Carbon prices for the next thousand years

Reyer Gerlagh and Matti Liski*

This draft: June 1, 2012

Abstract

Climate is a persistent asset, bar none: changes in climate-related stocks have consequences spanning over centuries or possibly millennia to the future. To reconcile the discounting of such far-distant impacts and realism of the shorter-term decisions, we consider hyperbolic time-preferences in a climate-economy model. Time-changing utility discount rates have unexplored general-equilibrium effects: carbon prices exceed the pure carbon externality costs —the Pigouvian tax level— potentially by multiple factors. We derive Markov equilibrium carbon prices dependent on climate system parameters, damage estimates, technology parameters, and both short- and long-term time preferences. The equilibrium time discount rate is endogenous, and it can justify high carbon taxes as advocated by Stern while maintaining the realism of the macroeconomic outcome, thus providing a solution for the dilemma centering the carbon tax-discount rate debate. The welfare ranking of the policy alternatives is unambigous: enforcing the Pigouvian tax decreases a consistently-defined welfare measure vis-a-vis the Markov equilibrium.

(JEL classification: H43; H41; D61; D91; Q54; E21. Keywords: carbon tax, discounting, climate change, inconsistent preferences)

1 Introduction

"Climate" is an extremely persistent asset, with a complicated and long delay structure of impacts involving atmospheric and ocean carbon dioxide diffusion, and land surface and

*Gerlagh <r.gerlagh@uvt.nl> is at the Economics Department of the Tilburg University. Liski <matti.liski@aalto.fi> is at the Aalto University School of Economics, Helsinki. We thank seminar participants at Toulouse TSE, Oxford OxCarre, Helsinki Hecer, and Stockholm IIES for many useful comments and discussions.

ocean temperature adjustments (e.g., Maier-Reimer and Hasselmann 1987, Hooss et al. 2001). The consequences of current changes in climate-related stocks span over centuries or possibly millennia into the future. The persistence of climate change is a central feature in applied climate-economy models, the so called integrated assessment models (IAMs) put forward by Peck and Teisberg (1992), Nordhaus (1993), and Manne and Richels (1995). However, policy evaluations of the climate-economy models ignore the persistent far-distant socioeconomic climate impacts, when holding a view on discounting that respects revealed preferences on shorter-term decisions such as the savings behavior (see, e.g., Nordhaus 2007, Weitzman 2007, Dasgupta 2008).

We take it as given that normative climate policy analysis should respect shorter term choices consistent with historical data (Nordhaus 2007) but also the evidence that the far-distant future should be treated differently. Weitzman (2001) surveyed 2,160 economists for their "best estimate of the appropriate real discount rate to be used for evaluating environmental projects over a long time horizon", and used the data to argue that the policy maker should use a discount rate that declines over time (reaching zero after 300 years). Rather than relying on experts, some studies invoke similarity as evidence for non-constant discounting: from the current perspective, generations living after 400 or, alternatively, after 450 years look the same. That being the case, no additional discounting arises from the added 50 years, while the same time delay commands large discounting in the near term (Rubinstein 2003, Karp 2005).

This paper reconciles the distinct short- and long-term views on time discounting in an integrated climate-economy model — while much has been said to motivate such preferences, their general equilibrium implications seem to have gone unnoticed.[1] Our approach is normative, without dictatorial objects, and it respects the order in which decision makers appear in the time line. In this setting, the short- and long-run discounting leads to a climate policy game between generations, and to an equilibrium utility discount factor, as opposed to one that is assumed *a priori*. This endogeneity of time preference allows partial de-linking of the equilibrium discount factor used for savings from the one used for the carbon price determination. We find that the equilibrium carbon prices

[1] The Stern review (2006) triggered a heated debate on how the policies of the climate-economy models could be made more sensitive to the long-term climate outcomes — essentially, the literature has sought for reasons to use lower long-term discount rates. In general, non-constant discount rates can result, e.g., from: aggregation over heterogenous individuals (Gollier and Zeckhauser, 2005; Lengwiler, 2005, Jackson and Yariv, 2011); uncertainty (Weitzman, 2000, Gollier, 2002). We discuss the motivations directly consitent with our structure in detail below.

exceed the Pigouvian tax level — unambiguously defined as the net present value of the future externality costs from current emissions — potentially by multiple factors. Apart from the time structure of preferences, the framework for quantitative analysis producing these results is a standard general-equilibrium growth framework with a climate module, following the Nordhaus' tradition and its recent gearing towards the modern macro traditions by Golosov, Hassler, Krusell, and Tsyvinsky (2011).

Our point of departure is a Markov equilibrium where each generation sets its self-interested savings and climate targets understanding how the future generations respond to current choices. We use a simple and tractable approach to climate impacts having thus the same virtues as recent approaches (Golosov et al. 2011), although we derive the climate delays from an explicit multi-layer climate system to obtain "carbon prices for the next thousand years". Under reasonable assumptions, we can analytically derive the Markov equilibrium savings and the carbon price dependent on multi-layer climate system parameters, damage estimates, production technology parameters, and both short and long-term time-preferences. The formula for the carbon price explicitly shows the interaction between the delay structure of the carbon-climate cycle and the time-inconsistent preference structure, having substantial implications for carbon pricing.

Persistent climate impacts allow partial commitment to long-run preferences through climate policies, and is therefore valued above the pure Pigouvian externality costs. This mechanism is similar to that delivering value for commitment devices in self-control problems (Laibson, 2007), although self-control at the individual level is not the interpretation of the "behavioral bias" in our economy, as we explain shortly. In the Markov equilibrium, the commitment value is larger the longer are the climate delays, and it can justify high carbon prices as suggested by the Stern Review (2006) while maintaining the realism of the macroeconomic performance. Table 1 contains the gist of the quantitative assessment, detailed at the end of the paper. The technology parameters of the model are calibrated to 25 per cent gross savings, when preferences are consistent and the annual time discount rate is 2 per cent. This is consistent with the Nordhaus' DICE 2007 baseline scenario (Nordaus, 2007), giving 9.7 Euros per ton of CO_2 as the optimal carbon price in year 2010 (i.e., 46 Dollars per ton C). When long-term preference parameters are chosen such that the long-term receives a higher weight (roughly consistent with Weitzman's (2001) survey results), short-term preferences can be matched so that the model remains observationally equivalent to Nordhaus in terms of macroeconomic performance, savings in particular. But carbon prices increase: for very low long-term discounting, ultimately carbon prices approach those suggested by Stern.[2] Note that the equilibrium outcome — high carbon prices and realistic savings — cannot happen in the first-best for such preferences, but only in the Markov equilibrium describing how the climate decisions are *de facto* made given the order of moves in the time line.

	discount rate			
	short-term	long-term	savings	carbon price
"Nordhaus"	.02	.02	.25	8.4
Markov	.027	.001	.25	116.9
"Stern"	.001	.001	.30	152.4

Table 1: Equilibrium carbon prices in EUR/tCO_2 year 2010.

What is then the first-best? The Markov equilibrium is not on the Pareto frontier because the relative weight given to subsequent periods changes over time: long-term time preferences are substituted for by short-term time preferences when control over decisions moves ahead to future generations. It is therefore natural to look for policy rules that can at least partially overcome time-inconsistencies and improve welfare. Can we improve welfare by imposing the Pigouvian externality pricing of carbon as an institutional constraint? We derive the formula for the Pigouvian tax, depending on the same parameters as the equilibrium carbon price, and then we impose this formula as a constraint on the equilibrium behavior — it can be thought of as a rule of behavior for an environmental agency scrutinizing the climate policies within each period. The result is striking: pricing carbon according to the common principle that each unit of emissions should pay the present-value marginal damages caused by that unit lowers welfare for all generations! That is, even if the present generation and all future generations could commit to follow this rule, the commitment would generate no social value, without additional policy measures.

We then look for policies that can actually improve welfare in a self-enforcing manner. Another striking result follows: such policies lead to a further departure above and beyond the Markov and thus Pigouvian carbon price levels. This result, arising from a larger set of available strategies than Markov, corroborates the finding that the welfare improving policies tend to disconnect variables determining macroeconomic outcomes from those determining carbon pricing. The changing time-preference opens the door

[2] Under "Stern" the capital-share of output is fully saved (30 per cent).

for our fundamental results, but their quantitative significance results from the unusual delays in climate change; in our explicit delay structure, the peak impact lags 70 years behind the date of emissions. The analytics allows us to decompose the contribution of the different layers of the climate system to the carbon price: ignoring the full delay structure — as in some recent carbon pricing formulas — misses the correct price levels by a factor of 2, even when preferences are consistent.

Why are non-constant pure time-preferences natural in the context of climate decisions? In this paper, we take the structure of preferences as given and focus on their general-equilibrium climate policy implications; however, multiple recent arguments can justify them. First, decision-makers may not be able to distinguish the welfare in future periods 100 and 101 as clearly as they can separate period 1 and 2. A decision procedure based on the long-run similarity implies a lower long-term discount factor than that for the short-term decisions (see Rubinstein 2003 for the procedural argument; and Karp 2005 for a climate application). Second, climate investments are public decisions requiring aggregation over heterogenous individual time-preferences, leading again to a non-stationary aggregate time-preference pattern, typically declining with the length of the horizon, for the group of agents considered (Zeckhouser and Gollier 2005; Jackson and Yariv 2011). We can also interpret Weitzman's (2001) survey on experts' opinions on discount rates as an aggregation of persistent views: the resulting social discount rate declines from 4 per cent for the immediate future (1-5 years) to 1 for the distant future (76-300 years), and then close to zero for the far-distant future. Third, the long-term valuations must by definition look beyond the welfare of the immediate next generation; any pure altruism expressed towards the long-term beneficiaries implies changing utility-weighting over time (Phelps and Pollak 1968 & Saez-Marti and Weibull 2005).[3]

Welfare analysis with incongruent preferences is potentially a complicated undertaking. Our assumption is that agents in different periods are distinct generations, and therefore the multi-generation Pareto optimality is a natural welfare concept.[4] The welfare analysis follows closely our working paper Gerlagh and Liski (2011), where we introduce an approach for recovering some welfare objective associated with an equilibrium — such an objective can be found if and only if the allocation is on the Pareto frontier.

[3]The first and third arguments for using lower discount rates in the long run are directly consistent with our formal model. For the second, we do not formally consider the aggregation of preferences, but Jackson and Yariv (2011) show that utilitarian aggregation leads necessarily to a present bias.

[4]See Bernheim and Rangel (2009) for an alternative concept, and its relationship to the Pareto criterion. The Pareto criterion may not be reasonable when the focus is on the behavioral anomalies at the individual level.

We follow a positive welfare approach: any policy that is consistent with some welfare objective is judged good. Thus, no dictatorial objective is imposed. The approach to welfare analysis is elementary but proves powerful in this setting. First, we can easily show that the benchmark Markov equilibrium is not consistent with any welfare objective. Second, the Pigouvian externality pricing restores productive efficiency in a very traditional sense, but the policy, as a stand-alone measure, can never generate a welfare objective as defined above. Finally, we can explicitly solve a welfare optimum, and show in our calibration section that a sophisticated policy comes close to this outcome.

The paper is organized as follows. The next section introduces a simplified version of the model in three periods to pinpoint the biases in the carbon prices, policy proposals, as well as the main idea of the welfare analysis. The section has several subsections as we want to clarify all stages of the main plot in three periods in order to avoid confusion in the main model. Section 3 then introduces the full infinite-horizon climate-economy model, and provides the first look at the numbers on carbon pricing. Section 4 calibrates the full model with climate system parameters from the scientific evidence to generate the climate-economy outcomes over the next thousand years — due to the unique delays in the system non-trivial effects remain over such horizons. Section 5 concludes.

2 A three-period model

2.1 Technologies and preferences

Consider three generations, living in periods $t = 1, 2, 3$. In each period, consumers are represented by an aggregate agent having a utility function and production technology. Our three-period model is a very reduced-form model that captures some essential elements of the climate change problem, and serves to illustrate the preference structure, some key assumptions, and the concepts for equilibrium and welfare analysis. In period one, production depends on the use of fossil fuels, with associated emissions. In period two, a substitute energy source has been developed, and consumption possibilities depend on the capital stock inherited. In period three, emissions from the first period have translated into a climate problem negatively affecting production. An allocation $(\mathbf{c}, \mathbf{k}, z) = (c_1, c_2, c_3, k_2, k_3, z) \in A \subseteq R^6_+$ (convex set) constitutes a consumption level for each generation c_t, the first-period use of fossil fuels z, which we also consider a proxy for the emissions of carbon dioxide emissions, and capital stocks k_2 and k_3 left for future agents (k_1 is given). Generations are assumed to have the following simple welfare representation

$$
\begin{aligned}
w_1 &= u_1(c_1) + \rho[u_2(c_2) + \theta u_3(c_3)] \quad (1)\\
w_2 &= u_2(c_2) + \sigma[u_3(c_3)] \quad (2)\\
w_3 &= u_3(c_3), \quad (3)
\end{aligned}
$$

where all utility functions u_t are assumed to be continuous and, in addition, strictly concave, differentiable, and satisfying $\lim_{c\to 0} u_t' = \infty$. Parameters $\rho, \theta, \sigma \in [0,1]$ are discount factors. To capture the idea of lower long-term discounting, we assume $\theta > \sigma$, so that the first agent gives a higher direct weight to the last period utility than the middle agent.[5] Here, we leave it open whether the short-term discounting differs across agents $\sigma = \rho$, or not. In three periods, parameters ρ, θ, σ can all take different values, but in the infinite-horizon setting we assume that each generation is born with the same short- and long-run preferences as in Phelps-Pollak-Laibson.[6]

In the first period, the consumption possibilities are determined by a strictly concave neoclassical production function $f_1(k_1, z)$, where k_1 is the capital stock, and z is the use of fossil fuels, or emissions of carbon dioxide, $\frac{\partial f_1}{\partial k} = f_{1,k}, \frac{\partial f_1}{\partial z} = f_{1,z} > 0$. The first generation starts with a capital stock k_1, and produces output using z, which can be used to consume c_1, or to invest in capital for the immediate next period k_2:

$$c_1 + k_2 = f_1(k_1, z). \quad (4)$$

For convenience we abstract from fossil fuel use in the second and third period, but first-period fossil fuel use enters, with a delay, as a negative externality in the third period. The second agent starts with the capital stock k_2, produces output using a strictly concave neoclassical production function $f_2(k_2)$, and can use its income to consume c_2, or to invest in capital for the third period k_3:

$$c_2 + k_3 = f_2(k_2). \quad (5)$$

[5]As in Phelps and Pollak (1968) or in Saez-Marti and Weibull (2005) this can be interpreted as pure altruism towards the last generation. Alternatively, as in Karp (2005), following the idea of Rubinstein (2003), $\theta > \sigma$ can reflect difficulties in making far-distant comparisons. Aggregation necessary for public policy making (e.g., Jackson and Yaari, 2011) is also consistent with $\theta > \sigma$, but modeling aggregation procedure is beyond our scope in this paper.

[6]We can obtain the common β, δ model as in Phelps-Pollak-Laibson if $\rho = \sigma$, by defining $\beta = \rho/\theta$ and $\delta = \theta$. Then, $w_1 = u_1 + \beta\delta u_2 + \beta\delta^2 u_3$ and $w_2 = u_2 + \beta\delta u_3$. Inconsistencies are identified by $\beta < 1$, corresponding to $\theta > \sigma$ in our case. For our purposes, it is slightly more straightforward to name the long-run weights as θ and σ. The weight ρ allows some freedom in terms of interpretations (e.g. the length of a period may be different for $t = 1, 2, 3$) but is inconsequential for the consistency of the preferences.

The third consumer derives utility from its consumption, which equals production. Past emissions now enter negatively, as damages, in the production function, $f_{3,k} > 0$, $f_{3,z} < 0$:

$$c_3 = f_3(k_3, z). \tag{6}$$

We assume that also this production function is strictly concave.

2.2 Equilibrium carbon price

Consider now the subgame-perfect equilibrium (SPE) of the game where generations choose consumptions and emissions in the order of their appearance in the time line, given the preference structure (1)-(3) and choice sets (4)-(6).

The third agent consumes all capital received and cannot influence past emissions. The second agent decides on the capital k_3 transferred to the third agent, given the capital inherited k_2 and the emissions z chosen by the first agent. We thus have a policy function $k_3 = g(k_2, z)$. The second generation solves

$$\max_{k_3} u_2(c_2) + \sigma u_3(f_3(k_3)), \tag{7}$$

leading to equilibrium condition

$$u_2' = \sigma u_3' f_{3,k} \Rightarrow 1 = \frac{R_{2,3}}{MRS^{\sigma}_{2,3}}, \tag{8}$$

where we use introduce the notation $R_{i,j}$ for the rate of return on capital from period i to j, and $MRS^{\sigma}_{2,3}$ for the absolute value of the marginal rate of substitution between consumptions in 2 and 3 for discount factor σ.

The strict concavity of utility implies consumption smoothing, and thus if the second agent inherits marginally more capital k_2, the resulting increase in output is not saved fully but rather split between the second and third generation:

Lemma 1 *Policy function g satisfies* $0 < g_k < R_{1,2}$.

Proof. Substitute the policy function $k_3 = g(k_2, z)$ in (8),

$$\sigma u_3'(f_3(g(k_2, z), z)) f_{3,k}(g(k_2, z), z) = u_2'(f_2(k_2) - g(k_2, z)). \tag{9}$$

Full derivatives with respect to k_2 lead to

$$\sigma g_k (u_3'' f_{3,k} f_{3,k} + u_3' f_{3,kk}) = u_2''(f_2' - g_k)$$

$$\Longrightarrow g_k = \frac{f_2' u_2''}{\sigma u_3'' f_{3,k} f_{3,k} + \sigma u_3' f_{3,kk} + u_2''} < f_2' = R_{1,2}. \tag{10}$$

as $u_t'', f_{3,kk} < 0$ and $f_{3,k}, u_3' > 0$. ■

Lemma 1 holds generally but for a tractable analysis of the effect of first-period emissions on the second-period policy g_z we need restrictions; these restrictions are common in the integrated assessment models, including ours and, e.g., Golosov el al. (2011), so we explicate them here. Taking the full derivatives of (9) with respect to z, we get

$$g_z = -\frac{\sigma(u_3'' f_{3k} f_{3,z} + u_3' f_{3,kz})}{u_2'' + \sigma u_3'' f_{3,k} f_{3,k} + \sigma u_3' f_{3,kk}}$$

The first term in the numerator captures the income effect of emissions and is positive. If the first generation emits more, the third generation has lower utility levels and the second generation will tend to save more, as the marginal utility of the third generation increases. The second term in the numerator captures the productivity effect and is negative. If the first generation emits more, productivity of capital in the third period will fall, and the return to investments in the second period will fall alongside. Assuming log utility, and that the production damage is multiplicative

$$u_t(c_t) = \ln(c_t) \tag{11}$$
$$f_3(k_3, z) = f_3(k_3)\omega(z), \tag{12}$$

where $\omega(z)$ is a strictly decreasing damage function, implies that the direct effect of emissions on savings vanishes, $g_z = 0$, as can be easily verified from (8).

The first agent decides on consumption and fossil fuel use, given the policy function g, to maximize its welfare

$$w_1 = u_1 + \rho[u_2(f_2(k_2) - g(k_2, z)) + \theta u_3(f_3(g(k_2, z), z)].$$

The choice for savings k_2 satisfies

$$\begin{aligned} u_1' &= \rho(f_{2,k} - g_k)u_2' + \rho\theta f_{3,k} g_k u_3' \\ \Rightarrow \quad 1 &= \frac{R_{1,2}}{MRS_{1,2}^{\rho}} + \left[\frac{MRS_{2,3}^{\sigma}}{MRS_{1,3}^{\rho,\theta}} - \frac{1}{MRS_{1,2}^{\rho}}\right] g_k. \end{aligned} \tag{13}$$

Here, we indicate by the discount factor whose MRS is in question, in line with preferences (1)-(3). When $\theta = \sigma$, preferences are consistent, and the term in brackets vanishes by standard envelop arguments; capital k is valued according to the standard consumption-based asset pricing equation $1 = R_{1,2}/MRS_{1,2}^{\rho}$. For $\theta > \sigma$, the second agent has a steeper indifference curve between consumptions in periods 2 and 3: the first-order effect in the bracketed term remains positive, leading to excessive capital returns, $1 < R_{1,2}/MRS^{\rho}_{1,2}$, when compared to first-best of generation 1 (obtained under $\theta = \sigma$). Consider then how the first generation chooses the carbon policy z:

$$u_1' f_{1,z} = \rho g_z u_2' - \rho\theta(f_{3,k} g_z + f_{3,z}) u_3'. \tag{14}$$

Throughout this paper, we impose (11)-(12) and thus $g_z = 0$, so that the carbon policy satisfies

$$MRS^{\rho,\theta}_{1,3} = \frac{MCD}{MCP} \tag{15}$$

where we let $MCP = f_{1,z}$ denote the marginal carbon product, and $MCD = -f_{3,z}$ denote the marginal carbon damages. If $\theta = \sigma$, then consumption choices imply $1 = R_{1,2}/MRS^{\theta}_{1,2} = R_{1,3}/MRS^{\rho,\theta}_{1,3}$, so that from (15) the carbon price becomes just equal to the damage, discounted with capital return:

$$MCP = \frac{MCD}{R_{1,3}}. \tag{16}$$

This is the general-equilibrium Pigouvian carbon price, under consistent preferences $\theta = \sigma$ (or when the first agent can commit to his first-best). However, if $\theta \neq \sigma$, in equilibrium, while (15) still holds, the market returns no longer reveal the decision-maker's preferred consumption trade-offs, $1 \neq R_{1,3}/MRS^{\rho,\theta}_{1,3}$. Rewriting (14), after substitution of (8) gives

$$MCP = f_{1,z} = \frac{-\frac{f_{3,z}}{f_{3,k}} - (1 - \frac{\sigma}{\theta}) g_z}{f_{2,k} - (1 - \frac{\sigma}{\theta})(f_{2,k} - g_k)}. \tag{17}$$

Under $g_z = 0$, the denominator falls short of $f_{2,k}$ if $\frac{\sigma}{\theta} < 1$.This together with Lemma 1 implies

$$MCP > \frac{MCD}{R_{1,3}} \text{ if and only if } \frac{\theta}{\sigma} > 1. \tag{18}$$

In equilibrium, the first agent establishes a higher carbon price, compared to the Pigouvian level, if and only if $\sigma < \theta$, i.e., when the first agent gives a higher weight to the long-term utility than the second agent. The result has a very simple intuition. The first consumer would like to transfer more wealth to the third consumer, compared with the preferred wealth transfer of the second consumer: the high capital returns reflect this distortion and therefore depress the present-value damage below the true valuation emissions reductions by the first consumer.The opposite distortion —too low carbon price— occurs if $\sigma > \theta$.

Proposition 1 *Assume (11)-(12). If preferences are inconsistent ($\sigma \neq \theta$), the first-period carbon price does not satisfy the Pigouvian pricing rule, i.e., $MCP \neq \frac{MCD}{R_{1,3}}$. The carbon price is exceeds the Pigouvian level if and only if $\sigma < \theta$.*

Proof. *Above.* ■

Note that in this general-equilibrium setting the market return $R_{1,3}$ depends both on the demand for savings (technology) and the supply for savings (preferences), so the Pigouvian carbon price $MCP = \frac{MCD}{R_{1,3}}$ is tied to the savings in equilibrium. This is the reason why in standard climate-economy models, the carbon pricing is dictated by the same parameters that are used to calibrate the model to match savings behavior. The result in (18) is the simplest possible illustration of the point in our paper: in the climate policy game, the carbon pricing is disconnected from the equilibrium capital return. This result opens a number of questions that will be subsequently analyzed. Since the result arises in a strategic interaction equilibrium, it clearly cannot reflect a fully efficient outcome. Would enforcing the Pigouvian rule $MCP = \frac{MCD}{R_{1,3}}$ improve welfare? More generally, we ask how an optimal policy would look like, or whether we can quantify the effects using a more detailed climate-ecomy model.

2.3 Welfare, efficiency, and the Pigouvian rule

To address welfare implications of policies considered we need to develop a consistently defined welfare measure.[7] Consider an allocation $(\mathbf{c}, \mathbf{k}, z)$ that is Pareto efficient for welfare levels (w_1^*, w_2^*, w_3^*) defined in (1)-(3). If we maximize w_1, subject to the constraints $w_2 \geq w_2^*$, and $w_3 \geq w_3^*$ and feasibility constraints (4)-(6), then we must find the same allocation, and non-negative Lagrange multipliers $(\alpha, \beta) \in R_+^2$ for the welfare constraints. That is, the Pareto efficient allocation is also the solution of a welfare program maximizing

$$W(\mathbf{c}, \mathbf{k}, z) = w_1 + \alpha w_2 + \beta w_3 \tag{19}$$

$$= u_1 + (\rho + \alpha) u_2 + (\rho\theta + \alpha\sigma + \beta) u_3 \tag{20}$$

subject to (4)-(6). The conclusion also holds the other way around: any solution to a welfare maximization program with some $(\alpha, \beta) \in R_+^2$ is Pareto efficient. Strict concavity of the production and utility functions ensures the uniqueness of the allocation. Therefore, we can associate any Pareto efficient allocation with a pair of non-negative welfare weights $(\alpha, \beta) \in R_+^2$, and also with a "Planner" whose objective function is the

[7] This development follows closely Gerlagh and Liski (2011).

corresponding $W(\mathbf{c}, \mathbf{k}, z)$ giving some non-negative weight for all generations. If and only if weights are non-negative, we say that the Planner is representative, and the allocation is Pareto efficient. The Planner is merely a tool for welfare analysis, and not intended to introduce dictatorial preferences over allocations: given the equilibrium allocation, we will see if parameters α, β and thus a representative Planner can be attached to it. If yes, the allocation satifies the Pareto criterion as defined.

Note that (20) defines weights on utilities (u_1, u_2, u_3) that go together with non-negative (α, β). In equilibrium, however, we do not directly observe (α, β) but only the weights on utilities, and so the welfare objective W may not exist even though there is an objective expressed in term of utilities. Consider some feasible equilibrium allocation $(\mathbf{c}, \mathbf{k}, z)$ implying a stream of utilities (u_1^*, u_2^*, u_3^*), such that the allocation maximizes

$$U(\mathbf{c}, \mathbf{k}, z) = u_1 + \alpha' u_2 + \beta' u_3 \tag{21}$$

for some positive utility weights $(\alpha', \beta') \in R_+^2$. If such positive weights exist, the allocation is observationally equivalent to a Planner's optimum with objective $U(\mathbf{c})$, or shortly, the equilibrium is Planner-equivalent — if and only if the utility weights are positive, we say that the allocation is Planner-equivalent.[8] Whether the Planner defined this way is also representative $(\alpha, \beta \geq 0)$ determines if the allocation is efficient.

Pigouvian carbon pricing goes together with efficiency (necessity) but does not imply it (sufficiency). We can now show this using the above concepts. For any Planner-equivalent allocation, we have $\alpha' > 0$ and $\beta' > 0$, and the first-order conditions for $\{k_2, k_3, c_1, c_2, c_3\}$ tell us that any Planner-equivalent allocation satisfies:

$$1 = [\frac{\alpha' u_2'}{u_1'}] f_{2,k} = [\frac{\beta' u_3'}{u_1'}] f_{2,k} f_{3,k} \tag{22}$$

$$\Rightarrow \quad 1 = \frac{R_{1,2}}{MRS_{1,2}} = \frac{R_{1,3}}{MRS_{1,3}}, \tag{23}$$

where we drop superscripts on MRS. For emissions in the first period z, the first-order condition requires $u_1' f_{1,z} + \beta' u_3' f_{3,z} = 0$, which we can, again, rewrite as

$$MCP = \frac{MCD}{MRS_{1,3}}, \tag{24}$$

which together with previous equations leads to the Pigouvian pricing rule

$$MCP = \frac{MCD}{R_{1,3}}. \tag{25}$$

[8] Since each period utility only depends on one consumer good, maximizing a weighted sum of u_t, is equivalent to maximizing a weighted sum of c_t, and we can interpret Planner-equivalence as being indicative of productive efficiency.

The Pigouvian principle is a necessary and sufficient test for the existence of a Planner (not yet representative).

Lemma 2 *An allocation with strictly positive consumption, capital, and fuel use is Planner-equivalent: it maximizes $U(\mathbf{c}, \mathbf{k}, z)$ with positive α' and β', if and only if the Pigouvian rule (25) is satisfied.*

Proof. Necessity of the cost-benefit rule has been established above. For sufficiency, we notice that given the allocation, we can construct positive weights α' and β' from (22) such that with these weights the Planner prefers not to deviate from $\{k_2, k_3, c_1, c_2, c_3\}$. Rule (25) then ensures that the first-order condition for z is also satisfied. ■

The equivalence will be instrumental in our equilibrium analysis. If the Pigouvian rule is not satisfied, the equilibrium allocation implies that no Planner can exist. We have seen that in equilibrium the Pigouvian rule will not hold, so the conclusion for efficiency is immediate:

Corollary 1 *The equilibrium described in Proposition 1 is not Planner-equivalent if $\sigma \neq \theta$.*

This conclusion follows from Lemma 2 which shows that the allocation can be interpreted as some Planner's allocation (including the representative Planner) if and only if the Pigouvian rule holds. Since the equilibrium deviates from the rule, we cannot find positive welfare weights that would support the equilibrium outcome as Pareto efficient.

Can we then obtain efficiency by enforcing the Planner-equivalence, i.e., the Pigouvian rule? We now impose such a restriction as an institutional constraint on the equilibrium behavior — it can be thought of as an environmental agency scrutinizing the climate policies within each period. The environmental agency has no preferences, and it simply enforces the Pigouvian taxes, without restricting the choices of each generation in any other way.[9]

Proposition 2 *Enforcing Pigouvian externality pricing leads to a Planner-equivalent but not to a representative (efficient) outcome if preferences are hyperbolic, $\sigma < \theta$.*

Proof. We have seen in Lemma 2 that the Pigouvian tax rule implements planner equivalence, so that from the first-order conditions for c_2, c_3 and k_3 we have

$$\alpha' u_2' = \beta' u_3' f_{3,k}.$$

[9] If preferences are time-inconsistent in the other direction, $\sigma > \theta$, the proof is much more tedious, and we leave the analysis of this case to the infinite horizon model.

As the second generation does not need fossil fuels, the policy function g does not change. Substituting the second generation policy rule (8) gives

$$\beta' = \sigma\alpha'.$$

Adding $\rho(\theta - \sigma) > 0$ to the right-hand side,

$$\beta' < \rho\theta + (\alpha' - \rho)\sigma.$$

However, in view of (20), welfare weights associated with utility weights α' and β', are positive if and only if

$$\alpha' \geq \rho \quad (26)$$

$$\beta' \geq \rho\theta + (\alpha' - \rho)\sigma. \quad (27)$$

■

While enforcing Pigouvian externality pricing does not lead to full efficiency, could we obtain at least a Pareto improvement? In three periods, it is immediate that the answer is negative. The Pigouvian tax will constrain only the first generation's choices for fossil fuel use. It follows that the first generation has lower welfare as its preferred investment-emission reduction policy is ruled out.

Proposition 3 *The Pigouvian tax does not imply a Pareto improvement vis-a-vis the equilibrium without it.*

The reason for this result is simple: the Pigouvian tax is only a constraint on the first generation, as it could have implemented the same allocation without the rule, or without consulting the later generations. Therefore, enforcing the Pigouvian tax must decrease welfare of the first generation if $\theta \neq \sigma$. If preferences are time-consistent, imposing the Pigouvian tax has no effect on the equilibrium, so the requirement becomes redundant.

2.4 Discussion

The main plot should be clear after this three-period analysis. The carbon price that is optimal from the current generation's perspective, given future strategies, can be disconnected from capital returns, due to the commitment value of climate policies; see Proposition 1. The full model below is designed to quantitatively assess this observation. The infinite-horizon model adds to the analysis in multiple ways. First, in three periods there is not much room for policy analysis. For example, generation 1 cannot benefit from the later generations' adherence to the Pigouvian tax, without having a longer sequence of generations. As we will see at the end of the next section, the infinite horizon potentially offers the option for welfare-improving policies. Second, as is well known, the delays of impacts in climate change are important, and, in our case even more so, since the commitment value of policies depends precisely on how persistent are the impacts of current choices over time. The infinite horizon model enables us to provide analytical results that prescribe the equilibrium carbon prices dependent on technology, climate, and preference parameters. Third, the infinite horizon model enables us to provide a quantitative assessment of the magnitudes using realistic parameters values.

3 Infinite horizon model

3.1 Technologies and preferences: general setting

Consider a sequence of periods $t \in \{1, 2, ...\}$ and a fossil-fuel use history

$$s_t = (z_0, z_1, ..., z_{t-2}, z_{t-1})$$

where z_t denotes fossil-fuel use at time t. We let production possibilities depend on the emission history (i.e., past fossil-fuel use), capital k_t, labour use l_t, and current fossil fuel use z_t, and denote the production function by $f_t(k_t, l_t, z_t, s_t)$. History s_t can enter for two reasons: first, it captures damages from historical emissions as in integrated-assessment models following the Nordhaus' tradition; and second, the fuel use can be linked to the energy resources whose availability and cost of use depends on the past usage. In the specific model below, we abstract from the latter type of history dependence.[10] Let $\delta \leq 1$ denote the capital depreciation factor. Then, the budget accounting equation between period t and $t+1$ is

$$c_t + k_{t+1} = f_t(k_t, l_t, z_t, s_t) + (1 - \delta)k_t, \tag{28}$$

where c_t is the total consumption and k_{t+1} is the capital left for the next period. In each period, the representative consumer makes the consumption, fuel use, and investment decisions. Let per-period utility be u_t and define generation t welfare generated by

[10] We could introduce finite fossil-fuel resources as in Golosov et al. (2011). However, since the long-term carbon pricing is described by energy-resource use where the cost structure does not depend on history, we abstract from this transitory phase.

sequence $\{c_t, z_t, k_t\}_{t=1}^{\infty}$ as

$$w_t = u_t + \rho \sum\nolimits_{\tau=t+1}^{\infty} \theta^{\tau-t-1} u_\tau \tag{29}$$

where we identify dynamically consistent preferences by $\theta = \rho$, so that each future period $\tau > t$ is discounted with the same discount factor $\theta^{\tau-t}$. Using the same motivations as in three-periods, decision-makers use lower discount rates for long-term evaluations than for short-term evaluations, $\rho < \theta$, but the formal analysis is not restricted to this quasi-hyperbolic setting; for interpretations, the quasi-hyperbolic case is the most natural to keep in mind, but we will state explicitly the formal results that require $\rho < \theta$. One interpretation is that each generation lives for one period, so that the immediate next generation wealth is discounted with factor ρ (children), while the subsequent generation is discounted at a lower additional rate captured through θ. As is well-known, if the current generation ignores all future generations' wealth but for that value coming through the immediately next generation's wealth, then the future utilities are discounted in a dynastic manner using the same effective utility discount factor ρ for all periods. Instead, if some direct weight is given to future generations' wealth, in addition to the weight given indirectly through the immediately next generation, the current generations expresses pure altruism towards them, and the discount factor must satisfy $\theta > \rho$; see Saez-Marti and Weibull (2005). For convenience we will interchangeably refer to periods and generations by t.

3.2 The basis climate-economy model

The production side of our specific climate-economy model follows Golosov et al. (2011) which marks an important deviation from Nordhaus' (e.g., 1993) approach to integrated-assessment modeling: abatement does not enter as a separate decision variable but it is only implied by the choices for energy production. However, our modeling of the climate dynamics and preferences depart substantially from both Golosov et al. and Norhaus. We pull together the production structure as follows:

$$y_t = k_t^{\alpha} A_t(l_{y,t}, e_t)\omega(s_t) \tag{30}$$

$$e_t = E_t(z_t, l_{e,t}) \tag{31}$$

$$l_{y,t} + l_{e,t} = l_t \tag{32}$$

$$\omega(s_t) = \exp(-\Delta_y D_t), \tag{33}$$

$$D_t = \sum_j [(1-\varepsilon_j)^t D_{j,0} + \sum_i \sum_{\tau=1}^{t-1} a_i b_j \pi \varepsilon_j \frac{(1-\delta_i)^\tau - (1-\varepsilon_j)^\tau}{\varepsilon_j - \delta_i} z_{t-\tau}] \tag{34}$$

Production. We have the following components of gross production: (i) the Cobb-Douglas capital part k_t^α; (ii) function $A_t(l_{y,t}, e_t)$ for the energy-labour composite in the final good sector, (iii) the energy production using fossil fuels and labour captured by $E_t(z_t, l_{e,t})$, and (iv) the damage part given by function $\omega(s_t)$ capturing the output loss of production depending on the history of emissions from the fossil-fuel use. The energy-part of production $E_t(z_t, l_{e,t})$ could be made dependent also on the full history of fossil-fuel use s_t, as fossil-fuel extraction costs can be explicitly modeled and their total availability can be linked to the cumulative fossil-fuel use. We abstract from such dependence as this allows describing the savings and emissions policies without the specification of functions A_t and E_t; we provide such a specification and describe the energy sector in detail in the quantitative analysis section 4.

We assume that capital depreciates in one period, $\delta = 1$ (one period will be 10 years in the quantitative section).

Damages and carbon cycle. The exponential form in $\omega(s_t)$ approximates the damage function that is used in many integrated assessment models with explicit variables for atmospheric CO_2 concentrations and temperature change. The exponential form with linear terms within the exponent has been used by Golosov et al. (2011), but we introduce a more detailed delay-structure for the emission impacts that leads to markedly different carbon price calculations, even without preference inconsistencies. The variable D_t is a proxy for the squared value of the global mean earth surface temperature rise. We explain now how expression (34) for D_t is derived.

Climate change dynamics can be described in reduced form through a multi-box model for atmospheric CO_2 stocks. This is a common approach in integrated assessment models to describe the carbon cycle as a diffusion process between various layers of the atmospheric, the ocean and the biosphere (Hasselmann et al. 1997).[11] Let there be multiple atmospheric CO_2 boxes, labeled i, such that share a_i of antropogenic emissions enter box i, $\sum_i a_i = 1$, while the depreciation rate of box i is δ_i. The stock S_i in box i develops according to

$$\begin{aligned} S_{i,t} &= (1-\delta_i) S_{i,t-1} + a_i z_{t-1} \\ S_t &= \sum_i S_{i,t} \end{aligned}$$

where z_t is fossil fuel energy use, expressed here in GtonCO_2.

[11] For example, a 3-box model almost perfectly fits the CO_2 dynamics in DICE (Nordhaus 2007). The illustration is available on request.

Given the atmospheric CO_2 stock, the temperature adjusts slowly. Similarly to the atmospheric CO_2 module, we model temperature adjustment through a multi-box system. Typically, temperatures are considered dependent on atmospheric CO_2 through a concave function (the logarithm is often used as an approximation), while damages depend on temperatures through a convex function, e.g. quadratic. Within the domain of relevant CO_2 concentrations up to 1000 ppmv,[12] the chain of the concave and convex function, leads to an almost linear relationship between stocks and damages.[13] We use the variable $D_{j,t}$ as a proxy for squared temperature for box j. Let π be the sensitivity of (squared) temperature for atmospheric CO_2, so that if S_t is constant then $D_t = \pi S_t$ follows as the stationary state of the climate, and ε_j is the adjustment speed of the $j-th$ box, with weight b_j, $\sum_j b_j = 1$:[14]

$$\begin{aligned} D_{j,t} &= D_{j,t-1} + \varepsilon_j (b_j \pi S_t - D_{j,t}) \\ D_t &= \sum\nolimits_j D_{j,t} \end{aligned}$$

Given these two layers of climate variables, it is a straightforward matter of verification that future damages depend on past emissions linearly as:

$$S_{i,t} = (1-\delta_i)^{t-1} S_{i,1} + \sum\nolimits_{\tau=0}^{t-1} a_i (1-\delta_i)^{\tau-1} z_{t-\tau} \tag{35}$$

$$D_{j,t} = (1-\varepsilon_j)^t D_{j,0} + \sum\nolimits_i \sum\nolimits_{\tau=1}^{t-1} a_i b_j \pi \varepsilon_j \frac{(1-\delta_i)^\tau - (1-\varepsilon_j)^\tau}{\varepsilon_j - \delta_i} z_{t-\tau}, \tag{36}$$

where expression (36) leads to (34). The two layers are an important part of the carbon-temperature-cycle model, as they introduce a time-lag between emissions and the peak in damages; the term within the summation signs peaks at a period between $1/\delta_i$ and $1/\varepsilon_j$. For the most simple 1-box representation, a typical estimate for the atmospheric CO_2 depreciation δ_1 is one per cent per year, while for the temperature adjustment ε_1 it is two per cent per year; the associated peak in temperature-response is after about 70 years. In other words, peak temperatures lag 70 years behind emissions, providing a rule of thumb for discounting damages: discounting future damages at a rate of r per cent per

[12]ppmv=parts per million by volume.

[13]Indeed, the early calculations by Nordhaus (1991) based on local linearization, are surprisingly close to later calculations based on his DICE model with a full-fledged carbon-cycle temperature module, apart from changes in parameter values based on new insights from natural sciences.

[14]The constant π is the climate sensitivity; stated differently, $1/\pi$ is the amount of atmospheric CO_2 that leads to a long-term temperature increase of 1 Kelvin. The estimation of the reduced-form carbon cycle model is based on complex carbon-cycle models, e.g. Maier-Reimer and Hasselman (1987) and Hooss et al. (2001), IPCC (2007).

year results in a discount factor of about 2^{-r} after 70 years. An annual 1, 2 or 3 per cent discount rate results in a discount factor of damages of 1/2, 1/4, or 1/8, respectively.

Golosov et al. (2011) specification can be understood as one where the temperature adjustment is immediate: $\varepsilon_j = 1$, an emission impulse leads to an immediate temperature shock that slowly decays. A reduced-form model that proxies the DICE model (Nordhaus 2007) shows an emissions-damage peak after 60 years, while the reduced-form model we employ here for $\omega(.)$, based on the natural sciences literature, produces a peak in the emission-damage response function after about 80 years. Additionally, it has a fat tail: about 21 per cent of emissions does not depreciate within the horizon of thousand years. The function $\omega(s_t)$ captures the essence of the climate change delays, and will be used both in the analytical and the quantitative section. The Appendix compares the response functions graphically for Nordhaus (2007), Golosov et al. (2011), and in our model.

Periodic utility. We assume that the utility function is logarithmic, and through a separable linear term we also include the possibility of intangible damages associated with climate change:

$$u_t = \ln(\frac{c_t}{l_t}) - \Delta_u D_t. \tag{37}$$

The utility loss $\Delta_u D_t$ is not necessary for the substance matter of this paper, but it proves usefu tol explicate how it enters the carbon price formulas; however, in calibration, we let $\Delta_u D_t = 0$ to maintain an easy comparison with the previous studies.[15] It is important to note that we write utility as an average in our analysis. Alternatively, we can write aggregate utility within a period by multiplying utility with population size, $u_t = l_t \ln(c_t/l_t) - l_t \Delta_u D_t$. The latter approach is feasible but it leads to considerable complications in the formulas below.[16] Scaling the objective with labor rules out stationary strategies — they become dependent on future population dynamics —, and also impedes a clear interpretation of inconsistencies in discounting; while the formulas in the lemmas depend on this assumption, the substance of the Propositions is not altered.

Strategies. The equilibrium outcome depends on the equilibrium concept, and on the restrictions on strategies that will be used. Our starting point is the symmetric and stationary Markov equilibrium. Symmetry has the meaning that if generations did not know the period where they live, they would agree on the policy rule used at each t — even though there is technological change and population growth, given objective (37), there

[15]When temperature rise is continuous for the next centuries, many ecosystems will not be able to adapt quickly enough and the loss of biodiversity will be inevitable, which may negatively and directly affect the future welfare.

[16]The expressions for this case are available on request

will be an equilibrium where the same policy rule will be used for all t. Markov restriction means that the policy does not condition on the history of past behavior. Moreover, we impose differentiability of the policy in the states (see Krusell et al. (2002) and Karp (2007) for implications following from eliminating the symmetry and differentiability restriction). In equilibrium, the policy will take the form $k_{t+1} = g_t(k_t, s_t)$, $z_t = h_t(k_t, s_t)$, where s_t enters as the weighted sum of past emissions as expressed in (34); history s_t matters for strategies through its effect on the current stocks S_t and D_t.[17] In a later section, we consider non-stationary strategies that support other than Markov savings and emissions rules.

Structure of equilibria. Given the policies $g_t(k_t, s_t)$ and $h_t(k_t, s_t)$, we can write the welfare in (29) as follows

$$\begin{aligned} w_t &= u_t + \rho W_{t+1}(k_{t+1}, s_{t+1}), \\ W_t(k_t, s_t) &= u_t + \theta W_{t+1}(k_{t+1}, s_{t+1}) \end{aligned}$$

where $W_{t+1}(k_{t+1}, s_{t+1})$ is the (auxiliary) value function. All equilibria considered in this paper, whether Markov, Planner-equivalent, or generated by a Pigouvian tax rule, will be of the form where constant $0 < g < 1$ of the gross output is invested,

$$k_{t+1} = g y_t, \tag{38}$$

whereas the climate policy defines fossil-fuel use and thus emissions are determined through a constant h satisfying

$$f_{t,z} = h(1-g)y_t, \tag{39}$$

where $f_{t,z} = MCP_t$ is the marginal product of fossil fuel use, the carbon price. Policies will be characterized simply by a pair of constants (g, h)! That a constant fraction of output is saved should not be surprising, given log utility and Cobb-Douglass production; even with inconsistent preferences, the stationary strategies take this form.[18] Condition (39) implies that the marginal carbon price per consumption is a constant, $h = f_{t,z}/c_t$ where $c_t = (1-g)y_t$, which may seem suprising given the complicated delay structure (34), and changing productivities in (30)-(34), and preference inconsistencies.[19]

[17]Strictly speaking (S_t, D_t) is the payoff-relevant state variable rather than s_t but it proves useful to work directly with s_t in the expressions below.

[18]See, e.g., Barro 1999, for the analysis of the one-capital good case.

[19]Golosov et al. find emission policies that have the same features; our policies are from the same class of functions, which explains the similarity.

Postponing the discussion on welfare and the verification that the policies actually take the above form, it proves useful to state the properties of the value function implied by a (g, h) policy (the proofs, unless helpful in the text, are in the Appendix).

Lemma 3 *(separability) Given the model (30)–(37), assuming that future policies $g_\tau(\cdot)$ and $h_\tau(\cdot)$ for $\tau = t+1, t+2, ...$ satisfy (38) and (39), then the value function is additive in capital and historical emissions*

$$W_{t+1}(k_{t+1}, s_{t+1}) = V_{t+1}(k_{t+1}) - \Omega(s_{t+1}).$$

with parametric form

$$\begin{aligned} V_{t+1}(k_{t+1}) &= \xi \ln(k_{t+1}) + \tilde{A}_{t+1} \\ \Omega(s_{t+1}) &= \sum_{\tau=1}^{t-1} \zeta_\tau z_{t+1-\tau}. \end{aligned}$$

where $\xi = \frac{\alpha}{1-\alpha\theta}$, $\zeta_1 = (\frac{\Delta_y}{1-\alpha\theta} + \Delta_u) \sum_i \sum_j \frac{a_i b_j \pi \varepsilon_j}{[1-\theta(1-\delta_i)][1-\theta(1-\varepsilon_j)]}$, and $\tilde{A}_{t+1}$ is independent of k_{t+1} and s_{t+1}.

The result shows that we can obtain the value of savings k_{t+1} and the costs from fossil-fuel use z_t separately.

3.3 Markov equilibrium carbon price

Consider first savings. Each generation takes the future policies, captured by constants (g, h) in (38)-(39), as given and chooses its current savings to satisfy

$$u'_t = \rho V'_{t+1}(k_{t+1}),$$

where function $V(\cdot)$ from Lemma 3 captures the continuation value implied by the equilibrium policy.

Lemma 4 *(savings) The equilibrium investment share $g = k_{t+1}/y_t$ is*

$$g^* = \frac{\alpha\rho}{1 + \alpha(\rho - \theta)}. \tag{40}$$

The proof of the Lemma is a straightforward verification exercise following from the first-order condition. The savings depend only on the capital share α and preference parameters; note that when preferences are consistent $\theta = \rho$, we obtain $g = \rho\alpha$, as is expected.

Consider then the equilibrium carbon price $f_{t,z}$, that is, the marginal product of the fossil-fuel use z_t. The first-order condition is

$$u'_t f_{t,z} = \rho \frac{\partial \Omega(s_{t+1})}{\partial z_t},$$

where function $\Omega(.)$ gives the future costs implied by the equilibrium policy. Given Lemma 3, the equilibrium carbon price and the fossil-fuel use is immediate:

Lemma 5 *Equilibrium emissions $z_t = z_t^*$ depend only on the current technology at period t as captured through $A_t(.)$ and $E_t(.)$. The carbon price satisfies (39) where $h^* = \rho\zeta_1$:*

$$\begin{aligned} MCP_t = f_{t,z} &= \rho\zeta_1(1-g)y_t \\ \rho\zeta_1 &= \sum_i \sum_j \frac{(\frac{\Delta_y}{1-\alpha\theta} + \Delta_u)\rho\pi a_i b_j \varepsilon_j}{[1-\theta(1-\delta_i)][1-\theta(1-\varepsilon_j)]} \end{aligned} \quad (41)$$

The Markov equilibrium carbon price depends, not surprisingly, on the delay structure in the carbon cycle captured by parameters δ_i and ε_j for each box. It is easy to see that carbon prices increase with climate sensitivity ($\partial\zeta_1/\partial\pi > 0$) and a slower carbon depreciation ($\partial\zeta_1/\partial\delta_i < 0$), and decrease with slower temperature adjustment ($\partial\zeta_1/\partial\varepsilon_j > 0$), while higher short- and long-term discount rates both decrease the carbon price ($\partial\zeta_1/\partial\rho > 0$; $\partial\zeta_1/\partial\theta > 0$).

3.4 The Pigouvian tax rule

The Pigouvian rule for carbon pricing states that all social costs of carbon emissions at time t should be imposed on polluters at t. It is of interest to see if and how the Markov policy deviates from this principle, and, then, if any welfare gains can be achieved by imposing the Pigouvian tax rule as an institutional constraint on the equilibrium. We explain first the deviation and explore the welfare implications in Section 3.6.

The social cost of current emissions is the present-value marginal damages caused by those emissions, and it thus depends on the discount factor used in the calculations. Which discount factor should be used? For policy rules satisfying (38)-(39), we obtain savings and emissions decisions separately (Lemma 3): we can first find savings, and use the implied utility discount factor for determining the present-value damages and thus for the Pigouvian emissions policy. Let $0 < \gamma < 1$ denote implied utility discount factor obtained, in equilibrium, from

$$u'_t = \gamma u'_{t+1} R_{t,t+1}$$

where $R_{t,t+1}$ is the capital return between t and $t+1$. Thus,

$$\gamma = \frac{u'_t}{u'_{t+1} R_{t,t+1}} = \frac{c_{t+1}}{c_t R_{t,t+1}} = \frac{c_{t+1}}{c_t} \frac{k_{t+1}}{\alpha y_{t+1}} = \frac{g}{\alpha}. \tag{42}$$

In Markov equilibrium where $g = g^*$, we have

$$\gamma^* = \frac{\rho}{1 + \alpha(\rho - \theta)}, \tag{43}$$

but γ can also deviate from this if the equilibrium is non-Markovian. Since we come back to such policies shortly, we label the Pigouvian tax by the superscript γ, as the formula provides the carbon price as a function of the utility discount factor γ.[20]

Lemma 6 *The net present value of marginal damages of emissions, referred to as the Pigouviant tax, τ_t^γ is proportional to gross output,*

$$\tau_t^\gamma = h^\gamma (1-g) y_t \tag{44}$$

$$h^\gamma = \sum_i \sum_j \frac{(\frac{\Delta_y}{1-g} + \Delta_u)\gamma \pi a_i b_j \varepsilon_j}{[1 - \gamma(1-\delta_i)][1 - \gamma(1-\varepsilon_j)]} \tag{45}$$

where y_t is gross production.

The Pigouvian tax policy imposes the rule that marginal productivity of emissions should equal the Pigouvian tax, $f_{t,z} = y_t \frac{A_{te}}{A_t} E_{t,z} = \tau_t^\gamma$. We can now compare the two carbon prices:

Proposition 4 *For $\theta > \rho$, the equilibrium carbon price strictly exceeds (falls short of) the Pigouvian carbon price if climate change delays are sufficiently long (short). That is, for γ defined by (43), if δ_i and ε_j are close to zero:*

$$f_{t,z} > \tau_t^\gamma$$

while $f_{t,z} < \tau_t^\gamma$ if δ_i and ε_j are close to one. For $\theta < \rho$ the inverse relation holds.

If $\theta > \rho$, preferences are quasi-hyperbolic, and if, in addition, the climate system is sufficiently persistent, then the current generation sees the climate asset as a commitment device, and therefore values external cost above the Pigouvian level.

[20] The formula has wider applicability than this positive context where the utility discount rate is derived from actual savings decisions: we could apply it to determine Pigouvian taxes based on normative considerations of equity, leading to higher or lower values for γ. We can also inversely determine the utility-discount factor that would be required to justify a certain carbon price.

3.5 First look at numbers

For the Markov and Pigouvian carbon prices in (44) and (41), we do not have to solve the full model: the initial income, savings, and the carbon cycle parameters allow obtaining the initial carbon price levels. We explicate the procedure for calibrating the carbon cycle in Appendix.[21]. We note here only that the model is decadal (10-year periods), and that our box presentation of the carbon cycle results in a slightly lower response of damages to emissions for the first 80 years, and substantially higher damages after 300 years.[22] That is, our parameters suggest a slightly slower and more persistent climate response.[23]

We seek to calibrate the model to match the case presented in Nordhaus (2007) as a benchmark. For example, assuming damages equivalent to 2.7 per cent of output at a temperature rise of 3 Kelvin, as in Nordhaus (2001), we obtain $\Delta_y = 0.003$; we set $\Delta_u = 0$. The annual pure time discounting of 2 per cent, leads to a 25 per cent gross savings rate and a Pigouvian carbon price of 8.4 Euro/tCO_2, equivalent to 46 USD/tC.[24] This is number for 2010 is very close to the starting level of the Nordhaus' Pigouvian tax.[25]

We can decompose the carbon price into three parts. First, consider the one-time costs if damages were immediate (ID) but only for one period,[26]

$$ID = (\frac{\Delta_y}{1-\alpha\theta} + \Delta_u)\pi(1-g)y_t,$$

where $1-\alpha\theta$ is replaced by $1-\alpha\gamma = 1-g$ for the Pigouvian tax rule. This value is multiplied by a factor to correct for the persistence of climate change, the persistence factor (PF),

$$PF = \sum_i \frac{a_i}{[1-\theta(1-\delta_i)]}$$

[21] Appendix "Calibration: carbon cycle"

[22] see Appendix "Comparisons of damage responses".

[23] The main reason for the deviation is that DICE assumes an almost full CO_2 storage capacity for the deep oceans, while large-scale ocean circulation models point to a reduced deep-ocean overturning running parallel with climate change (Maier-Reimer and Hasselman 1987). The positive feedback from temperature rise to atmospheric CO2 through the ocean release is essential to explain the large variability observed in ice cores in atmospheric CO_2 concentrations.

[24] Note that 1 tCO2 = 3.67 tC, and 1 Euro is about 1.3 USD.

[25] Minor differences are caused by a correction for the price index, and a somewhat more persistent damage structure in our reduced model

[26] the term for Δ_y is adjusted to account for the decrease in the future capital stock caused by a current drop in output.

which we then multiply by a factor to correct for the delay in the temperature adjustment, the delay factor (DF),

$$DF = \sum_j \frac{b_j \rho \varepsilon_j}{1-\theta(1-\varepsilon_j)}$$

Table 2 below presents the decomposition of the Pigouvian tax, as well as the Markov tax when the short-term annual discount rate is 2.55 and the long-term is rate .5 per cent; we discuss these choices shortly.

	ID	PF	DF	Carbon price
Pigou	7.61	2.44	.45	8.4
Markov	8.05	5.76	.63	29.4

Table 2: Decomposition of Carbon price, MCP [Euro/tCO2]. ID=immediate costs, PF=persistence factor, DF=delay factor, $MCP = ID \times PF \times DF$. Parameter values in the text.

Leaving out the time lag between CO_2 concentrations and temperature amounts to replacing the column DF by ρ: abstracting from the delay in temperature adjustments, as in Golosov et al. (2011), increases the Pigouvian carbon price by almost factor 2.

From Table 2, the Markov carbon prices stand at 29.4 Euro/tCO_2, three and half times the Pigouvian level (and well above the current carbon prices at the EU emissions trading system). How did we choose the short- and long-run discount rates underlying the Markov price? Weitzman's (2001) survey led to discount rates declining from 4 per cent for immediate future (1-5 years) to 3 per cent for near future (6-25 years), to 2 per cent for medium future (26-75 years), to 1 per cent for distant future (76-300), and then close to zero for far-distant future. Consistent with Weitzman and our 10-year length of one decision-period, we use the short-term discount rate close to 3 per cent, and the long-term rate below 1 per cent. This still leaves degrees of freedom in choosing the two rates. We use this freedom to match the savings rate of 25 per cent and thus the macroeconomic performance in the Norhaus (2007) case: we choose ρ and θ to maintain the equilibrium utility discount factor at $\gamma = 0.817$ (2 per cent annual discount rate) that keeps savings constant across the Markov equilibria considered. Moreover, since the equilibrium utility discount rate remains at 2 per cent, the Nordhaus case is the Pigouvian benchmark; using the 2 per cent rate to determine the present-value damages implies Pigouvian taxes as proposed by Nordhaus. See now Table 3.

The first row reproduces the Pigouvian case from Table 2 assuming consistent pref-

	annual discount rate						
	short-term	long-term	equilibrium	ID	PF	DF	Carbon price
"Nordhaus"	.02	.02	.02	7.61	2.44	.45	8.4
Markov	.0235	.01	.02	7.89	3.69	.55	16.1
Markov	.0255	.005	.02	8.05	5.76	.63	29.4
Markov	.0271	.001	.02	8.19	19.63	.73	116.9
"Stern"	.001	.001	.001	8.19	19.36	.95	152.4

Table 3: Disconnecting carbon pricing from equilibrium discounting. Parameter values as in Table 2

erences when the annual utility discount rate is set at 2 per cent: this row presents the carbon price under the same assumptions as in Nordhaus (2007), but using our carbon cycle model (which approximates his case very well, as discussed). Keeping the equilibrium time-preference rate at 2, thus maintaining savings at the level proposed by Nordhaus (reported also in Table 1 of the Introduction), we move to the Markov equilibrium by departing the short- and long-term discount rates in first and second columns. We obtain a radical increase in the carbon price as the long-term discounting decreases, while savings remain at the same level. The highest carbon tax, 116.9 EUR/tCO_2, corresponds to the case where the long-run discounting is as proposed by Stern (2006); this case also best matches the Weitzman's values. For reference, we report the Stern case where the long-term discounting holds throughout, the carbon price takes value 152.4 EUR/tCO_2, and gross savings cover about 30 per cent of income. Thus, our third Markov equilibrium closes considerably the gap between Stern and Nordhaus carbon prices, without having unrealistic by-products for the macroeconomy.[27]

We can also experiment with damage sensitivity vis-a-vis the time-preference structure. Let $\Delta_Y = 0.00144$ stand for the low-damage case where a 3K temperature increase leads to only 1.3 percent output loss. Let $\Delta_Y = 0.003$ plus $\Delta_U = 0.0081$ stand for a high-damage case when intangible damages associated to a 3K temperature rise amount to an equivalent of 7.3 per cent of consumption, so that total damages are equivalent to 10 per cent of consumption. The median damage is as before, and for the comparison we

[27]The deviation between the Markov (thus Nordhaus) and Stern savings can be made extreme by sufficiently increasing the capital share of the output that gives the upper bound for the fraction of y_t saved; close to all income is saved under Stern preferences as this share approaches unity (Weitzman, 2007). However, with reasonable parameters such extreme savings do not occur, as in Table 3.

choose the second Markov equilibrium where the long-term discount rate amounts to a half per cent annually. Table 4 shows that the Markov equilibrium tax for low damages are between the median and high damage tax of Nordhaus (i.e, Pigouvian). Both the damage estimates and the time preference structure are equally important to determine carbon prices.

Carbon Prices	low damages	median damages	high damages
“Nordhaus”	4.0	8.4	31.0
Markov equilibrium	14.2	29.4	108.9
“Stern”	73.4	152.4	564.3

Table 4: Carbon prices [Euro/tonCO2] dependence on structure of time preferences and damage estimates

3.6 Welfare and efficiency

Our welfare concept is the multi-agent Pareto optimality. As in three periods, we consider Planner-equivalence, and then if such a planner is representative (efficient). For the infinite horizon, an allocation $\{c_t, z_t, k_t\}_{t=1}^{\infty}$ is Planner-equivalent if it maximizes welfare of a (fictitious) Planner with objective function

$$\sum_{t=1}^{\infty} \alpha'_t u_t$$

where $\{\alpha'_t\}_{t=1}^{\infty}$ is some sequence of utility weights with bounded mass. Given the structure of our equilibria with constant savings, we focus directly on geometric weights, $\gamma = \alpha'_{t+1}/\alpha'_t$; we have already constructed such a geometric weight for the Markov equilibrium. As in three periods, the Planner is representative (efficient) if and only if the allocation implies non-negative welfare weights. The condition for efficiency turns out be simple (proof in Appendix but see also Gerlagh and Liski, 2011):

Lemma 7 *The Planner with γ is representative if and only if $\gamma \geq \max\{\rho, \theta\}$.*

If both short and long-term preference paramemeters were equal, say ρ (resp., θ), the utility discount factor γ would simply be ρ (resp., θ). If time preferences are inconsistent, $\theta \neq \rho$, then the equilibrium utility discount factor is bounded by the two contrafactuals:

by straightforward substitution, we can see that for $\theta \neq \rho$, the Markov-equilibrium discount factor satisfies

$$\min\{\rho, \theta\} < \gamma^* < \max\{\rho, \theta\} < 1.$$

The reasoning for this result is straightforward. The current generation cares more for total future welfare, and thus saves more, than a planner who would have consistent preferences with discount factor satisfying $\gamma = \rho$. Then, the current agent cares less, and saves less, compared to a representative planner who would have consistent preferences with $\gamma = \theta$. Clearly, the equilibrium savings must be somewhere between the extremes.

Proposition 5 *An equilibrium with utility discount factor γ (not necessarily γ^*) is Planner-equivalent if and only if the Pigouvian carbon pricing rule in Lemma 6 holds.*

Proof. Given utility discount factor γ, we can obtain the Planner's allocation as the one that follows in Markov equilibrium when $\rho = \theta = \gamma$. The Markov equilibrium implies the first-order conditions for capital investments, and these conditions are sufficient for efficient savings in this economy. The remaining efficiency condition is for emissions. Efficiency implies and is implied by the Pigouvian carbon price. ■

If we abstract from intangible climate change damages ($\Delta_u = 0$), implementing Pigouvian carbon prices has a clear justification as it implements productive efficiency: no consumption sequence exists that yields a strictly higher discounted utility for market utility discount rate γ^*. Yet, productive efficiency does not imply welfare efficiency, because if $\theta \neq \rho$, then $\gamma < \max\{\theta, \rho\}$ so that Lemma 7 is not satisfied.[28]

Proposition 6 *If $\rho \neq \theta$, the equilibrium with Pigouvian carbon prices is not Pareto efficient.*

While the Pigouvian carbon price does not restore full efficiency, it might be argued that the productive inefficiency removed produces at least a Pareto improvement. However, not even this can be achieved. To show this, and to identify policies that improve welfare, let us consider how future policies affect current welfare. Let preferences be quasi-hyberbolic $\theta > \rho$, and note that from the perspective of the current generation, future savings and emission levels are optimal if they are consistent with the long-term time preference θ, that is, if $g = \alpha\theta$ and $h^{\gamma=\theta}$ where h^γ is defined in Lemma 6; then future agents would behave as if they were consistent with present long-term preferences. Thus, this artificial thought-experiment gives a benchmark against which we can test various policy proposals affecting the current welfare through future policies.

[28]Note that this conclusion does not require $\Delta_u = 0$.

Lemma 8 *For $\theta > \rho$ and $\tau > t$,*

$$\frac{\partial w_t}{\partial g_\tau} > 0 \text{ iff } g_\tau = g < \alpha\theta$$
$$\frac{\partial w_t}{\partial h_\tau} > 0 \text{ iff } h_\tau = h < h^\theta.$$

Since the equilibrium policies satisfy the inequality conditions in the Lemma — savings are too low and emissions too high — any policy that manages to increase either one or both of the future decision variables from their Markov level increases the current welfare. It turns out that imposing the stand-alone Pigouvian tax principle implies a correction in the wrong direction.

Proposition 7 *Implementing Pigouvian carbon prices from period t onwards implies a welfare loss for generation t vis-a-vis the Markov equilibrium.*

Proof. By Lemma 3, the change to the Pigouvian carbon price does not affect policy g; thus, we can focus on the effect of future carbon policies on current welfare w_t. Let $\rho < \gamma < \theta$, and let climate change be a slow process such that $\tau_t^\theta > f_{t,z} > \tau_t^\gamma$; see Proposition 6. Imposing the Pigouvian carbon price will then decrease the future carbon price, taking it further away from τ_t^θ, decreasing current welfare as shown in Lemma 8. The same mechanism applies for other time preferences and/or fast climate change. For $\rho > \gamma > \theta$ and slow climate change, or for $\rho < \gamma < \theta$ and fast climate change, we have $\tau_t^\theta < f_{t,z} < \tau_t^\gamma$, and again implementing future Pigouvian carbon prices will decrease current welfare. Similarly, for $\rho > \gamma > \theta$ and fast climate change we have $\tau_t^\theta > f_{t,z} > \tau_t^\gamma$. Thus, imposing Pigouvian carbon prices on current policies must reduce welfare compared to the unrestricted Markov welfare. ■

The remarkable feature of the above proposition is that a Pigouvian carbon price policy strictly decreases welfare, not as a second-order effect, but as a first-order effect.

Lemma 8 suggests that we can look for welfare-improvements through self-enforcing policy rules; there is surplus to be created by increasing the policy choices (g, h) for all generations while maintaining their incentive constraints, coming from the threat to switch back to Markov equilibrium where no rules regarding future behavior apply. Consider a policy pair $(\widehat{g}, \widehat{h})$ that the current generation would like to propose for all generations, including itself. From Lemma 8, generation t would like to propose for all future generations efficient decision rules $g^{PE} = \theta\alpha$ and $h^{PE} = h^\theta$ but,[29] achieving this

[29]Lemma 7 states that the representative Planner must have a utility discount factor of θ or larger. Here, $\gamma = \theta$, and h^θ equals the Pigouvian tax, ensuring that that the allocation is Planner-equivalent (Proposition 5).

requires that these policies are followed also at t, which is ruled out by the current incentive constraints: (g^{PE}, h^{PE}) does not maximize w_t. But, also from Lemma 8, the current generation is willing to give up part of its consumption, by increasing g and h, beyond their Markov equilibrium level, anticipating that all subsequent decision-makers will follow suit when facing the same decision. The best self-enforcing policy pair supported by the Markov equilibrium is between the efficient and the Markov policy:

Proposition 8 *There exist a policy pair $g^{PE} > \widehat{g} > g^*$ and $h^{PE} > \widehat{h} > h^*$, such that this policy rule maximizes welfare of the first generation, and no future generation can benefit from switching back to the Markov equilibrium.*

We have seen that the Markov equilibrium allows us to disconnect utility discounting and carbon pricing. Proposition 8 implies that we can tighten the carbon policy further ($\widehat{h} > h^*$) while keeping savings at the Markov level ($\widehat{g} = g^*$).[30] Thereby, we can justify, from the positive welfare point of view, carbon prices that are not only higher than the Pigou tax but also higher than the Markov price, without increasing the equilibrium utility discount factor and savings. Recall that the discussion following Stern (2006) on the appropriate level of the carbon price is centered around the choice of the social discount factor. To justify the Stern's level for carbon prices, one may invoke ethical arguments or fundamental uncertainties (Weitzman, 2007). Our argument is completely different. When the short and long-term time-preferences differ, the self-enforcing policies can support stricter than Markov carbon policies, that close the gap to Stern even further, $\widehat{h} > h^*$. In our quantitative analysis below, we illustrate that the quantitative magnitude of this policy is significant.

The model can also justify Stern through high equilibrium utility discount factors: self-enforcing policies that target both savings and emissions imply simultaneously higher equilibrium discount factors, $\widehat{g}/\alpha > g^*/\alpha$, and lower emissions, $\widehat{h} > h^*$. Obviously, this maximizes the welfare potential of the policies considered, but can lead to savings rates close to α, which we calibrated in our numerical model below to 0.306. Such savings may be hard to justify, in particular if the capital share α is large, which is exactly the common critique on Stern. We concur with the practical relevance of this criticism, but note that the endogenous link between equilibrium discounting and welfare shows that the Stern proposal is intelligible as an equilibrium outcome. We come back to the numbers in the quantitative analysis.

[30] The argument for the self-enforcing policies holds for both policy variables separately, so this one-sided policy improvement is feasible.

4 Quantitative analysis

While the climate and savings policies can be obtained in closed form for a given state of the economy, for the future climate-economy adjustment path we need to further specify technologies, and make additional quantitative choices. In particular, we need a detailed structure for the energy sector.

4.1 Production and energy

Consider a production function as in (30) but further specified to

$$\begin{aligned} y_t &= k_t^{\alpha}[A_t(l_{y,t}, e_t)]^{1-\alpha}\omega(s_t) \\ A_t(l_{y,t}, e_t) &= \min\{A_{y,t}l_{y,t}, A_{e,t}e_t\} \end{aligned}$$

where the overall labor-energy composite $A_t(l_{y,t}, e_t)$ takes CES form with extremely low elasticity of subsitution between labor in the final-good sector $l_{y,t}$ and energy e_t, i.e., we take it as Leontief. Productivities $A_{y,t}$ and $A_{e,t}$ are calibrated and thus exogenous. The Leontief assumption avoids unrealistically deep immediate cuts of emissions; see also Hassler, Krusell and Olovsson (2011).[31] Energy e_t also uses labor: the core allocation problem on which we add detail here is how to allocate a given total labor l_t at time t between final output $l_{y,t}$, fossil-fuel energy, $l_{f,t}$, and non-carbon energy, $l_{n,t}$. Thus, importantly, the energy and climate policy steers the labor allocation $(l_{y,t}, l_{f,t}, l_{n,t})_{t\geq 0}$ and therefore the quantities of fossil-fuel, $e_{f,t}$, and non-carbon energy, $e_{n,t}$, summing up to the total energy input:

$$e_t = e_{f,t} + e_{n,t}.$$

We assume that $e_{f,t}$ can be produced with constant-returns to scale technology using labor $l_{f,t}$ and the fossil-fuel z_t,

$$e_{f,t} = \min\{A_{f,t}l_{f,t}, B_t z_t\},$$

where $A_{f,t}$ and B_t describe productivities. The fuel resource is not a fixed factor and commands no resource rent; by this assumption, our focus is on the "coal phase", as coined by Golosov et al. (2011), where the fuel resource relevant for long-term climate policies is in principle unlimited. In contrast, the non-carbon production is land-intensive

[31] We have performed the quantitative analysis also for a Cobb-Douglas form for $A_t(l_{y,t}, e_t)$. These results are available on request. See also the next footnote.

and subject to diminishing returns and land rents:

$$e_{n,t} = \frac{\varphi + 1}{\varphi}(A_{n,t} l_{n,t})^{\frac{\varphi}{\varphi+1}},$$

where $\varphi > 0$ describes the elasticity of supply from this sector, given the labor cost. Recall that savings and climate policies, characterized by (g, h), can be found separately from the labor allocation. Then, we can solve $(l_{y,t}, l_{f,t}, l_{n,t}, w_t)$, where w_t is wage at time t, as the competetive equilibrium at each t, given output price $p_t = u'((1 - g)y_t)$ and the carbon price imposed on per unit of carbon released in production of $e_{f,t}$. Note that this two-step procedure for each t works because the functional forms for utility (log) and capital in production (power function), pre-determine the savings share of output (g) and climate policy (through h) as a constant carbon price per income; the resulting temporal prices for output and emissions allow then the calculation of the labor market outcome separately for each period.[32]

We provide the equilibrium conditions for the labor-market allocation in the Appendix.

Our calibration progresses as follows.[33] When there is no carbon policy, $h = 0$, the labor market allocation can be solved in closed form; thus, we can invert the model to map from quantities path $(l, y, e_f, e_n)_{t\geq 0}$ to productivities $(A_y, A_e, A_f, A_n)_{t\geq 0}$.[34] We match the business-as-usual (BAU) quantities $(y, e_f, e_n)_{t\geq 0}$ with the A1F1 SRES scenario from the IPCC (2000). Population follows a logistic growth curve with parameters given by World Bank forecasts. Population in 2010 is set at 6.9 [billion], while the maximum population growth rate is chosen such that in 2010 the effective population growth rate per decade equals 0.12 [/decade]. The maximum expected population (reached at about 2200) is set at 11 [billion]. For the BAU calibration, we also need the following values: the price of energy at 2010; the World Gross value-added 2010.[35] We calibrate to 25

[32]It is now clear that our climate policy is essentially a labor market policy. The Leontief between the final-good and energy labor in $A_t(l_{y,t}, e_t)$ prevents unrealistic short-term reallocations of labor following emissions penalties. Note that, in principle, the Leontief does not rule out growth in the long run: the economy can be scaled up without limits with labor, if $h = 0$. Thus, it is the land-intensive carbon-free production that prevents the scale-up in the presence of carbon policies.

[33]We calibrate the model using Excel. We have available one file that pulls together all parameters and the calibration procedure for the carbon cycle as well as for the economic model. The Excel file also contains model simulations. Additionally, we run the model in GAMS for more elaborate analysis. Both files are available on request.

[34]We express all energy in carbon units; to obtain this, we set $B_t = 1$ and employ three distinct energy productivities (A_e, A_f, A_n).

[35]These values, as well as $(l, y, e_f, e_n)_{t\geq 0}$ and $(A_y, A_e, A_f, A_n)_{t\geq 0}$, are in the Excel file (sheet "calibra-

per cent gross savings. Under consistent preferences with 2 per cent annual pure time discount, the capital share becomes $\alpha = g/\rho = .306$. For the Markov equilibrium, we take $\rho = .7724$ and $\theta = .9511$, corresponding to 2.55 and .5 per cent annual discounting, respectively. These choices preserve $g = .25$. From the BAU calibration we proceed to policy experiments: Markov, Pigouvian, and more advanced policies. We turn next to these results.

4.2 Climate-economy adjustment paths

We use now the full model for four scenarios. The simulations are run through the years 2010-3000; the model is initiated at the period labeled '2010', representing 2006-2015.

The first scenario is the business as usual (BAU), where we assume the complete absence of climate policy, $h = 0$, while we set g to its Markov equilibrium level. The second scenario presents the Markov equilibrium when the Pigouvian tax rule is implemented. The third scenario removes the Pigouvian carbon price rule. The fourth and final scenario implements an advanced policy strategy, where g and h are set at the level that maximizes the welfare of the first generation, assuming that all future generations adhere to the same policy, as discussed above. This scenario implies a joint policy of savings and climate change, but alternatively, we could refrain from a savings policy if we want to consider optimal climate policy, separately from capital savings decisions.

The first Figure shows the carbon prices for the three policy scenarios. The Pigouvian tax rule has the lowest carbon prices, from 11 EUR/tCO_2 in 2020, increasing with output to 43 EUR/tCO_2 in 2100.[36] The Markov equilibrium implements a considerably higher carbon price, rising from 38 to 105 EUR/tCO_2 during the same time period. Finally, the advanced policy further increases the carbon price to 44 EUR/tCO_2 in 2020, arriving at 106 EUR/tCO_2 in 2100. Between the scenarios, welfare is, of course, the lowest for all generations in the BAU scenario and increases as we move to the Pigouvian scenario, and then to the Markov equilibrium, and, finally, the highest welfare is reached by all generations in the Advanced Policy scenario.

Figure 1 here: Carbon prices over 2020-2100

The second Figure shows the emissions associated with these scenarios. The key takeaway from this figure is that Pigouvian taxes cannot prevent emissions from increasing

tion economics").

[36]Multiply the values by factor 4.7 to arrive at the USD/tC values.

substantially beyond current levels, for the full two coming centuries. In contrast, the carbon prices in the Markov equilibrium trigger a complete switch towards carbon-free energy sources before the end of 2100, while the Advanced Policy has the most drastic implications for energy supply and use. The increasing cascade of carbon prices translates into lower cumulative emissions over the period 2006-2105. They range from 6588 $GtCO_2$ in BAU, via 5276 $GtCO_2$ for the Pigouvian scenario, and 1768 $GtCO_2$ for the Markov scenario, to 1413 $GtCO_2$ for the Advanced Policy scenario. We note that while the Pigou scenario matches the Nordhaus DICE 2007 emissions well, the model calculates a change in the energy system that is unrealistically fast, given that the existing capital structure determines future energy options for a long time.[37]

Figure 2 here: CO2 emissions 2000-2200

The third Figure shows the temperature rise associated with these scenarios. Due to the extreme persistence of climate, we show the temperatures up to the year 3000. There is large uncertainty as to the climate sensitivity, but based on best guesses from sciences, taking no policy action can reasonably be expected to lead to temperature rises above 10 Kelvin, changing earth into an environment that hardly resembles anything humans have encountered in history. Applying the Pigouvian carbon price rule curbs the temperature rise to a maximum temperature rise of 3.6 K, reached around 2220. Even after thousand years, global temperatures are hardly below 3K. Such a temperature path could already lead to the collapse of the Greenland ice sheet.[38] The Markov equilibrium where emissions over the coming century are cut by almost 90% vis-a-vis the Business as Usual just ensures the temperature rise to remain below 2 K. Finally the Advanced Policy scenario keeps the temperature slightly lower, but even in this drastic scenario, for the coming thousand years, temperatures will remain above levels seen for the past 400,000 years.

$***$Figure 3 here: Temperature rise 2000-3000$***$

The fourth Figure shows the effect of the policies on consumption per capita. The Pigouvian carbon prices decrease consumption at about 0.1 percentage, relative to the Business as Usual for the coming century, and prevents a downturn at the second half of

[37] Because of the added complexity, only a few integrated assessment models describe a vintage-based capital structure (e.g., van der Zwaan et al. 2002).

[38] See Krieglera et al. (2009), and Alley et al. (2010) with the quote "The evidence suggests nearly total ice-sheet loss may result from warming of [..] perhaps as little as 2°C or more than 7°C."

the millennium when temperature rises would cause severe damage. The Markov policy decreases consumption at about one percentage in addition to the Pigouvian tax scenario, for the coming century. In return, consumption increases from 2150 onwards, by up to 3 per cent between 2200 and 2300. The advanced policy takes it one step further, but the difference with the Markov equilibrium is small. We emphasize that these results do not follow from an unortohodox model, but are consistent with the experiences from numerous integrated assessment models. The main consequences of climate policies of the 21st century will be felt beyond 2200, and in our setting the effect is relatively pronounced as not all integrated climate-economy assessment models feature the same persistence of climate change. However, our parameters are based on rigorous scientific insights.[39]

∗ ∗ ∗Figure 4 here: Consumption per capita 2000-3000 ∗ ∗∗

The fifth and last Figure makes a nice comparison with Nordhaus' (2007) 'wrinkle experiment'. In his criticism on Stern, Nordhaus (2007) proposes a thought experiment. He assesses a project that can yield a small but perpetual gain in a distant future. In his example, he considers an 0.1 percent point gain in income 200 years from now onwards, and asks how much the current generation is willing to give up to achieve such a future gain. If we discount future gains at 0.1 per cent per year, the net present value of the project amounts to 80 per cent of one year of income. Nordhaus asserts that we certainly do not want to give up so much for such a small future gain, even if perpetual. The thought experiment that our figures suggest is similar, but has a different tone. Comparing the Pigou scenario with the Markov scenario, the project we consider brings back the maximal global average warming from 3.3 K to below 2 K, and a large part of this gap (0.9K) is almost perpetual. The question we must ask is whether the current generation is willing to give up one percentage of its income to implement that project. Under our parameter choices, the answer is affirmative to this project decision. A subsequent project, comparing the Advanced Policy with the Markov scenario, would shave off another 0.1K from global warming, from 2200 onwards, perpetually. The costs for this challenging policy would be an additional 0.1 per cent of our income for the coming century.[40] Under our parameter choices, the answer is again affirmative to this far-reaching project as well. The parameters may err on both sides. We may have

[39]Moreover, it is not possible to calculate the same scenarios in other models, as they do not allow for Markov type of equilibria.

[40]To calculate the costs of the climate policy on its own merits, we constructed an auxiliary scenario where we fixed the savings policy to the BAU levels, and determined the optimal carbon tax policy h, that would be self-enforcing.

considered too low discount rates for the long-term structure of time preferences. On the other hand, we may underestimate the damages of climate change severely. The most important consequences of climate change might not be related to loss of income as measured through industrial activities and services, but more likely future generations will measure their loss in terms of large ecosystems such as rainforests and coral reefs that collapsed when they could not adapt quickly enough to the rapidly changing global environment. Though global warming receives most attention, ocean acidification is increasingly considered a complementary consequence of anthropogenic CO2 emissions, and a major threat to marine biodiversity (Orr et al. 2005). Our play with the future is with nature's richness; our human capacity for industrial production deserves less of our concerns.

$***$Figure 5 here: Changes in consumption between scenarios 2000-3000 $***$

To decide on optimal climate policy, we are in need of two sources of information. First, we must get an understanding of the possible consequences of climate change, and we cannot stop after an estimate of tangible economic damages. We must include consequences of climate change on large ecosystems if there are people that care for these, admitting the uncertainty. Second, given the assessment of possible consquences, we should assess the willingness of people to give up part of their income to prevent part of these consequences from happening. If as a global society we find the future environment more important than our cost-benefit analyses based on returns for capital can account for, it is inappropriate to follow the Pigouvian carbon price rule. Such a rule may improve productive efficiency, but if at the same time it reduces welfare as our analysis above has shown; it is of no use to adhere to it.

5 Discussion

Our analysis closely links to the discussion on the discount rate, and specifically the editorial comment of Nordhaus (1997), where he discusses a benchmark climate change policy, comparable to our Pigouvian tax rule, and various alternatives to give more weight to our concerns for climate change.The first alternative applies a lower discount rate uniformly to both capital investments and climate change policy. The second alternative uses 'differential discounting', a market discount rate for capital investments, while a lower discount rate is used for climate change policy; the approach is based on Hasselmann et al. (1997). The third approach sets a long-term climate target, based on the variable that we believe matters most for ultimate climate chnage damages, such as limiting the global temperature rise for the year 2500. The fourth approach sets intermediate climate change targets, such as for emissions or atmospheric CO_2 concentrations. In his assessment of the Pigouvian tax rule and the alternative policies, Nordhaus finds that the Pigouvian tax rule outperforms all alternatives. That is, all alternatives result in strict welfare losses compared to the Pigouvian tax rule. The long-term climate target has the lowest welfare losses given the gain of climate stabilization, using intermediate climate variables as targets comes next, differential discounting comes third, and using lower discount rates for both capital and climate change policy is the most costly policy.

Our assessment of the above five policies is markedly different. Nordhaus' assessment requires two main assumptions. First, that social preferences are time-conistent and second that social preferences are consistent with observed investments in capital. In our analysis, we only relaxed the first assumption of time-consistent preferences. Proposition 8 establishes that when intergenerational preferences are not time-consistent, then maximal welfare for the first generation implies commitment to a policy with lower discount rates for both capital investments and climate change policy. The level of effective discounting need not be the same for both policies, making clear that differential discounting can be optimal from the current generations' point of view. The Pigouvian tax policy, where capital investments follow the Markov equilibrium path, performs strictly worse compared to a policy based on lower discounting or differential discounting. The climate target policies, meanwhile, suffer from another type of problem. They are infeasible without support of an intertemporal global commitment device. A major problem with policies based on climate targets is that they defer the costs of meeting these targets to the future. But when the costs cannot be delayed any longer, because the target becomes infeasible otherwise, the generation that has to decide at that time will carry out its own cost-benefit analysis, and while the costs in net present value terms has gone up considerable, the net present gains of a climate target per se will not have changed. Thus, the future generation will relax the targets, the standard problem of lack of commitment known from the literature. As the current generation understands that future generations will not keep up the targets set by the present, they prefer to contribute themselves to reaching the target.

We do not take any position on the desirability of more or less stringent climate policies. The social preferences may be such that preventing large and possibly semi-permanent shifts in our global climate is considered insufficiently important to warrant large investments in carbon-free energy sources, or more generally, a loss of consumption of say 1 per cent of income. But our analysis shows unambiguously that returns on capital cannot serve as a proxy for long-term intertemporal social preferences. There is no market for intergenerational welfare transfers, and therefore, market outcomes cannot reveal our preferences. This insight works in two directions. First, missing markets imply a potential source of inefficiency. Imposing long-term cost-benefit analysis and Pigouvian carbon prices based on short-term market variables can further deteriorate the market inefficiency. Second, as the market cannot reveal social preferences, and as these preferences are important to decide on the direction that our economy is to take, we have to think seriously about other means to assess these preferences. There is need for more climate literacy, both on the side of consequences of climate change, as on the side of consequences of climate policies.

Appendix

Lemma 3

The proof by is by induction. Assume that (38) and (39) hold for all future periods $t+1, t+2, ...$, and that the lemma holds for $t+2$. We can thus construct the value function for the next period, as

$$W_{t+1}(k_{t+1}, s_{t+1}) = u_{t+1} + \theta W_{t+2}(k_{t+2}, s_{t+2}).$$

Substitution of the investment decision at time $t+1$, $k_{t+2} = gy_{t+1}$ and emissions $z_{t+1} = z^*_{t+1}$, gives

$$W_{t+1}(k_{t+1}, s_{t+1}) = [\alpha \ln(k_{t+1}) + ... + \ln(\omega(s_{t+1}))] - \Delta_u D_{t+1} + \theta\xi[\alpha \ln(k_{t+1}) + ... + \ln(\omega(s_{t+1}))] + \theta\Omega(s_{t+2})$$

where "..." refer to constants, such as $\ln(1-g)$ that do not depend on the state variables k_{t+1} and s_{t+1}. Collecting the coefficients in front of $\ln(k_{t+1})$ yields the part of $V_{t+1}(k_{t+1})$ depending k_{t+1} with the recursive determination of ξ,

$$\xi = \alpha(1 + \theta\xi).$$

so that $\xi = \frac{\alpha}{1-\alpha\theta}$ follows.

Collecting the terms with s_{t+1} yields $\Omega(s_{t+1})$ through

$$\Omega(s_{t+1}) = \ln(\omega(s_{t+1}))(1 + \theta\xi) - \Delta_u D_{t+1} + \theta\Omega(s_{t+2}).$$

where $z_{t+1} = z^*_{t+1}$ appearing in $s_{t+2} = (z_1, ...z_t, z_{t+1})$ is independent of k_{t+1} and s_{t+1} (by Lemma 5 that holds by the induction hypothesis) so that we only need to consider the values for $z_1, ..., z_t$ when evaluating $\Omega(s_{t+1})$. The values for ζ_τ can be calculated by collecting the terms in which $z_{t+1-\tau}$ appear. Recall that $\ln(\omega(s_{t+1})) = -\Delta_y D_{t+1}$ so that

$$\zeta_\tau = ((1+\theta\xi)\Delta_y + \Delta_u) \sum_{(i,j)} a_i b_j \pi \varepsilon_j \frac{(1-\delta_i)^\tau - (1-\varepsilon_j)^\tau}{\varepsilon_j - \delta_i} + \theta\zeta_{\tau+1}$$

Substitution of the recursive formula, for all subsequent τ, gives

$$\zeta_\tau = (\frac{\Delta_y}{1-\alpha\theta} + \Delta_u) \sum_{(i,j)} \sum_{t=\tau}^{\infty} a_i b_j \pi \varepsilon_j \theta^{t-\tau} \frac{(1-\delta_i)^t - (1-\varepsilon_j)^t}{\varepsilon_j - \delta_i}$$

To derive the value of ζ_1, we consider

$$\begin{aligned} & \sum_{t=1}^{\infty} \theta^{t-1} \frac{(1-\delta_i)^t - (1-\varepsilon_j)^t}{\varepsilon_j - \delta_i} \\ = \; & \frac{\sum_{t=1}^{\infty}[\theta(1-\delta_i)]^t - \sum_{t=1}^{\infty}[\theta(1-\varepsilon_j)]^t}{\theta(\varepsilon_j - \delta_i)} \\ = \; & \frac{\frac{\theta(1-\delta_i)}{1-\theta(1-\delta_i)} - \frac{\theta(1-\varepsilon_j)}{1-\theta(1-\varepsilon_j)}}{\theta(\varepsilon_j - \delta_i)} \\ = \; & \frac{1}{[1-\theta(1-\delta_i)][1-\theta(1-\varepsilon_j)]} \end{aligned}$$

Q.E.D.

Lemma 5

The first-order conditions for fossil-fuel use z_t, and the labor allocations over the final goods $l_{y,t}$ and the energy sectors $l_{e,t}$ give:

$$u'_t \frac{\partial y_t}{\partial z_t} = \rho \frac{\partial \Omega_{t+1}}{\partial s_{t+1}} \frac{\partial s_{t+1}}{\partial z_t} \Rightarrow \frac{1}{1-g} \frac{1}{A_t} \frac{\partial A_t}{\partial e_t} \frac{\partial E_t}{\partial z_t} = \rho\zeta_1 \tag{46}$$

$$\frac{\partial A_t}{\partial l_{y,t}} = \frac{\partial A_t}{\partial e_t} \frac{\partial E_t}{\partial l_{e,t}} \tag{47}$$

The second part of the Lemma follows immediately from (46):

$$\frac{\partial y_t}{\partial z_t} = f_{t,z} = \rho\zeta_1(1-g)y_t.$$

The second equation equals the productivity of labor in the final good sector with the indirect productivity through energy production. We have thus four equations, energy production (31), labour market clearance (32), and the above two first-order conditions, that jointly determine four variables: $z_t, l_{y,t}, l_{e,t}, e_t$, only dependent on technology at time t through $A_t(l_{y,t}, e_t)$ and $E_t(z_t, l_{e,t})$. Thus, $z_t = z^*_t$ can be determined independent of the state variables k_t and s_t.

Lemma 6

The result can be proved by repeating the steps in the proof of Lemma 3 with $\rho = \theta = \gamma$, to obtain the marginal damage cost of emissions; this identifies the Pigouvian tax. We omit this repetition but, instead, find the Pigouvian pricing from Lemma 5 just by imposing $\rho = \theta = \gamma$. Note that Lemma 6 reports the tax for any given g and γ. Because the savings and climate policies are separable (Lemma 3), the expression for the Pigouvian tax holds whether g and γ correspond to the equilibrium values or not.

Lemma 7

This proof follows Gerlagh and Liski (2011). When the allocation maximizes the objective

$$W(\cdot) = \sum\nolimits_{t=1}^{\infty} \alpha_t w_t$$

with non-negative welfare weights α_t having a bounded mass $\sum_{t=1}^{\infty} \alpha_t < \infty$, then the allocation is Pareto efficient and the welfare aggregator $W(\cdot)$ is the objective of a representative Planner.

The lemma considers allocations represented through geometric utility weights $\alpha'_t = \gamma^{t-1}$. If $\gamma < \rho$ or $\gamma < \theta$ one cannot construct a sequence of non-negative welfare weights α_t consistent with a Planner maximizing under the sequence of utility weights α'_t. Suppose the contrary, that welfare weights $\alpha_t \geq 0$ consistent with α'_t exist. Then, using the definition of welfare, we see that for some $\tau > t$, the relationship between the two is $\alpha'_1 = \alpha_1$ and $\alpha'_\tau = \sum_{t=1}^{\tau} \alpha_t \rho \theta^{\tau-t-1}$ for $\tau > 1$. Expanding the latter gives

$$\alpha'_\tau = \alpha_1 \rho \theta^{\tau-2} + \alpha_2 \rho \theta^{\tau-3} + ... + \alpha_{\tau-1}\rho + \alpha_\tau. \tag{48}$$

If $\gamma < \theta$ and $\alpha_1 > 0$, we see that the equation cannot hold with $\alpha_\tau \geq 0$ for sufficiently large τ: $\alpha'_\tau - \alpha_1 \rho \theta^{\tau-2} < 0$ for some finite $\tau > 0$.

If $\gamma < \rho$, include only the last two terms on the right in (48) to obtain

$$\alpha'_{\tau+1} \geq \rho \alpha_\tau + \alpha_{\tau+1}.$$

Substitute $\alpha'_{\tau+1} = \gamma \alpha'_\tau$,

$$\gamma \alpha'_\tau - \rho \alpha_\tau \geq \alpha_{\tau+1}.$$

Since $\gamma < \rho$, this cannot hold with $\alpha_{\tau+1} \geq 0$ for any τ.

Consider now $\gamma \geq \max\{\rho, \theta\}$. We show that now one can construct the non-negative welfare weights. We construct an algorithm for finding the weights. Let $\widetilde{\alpha}_1 = \{\alpha^1_\tau\}_{\tau \geq 1}$,

$\widetilde{\alpha}_2 = \{\alpha^2_\tau\}_{\tau\geq 2}$, and so on. Define

$$\begin{aligned}\alpha^1_\tau &= \gamma^{\tau-1}, \tau \geq 1\\ \alpha^2_\tau &= \alpha^1_\tau - \alpha^1_1\theta^{\tau-t-1}, \tau \geq 2\\ &\ldots\\ \alpha^{t+1}_\tau &= \alpha^t_\tau - \alpha^t_t\theta^{\tau-t-1}, \tau \geq t.\end{aligned}$$

The value of α^t_τ measures the weight remaining for generation τ after all altruistic weights of generations 1 to $t-1$ have been subtracted. Note that the equilibrium implies utility weights $\widetilde{\alpha}_1$, and $\{\alpha^t_t\}_{t\geq 1}$ is the sequence of welfare weights consistent with $\widetilde{\alpha}_1$. The main intermediate result that we need, in order to prove that the sequence of welfare weights $\{\alpha^t_t\}_{t\geq 1}$ is non-negative, is that for all $\tau \geq t$:

$$\frac{\alpha^t_{\tau+1}}{\alpha^t_\tau} > \max\{\rho, \theta\}. \tag{49}$$

By construction, this condition is satisfied for $t = 1$. It implies that next sequence $\widetilde{\alpha}_2$, induced by the algorithm, is non-negative, as

$$\alpha^2_\tau = \gamma^{\tau-1} - \theta^{\tau-t-1} > \alpha^1_\tau\{(\max\{\rho,\theta\})^{\tau-1} - \rho\theta^{\tau-t-1}\} > 0, \tau \geq 2.$$

By induction, if the condition holds for $\widetilde{\alpha}_t$, the sequence $\widetilde{\alpha}_{t+1}$ is non-negative:

$$\alpha^{t+1}_\tau > \alpha^t_\tau\{(\max\{\rho,\theta\})^{\tau-t} - \rho\theta^{\tau-t-1}\} > 0, \tau \geq t.$$

Thus, we are done if we can show that condition (49) holds. Notice that

$$\alpha^{t+1}_{\tau+1} = \alpha^t_{\tau+1} - \alpha^t_t\rho\theta^{\tau-t} > \max\{\rho,\theta\}\alpha^t_\tau - \alpha^t_t\rho\theta^{\tau-t} \geq \theta\{\alpha^t_\tau - \alpha^t_t\rho\theta^{\tau-t-1}\} = \theta\{\alpha^{t+1}_\tau\}.$$

If $\theta > \rho$, this proves that $\alpha^{t+1}_{\tau+1} > \theta\{\alpha^{t+1}_\tau\} > \rho\{\alpha^{t+1}_\tau\}$. On the other hand, if $\theta < \rho$, we have

$$\alpha^{t+1}_{\tau+1} = \alpha^t_{\tau+1} - \alpha^t_t\rho\theta^{\tau-t} > \max\{\rho,\theta\}\alpha^t_\tau - \alpha^t_t\rho\theta^{\tau-t} \geq \rho\{\alpha^t_\tau - \alpha^t_t\rho\theta^{\tau-t-1}\} = \rho\{\alpha^{t+1}_\tau\},$$

which completes the proof.

Lemma 8

Consider a policy path $(g_t, z_t)_t$ given. We then look at variations in policies at time τ. We can apply the same analysis as for the proof of Lemma 3 and conclude that the lemma still holds. The only modification of the lemma is that we need to understand that the parameter $\widetilde{A}_t$ depends on the future policies. From the proof of Lemma 3, we also recall that $u_\tau + \theta W_{\tau+1}(k_{\tau+1}, s_{\tau+1})$ is concave in both $k_{\tau+1}$ and $z_{\tau+1}$, which implies that $u_\tau + \theta W_{\tau+1}(k_{\tau+1}, s_{\tau+1})$ is increasing in g_τ iff $g_\tau < \alpha\theta$, and increasing in z_τ iff $z_\tau < z^\theta_\tau$.

Proposition 4

We want to know the sign of $f_{t,z} - \tau_t^{Pigou}$. For this we divide the difference by $\Delta\beta\varepsilon$, and consider

$$\frac{1}{\varepsilon-\beta}[\frac{\rho(1-\beta)}{1-\theta(1-\beta)} - \frac{\rho(1-\varepsilon)}{1-\theta(1-\varepsilon)} - \frac{\gamma(1-\beta)}{1-\gamma(1-\beta)} + \frac{\gamma(1-\varepsilon)}{1-\gamma(1-\varepsilon)}].$$

For $\beta = e = 0$, the second term degenerates as it becomes zero. For $\beta \neq \varepsilon$ very small, we can then apply l'Hopital's rule, and approximate the term by looking at the first derivatives for β and ε evaluated at $\beta = \varepsilon = 0$:

$$\begin{aligned} & \frac{1}{\varepsilon-\beta}[\frac{\rho(\varepsilon-\beta)}{(1-\theta)^2} - \frac{\gamma(\varepsilon-\beta)}{(1-\gamma)^2}] \\ = \; & \frac{\rho}{(1-\theta)^2} - \frac{\gamma}{(1-\gamma)^2} \end{aligned}$$

This term is easily shown to be strictly negative. We substitute $\gamma = \frac{\rho}{1-\alpha(\theta-\rho)}$ (and $1-\gamma = \frac{1-\alpha\theta-(1-\alpha)\rho}{1-\alpha(\theta-\rho)}$) into this equation, divide by ρ, and get

$$\frac{1}{(1-\theta)^2} - \frac{(1-\alpha(\theta-\rho))}{(1-\alpha\theta-(1-\alpha)\rho)^2}$$

As the numerator of the second term is smaller then 1 iff $\theta > \rho$, and $1-\alpha\theta-(1-\alpha)\rho > 1-\theta$ iff $\theta > \rho$, so that the denominator is larger for the second term, it follows immediately that the sign is positive iff $\theta > \rho$. Using the same procedure for β and ε close to one, we find from l'Hopital's rule

$$\frac{1}{\varepsilon-\beta}(\varepsilon-\beta)(\rho-\gamma) = \rho-\gamma$$

and we immediately see that the gap is positive iff $\rho > \gamma$, that is, iff $\rho > \theta$.

Equilibrium labor market allocation in Section 4

This Appendix details the labor market allocation for the functional forms introduced in Section 4; this allocation is then numerically solved to obtain the overall climate-economy adjustment path in Section 4.2. As explained in the text, the allocation can be solved period-by-period taking the (i) productivity parameters, (ii) total labor, (iii) savings g, and (iv) carbon policies $h = \rho\zeta_1$ as given. Items (i)-(ii) change over time, implying reallocations of labor. But these reallocations satisfy equations (50)-(53) below. We drop the time subscript in the variables.

1. Consumers choose the fraction g of output y that is available for consumption. We normalize prices for the final good to equalize marginal utility, so that factor prices can be interpreted as marginal welfare per factor endowment:
$$p = \frac{1}{c} = \frac{1}{(1-g)y}.$$

2. Final-good producers of y take capital k, wages w, and the prices of energy q and output p as given. Since $y = k^\alpha[\min\{A_y l_y, A_e e\}]^{1-\alpha}\omega(s)$, factor compensation for labour and energy together receives a share $(1-\alpha)$ of the value of output py:
$$wl_y + qe = (1-\alpha)py$$
where $e = e_f + e_n$.

3. Fossil-fuel energy production combines labor and fuels, with technology $e_{f,t} = \min\{A_{f,t} l_{f,t}, B_t z_t\}$. Fossil fuel use and labour employed, $z, l_f \geq 0$, are strictly positive if q covers the factor payments, including the carbon price $\rho\zeta_1$
$$\left[q - \left(\frac{w}{A_f} + \frac{\rho\zeta_1}{B}\right)\right] \times l_f \leq 0.$$
The zero profit condition for fossil fuel energy allocates the value of fossil fuel energy to labour and emission permits; using the production identity we can express it in terms of labour employed.
$$qe_f = wl_f + \rho\zeta_1 z = (w + \frac{\rho\zeta_1 A_f}{B})l_f$$

4. Carbon-free energy inverse supply is given by the first-order condition
$$q = w\frac{\partial l_n}{\partial e_n} = \frac{w_t}{(A_n)^{\frac{\varphi}{\varphi+1}}}(l_n)^{\frac{1}{\varphi+1}}.$$
The value share of labour employed in the carbon-free energy sector equals $\varphi/(1+\varphi)$, so that the rent value is expressed in labour employed:
$$qe_n = (1 + \frac{1}{\varphi})wl_n$$

We obtain four equations in four unknowns l_y, l_f, l_n, w:

$$A_y l_y = A_e(A_f l_f + \frac{\varphi+1}{\varphi}(A_n l_n)^{\frac{\varphi}{\varphi+1}}) \quad (50)$$
$$wl + \frac{\rho\zeta_1 A_f}{B} l_f + \frac{1}{\varphi} wl_n = \frac{1-\alpha}{1-g} \quad (51)$$
$$\frac{w}{A_f} + \frac{\rho\zeta_1}{B} \geq \frac{w}{(A_n)^{\frac{\varphi}{\varphi+1}}}(l_n)^{\frac{1}{\varphi+1}} \perp l_f \geq 0 \quad (52)$$
$$l_y + l_f + l_n = l \quad (53)$$

Equation (50) follows since, for strictly positive input prices, $A_t(\cdot) = \min\{A_y l_y, A_e e\} \Rightarrow A_y l_y = A_e e$. Equation (51) allocates the value of output that is not attributed to capital (the right-hand side) to the labour, carbon emissions, and land rent for the non-carbon energy (where we latter two terms are expressed in labour units). Equation (52) compares the production costs for fossil fuel energy with non-carbon energy, and the last equation is the labor market clearing equation. Note that the solution depends on the state of the economy only through total labor l and productivities A_y, A_e, A_f, A_n.

In the absence of a carbon policy, $\zeta_1 = 0$, we can solve the allocation in closed-form:

$$l_{n,t} = \frac{A_{n,t}^{\varphi}}{A_{f,t}^{\varphi+1}} \tag{54}$$

$$w_t = \frac{1-\alpha}{1-g}\frac{\varphi}{\varphi l_t + l_{n,t}} \tag{55}$$

$$l_{y,t} = \frac{A_{e,t}}{A_{y,t} + A_{e,t}A_{f,t}}[A_{f,t}(l_t - l_{n,t}) + \frac{\varphi+1}{\varphi}(A_{n,t}l_{n,t})^{\frac{\varphi}{\varphi+1}}] \tag{56}$$

$$l_{f,t} = l_t - l_{y,t} - l_{n,t} \tag{57}$$

Here we include the time subsripts to emphasize the drivers of the solution. This business-as-usual allocation is used to calibrate the productivities as explained in the text.

Calibration

In this section, we pull together all carbon cycle and economic parameters used in the quantitative assessment. We provide also, on request, two independent program files (GAMS and Excel) that contain the parameters and can be used for policy experiments.

Calibration: Climate parameters

We take data from Houghton (2003) and Boden et al. (2011) for carbon emissions in 1751–2008. We choose a 3-box representation of the global biogeochemical cycles; the first box represents the atmosphere plus the upper ocean layer, the second box represents the biomass, while the third box represents the deep ocean. The boxes contain physical carbon stocks measured in $[TtCO_2]$. We calibrate the model to minimize the error between the atmospheric concentration prediction from the 3-box model and the Mauna Loa observations, while maintaining CO2 stocks in the various boxes and flows between the boxes consistent with scientific evidence as reported in Fig 7.3 from the IPCC fourth assessment report from Working Group I (Solomon et. al. 2007). There are 4 parameters we calibrate: (1) the CO2 absorption capacity of the "atmosphere plus upper ocean", (2) the CO2 absorption capacity of the biomass box relative to the atmosphere, while we fix the relative size of the deep ocean box at 4 times the atmophere, based on the IPCC special report on CCS, Fig 6.3 (Caldeira and Akai, 2005). We furthermore calibrate (3) the speed of CO2 exchange between the atmosphere and biomass and (4) between the atmosphere and the deep ocean. Subsequently, we transform this annual 3-box model into a decadal model adjusting the exchange rates within a period between the boxes and the shares of emissions that enter the boxes within the period of emissions. For the mathematical representation of the carbon cycle used in the climate-economy model, we linearly transform the physical decadal model into a fully equivalent model that is mathematically more convenient, but where the boxes lose their precise physical meaning: box 0 measures the amount of atmospheric carbon that never depreciates; box 1 contains the atmospheric carbon with a depreciation of about 9 per cent in a decade; while carbon in box 2 depreciates half per decade.[41] About 20 per cent of emissions enter either the upper ocean layer, biomass, or the deep ocean within the period of emissions. In the reduced-form model, they do not enter the atmospheric carbon stock, so that the shares a_i sum to 0.8. Our procedure provides an explicit mapping between the physical carbon cycle and the reduced-form model for atmospheric carbon with varying depreciation rates; the Excel file supporting the manuscript contains these steps and allows easy experimentation with the model parameters. The resulting boxes, their emission shares, and depreciation factors are:

$$\begin{aligned} S_{t=2005} &= (.309, .265, .236) \\ a &= (.166, .205, .429) \\ \delta &= (0, .088, .493). \end{aligned}$$

These coefficients are used to calculate the stock dynamics as given by (35) when the path of future emissions is given by the economy model. To transform stocks into damages as given by (36), we assume a 1-box damage model and choose the parameters as follows:

$$b = 1, \varepsilon = .183, \pi = 4.1$$

We thus have one box ($b = 1$). We inteprete D_t in (36) as the Global Mean Temperature (GMT) squared ($GMT_t = \sqrt{D_t}$); $\varepsilon = .183$ in the decadal model implies a temperature

[41] From a technical perspective, the decay rates in the final reduced-form model describe the eigenvalues of the original model.

adjustment speed of 2 per cent per year. Choice $\pi = 4.1$ $[K^2/GtCO_2]$ implies a climate sensitivity of 3 $Kelvin$ per $2.2TtCO_2$. These choices are within the ranges of scientific evidence (Solomon et al. 2007).

We seek to calibrate the damage parameters to match the case presented in Nordhaus (2007) as a benchmark. Assuming damages equivalent to 2.7 per cent of output at a temperature rise of 3 Kelvin, as in Nordhaus (2001), we obtain $\Delta_y = 0.003$; we set $\Delta_u = 0$, unless otherwise stated.

Calibration: Economic parameters

In Section 4 (see also Appendix on the labor allocation solution) we explained the business-as-usual calibration, that is, the mapping from quantities path $(l, y, e_f, e_n)_{t\geq 0}$ to productivities $(A_y, A_e, A_f, A_n)_{t\geq 0}$. Thus, we need to make choices that drive the quantities $(l, y, e_f, e_n)_{t\geq 0}$. Population is assumed to follow a logistic growth curve:

$$l_{t+1} = [1 + \gamma_L(1 - \frac{l_t}{l_{\max}})]l_t$$

with parameters given by the World Bank forecasts. Population in 2010 (L) is set at 6.9 [billion], while the maximum population growth rate γ_L is chosen such that in 2010 the effective population growth rate per decade equals 0.12 [/decade]. The maximum expected population (reached at about 2200) is set at 11 [billion].

Consider then the determinants of initial output $y_{t=2010}$. We take Gross Global Product as 600 Trillion Euro [$Teuro$] for the first decade, 2006-2015 (World Bank, using PPP). Fossil-fuel energy input e_f, measured in CO_2, is .318 [$TtCO_2$] for 2006-2015 (SRES IPCC 2000); we set e_n at 10 per cent of e_f. These are the raw quantities entering production function at $t = 2010$; now, we use the model structure to calibrate the remaining variables at $t = 2010$. There is only one energy sector parameter to be set: the elasticity of carbon-free supply; see Section 4. We set this parameter to $\varphi = 2$.

We calibrate the preference parameters to yield 25 per cent savings ($g = .25$), which together with $u(c) = \ln(c)$ and $y_{t=2010} = 600$, gives the price of output as $p = 1/((1 - g)600)$. The relative price of energy is taken to be $q/p = 50$ [$euro/tCO_2$]. In Appendix for the labor allocation, we derived the labor allocation as a function of productivities. We have now information on the endogenous variables: the energy outputs (e_n,e_f), total output y and the energy price q/p, so we can solve both the labor allocation (l_y, l_f, l_n) and the productivities (A_y, A_e, A_f, A_n) at $t = 2010$.

To progress to the next decade $t = 2020$, we take capital k given by savings, keep the energy price $q/p = 50$ [$euro/tCO_2$], and match y and (e_n,e_f) to the A1F1 SRES scenario from the IPCC (2000). This way the calibration procedure for productivities can be repeated for all future decades.

Finally, capital elasticity α follows from the assumed time-preference structure ρ and θ, and observed historic gross savings g.. As a base-case, we consider net savings of 25% ($g = .25$), and a 2 per cent annual pure rate of time preference ($\rho = \theta = 0.817$), resulting in $\alpha = g/\rho = 0.306$. For the Markov equilibrium, we take $\rho = .7724$ and $\theta = .9511$. These choices preserve $g = .25$.

Sea level rise in damages

Climate change does not stop at temperature changes. After temperature rises, the sea level will rise as well, and it may do so more or less proportionally to temperatures (Jevrejeva et al. 2011). When damages are proportional to output and quadratic in the level of sea-level rise, we can use SLR_t for the damages associated with sea level rise, and write for the dynamics of damages

$$SLR_t = SLR_{t-1} + \varepsilon_{SLR}(\pi_{SLR}D_t - SLR_t)$$

It is a tedious but straightforward matter to derive the resulting dynamics as

$$SLR_t = \sum_i \sum_j \sum_\tau a_i b_j \pi \pi_{SLR} \varepsilon_j \varepsilon_{SLR} \times \frac{(\varepsilon_j - \varepsilon_{SLR})(1-\delta_i)^{\tau+1} + (\varepsilon_{SLR} - \delta_i)(1-\varepsilon_j)^{\tau+1} + (\delta_i - \varepsilon_j)(1-\varepsilon_{SLR})^{\tau+1}}{(\varepsilon_j - \delta_i)(\varepsilon_j - \varepsilon_{SLR})(\varepsilon_{SLR} - \delta_i)} z_{t-\tau}$$

where we forego the terms associated with the initial conditions. Let Δ_{SLR} be the costs relative to output of 1m sea level rise. After some tedious substitutions, the Markov equilibrium carbon price is then calculated as

$$\rho\zeta_{SLR} = \sum_i \sum_j a_i b_j \pi \pi_{SLR} \varepsilon_j \varepsilon_{SLR} \rho \Delta_{SLR} \times \frac{(\varepsilon_j^2 - \delta_i^2)\varepsilon_{SLR} + (\varepsilon_{SLR}^2 - \varepsilon_j^2)\delta_i + (\delta_i^2 - \varepsilon_{SLR}^2)\varepsilon_j}{(\varepsilon_j - \delta_i)(\varepsilon_j - \varepsilon_{SLR})(\varepsilon_{SLR} - \delta_i)(1-\theta[1-\varepsilon_{SLR}])(1-\theta[1-\varepsilon_j])(1-\theta[1-\delta_i])}$$

For sea level rise, both the senstivity π_{SLR} and the speed of adjustment ε_{SLR} are both very uncertain, but the estimates for both parameters are strongly and negativel correlated: a higher sensitivity must be matched with a lower adjustment speed, to match the historically observed records, $\varepsilon_{SLR}\pi_{SLR} \in [0.02, 0.05]$ (per decade). Estimates for the sensitivity range from 0.2 to 5 meter sea level rise per W/m2 forcing increase (Jevrejeva 2011), where typically 1 W/m2 leads to a temperature rise just below 1 Kelvin, so that our parameters range would be $\pi_{SLR} \in [0.2, 4]$. A choice of $\pi_{SLR} = 1$ [m/K] and $\varepsilon_{SLR} = 0.04$ would represent a reasonable assumption. Yet, the resulting carbon prices will not deviate too much from the carbon prices presented in the main text. The literature does not provide estimates for damages associated with sea level rise that substantially exceed those for temperature rise, that is $\Delta_{SLR}\pi_{SLR} \ll \Delta_y$, so that, given the extended lag in sea level rise, the increase in the level of carbon prices associated will be small in relative terms.

Comparison of climate response functions

We compare our response function for damages, as percentage of output, resulting from emissions, with those in Nordhaus (2007) and Golosov et al. (2011). The GAMS source code for the DICE model provides a large variety of scenarios with different policies such as temperature stabilization, concentration stabilization, emission stabilization, the Kyoto protocol, a cost-benefit optimal scenario, and delay scenarios. For each of these scenarios we calculated the damage response function by simulating an alternative scenario with equal emissions, apart from a the first period when we decreased emissions by 1GtC. Comparison of the damages, in terms relative of output, then defines the response function for that specific scenario. It turns out that the response functions are very close, and we took the average over all scenarios. To interpret the response function in Nordhaus (2007), we notice that the average DICE carbon cycle and damage response can very accurately be described by our reduced form using the parameters $a = (0.575, 0.395, 0.029)$, $\delta = (0.310, 0.034, 0)$, which give a perfect fit for the carbon cycle of DICE2007, and $\varepsilon = 0.183$, $\pi = 4.09$ for the temperature delay. That is, the carbon-cycle in DICE (Nordhaus 2007) is characterized by a very large long-term uptake of CO2 in the oceans. The reduced model in Golosov et al. is represented by $a = (0.2, 0.486, 0.314)$, $\delta = (0, 0.206, 1)$, which implies a similar carbon cycle model to ours, but Golosov et al. have no temperature delay structure, $\varepsilon = 1$. The figure below presents the emissions damage response, $\frac{dD_{t+\tau}}{dz_t}$.

$* * *$Figure 6 here: Emissions-Damage responses $* **$

References

[1] Alley, Andrews, Brigham-Grette, et al. (2010), "History of the Greenland Ice Sheet: paleoclimatic insights", in Quaternary Science Reviews 29: 1728-1756.

[2] Barro R.J. (1999), Ramsey meets Laibson in the neoclassical growth model, Quarterly Journal of Economics 114: 1125-1152.

[3] Bernheim, D.B., and A. Rangel (2009), Beyond Revealed Preference: Choice-Theoretic Foundations for Behavioral Welfare Economics, The Quarterly Journal of Economics, MIT Press, vol. 124(1), pages 51-104.

[4] Boden, T.A., G. Marland, and R.J. Andres. 2011. Global, Regional, and National Fossil-Fuel CO2 Emissions. Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, U.S. Department of Energy, Oak Ridge, Tenn., U.S.A.

[5] Caldeira K. and M. Akai (eds) 2005, Ocean storage, Ch 6 in IPCC special report on carbon dioxide capture and storage, edited by Metz B., O. Davidson, H. de Coninck, M. Loos, and L. Meyer, Cambridge University Press.

[6] Caplin A, and Leahy J (2004), The social discount rate, Journal of Political Economy 112: 1257-1268.

[7] Dasgupta, P., Discounting Climate Change, Journal of Risk and Uncertainty, 2008, 37(2-3), 141-169.

[8] Fischer C. and R. Newell, 2008, Environmetnal and technology policies for climate mitigation, J of Environmental and Economic Management 55:142-162.

[9] Gerlagh, R., and M. Liski, Public investment as commitment, CESifo Working Paper Series No. 3330, available at hse-econ.fi/liski/papers/pubinv.pdf

[10] Gollier, C., and R. Zeckhauser (2005), Aggregation of Heterogeneous Time Preferences, Journal of Political Economy, 2005, vol. 113, issue 4, ppp. 878-896.

[11] Golosov, M., J. Hassler, P. Krusell, A. Tsyvinski (2011), Optimal taxes on fossiel fuel in general equilibrium, NBER WP 17348.

[12] Hasselmann, K., S. Hasselmann, R Giering, V Ocana, H v Storch (1997), Sensitivity of optimal CO_2 emissions paths using a simplified structural integrated assessment model (SIAM), Climatic Change 37: 345-387

[13] Hooss G, R. Voss, K Hasselmann, E Maier-Reimer, F Joos (2001), A nonlinear impulse response model of the coupled carbon cycle-climate system (NICCS), Climate Dynamics 18: 189-202

[14] Houghton, R.A. 2003.Revised estimates of the annual net flux of carbon to the atmosphere from changes in land use and land management 1850-2000. Tellus55B(2):378-390.

[15] IPCC, Intergovernmental Panel on Climate Change (2000), Special Report on Emissions Scenarios, edited by N. Nakicenovic and R. Swart, Cambridge Univ. Press, Cambridge, U.K.

[16] Jackson, O. M., and L. Yariv, Collective Dynamic Choice: The Necessity of Time Inconsistency, WP-2011

[17] Jevrejeva, S., J.C. Moore, A. Grinsted (2011), Sea level projections to AD2500 with a new generation of climate change scenarios, Global and Planetary Change 80-81:14-20.

[18] Karp L. (2005), Global warming and hyperbolic discounting, Journal of Public Economics 89: 261-282.

[19] Karp L. (2007), Non-constant discounting in continuous time, Journal of Economic Theory 132: 557-568.

[20] Krieglera, Hall, Helda, Dawson, and Schellnhuber (2009), "Imprecise probability assessment of tipping points in the climate system", Proceedings of the National Academy of Sciences, vol 106: 5041-5046.

[21] Krusell P., Kuruscu B, Smith A.A. (2002), Equilibrium welfare and government policy with quasi-geometric discounting, Journal of Economic Theory 105: 42-72.

[22] Laibson D. (1997), Golden eggs and hyperbolic discounting, Quarterly Journal of Economics 112: 443-477.

[23] Maier-Reimer E. and K.Hasselman (1987), Transport and storage of CO2 in the ocean - an inorganic ocean-circulation carbon cycle model, Climate Dynamics 2, 63-90.

[24] Manne A.S., R.Mendelsohn, and R.Richels (1995), MERGE, A model for evaluating regional and global effects of GHG reduction policies, Energy Policy 23, 17-34.

[25] Nordhaus, W. D.(1993), Optimal greenhouse-gas reductions and tax policy in the "DICE" model, The American Economic Review, 83 (2), pp. 313-317.

[26] Nordhaus, W.D. (1997), Discounting in economics and climate change, an editorial, in Climatic Change 37: 315-328.

[27] Nordhaus, W. D.(2007), A Review of The Stern Review on the Economics of Climate Change, Journal of Economic Literature, 45 (3), 686-702.

[28] Nordhaus, W.D, and J. Boyer (2000), Warming the World: Economic Modeling of Global Warming, MIT Press, Cambridge, MA.

[29] Orr, J.C., V.J. Fabry, O. Aumont, et al. (2005), Anthropogenic ocean acidification over the twenty-first century and its impacts on calcifying organisms, Nature 437: 681-686.

[30] Phelps E.S. and R.A. Pollak (1968), On second-best national saving and game-equilibrium growth, Review of Economic Studies 35(2): 185-199.

[31] Saez-Marti M. and J.W. Weibull (2005), Discounting and altruism to future decision makers, Journal of Economic Theory 122: 254-266

[32] Solomon, S., D. Qin, M. Manning, Z. Chen, M. Marquis, K.B. Averyt, M. Tignor and H.L. Miller (eds), "Climate Change 2007: The Physical Science Basis", Technical Summary, Table TS2.5, footnote b.

[33] Stern, N. (2006), "The economics of climate change: the Stern review", Cambridge, UK: Cambride University Press.

[34] Strotz, R. H. (1956), Myopia and Inconsistency in Dynamic Utility Maximization, The Review of Economic Studies, 23(3), pp. 165-180.

[35] Peck S.C. and T.J.Teisberg (1992), CETA: a model for carbon emissions trajectory assessment, Energy Journal 13, 55-77.

[36] Weitzman, M. (2001). Gamma Discounting, American Economic Review, Vol. 91, No. 1, pp. 260-271.

[37] Weitzman, M. (2007). A Review of The Stern Review on the Economics of Climate Change, Journal of Economic Literature, 45 (3), 703-724.

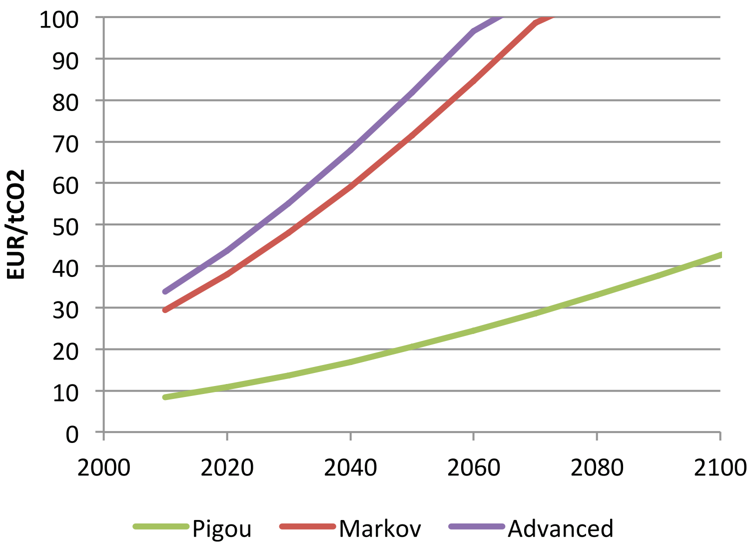

Figure 1. Carbon prices in 3 scenarios

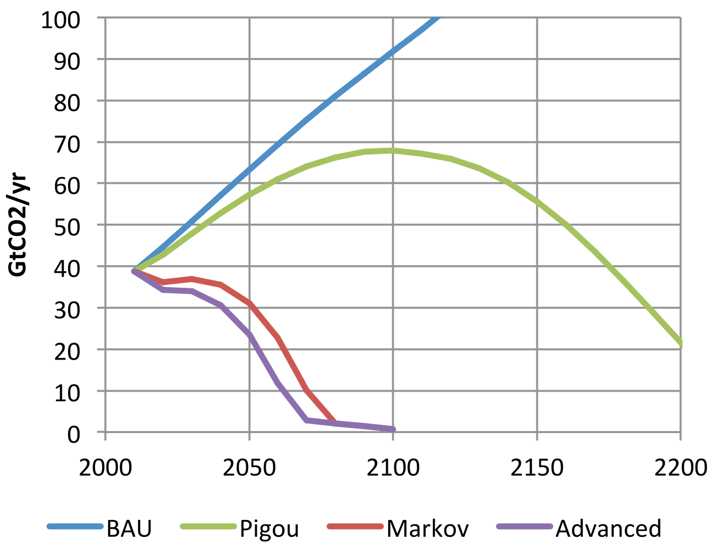

Figure 2. CO2 emissions, per year, in 4 scenarios

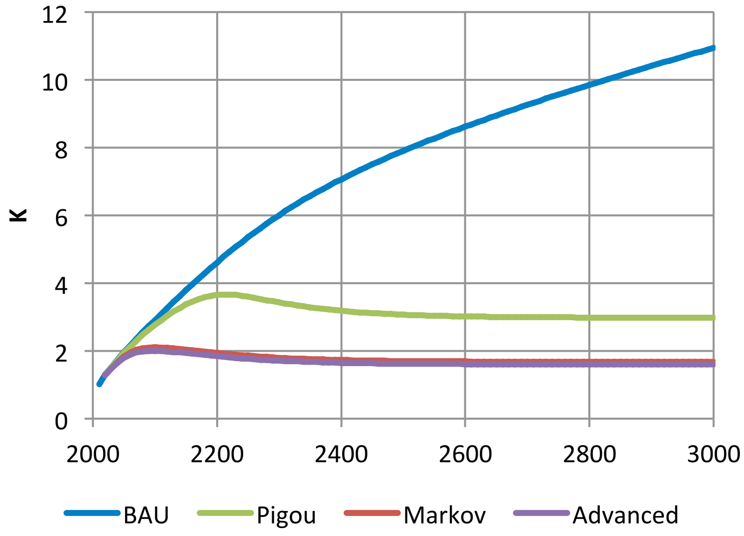

Figure 3. Temperature rise in 4 scenarios

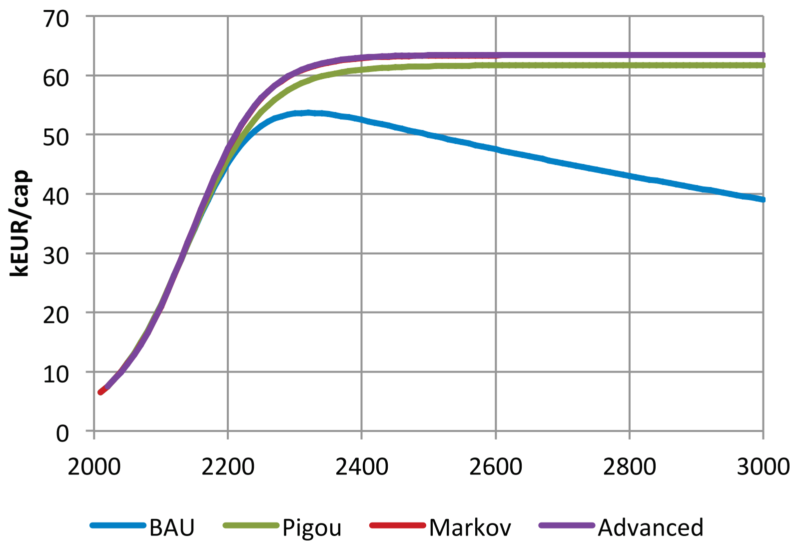

Figure 4. Per capita consumption levels in 4 scenarios

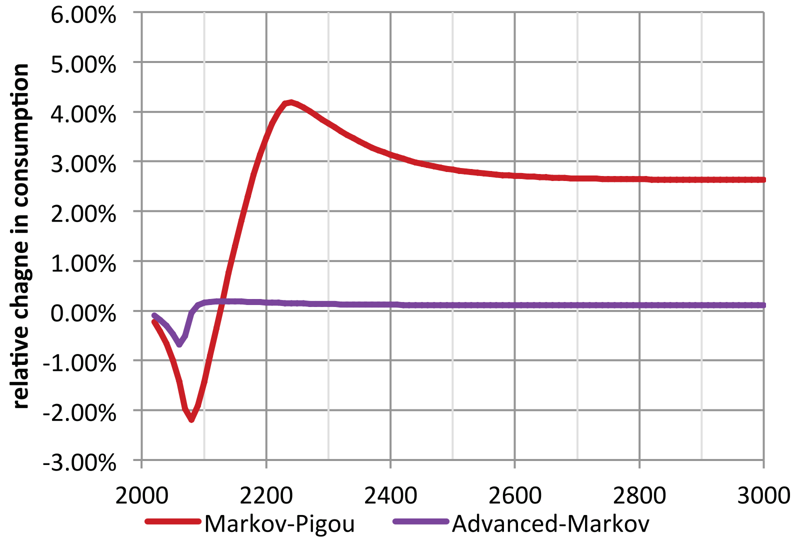

Figure 5. Change in consumption, Markov vs. Pigou and Advanced vs Markov

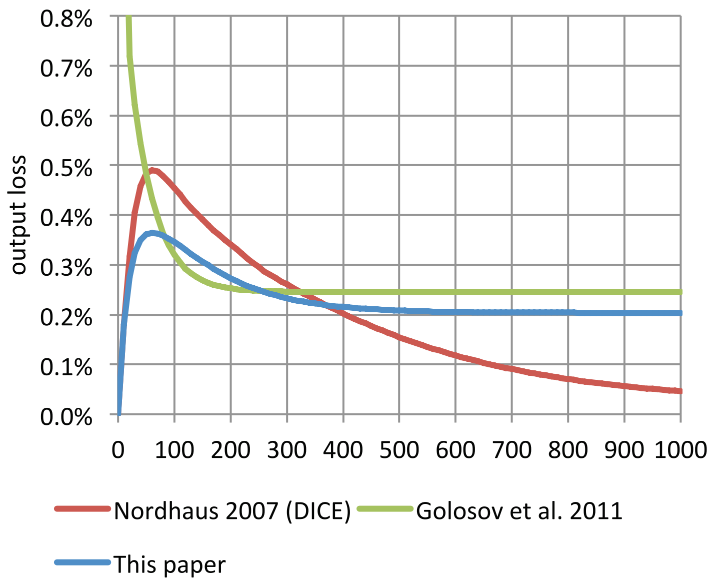

Figure 6. Emissions-Response function in 3 models. Damages as share of output for a 1 TtCO2 impulse.