On Competitive Nonlinear Pricing

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Introduction

Motivation

In competitive environments, private information may have a dramatic impact on market outcomes.

Theory has mostly focused on the exclusive case in which agents can trade with at most one partner.

This assumption is often unrealistic for the modelling of financial or insurance markets.

We study the impact of private information on equilibrium outcomes when competition is nonexclusive.

How Prices Are Formed under Nonexclusivity ?

The formation of prices on nonexclusive markets is by nature a decentralized process.

Bilateral negotiations allow to tailor contracts at will, unlike contracts that are normalized for quotation.

The balance of supply and demand may be ensured not by a single price, but by nonlinear tariffs.

A Bestiary of Equilibrium Outcomes

Average-cost linear pricing : $\mathbf{E}\,[(p - c_i)D_i(p)] = 0$ (Pauly 1974, Sheshinski 2008, Attar, Mariotti, and Salanié 2011).

Marginal-cost nonlinear pricing : $p_j = \mathbf{E}\,[c_i \,|\, i \geq j]$ (Jaynes 1978, Hellwig 1988, Glosten 1994, Dubey and Geanakoplos 2001).

Cournot nonlinear pricing : $p_j > \mathbf{E}\,[c_i \,|\, i \geq j]$ (Biais, Martimort, and Rochet 2000, 2013, Back and Baruch 2013).

Degenerate linear pricing : $p = \max\{c_i\}$ (Attar, Mariotti, and Salanié 2013, Ales and Maziero 2013).

What We Do in This Paper

We study a model of trade under uncertainty in which a privately informed buyer trades a divisible good with several sellers.

There are finitely many states, in which the buyer has strictly convex preferences ordered by single crossing.

Sellers compete to provide this good by offering (non-)linear tariffs from which the buyer then chooses.

We characterize aggregate outcomes in equilibria with convex tariffs (sequences of limit orders).

Theoretical Implications

We show that a major market failure arises when there is adverse selection or sellers have increasing marginal costs.

If an equilibrium exists at all, only the type who is the most eager to trade may do so in equilibrium.

Our results point at an inherent fragility of organized exchanges such as limit-order books.

Alternatively, they cast doubts about the robustness of existing strategic models of such exchanges.

The Model

The Trading Environment

Trade takes place between a single buyer and a finite number of sellers $k \in \{1, \dots, K\}$, $K \geq 2$.

A feasible trade is an arbitrary vector $((q^1, t^1), \dots, (q^K, t^K))$ in $(\mathbb{R}_+ \times \mathbb{R})^K$.

The buyer is privately informed of her type $i \in \{1, \dots, I\}$; denote by $m_i > 0$ the probability that she is of type i, $\sum_i m_i = 1$.

The Buyer's Preferences

A type-i buyer has utility $u_i(Q,T)$, where $Q \equiv \sum_k q^k$ and $T \equiv \sum_k t^k$ are aggregate quantities and transfers ; u_i is continuous, strictly quasiconcave, and strictly decreasing in transfers.

We suppose that the functions u_i satisfy a strict single-crossing property : for all $i < i'$, $Q < Q'$, T, and T'

$$u_i(Q,T) \leq u_i(Q',T') \ \Rightarrow \ u_{i'}(Q,T) < u_{i'}(Q',T').$$

Thus $D_i(p) \equiv \arg\max_{Q \in \mathbb{R}_+ \cup \{\infty\}} \{u_i(Q,pQ)\}$ is nondecreasing in i for all $p \in \mathbb{R}$. We further assume that for all $i < i'$ and p

$$0 < D_i(p) < \infty \ \Rightarrow \ D_i(p) < D_{i'}(p).$$

The Sellers' Preferences

If a seller trades (q, t) with a type-i buyer, he obtains a profit $t - c_i(q)$, where c_i is a strictly increasing convex cost function.

We assume that sellers weakly prefer to trade at the margin with a low- than with a high-type buyer : for all $i < i'$ and $q < q'$

$$\partial^+ c_i(q) \leq \partial^- c_{i'}(q').$$

This is consistent with constant- or increasing marginal costs and private- or common-value environments.

Competition

Competition is nonexclusive, that is, bilateral and private, and sellers compete in tariffs. The timing of the game is :

1. Each seller k posts a tariff τ^k. We let $\tau^k(0) \equiv 0$ and $\tau^k(q) \equiv \infty$ if seller k does not offer the quantity q.

2. After learning her type, the buyer purchases a quantity q^k from each seller k, for which she pays in total $\sum_k \tau^k(q^k)$.

Strategies

A strategy for seller k is a lower semicontinuous mapping $\tau^k : \mathbb{R}_+ \to \mathbb{R} \cup \{\infty\}$ such that $\tau^k(0) = 0$ and $\{q \in \mathbb{R}_+ : \tau^k(q) < \infty\}$ is compact.

A strategy for the type-i buyer is a mapping σ_i associating to each tariff profile $(\tau^1, \ldots, \tau^K)$ a vector $(q^1, \ldots, q^K) \in \mathbb{R}_+^K$.

Given our assumptions on u_i and on the admissible tariffs τ^k,

$$\max \left\{ u_i \left(\sum_k q^k, \sum_k \tau^k(q^k) \right) \right\}$$

has a solution for any type i and tariff profile $(\tau^1, \ldots, \tau^K)$.

Equilibria with Convex Tariffs

In line with Biais, Martimort, and Rochet (2000, 2013) and Back and Baruch (2013), we focus on pure-strategy equilibria in which sellers post convex tariffs, that is, sequences of limit orders.

$\Rightarrow$ Because the utility function u_i is strictly quasiconcave, a type-i buyer has uniquely determined aggregate equilibrium demand Q_i and transfer T_i.

$\Rightarrow$ Convexity of equilibrium tariffs is preserved under aggregation. In particular, the function defined by

$$T^{-k}(Q^{-k}) \equiv \min\left\{\sum_{k'\neq k} \tau^{k'}(q^{k'}) : \sum_{k'\neq k} q^{k'} = Q^{-k}\right\}$$

is convex.

Example I : Trade with Quasilinear Utilities

The seller's utility is quasilinear :

$$u_i(Q,T) = v_i(Q) - T, \quad \partial v_{i'} > \partial v_i' \text{ if } i' > i.$$

In Biais, Martimort, and Rochet (2000, 2013),

$$c_i(Q) = \theta_i Q - \frac{\gamma\sigma}{2} Q^2, \quad \theta_{i'} > \theta_i \text{ if } i' > i.$$

Example II : Insurance

As in Rothschild and Stiglitz (1976), the buyer faces a binary lottery $\pi_i W_G \oplus (1-\pi_i) W_B$ over her wealth and she has utility

$$\pi_i u(W_G - P) + (1-\pi_i) u(W_B - P + R).$$

Profits $p-(1-\pi_i)r$ can be rewritten as $t - c_i q$ if $t \equiv p$, $c_i \equiv 1-\pi_i$, and $q \equiv r$, so that $\pi_i > \pi_{i'}$ if and only if $c_{i'} > c_i$ or $i' > i$, and

$$u_i(Q,T) = \pi_i u(W_G - T) + (1-\pi_i) u(W_B - T + Q).$$

$MRS_i(Q,T)$ is strictly decreasing in i for all (Q,T) so that the buyer's single-crossing property is satisfied.

Constant Marginal Costs

The Main Result

Theorem 1 *Suppose constant marginal costs, $c_i(q) = c_i q$ for all i and q, with $c_{i'} \geq c_i$ for all $i < i'$, and let $(\tau^1, \ldots, \tau^K, \sigma_1, \ldots, \sigma_I)$ be an equilibrium with convex tariffs. If some trade takes place in equilibrium, then*

(i) All trades take place at unit price c_I and each type-i buyer purchases $D_i(c_I)$ in the aggregate.

(ii) If $D_i(c_I) > 0$, then $c_i = c_I$. Thus each seller earns zero profit on each trade.

Discussion

Linear pricing arises endogenously, reflecting the disciplinary role or competition.

Cross-subsidies between types are ruled out in any equilibrium with convex tariffs.

There cannot exist two types $i < i'$ with $c_i < c_{i'}$ who both trade positive quantities.

There is a lack of lower hemicontinuity when one moves to the continuum case.

The Buyer's Indirect Utility Function

From the viewpoint of seller k, the aggregate tariff T^{-k} of his opponents can be computed anticipating their tariff offers.

In turn, T^{-k} determines how each type evaluates any bundle (q, t) she may trade with seller k. Define the indirect utility function

$$z_i^{-k}(q, t) \equiv \max_{Q^{-k} \in \mathbb{R}_+} \{u_i(q + Q^{-k}, t + T^{-k}(Q^{-k}))\}.$$

The Buyer's Behavior

Fix an equilibrium $(\tau^1, \ldots, \tau^K, \sigma_1, \ldots, \sigma_I)$ with convex tariffs.

Lemma 1 *Each type i has a best response $(q_i^1, \ldots, q_i^K)$ to the tariff profile $(\tau^1, \ldots, \tau^K)$ such that $q_{i+1}^k \geq q_i^k$ for all k and $i < I$.*

Lemma 2 *For all k, $i < i'$, $q < q'$, t, and t',*

$$z_i^{-k}(q,t) \leq z_i^{-k}(q',t') \;\Rightarrow\; z_{i'}^{-k}(q,t) \leq z_{i'}^{-k}(q',t').$$

$$z_i^{-k}(q,t) < z_i^{-k}(q',t') \;\Rightarrow\; z_{i'}^{-k}(q,t) < z_{i'}^{-k}(q',t').$$

How the Sellers Can Break Ties

A menu $\{(0,0),\ldots,(q_i,t_i),\ldots\}$ is incentive feasible if for all i, i'

$$IC_{i,i'} \quad z_i^{-k}(q_i,t_i) \geq z_i^{-k}(q_{i'},t_{i'}).$$

$$IR_i \qquad z_i^{-k}(q_i,t_i) \geq z_i^{-k}(0,0).$$

Lemma 3 *Seller k's equilibrium profit is no less than*

$$V^k(\boldsymbol{\tau}^{-k}) \equiv \sup\left\{\sum_{i=1}^{I} m_i(t_i - c_i q_i)\right\}$$

over all incentive-feasible menus with nondecreasing quantities.

Binding Constraints

Lemma 4 *Suppose that the equilibrium* $(\tau^1, \ldots, \tau^K, \sigma_1, \ldots, \sigma_I)$ *has nondecreasing individual quantities. Then, for any seller* k*, if for some type* i *the individual-rationality constraint*

$$IR_i(\boldsymbol{\tau}) \qquad z_i^{-k}(q_i^k, \tau^k(q_i^k)) \geq z_i^{-k}(0, 0)$$

is slack, one has $i \geq 2$ *and the incentive-compatibility constraint*

$$IC_{i,i'}(\boldsymbol{\tau}) \quad z_i^{-k}(q_i^k, \tau^k(q_i^k)) \geq z_i^{-k}(q_{i-1}^k, \tau^k(q_{i-1}^k))$$

binds.

The Linear-Pricing Result

Lemma 5 *Suppose that the equilibrium* $(\tau^1, \ldots, \tau^K, \sigma_1, \ldots, \sigma_I)$ *has nondecreasing individual quantities and that some trade takes place in equilibrium. Then there exists* $p \in \mathbb{R}$ *such that in equilibrium all trades take place at unit price* p *and each type* i *purchases* $D_i(p)$ *in the aggregate.*

The No-Cross-Subsidization Result

Lemma 6 *Suppose that the equilibrium* $(\tau^1, \ldots, \tau^K, \sigma_1, \ldots, \sigma_I)$ *has nondecreasing individual quantities and that some trade takes place in equilibrium. Then, for* p *defined as in Lemma 5, we have* $p = c_i = c_I$ *for any type* i *who actively trades.*

Other Equilibrium Outcomes

Define

$$\underline{V}^k(\boldsymbol{\tau}) \equiv \text{sup}\left\{\sum_{i=1}^{I} m_i[\tau^k(q_i) - c_i q_i]\right\}.$$

over all nondecreasing family $(q_1, \ldots, q_I)$ satisfying $IR_i(\boldsymbol{\tau})$ and $IC_{i,i'}(\boldsymbol{\tau})$. By Lemma 3, equilibrium profits v^k satisfy

$$v^k \geq V^k(\boldsymbol{\tau}^{-k}) \geq \underline{V}^k(\boldsymbol{\tau}) \geq v^{k\prime},$$

where $v^{k\prime}$ is seller k's profit if the buyer's best response has nondecreasing individual quantities, as in Lemma 1.

Because

$$\sum_k v^k = \sum_i m_i(T_i - c_i Q_i) = \sum_k v^{k\prime},$$

we thus get that $v^k = V^k(\boldsymbol{\tau}^{-k})$ for all k : no seller gets more than if the buyer sticks to nondecreasing quantities.

Lemma 7 *If* $(\tau^1, \ldots, \tau^K, \sigma_1, \ldots, \sigma_I)$ *is an equilibrium with convex tariffs, then there exists a strategy profile* $(\hat{\sigma}_1, \ldots, \hat{\sigma}_I)$ *for the buyer such that* $(\tau^1, \ldots, \tau^K, \hat{\sigma}_1, \ldots, \hat{\sigma}_I)$ *is an equilibrium with nondecreasing individual quantities that yields the same profit to each seller.*

Increasing Marginal Costs

The Main Result

Theorem 2 *Suppose strictly increasing marginal costs, and let* $(\tau^1, \ldots, \tau^K, \sigma_1, \ldots, \sigma_I)$ *be an equilibrium with convex tariffs. If some trade takes place in equilibrium, then there exists* $p \in \mathbb{R}$ *solution to*

$$p \in \partial c_I \left(\frac{D_I(p)}{K} \right)$$

and such that

(i) All trades take place at unit price p *and each type-*i *buyer purchases* $D_i(p)/K$ *from each seller.*

(ii) Only the type-I *buyer actively trades in equilibrium.*

Novel Insights

Even under private values, in equilibrium only type-I buyers can trade, at a price equal to the competitive equilibrium price.

The convexity of the cost functions creates an aggregate risk-sharing problem even if there is no adverse selection.

Proof Outline

Lemmas 1–5 (except the zero-profit result) remain unchanged, or only necessitate minor changes.

The analogue of Lemma 6 (Theorem 2(i)) also relies on a limit-order deviation.

To deal with other equilibrium outcomes, one shows that the buyer has a best response to the tariff profile $(\tau^1, \ldots, \tau^K)$ that has nondecreasing individual quantities and yields the sellers an aggregate profit

$$V^0(\tau^1, \ldots, \tau^K) \equiv \sum_i m_i T_i - \inf \left\{ \sum_k \sum_i m_i c_i(q_i^k) \right\}.$$

Concluding Remarks

Summary

Together with adverse selection or increasing marginal costs, nonexclusivity leads to a strong form of market failure.

Each buyer's inability to control the seller's trades with the other buyers introduces an additional moral-hazard element.

This makes screening more costly, and implies that the buyer either trades efficiently, or does not trade at all.

Bertrand versus Cournot

Indifferences and discontinuities play a crucial role in our analysis, as they do in the standard Bertrand model.

This contrasts with the continuous-type case studied by Biais, Martimort, and Rochet (2000, 2013).

Current Work

To destabilize candidate equilibria with convex tariffs, we use direct revelation mechanisms, that is, deviations that are not themselves convex.

We conjecture that under suitable assumptions on the buyer's preferences, our results continue to hold in the case where only convex tariffs are admissible.

This would cast doubts on the inherent stability of the limit-order book (Glosten 1994, Back and Baruch 2013), even if side trades are not feasible.