Dynamic Managerial Compensation: On the Optimality of Seniority-based Schemes*

Daniel Garrett — Toulouse School of Economics

Alessandro Pavan — Northwestern University

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Abstract

We characterize the firm's optimal contract for a manager who faces costly effort decisions and whose ability to generate profits for the firm changes stochastically over time. We focus on how the power of managerial incentives (i.e., the sensitivity of pay to performance) and effort change over time. When the manager is risk neutral, we find that the power of incentives and effort typically increase with the manager's tenure in the firm. This conclusion is, however, reversed when the manager is sufficiently risk averse.

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1 Introduction

The contracts that successful firms offer to their top employees are designed taking into account that managerial ability to generate profits for the firm is bound to change over time. While firms can anticipate these changes in advance, their effect on the profitability of the firm often remains managers' private information. How a manager responds to changes in his productivity, and thus the effect of such changes on the firm's profits, ultimately depend on the remuneration package. In this paper we ask the following questions: How should compensation be designed in anticipation of shocks to managerial productivity? Should managers be induced to work harder when their productivity increases or rather when they experience a negative productivity shock? Related to the above questions, how should a manager's "pay for performance" change with time, i.e., with the length of the employment relationship?

We consider an environment where the manager possesses private information about his productivity (i.e., his ability to generate cash flows for the firm) even prior to joining the firm. This private information may originate in tasks performed in previous employment relationships, as well as in personal traits that are not directly observable by the firm. We examine the implications of the manager's private information, both as to initial and subsequent productivity, for optimal incentive schemes.

In the environment described, a firm finds it expensive to ask a manager to exert more effort for three reasons. First, higher effort is costly for the manager and hence must be compensated. Second, asking higher effort from a manager of given productivity requires increasing the compensation promised to all managers of higher productivity. This increase must be provided even if the firm does not ask higher effort from any of these more productive managers. It is necessary to discourage such managers from mimicking the less productive ones. The "rent" that a highly productive manager obtains when pretending to be a less productive "type" comes from the possibility of generating the same profits as the less productive type by working less, thus economizing on the disutility of effort. The size of this rent is increasing in the level of effort asked of the less productive type, because such effort in turn determines the compensation that must be provided by the firm. Third, a higher effort requires more sensitivity of pay to performance, which, in turn, exposes the manager to more volatility in his compensation. When the manager is risk averse, such an increase in volatility reduces the manager's expected payoff, in turn requiring higher compensation by the firm.

The above effects of effort on compensation shape the way the firm induces its managers to respond to shocks to productivity over time. Importantly, note that the first two effects are present even if the manager is risk neutral.[1] In this case, the firm distorts downward (relative to the first-

[1] The reason why in this case the firm cannot extract all the surplus by "selling out" the business to the manager is that the latter possesses private information about his ability to generate cash flows and hence about his value for the firm's business.

best) the level of effort asked to those managers whose initial productivity is low so as to reduce the rents that must be paid to those managers whose initial productivity is high. When the effect of the initial productivity on future productivity is expected to decline with time, the firm then gradually raises the power of incentives in the contracts offered to those managers of initial low productivity so as to induce them to provide more effort as the length of the employment relationship grows.[2] Interestingly, the increase in effort over time is most pronounced for those managers whose initial productivity is lowest, since the distortions in these managers efforts are initially greatest.

Next consider the case of a risk-averse manager. Mitigating the volatility of compensation calls for reducing the power of incentives later in the relationship. The reason is that the manager's initial productivity is a fairly good predictor of his ability to generate cash flows early on, but only a noisy predictor at a more distant future. Whether the rent effect or the risk effect prevail as the length of the employment relationship grows then depends on factors such as the manager's initial productivity at the time of contracting, the persistence of productivity, and the manager's degree of risk aversion.

In this paper we develop a simple, yet flexible, framework that permits us to investigate the implications of the above trade-offs both for optimal effort and for the sensitivity of pay to performance under the optimal contract. When the manager is risk averse, the relationship between effort and pay for performance need not be straightforward. One reason is that the manager's responsiveness to incentives depends also on the *level* of his pay. A risk-averse manager who expects to be paid a lot, for instance due to good past performance, is less willing to exert high effort for any given sensitivity of pay to performance. This is simply because he values additional compensation less. A related difficulty is that a manager may be rewarded for high cash flows both through contemporaneous and future payments. Future payments can depend in turn on the manager's future actions and productivity realizations. This potentially blurs the relationship between pay-for-performance and effort.

To avoid these complications, we consider simple compensation schemes where in each period the payments depend possibly on the entire history of reports about current and past productivity, but only on current cash flows.[3] Importantly, note that, in our environment, restricting the firm to

[2] This property of declining distortions is quite common in the literature on dynamic contracting with adverse selection (see, e.g., Baron and Besanko, 1984, Besanko, 1986, Courty and Li 2000, Battaglini, 2005, among others). As explained in Pavan, Segal, and Toikka (2012), the precise statistical property that is responsible for this result is the property that the "impulse responses of future productivity to initial productivity" decline over time, as opposed to other properties of the productivity process such as the degree of correlation between initial and future productivity, or the predictive power of initial productivity, as measured by the volatility of the forecast error of future productivity given initial productivity. One of the key contributions of the current paper is to study the extent to which such results are robust to the possibility that the agent is risk averse.

[3] In effect, the manager is allowed to propose changes to his reward package in response to variations in his productivity. That managers propose changes to compensation over the relationship is consistent with an observed practice

offering such schemes is always without loss of optimality for the firm: while richer schemes may also be optimal, they never improve upon the simple ones we consider.

In addition to their simplicity, the advantage of the schemes we consider is that they allow a ready definition of the "power of incentives". When the scheme is linear, it is customary to identify the power of incentives with the rate at which payments increase with current cash flows (see, e.g., Gibbons and Murphy (1991)). We extend this definition to incentive schemes which are nonlinear in cash flows by defining the "power of incentives" to be the sensitivity of pay to performance around the performance level that is expected in equilibrium. This is a necessary generalization given that linear schemes need not be optimal when the manager is risk averse.

It turns out that characterizing the dynamics of the power of incentives under optimal contracts is easier than characterizing the dynamics of effort choices, which, except in special cases, is notoriously difficult.[4] We provide a new set of Euler equations that jointly determine how the (ex-ante expectation of) the power of incentives evolves with the length of the employment relationship. We characterize these equations by using variational arguments that preserve incentive compatibility. In other words, we first identify perturbations of the proposed effort and compensation policies that, when applied to an incentive compatible contract, guarantee that the new contract continues to be incentive compatible for the manager. We then identify properties of the proposed policies that guarantee that no such perturbations can improve upon the firm's expected profits. This approach permits us to bypass many of the difficulties encountered in the literature. While this approach does not always permit us to characterize how effort responds to all possible contingencies (which, as mentioned above, is sensitive to details), it does permit us to make fairly robust predictions as to how, on average, effort and the power of incentives evolve over time under fully optimal contracts.

Our approach thus differs from the one typically followed in the literature, which involves replacing "global" incentive constraints with certain "local" constraints and then identifying restrictions on the primitive environment that validate the relaxed approach. In our environment, the advantage of the "relaxed" approached (when properly validated, which is possible only in special cases) is that it permits one to replace predictions about the dynamics of the *average* power of incentives (where the average is taken with respect to productivity histories) with predictions that depend on the realized history. Because, in practice, productivity is hard to measure, confining attention to the average power of incentives seems reasonable in many cases of interest. It also permits us to dispense with restrictions as to the primitives of the environment (in particular, the process for productivities and

that appears to have become more frequent in the last decade (see, among others, Kuhnen and Zwiebel, 2008, and Bebchuck and Fried, 2004).

[4]For instance, Edmans and Gabaix (2011) argue that "the full contracting problem is usually intractable as there is a continuum of possible effort choices." Their approach then consists in focusing on environments where a careful balancing of the costs and benefits of additional effort is unnecessay, for the optimal effort is constant over time and equal to the maximally feasible level.

degree of managerial risk aversion) that one must otherwise impose to validate the relaxed approach. In other words, the predictions we deliver are weaker than those typically considered in the dynamic contracting literature, but more robust.

Implications for empirical work. Our results are relevant to a debate about how managerial incentives change over a manager's tenure. The empirical literature often focuses on a measure of "pay-for-performance" proposed by Jensen and Murphy (1990). This is the responsiveness of CEO pay to changes in shareholder wealth. By focusing on the manager's contemporaneous rewards for higher cash flows, our definition of the power of incentives thus mirrors the measures typically used in the empirical literature.

The evidence of how pay–for-performance varies with managerial tenure is mixed. More recent work finds that the sensitivity of pay to performance increases with tenure. Murphy (1991), Lippert and Porter (1997), and Cremers and Palia (2010) support this view, while Murphy (1986) and Hill and Phan (1991) find evidence of the opposite. A number of theories have been proposed to explain these patterns. Gibbons and Murphy (1991) provide a model of career concerns to suggest that explicit pay-for-performance ought to increase closer to a manager's retirement. Edmans et al. (2012) suggest a similar conclusion but based on the idea that, with fewer remaining periods ahead, replacing current pay with future promised utility becomes more difficult to sustain. Arguments for the opposite finding have often centered on the possibility that managers capture the board later into their tenure (see, e.g., Hill and Phan (1991) and Bebchuk and Fried (2004)), while Murphy (1986) proposes a theory based on market learning about managerial quality over time.

Our paper contributes to this debate by indicating that the key determinant for whether the power of incentives should be expected to increase or decrease over time may lie in the manager's degree of risk aversion. Another prediction of our model is that the increase in the power of incentives over time is most pronounced (equivalently, the decrease is slower) for those managers whose initial productivity is low. Because productivity is correlated with performance, our theory predicts a form of negative correlation between early performance and the variation in the power of incentives over the employment relationship. This prediction about the role of early performance for the dynamics of the power of incentives seems a distinctive feature of our theory, although, to the best of our knowledge, has not been tested yet.

Organization of the paper. The rest of the paper is organized as follows. We briefly review some related literature in the next section. Section 3 then describes the model while Section 4 characterizes the firm's optimal contract. Section 5 concludes. All proofs are in the Appendix at the end of the manuscript.

2 Related literature

The literature on managerial compensation is obviously too large to be summarized within the context of this paper. We refer the reader to Prendergast (1999) for an excellent overview and to Edmans and Gabaix (2009) for a survey of some recent developments. Below, we limit our discussion to the work that is mostly related to our paper.

First, obviously related is the literature on "dynamic moral hazard" and its application to managerial compensation. Seminal works in this literature include Lambert (1983), Rogerson (1985), and Spear and Srivastava (1987). These works provide qualitative insights about the optimal policy, but do not provide a full characterization. This has been possible only in restricted settings: Phelan and Townsend (1991) characterize optimal policies numerically in a discrete-time model, while Sannikov (2008) characterizes the optimal policy in a continuous-time setting with Brownian shocks. In contrast to these works, Holmstrom and Milgrom (1987) show that the optimal contract has a simple structure when (a) the agent does not value the timing of payments, (b) noise follows a Brownian motion and (c) the agent's utility is exponential and defined over consumption net of the disutility of effort. Under these assumptions, the optimal contract takes the form of a simple linear aggregator of aggregate profits.

Contrary to the above works, in the current paper we assume that, in each period, the agent observes the shock to his productivity before choosing effort.[5] In this respect, the paper is closely related to Laffont and Tirole (1986) who first proposed this alternative timing. This alternative approach permits one to use techniques from the mechanism design literature to solve for the optimal contract. As noted above, the same approach has been recently applied to dynamic managerial compensation by Edmans and Gabaix (2011) and Edmans et al. (2012). Our model is similar in spirit, although a key distinction is that we assume that the manager is privately informed about his initial productivity before signing the contract; this is what drives the result that the managers must be given a strictly positive share of the surplus. As discussed above, another key difference with respect to these papers is that we characterize how effort and the power of incentives in the optimal contract evolve over time.

The paper is also related to our previous work on managerial turnover in a changing world (Garrett and Pavan, 2012). In that paper we assume that all managers are risk neutral and focused on the dynamics of retention decisions. In contrast, in the present paper, we abstract from retention (i.e., assume a single manager) and focus instead on the effect of risk aversion on the dynamics of the power of incentives and effort.

[5] While we abstract from the possibility that performance is also affected by transitory noise that occurs after the manager chooses his effort, in many cases of interest, the contracts we characterize continue to implement the desired effort policies even when performance is affected by transitory shocks that are observed by the agent only after committing his effort.

A growing number of papers study optimal financial instruments in dynamic principal-agent relationships. For instance, DeMarzo and Sannikov (2006), DeMarzo and Fishman (2007), Sannikov (2007),[6] and Biais et al. (2010) study optimal financial contracts for a manager who privately observes the dynamics of cash-flows and can divert funds from investors to private consumption. In these papers, it is typically optimal to induce the highest possible effort (which is equivalent to no stealing/no saving); the instrument which is then used to create incentives is the probability of terminating the project. One of the key findings is that the optimal contract can often be implemented using long-term debt, a credit line, and equity. The equity component represents a linear component to the compensation scheme which is used to make the agent indifferent as to whether or not to divert funds to private use. Since the agent's cost of diverting funds is constant over time and output realizations, so is the equity share. In contrast, we provide an explanation for why and how this share may change over time. While these papers suppose that cash-flows are i.i.d., Tchistyi (2006) explores the consequences of correlation and shows that the optimal contract can be implemented using a credit line with an interest rate that increases with the balance.[7]

From a methodological standpoint, we draw in this paper from recent results in the dynamic mechanism design literature. In particular, the approach here builds on the techniques developed in Pavan, Segal, and Toikka (2012). That paper provides a general treatment of incentive compatibility in dynamic settings. It extends previous work by Besanko (1985), Courty and Li (2000), Battaglini (2005), Eso and Szentes (2007), and Kapicka (2008), among others, by allowing for more general payoffs and stochastic processes and by identifying the role of impulse responses as the key driving force for the dynamics of optimal contracts.[8] An important dimension in which the current paper makes progress is the characterization of optimal dynamic mechanisms for risk-averse agents with information that is correlated over time.[9] In this respect, the paper is also related to the literature on optimal dynamic taxation (also known as Mirrleesian taxation, or new public finance). Some recent contributions to this literature include Battaglini and Coate (2008), Zhang (2009), Farhi and Werning (2013) and Golosov, Troshkin, and Tsyvinski (2012). As mentioned already, an important novelty of our work relative to these papers is in the approach we follow to identify properties of

[6] As in our work, and contrary to the other papers cited here, Sannikov (2007) allows the agent to possess private information prior to signing the contract. Assuming the agent's initial type can be either "bad" or "good", he characterizes the optimal separating menu where only good types are funded.

[7] Other recent papers that consider persistent private information are He (2008), Edmans, Gabaix, Sadzik, and Sannikov (2011), Strulovici (2012), and Williams (2011). Contrary to these papers and the current one, DeMarzo and Sannikov (2008), Bergemann and Hege (1998, 2005), Horner and Samuelson (2012), and Garfagnini (2011), consider an environment in which both the investors and the agent learn about the firm's true productivity over time and where the agent's beliefs about the likely success of the project differ from the investors' only in case the agent deviates, in which case the divergence in beliefs may be persistent.

[8] We refer the reader to that paper for a more extensive review of the dynamic mechanism design literature.

[9] For static models with risk aversion, see Salanie (1990), and Laffont and Rochet (1998).

optimal contracts. While these papers consider a relaxed approach and then impose restrictions on the primitives to validate its solution, here we identify perturbations that leave the proposed policies implementable. We then use variational arguments to identify properties of the dynamics of effort and the power of incentives under optimal contracts, without having to solve completely for how these policies respond to all possible contingencies.

3 The Model

3.1 The environment

Players, actions, and information. The firm's shareholders (hereafter referred to as the principal) hire a manager to work on a project for two periods. (In an earlier version (Garrett and Pavan, 2011) we considered the case of a relationship lasting T periods, with T either finite or infinite; for simplicity, here we consider only two periods, which suffices to illustrate the key ideas and the main results.) In each period $t = 1, 2$, the manager receives some private information $\theta_t \in \Theta_t = \left[\underline{\theta}_t, \bar{\theta}_t\right]$ about his ability to generate cash flows for the firm (his type). After observing θ_t, he then commits effort $e_t \in E = \mathbb{R}$.[10] The latter, combined with the manager's productivity θ_t, then leads to cash flows π_t according to the simple technology $\pi_t = \theta_t + e_t$.

Both $\theta \equiv (\theta_1, \theta_2)$ and $e \equiv (e_1, e_2)$ are the manager's private information. Instead, the cash flows $\pi \equiv (\pi_1, \pi_2)$ are verifiable, and hence can be used as a basis for the manager's compensation.

Payoffs. For simplicity, we assume no discounting.[11] The principal's payoff is the sum of the firm's cash flows in the two periods, net of the manager's compensation, i.e.

$$U^P(\pi, c) = \pi_1 + \pi_2 - c_1 - c_2,$$

where c_t is the period-t compensation to the manager and where $c \equiv (c_1, c_2)$. The function U^P is also the principal's Bernoulli utility function used to evaluate possible lotteries over (π, c).

By choosing effort e_t in period t, the manager suffers a disutility $\psi(e_t)$. The manager's Bernoulli utility function is then given by[12]

$$U^A(c, e) = v\left(c_1\right) + v\left(c_2\right) - \psi(e_1) - \psi\left(e_2\right), \tag{1}$$

where $e \equiv (e_1, e_2)$ and where $v : \mathbb{R} \to \mathbb{R}$ is a strictly increasing, weakly concave, surjective, Lipschitz continuous, and differentiable function. The case where v is linear corresponds to the case where the manager is risk neutral, while the case where v is strictly concave corresponds to the case where he is risk averse. Note that the above payoff specification also implies that the manager has preferences for

[10] That effort can take any real value is only for simplicity.

[11] None of the results hinge on this assumption.

[12] As is common in the literature, we equate the manager's period-t consumption c_t with the period-t compensation. In other words, we abstract from the possibility of secret private saving.

consumption smoothing. This assumption is common in the dynamic moral hazard (and taxation) literature (a few notable exceptions are Holmstrom and Milgrom (1987) and more recently Edmans and Gabaix (2011)).[13] We denote the inverse of the felicity function v by w (i.e., $w \equiv v^{-1}$).

Productivity process. The manager's first-period productivity, θ_1, is drawn from an absolutely continuous c.d.f. F_1 with density f_1 strictly positive over Θ_1. His second-period productivity is drawn from an absolutely continuous c.d.f. $F_2(\cdot|\theta_1)$ with density $f_2(\cdot|\theta_1)$ strictly positive over a subset $\Theta_2(\theta_1) = \left[\underline{\theta}_2(\theta_1), \bar{\theta}_2(\theta_1)\right]$ of Θ_2.[14]

While our results extend to more general processes, for simplicity, we will focus here on the case where θ_t follows a simple autoregressive process. That is, we will assume that $\theta_2 = \gamma\theta_1 + \varepsilon$, with ε drawn from a continuously differentiable c.d.f. G with support $[\underline{\varepsilon}, \bar{\varepsilon}]$. We assume that $\gamma \geq 0$, so that higher period-1 productivity leads to higher period-2 productivity in the sense of first-order stochastic dominance. We will refer to $\gamma = 1$ as to the case of "fully persistent productivity" (meaning that, holding effort fixed, the effect of any shock to period-1 productivity on the firm's average cash flows is constant over time). We will be primarily interested in the case where $\gamma \in [0, 1]$.

Effort disutility. As is standard in the literature (see, e.g., Laffont and Tirole (1986)), we assume that there exists an arbitrarily large threshold $\bar{e} > 0$ such that ψ is thrice continuously differentiable over $(0, \bar{e})$ with $\psi'(e), \psi''(e) > 0$ and $\psi'''(e) \geq 0$ for all $e \in (0, \bar{e})$, and that $\psi'(e) > 1$ for all $e > \bar{e}$. We then further restrict ψ to be quadratic over the relevant range $[0, \bar{e}]$ and linear elsewhere. More precisely, we assume that $\psi(e) = 0$ for all $e \leq 0$, $\psi(e) = e^2/2$ for all $e \in [0, \bar{e}]$, and $\psi(e) = \bar{e}e - \bar{e}^2/2$ for all $e > \bar{e}$. That the disutility is quadratic over $[0, \bar{e}]$ is important for certain technical reasons discussed below, and also helps to ease exposition. The reason for assuming that ψ is linear above $\bar{e}$ is that this ensures that ψ is Lipschitz continuous. The last property in turn guarantees that both the efficient and the profit-maximizing effort levels are interior, while also ensuring that the manager's payoff under any incentive-compatible contract is equi-Lipschitz continuous in his productivity and hence can be conveniently expressed through a differentiable envelope formula (more below).[15]

3.2 The principal's problem

The principal's problem is to choose a contract specifying for each period a recommended effort choice along with a compensation that conditions on the observed cash-flows. It is convenient to think of such a contract as a mechanism $\Omega \equiv \langle \xi, x \rangle$ comprising a recommended *effort policy*

[13] As is standard, this specification also presumes time consistency. This means that, in both periods, the agent maximizes the expectation of U^A, where the expectation clearly depends on all available information.

[14] In other words, while Θ_2 is the support of the marginal distribution of θ_2, the support of the conditional distribution is allowed to be a subset of Θ_2.

[15] We suppose throughout that the principal chooses to induce effort below $\bar{e}$. We assume that $\bar{e}$ is taken so large that the constraint does not bind; i.e., the restriction does not affect the principal's payoff.

$\xi \equiv (\xi_1(\theta_1), \xi_2(\theta))$ and a *compensation scheme* $x \equiv (x_1(\theta_1, \pi_1), x_2(\theta, \pi))$. The effort that the firm recommends in period one is naturally restricted to depend on the manager's self-reported productivities $\theta = (\theta_1, \theta_2)$ only through θ_1. This property reflects the assumption that the manager learns his second-period productivity only at the beginning of the second period, as explained in more detail below. The effort that the firm recommends in the second period does not depend on the first-period cash flow. This can be shown to be without loss of optimality for the firm, a consequence of the assumption that cash flows are deterministic functions of effort and productivity. The compensation paid in each period naturally depends both on the reported productivities and the observed cash flows.[16]

Importantly, we assume that the firm offers the manager the contract after he is already informed about his initial productivity $\theta_1 \in \Theta_1$. After receiving the contract, the manager then chooses whether or not to accept it. If he rejects it, he obtains an outside continuation payoff which we assume to be equal to zero for all possible types. If, instead, he accepts it, he then reports his productivity $\hat{\theta}_1 \in \Theta_1$ and is recommended effort $\xi_1(\hat{\theta}_1)$. The manager then privately chooses effort e_1 which combines with the manager's productivity θ_1 to give rise to the period-1 cash flows $\pi_1 = \theta_1 + e_1$. After observing the cash flows π_1, the firm then pays the manager a compensation $x_1(\hat{\theta}_1, \pi_1)$.

The functioning of the contract in period two parallels the one in period one. At the beginning of the period, the manager learns his new productivity θ_2. He then updates the principal by sending a new report $\hat{\theta}_2 \in \Theta_2$. The contract then recommends effort $\xi_2(\hat{\theta})$ which may depend on the entire history $\hat{\theta} \equiv (\hat{\theta}_1, \hat{\theta}_2)$ of reported productivities. The manager then privately chooses effort e_2 which, together with θ_2, leads to the cash flows π_2. After observing π_2, the firm then pays the manager a compensation $x_2(\hat{\theta}, \pi)$ and the relationship is terminated.

As usual, we will restrict attention to contracts that are accepted by all types and that induce the manager to report truthfully and follow the principal's recommendations in each period.[17] We will refer to such contracts as *individually rational* and *incentive compatible*.[18]

[16] One can think of the reports by the manager as playing the role of selecting a clause in the contract affecting the sensitivity of the compensation to the observed cash flows. Also note that, while it is convenient to think of the contract as a direct revelation mechanism, the allocations sustained under the optimal contract will typically be sustainable also without the need for direct communication between the manager and the firm (this is true, in particular, when there is a one-to-one mapping from the equilibrium cashflows to the manager's productivity). However, that the manager explicitly updates the board about his ability to generate cash flows seems not only plausible but consistent with observed practice (see footnote 3).

[17] Note that, because the environment is Markov, restricting attention to contracts that induce the manager to follow a truthful and obedient strategy in period two also after having departed from truthful and obedient behavior in period one is without loss of optimality.

[18] As explained above (see footnote 1), because the manager is privately informed at the time he is offered the contract, selling out the firm to the manager is not optimal. This is true even when the manager is risk neutral, and even in the absence of cash constraints.

4 Profit-maximizing Contracts

4.1 Representation of the firm's profits

We begin by deriving an expression for the firm's profits under any individually rational and incentive compatible contract that will prove instrumental to the characterization of the optimal policies. When the agent is risk neutral (v is linear), the firm's profits can be expressed entirely in terms of the effort policy ξ. When, instead, the agent is risk averse, the firm's profits will depend also on the first-period compensation; how the firm compensates each manager in the second period is then determined entirely by the effort policy and the period-one compensation.

We start by representing the manager's equilibrium payoff under any individually rational and incentive compatible contract Ω. Let $\pi_t(\theta) = \theta_t + \xi_t(\theta)$ and $c_t(\theta) = x_t(\theta, \pi(\theta))$ denote, respectively, the firm's period-t equilibrium cash flows and compensation to a manager whose productivity history is θ (by "equilibrium", we mean under a truthful and obedient strategy for the agent).[19] Then let $V(\theta) = v(c_1(\theta_1)) + v(c_2(\theta)) - \psi(\xi_1(\theta_1)) - \psi(\xi_2(\theta))$ denote the manager's equilibrium payoff in state θ. We then have the following result.

Lemma 1 *Suppose that the contract $\Omega = \langle \xi, x \rangle$ is individually rational and incentive compatible. The compensation that the firm provides in equilibrium to each manager whose lifetime productivity is $\theta = (\theta_1, \theta_2)$ must satisfy*[20]

$$v\left(c_1\left(\theta_1\right)\right) + v\left(c_2\left(\theta\right)\right) = W\left(\theta;\xi\right) + K, \text{ where} \tag{2}$$

$$\begin{aligned} W\left(\theta;\xi\right) \quad &\equiv \quad \psi(\xi_1(\theta_1)) + \psi\left(\xi_2(\theta)\right) + \int_{\underline{\theta}_1}^{\theta_1} \left[\psi'\left(\xi_1\left(s\right)\right) + \gamma \mathbb{E}^{\tilde{\theta}_2|s}\left[\psi'(\xi_2(s,\tilde{\theta}_2))\right]\right] ds \\ &+ \int_{\underline{\theta}_2}^{\theta_2} \psi'(\xi_2(\theta_1,s))ds - \mathbb{E}^{\tilde{\theta}_2|\theta_1}\left[\int_{\underline{\theta}_2}^{\tilde{\theta}_2} \psi'(\xi_2(\theta_1,s))ds\right], \end{aligned} \tag{3}$$

and where K is a nonnegative scalar.

The result in the Lemma is obtained by combining period-2 necessary conditions for incentive compatibility (as derived, for example, in Laffont and Tirole (1986)) with period-1 necessary conditions for incentive compatibility (as derived, for example, in Pavan, Segal and Toikka (2012); see also Garrett and Pavan (2012) for a similar application to a model of managerial turnover).

The result in the lemma highlights how the surplus that the firm must leave to each manager depends on the effort policy ξ. Consider a manager of initial productivity θ_1. His expected payoff

[19] As explained above, ξ_1 and x_1 naturally depend on (θ, π) only through the period-1 observations. We abuse notation here to ease the exposition.

[20] To be precise, Condition (2) must hold with probability one; that is, it can be violated but only over a set of zero-probability measure.

under any individually-rational and incentive-compatible contract must satisfy

$$\mathbb{E}^{\tilde{\theta}|\theta_1}\left[V(\tilde{\theta})\right] = \mathbb{E}^{\tilde{\theta}|\underline{\theta}_1}\left[V(\tilde{\theta})\right] + \int_{\underline{\theta}_1}^{\theta_1}\left[\psi'\left(\xi_1\left(s\right)\right) + \gamma\mathbb{E}^{\tilde{\theta}_2|s}\left[\psi'(\xi_2(s,\tilde{\theta}_2))\right]\right]ds. \tag{4}$$

The surplus that a type θ_1 expects above the one expected by the lowest period-1 type $\underline{\theta}_1$ is thus increasing in the effort that the firm asks from the managers with initial productivities $\theta_1' \in (\underline{\theta}_1, \theta_1)$ in each of the two periods. This surplus is necessary to dissuade type θ_1 from mimicking the behavior of these lower types. Such mimicry would involve, say, reporting a lower type in the first period and then replicating the distribution of that type's productivity reports in the second period. By replicating the same cash flows expected from a lower type, a higher type can obtain the same compensation while working less, thus economizing on the disutility of effort.

The effort that a higher type can save by mimicking a lower type is clearly increasing in the cash flows the firm expects from the lower type. Consider the effect of asking higher effort of a low period-1 type on the surplus that the firm must leave to those managers whose initial productivity is higher. If productivity is only partially persistent (in our autoregressive model, suppose that $\gamma < 1$), then asking for this increase in effort at period 2 has a smaller effect than asking for it at period 1. The reason is that the amount of effort that the higher period-1 type expects to be able to save relative to the lower period-1 type he mimics is smaller in the second period, reflecting that the initial productivity is imperfectly persistent. As we will see below, this property plays an important role in shaping the dynamics of effort and the power of incentives under the optimal contract.

Now note that the scalar K in Lemma 1 corresponds to the expected payoff $\mathbb{E}^{\tilde{\theta}|\underline{\theta}_1}\left[V(\tilde{\theta})\right]$ of the lowest period-1 type. Using (4), it is easy to see that, if the lowest period-1 type finds it optimal to accept the contract (i.e., if $K \geq 0$), then so does any manager whose initial productivity is higher. Furthermore, it is easy to see that, fixing the effort policy ξ, the firm's profits are always maximized by setting $K = 0$, as we do throughout the rest of the paper. The above observations then permit us to express the firm's expected profits as a function of the effort policy ξ and the period-1 equilibrium compensation c_1.

Lemma 2 *The firm's expected profits under any individually rational and incentive compatible contract $\Omega \equiv \langle \xi, x \rangle$ yielding zero surplus to a manager of lowest period-1 productivity is given by*

$$\mathbb{E}\left[U^P\right] = \mathbb{E}\left[\tilde{\theta}_1 + \xi_1(\tilde{\theta}_1) + \tilde{\theta}_2 + \xi_2(\tilde{\theta}) - c_1(\tilde{\theta}_1) - w\left(W(\tilde{\theta};\xi) - v\left(c_1(\tilde{\theta}_1)\right)\right)\right] \tag{5}$$

where $c_1(\theta_1) = x_1(\theta_1, \pi_1(\theta_1))$ is the compensation given in equilibrium to a manager whose period-1 productivity is θ_1 and where $\pi_1(\theta_1) = \theta_1 + \xi_1(\theta_1)$ are the equilibrium cash flows generated by the same manager.

Note that, when the manager is risk neutral ($v(y) = w(y)$ for all y), the result in Lemma 2 implies that the firm's payoff is equal to the entire surplus of the relationship, net of a term that corresponds to the surplus that must be left to the manager and which depends only on the effort policy ξ :

$$\mathbb{E}\left[U^P\right] = \mathbb{E}\left[\begin{array}{c} \tilde{\theta}_1 + \xi_1(\tilde{\theta}_1) - \psi(\xi_1(\tilde{\theta}_1)) + \tilde{\theta}_2 + \xi_2(\tilde{\theta}) - \psi(\xi_2(\tilde{\theta})) \\ -\int_{\underline{\theta}_1}^{\tilde{\theta}_1} \left[\psi'\left(\xi_1\left(s\right)\right) + \gamma \mathbb{E}^{\tilde{\theta}_2|s}\left[\psi'(\xi_2(s,\tilde{\theta}_2))\right]\right] ds \end{array}\right]. \tag{6}$$

The expression in (6) is what in the dynamic mechanism design literature is referred to as "dynamic virtual surplus".

As one should expect, when instead the agent is risk averse, the firm's payoff depends not only on the effort policy but also on the way the compensation is spread over the two periods. The value of the result in Lemma 2 comes from the fact that the choice over such compensation can be reduced to the choice over the period-1 compensation. The result in the lemma uses the property that any two contracts implementing the same effort policy ξ must give the manager the same utility of compensation not just in expectation, but at (almost) every productivity history $\theta = (\theta_1, \theta_2)$. This equivalence result (which is the dynamic analog in our non-quasilinear environment of the celebrated "revenue equivalence" for static quasilinear problems) plays an important role below in the characterization of the optimal policies.

4.2 Optimal policies

We now consider the question of which contracts maximize the firm's expected profits. Note that the firm's profits depend on the contract $\Omega \equiv \langle \xi, x \rangle$ only through (i) the effort policy ξ and (ii) the equilibrium period-1 compensation c_1, as defined in Lemma 2. Given ξ and c_1, the period-2 equilibrium compensation $c_2(\theta) = x_2\left(\theta, \pi(\theta)\right)$ is then uniquely determined by the need to provide the manager a lifetime utility of monetary compensation equal to the level required by incentive compatibility, as given by (2). That is,

$$c_2\left(\theta\right) = w\left(W\left(\theta; \xi\right) - v\left(c_1\left(\theta_1\right)\right)\right).$$

The payment scheme $x \equiv \left(x_1\left(\theta_1, \pi_1\right), x_2\left(\theta, \pi\right)\right)$, however, is not uniquely determined for values of the cash flows inconsistent with truthful and obedient behavior by the manager; that is, for $\pi_t \neq \pi_t\left(\theta\right) \equiv \theta_t + \xi_t\left(\theta\right)$. The choice of how the payment schemes must be extended off-path to induce the manager to behave as prescribed by the firm is considered in detail below. Before doing so, we first characterize properties of the effort and (on-path) compensation that are sustained under any optimal contract.

As anticipated in the Introduction, the approach typically followed in the dynamic mechanism design literature to identify the optimal policies is the following. First, consider a *relaxed program* that replaces all relevant incentive-compatibility constraints with the envelope conditions that permit one to express the principal's payoff as "dynamic virtual surplus". In our environment, this means choosing policies (ξ_1, ξ_2, c_1) so as to maximize the expression for the firm's profits in (5). The envelope conditions are necessary but not sufficient for incentive compatibility. Therefore, one must typically identify auxiliary assumptions on the primitives of the problem that guarantee the solution to the relaxed program can indeed be sustained in a contract that is truly individually rational and incentive compatible. Identifying such primitive conditions is not always simple but often possible when the agent is risk neutral. It is typically prohibitively complex when the agent is risk averse.

The approach we follow here is therefore different. Because the firm's profits under any individually rational and incentive compatible contract must be consistent with the representation in (5), we use this expression to evaluate the performance of different contracts. Not all policies (ξ_1, ξ_2, c_1) are implementable by an appropriate contract, however, and this may be the case for policies that maximize (5). For this reason, we do not aim at maximizing this expression directly. Instead, we use simple variational arguments to identify the properties of policies sustained under optimal contracts. More precisely, we identify perturbations that, when applied to policies that can be implemented under a contract that is individually rational and incentive compatible for the manager, guarantee that the new perturbed policies remain implementable under a contract with the same properties (that is, the new perturbed policies continue to be implementable under a contract that is individually rational and incentive compatible for the manager). For the candidate policies to be sustained under an optimal contract, it then must be the case that no such perturbations increase the firm's profits, as expressed in (5).

The hurdle is in identifying perturbations that preserve the manager's incentives. As we show in the Appendix, a family of such perturbation is obtained by (i) perturbing the effort at each history so that the marginal disutility of effort is scaled by a constant uniformly across histories, and then (ii) adjusting the compensation so that the net payoffs continue to satisfy the conditions in Lemma 1 while maintaining $\mathbb{E}^{\tilde{\theta}|\underline{\theta}_1}\left[V(\tilde{\theta})\right] = 0$. Under our simplifying assumption that the disutility of effort is quadratic over the relevant range, such perturbations amount to scaling directly the effort policy by a uniform constant. The requirement that such perturbations do not increase profits the yields the following result.

Proposition 1 *Suppose that an optimal contract exists. Then the effort and compensation policies defined by the conditions below are sustained under an optimal contract:*

$$\mathbb{E}\left[\psi'\left(\xi_1^*\left(\tilde{\theta}_1\right)\right) w'\left(v\left(c_1^*\left(\tilde{\theta}_1\right)\right)\right)\right] = 1 - \mathbb{E}\left[\frac{\psi''\left(\xi_1^*\left(\tilde{\theta}_1\right)\right)}{f_1\left(\tilde{\theta}_1\right)}\int_{\tilde{\theta}_1}^{\bar{\theta}_1} w'\left(v\left(c_1^*\left(r\right)\right)\right) f_1\left(r\right) dr\right] \tag{7}$$

$$\mathbb{E}\left[\psi'\left(\xi_2^*\left(\tilde{\theta}\right)\right) w'\left(v\left(c_2^*\left(\tilde{\theta}\right)\right)\right)\right] = 1 - \gamma\mathbb{E}\left[\frac{\psi''\left(\xi_2^*\left(\tilde{\theta}\right)\right)}{f_1\left(\tilde{\theta}_1\right)}\int_{\tilde{\theta}_1}^{\bar{\theta}_1} w'\left(v\left(c_1^*\left(r\right)\right)\right) f_1\left(r\right) dr\right] \tag{8}$$

$$-\mathbb{E}\left[\frac{\psi''\left(\xi_2^*\left(\tilde{\theta}\right)\right)}{f_2\left(\tilde{\theta}_2|\tilde{\theta}_1\right)}\int_{\tilde{\theta}_2}^{\bar{\theta}_2}\left\{w'\left(v\left(c_2^*\left(\tilde{\theta}_1, r\right)\right)\right) - w'\left(v\left(c_1^*\left(\tilde{\theta}_1\right)\right)\right)\right\} f_2\left(r|\tilde{\theta}_1\right) dr\right]$$

with

$$w'\left(v\left(c_1^*(\theta_1)\right)\right) = \mathbb{E}^{\tilde{\theta}_2|\theta_1}\left[w'\left(v\left(c_2^*\left(\theta_1, \tilde{\theta}_2\right)\right)\right)\right] \tag{9}$$

for almost all θ_1, and

$$c_2^*(\theta) = w\left(W\left(\theta;\xi\right) - v\left(c_1^*\left(\theta_1\right)\right)\right)$$

almost surely, where $W(\theta;\xi)$ is the total utility of compensation, as given by (3). The effort policy implemented under an optimal contract is essentially unique.[21] *If v is strictly concave, the compensation policy implemented under an optimal contract is also essentially unique.*

Conditions (7) and (8) capture how the firm optimally solves the trade-off between increasing the manager's effort on the one hand and reducing the payments to the manager on the other. In addition to these conditions, when the manager has preferences for consumption smoothing, his compensation must be appropriately distributed over time according to the well-known inverse Euler equation first suggested by Rogerson (1985).

It is worth commenting on where our approach is similar to the one in the existing literature and where it departs. Condition (9) is obtained by considering perturbations in the compensation policy that leave the manager's payoffs unchanged. Such perturbations can be chosen so as to preserve individual rationality and incentive compatibility of the contract. Indeed, we can consider variations in period-1 compensation coupled with adjustments to period-2 compensation chosen so that the total utility the manager derives from life-time compensation continues to satisfy Condition (2). Under any optimal contract, such perturbations must not increase the firm's expected profits. For this to be the case, the proposed compensation scheme must satisfy Condition (9), which is the same inverse Euler condition

$$\frac{1}{v'(c_1^*(\theta_1))} = \mathbb{E}^{\tilde{\theta}_2|\theta_1}\left[\frac{1}{v'(c_2^*(\theta_1, \tilde{\theta}_2))}\right]$$

[21] Bty essentially unique, we mean except over a zero-measure set of productivity histories.

first identified by Rogerson (1985). The only novelty relative to Rogerson is that here the total utility from compensation is required to satisfy Condition (2).

The point where our analysis departs from the rest of the literature is in the derivation of Conditions (7) and (8). As mentioned above, these conditions are obtained by considering perturbations of the effort policy ξ that preserve incentive compatibility. Contrary to the perturbations of the compensation scheme that lead to Condition (9), these perturbations necessarily change the manager's expected payoff, as one can readily see from (4). For these perturbations not to increase the firm's expected profits, it must be the case that the policies implemented under the optimal contract satisfy Conditions (7) and (8) in the proposition. We will come back to the interpretations of these conditions in a moment. Before doing so, we first discuss the implications of the above results for the dynamics of the power of incentives.

4.3 Dynamics of the power of incentives

The compensation schemes we consider below are defined by two properties. First, the payments in each period depend on the history of observed cash flows only through the contemporaneous observations (that is, $x_t(\theta, \pi)$ depends on π only though π_t, all t). Second, each payment $x_t(\theta, \pi)$ is differentiable in the contemporaneous cash flows π_t. The reason for focusing on such schemes is twofold. First, they favor a definition of *power of incentives* formally given in Definition 1 below that parallels the one typically considered in the literature that restricts attention to linear schemes (see, e.g., Lazear (2000)). Second, such schemes offer a valuable form of robustness. Suppose that the manager is concerned that, after committing his effort, Nature perturbs locally the cash flows with some transitory noise. The schemes proposed above can be constructed so that the manager finds it optimal to choose the level of effort prescribed by the firm if he expects such perturbation to be sufficiently small.[22] Importantly, note that such schemes *virtually* implement the optimal policies. What we mean is the following. Let (ξ^*, c^*) be policies satisfying the conditions of Proposition 1. For any $\varepsilon > 0$ there exists a compensation scheme x satisfying the above two properties along with a pair of effort and consumption policies (ξ, c) such that the following are true: (i) the contract $\Omega \equiv \langle \xi, x \rangle$ is individually rational and incentive compatible for the manager; (ii) in each state θ the compensation the agent receives under Ω is c; (iii) with probability one $|\,(\xi(\theta), c(\theta)) - (\xi^*(\theta), c^*(\theta))\,| \leq \varepsilon$. In other words, the firm can always implement policies arbitrarily close to the fully optimal ones using differentiable schemes. Hereafter, when we say that a compensation scheme x is optimal, we then mean that it virtually implements a pair of policies (ξ^*, c^*) satisfying the conditions of Proposition 1, with ε arbitrarily small.[23]

[22] In some special cases, the perturbation can actually be taken arbitrarily large (see the conditions for implemem-ntability in quasiliear schemes in the working-paper version of the manuscript).

[23] In many, but not all, cases, ε can actually be take to be equal to zero.

We then define the power of incentives under a differentiable scheme as follows.

Definition 1 (Power of incentives) *Let x be a compensation scheme implementing the effort policy ξ and such that, in each period $t = 1, 2$, (i) x_t depends on the history of observed cash flows only through the contemporaneous observation π_t, and (ii) x_t is differentiable in π_t. The* power of incentives *in each period $t = 1, 2$ is then defined to be the local sensitivity*

$$\frac{\partial x_t(\theta, \pi_t)}{\partial \pi_t}|_{\pi_t = \pi_t(\theta)}$$

of the period-t compensation to the period-t cash flows, evaluated around the equilibrium level $\pi_t(\theta) = \theta_t + \xi_t(\theta)$.

The power of incentives in each period t is thus measured by the rate by which the period-t compensation changes with the period-t cash flows around the equilibrium level. The reason why the power of incentives are defined only locally is that, in general, restricting attention to linear schemes need not be without loss of optimality. Furthermore, in general, the power of incentives is not uniquely defined for cash flows arbitrarily far away from the equilibrium levels.

Now note that a necessary condition for the payment scheme x to (virtually) implement the policies ξ^* and c^* of Proposition 1 is that, for any θ,

$$\begin{aligned}
\frac{\partial x_1(\theta_1, \pi_1)}{\partial \pi_1}|_{\pi_1 = \pi_1^*(\theta_1)} &= \psi'\left(\xi_1^*\left(\theta_1\right)\right) w'\left(v\left(c_1^*\left(\theta_1\right)\right)\right), \text{ and} \\
\frac{\partial x_2(\theta, \pi_2)}{\partial \pi_2}|_{\pi_2 = \pi_2^*(\theta)} &= \psi'\left(\xi_2^*\left(\theta\right)\right) w'\left(v\left(c_2^*\left(\theta\right)\right)\right)
\end{aligned}$$

with $\pi_t^*(\theta) = \theta_t + \xi_t^*(\theta)$. Interestingly, then note that the left-hand sides of the Euler conditions (7) and (8) in Proposition 1 are the (ex-ante) expectations of the power of incentives. We can therefore use (7) and (8) to examine how the expected power of incentives evolves over time, under any optimal contract. In particular, we seek to understand how the dynamics of the power of incentives depends on the persistence of the manager's productivity (here captured by γ) and the manager's degree of risk aversion.

Risk neutrality. When the manager is risk neutral (v is equal to the identity function), the power of incentives is simply the marginal disutility of effort, evaluated at the prescribed effort level $\xi_t^*\left(\theta\right)$. In this case, the Euler conditions (7) and (8) pin down not only the dynamics of the power of incentives, but also the dynamics of effort. In particular, when the disutility of effort is quadratic over the relevant range, as we assumed here, then

$$\mathbb{E}\left[\xi_1^*\left(\tilde{\theta}_1\right)\right] = 1 - \mathbb{E}\left[\frac{1 - F_1\left(\tilde{\theta}_1\right)}{f_1\left(\tilde{\theta}_1\right)}\right] \text{ and} \tag{10}$$

$$\mathbb{E}\left[\xi_2^*\left(\tilde{\theta}\right)\right] = 1 - \gamma\mathbb{E}\left[\frac{1 - F_1\left(\tilde{\theta}_1\right)}{f_1\left(\tilde{\theta}_1\right)}\right]. \tag{11}$$

Conditions (10) and (11) highlight how the dynamics of expected effort depends on the persistency parameter γ: expected effort is higher in the second period than in the first one when $\gamma < 1$ and is the same in both periods when $\gamma = 1$. Note that these conditions hold true irrespective of the shape of the period-1 distribution. The result that, under risk neutrality, on average, effort increases with tenure is thus independent of whether or not one assumes that the period-1 distribution is log-concave, an assumption that is typically made in the mechanism design literature to validate the relaxed approach.[24]

To interpret the result, note that the left-hand side of Conditions (10) and (11) represents the expected marginal cost of higher effort, in terms of extra disutility for the manager. The right-hand side represents the expected marginal benefit for the firm (stemming from the increase in cash flows), less a term which captures the effect of higher effort on the surplus that the firm must leave to the manager to induce him to reveal his productivity (this surplus is over and above the minimal compensation required to compensate the manager for his disutility of effort, as one can see by inspecting (4)).

To understand why, under risk neutrality, the power of incentives (and hence effort), on average, increases over time, recall that the reason why the firm distorts downward the effort asked to those managers whose initial productivity is low is to reduce the rents it must leave to those managers whose initial productivity is high. When productivity is not fully persistent, the ability of a manager of high initial productivity to replicate the cash flows of a manager of lower initial productivity by working less is expected to decline over time. Asking those managers of low initial productivity to work harder in the distant future is thus less costly for the firm than asking the same managers to work harder in the early periods. This explains why, on average, the power of incentives increases with tenure. Furthermore, the increase is most pronounced precisely when productivity is expected to be little persistent. In the limit where productivity is expected to be independent over time (γ close to zero), the effort the firm asks to each manager in the second period converges to the first-best level ($e^{FB} = 1$) irrespective of the initial productivity.

Risk aversion. To understand how risk aversion affects the above conclusions, consider utility functions given by $v_\rho(c) = \frac{(c+z)^{1-\rho}}{1-\rho}$ for $z, \rho \geq 0$. We view z as a level of consumption guaranteed by other income sources, so that the manager has constant-relative-risk-aversion preferences. We renormalize the value of the manager's outside option to $\frac{z^{1-\rho}}{1-\rho}$. We consider $\rho \leq \bar{\rho}$ for some $\bar{\rho} < +\infty$, and fix z sufficiently large to guarantee that $c + z$ remains positive over these values of ρ.[25] We then have the following result.

[24] The reason why here we could dispense with such an assumption is that our result applies to the dynamics of the average effort as opposed to the dynamics of effort along any productivity history.

[25] This replaces our assumption that $v(\cdot)$ is onto $\mathbb{R}$, which is clearly stronger than required. For the purposes of Proposition 2 below, it suffices to take $z \geq L(\bar{\varepsilon} - \underline{\varepsilon})$, where L is the Lipschitz constant for ψ.

Proposition 2 *Fix the productivity distributions F_1 and G and the disutility of effort function ψ, as in the model set-up. Suppose that the manager's preferences for consumption in each period are represented by the isoelastic function $v_\rho(\cdot)$, as defined above. For any level of persistence $\gamma < 1$ of the manager's productivity, there exists a critical level of risk aversion $\bar{\rho} > 0$ such that, for all $\rho \in [0, \bar{\rho}]$, the expected power of incentives under any optimal compensation scheme is higher in period two than in period one.*

The result in Proposition 2 extends to moderately low degrees of risk aversion the result established above for the case of a risk-neutral manager that, when productivity is less than fully persistent, the power of incentives, on average, increases with the manager's tenure in the firm. The levels of risk aversion for which the result holds (i.e., how large one can take $\bar{\rho}$ in Proposition 2) naturally depends on how persistent productivity is, that is on γ. For a fixed level of risk aversion, if γ is close to 1, i.e., if the initial productivity is highly persistent, then the above result about the dynamics of the power of incentives is *completely reversed*: the power of incentives is higher in period one than in period two, as stated in the next proposition.

Proposition 3 *Fix the productivity distributions F_1 and G and the disutility of effort function ψ, as in the model set-up. For any strictly concave function v, there exists a critical level of persistence $\bar{\gamma} \leq 1$ such that, for all $\gamma \geq \bar{\gamma}$, the expected power of incentives under any optimal compensation scheme is weakly higher in period one than in period two. Provided that $c_2^*(\theta)$ is not constant in θ_2 for (almost) all θ_1, then $\bar{\gamma}$ can be taken* strictly *less than 1 and the expected power of incentives is* strictly *higher in period one than in period two.*[26]

To understand the result, recall that, when the manager is risk neutral and γ is close to 1, then the effect of higher period-2 effort on the surplus that the firm must leave to each manager is almost the same as the effect of higher period-1 effort (it is exactly the same when $\gamma = 1$). As a result, the power of incentives, on average, is almost the same across the two periods. Things are different in the case of a risk-averse manager. Recall that $\gamma = 1$ means that the effect of each manager's initial productivity on the firm's expected cash flows is constant over time. This does not mean though that the manager's productivity is the same in both periods (unless G is degenerate). In turn this means that, holding constant the power of incentives (equivalently, effort) over time, the manager faces more volatility in his period-2 compensation than in his period-1 compensation. Because ultimately the cost of such volatility is borne by the firm, it is optimal for the latter to reduce the power of incentives over time so as to reduce the manager's exposure to compensation risk.

To see this more formally, consider the Euler conditions (7) and (8) in Proposition 1 and focus on the case where productivity is fully persistent (i.e., $\gamma = 1$). When the disutility of effort is quadratic,

[26] We expect that this condition holds in all but "knife-edge" cases. A sufficient condition, for instance, is that the inverse hazard rate $\frac{1-F_1(\theta_1)}{f_1(\theta_1)}$ is decreasing and that the manager's risk aversion is not too large.

the first two terms on the right-hand sides of these equations are identical. The key difference across the two periods comes from the third term in the right-hand side of (8) which is always negative and captures the effect of the volatility in period-2 compensation on the surplus the firm must give to the manager to induce him to participate. Such volatility originates in the need to make period-2 compensation sensitive to period-2 performance to incentivize period-2 effort. As a result, such volatility can be reduced by lowering the power of incentives in period two. Under any optimal contract, the firm thus reduces the power of incentives over time to reduce the manager's exposure to compensation risk. The same conclusions clearly apply to the case where productivity is highly but not fully persistent. Once again, how persistent productivity must be for the result to obtain depends on the details of the productivity distribution (that is on G and F_1).

One further way to understand why the expected power of incentives declines over time when the manager is risk averse and productivity is sufficiently persistent is to consider the manager's period-2 compensation when the period–2 effort is restricted to depend only on the first-period productivity (that is, when both ξ_1 and ξ_2 depend only on θ_1). The manager's period-2 compensation can then be written as

$$w\left(\begin{array}{c}\psi(\xi_1(\theta_1))+\psi\left(\xi_2(\theta_1)\right)+\int_{\underline{\theta}_1}^{\theta_1}\left[\psi'\left(\xi_1\left(s\right)\right)+\gamma\psi'(\xi_2(s))\right]ds\\ +\left(\theta_2-\mathbb{E}^{\tilde{\theta}_2|\theta_1}\left[\tilde{\theta}_2\right]\right)\psi'(\xi_2(\theta_1))-v\left(c_1\left(\theta_1\right)\right)\end{array}\right).$$

It is then easy to see that the volatility of period-2 compensation is decreasing in the period-2 effort $\xi_2(\theta_1)$. When the manager is risk averse w is strictly convex. By reducing ξ_2, the firm then reduces the expected period-2 compensation, for any level of the period-1 productivity.[27] When this effect is strong, the firm may then find it optimal to reduce the power of incentives over time.

4.4 Further discussion of optimal policies

Conditions (7) and (8) yield important insights about the effect of seniority on the power of incentives under optimal contracts. These conditions were obtained by maximizing firm's profits over all possible implementable policies. As noted above, an alternative approach typically followed in the dynamic mechanism design literature consists in maximizing the firm's profits subject only to certain "local incentive constraints". In our environment, this amounts to maximizing (5) over all possible effort and compensation policies, thus ignoring the possibility that policies that maximize (5) cannot be implemented by a contract which is individually rational and incentive compatible for the manager.

[27] If we restrict attention to effort policies that depend only on period-1 productivity, then the result in Proposition 3 applies not only to the dynamics of the power of incentives but also to the dynamics of expected effort: i.e., expected effort declines over time under the assumptions of the proposition. When we do not impose this restriction, however, we have been unable to untangle the effect of risk aversion on expected effort from its effect on the expected power of incentives. This appears difficult because of the need to control for the correlation between second-period compensation and second-period effort, conditional on the period-1 productivity realization.

This second approach is called the "relaxed approach". Whether this relaxed approach yields policies that can indeed be implemented under an incentive-compatible contract is something that is verified ex-post, once the solution to such approach is in hand. One advantage of this approach is that (when validated) it facilitates a more precise characterization of the optimal policies. In our environment, such approach would permit us to obtain conditions analogous to (7) and (8), but which hold ex-post, i.e. for each possible productivity history, as opposed to in expectation.

Proposition 4 *Let $\left(\xi^R, c^R\right)$ be effort and compensation policies that maximize (5) (note that these policies need not be implementable under an incentive-compatible contract). Then $\left(\xi^R, c^R\right)$ must satisfy Condition (9) for almost all θ_1 and Condition (2) for almost all θ. In addition, $\left(\xi^R, c^R\right)$ must satisfy the following two conditions:*[28]

$$\psi'\left(\xi_1^R(\theta_1)\right) w'\left(v\left(c_1^R(\theta_1)\right)\right) = 1 - \frac{\psi''\left(\xi_1^R(\theta_1)\right)}{f_1(\theta_1)} \int_{\theta_1}^{\bar{\theta}_1} w'\left(v\left(c_1^R(r)\right)\right) f_1(r)\, dr, \tag{12}$$

and

$$\begin{aligned}\psi'\left(\xi_2^R(\theta)\right) w'\left(v\left(c_2^R(\theta)\right)\right) &= 1 - \psi''(\xi_2^R(\theta))\frac{\gamma}{f_1(\theta_1)}\int_{\theta_1}^{\bar{\theta}_1} w_1'\left(v\left(c_1^R(r)\right)\right) f_1(r)\, dr \\ &-\frac{\psi''(\xi_2^R(\theta))}{f_2(\theta_2|\theta_1)}\int_{\theta_2}^{\bar{\theta}_2}\left\{w'\left(v\left(c_2^R(\theta_1, r)\right)\right) - w'\left(v\left(c_1^R(\theta_1)\right)\right)\right\} f_2(r|\theta_1)\, dr.\end{aligned} \tag{13}$$

The policy ξ^R is essentially unique. If v is strictly concave, then c^R is also essentially unique.

Note that, when the manager is risk neutral and the disutility of effort is quadratic, the policy ξ^R is given by

$$\xi_1^R(\theta) = 1 - \frac{1 - F(\theta_1)}{f(\theta_1)} \tag{14}$$

$$\xi_2^R(\theta) = 1 - \gamma\frac{1 - F(\theta_1)}{f(\theta_1)}. \tag{15}$$

If the hazard rate $\frac{f(\theta_1)}{1-F(\theta_1)}$ is increasing, as is often assumed in the literature (a property which is satisfied for a wide class of distributions), then one can show that there exists a (linear) compensation scheme that implements the above policies (see the working paper version for details). We then have the following corollary.

Corollary 1 *Suppose that the manager is risk neutral and that the hazard rate of the period-1 distribution is (weakly) increasing. Then the effort policy ξ^R that solves the relaxed program (that is, that maximizes (5)), as given by (14) and (15), is implemented under any optimal contract.*

[28] Again, these conditions must hold with probability one.

An implication of the above result is that managers whose initial productivity is high are offered higher powered incentives than those managers whose initial productivity is low. The reason for this finding relates once again to the effect of effort on managerial rents. When the hazard rate of the period-1 distribution is increasing, the weight the firm assigns to rent extraction relative to efficiency (as captured by the inverse hazard rate $[1 - F_1(\theta_1)]/f_1(\theta_1)$) is smaller for higher types (recall that asking type θ_1 to exert more effort requires increasing the rent of all types $\theta_1' > \theta_1$). As a result, the firm offers more high-powered incentives to those managers whose initial productivity is high. When it comes to the dynamics of the power of incentives, we then have the following comparison across types.

Corollary 2 *Suppose that the manager is risk neutral and that the hazard rate of the period-1 distribution is (weakly) increasing. Then there exists a linear compensation scheme that implements the optimal effort and consumption policies. Furthermore, under any differentiable scheme implementing the optimal policies, the increase in the power of incentives over time is more pronounced for those managers whose initial productivity is low.*

The result reflects the fact that period-1 effort is more distorted for those managers whose initial productivity is low, implying that, over time, the correction is larger for those managers. The result in the previous Corollary thus yields another testable prediction: because productivity is positively correlated with performance, the econometrician should expect to find a negative relation between early performance and the increase in the power of incentives over time. Note that this prediction is not shared by alternative theories of dynamic managerial compensation that explain possible increases in the power of incentives over time by a decline in career concerns (see, e.g., Gibbons and Murphy (1992)).

Next, consider the case of a risk-averse manager. In this case, verifying that the policies $\left(\xi^R, c^R\right)$ that solve the relaxed program can be implemented under a contract that is incentive compatible for the manager (and hence that such policies are sustained under an optimal contract) is more difficult. We do so for numerical examples on a case-by-case basis. To illustrate, consider a manager with CRRA preferences with risk aversion parameter $\rho = 1/2$ and $z = 0$ (meaning that $v_\rho(c) = 2\sqrt{c}$). Further assume that both θ_1 and ε are uniformly distributed over the unit interval. Figure 1 below shows how the power of incentives in period 1 and the expected power of incentives in period 2 vary with the initial productivity θ_1, for different levels of persistence ($\gamma = 1$, $\gamma = 1/2$ and $\gamma = 0$). It also plots the corresponding (expected) effort levels.

When productivity is fully persistent ($\gamma = 1$), the expected power of incentives is higher in period 1 than in period 2, irrespective of the initial productivity. The same is true for expected effort. This result thus parallels the one in Proposition 3 but without averaging across different productivity levels. For smaller values of γ (in the figure, for $\gamma = 1/2$ and $\gamma = 0$), whether the expected power of incentives increases or decreases over time depends on the initial productivity. For high initial productivities, the power of incentives declines over time, whereas the opposite is true for lower productivity levels. Again, a similar result applies to expected effort. These findings reflect the trade-off between reducing the manager's exposure to risk, which calls for reducing both the power of incentives and effort at later periods, and reducing the manager's expected rents, which calls for low-powered incentives early on followed by higher-powered incentives later in the relationship. The effect of the power of incentives on expected rents is similar across the two periods when either (i) productivity is persistent ($\gamma = 1$), or (ii) the initial productivity is high, in which case the effect of effort on rents is negligible. In these cases, the firm optimally reduces the power of incentives over time so as to reduce the risk the manager faces when it comes to his future compensation.

Note that, in general, the power of incentives (and effort) need not be monotone in the manager's productivity. While this can be true also for a risk-neutral manager (in particular, when the inverse hazard rate $\frac{1-F_1(\cdot)}{f_1(\cdot)}$ is not monotone), risk aversion provides an additional reason. For example, even when the power of incentives is increasing in the manager's productivity, his effort may be decreasing. For instance, in Case 2, the expected second-period effort is decreasing, while the expected power of incentives is increasing. The reason for this possibility is that when the manager has a higher productivity, he expects to be paid more, and is therefore more difficult to motivate. That is, for the same power of incentives, the effort the manager is willing to choose is lower because his marginal value for additional consumption is lower. This illustrates a more general principle that, for a risk-averse manager, the power of incentives and effort, while related, must be considered separately.

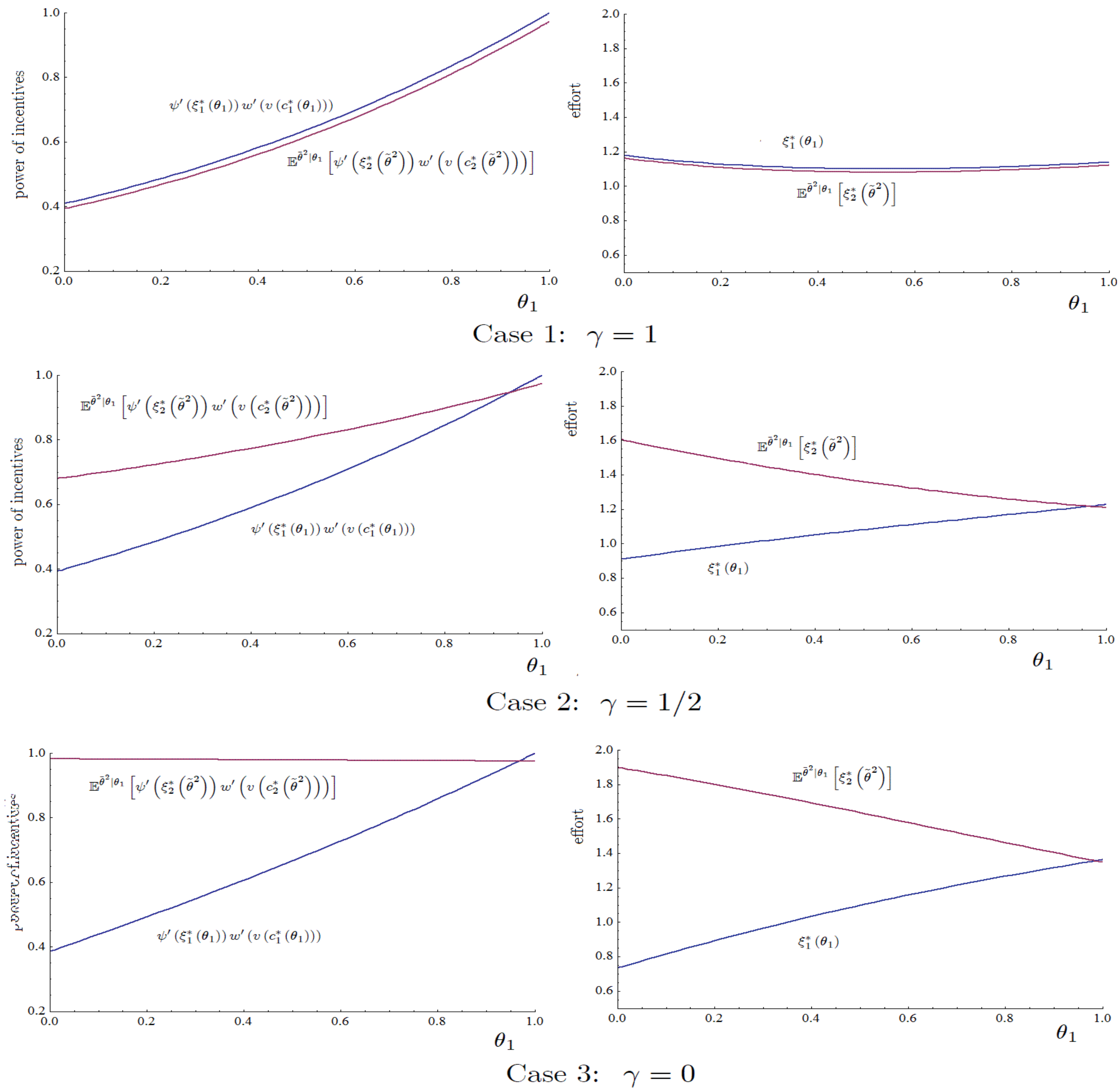

Figure 1. Expected power of incentives and effort conditional on date-1 productivity

5 Conclusions

We investigated the properties of optimal compensation schemes in an environment in which managerial ability to generate profits changes over time and is the managers' private information. We provided a definition of the "power of incentives" which seems appealing in a dynamic setting where payment schemes may not be linear in the cash flows. We derived properties of the power of incentives by applying an approach which, to the best of our knowledge, is new to the dynamic contracting literature. Finally, we shed light on the dynamics of effort under optimal contracts.

When the manager is risk-neutral, we showed that it is typically optimal for the firm to offer a reward package whose power of incentives increases over time, thus inducing the manager to exert, on average, more effort as his tenure in the firm grows. This result hinges on the joint assumptions that (i) the manager has private information about his productivity at the time he is hired, and (ii) that the effect of the manager's initial productivity on his productivity in the subsequent periods declines over time. Both assumptions seem plausible. The first assumption implies that the firm optimally reduces the power of incentives in the contracts offered to those managers whose initial productivity is low so as to discourage more productive types from mimicking. The second assumption implies that asking a low effort to those managers whose initial productivity is low so as to reduce the rents of those managers whose initial productivity is high is more effective when done in the early periods, when the effect of the initial productivity is still pronounced, than in the distant future, when such an effect has become small.

Building on the results for the risk-neutral case, we then showed that risk aversion reduces, and in some cases can even reverse, the profitability of payment schemes whose power of incentives increases over time. The reason is that high-powered incentives in the distant future expose the manager to large compensation risks, for the sensitivity of pay to performance is then high precisely when the manager is less certain about his ability to generate cash flows for the firm. While we illustrated these findings in a very simple two-period model with specific structural assumptions, the insights appear to extend to more general environments. In future work it would be interesting to calibrate the model so as to quantify the relevance of the effects we identified and to derive specific predictions about the dynamic combination of stocks, options, and fixed pay that induce the optimal sensitivity of compensation to performance.

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6 Appendix

Proof of Lemmas 1 and 2. The proof for both results follows from the fact that a necessary condition for a contract $\Omega = \langle \xi, x \rangle$ to be incentive-compatible for the manager is that the latter prefers to follow a truthful and obedient strategy in each period to lying about his productivity and then adjusting his effort choice so as to hide the lie (i.e., so as to generate the same cash flows as the type been mimicked). This step turns the problem into one of pure adverse selection, as first suggested by Laffont and Tirole (1986). Using results from the recent dynamic mechanism design literature, one can then show that, when the payoff structure satisfies the properties in the model setup (in particular, when the disutility of effort is differentiable and Lipschitz continuous), then a necessary condition for incentive compatibility is that, for any (θ_1, θ_2), the manager's ex-post payoff satisfies

$$V(\theta_1, \theta_2) = V(\theta_1, \underline{\theta}_2(\theta_1)) + \int_{\underline{\theta}_2(\theta_1)}^{\theta_2} \psi'(\xi_2(\theta_1, s))ds \tag{16}$$

and that his period-1 expected payoff satisfies (4). The first condition is analogous to the static one in Laffont and Tirole (1986). The necessity of (4) follows from adapting to the environment under examination the result in Theorem 1 in Pavan, Segal, and Toikka (2012)—see also our previous work, Garrett and Pavan (2012) for a derivation of a similar condition in a model of managerial turnover.

Combining (16) with (4) then implies that, under any contract that is individually rational and incentive compatible, the utility that each manager derives from his lifetime compensation must satisfy (2) with $K = \mathbb{E}^{\tilde{\theta}|\underline{\theta}_1}\left[V(\tilde{\theta})\right] \geq 0$, as stated in Lemma 1. That the principal's payoff in turn satisfies the representation in Lemma 2 follows from the observations above along with the fact that, under any optimal contract, $K = 0$. Q.E.D.

Proof of Proposition 1. The proof is in three steps. Step 1 identifies necessary and sufficient conditions for any given effort policy to be implementable. Step 2 identifies a family of perturbations that preserve incentive compatibility and then uses this family to identify necessary conditions for the proposed effort and compensation policies (ξ^*, c^*) to be sustained under an optimal contract. Finally, Step 3 shows that the policies (ξ^*, c^*) that satisfy conditions (7), (8) and (9) in the proposition are essentially unique (i.e., unique up to a zero-measure set of possible productivity histories).

Step 1 (Necessary and sufficient condition for implementability of effort policies). We start with the following result.

Lemma 3 *The following conditions are necessary and sufficient condition for an effort policy ξ to be implementable: (a) for all $\theta_1, \hat{\theta}_1 \in \Theta_1$,*

$$\begin{aligned}&\int_{\hat{\theta}_1}^{\theta_1} \psi'\left(\xi_1(\hat{\theta}_1) - (s - \hat{\theta}_1)\right) + \gamma\mathbb{E}^{\tilde{\theta}_2|s}\left[\psi'\left(\xi_2\left(\hat{\theta}_1, \tilde{\theta}_2\right)\right)\right] ds \\ \leq \; &\int_{\hat{\theta}_1}^{\theta_1} \left[\psi'\left(\xi_1\left(s\right)\right) + \gamma\mathbb{E}^{\tilde{\theta}_2|s}\left[\psi'\left(\xi_2\left(s, \tilde{\theta}_2\right)\right)\right]\right] ds,\end{aligned} \tag{17}$$

and (b) for all θ_1, $\pi_2(\theta_1, \theta_2) \equiv \xi_2(\theta_1, \theta_2) + \theta_2$ is nondecreasing in θ_2.

Proof. Given the policies $\langle \xi, c \rangle$, let x be the compensation scheme defined as follows. In each period t, given the reports θ, the manager is assigned a "target" $\pi_t(\theta) = \theta_t + \xi_t(\theta)$. He is then paid a fixed compensation equal to $c_t(\theta)$ if $\pi_t \geq \pi_t(\theta)$ and else is charged a large fine. It is easy to see that any pair of policies $\langle \xi, c \rangle$ that is implementable under some compensation scheme $\hat{x}$ is also implementable under the compensation scheme x defined above. Hereafter, we thus confine attention to contracts where the compensation scheme satisfies this property.

Next note that the following are necessary and sufficient conditions for the manager to find it optimal to follow a truthful and obedient strategy in period two, irrespective of the period-1 type and the period-1 cash flows (see, in particular, Laffont and Tirole, 1986)): (a) for any θ_1, $\pi_2(\theta_1, \theta_2) \equiv \xi_2(\theta_1, \theta_2) + \theta_2$ is nondecreasing in θ_2, and (b) for any $\theta = (\theta_1, \theta_2)$,

$$c_2(\theta) = w\left(\psi(\xi_2(\theta)) + K_2(\theta_1) + \int_{\underline{\theta}_2}^{\theta_2} \psi'(\xi_2(\theta_1, s))\, ds\right)$$

where $K_2(\cdot)$ is an arbitrary function of θ_1.

Now consider any contract Ω such that (i) the manager's equilibrium payoff satisfies the conditions in Lemma 1 (recall that these conditions are necessary for incentive compatibility; also observe that these conditions imply property (b) above) and (ii) $\pi_2(\theta)$ is nondecreasing in θ_2, for any θ_1. Given this contract, let $U^{\Omega}(\theta_1, \hat{\theta}_1)$ be the payoff that a manager whose period-1 productivity is θ_1 obtains when he reports $\hat{\theta}_1$, then chooses period-1 effort optimally (which means attaining the target $\pi_1(\hat{\theta}_1)$), and then behaves optimally in period 2 (which means following a truthful and obedient strategy). Then

$$\begin{aligned} U^{\Omega}\left(\theta_1, \hat{\theta}_1\right) &= U^{\Omega}\left(\hat{\theta}_1, \hat{\theta}_1\right) + \psi\left(\xi_1\left(\hat{\theta}_1\right)\right) - \psi\left(\xi_1\left(\hat{\theta}_1\right) - \left(\theta_1 - \hat{\theta}_1\right)\right) \\ &\quad + \mathbb{E}^{\tilde{\theta}_2|\theta_1}\left[\int_{\underline{\theta}_2}^{\tilde{\theta}_2} \psi'\left(\xi_2\left(\hat{\theta}_1, s\right)\right) ds\right] - \mathbb{E}^{\tilde{\theta}_2|\hat{\theta}_1}\left[\int_{\underline{\theta}_2}^{\tilde{\theta}_2} \psi'\left(\xi_2\left(\hat{\theta}_1, s\right)\right) ds\right] \\ &= U^{\Omega}\left(\hat{\theta}_1, \hat{\theta}_1\right) + \int_{\hat{\theta}_1}^{\theta_1} \psi'\left(\xi_1\left(\hat{\theta}_1\right) - \left(s - \hat{\theta}_1\right)\right) + \gamma \mathbb{E}^{\tilde{\theta}_2|s}\left[\psi'\left(\xi_2\left(\hat{\theta}_1, \tilde{\theta}_2\right)\right)\right] ds. \end{aligned}$$

Because the contract Ω is constructed so as to satisfy the conditions in Lemma 1

$$U^{\Omega}(\theta_1, \theta_1) = U^{\Omega}\left(\hat{\theta}_1, \hat{\theta}_1\right) + \int_{\hat{\theta}_1}^{\theta_1} \left[\psi'(\xi_1(s)) + \mathbb{E}^{\tilde{\theta}_2|s}\left[\psi'\left(\xi_2\left(s, \tilde{\theta}_2\right)\right)\right]\right] ds$$

It follows that a necessary and sufficient condition for $U^{\Omega}\left(\theta_1, \hat{\theta}_1\right) \leq U^{\Omega}(\theta_1, \theta_1)$ for all $\theta_1, \hat{\theta}_1$ is that ξ satisfies the integral monotonicity condition (17). Combining the above results, we then have that the conditions in the lemma are both necessary and sufficient for the policy ξ to be implementable. ■

Step 2 (Euler Equations). We now establish that the Euler Conditions (7), (8), and (9) are necessary conditions for the policies ξ^* and c^* to be implemented under an optimal contract. To see this, consider the perturbed effort policy $\xi = (\xi_1^*(\cdot) + a, \xi_2^*(\cdot) + b)$ for some constants $a, b \in \mathbb{R}$. Then consider the perturbed compensation policy c given by $c_1(\theta_1) = c_1^*(\theta_1)$ and $c_2(\theta) = w(W(\theta; \xi) - v(c_1^*(\theta_1))$ all θ. It is easy to see that, if the policy ξ^* satisfies the conditions of Lemma 3, so does the perturbed policy ξ. Furthermore, because the compensation policy c is constructed so that the manager's payoff under a truthful and obedient strategy continues to satisfy the conditions in Lemma

1, it is easy to see that, if the policies ξ^* and c^* are implementable, so are the perturbed policies $\langle \xi, c \rangle$.

Now consider the firm's expected profits under the perturbed policies. For the original policies to be optimal, the expected profits must be maximized at $a = b = 0$. Using (5), we then have that the derivative of the firm's profits with respect to a, evaluated at $a = b = 0$, vanishes only if the policies ξ^* and c^* satisfy Condition (7) (to see this, it suffices to take the derivative of $\mathbb{E}\left[U^P\right]$ with respect to a and then integrate by parts). Likewise, the derivative of $\mathbb{E}\left[U^P\right]$ with respect to b, evaluated at $a = b = 0$, vanishes only if the policies satisfy (8).

The argument for the necessity of (9) is similar. Fix the effort policy ξ^* and consider a perturbation of the period-1 consumption policy so that the new policy satisfies $v(c_1(\theta_1)) = v\left(c_1^*(\theta_1)\right) + a\eta(\theta_1)$ for a scalar a and some measurable function $\eta(\cdot)$. In other words, $c_1(\theta_1) = w\left(v\left(c_1^*(\theta_1)\right) + a\eta(\theta_1)\right)$. Then adjust the period-2 compensation so that $c_2(\theta) = w(W(\theta;\xi) - v(c_1(\theta_1))$ all θ. It is easy to see that the pair of policies $\langle \xi^*, c \rangle$ continues to be implementable. The firm's expected profits under the perturbed policies are

$$\mathbb{E}\left[U^P\right] = \mathbb{E}\left[\begin{array}{c} \tilde{\theta}_1 + \xi_1^*(\tilde{\theta}_1) + \tilde{\theta}_2 + \xi_2^*(\tilde{\theta}) \\ -w\left(v\left(c_1^*\left(\tilde{\theta}_1\right)\right) + a\eta\left(\tilde{\theta}_1\right)\right) - w\left(W(\tilde{\theta};\xi) - v\left(c_1(\tilde{\theta}_1)\right) - a\eta(\theta_1)\right) \end{array}\right].$$

Optimality of c^* then requires that the derivative of this expression with respect to a vanishes at $a = 0$ for all measurable functions η. This is the case only if Condition (9) holds.

Step 3 (Uniqueness of the optimal policies). We first show that the optimal effort policy is essentially unique. Suppose, towards a contradiction, that there exist two optimal contracts $\Omega^{\#}$ and $\Omega^{\#\#}$ implementing the policies $\langle \xi^{\#}, c^{\#} \rangle$ and $\langle \xi^{\#\#}, c^{\#\#} \rangle$ respectively. Now suppose that $\xi^{\#}$ and $\xi^{\#\#}$ prescribe different effort levels over a set of productivity histories of strictly positive probability. Let $\alpha \in (0,1)$ and define $\xi^{\alpha} \equiv \left(\alpha\xi^{\#} + (1-\alpha)\xi^{\#\#}\right)$, $c_1^{\alpha}(\theta_1) \equiv w(\alpha v\left(c_1^{\#}(\theta_1)\right) + (1-\alpha) v\left(c_1^{\#\#}(\theta_1)\right))$ and $c_2^{\alpha}(\theta) \equiv w(W(\theta;\xi^{\alpha}) - v(c_1^{\alpha}(\theta_1))$. Note that the pair of policies $\langle \xi^{\alpha}, c^{\alpha} \rangle$ is implementable (to see this, note that (i) the effort policy ξ^{α} satisfies the conditions of Lemma 3 and (ii) the equilibrium payoffs under the policies $\langle \xi^{\alpha}, c^{\alpha} \rangle$ satisfy the conditions in Lemma 1).

Next, note that (5) is strictly concave in the effort policy ξ (recognizing that the policy ξ enters (5) also through $W(\theta;\xi)$ as defined in (3)) and weakly concave in c_1.[29] This means that the firms's expected profits $\mathbb{E}\left[U^P\right]$ under the new policy $\langle \xi^{\alpha}, c^{\alpha} \rangle$ are strictly higher than under either $\langle \xi^{\#}, c^{\#} \rangle$ or $\langle \xi^{\#\#}, c^{\#\#} \rangle$, contradicting the optimality of these policies.

By the same arguments, if v is strictly concave and $c_1^{\#}(\theta_1) \neq c_1^{\#\#}(\theta_1)$ over a set of positive probability, then the new policies $\langle \xi^{\alpha}, c^{\alpha} \rangle$ constructed above yield strictly higher profits than those

[29] By *strict* concavity we mean with respect to the equivalence classes of functions which are equivalent if they are equal almost surely.

sustained under $\Omega^{\#}$ and $\Omega^{\#\#}$, irrespective of whether or not $\xi^{\#} \neq \xi^{\#\#}$. This in turn implies that, when v is strictly concave, the optimal compensation policy is also (essentially) unique. Q.E.D.

Proof of Proposition 2. Let $\xi_\rho^* = \left(\xi_{\rho,1}^*, \xi_{\rho,2}^*\right)$ be the (essentially unique) effort policy for parameter ρ, and let $c_\rho^* = \left(c_{\rho,1}^*, c_{\rho,2}^*\right)$ be the consumption policy which is essentially unique whenever $\rho > 0$. Suppose with a view to contradiction that the result is not true. Then, for any number $n \in \mathbb{N}$, we can find a $\rho_n \in \left(0, \frac{1}{n}\right)$ such that

$$\mathbb{E}\left[\psi'\left(\xi_{\rho_n,1}^*\left(\tilde{\theta}_1\right)\right) w'_{\rho_n}\left(v_{\rho_n}\left(c_{\rho_n,1}^*\left(\tilde{\theta}_1\right)\right)\right)\right] \geq \mathbb{E}\left[\psi'\left(\xi_{\rho_n,2}^*\left(\tilde{\theta}\right)\right) w'_{\rho_n}\left(v_{\rho_n}\left(c_{\rho_n,2}^*\left(\tilde{\theta}\right)\right)\right)\right],$$

where w_{ρ_n} is the inverse of v_{ρ_n}. On the other hand, we have $\mathbb{E}\left[\psi'\left(\xi_{0,1}^*\left(\tilde{\theta}_1\right)\right)\right] < \mathbb{E}\left[\psi'\left(\xi_{0,2}^*\left(\tilde{\theta}\right)\right)\right]$, as is immediate from (10) and (11). Using (9), one can easily check that compensation $c_{\rho,1}^*$ and $c_{\rho,2}^*$ remains uniformly bounded (almost everywhere), uniformly across ρ in a neighborhood of zero. Hence, by our assumption that optimal effort is bounded below $\bar{e}$, the following must be true: there exists $\varepsilon > 0$ and $N \in \mathbb{N}$ such that, for all $n \geq N$, $\Pr\left\{\left\|\xi_{\rho_n}^*\left(\tilde{\theta}\right) - \xi_0^*\left(\tilde{\theta}\right)\right\| \geq \varepsilon\right\} \geq \varepsilon$.

Denote by $h(\xi)$ the principal's expected profits (5) when the manager is risk neutral (i.e., $\rho = 0$) and the effort policy is ξ. The convexity of ψ (i.e., of $e^2/2$) over effort levels in $[0, \bar{e}]$ implies the existence of a function $\kappa : \mathbb{R}_+ \to \mathbb{R}_{++}$ such that the following holds. If ξ' and ξ'' are two policies taking values in $(0, \bar{e})$ and satisfying $\Pr\left\{\left\|\xi'\left(\tilde{\theta}\right) - \xi''\left(\tilde{\theta}\right)\right\| \geq \varepsilon\right\} \geq \varepsilon$ for some $\varepsilon > 0$, then $h\left(\frac{1}{2}\xi' + \frac{1}{2}\xi''\right) - \left(\frac{1}{2}h(\xi') + \frac{1}{2}h(\xi'')\right) > \kappa(\varepsilon)$. Hence, there exists $\varepsilon > 0$ and $N \in \mathbb{N}$ such that, for all $n \geq N$, $h\left(\frac{1}{2}\xi_{\rho_n}^* + \frac{1}{2}\xi_0^*\right) - \left(\frac{1}{2}h\left(\xi_{\rho_n}^*\right) + \frac{1}{2}h(\xi_0^*)\right) > \kappa(\varepsilon)$.

Note that, for any $\bar{y} > 0$ and $\bar{\rho} \in (0,1)$, $v_\rho(y)$ and $w_\rho(y)$ are continuous in $\rho \in [0, \bar{\rho}]$, uniformly over $y \in [0, \bar{y}]$. Hence, the principal's objective is continuous in ρ in a small enough neighborhood of $\rho = 0$, uniformly over bounded effort and consumption policies. Therefore, $h\left(\xi_{\rho_n}^*\right) \to h(\xi_0^*)$ as $n \to +\infty$. This, together with the previous observation, implies $h\left(\frac{1}{2}h\left(\xi_{\rho_n}^*\right) + \frac{1}{2}h(\xi_0^*)\right) > h\left(\xi_{\rho_n}^*\right) + \kappa(\varepsilon) > h(\xi_0^*)$, whenever n is sufficiently large, contradicting the optimality of ξ_0^*. Q.E.D.

Proof of Proposition 3. Note that, for all θ_1, incentive compatibility requires $c_2^*(\theta_1, \cdot)$ is nondecreasing. Therefore, using (9), we conclude that

$$\mathbb{E}\left[\frac{1}{f_2\left(\tilde{\theta}_2|\tilde{\theta}_1\right)}\int_{\underline{\theta}_2}^{\bar{\theta}_2(\tilde{\theta}_1)}\left\{w'\left(v\left(c_2^*\left(\tilde{\theta}_1, r\right)\right)\right) - w'\left(v\left(c_1^*\left(\tilde{\theta}_1\right)\right)\right)\right\} f_2\left(r|\tilde{\theta}_1\right) dr\right], \tag{18}$$

is weakly positive. Provided that $c_2^*(\theta)$ is not constant in θ_2 for almost all θ_1, then it is strictly positive. Thus the third term on the right-hand side of (8) is negative (and possibly strictly negative). Since $\psi'' = 1$, the right-hand sides of (7) and (8) are identical except for the third term of (8). We conclude that the expected power of incentives is weakly smaller at date 2 than at date 1, and strictly smaller in case $c_2^*(\theta)$ is not constant in θ_2 for almost all θ_1. It is easy to see that the same conclusion holds for all values $\gamma > 1$. The result then follows from continuity of the expected power of incentives in γ, following arguments analogous to those in the proof of Proposition 2. Q.E.D.

Proof of Proposition 4. The necessity of (12) and (13) follow from evaluating (5) at the perturbed effort policy $\xi_1^R(\theta_1) + a\nu(\theta_1)$ and $\xi_2^R(\theta) + b\omega(\theta)$ for scalars a and b and measurable functions $\nu(\cdot)$ and $\omega(\cdot)$, differentiating with respect to a and b respectively, and then evaluating at $a = b = 0$. These derivatives must be equal to zero for all measurable functions $\nu(\cdot)$ and $\omega(\cdot)$, which is true only if (12) and (13) hold almost everywhere. Uniqueness of ξ^R and c^R, as well as the necessity of (9), follow from the same arguments as in the proof of Proposition 1.