model and the geometric mortgage model.

### B.3.1 Step 1: A simple MBS pricing model

Changes in duration of the Barclays MBS index are often driven by changes in the index composition. As mortgages are prepaid and new ones are issued with different coupons, both the weighted-average-life and weighted-average-coupon of the Index change significantly. Any model that wants to have a chance at matching the observed durations must account of these compositional changes.

For simplicity, we assume that all mortgages are 30-year fixed-rate mortgages. We construct a portfolio of MBS with *remaining* maturities ranging from 1 to 360 months. Each month, a fraction of each MBS prepays. We assume that the prepayment rate is given by a function $CPR(c-r)$ which depends on the "prepayment incentive" of that particular MBS, defined as the difference between the original coupon rate of that mortgage and the current mortgage rate. We assume that every prepayment is a refinancing: a dollar of mortgage balance prepaid result in a dollar of new mortgage balance originated at the new mortgage rate. In addition, each period an exogenously given amount of new mortgages are originated with a coupon equal to that month's mortgage rate to reflect purchase originations (as opposed to refinancing originations).

In a given month $t$, each mortgage $i$ has starting balance $bal_t^i$, pays a monthly mortgage $pmt_t^i$ of which $int_t^i$ is interest and $prin_t^i$ is scheduled principal, where $i$ is the remaining maturity of the mortgage, i.e., the mortgage was originated at time $t-(360-i)-1$. Denote the unscheduled principal payments, or prepayments, by $prp_t^i$. Let $SMM_t^i$ be the prepayment rate in month $t$ on that mortgage. The evolution equations for actual mortgage cash flows are:

$$\begin{aligned}
int_t^i &= \frac{c_{t-(360-i)-1}}{12} \times bal_t^i \\
prin_t^i &= pmt_t^i - int_t^i \\
prp_t^i &= SMM_t^i (bal_t^i - prin_t^i) \\
bal_{t+1}^{i-1} &= (1 - SMM_t^i)(bal_t^i - prin_t^i) \\
pmt_{t+1}^{i-1} &= (1 - SMM_t^i) pmt_t^i
\end{aligned}$$

The initial payment is given by the standard annuity formula, normalizing the amount borrowed to 1.

$$\begin{aligned}
pmt_t^{360} &= \frac{\frac{c_{t-1}}{12}}{1-(1+c_{t-1}/12)^{-360}} \\
bal_t^{360} &= 1 + \sum_{i=1}^{360} prp_{t-1}^i
\end{aligned}$$

The last equation says that the initial balance of new 30-year FRMs is comprised on 1 unit of purchase originations, an exogenously given flow of originations each period, plus refinancing originations which equal all prepayments from the previous period.

Furthermore, at every month $t$ we compute *projected* cash flows on each mortgage assuming mortgage rates stay constant from $t$ until maturity $i$. These projected cash flows follow the same evolution equations as presented above. Denote these projected cash flows with a tilde over the variable.

We can then compute the price $P_t$, (modified) duration $Dur_t$, and weighted-average-life $WAL_t$ of the MBS *portfolio* comprised of all vintages:

$$P_t = \sum_{i=1}^{360} \sum_{s=0}^{i} \frac{\tilde{pmt}_{t+s}^{i-s} + \tilde{prp}_{t+s}^{i-s}}{(1 + r_t/12)^s}$$

$$Dur_t = \frac{1}{1 + \frac{r_t}{12}} \sum_{i=1}^{360} \frac{1}{P_t^i} \sum_{s=0}^{i} \frac{\tilde{pmt}_{t+s}^{i-s} + \tilde{prp}_{t+s}^{i-s}}{(1 + r_t/12)^s} s$$

$$WAL_t^i = \sum_{i=1}^{360} \frac{\sum_{s=0}^{i} (\tilde{pmt}_{t+s}^{i-s} + \tilde{prp}_{t+s}^{i-s}) s}{\sum_{i=1}^{360} \sum_{s=0}^{i} (\tilde{pmt}_{t+s}^{i-s} + \tilde{prp}_{t+s}^{i-s})}$$

What remains to be specified is our prepayment model delivering the single-month mortality $SMM_t^i$ used above. Following practice, we assume an annual constant prepayment rate (CPR) which is a S-shaped function of the rate incentive: $CPR_t^i = CPR(r_t - c_{t-(360-i)-1})$:

$$CPR(x) = \underline{CPR} + (\overline{CPR} - \underline{CPR}) \left(1 - \frac{\exp(\psi(x - \bar{x})}{1 + \exp(\psi(x - \bar{x}))}\right)$$

The annual CPR implies a monthly SMM $SMM_t^i = factor_i \times (1 - (1 - (CPR_t^i)^{1/12})$. The multiplicative $factor_i$ allows us to deal with slow prepayments early in the life of the mortgage (the "ramp-up" phase) and late in the life of the mortgage (the "burn-out" phase). For simplicity, we make $factor_i$ linearly increasing from 0 in month 1 (when $i = 360$) to 1 in month 30, flat at 1 between month 30 and month 180 and linearly decreasing back to 0 between months 180 and month 360. We choose the CPR curve parameters $\{\underline{CPR}, \overline{CPR}, \psi, \bar{x}\}$ to minimize the sum of squared errors between the time series of model-implied duration $\{Dur_t\}$ and observed duration on the Barclays index.

To produce the time-series of model-implied duration $\{Dur_t\}$, we feed in the observed 30-year conventional fixed rate mortgage rate (MORTGAGE30US in FRED), $\{r_t\}$. We initialize the portfolio many years before the start of our time series data to ensure that the model is in steady state by the time our time series for the Barclays index starts. Specifically, we start the computation in April 1903 by issuing 1 MBS. By March 1933, we have a complete portfolio of 360 fixed-rate amortizing mortgages, maturing any month from April 1933 to March 1963.

The left panel of Figure 4 shows the observed time series of duration on the Barclays MBS index plotted against the model-implied duration on the MBS pool. The two time series track each other quite closely despite several strong modeling assumptions. The resulting CPR curve looks close to historical average prepayment behavior on agency MBS, as prepayment data from SIFMA indicate. CPR is slightly above 40% when the rate incentive is 200 basis points or more, about 15% when the rate incentive is zero, and slightly above 5% when the rate incentive is below -200 basis points.

### B.3.2 Step 2: Matching MBS pool to perpetual mortgage in our model

With a well-calibrated auxiliary model for a MBS pool, we now proceed to match key features of that auxiliary model's MBS pool to the mortgage in our model, which is a geometrically declining perpetuity.

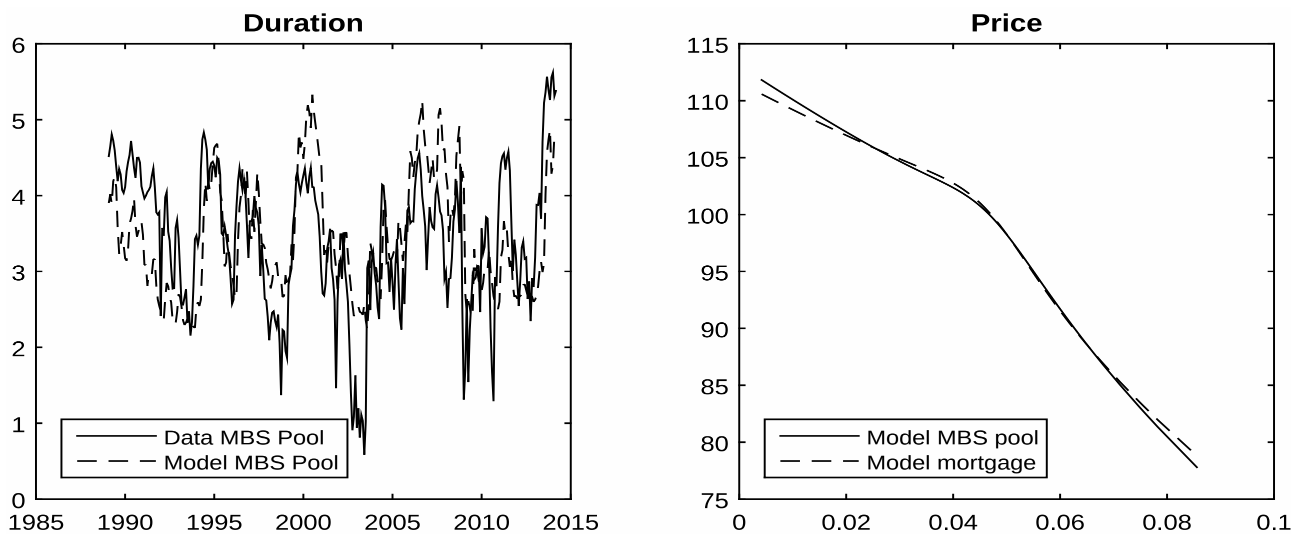
We start by computing the price $P(r)$ of a fixed-rate MBS with maturity $T$ and coupon $c$ as a function of the current real MBS rate $r$, using the constant prepayment rate function $\hat{CPR}(r) = CPR(r - c)$ obtained from step 1. For $T$ and $c$ we use the time-series average of the weighted-average maturity and weighted-average real coupon, respectively, from the model-implied MBS pool obtained in step 1.[27]

We can write the steady-state price of a guaranteed geometric mortgage with parameters $(\delta, F)$ and a per-

[27] To get real mortgage rates from nominal mortgage rates, we subtract realized inflation over the following year. To get real coupons and MBS rates from real mortgage rates, we subtract 50 bps to account for servicing and guarantee fees.

Figure 4: Matching Mortgages in Model to Data

The left panel plots the observed time series of duration on the Barclays MBS index (solid line) plotted against the duration on the model-implied MBS pool (dashed-line). The right panel plots the mortgage price-interest rate relationship for the model-implied MBS pool (solid line) and the model-implied geometrically declining perpetual mortgage (dashed line). Prices on a \$100 face value mortgage are on the vertical axis, while interest rates are on the horizontal axis. The Barclays MBS index data are from Bloomberg for the period 1989 until 2014 (daily frequency). The calculations also use the 30-year fixed-rate mortgage rate from FRED.




period fee $\gamma$ paid for the life of the loan recursively as:

$$Q(r,\gamma)+\gamma=\frac{1}{1+r}\left(1+C\hat{P}R(r)\delta F+(1-C\hat{P}R(r))\delta(Q(r,\gamma)+\gamma)\right)$$

Solving for $Q(r,\gamma)$, we get

$$Q(r,\gamma)=\frac{1+C\hat{P}R(r)\delta F}{1+r-\delta(1-C\hat{P}R(r))}-\gamma. \tag{63}$$

Note that the fee $\gamma$ in equation 63 is quoted in units of the guaranteed bond's price. However, in the data MBS pool we observe a guarantee and servicing fee of approximately 50 bp on average that is charged as a spread on top of a bond's yield. During the calibration, we thus need to use the net-of-fees rate for the MBS pool and the gross-of-fees rate for the geometric bond.

The stage 2 calibration determines how many units $X$ of the geometric mortgage with parameters $(\delta, F)$ one needs to sell to hedge one unit of the MBS against parallel shifts in interest rates, across the range of historical mortgage rates:

$$\min_{\delta,F,X,\gamma}\int[P(r)-XQ(r+0.005,\gamma)]^2dr,$$

subject to

$$\log\left(\frac{1}{Q}+\delta\right)=\log\left(\frac{1}{Q+\gamma}+\delta\right)+0.005. \tag{64}$$

The equality constraint 64 determines the price-fee $\gamma$ that corresponds to the 50 bps rate-fee. The LHS is the gross-of-fees mortgage rate and the RHS is the equivalent net-of-fees mortgage rate plus the 50 bps fee[28]. Generally the equivalent price-fee will depend on the level of the price, which is endogenous to the minimization problem. Thus the constraint determines $\gamma$ as the equivalent price-fee when the MBS trades at par (with price 1) so that $Q=1/X$.

[28]The yield of a geometric bond with price $Q$ and duration parameter $\delta$ is $r=\log\left(\frac{1}{Q}+\delta\right)$.

We estimate values of $\delta = 0.948$, $F = 9.910$, which implies $\alpha = 0.520$, and $X = 0.1080$. For the model calibration, we only need $\delta$ and $\alpha$. The right panel of Figure 4 shows that the fit is excellent. The average error is only 0.34% of the MBS pool price.

In conclusion, despite its simplicity, the perpetual mortgage in the model captures all important features of real life mortgages (or MBS pools). The relationship between price and interest rate is convex when rates are high and concave ("negative convexity") when rates are low, which is when the prepayment option is in the money. It matches the interest rate risk (duration) of real-life mortgages, for different interest rate scenarios.