Strategic Foundations for Efficient Competitive Markets with Adverse Selection*

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Abstract

I model a competitive insurance market with adverse selection as an "*informed-principal game*" . The informed buyer offers a set of contracts to all uninformed sellers, who accept or reject. If all sellers reject, then there is no trade. Otherwise, each one of the sellers who accepted has the right to add more contracts to the already existing offer if he wishes so. The buyer can choose one contract from one seller at the last stage of the game. I characterise the perfect Bayesian equilibria of this game. I show that the well-known Rothschild-Stiglitz-Wilson (RSW) allocation places a lower bound in the equilibrium payoff of each type and is the unique equilibrium allocation (or a "*strong solution*") when it is not Pareto dominated. Every interim incentive efficient allocation, that Pareto dominates the RSW allocation, is an equilibrium allocation and corresponds to a "*neutral optimum*". Bertrand-type competition among sellers drives expected profits to zero and demands every equilibrium allocation to be interim incentive efficient. The approach extends to any finite number of types and states, and to other similar markets like credit, labour or informed seller markets.

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JEL classification: *D04, D47, D60, D82, D86*

1 Introduction

One of the main open questions in information economics regards the right game-theoretic modelling of efficient competition in markets with asymmetric information. Early contributions in this literature[1] highlighted that in this type of markets the "*usual*

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[1]The most important contributions in this literature are Akerlof [1], Spence [33] and Rothschild and Stiglitz [32].

modeling" of price or/and quantity competition is not sufficient; equilibria may not exist or may be Pareto dominated using these models. The present paper builds on the seminal contributions of Myerson [25] and Maskin and Tirole [22] to construct a novel noncooperative game that possesses all the nice properties of a competitive market: (i) Equilibria always exist, (ii) competitors earn zero expected profits in equilibrium, and (iii) all equilibrium allocations are interim incentive efficient, in the sense of Holmstrom and Myerson [17].

To formalise this argument, I employ a standard adverse selection insurance market like the one analysed in Rothschild and Stiglitz [32] and Wilson [35]. A risk-averse privately informed buyer with a random endowment seeks for insurance, which is supplied in the market by uninformed risk-neutral sellers. The market takes the following simple extensive form: Initially, the buyer proposes a set of contracts to all sellers, who accept or reject. If all sellers reject, there is no trade. Otherwise, all those sellers who accepted have the right to add a new set of contracts in the already existing offer. A menu of contracts between the buyer and seller X is defined as the union of the set of contracts proposed by both parties. After the buyer observes all the menus of contracts that have been formed in the market, he can choose any of the available contracts contained in any of the menus. He is restricted to contract with only one seller- i.e. contracts are exclusive.[2]

I characterise the set of perfect Bayesian equilibria (PBE) of this game. The well-known Rothschild-Stiglitz-Wilson (or RSW) allocation plays a crucial role in the analysis. It is defined as the allocation that maximises the payoff of each type of the buyer within the set of incentive compatible allocations that make positive profits, irrespective of the beliefs of the sellers. The main results of the paper are the following: First, the RSW allocation is the unique equilibrium allocation when it is contained in the set of interim incentive efficient allocations.[3] Second, any interim incentive efficient allocation that strictly Pareto dominates the RSW allocation is an equilibrium allocation. Last, only interim incentive efficient allocations can be sustained as equilibrium allocations in pure strategies.[4]

The three main elements in the game that drive the result are the following: (i) The stage in which the buyer proposes a set of contracts. (ii) The stage in which the competing uninformed sellers have the right to add new sets of contracts. (iii) The fact that every menu of contracts is the union of the set of contracts proposed by both parties, and specifically it must contain the set of contracts proposed by the buyer as an option.

[2] I will use masculine pronounces (he or him) for the buyer and feminine pronounces (she or her) for the sellers. Moreover, for notational convenience, I depart from the usual model of a continuum of buyers and I employ a single-buyer model. This is without loss of generality.

[3] Unless otherwise stated, interim incentive efficiency is defined with respect to the prior beliefs of the sellers about the type of the buyer.

[4] "Pure strategies" are strategies where the buyer and sellers do not randomise over their proposals.

To begin with, following Maskin and Tirole [22], by allowing the informed party to propose a set of contracts in the first stage, he can always guarantee his RSW allocation in any equilibrium. In fact, the RSW allocation is the unique equilibrium allocation, or a *"strong solution"* in the sense of Myerson [25], when it is interim incentive efficient. Moreover, this stage along with the fact that every menu of contracts must contain the set of contracts proposed by both the buyer and any seller is important for the existence of equilibria. I show that any interim incentive efficient allocation that Pareto dominates the RSW allocation is an equilibrium allocation and corresponds to a *"neutral optimum"* in the sense of Myerson [25].[5] The intuition behind this result is as follows: Assume that in equilibrium, every type proposes the same set of contracts that consists of an interim incentive efficient allocation, and contracts with seller X. In case some other seller Y proposes any other set of contracts, then according to the equilibrium strategies, all types contract with seller Y. Note that all types are as well off as they are by contracting with seller X, because every menu of contracts must include the set of contracts proposed by the buyer. In this case, given that the proposal made by the buyer was interim incentive efficient, any possible *"cream-skimming"* offer attracts all types and must therefore be loss-making. For an appropriate set of off-the-equilibrium path beliefs, no type has an incentive to deviate either and therefore any interim incentive efficient allocation that strictly Pareto dominates the RSW allocation can always be sustained in equilibrium.

Lastly, The stage in which the uninformed sellers can add contracts in the already existing set of contracts proposed by the buyer is used to exploit Bertrand-type competition among sellers and eliminate profits and allocations that are not incentive efficient. Indeed, as I show, competition any equilibrium allocation must be interim incentive efficient.

Related Literature. My work is related to several strands in the literature. To begin with, the seminal paper on competitive screening markets with adverse selection is Rothschild and Stiglitz [32]. They analyse an insurance market with adverse selection and show that for some parameter values a "competitive" equilibrium fails to exist. Wilson [35] and Riley [30] place restrictions on the set of possible contracts insurance firms can offer and show that an equilibrium always exists. Miyazaki [24] extends the idea of Wilson [35] to a model where insurance firms can offer menus of contracts (instead of single contracts) and proves that an equilibrium always exists and the equilibrium allocation is always constrained efficient. From those two authors, this allocation is often called the Miyazaki-Wilson (or MW) allocation. Hellwig [16] provides a game-theoretic foundation for the idea of Wilson [35]. Along with the equilibrium allocation of Wil-

[5] The RSW allocation is itself a neutral optimum if it is contained in the set of interim efficient allocations.

son [35], he shows that there is a continuum of other equilibrium allocations.[6] Engers and Fernadez [11] also propose a game with an infinite number of moves to provide foundations for Riley's equilibrium allocation. Similarly to Hellwig [16], a continuum of other allocations can be supported as equilbria. Another strand in the literature that analyses the existence of mixed strategy equilibria in the elementary Bertrand game of Rothschild and Stiglitz [32] is Rosenthal and Weiss [31], and Dasgupta and Maskin [7, 8].

Netzer and Scheuer [26] propose the following three-stage game: In the first stage, insurance firms offer menus of contracts. In the second stage, each firm decides either to stay in the market, or become inactive, in which case it must pay an exogenously-given withdrawal cost. In the last stage, insurees select from the set of contracts offered by all active firms. Depending on the value of the withdrawal cost, there may be an equilibrium, where the equilibrium allocation coincides with the MW, or not. Mimra and Wambach [23] examine a game in which in the first stage insurance firms offer menus of contracts and there is an infinite number of rounds in the second stage in which each firm can withdraw contracts out of those it has proposed. Insurees choose from the set of contracts that have not been withdrawn after the end of this process. Without further restrictions, the equilibrium set of this game contains every incentive compatible and positive profit allocation. If, however, there are firms ready to enter the market after all incumbent firms have made their moves, the equilibrium allocation coincides with MW. Diasakos and Koufopoulos [9] adopt a similar approach to Hellwig [16] but introduce endogenous commitment in the first stage. It is claimed that the MW allocation is the unique equilibrium of the game. However, neither the action/strategy space nor the contract space are well-defined in this paper.

Asheim and Nilssen [2], Faynzilberg [12] and Picard [27] also examine different games and show that the equilibrium allocation coincides with MW. For instance, in Asheim and Nilssen [2] it is possible for insurance firms to renegotiate the contracts they have signed with their customers, imposing the constraint that they can not discriminate among the different types in the renegotiation stage. Faynzilberg [12] examines a model in which insurance firms can become insolvent, which introduces an externality between agents in a contract. Picard [27] examines a similar externality model in which insurance firms can offer "participating contracts" such that any insuree who signs a contract needs to "participate" in the profits of the firm who offered it. It seems that these models significantly depart from the original formulation of Rothschild and Stiglitz [32] by imposing hardly justifiable theoretical assumptions. Moreover, apart from Picard [27], all the above papers analyse the case of two types. In this paper, I analyse a more general environment with any finite number of types and states of nature, and the

[6] The set of equilibria of Hellwig's game coincides with the set of equilibria of a signaling game in which the informed party moves first and can propose a unique contract that is accepted or rejected by some firm. See also Maskin and Tirole[22] pp. 30.

assumptions imposed are easier to be justified. Moreover, the equilibrium set of this paper significantly differs from the equilibrium set of all the aforementioned papers.

The seminal papers on informed principal models are Spence [33], Myerson [25] and Maskin and Tirole [22]. Myerson [25] examines a general environment in which a principal with private information designs a mechanism to coordinate his subordinates. The focus of the paper is on the development of a theory of inscrutable mechanism selection for the principal, and what axioms desirable mechanisms must satisfy. The principal's neutral optima are defined as the smallest possible set of "*unblocked*" mechanisms. Spence [33] examines a labour market in which workers can acquire costly education before applying for jobs. He shows that a continuum of equilibria exist most of them Pareto dominated. This multiplicity of equilibria gave rise to an extensive literature examining possible equilibrium refinements. These refinements tried to put restrictions on the off-the-equilibrium path beliefs that are used to support undesirable equilibria. Among others, the most i well-known refinements are Kohlberg and Mertens [19], Cho and Kreps [5] and Banks and Sobel [3]. Maskin and Tirole [22] analyse an informed principal environment with an extended set of contracts (or mechanisms). They consider a three stage game (proposal- acceptance/rejection- execution) and, similarly to this paper, they show that in any equilibrium of the game, the informed principal can guarantee his RSW allocation in contrast to Spence [33]. Naturally, a wealth of other allocations that weakly Pareto dominate the RSW contract can be supported in equilibrium for some set of beliefs.

Lastly, this paper is also related to the literature in general equilibrium with adverse selection starting from Prescott and Townsend [29, 28]. Gale [13, 14, 15], and Dubey and Geanakoplos [10] explore different notions of competition to prove existence of equilibria, and propose refinements of beliefs to pin down the equilibrium set. Bisin and Gottardi [4] also analyse a Walrasian market with adverse selection and introduce markets for property rights that agents can trade. They show that property rights help the implementation of constrained efficient allocations. Citanna and Siconolfi [6] also provide a different notion of competition and prove, under mild restrictions, that for any finite number of types an equilibrium always exists and it is constrained efficient. In fact, their analysis is closely related to my paper, even though it is more general and it is applied to a Walrasian market.

In Section 2, I present the model. In Section 3, I provide some preliminary properties of interim incentive efficient allocations that prove to be important for the analysis. I also provide an algorithm to characterise the RSW allocation which plays an important role in this paper. In Section 4, I define the game- i.e. market structure, contract space and strategies of the players. Moreover, I give a definition of a perfect Bayesian equilibrium. In Section 5, the main results of the paper are presented. Lastly, in Section 6 some extensions of the model are suggested.

2 The Model

There is a risk-averse buyer with a finite number of possible types $t = 1, ..., T$. There is a finite number of possible states $\omega = 0, 1, ..., \Omega$. The endowment of the buyer is risky and is denoted by $e = (W - d_0, W - d_1, ..., W - d_\Omega)$, where $d_0 = 0$ and $d_\omega > 0$, for any $\omega \geq 1$. For simplicity, I assume that the endowment is type-invariant. Type t's objective probability distribution over the states is denoted by $\theta^t = (\theta^t_0, ..., \theta^t_\Omega)$. The type of the buyer is his private information. Assume that $\sum_{\omega=0}^{\Omega} \theta^s_\omega d_\omega < \sum_{\omega=0}^{\Omega} \theta^{s'}_\omega d_\omega$ for any $s > s'$; the expected endowment is increasing in the index of types. The prior beliefs about the type of the buyer are $\lambda^{\cdot}_0 = \{\lambda^t_0\}^T_{t=1}$, with $\sum_{t=1}^{T} \lambda^t_0 = 1$. Furthermore, I assume that the state of nature is perfectly observable and verifiable by a court of law. This is the minimum requirement for contracts to be enforceable. The von Neuman-Morgenstern utility index of all types is state- and type- independent and is represented by $u : X \to \mathbb{R}$, where u is continuous, strictly increasing and strictly concave.

Sellers are denoted by $i \in N$, where $N \geq 2$ is also the number of sellers in the market. They are all risk-neutral, expected utility maximisers and they have enough wealth in order to provide insurance to the buyer if he wishes so. The number of sellers must be at least two for competition to exist.[7] Denote as V^i the expected utility of seller i.

An insurance contract is denoted by $\psi = (p, b_1, ..., b_\Omega) \in \mathbb{R}^{\Omega+1}_+$ with p denoting the premium paid and b_ω the benefit received by the buyer in state ω. The space of *feasible insurance contracts* is given by $\Psi = \{(p, b_1, ..., b_\Omega) : 0 \leq p \leq \min\{W, W - d_\omega + b_\omega\}, b_\omega \geq 0, \forall\ \omega\}$. The expected utility of type t from insurance contract ψ is given by: $U^t(\psi) = \sum_{\omega=0}^{\Omega} \theta^t_\omega u(W - d_\omega - p + b_\omega)$. Denote the null contract by $\psi_o = (0, ..., 0)$ and the status quo utility of type t as: $\underline{U}^t = \sum_{\omega=0}^{\Omega} \theta^t_\omega u(W - d_\omega)$. The net expected profit (cost) of insurance contract ψ when taken up by type t is given by $\pi^t(\psi) = p - \sum_{\omega=1}^{\Omega} \theta^t_\omega b_\omega$. Denote by $\Pi(\psi^{\cdot}) = \sum_{t=1}^{T} \lambda^t_0 \pi^t(\psi^t)$ the expected profit of allocation $\psi^{\cdot}$.

Single-Crossing Assumption: Whenever ψ, ψ' are such that $U^s(\psi) \geq U^s(\psi')$ and $U^{s+1}(\psi') > U^{s+1}(\psi)$, then $U^{s+h}(\psi') > U^{s+h}(\psi)$ and $U^{s-h}(\psi) > U^{s-h}(\psi')$ for any $h > 1$.

In words, single crossing says that for any two contracts ψ and ψ', if some type s weakly prefers ψ to ψ' and the immediate successor type $s + 1$ strictly prefers ψ' to ψ, then all types lower in the rank from type s strictly prefer ψ to ψ' and those types

[7] The assumption of one buyer and many sellers is without loss of generality. Usually in this type of model, it is assumed that there is a continuum of informed parties (buyers). The reason I chose to analyse a single buyer model is that it greatly simplifies the notation and analysis and it is closer to the contract theory framework. The results would remain the same even if we assumed that there was a continuum of buyers and no aggregate uncertainty. In this model, insurance would be provided by sellers who could freely start operating insurance firms.

higher in the rank from type $s+1$ strictly prefer ψ' and ψ. Note that when there are only two states of nature $\Omega = 1$, this condition is vacuously satisfied.

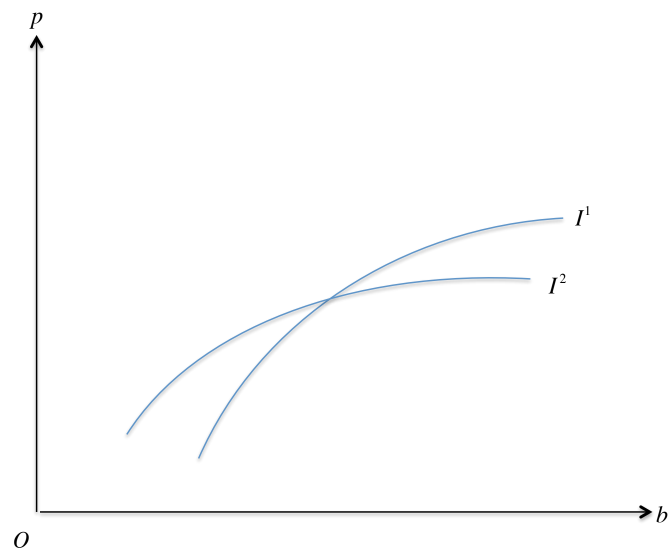

Figure 1: A graphical illustration of Wilson's model for $T = 2$.

A special case of the model when $\Omega = 1$ is known in the literature of competitive insurance markets with adverse selection as *Wilson's model*, from WIlson [35]. In this model, there are only two states $\omega = 0, 1$ which can be interpreted as no accident and accident respectively. A graphical illustration of this model is provided in Figure 1. In this figure, the benefit is represented on the horizontal axis and the premium on the vertical axis. The indifference curve of type t is labeled by I^t. In the figure, there are only two possible types. Although not depicted, the profit functions are upward straight lines. The zero-profit line for type t is the line with slope θ_1^t passing through the origin. Because of the single-crossing property, the indifference curve of type t is always steeper than the one of type $t+1$ for any b. Utility increases as we move south-east and profit increases as we move north-west for all types. We can easily establish that all indifference curves are tangential to the zero-profit lines at the same point $b^t = d$ (the point of full insurance), for all t with a higher premium for the high-risk types.[8] This is because the utility indexes are type- and state- independent.

Definition: An allocation $\psi^{\cdot} = \{\psi^t\}_{t=1}^T$ is a set of contracts one for each type of the buyer. Denote by $\mathbf{\Psi} \subseteq \mathbb{R}_+^{T\times(\Omega+1)}$ the set of all allocations.

Definition: An allocation $\psi^{\cdot} = \{\psi^t\}_{t=1}^T$ is *incentive compatible* (IC) if and only if: $U^s(\psi^s) \geq U^s(\psi^{s'}), \forall\ s, s'$. Denote by $\mathbf{\Psi}^{IC}$ the set of all incentive compatible allocations.

[8]For a detailed exposition of Wilson's model see Jehle and Reny [18].

DEFINITION: An allocation $\psi^{\cdot} = \{\psi^t\}_{t=1}^T$ is *interim individually rational* (IR) if and only if: (i) $U^t(\psi^t) \geq \underline{U}^t, \quad \forall \quad t = 1, ..., T$, and, (ii) $\Pi(\psi^{\cdot}) \geq 0$. Denote the set of all incentive compatible and interim individually rational allocations as $\mathbf{\Psi}^{ICR}$.

DEFINITION: An alocation $\psi^{\cdot} = \{\psi^t\}_{t=1}^T$ weakly Pareto dominates allocation $\hat{\psi}^{\cdot} = \{\hat{\psi}^t\}_{t=1}^T$ if $U^t(\psi^t) \geq U^t(\hat{\psi}^t)$ for all $t = 1, ..., T$ with the inequality being strict for at least one t. Strict Pareto dominance is defined by taking all inequalities to be strict.

DEFINITION: An allocation $\psi^{\cdot} = \{\psi^t\}_{t=1}^T$ is weakly (strongly) interim incentive efficient (IE), in the sense of Holmstrom and Myerson [17], if and only if $\psi^{\cdot} \in \mathbf{\Psi}^{ICR}$, and there exists no other $\tilde{\psi}^{\cdot} = \{\tilde{\psi}^t\}_{t=1}^T \in \mathbf{\Psi}^{ICR}$ that strictly (weakly) Pareto dominates $\psi^{\cdot}$.

Note that interim efficiency is defined with respect to the buyer's payoff and the prior beliefs of the sellers. In fact, a IE allocation maximises the welfare of all types of the buyer guaranteeing at the same time that no seller is worse off than not participating in any transaction with the buyer. The efficiency concept demands only that allocations are individually rational on average and not necessarily ex post individually rational.

It is a well-known result in this type of economies (with type independent utility indexes) that the sets of weak and strong interim incentive efficient allocations coincide; all weak interim incentive efficient allocations are also strong.[9] Therefore, denote as $\mathbf{\Psi}^{IE}$ the set of strongly interim efficient allocations. Evidently, $\mathbf{\Psi}^{IE} \subseteq \mathbf{\Psi}^{ICR}$.

3 Preliminary Properties

In this section, I provide some preliminary properties of the set of interim incentive efficient allocations that will prove useful in the analysis of the equilibria of the game. A special case of the model is the Rothschild and Stiglitz [32] model, where $T = 2$ and $\Omega = 1$. Type 1 is the high-risk type and type 2 is the low-risk type. To better illustrate the argument, I will often employ this special case. Figure 2 provides a graphical illustration of the Rothschild and Stiglitz [32] model. In the same figure, we can also see the well-known Rothschild and Stiglitz [32] allocation, often called the Rothschild and Stiglitz [32] equilibrium. A generalisation of the Rothschild and Stiglitz [32] allocation will be called from now on the RSW allocation; an acronym for Rothschild-Stiglitz-Wilson.

DEFINITION: The RSW allocation is denoted as $\psi^{\cdot}_{RSW}$ and can be derived by solving

[9] See Citanna and Siconolfi [6] for a discussion.

the following lexicographic maximisation program which I call Program RSW:

$$\text{Lex} \max_{\psi^1,...,\psi^T} \quad (U^1(\psi^1), ..., U^T(\psi^T))$$

$$U^s(\psi^s) \geq U^s(\psi^{s'}) \quad \text{for all} \quad s, s'$$

$$\pi^t(\psi^t) \geq 0, \quad \text{for all} \quad t = 1, ..., T$$

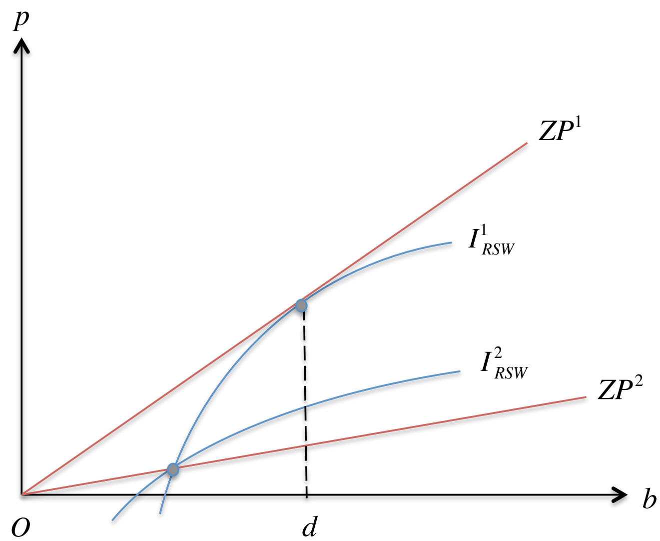

Figure 2: The RSW allocation for the special case of $T = 2$ and $\Omega = 1$.

Denote as ψ^t_{RSW} the RSW insurance contract of type t. In the optimisation program, the constraint set is a closed subset of a compact set and therefore is compact. Note that the order of types in the lexicographic maximisation program is irrelevant. All that matters is that the utility of every type is maximised inside the feasible set.

The RSW allocation plays a significant role in this paper as well as in any competitive market with adverse selection. This is because it is the only incentive compatible allocation that maximises the utility of all types and it is also ex post individually rational. In the spirit of Myerson [25], the RSW allocation is a "*safe*" or incentive compatible "*type-by-type*" allocation (or mechanism). A safe mechanism is one which would be incentive compatible even if the sellers knew the type of the buyer. In Maskin and Tirole [22], the RSW is the only allocation that all types of the buyer can guarantee in any equilibrium.

Many of the proofs will be based on the following important property of IC allocations.

Lemma 3.1. *For every $\psi^{\cdot} \in \mathbf{\Psi}^{ICR}$ and $\varepsilon > 0$, there exists $\tilde{\psi}^{\cdot} \in \mathbf{\Psi}^{ICR}$ that strictly Pareto dominates $\psi^{\cdot}$ and $\Pi(\tilde{\psi}^{\cdot}) = \Pi(\psi^{\cdot}) - \varepsilon$.*

Proof: Take allocation $\psi^{\cdot} \in \mathbf{\Psi}^{IC}$ with $\psi^t = (p^t, b_1^t, ..., b_\Omega^T)$, for all $t = 1, ..., T$ and let $\Pi(\psi^{\cdot})$ be the profit. For any $\varepsilon^t > 0$, for any $t = 1, ..., T$ define $\tilde{\psi}^t = (p^t - \varepsilon^t, b_1^t, ..., b_\Omega^T)$. Because only local incentive constraints bind (single-crossing), and the continuity and strict monotonicity of the utility functions, we can always find $\varepsilon^1, ..., \varepsilon^T$ arbitrarily small which define $\tilde{\psi}^{\cdot} = (\tilde{\psi}^1, ..., \tilde{\psi}^T)$ such that $\tilde{\psi}^{\cdot} \in \mathbf{\Psi}^{IC}$ and $\Pi(\tilde{\psi}^{\cdot}) = \sum_{t=1}^T \lambda_0^t \pi^t(\tilde{\psi}^t) = \sum_{t=1}^T \lambda_0^t \pi^t(\psi^t) - \sum_{t=1}^T \lambda_0^t \varepsilon^t = \Pi(\tilde{\psi}^{\cdot}) - \sum_{t=1}^T \lambda_0^t \varepsilon^t$. By defining $\varepsilon \equiv \sum_{t=1}^T \lambda_0^t \varepsilon^t$ we obtain the result. Q.E.D.

If we recall the definition of IE allocations, it is not hard to see that, under single-crossing, risk-neutrality of the sellers, and Lemma 3.1, every such allocation must be zero profit.

Lemma 3.2. *Every* $\psi^{\cdot} \in \mathbf{\Psi}^{IE}$ *is such that* $\Pi(\psi^{\cdot}) = 0$.

Proof: Assume that there exists some $\psi^{\cdot} \in \mathbf{\Psi}^{IE}$, with $\psi^t = (p^t, b_1^t, ..., b_\Omega^t)$, for all $t = 1, ..., T$, such that $\Pi(\psi^{\cdot}) > 0$. From Lemma 3.1, there exists $\varepsilon > 0$ arbitrarily small and $\tilde{\psi}^{\cdot} \in \mathbf{\Psi}^{IC}$ such that $\Pi(\tilde{\psi}^{\cdot}) = \Pi(\psi^{\cdot}) - \varepsilon > 0$. Therefore, $\tilde{\psi}^{\cdot} \in \mathbf{\Psi}^{ICR}$. However, $\tilde{\psi}^{\cdot}$ is such that $\tilde{\psi}^t = (p^t - \varepsilon^t(\varepsilon^T), b_1^t, ..., b_\Omega^t)$, for all $t = 1, ..., T$, which means that the utility of all types is higher under $\tilde{\psi}^{\cdot}$ or $U^t(\tilde{\psi}^t) > U^t(\psi^t)$ contradicting that $\psi^{\cdot} \in \mathbf{\Psi}^{IE}$. Q.E.D.

4 The Game

In this section, I describe the interaction of economic agents in the market. I specify a noncooperative game- i.e. a *market mechanism*- in which all economic participants meet and exchange contracts. The most important element is that all agents act competitively.

In the market, Nature moves first and decides the buyer's type. This is neither observable nor verifiable by any third party. Then, the buyer makes a first proposal to all sellers in the form of a set of a finite number of contracts.[10] Denote the set of contracts proposed by the buyer as $\mu_b^{\cdot}$. After the buyer's proposal, each one of the sellers decides whether to participate in the game or not (accept or reject the proposal). Any seller who decides to participate can propose a new set of a finite number of contracts. Denote the set of contracts proposed by seller i as $\mu_i^{\cdot}$. A menu of contracts between the buyer and some seller i is defined as the union of the proposals of the set of contracts from the buyer and seller i. Denote such a menu as $m^i = (\mu_i^{\cdot}, \mu_b^{\cdot})$. The buyer has access to any menu traded in the market. Note that one of the most important features of the game is that every menu of contracts between the buyer and seller i must always include as options both the buyer's as well as the seller's proposals. If any of the sellers

[10] For technical reasons any set of contracts can only contain a finite number of contracts. This is without loss of generality.

decides not to participate, then she cannot make any proposal and she is excluded from the game irreversibly.

An allocation can be formed by combining contracts from all (or some) menus of contracts in the market or out of a single menu. Naturally, every equilibrium allocation must be incentive compatible. The buyer can sign only one contract with only one seller. This is one of the most common and most important assumptions used in this literature. Relaxing this assumption leads to *common agency* problems. See Martimort [20] and Stole [34]. Denote the "winning" seller- i.e. the seller who contracts with the buyer- by i^*.

To formally describe the timing of events, the market is formulated as an extensive form game, denoted as Γ^e. To simplify notation, denote as a_k^j, $j = \{N, b, i\}$, $k = 0, 1, 2, 3, 4$, an action from nature, the buyer, or seller i in stage k:

Stage 0: Nature decides the type of the buyer: $a_0^N \in \{1, ..., T\}$.

Stage 1: The buyer proposes a set of contracts $a_1^b = \mu_b^{\cdot}$.

Stage 2: Each seller $i \in \mathbb{N}$ accepts or rejects the proposal of the buyer, $a_2^i = \{1, 0\}$, for all $i \in \mathbb{N}$. $\dot{N}$= {set of sellers who accepted}. If $\dot{N} = \{\emptyset\}$, the game ends with payoffs $V^i = 0$, for each $i \in \mathbb{N}$ and $\underline{U}^t$ for type $t = 1, ..., T$. If $\dot{N} \neq \{\emptyset\}$, the game moves to Stage 3. For each $i \notin \dot{N}$, $V^i = 0$.

Stage 3: Each seller $i \in \dot{N}$ proposes a new set of contracts $a_3^i = \mu_i^{\cdot}$. A menu of contracts between the buyer and some seller i is formed by taking the union of the proposals of the set of contracts proposed by the buyer and seller i. Denote such a menu as $m^i = (\mu_i^{\cdot}, \mu_b^{\cdot})$.

Stage 4: The buyer signs one of the available contracts $a_4^b \in \cup_{i \in \dot{N}} m^i$. If $a_4^b = \psi^s$, for some s, the game ends with payoffs $U^t(\psi^s)$ for the buyer of type t, $V^{i^*} = \pi^t(\psi^s)$ for seller i^*.

A strategy for the buyer is denoted by s^b and consists of a proposal of a set of contracts $\mu_b^{\cdot}$ in Stage 1 and, for any possible menu of contracts, a choice of a contract proposed by the sellers or a contract out of his proposed set of contracts in Stage 4. A strategy for seller i is denoted by s^i and, for any possible proposal of a set of contracts by the buyer, consists of a decision (accept/ reject) in Stage 2 and, a proposal of a new set of contracts in Stage 3 (conditional on acceptance). A mixed strategy is defined by taking probability measures over the set of pure strategies.

After Stage 1 each seller observes an action from the buyer. This action may disclose some information regarding the buyer's type. Therefore, all sellers revise their beliefs appropriately after the buyer's action. The *common posterior beliefs* of all sellers after Stage 1 are denoted as $\lambda_1^{\cdot} = \{\lambda_1^t\}_{t=1}^T$.

I will be interested in the *perfect Bayesian equilibria* of the overall game. Given the dynamic nature of the game and the fact that sellers take actions after observing a move from the buyer, this is a plausible assumption. A perfect Bayesian equilibrium is a vector of strategies for the buyer and all the sellers and a vector of beliefs at each information set such that (i) the strategies of the players are optimal at every node of the game tree (sequential rationality), (ii) interim beliefs about the type of the buyer are the same in nodes where he does not take an action and are derived by Bayes' rule from the strategies of the players (Bayesian updating).

5 Equilibria and their Properties

We can now turn our interest to the perfect Bayesian equilibria of game Γ^e. In Section 3, we characterised properties of allocations with no reference to any game or market mechanism. All the properties in Section 3 hold even if there is central planning (a mechanism designer) collecting information and implementing allocations. In fact, the main goal of this paper is to examine the relationship between IE allocations and the set of equilibrium allocations for game Γ^e. We will only be interested in the set of pure strategy PBE refraining from examining strategies in which agents mix over pure strategies.

First, note that Game Γ^e has a strong signaling feature.[11] This is because the informed party offers a set of contracts which may perhaps reveal some of his information to the uninformed parties. In fact, the game is an enriched "*Informed Principal Game*", in which the buyer is the informed principal and the sellers the subordinates. The difference from the usual informed principal model of Myerson [25] and Maskin and Tirole [22] is that the subordinates have the right to influence the mechanism with their offers. A significant observation is that Myerson's [25] *inscrutability principle* holds in game Γ^e. According to this principle, the informed party (buyer) never needs to disclose his type with his proposal. The justification for this claim is that the buyer never needs to communicate any information to the sellers by his choice (of a mechanism), because he can always build such communication into the process of the mechanism itself. What it is meant by this is that for every possible equilibrium in which there is partial revelation of information- e.g. an equilibrium in which partitions of different types offer the same set of contracts- there is another equilibrium in which all types offer the same set of contracts and the equilibrium payoffs are equivalent.[12] Therefore, there is no loss of

[11]In pure signaling games, there exist *pooling* equilibria, in which no information is transmitted from the informed to the uninformed parties, as well as *separating* equilibria in which all relevant information is transmitted. In these games however, interaction among the uninformed and the informed parties ends after the transmission of this information. Moreover, the mechanisms that are allowed to be traded in these models are very limited- e.g. in Spence [33]; every worker can propose only one contract. Our game differs from these models, because the uninformed parties have the right to make a proposal of contracts to the informed party and the informed party can offer richer mechanisms. Therefore, the set of equilibria has a different structure.

[12]Note that full revelation is never a possible equilibrium scenario. The intuition behind this result is the following: Assume (for simplicity) that there exists an equilibrium in which every different type makes a distinct proposal and his type is fully revealed. It must necessarily be (because of sequential rationality) that the proposal of any type must be utility maximising for this type (or incentive compatible). In other words no type must have an incentive to propose something that some other type

generality in assuming that the sellers make their proposals without having updated their beliefs about the type of the buyer. In other words, there is no loss of generality to assume that $\lambda_1^{\cdot} \equiv \lambda_0^{\cdot}$ in any equilibrium. The first result of the paper is stated as follows:

Proposition 5.1. *If $\hat{\psi}^{\cdot}$ is an equilibrium allocation of game Γ^e, then $\Pi(\hat{\psi}^{\cdot}) = 0$.*

Proof: It is trivial to see that there cannot exist a strictly negative profit equilibrium allocation. Assume therefore that there exists an equilibrium allocation $\hat{\psi}^{\cdot}$ such that $\Pi(\hat{\psi}^{\cdot}) > 0$. Assume, for simplicity, that all sellers have accepted the offer of the buyer: $\dot{N} \equiv N$. One can easily prove that there exists at least one seller $i' \in N$ such that $\hat{V}^{i'} < \Pi(\hat{\psi}^{\cdot})$, where $\hat{V}^{i'}$ is the equilibrium payoff of Seller i'. From Lemma 3.1, for any $\varepsilon > 0$, there exists $\tilde{\psi}^{\cdot}$ that strictly Pareto dominates $\hat{\psi}^{\cdot}$ and $\Pi(\tilde{\psi}^{\cdot}) = \Pi(\hat{\psi}^{\cdot}) - \varepsilon > 0$. Therefore, there exists ε' (arbitrarily small) such that $\Pi(\tilde{\psi}^{\cdot}) = \Pi(\hat{\psi}^{\cdot}) - \varepsilon' > \hat{V}^{i'}$ which simply means that seller i' can increase his payoff by proposing $\tilde{\mu}_{i'}^{\cdot} = \tilde{\psi}^{\cdot}$ such that a new menu $\tilde{m}^{i'} = (\hat{\mu}_b^{\cdot}, \tilde{\mu}_{i'}^{\cdot})$ is formed. In this case, all types must contract with seller i', given that $\tilde{\psi}^{\cdot}$ strictly Pareto dominates $\hat{\psi}^{\cdot}$, otherwise the equilibrium fails to be sequentially rational- an immediate contradiction. i' makes strictly higher profits than when offering allocation $\hat{\psi}^{\cdot}$ which contradicts the thesis that $\hat{\psi}^{\cdot}$ is an equilibrium allocation. Q.E.D.

This result highlights the competitive identity of the game. Because of the presence of many competing sellers equilibria are compatible with the zero-profit condition (due to constant-returns-to-scale). The most important assumption guaranteeing this is Single-Crossing. This assumption allows sellers to "*Bertrand compete*" in order to eliminate any strictly positive profits if there exist. Note however that single-crossing is only a sufficient condition and not a necessary one.

The following result is one of the main results of the paper:

Theorem 5.2. *If $\hat{\psi}^{\cdot}$ is an equilibrium allocation of game Γ^e, then $\hat{\psi}^{\cdot} \in \mathbf{\Psi}^{IE}$.*

Proof: Assume $\hat{\psi}^{\cdot} \notin \mathbf{\Psi}^{IE}$ is an equilibrium allocation. From Proposition 5.1, we know that $\Pi(\hat{\psi}^{\cdot}) = 0$. Assume, again for simplicity, that all sellers have accepted the offer of the buyer: $\dot{N} \equiv N$, and all of them make zero profits in this equilibrium. Because $\hat{\psi}^{\cdot} \notin \mathbf{\Psi}^{IE}$, there exists a continuum of ICR allocations that strictly Pareto dominate $\hat{\psi}^{\cdot}$. In this case, from Lemma 3.1, there exists $\varepsilon \equiv \sum_{t=1}^{T} \lambda_0^t \varepsilon^t > 0$ arbitrarily small for some $(\varepsilon^1, ..., \varepsilon^T) >> 0$ that define $\tilde{\psi}^{\cdot} \in \mathbf{\Psi}^{ICR}$ which strictly Pareto dominates $\hat{\psi}^{\cdot}$ and is

proposes in equilibrium. Given that the buyer's type becomes publicly known after his proposal, in the continuation of the game, and because of Bertrand competition, at least one seller must propose to this type a menu that contains as a contract, the contract that this type would get under complete information. Otherwise, the equilibrium fails to be sequentially rational. Note that that with state and type independent utility functions the first-best contract of type t is $(\sum_{\omega=1}^{\Omega} \theta_\omega^t d_\omega, d_1, ..., d_\Omega)$. Every type is perfectly insured. Because by assumption $\sum_{\omega=1}^{\Omega} \theta_\omega^T d_\omega < \sum_{\omega=1}^{\Omega} \theta_\omega^t d_\omega$ for any $t = 1, ..., T$, all types strictly prefer the first-best contract of type T over their contract. Therefore, there is a contradiction with the assumption that the first initial offer was utility maximising for all types.

such that $\tilde{\psi}^t = (\hat{p}^t + \varepsilon^t, \hat{b}_1^t, ..., \hat{b}_\Omega^t)$, for any $t = 1, ..., T$. Therefore, at least one seller i' can offer allocation $\tilde{\psi}^{\cdot}$ that makes strictly positive profits. All types must contract with seller i' because of sequential rationality. This contradicts the thesis that all types sign a contract from allocation $\hat{\psi}^{\cdot}$. Therefore, if there exists an equilibrium then the only possible equilibrium allocation must be such that $\hat{\psi}^{\cdot} \in \psi^{IE}$. Q.E.D.

Not only equilibrium allocations must be zero-profit, but also, from Theorem 5.2, there cannot be Pareto dominated by other allocations. The stage where sellers can propose contracts is critical for both Proposition 5.1 and Theorem 5.2; it allows Bertrand-type competition among sellers in order to eliminate any strictly positive profits and to force allocations to be IE. This seems to be an important departure from all the relevant papers in the literature since in their main parts, they are unable to exclude allocations that are not interim incentive efficient.

The following proposition is the third result concerning the set of equilibrium allocations of game Γ^e.[13] It states a minimum bound in the equilibrium payoff of each type.

Proposition 5.3. *If $\hat{\psi}^{\cdot}$ is an equilibrium allocation of game Γ^e, then $U^t(\hat{\psi}^t) \geq U^t(\psi^t_{RSW})$, for all $t = 1, ..., T$.*

Proof: Assume that there exists an equilibrium in which the equilibrium allocation $\hat{\psi}^{\cdot}$ is such that $U^t(\hat{\psi}^t) < U^t(\psi^t_{RSW})$ for some t. From Proposition 5.1, we know that $\Pi(\hat{\psi}^{\cdot}) = 0$. Assume that the set of sellers participating is $\dot{N}$. Take $\varepsilon > 0$ small enough. Let the buyer of type t propose the following set of contracts in Stage 1: $\mu_b^{\cdot} = \tilde{\psi}_b^{\cdot} \in \mathbf{\Psi}^{IC}$ such that $\tilde{\psi}_b^{\cdot}$ is the solution of the perturbed program RSW with $U^s(\psi^s) \geq U^s(\psi^{s'}) + \varepsilon$ and $\pi^s(\psi^s) \geq \varepsilon$. Under single-crossing, these constraints can always be satisfied for $\varepsilon > 0$ small enough. Let ε be such that $U^t(\tilde{\psi}^t) > U^t(\hat{\psi}^t)$. The buyer has the discretion to choose one of these contracts in Stage 4. Because $\pi^t(\tilde{\psi}_b^t) > 0$, for all $t = 1, ..., T$, no seller is worse off including the allocation proposed by the buyer in any menu of contracts since, given that $\tilde{\psi}_b^{\cdot}$ is strictly incentive compatible, the buyer will choose the right contract out of this allocation if he does so. In fact, this allocation gives strictly positive profit regardless of the beliefs and therefore at least one seller $i \in \dot{N}$ has to accept the proposal of the buyer. Therefore, at any continuation of the game, type t's payoff is at least $U^t(\tilde{\psi}^t)$. This contradicts the initial hypothesis that $\hat{\psi}^{\cdot}$ was an equilibrium allocation. Because this is true for any $\varepsilon > 0$, a lower bound in the utility of any type t is $U^t(\psi^t_{RSW})$. Q.E.D.

Corollary 5.4. *If $\psi^{\cdot}_{RSW} \in \mathbf{\Psi}^{IE}$, then $\psi^{\cdot}_{RSW}$ is the unique equilibrium allocation.*

[13] Note that the equilibrium allocation may be a combination of contracts from $\mu_b^{\cdot}$ and $\mu_i^{\cdot}$ for any $i \in \dot{N}$. This is the case for example when the RSW allocation is interim incentive efficient. The equilibrium allocation must always be incentive compatible. There exist equilibria in which some seller serves all types.

Proposition 5.3 (and its proof) is similar to Proposition 5 of Maskin and Tirole [22]. There, it is proven that when the informed party is the one who makes the contract offer, he can always guarantee his RSW allocation in any equilibrium provided that the contract space is rich enough.[14] Maskin and Tirole [22] examine a general informed principal model as a three-stage noncooperative game (contract proposal-acceptance/rejection- execution). The model in this paper differs in at least two respects from Maskin and Tirole [22]. First, in this paper, there are multiple uninformed parties who compete for the same informed party in exclusive contracts, unlike Maskin and Tirole [22] where there is only one informed and one uninformed party. Moreover, after the buyer's (the informed party) proposal, the sellers (uninformed parties) have the right to also propose a set of contracts. This dramatically changes the set of equilibria as it is proven in Proposition 5.1 and Theorem 5.2. In fact, in Γ^e only IE allocations can be supported in equilibrium, unlike Maskin and Tirole [22], where any allocation that weakly Pareto dominates the RSW allocation can be supported as an equilibrium allocation for some set of beliefs.

In the spirit of Myerson [25], the RSW allocation is the only "*strong solution*" when it is interim incentive efficient. A strong solution is an allocation that is safe and undominated. When a strong solution exists, then it must always be the equilibrium allocation.

Proposition 5.3 provides some lower bound in the payoff of the buyer at any equilibrium. This seems to be an important departure from Spence [33], Hellwig [16] as well as other signaling models. For, under their specification, the buyer is not able to guarantee his RSW allocation, because the threat of withdrawal of a contract in the third stage of the game turns out to be self-fulfiling. In other words, there is a wealth of out-of-equilibrium beliefs sellers can entertain, which is enough to support "non-competitive" positive-profit equilibria.[15] The buyer can always guarantee his RSW allocation because, for any possible beliefs sellers may entertain about his type, after his proposal, letting the buyer choose from the RSW allocation in Stage 4 is proved to be harmless.

Besides their importance, Proposition 5.1 and Theorem 5.2 cannot have a bite unless we prove that equilibria exist for all possible parameter values. In Proposition 5.3, it is proven that the RSW allocation is an equilibrium allocation if and only if it is incentive efficient relative to the prior beliefs. Note that the same results hold even in the elementary model examined by Rothschild and Stiglitz [32]. However, one of the main difficulties in these environments is that incentive efficiency sometimes requires cross-subsidisation. This simply means that to increase the payoff of some type(s), some contracts must become loss-making- i.e. to violate the ex post individual rationality

[14] By rich enough, Maskin and Tirole [22] consider the space which contains a contract for every type of the seller.

[15] Note that in Hellwig's game as well as the equivalent signaling game, type 1 can always guarantee his RSW contract in any equilibrlium. This is because for all possible beliefs, this contract makes nonnegative profits.

constraints of the sellers. In fact, that was the initial problem pointed out by Rothschild and Stiglitz [32], who showed that in this case, there are robust regions in which a pure strategy equilibrium fails to exist. That was because, according to their definition of competition, some new seller (insurance firm in their jargon) could enter and "*skim the cream*" in the market by attracting only the low-risk types, creating losses for other sellers. Therefore the main interest is to examine whether pure strategy equilibria exist even when the RSW is not IE. Note that, in this range of parameters, Maskin and Tirole [22] find a continuum of equilibrium allocations; some of them strictly Pareto dominated. However, as we showed in Theorem 5.2, this is never the case in game Γ^e; Bertrand-type competition eliminates any strictly positive profits and forces equilibrium allocations to be IE. It only remains to show that equilibria always exist when the RSW allocation does not belong to the set of IE allocations.

The key to construct equilibria is to notice that every seller who accepts in Stage 2, implicitly accepts to offer every contract contained in the set of contracts proposed by the buyer. Noticing so allows us to construct equilibria in which the buyer always offers an interim incentive efficient allocation that strictly Pareto dominates the RSW allocation and some seller accepts. Cream-skimming is not possible in Γ^e because there are strategies for the buyer according to which all types contract with any entrant trying to skim the cream, and because of the nature of the set of contracts that have been proposed by the buyer, any entrant makes negative profits. The following theorem is the first main result of this paper.

Theorem 5.5. *Any $\hat{\psi}^{\cdot} \in \mathbf{\Psi}^{IE}$ that strictly Pareto dominates $\psi^{\cdot}_{RSW}$ is an equilibrium allocation of game Γ^e.*

Proof: Consider the following candidate equilibrium strategies: "The buyer, regardless his type, proposes a set contracts $\hat{\mu}^{\cdot}_b$ such that $\hat{\mu}^{\cdot}_b = \hat{\psi}^{\cdot}$, allocation $\hat{\psi}^{\cdot} \in \mathbf{\Psi}^{IE}$ and strictly Pareto dominates $\psi^{\cdot}_{RSW}$. Agent $i = 1$ accepts the proposal and proposes $\hat{\mu}^{\cdot}_1 = \psi^{\cdot}_o$- i.e. all the contracts being the null contracts. No other seller accepts the proposal. The menu formed between the buyer and seller 1 is denoted by $\hat{m}^1 = (\hat{\mu}^{\cdot}_b, \hat{\mu}^{\cdot}_1)$. If the buyer proposes any different set of contracts, the posterior beliefs are updated to $\tilde{\lambda}^{\cdot}_1$. If any other seller $i' \in \mathbb{N}/\{1\}$ decides to enter the market and makes a proposal, then all types contract with seller i'."

First, recall that any seller who enters the market has to include in any menu the set of contracts proposed by the buyer. Let us examine first all the possible subgames resulting after the buyer's "equilibrium proposal". If no seller, other than seller 1, enters the market, then given that allocation $\hat{\psi}^{\cdot}$ is incentive compatible, each type $t = 1, ..., T$ gains maximal payoff by sticking to contract $\hat{\psi}^t$ in Stage 4. Because $\hat{\psi}^{\cdot} \in \mathbf{\Psi}^{IE}$, by Lemma 3.2, $\Pi(\hat{\psi}^{\cdot}) = 0$ so seller 1 is indifferent between participating in the game or not. Given the equilibrium strategies, no other seller has an incentive to enter the market and offer a different set of contracts (form a different menu). For assume not. Let seller 2 form a new menu $\tilde{m}^2 = (\hat{\mu}_b^{\cdot}, \tilde{\mu}_2^{\cdot})$ where $\tilde{\mu}_2^{\cdot}$ contains contracts that are strictly preferred by some types and makes strictly positive profits. Given the equilibrium strategies, all types must contract with seller 2. This is sequentially rational for all types since, by construction of the game, they can always guarantee by any entrant the menu of contracts they have proposed in Stage 1. Because of Lemma 3.2, and given that $\hat{\psi}^{\cdot} \in \mathbf{\Psi}^{IE}$, any allocation that Pareto dominates $\hat{\psi}^{\cdot}$ must make strictly negative expected profits which contradicts the thesis that seller 2 can make strictly positive profits by forming a new menu.

All that is left is to construct appropriate beliefs $\tilde{\lambda}_1^{\cdot}$, and continuation payoffs such that $\tilde{U}^t \leq U^t(\psi_{RSW}^t)$ for all $t = 1, ..., T$. Consider the following candidate off-the-equilibrium path beliefs $\tilde{\lambda}_1^{\cdot} = (1-(T-1)\delta, \delta, ..., \delta)$, for some $\delta > 0$ small enough. For these beliefs, no set of contracts can be accepted by any of the sellers if it contains an element (contract) $\tilde{\psi}$ such that $U^1(\tilde{\psi}) > U^1(\psi_{RSW}^1)$. This is because it must necessarily make negative profits under $\tilde{\lambda}_1^{\cdot}$. From the definition of ψ_{RSW}^1 from Program RSW, under single-crossing, $U^t(\psi_{RSW}^t) < U^t(\psi_{RSW}^1)$ for any $t = 2, ..., T$. This means that after a deviation by the buyer of any type, at any continuation of the game given beliefs $\tilde{\lambda}_1^{\cdot}$, the maximal payoff any type can approximate the payoff he could have from ψ_{RSW}^1, which by definition is worse than the equilibrium payoff under $\hat{\psi}^{\cdot}$ given that the latter by assumption strictly Pareto dominates $\psi_{RSW}^{\cdot}$. Q.E.D.

An equilibrium always exists because of the nature of the menus of contracts allowed in the market. The key fact is that each menu of contracts must always include the set of contracts the buyer proposed in the first stage of the game. This is enough to create a credible threat for potential entrants who try to skim the cream in the market. Any equilibrium allocation of game Γ^e is a *neutral optimum* in the sense of Myerson [25]. No type can ever block such an allocation.

In Figure 3, a candidate equilibrium allocation is illustrated. This allocation provides full insurance to both types, strictly Pareto dominates the RSW allocation and makes zero-expected profits on average.

6 Extensions

In this section, I show how my results can be automatically extended to other similar environments with adverse selection. I formulate two different models that are usually exploited in the literature of contract theory: (i) A managerial compensation model, (ii) A model of credit relationship, and (iii) An informed seller model.

6.1 Managerial Compensation

A manager with a finite number of possible types $t = 1, ..., T$ bargains with potential identical employers. The productivity of the manager depends on his type θ^t with $\theta^t >$

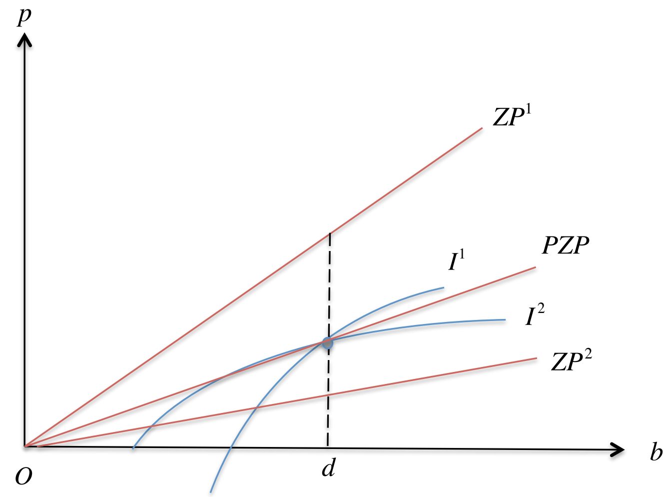

Figure 3: A candidate equilibrium for the special case of $T = 2$ and $\Omega = 1$.

θ^s, for any $t > s$. A manager of type t has utility function: $u^t(m, q) = y - \frac{1}{\theta^t} c(q)$, where y is money and q is observable output. Assume that $c' > 0$, $c'' > 0$, $\lim_{z\to\infty} c'(z) = 0$, $\lim_{z\to 0} c'(z) = \infty$. All employers have the same linear utility function by employing a manager of type t: $\theta^t q - y$. Everything else is similar to the basic insurance model. A contract is denoted by $\psi = (y, q) \in X \subset \mathbb{R}^2_+$. Everything else is defined similarly to the insurance model.

6.2 Credit Markets

There are only two periods and one consumption-investment good which is perishable.[16] Agents are categorised into two groups: entrepreneurs and bankers (or banks).

Each entrepreneur can be one of two possible types $t = 1, 2$. A set of measure λ_0^t of entrepreneurs is of type t. Their initial wealth is zero. They own stochastic investment technologies which take the following form: By investing z units of the consumption good in period zero an entrepreneur of type t can realize $\gamma^t f(z)$ units in period one with probabiliy θ^t and zero otherwise. Uncertainty is purely idiosyncratic, therefore states are individual. Assume that $f(\cdot)$ is twice differentiable, strictly increasing, concave and satisfies Inada conditions namely $\lim_{z\to 0} f'(z) = \infty$ and $\lim_{z\to\infty} f'(z) = 0$. $f(0) = 0$.

Types are distinguished in the following way: $\pi^2 > \pi^1$, $\gamma^2 < \gamma^1$ but $\pi^2\gamma^2 > \pi^1\gamma^1$. The investment technology of type 2 second-order stochastically dominates this of type 1.[17] $f(\cdot)$ is a normal decreasing-returns-to-scale production function. Inada conditions

[16]The economy is similar to that of Martin (2009) [21] with the difference that entrepreneurs do not possess any initial wealth at the time of contracting. This is only for simplicity and without loss of generality.

[17]All these assumptions are very common in the credit rationing literature.

are necessary to guarantee interior solutions.

Assume that only the individual state is observable by the banks and the government and verifiable by a court of law. That way, entrepreneur's type is not observable, before or after production has taken place, by either the banks or the government. Lastly, all entrepreneurs are risk neutral and indifferent between consuming in period.

Bankers are risk-neutral and they have enough wealth to finance all the projects in the market. A loan contract takes the following form: (z, R_s, R_f), where z is the amount of loan, R_s is the per unit of loan payment in case of success and R_f is the per unit of loan payment in case of failure. The rest is similar to the basic insurance model.

6.3 The Informed Seller Model

There is one informed seller endowed with one object. There is a large number of potential buyers who are interested in acquiring the object at some price. Assume for simplicity that there are no financial constraints. The seller has a finite number of types $t = 1, ..., T$. In addition, he can invest in some technology in order to improve the quality of the object if he wishes so. Investment is perfectly observable and verifiable and therefore it can be used as a contractible variable along with the selling price. The preferences of the seller of type t are represented by: $U^t(p, y)$, where p is money (the selling price) and y investment. U^t is strictly increasing in p and strictly decreasing in y. Moreover, assume that the single-crossing property holds. Buyers have linear preferences represented by $V^t(p, y) = \theta^t y - p$. θ^t is the marginal valuation of a buyer if he trades with a seller of type t. Assume, furthermore, that θ^t is increasing in t, so marginally, the buyer always prefers to trade with a seller of higher type. A contract in this model is denoted by $\psi = (p, y)$ and specifies a price for the object and a level of investment.[18] The rest is similar to the basic insurance model.

7 Discussion

In this paper, I studied a market with multiple competing uninformed risk-neutral sellers, one informed risk-averse buyer with risky endowment and common values. The buyer could be any of a finite number of types and this was his private information. I proposed a game form (market mechanism or platform) to model the interaction among the buyer and sellers. In particular, I allowed the buyer to propose a set of contracts in the first stage and the sellers to also propose sets of contracts if they accepted the offer. After the proposals, a menu of contracts was formed between the buyer and each one of the sellers who decided to participate consisting of all the contracts proposed in

[18] Note that in case there is no technology to improve the quality of the object, this model is identical to *Akerlof's lemons market*. Even in this case, the market mechanism is able to produce only efficient allocations.

both sets. The buyer had the discretion to choose any contract out of any mechanism in the market.

The main findings of the paper could be summed up as follows: First, the RSW allocation is an equilibrium allocation if and only if it is interim incentive efficient. Indeed, it is the unique strong solution. Second, the buyer is inscrutable in the sense that he never needs to reveal his type with his proposal. Third, only interim incentive efficient allocations can be possible equilibrium allocations of the game studied. Last, any interim incentive efficient allocation that strictly Pareto dominates the RSW allocation can be an equilibrium allocation and it corresponds to a neutral optimum. Surprisingly, no refinement was needed in out-of-equilibrium beliefs to eliminate unwanted equilibria.

As I claimed, the extensive form game is not restricted only to the basic insurance model but could potentially extend in other competitive models such as managerial compensation models, credit models, franchising models, informed seller models etc.

An interesting path of future research is to study more general markets with adverse selection with more general preferences for buyers and sellers and to see the limitations of the current formulation. Perhaps one good research strategy would be to examine an informed seller model as the one in Subsection 2.6.3 with multiple objects and non-linear utilities for the buyers. Is the market mechanism proposed in this paper sufficient to implement only interim efficient allocations as equilibria and under what conditions?

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