Capital Structure and Investment Dynamics with Fire Sales

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Introduction

- Corporate finance and the Modigliani Miller theorem
- Indeterminacy of capital structure
- Tradeoff between taxes and bankruptcy cost
- Our objective: understand GE effects of debt finance on investment and growth
- Competitive equilibrium when markets are incomplete
- Distortionary taxes and transfers
- Second best policies
- Related concern: capital adequacy regulation

Results

- The optimal capital structure is determinate, although individuals are indifferent between debt and equity
- Introducing a corporate income tax *implies* bankruptcy is (a) costly and (b) occurs with positive probability
- Equilibrium with a corporate income tax is constrained inefficient: welfare would be higher with
  - a higher level of investment, other things being equal
  - or a higher probability of default (in the limit, higher debt-equity ratio)
- In fact, we can approach the first best by imposing near 100% debt finance
- The introduction of a safe technology to increase the liquidity of the asset market can make everyone worse off

Outline

- The planner's problem
- An economy with frictions, debt, and equity
- Equilibrium
- Steady states
- Transition dynamics
- Constrained inefficiency
- Inefficient hedging

The Planner's Problem

Production of capital

- Time is discrete and divided into a sequence of dates $t = 0, 1, ...$
- There is a single perishable good which can be used for consumption or production
- Capital is produced using the good as the sole input
- An input of $I \geq 0$ units of the good produces $\varphi(I) \geq 0$ units of capital goods at the same date, where φ is C^2 and satisfies

$$\varphi'(I) > 0 \text{ and } \varphi''(I) < 0, \text{ for any } I \geq 0$$

- Investment is irreversible (capital cannot be consumed)

Production of consumption goods

- The good is produced using capital as the sole input, subject to constant returns to scale
- *Time to build*: one unit of capital produced at date t is available to produce $A > 0$ units of goods at date $t + 1$
- *Depreciation*: one unit of capital used for production at date t is transformed into $\bar{\theta}$ units of capital available for production at date $t + 1$
- *Boundedness*: There exists a constant $0 < \hat{k} < \infty$ such that

$$\varphi(Ak) < \left(1 - \bar{\theta}\right) k, \text{ for every } k > \hat{k}$$

- There is an initial endowment of $\bar{k}_0 > 0$ units of capital at date 0

Consumption

- There is a unit mass of identical, infinitely-lived consumers
- A consumption stream $\mathbf{c} = (c_0, c_1, ...) \geq \mathbf{0}$ generates utility

$$U(\mathbf{c}) = \sum_{t=0}^{\infty} \delta^t u(c_t),$$

where $0 < \delta < 1$

- The function $u(\cdot)$ has the usual properties: $u(c)$ is C^2 and satisfies

$$u'(c) > 0 \text{ and } u''(c) < 0 \text{ for any } c \geq 0.$$

The planner's problem

Choose $\{(c_t, k_t, I_t)\}_{t=0}^{\infty}$ to maximize

$$\sum_{t=0}^{\infty} \delta^t u(c_t)$$

subject to the constraints

$$c_t + I_t = Ak_t, \qquad t = 0, 1, ...,$$

$$k_{t+1} = \bar{\theta} k_t + \varphi\left(I_t\right), \qquad t = 0, 1, ...,$$

$$\left(c_t, k_t, I_t\right) \geq \mathbf{0}, \qquad t = 0, 1, ...,$$

and

$$k_0 = \bar{k}_0$$

The planner's solution

Suppose that $\{(c_t^*, k_t^*, I_t^*)\}_{t=0}^{\infty}$ is a solution to the planner's problem and suppose that

$$(c_t^*, k_t^*, I_t^*) \gg \mathbf{0}, \qquad t = 0, 1, \ldots.$$

Then there exist non-negative multipliers $\{(\lambda_t^*, \mu_t^*)\}_{t=0}^{\infty}$ such that

$$\delta^t u'(c_t^*) = \lambda_t^*, \qquad t = 0, 1, \ldots,$$

$$\lambda_{t+1}^* A + \mu_{t+1}^* \bar{\theta} = \mu_t^*, \qquad t = 0, 1, \ldots,$$

$$\mu_t^* \varphi'(I_t^*) = \lambda_t^*, \qquad t = 0, 1, \ldots.$$

Tranversality

- Feasibility and Boundedness imply that

$$\varphi(I_t) \leq \varphi(Ak_t) < \left(1 - \bar{\theta}\right) k_t, \ \forall k_t > \hat{k}$$

- Hence, the law of motion

$$\begin{aligned} k_{t+1} &= \bar{\theta} k_t + \varphi(I_t) \\ &\leq k_t, \ \forall k_t > \hat{k} \end{aligned}$$

implies that $\{k_t\}$ is bounded and

$$\lim_{t \to \infty} \sum_{s=t}^{\infty} \delta^s u(c_s) = 0$$

Steady states

- A steady-state solution satisfies

$$\left(\lambda_t^*, \mu_t^*, k_{t+1}^*, c_t^*, I_t^*\right) = \left(\delta^t \lambda^*, \delta^t \mu^*, k^*, c^*, I^*\right), \qquad \forall t = 0, 1, \ldots.$$

and

$$u'\left(c^*\right) = \lambda^*,$$
$$\delta \lambda^* A + \delta \mu^* \bar{\theta} = \mu^*,$$
$$\mu^* \varphi'\left(I^*\right) = \lambda^*$$

- The second and third conditions can be rewritten as

$$\frac{\delta A}{1 - \delta \bar{\theta}} = \frac{\mu^*}{\lambda^*} = \frac{1}{\varphi'\left(I^*\right)}$$

Efficient steady states

- The feasibility conditions become

$$c^* + I^* = Ak^*$$

$$k^* = \bar{\theta}k^* + \varphi\left(I^*\right)$$

- Thus, the efficient steady state capital stock is

$$k^* = \frac{\varphi\left(I^*\right)}{1-\bar{\theta}}$$

where I^* is determined by

$$\varphi'\left(I^*\right)\frac{\delta A}{1-\delta\bar{\theta}} = 1$$

An Economy with Frictions, Debt, and Equity

An economy with frictions

In a frictionless world, the planner's solution could be decentralized in the usual way; to make things more interesting, we impose a number of frictions

- Markets are *incomplete*: there are only spot markets for goods and assets
- Firms are financed using (short-term) *debt* and *equity*
- Firms pay a *distortionary* corporation tax but interest income is exempt
- In the event of default, firms are forced into bankruptcy and their assets are *liquidated*
- Liquidation is subject to a *finance constraint*

Firms

- The capital goods sector consists of a continuum of producers with identical technologies $\varphi(I)$
- Producers maximize profits and pay dividends to consumers at the end of each date
- Since production is instantaneous, finance is not required
- The consumption goods sector consists of a continuum of producers with identical technologies Ak
- Capital is subject to random depreciation, where $\theta_t \backsim F(\theta)$ is i.i.d. across producers and $\int \theta_t dF = \bar{\theta}$ (no aggregate uncertainty)
- Capital is long lived so goods producers finance capital purchase with (short-term) debt and equity

Bankruptcy, liquidation and settlement

Each date t is divided into three sub-periods, labeled A, B, and C.

A. In sub-period A, a firm either renegotiates the debt (rolls it over) or defaults and declares bankruptcy

B. In sub-period B, bankrupt firms sell their capital at the market-clearing price

C. In sub-period C, firms issue debt and equity to purchase newly produced capital goods; they choose the capital structure so as to maximize the market value of the firm

Sub-period A: The renegotiation game

The renegotiation game at date t consists of two stages:

1. The entrepreneur makes a "take it or leave it" offer to the bond holders to rollover the debt, replacing the maturing debt with face value d_t with new assets (a combination of equity and debt maturing at the following date with face value d_{t+1}).
2. The creditors simultaneously accept or reject the firm's offer.

The renegotiation succeeds if a majority of creditors accept and those who do not are paid d_t

There are multiple equilibria; we focus on equilibria in which renegotiation succeeds whenever possible

Sub-period A: The renegotiation game

- Without loss of generality, we consider the behavior of a representative firm with one unit of capital.
- If renegotiation succeeds, a creditor can obtain at least d_t / q_t units of capital
- The firm's manager can ensure the firms ends the period with $A / q_t + \theta_t$ units of capital
- Thus, the manager can make an acceptable offer if and only if

$$d_t \leq A + q_t \theta_t$$

- Renegotiation succeeds if and only if $\theta_t \geq z_t$, where the breakeven value z_t is defined by

$$d_t = A + q_t z_t$$

Sub-period B: Liquidation

- In sub-period B, the supply of capital from bankrupt firms is

$$\int_0^{z_t} \theta_t k_t dF$$

- The amount of "cash" available to purchase this capital is

$$A \int_{z_t}^1 k_t dF = A \left(1 - F\left(z_t\right)\right) k_t$$

- The market clears at a price q_t, where

$$q_t \int_0^{z_t} \theta_t k_t f\left(\theta_t\right) d\theta_t \leq A \left(1 - F\left(z_t\right)\right) k_t,$$

  and equality holds if $q_t < v_t$

Sub-period C: The capital market

- A firm in the capital goods sector chooses $I_t \geq 0$ to maximize $v_t \varphi(I_t) - I_t$, where v_t is the price of capital
- A firm in the consumption goods sector chooses z_t to maximize the value of the firm
- The goods market clears if

$$c_t + I_t = Ak_t$$

- The capital market clears if household wealth w_t is sufficient to purchase all of the securities issued by firms $v_t \left(\bar{\theta} k_t + \varphi(I_t)\right)$ plus the consumption c_t

$$c_t + v_t \left(\bar{\theta} k_t + \varphi(I_t)\right) = w_t$$

Taxes

- Tax base = value of the firm - renegotiated value of the debt:

$$v_t\left(\frac{A}{q_t}+\theta_t\right)-v_t\left(\frac{d_t}{q_t}\right)$$

- The tax rate $\tau > 0$ and the tax on equity at date t is then:

$$\tau \max\left\{\frac{v_t}{q_t}\left(A+q_t\theta_t-d_t\right),0\right\}$$

- Let z denote the breakeven point defined by $A+q_t z_t=d_t$; then tax on equity is

$$\tau \max\left\{v_t\left(\theta_t-z_t\right),0\right\}$$

- Taxes are returned to consumers as a lump sum transfer T_t in sub-period C

Asset pricing

- Let v_t^e and v_t^b denote the market price of this equity and debt at t
- The return on diversified equity and debt is known with certainty at date t because there is no aggregate uncertainty
- The one-period returns on debt and equity are equal:

$$1 + r_t = \frac{\int_0^1 \min\left\{A + q_{t+1}\theta_{t+1}, \frac{v_{t+1} d_{t+1}}{q_{t+1}}\right\} f\left(\theta_{t+1}\right) d\theta_{t+1}}{v_t^b} =$$

$$\frac{\int_0^{z_{t+1}} \left(\frac{v_{t+1}}{q_{t+1}} - \tau\right) \left(A + q_{t+1}\theta_{t+1} - d_{t+1}\right) f\left(\theta_{t+1}\right) d\theta_{t+1}}{v_t^e}.$$

The optimal capital structure

- In a symmetric equilibrium,
  - 100% debt finance implies firms default with probability one and $q_t = 0$
  - 100% equity finance implies default with probability zero and $q_t = v_t$
- Thus, if $\tau > 0$, equilibrium requires *costly* default

$$q_t < v_t$$

and equilibrium default occurs with probability *strictly* between zero and one:

$$0 < F(z_t) < 1.$$

Equilibrium

Profit maximization in the capital goods sector

In what follows, we consider only *symmetric* equilibria

- In the capital goods sector, the firm's decision is trivial: firms simply choose $I_t \geq 0$ to maximize profits

$$v_t \varphi\left(I_t\right) - I_t$$

- The optimal output is determined uniquely by the first-order condition

$$v_t \varphi'\left(I_t\right) \leq 1,$$

  with strict equality if $I_t > 0$
- The profits $\pi_t\left(v_t\right) = \max\left\{v_t \varphi\left(I_t\right) - I_t\right\}$ are paid to consumers in sub-period C

Value maximization in the consumption goods sector

- Suppose a firm has one unit of capital and the breakeven level z_{t+1}
- If $\theta_{t+1} < z_{t+1}$ the firm is bankrupt at $t+1$ and the liquidated value is

$$A + q_{t+1}\theta_{t+1}$$

- If $\theta_{t+1} > z_{t+1}$ the firm is solvent and, w.l.o.g., the firm retains all its earnings, uses them to purchase liquidated capital, and the firm's value in sub-period C is

$$v_{t+1}\left(\frac{A}{q_{t+1}} + \theta_{t+1}\right)$$

- The firm pays the corporation tax $\tau v_{t+1}\left(\theta_{t+1} - z_{t+1}\right)$ in sub-period C if it is solvent

Optimal capital structure

- The expected future value of the firm is

$$\int_0^{z_{t+1}} \left(A + q_{t+1}\theta_{t+1}\right) dF + \int_{z_{t+1}}^1 \left(\frac{v_{t+1}}{q_{t+1}} \left(A + q_{t+1}\theta_{t+1}\right) - \tau v_{t+1} \left(\theta_{t+1} - z_{t+1}\right) \right) dF \Bigg\}$$

- The breakeven level z_{t+1} is chosen to solve

$$v_t = \max_{z_{t+1}} \frac{\delta u'\left(c_{t+1}\right)}{u'\left(c_t\right)} \left\{ \int_0^{z_{t+1}} \left(A + q_{t+1}\theta_{t+1}\right) dF + \int_{z_{t+1}}^1 \left(\frac{v_{t+1}}{q_{t+1}} \left(A + q_{t+1}\theta_{t+1}\right) - \tau v_{t+1} \left(\theta_{t+1} - z_{t+1}\right) \right) dF \right\}.$$

Optimal capital structure

- The derivative of the firm's value with respect to z_{t+1} is

$$\frac{dv_t}{dz_{t+1}} = \frac{\delta u'\left(c_{t+1}\right)}{u'\left(c_t\right)} \left\{ \left(A + q_{t+1} z_{t+1}\right) f\left(z_{t+1}\right) - \frac{v_{t+1}}{q_{t+1}} \left(A + q_{t+1} z_{t+1}\right) \right.$$
$$\left. \tau v_{t+1} \left(\theta_{t+1} - z_{t+1}\right) f\left(z_{t+1}\right) + \tau v_{t+1} \left(1 - F\left(z_{t+1}\right)\right) \right\}$$

- The first-order condition can be written as

$$\left(1 - \frac{q_{t+1}}{v_{t+1}}\right) \left(A + q_{t+1} z_{t+1}\right) \frac{f\left(z_{t+1}\right)}{1 - F\left(z_{t+1}\right)} = \tau.$$

- The solution to this equation will be unique and satisfy $0 < z < 1$ if the hazard rate is increasing in z_{t+1} and $\left(1 - \frac{q_{t+1}}{v_{t+1}}\right) Af\left(0\right) < \tau$.

The consumption decision

- Consumers maximize

$$\sum_{t=0}^{\infty} \delta^t u(c_t)$$

subject to

$$\sum_{t=0}^{\infty} p_t c_t = v_0 k_0 + \sum_{t=0}^{\infty} p_t (T_t + \pi_t),$$

where

$$p_t = \prod_{s=1}^{t} \left(\frac{1}{1 + r_s} \right),$$

the initial capital stock is k_0, π_t are the profits of firms in the capital sector and T_t are lump sum transfers

Market clearing

- In sub-period B, market clearing requires

$$q_t \int_0^{z_t} \theta_t k_t f(\theta_t)\, d\theta_t = A(1 - F(z_t))\, k_t,$$

and equality holds if $q_t < v_t$.

- In sub-period C, the goods market clears if

$$c_t + I_t = Ak_t$$

- By Walras' Law, the securities market clears if the goods market clears

Equilibrium conditions

1. $\{c_t^*\}_{t=0}^{\infty}$ maximizes the consumers discounted utility subject to the budget constraint
2. For every t, $v_t^* \varphi' \left(I_t^*\right) \leq 1$ and equality holds if $I_t^* > 0$
3. For every t, z_t^* maximizes the value of the consumption-producing firm
4. For every t, the asset market clears in sub-period B:

$$q_t^* \int_0^{z_t^*} \theta_t f(\theta_t) d\theta_t = A \int_{z_t^*}^1 d\theta_t,$$

5. For every t, the goods market clears in sub-period C

$$Ak_t^* = c_t^* + I_t^*$$

6. For every t, $\{k_t^*\}$ satisfies the law of motion

$$k_{t+1}^* = \bar{\theta} k_t^* + \varphi \left(I_t^*\right) \text{ and } k_0^* = \bar{k}_0$$

Steady States

Steady-state equilibrium conditions

- Optimal consumption requires the first-order condition

$$\frac{1}{1+r^*} = \delta \frac{u'(c^*)}{u'(c^*)} = \delta,$$

and the budget constraint

$$\frac{1}{1-\delta} c^* = v^* k^* + \frac{1}{1-\delta} \left(\tau k^* q^* \int_{z^*}^{1} (\theta - z^*) \, dF + v^* \varphi (I^*) - I^* \right)$$

- Profit maximization requires the first-order condition

$$v^* \varphi' (I^*) = 1$$

Steady-state equilibrium conditions

- Value maximization requires

$$v^* = \delta \left\{ \int_0^{z^*} (A + q^* \theta)\, dF + \int_{z^*}^1 \left(\frac{v^*}{q^*} (A + q^* \theta) - \tau v^* (\theta - z^*) \right) dF \right\}$$

and the first-order condition

$$\left(1 - \frac{q_{t+1}}{v_{t+1}}\right) (A + q_{t+1} z_{t+1}) \frac{f(z_{t+1})}{1 - F(z_{t+1})} = \tau.$$

Steady-state equilibrium conditions

- The asset-market clears

$$q^* \int_0^{z^*} \theta dF = A \int_{z^*}^1 dF$$

- The goods-market clears

$$Ak^* = c^* + I^*.$$

- The law of motion

$$k^* = \bar{\theta} k^* + \varphi (I^*)$$

is satisfied, where $k^* = \bar{k}_0$

Existence and uniqueness

Theorem

There exists a unique steady-state equilibrium, obtained as a solution to the system of equations:

$$q^* = \frac{A\left(1 - F\left(z^*\right)\right)}{\int_0^{z^*} \theta dF},$$

$$v^* = \frac{\delta A}{1 - \delta\bar{\theta} + \tau \int_{z^*}^{1} \left(\theta - z^*\right) dF}$$

$$\left(1 - \frac{q_{t+1}}{v_{t+1}}\right)\left(A + q_{t+1} z_{t+1}\right) \frac{f\left(z_{t+1}\right)}{1 - F\left(z_{t+1}\right)} = \tau$$

Transition Dynamics

Non-steady-state paths

- Suppose that consumers are risk neutral, $u(c) \equiv c$; then the equilibrium conditions reduce to

$$v_t \varphi'(I_t) = 1$$

$$q_t = \frac{A(1 - F(z_t))}{\int_0^{z_t} \theta dF}$$

$$v_t = q_t + \tau (q_t)^2 \frac{1 - F(z_t)}{f(z_t)(A + q_t z_t)}$$

- These equations yield solutions for I_t, q_t and v_t as a function of z_t
- We are left with a of two-equation system

$$v(z_t) = \delta \left[A + v(z_{t+1})\bar{\theta} - \tau v(z_{t+1}) \int_{z_{t+1}}^{1} (\theta - z_{t+1}) dF \right] \quad (1)$$

$$k_{t+1} = \bar{\theta} k_t + \varphi(I(z_t)) \quad (2)$$

where $v_t = v(z_t)$ is the solution for v_t as a function of z_t and equation (1) can be solved for z_{t+1} in terms of z_t

Instability and uniqueness

- If we let $w(z)$ denote the right hand side of (1), then we are interested in the behavior of the dynamical system

$$v(z_t) = w(z_{t+1})$$

- We note that $v(z_t)$ and $w(z_{t+1})$ have the properties

$$\lim_{z\to 1} v(z_t) = 0 \text{ and } \lim_{z\to 1} w(z_{t+1}) = \delta A$$

and

$$\lim_{z\to 0} v(z_t) = \lim_{z\to 0} w(z_{t+1}) = \infty$$

- We can show that, for any z,

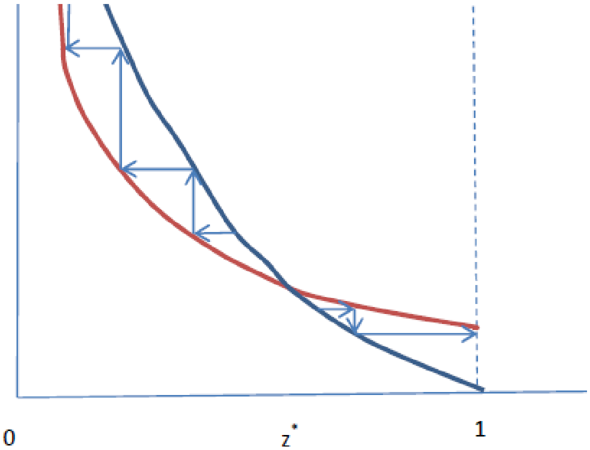
$$v'(z) < w'(z) < 0$$

and, as before, there is a unique solution $0 < z^* < 1$ to the equation

$$v(z^*) = w(z^*)$$

Instability and uniqueness

These properties are illustrated below, where the blue line is $v(z)$ and the red line is $w(z)$



Instability and uniqueness

- From the picture, it is clear that $z_t > z^*$ implies that for some $T > t$,

$$v(z_T) < w(1)$$

  so the equilibrium condition (1) cannot be satisfied at T
- For $z_t < z^*$, the equilibrium condition (1) can be satisfied for all t, however, $z_t \to 0$ which implies $v(z_t) \to \infty$ and $q(z_t) \to \infty$
- Now $v(z_t) \to \infty$ implies $I_t \to \infty$ and $k_t \to \infty$ but this is clearly in violation of the boundedness of $\{k_t\}$
- So any equilibrium must satisfy $z_t = z^*$ for all t

Constrained Inefficiency

The First Best

- From the planner's problem we know that

$$k^* = \frac{\varphi\left(I^*\right)}{1-\bar{\theta}}$$

$$\varphi'\left(I^*\right)\frac{\delta A}{1-\delta\bar{\theta}} = 1$$

- If $\tau = 0$ then $z^* = 1$ and

$$v^* = \delta\left\{A + v^*\bar{\theta}\right\} = \frac{\delta A}{1-\delta\bar{\theta}}$$

- Then the equilibrium condition $v^*\varphi'\left(I^*\right) = 1$ implies that

$$\varphi'(I^*)\frac{\delta A}{1-\delta\bar{\theta}} = 1$$

Temporary increase in investment

- Suppose $dI_t = \varepsilon > 0$ and $dI_\tau = 0$ for $\tau \neq t$
- The effect on welfare of the policy change is given by

$$\left(\varphi'(I^*) A\delta \sum_{t=0}^{\infty} \left(\delta\bar{\theta}\right)^t - 1 \right) u'\left(c^*\right)\varepsilon + o\left(\varepsilon\right)$$

- The term in parenthesis is positive iff

$$A\frac{\delta}{1-\delta\bar{\theta}} > \frac{1}{\varphi'(I^*)} = v^*$$

- But this follows from the equilibrium condition

$$v^* = \frac{\delta}{1-\delta\bar{\theta}} \left(A - \int_{z^*}^{1} \tau q^*(\theta - z^*) f(\theta) d\theta \right)$$

if $\tau > 0$ and $z^* < 1$

Increase in debt-equity ratio

- Consider a permanent increase starting from some date $t+1$
- The future value of q_τ is constant and future values of v_τ are given by a first-order difference equation that is divergent, so the only admissible solution is the new steady-state value
- The remaining variables are determined by

$$k_{t+1} = \bar{\theta} k^* + \varphi\left(I_{t+1}\right),$$
$$v_{t+1}\varphi'\left(I_{t+1}\right) = 1.$$

- Then I_τ is constant and greater than I^* iff $v_\tau > v^*$ and we can show that

$$\frac{dv_\tau}{dz} > 0$$

Increase in debt-equity ratio

- What happens as $z \longrightarrow 1$ (equilibrium is not defined when $z = 1$)?
- From the market-clearing condition, $z \longrightarrow 1$ implies $q \longrightarrow 0$
- As $q \longrightarrow 0$, we see that

$$v \longrightarrow \frac{\delta A}{1 - \delta\bar{\theta}} = v^{FB}$$

and hence $I \longrightarrow I^{FB}$

- Also, as $z \longrightarrow 1$ we can show that

$$\frac{v^b}{v^e} \longrightarrow \infty$$

so the debt-equity ratio is increasing in the limit.

Intuition

- Equilibrium is constrained inefficient because agents take prices as given and ignore the pecuniary externality of changes in v and q
- At the margin, z balances the two costs but the 'tax' on investment distorts I
- An exogenous increase in z increases the v and decreases q but the tax on equity

$$\tau v \int_z^1 (\theta - z)\, dF$$

  declines
- In fact, it is the lower tax that explains the increase in v; the fall in q does not affect v
- An increase in v increases k and hence increases welfare

Inefficient Hedging

Introducing a safe technology

- What is the effect of introducing a *safe* technology?
- Suppose there is an alternative technology for producing the consumption good: one unit of capital produces B units of the good and leaves $\bar{\theta}$ units of capital
- The risky technology is dominated unless $B < A$
- It is never optimal for firms to use both technologies
- A fraction ℓ use the risky technology and $1 - \ell$ use the safe technology

Equilibrium with a choice of risk

- Risky firms choose z^* to maximize the value of the firm v^*
- Safe firms use 100% debt financing
- Firms are indifferent between the two technologies if

$$v^* = \delta \frac{v^*}{q^*} \left(B + q^* \bar{\theta} \right) \iff q^* = \frac{\delta B}{1 - \delta \bar{\theta}}$$

- If ℓ^* is the fraction of the capital stock devoted to the risky technology, the market-clearing condition at sub-period B is

$$q^* \ell k^* \int_0^{z^*} \theta dF = A \ell^* k^* \int_{z^*}^1 \theta dF + B \left(1 - \ell^*\right) k^*$$

and the market-clearing condition at sub-period C is

$$c^* = A \ell^* k^* + B \left(1 - \ell^*\right) k^* - I^*$$

Welfare

- Let $(k^*, c^*, z^*, q^*, v^*)$ be the steady-state equilibrium with only the risky technology and let $\bar{B}$ satisfy

$$q^* = \frac{\delta B}{1 - \delta\bar{\theta}}$$

- Then $\ell^* = 1$ is an equilibrium with two technologies for $B \leq \bar{B}$ and $0 < \ell^* < 1$ for $\bar{B} < B < A$.
- A small increase in B at $\bar{B}$ will lead to the allocation of a small amount of capital to the safe technology
- Under the usual assumptions of risk neutrality and uniform distribution of θ,

$$\left.\frac{dv^*}{dB}\right|_{B=\bar{B}} < 0$$

- The change in c^* evaluated at $B = \bar{B}$ and $\ell^* = 1$ is

$$\left.\frac{dc^*}{dB}\right|_{B=\bar{B}} = 4A\frac{dv^*}{dB} - 2v^*\frac{dv^*}{dB} + (A - B)4v^*\left.\frac{d\ell^*}{dB}\right|_{B=\bar{B},\ell^*=1}$$

Welfare

- Assume that

$$\varphi(I) = 2I^{\frac{1}{2}}$$

- Then $v^* \varphi'(I) = 1$ implies that

$$I^* = (v^*)^2$$

and

$$k^* = \frac{\varphi(I^*)}{1-\bar{\theta}} = 4v^*$$

- Then

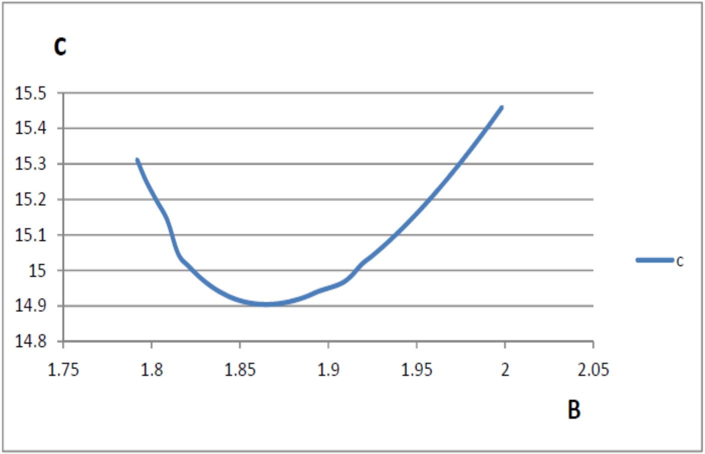
$$\begin{aligned} \left.\frac{dc^*}{dB}\right|_{B=\bar{B}} &= \left. 4A\frac{dv^*}{dB} - 2v^*\frac{dv^*}{dB} + (A-B)4v^*\frac{d\ell^*}{dB}\right|_{B=\bar{B},\ell^*=1} \\ &= \left.(4A-2v^*)\frac{dv^*}{dB} + (A-B)2v\frac{d\ell^*}{dB}\right|_{B=\bar{B},\ell^*=1} \end{aligned}$$

- Then $\frac{dc^*}{dB} < 0$ since $\frac{dv^*}{dB} < 0$ and $\frac{d\ell^*}{dB} \leq 0$ and

$$v^* < \frac{\delta}{2-\delta}2A$$

Numerical example

Parameters: $A = 2$, $\delta = 0.9$, $\tau = 0.35$, $\theta \backsim U[0, 1]$



Conclusion

- Introducing a corporate income tax *implies* that bankruptcy is costly and occurs with positive probability
- The optimal capital structure is determinate, although individuals are indifferent between diversified debt and equity
- Equilibrium with a corporate income tax is constrained inefficient: welfare would be higher with
  - a higher level of investment, other things being equal
  - or a higher probability of default (in the limit, higher debt-equity ratio)
- In fact, we can approach the first best by imposing near 100% debt finance
- If firms have a choice of a safe and a risky technology, the use of the safe technology makes them worse off