Becker meets Ricardo

Multisector matching with social and cognitive skills

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Introduction

- Individuals participate in education, labor and marriage
  - There is heterogeneity in outcomes across individuals and individual outcomes are correlated across sectors.
  - Cognitive and non-cognitive skills affect individual outcomes
- Evidence on importance of social skills
  - Direct data that market participants value and screen for social skills

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- Evidence on importance of social skills
  - Direct data that market participants value and screen for social skills
  - Using factor models, psychologists and economists show that social factors affect individual outcomes in many sectors
  - In a one factor (cognitive skill) world, Larry Summers would have been president of the United States and George Bush Jr. would have been a nobody.

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1. A model of social skills is one of social interaction where individuals have heterogenous social skills.
2. The model should differentiate cognitive skills from social skills

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1. The problem of modelling social and cognitive skill interaction becomes: How does cognitive and social skills affect team output?

2. Given efficient team prduction, the problem of matching within and across sectors is a linear programming problem. This is great for simulation and estimation.

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- Individuals have different communication costs (social skills)
- Task assignment, based on comparative advantage (Ricardo), takes into account how much remaining time each individual have for task completion.

Results Overview

- Full tasks specialization in labor and education, but partial specialization in marriage
- Many-to-one matching in teams in the labor market, a commonly observed organizational form
- Matching patterns differ across sectors:
  - Labor market: managers and workers sort by cognitive skills
  - Marriage market: spouses sort by both social and cognitive skills
  - Education market: students with different social and cognitive skills attend the same school

Observable Predictions and Simulation Results

- Observable predictions:
  - Individuals with higher social skills are more likely to become managers/teachers
  - Conditional on cognitive skills, managers with better social skills are better paid
  - Educational gap between workers and managers/teachers
  - Teachers of managers earn higher wages than teachers of workers
- Simulations
  - Wage distribution qualitatively lognormal, with bivariate uniform skill distribution
  - Teachers of future managers/teachers acquire discretely more cognitive skills than teachers of workers
  - Wage inequality increases as communication cost decreases

Predecessors

- Garicano, Garicano and Rossi Hansberg study how communication costs affect organization design, time use, occupation choice, team matching & human capital investments at work where individuals differ by cognitive skills.
- Using a different production technology, we extend them by:
  - Add another dimension of individual heterogeneity: communication costs.
  - Studying multisector (school, work and marriage) matching

Model Setup

- Risk-neutral individuals live for two periods
  - Enter education market as students, and then work and marry as adults
  - One unit of time endowment for each sector
  - Free entry of firms and schools
- Individuals are heterogenous in two dimensions
  - (fixed) gross social skill η, with $\eta \in \left[\underline{\eta}, \overline{\eta}\right]$
  - initial cognitive ability a, with $a \in [\underline{a}, \overline{a}]$
  - education transforms a into adult cognitive skill k, with $k \in \left[\underline{k}, \overline{k}\right]$
- Individuals' net payoff: wage (ω) + marriage payoff (h) − tuition (τ)
  - individual decision: who to match with in each sector

Single Agent Production in each Sector

- Output is produced by completion of two tasks, I and C
  - θ_i^I, θ_i^C: times i spent on task I and C respectively
  - time constraint in each sector: $\theta_i^I + \theta_i^C \leq 1$
- Single agent production:

$$\beta k_i \min \left\{ \theta_i^I, \gamma \theta_i^C \right\}, \beta < 1; \gamma > 1 \tag{Single}$$

  - no need for coordination: gross social skill η_i does not enter production

Team versus Single Agent Production

Single Agent Production

$$\beta k_i \min\left\{\theta_i^I, \gamma\theta_i^C\right\}, \beta < 1; \gamma > 1$$

Team Production

- Consider a two-person team with (η_i, k_i) and (η_j, k_j)
  - θ_i^I, θ_j^C: times i and j spend on task I and task C respectively
- Specialization needs coordination
  - Individual on task C bears (one-sided) coordination cost
- Team output:

$$\sqrt{k_i k_j} \min\left\{\theta_i^I, \gamma\eta_j\theta_j^C\right\}, \eta_j < 1 \qquad \text{(Team)}$$

- Assume team production is always superior to working alone

Specialization in the Labor Market

- Let social skill n: $n \equiv \gamma\eta$. Team output: $\sqrt{k_i k_j} \min\left\{\theta_i^I, n_j\theta_j^C\right\}$
- $\omega(n_I, k_I)$ is total earnings of type I employee with skills (n_I, k_I).
- Let a firm hires i and j to produce δ units of output where δ is small.
- Need to allocate $\delta(k_i k_j)^{-\frac{1}{2}}$ units of effective time to tasks I and C.
- Let θ_i^I and $(\delta(k_i k_j)^{-\frac{1}{2}} - \theta_i^I)$ be times i and j allocate to task I respectively. Then $\theta_i^C = (\delta(k_i k_j)^{-\frac{1}{2}} - \theta_i^I)n_i^{-1}$ and $\theta_j^C = \theta_i^I n_j^{-1}$ are times i and j allocate to task C respectively.

- **Firm pays for time used. Leftover time sold elsewhere.**
- The expenditure of the firm's allocations is:

$$\omega(n_i, k_i)(\theta_i^I + (\frac{\delta}{\sqrt{k_i k_j}} - \theta_i^I)\frac{1}{n_i}) + \omega(n_j, k_j)(\frac{\delta}{\sqrt{k_i k_j}} - \theta_i^I + \theta_i^I \frac{1}{n_j})$$

- The firm chooses θ_i^I to minimize above.
- Proposition. Full task specialization is optimal, i.e., an individual is assigned to task I or C.
- Corollary.(Ricardo): Specialization is by comparative advantage. Let i do I if

$$\frac{\omega(n_i, k_i)}{\omega(n_j, k_j)} \leq \frac{1 - n_j^{-1}}{1 - n_i^{-1}}$$

- Many-to-one matching in teams: one member (manager) on task C and several members (workers) on task I
- Manager social skill: span of control or leadership
- Workers' social skills have no value for team production. So $\omega(n_i, k_i) = w(k_i)$.
- Proposition. There is a cutoff $\widehat{n}(k)$ such that a type-(n, k) individual does task C if and only if $n \geq \widehat{n}(k)$.
    - Managers are individuals with better social skills
    - Occupation choice is based on cognitive and social skills

PAM in the Labor Market

- Consider a type-(n_m, k_m) manager who chooses n_m workers of type (n_i, k_i) at wage $\omega(k_i)$ to maximize

$$\max_{(k_1,\ldots,k_{n_m})} \sum_{i=1}^{n_m} \left[\sqrt{k_m k_i} - \omega(k_i)\right]$$

  - In optimum, workers have the same k_w
  - Can be rewritten as $\phi(k_m) = \max_{k_w} \left[\sqrt{k_m k_w} - \omega(k_w)\right]$
  - Define equilibrium matching $\mu(k_m) \in \arg\max_{k_w} \left[\sqrt{k_m k_w} - \omega(k_w)\right]$
- Proposition.(Becker) Equilibrium exhibits positive assortative matching (PAM) along cognitive skills: $\mu'(k) > 0$

Equilibrium Wages in the Labor Market

- Due to free entry of firms, a type-(n, k) manager earns $n\phi(k)$ with

$$\phi(k) = \max_{k'} \left[\sqrt{kk'} - \omega(k') \right]$$

- First-order condition

$$\frac{d\omega(k')}{dk'}\Big|_{k'=\mu(k)} = \frac{1}{2}\sqrt{\frac{k}{k'}} \Leftrightarrow \frac{d\omega(k)}{dk} = \frac{1}{2}\sqrt{\frac{\mu^{-1}(k)}{k}}$$

- Envelope condition

$$\frac{d\phi(k)}{dk} = \frac{1}{2}\sqrt{\frac{\mu(k)}{k}}$$

Time Allocation in the Marriage Market

- Monogamy between i and j means they spend all their time with each other (**cannot sell time to outside market**)
  - **Assuming that it is not efficient to each produce alone**, choose θ_i^I ($\theta_i^C = 1 - \theta_i^I$) and θ_j^I ($\theta_j^C = 1 - \theta_j^I$) to maximize marital output

$$\sqrt{k_i k_j}\left(\min\left\{\theta_i^I, n_j(1-\theta_j^I)\right\} + \min\left\{\theta_j^I, n_i(1-\theta_i^I)\right\}\right)$$

  - Optimal solution

$$\theta_i^I = \frac{n_i n_j - n_j}{n_i n_j - 1}, \text{ and } \theta_j^I = \frac{n_i n_j - n_i}{n_i n_j - 1}.$$

- Optimal marital output

$$\sqrt{k_i k_j}\frac{2n_i n_j - n_j - n_i}{n_i n_j - 1}.$$

Partial Specialization and PAM in the Marriage Market

- Proposition. Full specialization is not optimal; equilibrium exhibits PAM along both n and k.
- Monogamy limits specialization
- Equilibrium sorts in two dimensions: individuals marry their own type
- Equilibrium total marital output for a (n, k, n, k) marriage:

$$\frac{2n}{n+1}k$$

Education Market

- Task assignment is exogenous
  - teachers do task C
  - students do task I
- Team production function: $\sqrt{a_i k_t} \min \left\{ \theta_i^I, n_t \theta_t^C \right\}$
  - in equilibrium, a type-(n_t, k_t) teacher can manage n_t students
  - input: student's initial cognitive skill a_i
  - output: student's adult cognitive skill k_i
- Tuition and teacher wage
  - tuition $\tau(k_t)$ depends on school quality – teacher's cognitive skill k_t
  - teacher wage: $n_t \tau(k_t)$

Equilibrium Education Choice

- Education choices maximize future net payoff

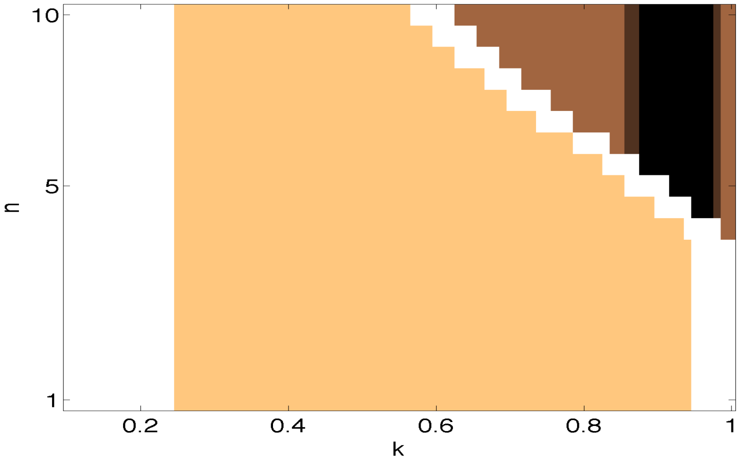
$$\max_{k_t} \left\{ n_s \phi(\sqrt{a_s k_t}), \omega(\sqrt{a_s k_t}), n_s \tau(\sqrt{a_s k_t}) \right\} + \frac{n_s}{n_s + 1} \sqrt{a_s k_t} - \tau(k_t)$$

- Conditional on occupation choice: equilibrium exhibits PAM
  - future managers/teachers choose higher k_t if a_s and n_s are larger: endogenous positive correlation between social and cognitive skills
  - future workers choose more k_t if a_s and n_s are larger
- Proposition. There is an educational gap: a student who has marginally more a_s or n_s and switches from being a worker to being a teacher/manager will discretely increase his or her schooling investment

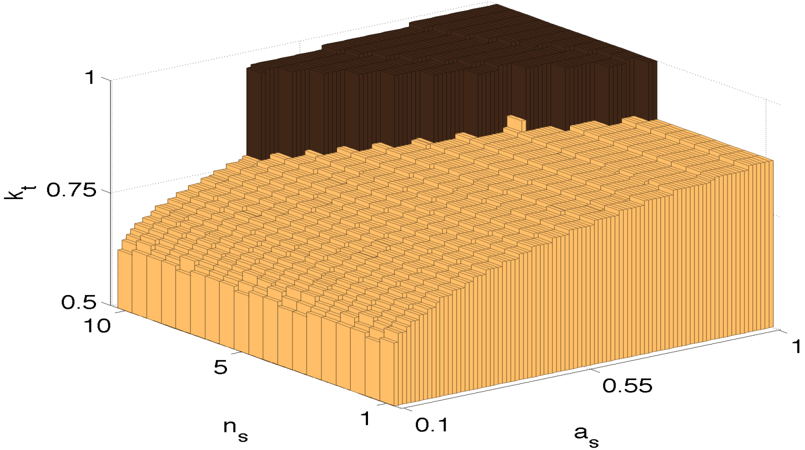
General Equilibrium

- All markets clear
  - labor market clears: equilibrium wages for workers, managers, and teachers equal demand with supply for each type of adult
  - marriage market clears: trivial with a sex ratio of one
  - education market clears: school tuitions are set such that available slots in schools equal the total supply for each type of student
- Equilibrium equivalent to a utilitarian social planner solving a linear programming problem

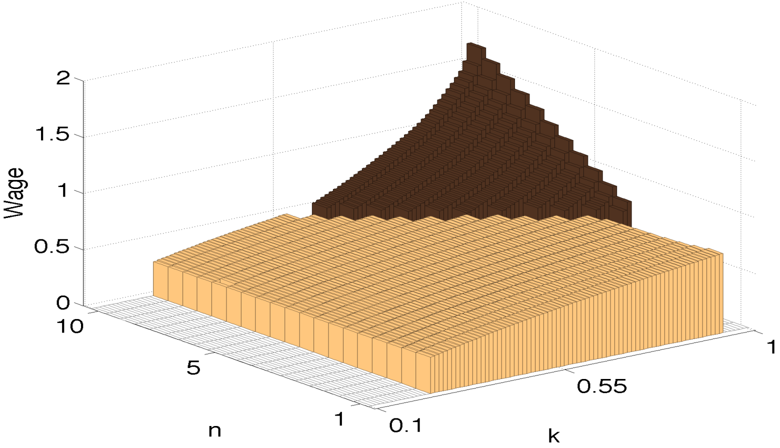
Numerical Simulation: Occupation Choice



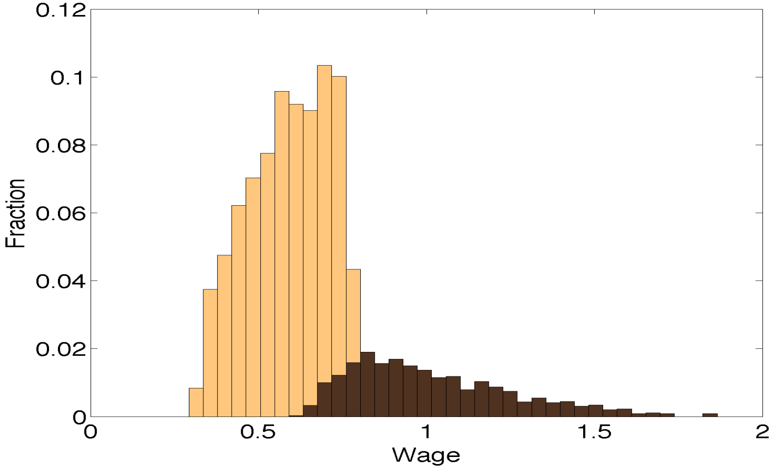
Numerical Simulation: Education Choice



Numerical Simulation: Equilibrium Wage



Numerical Simulation: Wage Distribution



Linear Programming

- Social planner chooses number (measure) of (n_m, k_m, n_w, k_w) firms and number of (n_t, k_t, n_s, a_s) schools to maximize:

$$\sum_{\text{firm types}} \# \text{ firm type } (n_m, k_m, n_w, k_w) \times \left(n_m \sqrt{k_m k_w}\right)$$
$$+ \sum_{\text{marriage types}} \# \text{ marriage type } (n, k, n, k) \times \left(\frac{2n}{n+1} k\right)$$

subject to, for each adult type (n, k),

$$\text{demand by firms} + \text{schools} \leq \text{supply of adults}$$

and for each student type (n, a),

$$\text{school slots for students} \leq \text{supply of students}$$

- wages and student payoffs: multipliers attached to the constraints
- infinite dimensional, dual program

Related Literature (Partial List)

- Importance of non-cognitive (including social) skills
  - Almlund, Duckworth, Heckman and Kautz (2011), Heckman (2011) ...
- Frictionless transferable utility model of marriage
  - one factor: Becker (1973,1974) ...
  - two factors: Anderson (2003), Chiappori, Oreffice and Quintana-Domeque (2010)
- Task assignment and hierarchies
  - Roy (1951), Sattinger (1975) ...
  - Lucas (1978), Rosen (1978, 1982), Eeckhout and Kircher (2011) ...
- Communcation costs, organization design, time use, occupational choice, human capital investment, one factor labor market matching
  - Garicano (2000), Garicano and Rossi Hansberg (2004,2006)
- Linear programming model of frictionless multifactor marriage matching model
  - Chiappori, McCann and Nesheim (2010)

Two sided communication costs

- Let i and j spend θ_i^I on task I and θ_j^C on task C respectively. Team output:

$$\sqrt{k_i k_j} \min \left\{ \eta_i \theta_i^I, \gamma \eta_j \theta_j^C \right\}$$

- N_{wm} is a set of workers managed by manager m where

$$\sum_{N_{wm}} \eta_w = \gamma \eta_m$$

- Then the manager solves:

$$\phi(k_m) = \max_{N_{wm}} \sum_{N_{wm}} \eta_w \sqrt{k_m k_w} - \omega(\eta_w, k_w)$$

- Some results:

$$\omega(\eta_w, k_w) = \eta_w w(k_w)$$
$$\omega(\eta_m, k_m) = \eta_m \phi(k_m)$$

  - No matching between social skills of manager and her workers.
  - Future workers will also make schooling investments based on their social and cognitive abilities.

Conclusion

- We present a tractable framework for multisector matching
    - all three sectors share qualitatively the same team production function
    - team production function incorporates specialization and task assignment
    - specify an explicit role for social skills in production
- Capture matching patterns in each of the three sectors
- Generate predictions consistent with empirical observations
- A first pass theory of social and cognitive skills
    - many possible extensions