INSIDER TRADING, STOCHASTIC LIQUIDITY, AND EQUILIBRIUM PRICES

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Do measures of stock liquidity reveal the presence of informed traders?

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- For example, Kyle (1985) proposes seminal model of insider trading:

  Insider knows terminal value of the firm that will be revealed to all at $T$. Market maker sets price equal to expected value given total order flow which is the sum of uninformed noise trader demand and insider trades. 

  $\Rightarrow$ Insider trades proportionally to difference between private valuation and price, and inversely related to time and price impact. 

  $\Rightarrow$ In equilibrium, price responds to order flow linearly. 

  $\Rightarrow$ In cross-section, Kyle’s $\lambda$, which can be estimated from a regression of price changes on order flow, should be higher for stocks with more informed trading (relative to liquidity/noise trading).

Several empirical measures of adverse selection proposed in the literature. (e.g., Glosten, 1987; Glosten and Harris, 1988; Hasbrouck, 1991)

Question: how well do these measures perform at picking up the presence of informed trading?
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- Find that measures of adverse selection are lower on days with informed trading
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Find that measures of adverse selection are lower on days with informed trading.
Two month excess return is around 9%
## Do informed trades move stock prices?

<table>
<thead>
<tr>
<th></th>
<th>days with informed trading</th>
<th>days with no informed trading</th>
<th>difference</th>
<th>t-stat</th>
</tr>
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<tr>
<td>excess return</td>
<td>0.0064</td>
<td>-0.0004</td>
<td>0.0068***</td>
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<td>turnover</td>
<td>0.0191</td>
<td>0.0077</td>
<td>0.0115***</td>
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Vyacheslav (Slava) Fos, UIUC
## Is adverse selection higher when informed trade?

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<tr>
<td><strong>Adverse Selection Measures</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda \times 10^6$</td>
<td>14.3311</td>
<td>20.1644</td>
<td>-5.8334***</td>
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<tr>
<td>$pimpact$</td>
<td>0.0060</td>
<td>0.0064</td>
<td>-0.0004**</td>
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<tr>
<td>$cumir$</td>
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<td>0.0015</td>
<td>-0.0002**</td>
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<tr>
<td>$trade - related$</td>
<td>0.0654</td>
<td>0.0673</td>
<td>-0.0019</td>
</tr>
<tr>
<td><strong>Other Liquidity Measures</strong></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$rspread$</td>
<td>0.0081</td>
<td>0.0089</td>
<td>-0.0008***</td>
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<tr>
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However, we find that measures of information asymmetry and liquidity indicate that stocks are more liquid when informed trades take place. This evidence seems at odds with our intuition. Biais, Glosten, and Spatt (2005): “As the informational motivation of trades becomes relatively more important, price impact goes up.”

⇒ The endogeneity issue seems more problematic than the literature may have previously recognized.
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Abnormal Share Turnover - Revisited

 Vyacheslav (Slava) Fos, UIUC

Do prices reveal the presence of informed trading?
We extend Kyle’s (insider trading) model to allow for general stochastic changes in volatility of uninformed order flow. Main results:

- Equilibrium price displays (endogenous) stochastic volatility if noise trader vol is predictable.
- Price impact (Kyle’s lambda) is stochastic: lower (higher) when noise trading volatility is higher (lower).
- Price impact (Kyle’s lambda) is submartingale: execution costs are expected to deteriorate over time.
- Informed trade more aggressively when noise trading volatility is higher and when measured price impact is lower.
- More information makes its way into prices when noise trading volatility is higher.
- Aggregate adverse selection execution costs for uninformed noise traders can be higher when noise trading is higher (and lambda is lower).
Theoretical Contribution

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We follow Back (1992) and develop a continuous time version of Kyle (1985)

Risk-neutral insider’s maximization problem:

$$\max_{\theta_t} \mathbb{E} \left[ \int_0^T (\nu - P_t) \theta_t dt \mid \mathcal{F}_t^Y, \nu \right]$$

(1)

As in Kyle, we assume there is an insider trading in the stock with perfect knowledge of the terminal value $\nu$

It is optimal for the insider to follow absolutely continuous trading strategy (Back, 1992).

Related work: Back and Pedersen (1998), Admati Pfleiderer (1988) and others...
Market Maker

- The market maker is also risk-neutral, but does not observe the terminal value. Instead, he has a prior that the value $\nu$ is normally distributed $N(\mu_0, \Sigma_0)$.

- The market maker only observes the total order flow:
  \[ dY_t = \theta_t dt + \sigma_t dZ_t \]

- where $\sigma_t$ is the stochastic volatility of the uninformed order flow:
  \[ d\sigma_t = m(t, \sigma^t) dt + \nu(t, \sigma^t) dM_t \]

  and $M_t$ is orthogonal (possibly discontinuous) martingale.

- Since the market maker is risk-neutral, equilibrium imposes that
  \[ P_t = E \left[ \nu \mid F_t^Y \right] \]

- We assume that the market maker and the informed investor observe $\sigma_t$. 
This may seem like a trivial extension of the Kyle (1985) model, as one might conjecture that one can simply ‘paste’ together Kyle economies with different noise-trading volatilities . . . . . . But, not so!

The insider will optimally choose to trade less in the lower liquidity states than he would were these to last forever, because he anticipates the future opportunity to trade more when liquidity is better and he can reap a larger profit.

Of course, in a rational expectations’ equilibrium, the market maker foresees this, and adjusts prices accordingly. Therefore, if noise trader volatility is predictable, price dynamics are more complex than in the standard Kyle model:

- Price displays stochastic volatility
- Price impact measures are time varying and not necessarily related to informativeness of order flow.
First, we conjecture a trading rule followed by the insider:

\[ \theta_t = \beta(t, \sigma^t, \Sigma_t)(v - P_t) \]

Second, we derive the dynamics of the stock price consistent with the market maker’s filtering rule, conditional on a conjectured trading rule followed by the insider:

\[ dP_t = \lambda(t, \sigma^t, \Sigma_t)dY_t \]

Then we solve the insider’s optimal portfolio choice problem, given the assumed dynamics of the equilibrium price.

Finally, we show that the conjectured rule by the market maker is indeed consistent with the insider’s optimal choice.
Price impact is stochastic:

\[ \lambda_t = \sqrt{\frac{\Sigma_t}{G_t}} \]  

(4)

where \( \Sigma_t \) is remaining amount of private information

\[ \Sigma_t = E\left[(v - P_t)^2 | F_t^Y\right] \]  

(5)

and \( G_t \) is remaining amount of uninformed order flow variance, solves the Backward stochastic differential equation (BSDE):

\[ \sqrt{G_t} = E\left[\int_t^T \frac{\sigma_s^2}{2\sqrt{G_s}} ds | \sigma_t^t\right] \]  

(6)

Optimal strategy of insider is:

\[ \theta_t = \frac{1}{\lambda_t} \frac{\sigma_t^2}{G_t} (v - P_t) \]  

(7)

⇒ Insider trades more aggressively when

- the ratio of private information \( (\sigma_t) \) to ‘equilibrium-expected’ noise trading volatility \( (G_t) \) is higher, and
- when price impact \( \lambda_t \) is lower.
General Features of Equilibrium

- Stock price displays time-varying volatility:
  \[ dP_t = \left( \frac{v - P_t}{G_t} \right) \sigma_t^2 dt + \sqrt{\frac{\Sigma_t}{G_t}} \sigma_t dZ_t \]  
  \[ (8) \]

- Note, that information asymmetry is necessary for price process to be non-constant.

- \( G_t \) is the crucial quantity to characterize equilibrium. Its BSDE solution satisfies:
  \[ G_t \leq E\left[ \int_t^T \sigma_s^2 ds \right] \]

- If \( \sigma \leq \sigma_t \leq \bar{\sigma} \) then we can show that there exists a maximal bounded solution to the recursive equation for \( G \) with:
  \[ \sigma^2 (T - t) \leq G_t \leq \bar{\sigma}^2 (T - t) \]  
  \[ (9) \]

- For several special cases we can construct an explicit solution to this BSDE:
  - \( \sigma_t \) deterministic.
  - \( \sigma_t \) general martingale.
  - \( \log \sigma_t \) Ornstein-Uhlenbeck process.
  - \( \sigma_t \) continuous time Markov Chain.
General Features of Equilibrium

- \( \lim_{t \to T} P_t = \nu \) ‘Stochastic bridge’ property of price in insider’s filtration.

- Market depth \((1/\lambda_t)\) is martingale.

- Price impact \((\lambda_t)\) is a submartingale (liquidity is expected to deteriorate over time).

- \(d\Sigma_t = -dP_t^2\) (stock price variance is high when information gets into prices faster, which occurs when noise trader volatility is high).

- Total profits of the insider are equal to \( \sqrt{\Sigma_0 G_0} \).

- Realized execution costs of uninformed can be computed pathwise as

\[
\int_0^T (P_{t+dt} - P_t)\sigma_t dz_t = \int_0^T \lambda_t \sigma_t^2 dt
\]

- Unconditionally, expected aggregate execution costs of uninformed equal insider’s profits.
We assume that uninformed order flow volatility is unpredictable (a martingale):

\[
\frac{d\sigma_t}{\sigma_t} = \nu(t, \sigma^t) dM_t,
\]

(10)

We can solve for \( G(t) = \sigma_t^2 (T - t) \),

Then market depth is a martingale:

\[
\frac{1}{\lambda_t} = \frac{\sigma_t}{\sigma_\nu},
\]

where \( \sigma_\nu^2 = \frac{\Sigma_0}{T} \) is the annualized initial private information variance level.

The trading strategy of the insiders is \( \theta_t = \frac{\sigma_t}{\sigma_\nu (T - t)} (\nu - P_t) \)

Equilibrium price dynamics are identical to the original Kyle (1985) model:

\[
dP_t = \frac{(\nu - P_t)}{T - t} dt + \sigma_\nu dZ_t.
\]

(11)
This example shows we can extend Kyle’s equilibrium by simply ‘plugging-in’ stochastic noise trading volatility:

- Market depth varies linearly to noise trading volatility,
- Insider’s strategy is more aggressive when noise trading volatility increases,
- Both effects offset perfectly so as to leave prices unchanged (relative to Kyle):
  - Prices display constant volatility.
  - Private information gets into prices linearly and independently of the rate of noise trading volatility (as in Kyle).

⇒ In this model empirical measures of price impact will be time varying (and increasing over time on average), but do not reflect any variation in asymmetric information of trades.
Suppose that volatility follows a strictly positive process of the form:

\[
\frac{d\sigma_t}{\sigma_t} = m(t, \sigma_t)dt + \nu(t, \sigma_t)dW_t 
\]  

If the expected growth rate of noise trading volatility follows a deterministic process \( m_t \):

- \( G(t) \) admits the solution: \( G(t) = \sigma_t^2 \int_t^T e^{\int_u^t 2m_s ds} du \)
- Private information enters prices at a deterministic rate
- Equilibrium price volatility is deterministic

For the insider to change his strategy depending on the uncertainty about future noise trading volatility, the growth rate of noise trading volatility \( m_t \) has to be stochastic.
We assume that uninformed order flow volatility follows a geometric Brownian Motion:

\[ \frac{d\sigma_t}{\sigma_t} = mdt + \nu dW_t, \]  

(13)

We can solve for \( G(t) = \sigma_t^2 B_t \) where \( B_t = \frac{e^{2m(T-t)} - 1}{2m} \).

Then market depth is \( \frac{1}{\lambda_t} = e^{-mt} \sigma_t \sqrt{\frac{B_0}{\Sigma_0}} \).

Equilibrium price dynamics follow a one-factor Markov non-homogenous bridge process:

\[ dP_t = \left( \frac{\nu - P_t}{B_t} \right) dt + e^{mt} \sqrt{\frac{\Sigma_0}{B_0}} dZ_t. \]  

(14)
As soon as there is predictability in noise trader volatility, equilibrium prices change (relative to Kyle):

- Price volatility increases (decreases) deterministically with time if noise trading volatility is expected to increase (decrease).
- Private information gets into prices slower (faster) if noise trading volatility is expected to increase (decrease).

Interesting separation result obtains:

- Strategy of insider and price impact measure only depends on current level of noise trader volatility.
- Equilibrium is independent of uncertainty about future noise trading volatility level ($\nu$).
- As a result, equilibrium price volatility is deterministic
- Private information gets into prices at a deterministic rate, despite measures of price impact (and the strategy of the insider) being stochastic!
Implications of constant growth rate

The Trading Strategy of the Insider

Figure: The Trading Strategy of the Insider
A two-state Continuous Markov Chain example

- Assume uninformed order flow volatility can take on two values $\sigma(0) < \sigma(1)$ where regime indicator $S_t \in [0, 1]$ follows:

$$dS_t = (1 - s_t) dN_0(t) - s_t dN_1(t),$$

where $N_i(t)$ is a standard Poisson counting process with jump intensity $\eta_i$ respectively.

- The solution is $G(t, s_t) = 1_{\{s_t=0\}} G^0(T - t) + 1_{\{s_t=1\}} G^1(T - t)$, where the deterministic functions $G^0, G^1$ satisfy the system of ODE (with boundary conditions $G^0(0) = G^1(0) = 0$):

$$G^0_\tau(\tau) = \sigma(0)^2 + 2\eta_0(\sqrt{G^1(\tau)G^0(\tau)} - G^0(\tau))$$

$$G^1_\tau(\tau) = \sigma(1)^2 + 2\eta_1(\sqrt{G^1(\tau)G^0(\tau)} - G^1(\tau))$$

- We compute execution costs of uninformed numerically in this case.

- Show that uninformed execution costs can be higher when noise trading volatility is higher (and Kyle lambda is actually lower).


**Figure:** $G$ function in high and low state
Figure: Four Private information paths
Empirical Motivation
Summary
Extension of Kyle’s model
Examples
Conclusion

Martingale noise trading volatility
General Diffusion Dynamics
Constant expected growth rate
Two State Markov Chain

Figure: Four paths of price impact $\lambda_t$
Figure: Four paths of Stock price volatility
**Figure:** Four paths of uninformed traders execution costs
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<td>Average price impact ($\int_0^T \lambda_t dt$)</td>
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<td>Execution costs ($\int_0^T \lambda_t \sigma_t^2 dt$)</td>
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<td>0.017</td>
<td>0.054</td>
<td>0.057</td>
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<tr>
<td>Normalized execution costs ($\frac{\int_0^T \lambda_t \sigma_t^2 dt}{\int_0^T \sigma_t^2 dt}$)</td>
<td>0.487</td>
<td>1.740</td>
<td>0.636</td>
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Average price impact is not informative about execution costs to uninformed traders. Normalizing by ‘abnormal’ trading volume is crucial. Even so, average execution costs to uninformed are path-dependent.
### Noise trading volatility paths

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<td>0.078</td>
<td>0.017</td>
<td>0.054</td>
<td>0.057</td>
</tr>
<tr>
<td>Normalized execution costs ($\frac{\int_0^T \lambda_t \sigma_t^2 dt}{\int_0^T \sigma_t^2 dt}$)</td>
<td>0.487</td>
<td>1.740</td>
<td>0.636</td>
<td>0.671</td>
</tr>
</tbody>
</table>

- Average price-impact is not informative about execution costs to uninformed traders.
- Normalizing by ‘abnormal’ trading volume is crucial.
- Even so, average execution costs to uninformed are path-dependent.
Recent empirical paper finds that standard measures of adverse selection and stock liquidity fail to reveal the presence of informed traders.

Propose extension of Kyle (1985) to allow for stochastic noise trading volatility. Seems more consistent with evidence:
- Insider conditions his trading on ‘liquidity’ state.
- Price impact measures are time-varying, and not necessarily higher when more private information flows into prices.
- Execution costs can be higher when measured price impact is lower.
- Generates stochastic price volatility.

Future work:
- Better measure of liquidity/adverse selection?
- Model of activist insider trading with endogenous terminal value. Why the 5% rule?
- Absence of common knowledge about informed presence.