Strategic oil supply and gradual development of substitutes

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Abstract

A dynamic game between oil exporters choosing when to sell a stock of oil, and oil importers able to gradually lower the cost of substitutes, is developed. The desire to lower R&D costs by developing the substitutes gradually explains why R&D into clean fuels begins before the substitutes are competitive. Oil supply decisions are constrained by the ever-improving substitute technologies. Supply is non-monotonic, initially falling, then forced up by competition from the substitute. The threat of climate change causes substitute development to slow down, as rapid development forces the exporter to extract oil faster, so aggravating pollution. If oil extraction becomes more expensive as supplies are depleted, the importer switches into clean fuels once these price oil out of the market; technological development will eventually be hastened to leave more of the oil locked underground. With multiple countries, importers have an incentive to free ride on each other's R&D efforts as these are sufficient to lower the price of oil.

Keywords: exhaustible resources, oil, alternative fuels, limit pricing, climate change
JEL Classification: D42, O32, Q31, Q40, Q54

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1 Introduction

Developed economies have many reasons to worry about their dependence on cheap oil, a resource increasingly controlled by a cartel (OPEC). The market power of the suppliers means oil importers may feel they are getting a bad deal. Dependence on a commodity found primarily in a politically volatile region has geopolitical and security ramifications. There are worries that the resource will run out suddenly, leading to a severe economic shock. And, finally, as oil is a fossil fuel, there are concerns over its environmental impacts, particularly climate change.

Importing countries may try to reduce their dependence on cheap oil—for example, by subsidising research into the development of an electrified transport infrastructure, or into third-generation biofuels. This is something the oil-producing countries would like to avoid, in order to maximise the value of their oil resources. Such conflicting interests mean there is a strategic dimension to thinking about oil dependence. Importing countries want to develop alternatives to oil, especially is oil is felt to be very expensive, polluting, or about to run out. Oil producers will want to preclude this by convincing their customers oil prices will remain sufficiently low. Indeed, OPEC and Saudi-Arabian decisionmakers often publicly say they are intent on maintaining a 'fair' oil price, one which does not lead to 'demand destruction'. [2]

In this paper, I consider strategic competition between an oil exporter and an importing country (or a group a cooperating countries) which are able to gradually reduce the costs of substitute technologies. The presence of these substitutes curtails the oil cartel's market power: first, because the cartel must provide importing countries with sufficient cheap oil, so as to discourage aggressive R&D programs; and, eventually, due to the fact that substitutes will impose a price ceiling on oil. Substitute development starts immediately, as the importer seeks to spread the cost of research over time. The supply of oil may initially fall, but will eventually be forced up as substitutes get cheaper. The substitute will only be used once oil is exhausted.

Once the exporter is forced to price just below the substitutes, the importer's R&D expenditures effectively determine the supply of oil: oil prices fall in lockstep with the price of the substitutes. When oil is polluting but the substitute is clean, cheaper substitutes will just lead to more oil supplied, and more pollution. Climate concerns induce the importer to slow down R&D efforts (assuming carbon pricing is not feasible, say, for political reasons). If exhaustion of oil oc-

[2] For example, "Saudi Arabia targets $ 100 crude price", *Financial Times*, Jan 16th 2012; "OPEC says pumping hard to bring oil price down", *Bloomberg.com*, May 3rd 2012.

curs due to increasing marginal costs of extraction, rather than by the entire physical stock being depleted, this result changes. In this more realistic case, the importer will eventually speed up substitute development, in order to shut a greater fraction of the oil reserves permanently out of the market (so preventing the embodied carbon from entering the atmosphere). If oil stocks will last for a long time, and near-term climate damages are substantial, there may still initially be a period in which the climate problem is tackled optimally by conducting less research.

The paper extends the Hoel (1978) model of a limit-pricing monopolist into a dynamic game, with the R&D process involving convex (per period) costs. This means that research will be undertaken gradually: it will be optimal to spread the costs of R&D over time, even if the substitute will not initially be competitive against oil. In other words, research into substitutes takes place at all times, certainly before the substitutes are used, and even before they are competitive against the resource. Accumulated knowledge also acts as a commitment device: a more advanced substitute technology means it is less costly to conduct the research required to make the technology competitive against oil.

The existing literature on substitute development has tended to focus on cases in which the R&D process consists of the optimal timing of when to adopt an alternative, fixed technology at a given exogenous cost. Gerlagh and Liski (2011) model a deterministic game in which the importer can trigger a process which ends with the introduction of the substitute. The delay between the decision to develop the substitute, and the arrival of the technology, acts as a commitment device: supposing the decision has not been made by a given period, the resource importer is committed to consuming the resource for at least an interval of length equal to the delay. The less resource remains, the more costly will this interval be, and the resource owner is forced to 'bribe' the importer into not switching by increasing supply of the resource as stocks fall. Earlier papers look at a similar situation without the adoption delay, with various assumptions on the ability to commit and the timing of moves (e.g. Dasgupta et al. (1983), Gallini et al. (1983), Olsen (1993)). These papers assume that the backstop technology is of some fixed quality once established, and thus focus on the timing of entry of some given technological innovation.

Harris and Vickers (1995) model a probabilistic R&D process in which a new innovation, once it arrives, makes the resource obsolete overnight. Thus, R&D produces discrete results, even though it takes place continuously. A particularly simple modification of the Hotelling rule characterises the resource owner's extraction rate, incorporating the strategic effect resource extraction has on the R&D efforts of the importer.

In all of the above studies, the substitute technology is essentially introduced overnight; in all but the last paper, the R&D decision is to determine the optimal date of the transition. The present paper seeks to model a more gradual R&D process, in which the accumulated stock of knowledge determines how competitive the substitute is.

An incremental process of backstop development has been considered by Tsur and Zemel (2003). However, they do not consider strategic issues, focusing only on the socially optimal case. They also do not consider increasing (per-period) marginal costs of R&D, as in the present paper, but rather impose an exogenous cap on the R&D rate. Thus the social planner will steer the economy to the steady-state process as quickly as possible, with maximal R&D efforts until this process is reached. In the present paper, R&D efforts are limited by increasing marginal costs.

Van der Ploeg and Withagen (forthcoming) note that, with monopolistic supply of a polluting resource, lowering the cost of a substitute may lead to more of the resource being left unused. This result is obtained for some exogenous change in the backstop price. In a sense, the present paper gives the same result but with an endogenous, optimal R&D process.[3]

The paper is structured as follows. I will first develop the basic model with physical exhaustion. This will serve to illustrate the basic structure of the problem, as well as reminding the reader of the model of Hoel (1978). Section 2 sets up the model and the social optimum is solved as a benchmark. Section 3 develops the non-cooperative equilibrium of this model. Section 4 extends the model to include a stock pollutant and extraction costs. Section 5 concludes.

[3] The 'Green Paradox' of Sinn (2008) refers to supply-side effects of 'green' policies in exhaustible resource markets. Specifically, any policy which tends to depress future demand *relative* to current demand will lead to resource suppliers reoptimising to extract their resources faster, hence expediting emissions and exacerbating the environmental problem. Thus the correct policy should aim the depress current demand more, for example by a decreasing ad valorem tax.

In the present paper, a supply-side effect is shown to imply that the environmental problem should lead to less intensive development of substitutes. However, the cause of this effect is not exhausibility of fossil fuels. A monopolist will cut back on output of any good. When consumption of this good is associated with an externality, the inefficiency related to market power is, at least partially, offset. Curtailing market power thus has an additional cost: higher consumption causes more severe external effects.

2 The social optimum

An economy uses a natural resource—think of fossil fuels—the flow of which is denoted by $q_F(t)$. This resource is exhaustible, with remaining stock denoted by $S(t)$, and the (given) initial stock by S_0. Resource extraction is costless.

There is a perfect substitute for the resource—for example, solar energy, biofuels, or coal-fired power with carbon capture and storage—called the backstop resource[4]. This substitute is produced perfectly competitively at a unit cost x, and the production rate is denoted $q_B(t)$. In fact, the backstop production cost depends on the accumulated knowledge of technologies used in backstop production.

Assumption 1. Backstop technology. The backstop production cost is a function of accumulated knowledge $K(t)$: $x = x(K(t))$, $x' < 0$, $x'' \geq 0$. The knowledge stock is normalised so that $K(0) = 0$, and the initial price is denoted $\overline{x} \equiv x(0)$. There exists a strictly positive lower bound to the backstop price: $\lim_{K\to\infty} x(K) = \underline{x} > 0$. If this bound is attained at $\overline{K}$, then $x'(K) = 0$ for $K > \overline{K}$.

Assumption 2. R&D process. R&D investment reduces the price of the resource incrementally. The rate of this research is denoted $d(t)$ and it builds up the knowledge stock according to $\dot{K} = d$. There are strictly convex monetary costs to conducting research[5] $c(d)$: $c \geq 0$, $c' \geq 0$, $c'' > 0, c(0) = 0$, $c'(0) = 0$. Knowledge does not depreciate.[6]

The representative consumer has a quasilinear felicity function $v(q_F, q_B, M) = u(q_F + q_B) + M$, with $u' > 0$, $u'' < 0$. I assume that using the backstop resource is always preferable to zero resource use: $\lim_{q\to 0} u'(q) > \overline{x}$. M denotes money, normalised so that the exogenously given money income is zero. This yields the inverse demand curve for the exhaustible resource or the substitute:

$$p(q_F, K) = \min\{u'(q_F), x(K)\} \tag{1}$$

Inverse demand is depicted in Figure 1. The backstop is supplied to satisfy the balance of the demand:

$$q_B(K) = {u'}^{-1}(p) - q_F \tag{2}$$

[4]This is stretching the sense in which the word 'resource' is usually applied in economics, but makes it easier to refer to consumption of *either* the exhaustible resource or the substitute.

[5]Relaxing the assumption of zero marginal cost at $d = 0$ is straightforward but yields no further intuition.

[6]This incremental research effort could perhaps be thought of as more like development and deployment investment. I will refer to it, for brevity, as 'research' or 'R&D'.

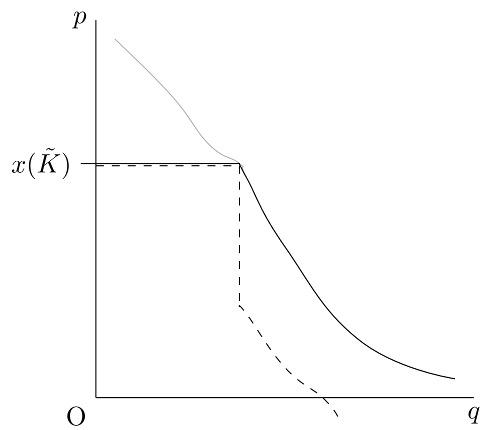


Figure 1: Inverse demand curve (gray); inverse demand for the exhaustible resource, given a backstop price $x(\tilde{K})$ (solid); and the corresponding marginal revenue curve (dashed).

I assume the utility function is such that $u'''(q)q + 2u''(q) < 0$ for all q. This assures concave revenues, and so the existence of a unique optimum to the monopolist's problem later on.

All agents in the economy live forever and discount the future at the common rate ρ. I omit notation to indicate the dependence of all variables on time.

Consider the social planner's problem:

$$\begin{aligned} &\max_{q_F, q_B, d} \int_0^\infty e^{-\rho t} \left(u(q_F + q_B) - x(K) q_B - c(d) \right) \mathrm{d}t \\ &\text{s.t. } \dot{S} = -q_F, \quad S(0) = S_0, \quad S \geq 0 \\ &\qquad \dot{K} = d, \quad K(0) = 0 \end{aligned} \tag{3}$$

Assuming an optimum exists, the problem is solved using Pontryagin's maximum principle. Denoting the costate variables on the resource stock and the knowledge stock, respectively, by λ_S and λ_K, the necessary conditions are

$$u'(q_F + q_B) \leq \lambda_S, \qquad q_F \geq 0, \quad \text{C.S.} \tag{4a}$$

$$u'(q_F + q_B) \leq x(K), \qquad q_B \geq 0, \quad \text{C.S.} \tag{4b}$$

$$c'(d) \leq \lambda_K, \qquad d \geq 0, \quad \text{C.S.} \tag{4c}$$

$$\dot{\lambda}_S = \rho \lambda_S \tag{4d}$$

$$\dot{\lambda}_K = \rho \lambda_K + q_B x'(K) \tag{4e}$$

$$\lim_{t \to \infty} e^{-\rho t} \lambda_S(t) S(t) = 0 \tag{4f}$$

$$\lim_{t \to \infty} e^{-\rho t} \lambda_K(t) K(t) = 0 \tag{4g}$$

These conditions are easily interpreted. The marginal utility of consuming an energy resource must be equal to its marginal cost, in the case of the fossil resource the scarcity rent ((4a) and (4b)). The marginal cost of research into the backstop technology has to equal the marginal benefit: the value of the marginal unit of knowledge ((4c)). As there are no extraction costs, the scarcity rent of the resource is constant in present value terms ((4d). The marginal value of the knowledge stock rises at the rate of interest plus capital gains ((4e)). The transversality conditions (4f) and (4g) indicate that the stocks of the resource and knowledge have to be used or built up so that the stock value as $t \to \infty$, in present value terms, is zero.

Definition 1. The *terminal path* refers to the optimal R&D process when the exhaustible resource is not used. It is the trajectory of R&D intensity $d(t)$, the R&D stock $K(t)$ and the associated costate variable $\lambda_K(t)$ which solve the social planner's problem for $S(0) = 0$. This solution is unique (see Proposition 1). As $K(t)$ is weakly monotonic and the optimisation problem is autonomous (not dependent on the starting date), I can denote the terminal path as the triplet $\{K, d^\infty(K), \lambda_K^\infty(K)\}$[7].

Proposition 1. The terminal path is unique and satisfies $\lim_{K \to \infty} \lambda_K^\infty(K) = \lim_{K \to \infty} d^\infty(K) = 0$.

Proof. All proofs are in the Appendix. □

The terminal path (Figure 2) describes the optimal R&D process once resource use stops, as a function of K. Even though defined here as the socially optimal path, it will appear also in the non-cooperative models. Note that the R&D intensity may behave non-monotonically. The marginal benefit of knowledge λ_S is just the present value of the stream of future cost reductions it yields.

[7] For finite $\overline{K}$, $K \geq \overline{K}$, $d^\infty(K) = \lambda_K^\infty(K) = 0$.

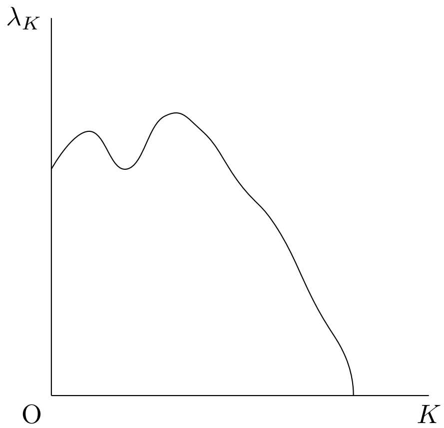


Figure 2: The terminal path in (K, λ_K^∞)-space. d^∞ increases monotonically with λ_K^∞. The economy moves to the right along the path at a rate increasing with λ_K.

At any moment, the total reduction in the cost flow is the marginal reduction in backstop cost, multiplied by the quantity of the substitute consumed. Thus, capital gains may be low if the backstop is consumed in small amounts, *or* if a marginal unit of knowledge only reduces the costs a little. When capital gains are low, the shadow value mostly represents future benefits and will fall more slowly, or even rise. The precise behaviour depends on the interaction of the demand for the resource and the effectiveness with which cumulative R&D effort reduces the backstop cost.

Proposition 2. The social optimum is characterised by two stages:

Stage I. $t \in [0, t^*)$, $t^* > 0$. Initially, only the exhaustible resource is used, with rate of extraction decreasing monotonically. The resource is fully used up by the switching date t^*. R&D intensity is strictly positive and increases monotonically.

Stage II. $t \in [t^*, \infty)$. In the second stage, the economy uses only the substitute and moves along the terminal path. Substitute use increases monotonically as the unit costs falls, until the date t^{**} (if finite) when the lower bound on the backstop cost is attained. Research effort is strictly positive until this date. Ultimately R&D effort falls to zero: $\lim_{t\to\infty} d(t) = 0$.

Thus, in the social optimum, initial resource use is sufficiently high so that, by the time the marginal utility of resource use (denoted p_F) rises to the backstop price, exhaustible resource use stops as the stock is fully depleted. This will not hold in the non-cooperative equilibrium. Figure 3 illustrates the social optimum for a case in which the lower bound on the backstop cost is attained in finite time.

Proposition 3. For the social optimum, an increase in impatience (a rise in the discount rate ρ) implies the backstop price at the moment of the switch will be higher: $\frac{\mathrm{d}x(t^*)}{\mathrm{d}\rho} > 0$. Either the initial extraction rate will rise, or the initial R&D intensity fall, or both. Effect on the timing of the switch is ambiguous: an earlier (later) switch implies that the initial resource extraction rate rises (R&D intensity falls), but the effect on initial R&D intensity (extraction rate) is ambiguous:

$$\frac{\mathrm{d}t^*}{\mathrm{d}\rho} < 0 \Rightarrow \frac{\mathrm{d}q_F(0)}{\mathrm{d}\rho} > 0$$
$$\frac{\mathrm{d}t^*}{\mathrm{d}\rho} > 0 \Rightarrow \frac{\mathrm{d}d(0)}{\mathrm{d}\rho} < 0.$$

Proposition 3 says that an increase in impatience will lead to at least one type of asset falling in valuation. Two assets exist in the economy: the exhaustible resource and knowledge. An increase in impatience will either increase the depletion of the former, or slow the accumulation of the latter, or both. Which of these effects dominates determines what happens to the timing of the switch: the switch will occur earlier if the incentives to conserve the resource are more responsive to time preference than the incentives to accumulate knowledge, so that faster consumption of the resource necessitates more intensive R&D to prepare for the switch. Conversely, a later switch may occur if the higher discount rate leads to much slower knowledge accumulation, thus requiring some conservation of the resource.

Proposition 4. An increase in the initial resource stock S_0 implies a higher initial extraction rate, a lower initial R&D rate, and a delay in introducing renewables: $\frac{\mathrm{d}q_F(0)}{\mathrm{d}S_0} > 0$, $\frac{\mathrm{d}d(0)}{\mathrm{d}S_0} < 0$, $\frac{\mathrm{d}t^*}{\mathrm{d}S_0} > 0$. An increase in the initial knowledge stock implies a higher initial extraction rate ($\frac{\mathrm{d}q_F(0)}{\mathrm{d}K_0} > 0$); the effect on the initial R&D rate and on the date of switch into renewables t^* is ambiguous.

Thus, a higher resource stock makes the problem of substitute development less pressing and will allow the social planner to share the benefits between higher resource consumption and being able to develop the substitute at a more leisurely pace. A more advanced technological state (initial knowledge stock) will also allow higher resource consumption. The effect on the R&D programme is indeterminate as it depends on the R&D profile along the terminal path.

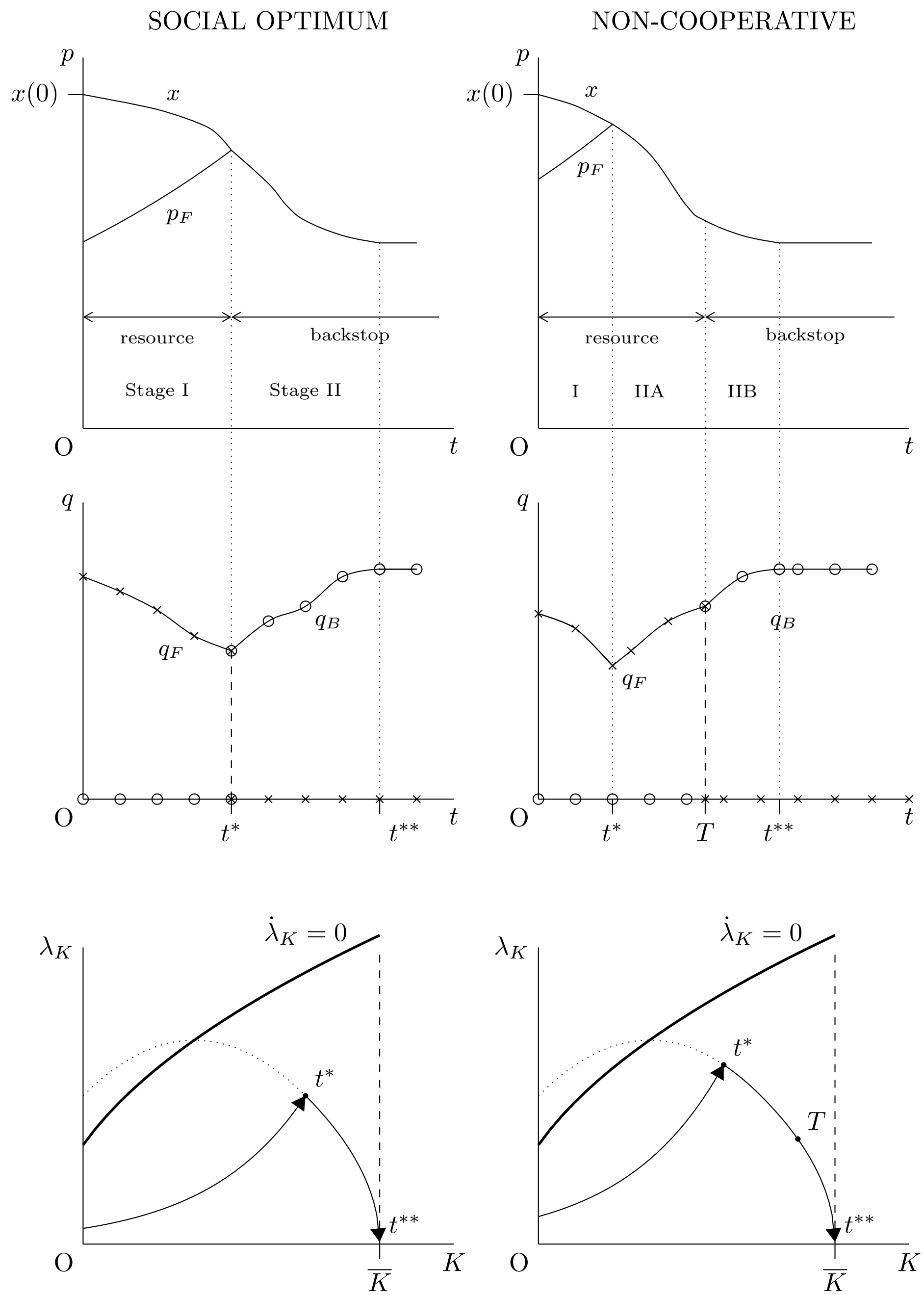


Figure 3: (*top*) Time paths of the backstop and resource prices under the social optimum (*left*) and the non-cooperative equilibrium (*right*); (*middle*) Quantities consumed of the exhaustible resource (crosses) and the backstop resource (dots); (*bottom*) Trajectories in (K, λ_K)-space (solid line) and the terminal path (dotted line).

3 The non-cooperative equilibrium

3.1 Equilibrium with commitment

Consider now setting up the above problem as a non-cooperative differential game, in which one agent (the exporter, indexed by E) owns the resource stock; and a second agent (the importer, indexed by I) buys the resource for consumption, and strategically develops and deploys the backstop technology. R&D is not conducted here by firms, but by the importing government. This might be because the government wants to coordinate R&D spending, or because the benefits due to R&D are not appropriable and hence R&D has funded by the government.

For purposes of intuition, I will first consider an equilibrium in the case in which commitment is possible; i.e. an equilibrium in open-loop strategies. Open-loop strategies are entire time paths of the choice variables. Hence the exporter is optimising extraction given a path for $d(t)$, and thus for the substitute cost $x(t)$. The importer, on the other hand, is trying to optimise $d(t)$, and so $x(t)$, given a time path of the extraction rate.

The open-loop equilibrium is intended to illustrate the qualitative features of the closed-loop equilibrium. For some initial states, the two equilibria coincide, and the open-loop equilibrium also serves as a check on the closed-loop solution. In the next section, I will show numerically that the two equilibria are very similar both qualitatively and quantitatively.

The limit-pricing argument discovered by Hoel (1978) is at the heart of the strategic equilibrium. Consider a monopolist supplying a resource, for which there exists a competitively supplied perfect substitute with a fixed, constant price. The monopolist will eventually start selling the resource at a price only just undercutting the marginal cost of the substitute, satisfying the entire demand at this price. Initially, the resource may be optimally priced below the backstop price. If resource demand is elastic, the resource owner has to choose between selling the marginal unit of the stock immediately, possibly depressing revenue earned for the inframarginal units, or at the time of exhaustion at the backstop price. If the initial resource stock is large, exhaustion may occur a long time in the future and immediate sale is preferred. It is straightforward to show that the same result holds for a given decreasing backstop price path.

The exporter maximises the discounted revenue stream

$$\max_{q_F} \int_0^\infty e^{-\rho t} \left(R(q_F; K) \right) \, \mathrm{d}t \tag{5}$$

where $R(q;K) \equiv p(q;K)q$ denotes revenue, with inverse demand given by (1). I will from now on omit the dependence on K. The problem is solved subject to the path of R&D spending $d(t)$, taken as given; to the resource constraint; and to the law of motion for the resource stock. Note that I rule out carbon taxes.[8]

The importer maximises the discounted stream of utility of the representative consumer, i.e. utility from resource consumption less spending on purchasing the exhaustible resource and R&D activities:

$$\max_{d} \int_0^\infty e^{-\rho t} \left(u(q_F + q_B) - p(q_F)q_F - x(K)q_B - c(d) \right) \mathrm{d}t \tag{6}$$

subject to the exhaustible resource supply path $q_F(t)$, taken as given; to the law of motion of the knowledge stock; and assuming that the representative consumer maximises utility, taking prices as given. I omit any tax or tariff instruments, to focus solely on the effect of technological development.

The perfect substitutability of the two resources affects the importer's problem too. For any given q_F, the demand for the backstop resource, and the resource price, are not differentiable with respect to the backstop price (equations (1) and (2)). This complication means that, in the limit-pricing stage, the equilibrium path is not uniquely defined:

Proposition 5. A continuum of open-loop equilibria exist. Any open-loop equilibrium features three stages:

Stage IA. $t \in [0, t^*)$, $t^* \geq 0$. Initially, only the exhaustible resource is used, with rate of extraction decreasing monotonically. A strictly positive quantity of the resource is left at the date t^*. Resource price is strictly below the unit cost of the backstop and follows the monopolist's Hotelling Rule, with marginal revenue rising at the discount rate. R&D intensity is strictly positive and increases monotonically, with the marginal cost increasing at the discount rate.

Stage IB. $t \in [t^*, T]$. Only the exhaustible resource is used, with the monopolist limit pricing at the backstop price. Resource use increases monotonically, and T is determined by the date at which the stock is fully exhausted. R&D intensity is initially strictly positive. It may behave non-monotonically. The marginal cost of R&D satisfies

$$\frac{\frac{\mathrm{d}c'(d)}{\mathrm{d}t}}{c'(d)} \in [\rho + \frac{q_F x'(K)}{c'(d)}, \rho]$$

If the date t^{**} at which the lower bound on the backstop cost is attained is

[8] This assumption is for analytical convenience. It could be justified by the observed difficulty of agreeing to a globally binding agreement on carbon pricing.

less than T, then R&D intensity is zero following this date and resource use is constant.

Stage II. $t \in [T, \infty)$. In the final stage, the economy uses only the substitute and follows the terminal path.

The indeterminacy of the equilibrium outcome in Proposition 5 results, in a sense, from dual limit-pricing. Given a path of the backstop price, the resource exporter will eventually seek to price just below the backstop. However, for a continuum of paths of resource extraction, the resource importer is similarly happy to develop the backstop technology so that it remains 'only just' uncompetitive vis-a-vis oil: tracking the resource price, but without an incentive to conduct R&D faster or slower. The capital gains to knowledge must lie somewhere between zero (the capital gains when the backstop is not used at all) and $q_F x'(K)$ (the capital gains when the backstop supplies the entire demand).[9]

In the next section I will show that the time-consistent equilibrium will feature terminal path R&D following the start of limit pricing. As my intention is to use the open-loop equilibrium only to illustrate the qualitative features of the closed-loop case, I will from now focus on this equilibrium only.

Intuition suggests that the non-cooperative equilibrium would feature excessively low extraction, as the exporter seeks to push up revenues, and too intensive R&D effort, as the importer wants to force the exporter to sell the resource faster. Again, at this level of generality, it is difficult to confirm this. However, if the elasticity of resource demand $\epsilon(q) \equiv \left|\frac{p(q)}{qp'(q)}\right|$ is weakly monotonic with respect to quantity, it is straightforward to verify the following:

Proposition 6. If $\epsilon'(q) \geq 0$, the open-loop equilibrium will feature inefficiently high initial R&D effort $d(0)$. If $\epsilon'(q) \leq 0$, then initial resource extraction rate $q_F(0)$ will be inefficiently low. With isoelastic utility ($\epsilon'(q) = 0$), both hold; the substitute becomes competitive inefficiently early.

Thus, under the assumption of isoelastic utility, the open-loop equilibrium will indeed imply excessively low initial resource extraction rates, as the monopolist cuts extraction from the socially optimal level, and excessively high R&D rates, as the importer starts benefiting from low backstop costs earlier, at the time limit pricing begins.

[9] Were the importer to conduct faster R&D, accounting for lower current capital gains, it would take control of the resource price immediately and the capital gains would jump to the upper bound. Were the importer to slacken R&D, raising accounted capital gains, it would price strictly above the backstop price, so that capital gains would fall to zero and R&D would immediately pick up again.

Proposition 7. With isoelastic demand, an increase in the initial resource stock S_0 increases initial equilibrium oil supply $q_F(0)$, lowers initial R&D efforts $d(0)$ and leads to a delay in the substitute becoming competitive (t^* rises).

Hence, having more of the exhaustible resource has similar effects as in the socially optimal case: the benefits are shared between higher oil supply, a reduced need to conduct costly R&D, and a delay in the substitute becoming competitive. It is more difficult to sign the effects of a higher initial knowledge stock.

3.2 Non-cooperative case without commitment

I will now turn to the equilibrium in the absence of commitment, limiting myself to Markovian strategies—strategies which are functions of the current state of the system only—and thus to the Markov-perfect Nash equilibrium concept (MPNE).

The Bellman equations for the exporter's and importer's problems, respectively, are

$$\rho V^E(K,S) = \max_{q_F} \left\{ R(q_F) + \tilde{d}(K,S)V_K^E(K,S) - q_F V_S^E(K,S) \right\} \tag{7}$$

$$\begin{aligned} \rho V^I(K,S) = \max_{d} \Big\{ & u(\tilde{q}_F(K,S) + q_B) - R(\tilde{q}_F(K,S)) - x(K)q_B - c(d) \\ & + dV_K^I(K,S) - \tilde{q}_F(K,S)V_S^I(K,S) \Big\} \end{aligned} \tag{8}$$

where p_F and q_B are given by (1) and (2).

To obtain further intuition, I will first obtain a result pertaining to open-loop equilibria in which the initial knowledge stock, denoted by $K_0 \geq 0$, is now allowed to vary. I will index the equilibria by their initial state (K_0, S_0).

Lemma 1. Consider the set of open-loop equilibria such that limit-pricing begins immediately, i.e. that satisfy

$$\Phi = \{(S_0, K_0) : MR(p^{-1}(x(K_0))) \leq e^{-\rho(T-t^*)}x(K(T))\}$$

where $MR(\cdot)$ denotes marginal revenue, and $p^{-1}(x(K))$ is inverse demand at the backstop price. The upper boundary of this set is given by $S_0 = \phi(K_0)$, along which the above holds as an equality, and satisfying $\phi' > 0$.

In words, under commitment, limit-pricing begins immediately for sufficiently low S_0, given any K_0. As the initial knowledge stock goes up, limit-pricing begin at higher resource stocks.

Proposition 8. In the set Φ, the open-loop equilibrium coincides with a Markov-perfect Nash equilibrium.

In other words, following the open-loop strategies (synthesised as functions of the state variables) is time-consistent once limit-pricing has started. The importer's strategy is not a function of the resource stock, and so the exporter cannot influence the importer's future actions. The exporter, on the other hand, will always limit-price, in which case the importer optimally develops the substitute technology as if the substitute did not exist.

I will now focus on a particular MPNE, one which indeed coincides with the open-loop equilibrium in the set Φ. There is potentially a large set of equilibria which satisfy this condition. I will proceed to find one which is continuously differentiable in terms of the value functions outside the set Φ. In other words, I am ruling out equilibria which feature coordinated jumps in strategies in the non-limit pricing stage.

Note that the payoffs along the locus $S = \phi(K)$ are easy to calculate as they coincide with the open-loop case. It is then possible to reformulate the problem as a dynamic game in which the terminal time is the moment at which the economy enters the set Φ, with the corresponding terminal payoffs. For either player's problem, if one now obtains a continuously differentiable function which satisfies the Bellman equation (with an interior solution) at all points outside the set Φ, and which approaches the terminal value at all points along the locus $S = \phi(K)$, then the Bellman equation also yields the optimal strategies (Theorem 5.3 in Başar and Olsder (1999)). Note that, in particular, smooth pasting conditions are not required.

I investigate the closed-loop case outside the set Φ numerically. In fact, the value functions turn out to be nondifferentiable at the regime boundary $S = \phi(K)$. Hence, the general method of discretisation with respect to time, followed by value function iteration using Chebyshev polynomials, will not work. Using B-splines to approximate the function produces a solution of poor quality near the regime boundary. Furthermore, it is more satisfactory to work in continuous time as the date of exhaustion is endogenous, and as the open-loop problem has been solved for the continuous-time case.

The first-order conditions to the above problems, with limit pricing not binding, are

$$d^* \equiv d^*(V_K^I) = c'^{-1}(V_K^I)$$
$$q_F^* \equiv q_F^*(V_S^E) = MR^{-1}(V_S^E)$$

Using these, the solution will satisfy

$$\begin{aligned}\rho V^I &= u(q_F^*) - q_F^* p(q_F^*) - c(d^*) + V_K^I d^* - V_S^I q_F^* \\ \rho V^E &= q_F^* p(q_F^*) + V_K^E d^* - V_S^E q_F^*\end{aligned} \tag{9}$$

where I have omitted the dependence of q_F^* and d^* on V_S^E and V_K^I, respectively. I will thus have to solve a system of two nonlinear partial differential equations. The boundary conditions will be given by the continuity of the value functions at the upper boundary of the set $\phi(K)$.

I solve the system as a functional problem using the collocation method: that is, I find n-dimensional approximations $\tilde{V}^I$, $\tilde{V}^E$ which satisfy the above system at n points (Judd (1998)). I am imposing smoothness in the set $\Phi^{-1} \equiv [0, \overline{K}] \times [0, \overline{S}] \backslash \Phi$ (for some $\overline{S} > \phi(\overline{K})$). Thus, Chebyshev collocation with Chebyshev nodes should yield good results. In order to be able to use this, I transform the set Φ^{-1} into a rectangle in (K, s) by using

$$s \equiv \frac{S - \phi(K)}{\overline{S} - \phi(K)} \tag{10}$$

implying $s \in [0, 1]$.

I will thus approximate transformed value functions $v^I(K, s)$, $v^E(K, s)$, the partial derivatives of which satisfy, for $i \in I, E$,

$$\begin{aligned}v_K^i &= V_K^i(K, S) + V_S^i(K, S)(1 - s)\phi'(K) \\ v_s^i &= V_S^i(K, S)(\overline{S} - \phi(K))\end{aligned} \tag{11}$$

The function approximations will be of the form

$$\tilde{v}^i(K, s) = V^{\phi, i}(K) + sA^i G(s, K)$$

where $V^\phi(K)$ is the relevant value function at the limit-pricing boundary $S = \phi(K)$, A is a coefficient matrix with dimensions (nm, nm), and $g(\cdot)$ is a (nm) vector of Chebyshev polynomials. Note that the boundary condition will be satisfied.

I now choose functional forms. Let utility be of the standard isoelactic form, $u(q) = \frac{q^{1-\frac{1}{\eta}}}{1-\frac{1}{\eta}}$. Let the backstop cost be given by $x(K) = \underline{x} + \frac{\gamma}{2}(\overline{K} - K)^2$. Let R&D costs be quadratic also: $c(d) = \frac{\xi}{2} d^2$. To illustrate the qualitative results, I parameterise arbitrarily with $\eta = 2$, $\xi = .01$, $\gamma = 1.6(-4)$.

For the approximation, I choose a 400-degree Chebyshev approximation, with 20 basis functions in each dimension. This yields a system with 800 equations and unknowns. I obtain the coefficients for the open-loop solution to use as my initial guess.

The system is solved rapidly by a standard non-linear rootfinding algorithm, probably largely due to the good initial guess. For initial guesses 'near' the open-loop equilibrium values, the system converges to effectively identical results; for very different initial guesses, convergence does not occur. Euler equation errors are small, of the order of 10^{-6} relative to the Euler equation values (Figure 4)[10].

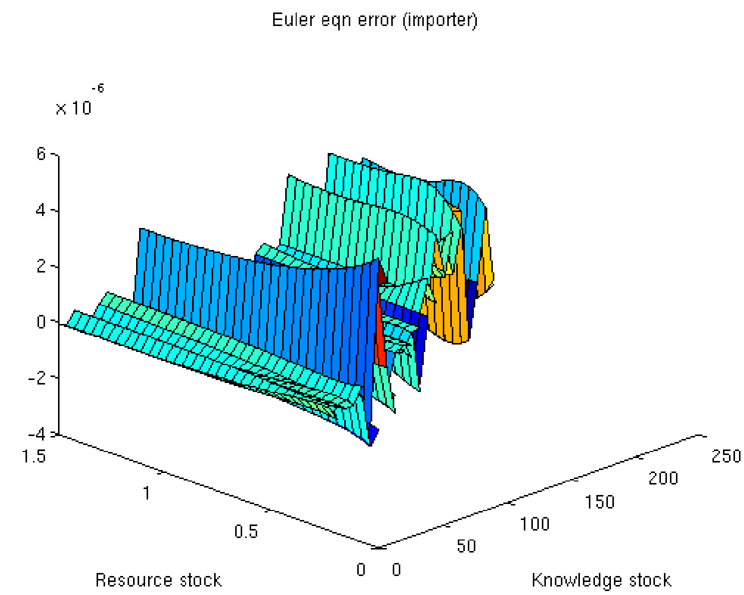
The results are displayed in Figures 5 to 8. As the system evolves, the economy travels towards the bottom right in the state space. Importer value (Figure 5) of course does not depend on the resource stock under limit pricing. Where limit pricing does not occur, higher initial resource stock implies higher value, as the exporter seeks to sell more of the plentiful resource early on. Importer value increases with the knowledge stock, reaching a maximum (corresponding to a permanent stream of constant resource use) when the backstop cost reaches its minimum (here at $K = 250$).

Exporter value (Figure 6) increases with resource stocks, being zero when no resource exists. Higher knowledge stocks reduce value, up until the lower bound on backstop cost (although, with the parameterisation used, this effect is hard to distinguish in the figure).

Optimal actions, as functions of the state, are shown in Figures 7 and 8. When limit pricing is active, R&D intensity of course coincides with the terminal path. It is also constant with respect to the resource stock. When limit pricing is not active, R&D intensity is lower (as in the open-loop case). There is a discontinuous jump in the actions at the locus where the regime switches into limit pricing. Immediately prior to the switch, a marginal unit of knowledge induces the exporter to sell more oil. This yields a marginal unit of surplus to the importer, but also depresses future value as the resource stock falls. The net impact is to lower the marginal value of R&D, and so the importer slows down R&D immediately prior to the switch, relative to the case under commitment. As soon as limit pricing begins, this effect disappears and R&D investment leaps up.

When limit pricing, the quantity of the resource sold is a function only of the knowledge stock. Before the start of limit pricing, resource sales are higher. Again, a discontinuity exists. Prior to the regime switch, oil extraction induces higher R&D efforts from the importer, which is costly to the exporter. Thus, the marginal value of the resource is lower, and the resource owner would extract more of it. Note that the model implicitly assumes that oil cannot be stored;

[10] One initial guess converged to a solution which was ruled out based on very large Euler equation errors.



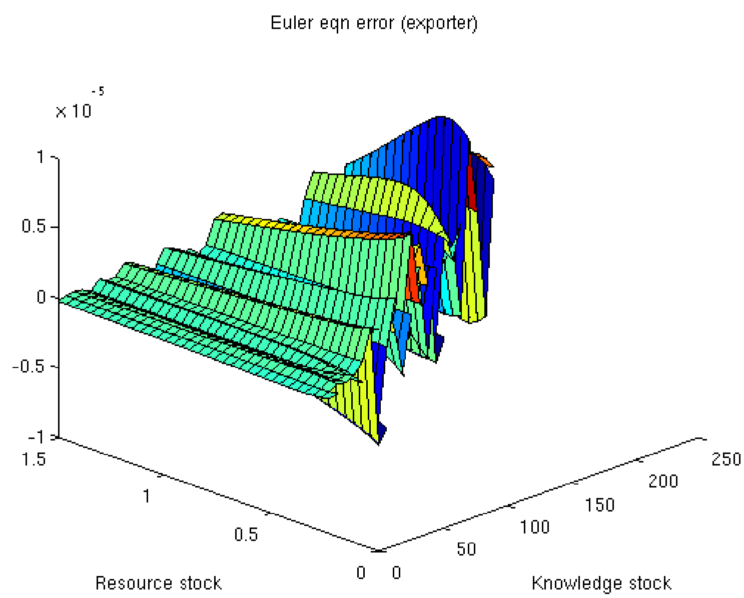


Figure 4: Euler equation errors for the two players, outside the collocation nodes, relative the the Euler equation LHS. The errors are less than one thousanth of a percent.

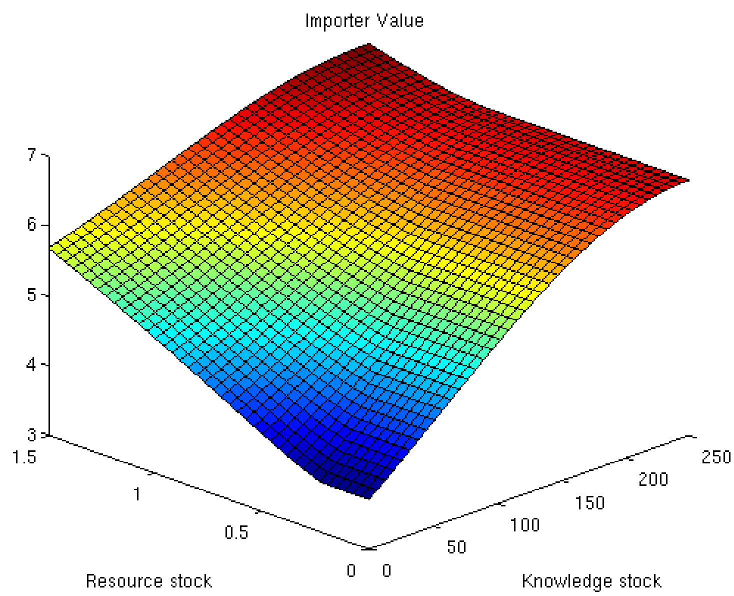


Figure 5: Resource importer value increases with knowledge, up to $K = 250$ at which substitute cost achieves its minimum value. Under limit pricing, stocks of the exhaustible resource make the importer no better off. With oil stocks high relative to knowledge stocks, the resource is initially priced strictly below substitute and the importer value increases as more oil is supplied.

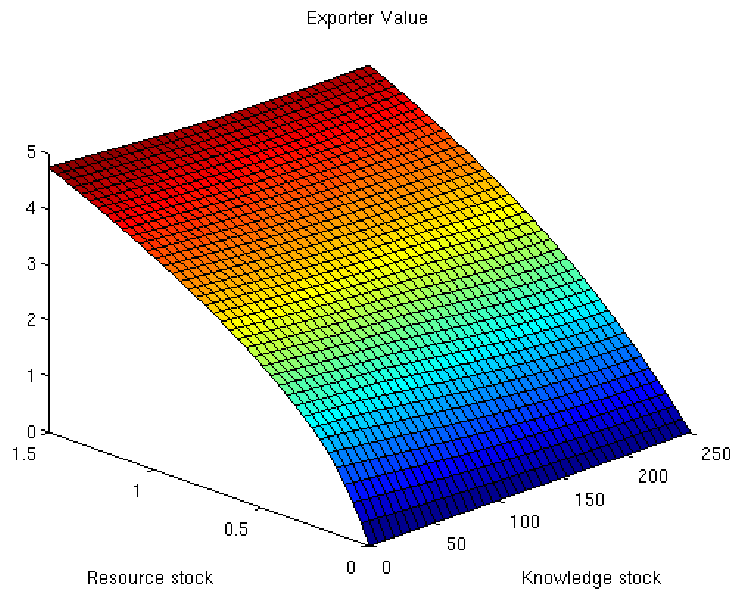


Figure 6: Resource exporter value increases with oil stocks. More competitive substitute (more knowledge) decreases value, up to the level after which substitute cost no longer falls (here $K = 250$).

with storage, markets would arbitrage away jump in the oil price. With plentiful resource stocks, the exporter would rather conserve the resource. The resource has higher marginal value, in that conserving the stock will lower the R&D rate until limit pricing begins. This can be seen by using the envelope theorem on the Bellman equation to obtain

$$\rho V_S^E = V_{KS}^E d + V_K^E \frac{\partial d}{\partial S} - V_{SS}^E q_F = \frac{\mathrm{d}V_S}{\mathrm{d}t} + V_K^E \frac{\partial d}{\partial S}$$

and then integrating between t and the date at which limit-pricing begins, t^*, to get

$$V_S^E\big|_t = e^{-\rho(t^*-t)}\ V_S^E\big|_{t^*} + \int_t^{t^*} e^{-\rho(z-t)}\ V_K^E\big|_z \frac{\partial d}{\partial S}\,\mathrm{d}z$$

where the derivative of the value function at t^* is understood to refer to the left-hand side limit of the derivative. Note that the integral will, for the results obtained, be positive: this is the cumulative value of the marginal unit of the resource in terms of deterring R&D investment.

I will now consider the differences between the equilibria with and without commitment. The excess values when commitment is not possible, compared

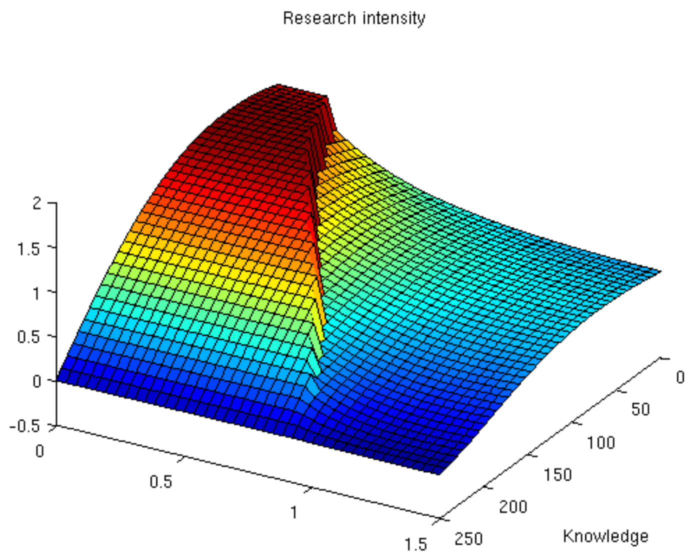


Figure 7: R&D intensity. Note that the axes have been reversed. Under limit pricing, the importer conducts R&D as per the terminal path (the concave part). For high oil stocks relative to knowledge stocks, oil is initially priced strictly below the substitute and the importer relaxes R&D efforts.

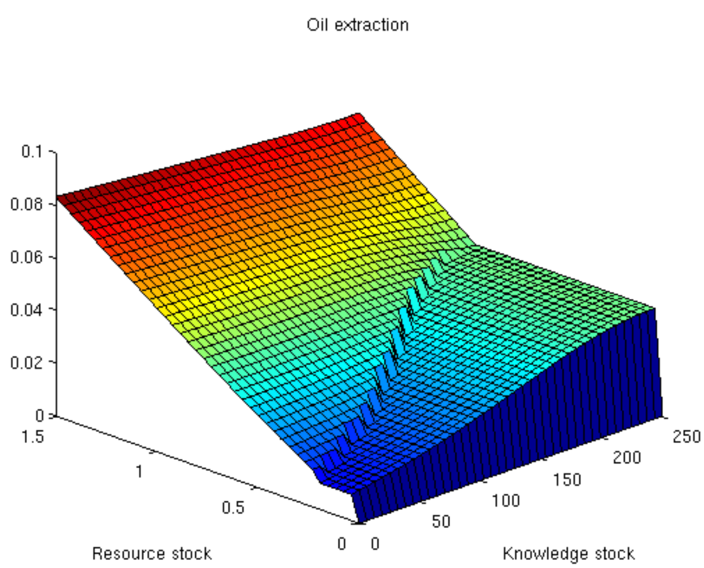


Figure 8: Exhaustible resource sales. Under limit pricing, extraction is determined by the substitute cost. For high oil stocks, relative to knowledge, exporter initially sells strictly more than the limit-pricing quantity.

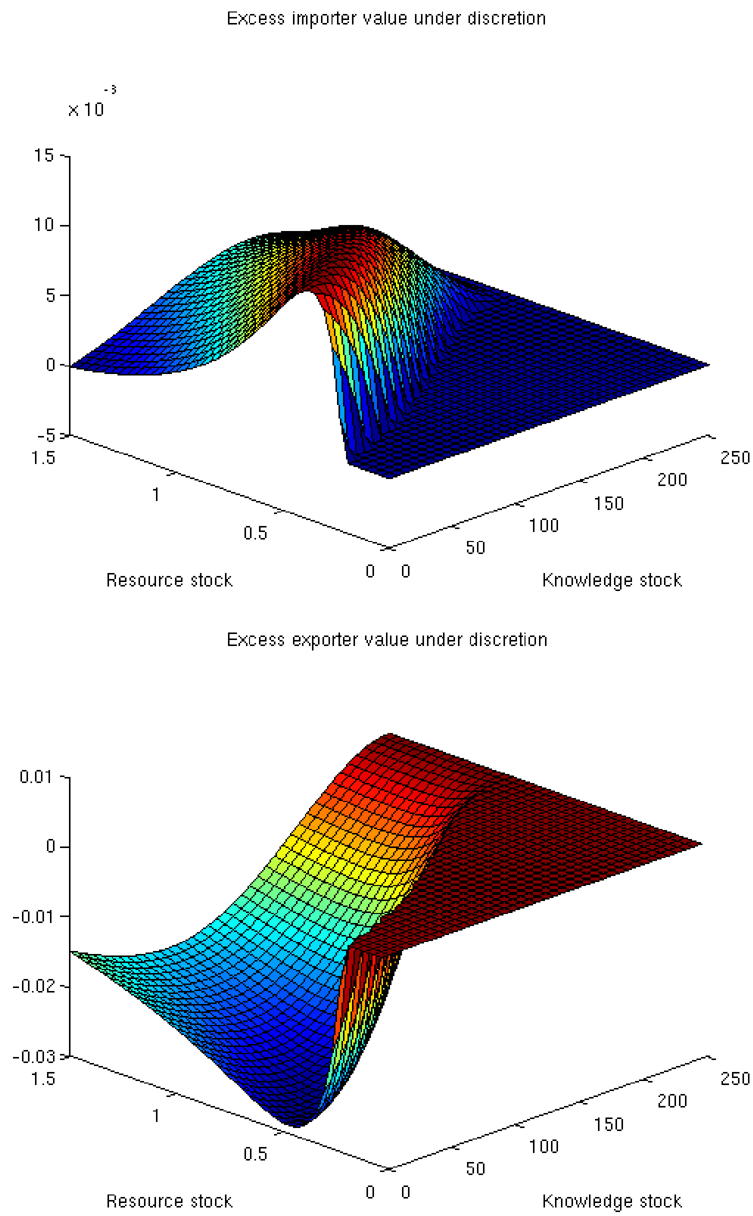


Figure 9: The importer gains when commitment is not possible (*top*), but the change in the value is at best just over 1%. The exporter loses by up to 3%.

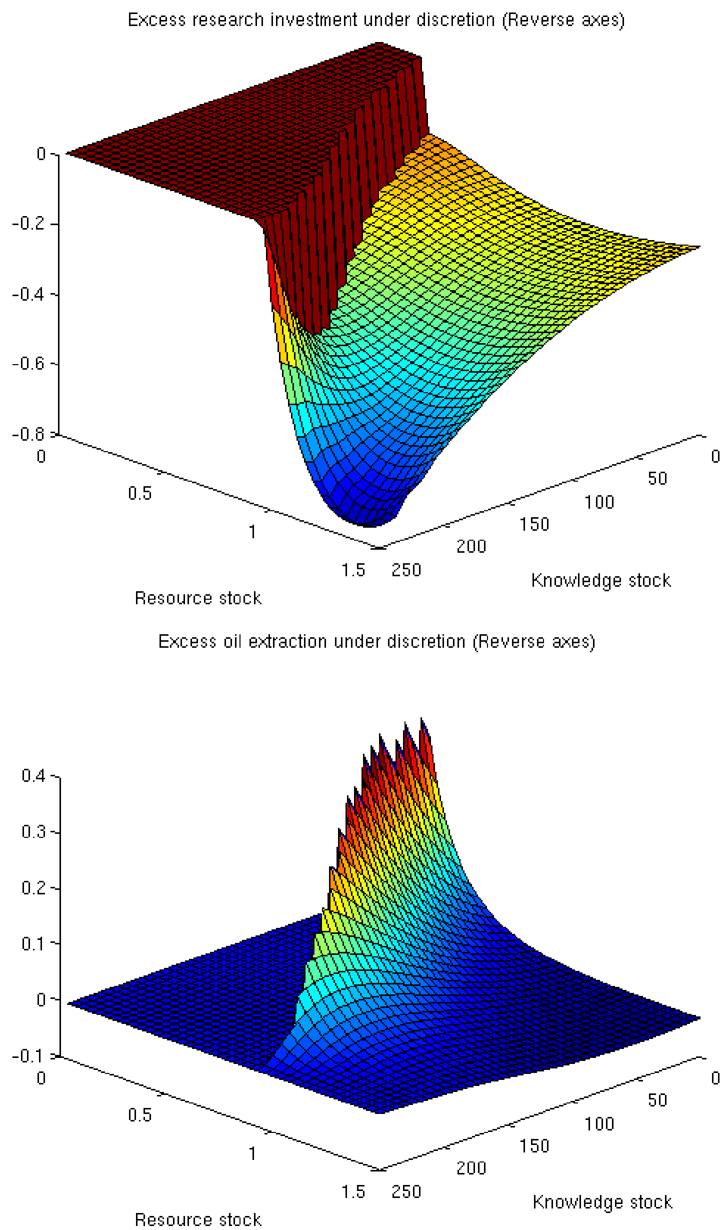


Figure 10: Euler equation errors for the two players, outside the collocation nodes, relative the the Euler equation LHS. The errors are less than one thousanth of a percent.

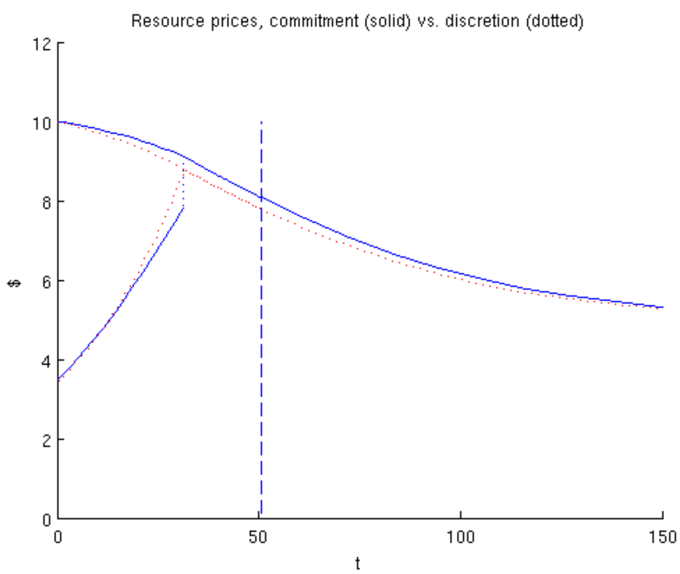


Figure 11: Price paths of the backstop resource (always decreasing) and the exhaustible resource (initially increases), for the commitment (open-loop) outcome (*dotted red*) and the discretionary (closed-loop) outcome (*solid blue*). Times of exhaustion are very close together and indicated by the dashed vertical line. Under discretion, initial resource prices are higher as the exporter tries to conserve the resource, in order to motivate lower R&D activity.

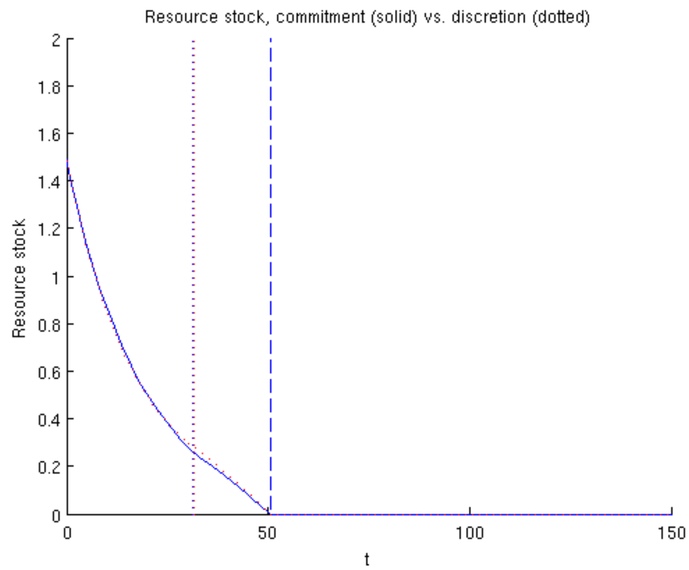


Figure 12: Resource stocks for the commitment (open-loop) outcome (*dotted red*) and the discretionary (closed-loop) outcome (*solid blue*). Times of exhaustion are very close together and indicated by the dashed vertical line; the dotted vertical lines indicate start of limit pricing.

to when it is, are shown in Figure 9. The importer gains is commitment is not possible, while the exporter loses, as the exporter is induced to sell more of the resource while the importer can relax R&D efforts (as explained above, and shown in Figure 10). Note that for high resource stocks, relative to knowledge, the exporter in fact reduces their extraction, compared to the commitment case.

Time paths of resource and backstop prices, and of the resource stock, are compared in Figures 11 and 12.

4 Stock pollution and economic exhaustion

I will now extend the model to take into account economic, rather than physical, exhaustion of the resource (Heal (1976)). Economic exhaustion occurs when resource extraction stops due to increasing extraction costs, rather than physical (total) depletion of reserves. This substantially increases the realism of the present model. I will also introduce a stock externality related to the cumulative use of the resource; the obvious motivation is climate change, resulting from the use of fossil fuels. Jointly, the two assumptions introduce interesting new dynamics to the model.

I will illustrate the basic dynamics by solving the special case in which limit pricing begins immediately at the start of the game.

Consider first the inclusion of stock-dependent extraction costs for the exporter. The objective function now becomes

$$\max_{p_F} \int_0^\infty e^{-\rho t} q_F(p_F; K) p_F - q_F(p_F; K) C(S) \, \mathrm{d}t \tag{12}$$

where $C' < 0$; in other words, the unit extraction cost increases as the remaining resource stock falls. The exporter's control variable is now the oil price; this allows the importer to affect resource supply by pricing the exporter out of the market.

The importer's problem becomes

$$\max_{d(t)} \int_0^\infty e^{-\rho t} \left(u(q_F + q_B) - p(q_F) q_F - x(K) q_B - c(d) - Z(G) \right) \mathrm{d}t \tag{13}$$

$$\dot{G} = q_F, G(0) = G_0 \tag{14}$$

$$\dot{K} = d, K(0) = K_0 \tag{15}$$

where $G(t)$ denotes the stock of greenhouse gases in the atmosphere. I assume away natural decay of the atmospheric stock; this simplifies the problem and is a fair approximation of reality, with drawdown of carbon dioxide into the deep oceans and eventual mineralisation occurring at timescales much longer than those typically considered in economic problems. $Z(\cdot)$ yields damages due to climate change impacts, which enter welfare additively as a function of the greenhouse gas stock (implicitly proxying changes in the climate system).[11]

Note that the models below are not open-loop equilibria, but rather control problems where the importer recognises the exporter will limit price. Solving the actual open-loop equilibrium is more involved, as the Hamiltonian is discontinuous in the limit-pricing stage and standard theorems in control theory do not apply. However, the control problem as formulated here illustrates the mechanisms underlying the feedback equilibrium.

4.1 Equilibrium with immediate limit pricing

To retain tractability, I will focus on cases in which limit pricing begins immediately, i.e. $t^* = 0$. As before, limit pricing eliminates any strategic dimensions to the game.

[11] This is a common assumption in the integrated assessment literature. However, damages could be argued to depend more on the rate of change of climate, rather than of the degree of change over preindustrial.

Definition 2. Given some instance of the model, the *reference equilibrium* is the equilibrium of the same instance absent the externality. Note that the R&D process in the reference equilibrium will follow the terminal path at all dates.

Proposition 9. With no extraction costs, and in the case $t^* = 0$, taking the externality into account reduces the optimal R&D rate (for any level of knowledge).

If the resource is supplied by a limit-pricing monopolist, it is optimal to slow down the development of substitutes to the polluting resource. The pollutant introduces an extra cost to investing in substitutes: a fall in the substitute price forces the monopolist to supply larger amounts of the polluting resource while stocks remain positive, thus raising the damage costs due to near-term pollution impacts.

With extraction costs present, the choice of R&D intensity has a third effect: it also influences the ultimate fraction of the resource extracted. A marginal unit of knowledge will imply exhaustion at a lower level of cumulative extraction, as it lowers the unit cost of producing the backstop, relative to the unit extraction cost of the exhaustible resource. This effect will encourage faster development of the backstop. However, the prospect of higher near-term pollution will still tend to deter it. The overall effect on the R&D process will depend on the balance of these effects.

Prior to exhaustion, the exporter will simply satisfy the entire inverse demand at the backstop price: $q_F = p^{-1}(x)$, $q_B = 0$. Exhaustion occurs when extraction becomes unprofitable, i.e. the unit extraction cost equals the price:

$$C(S(T)) = x(K(T)) \tag{16}$$

Following exhaustion, the stock externality presents a constant burden on welfare but does not affect incentives to conduct R&D. The economy will thus follow the terminal path. Hence, the importer's problem is to solve

$$\max_{d(t),T} \int_0^T e^{-\rho t} \left(u(p^{-1}(x)) - xp^{-1}(x) - c(d) - Z(G) \right) \mathrm{d}t + e^{-\rho T} \left(\pi^\infty(K(T)) - \frac{Z(G(T))}{\rho} \right)$$

where $\pi^\infty(K)$ denotes the welfare obtained from resource use following the terminal path after exhaustion. The choice of T is constrained by equation (16). I will focus on cases in which exhaustion is, indeed, economic: $S(t)$ is always strictly positive. Denoting the optimal values by an asterisk:

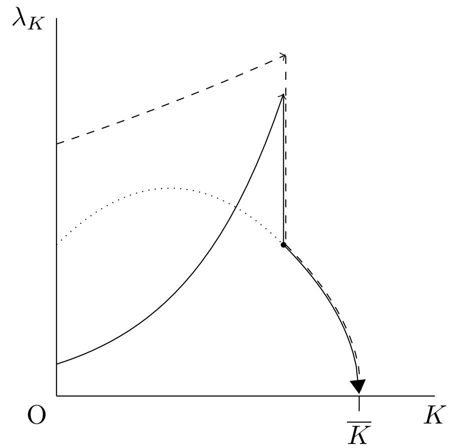


Figure 13: Alternative trajectories in (K, λ_K)-space. Note that R&D intensity is monotonic with respect to λ_K. If the concern for long-term damages outweighs the near-term pollution impacts (dashed line), initial R&D intensity is higher than in the case without the externality (terminal path; dotted line). High near-term damages can lead to initial R&D intensity being lower than without the externality (solid line), although eventually it will become optimal to start intensive R&D to halt resource use. Both trajectories feature a discontinuous fall in R&D intensity as soon as economic exhaustion occurs.

Proposition 10. Initial R&D intensity may be higher or lower in the equilibrium with the externality and extraction costs, compared to the reference equilibrium: $d^*(0) \lesseqgtr d_{\text{ref}}(0)$. Immediately preceding exhaustion, R&D intensity is higher for the given level of accumulated knowledge than in the reference equilibrium, as the importer drives down the backstop price in order to shut the polluting resource out of the market: $\lim_{t \to T^{*+}} d^*(K^*(t)) > d^\infty(K^*(T^*))$. If initial intensity is lower than in the reference equilibrium, it will equal the reference equilibrium rate just once, being lower always before and higher always after. As soon as exhaustion occurs, the R&D rate jumps discretely down to the terminal path.

In words, in the run-up to exhaustion, the importer will always race to drive the polluting resource out of the market (Figure 13). This way, the importer avoids the marginal damages due to long-term pollution (suffered in perpetuity). Of course, R&D also makes energy cheaper as in the case without the externality. Once oil is rendered uncompetitive, R&D intensity falls discretely to the terminal path: the additional marginal value to R&D, associated with the prospect of shutting out the polluting resource, has already beem realised.

If marginal damages at low levels of pollution are fairly significant, relative to the long-term damages, and the resource is plentiful, then early R&D efforts may be below the reference rate: the importer wants to delay short-term damages by delaying R&D (thus keeping short-term extraction rates low). In this case, there will come a unique point in time at which the importer starts to focus more on long-term concerns, beginning the crash programme; after this moment, R&D rates exceed the corresponding rates without the pollution problem, until the resource is exhausted.

5 Conclusions

I have analysed strategic competition between a resource exporter, selling an exhaustible resource, and a resource-consuming country, able to gradually improve, with convex per-period costs, a perfect substitute to this. Per-period convex costs imply that the cost of developing the resource are optimally spread out across time. With incremental technological progress, the non-cooperative outcome features three stages. Initially, the resource is priced strictly below the substitute cost, with decreasing resource use (thus increasing resource price) over time. After the substitute becomes competitive, the resource exporter will price oil just below the substitute, in order to keep the substitute off the market. As technological progress keeps making substitutes cheaper, the resource exporter is forced to supply increasing quantities. The path of resource extraction is thus non-monotonic. Finally, once the resource is depleted, the importer switches to the backstop technology. Unlike most other models of resource extraction and substitute development, the present model explains why R&D is undertaken even when the substitutes are far from being competitive against the resource.

When use of the exhaustible resource results in a stock pollution externality—as climate change follows from consumption of a fossil fuel such as oil—limit-pricing behaviour implies that, in the absence of carbon prices, it will be optimal to slow down research. The importer effectively controls oil supply; aggressive R&D programs will just result in the oil stock being depleted faster, leading to greater emissions. With oil extraction costs increasing as supplies dwindle, there is a third effect: R&D can make oil obsolete, actively bringing the oil age to a close with a part of the resource remaining unused. I have shown that this effect will always eventually dominate. As exhaustion looms close, the importer will race to drive the polluting resource out of the market.

These findings are important, as they inform the public debate over whether technological programs would prove to be a workable climate policy instrument, if carbon pricing remains politically difficult. Aggressive R&D subsidies can be used to wean economies off oil, provided that the moment of (economic) exhaustion is relatively close. However, if oil can be expected to remain competitive with the substitutes for a long time, more aggressive R&D may only result in greater near-term emissions, possibly aggravating climate change. Hence, the optimal response may still be to initially slow down R&D efforts. These results are necessarily indicative only, due to the simplicity of the model (Hart and Spiro (2011)). Nevertheless, they give partial intuition to a particular outcome of climate policy which has not been considered previously.

In the present paper, research into substitutes to oil has been a function of the government. An obvious extension of the model would be to consider what kinds of market incentives could yield a decentralised backstop development process. This remains work in progress.

Appendices

A The cooperative outcome

Proof of Proposition 1. EXISTENCE: TO BE WRITTEN.

If the upper bound $\overline{K}$ is reached in finite time t^{**}, $\lambda_K(t) = 0$ for $t \geq t^{**}$. Then it is immediate from the theory of ordinary differential equations that a unique solution exists to the initial value problem given by equations (4c) and (4e), with $K(t^{**}) = \overline{K}$, $\lambda_K(t^{**}) = 0$.

If the upper bound is never reached, i.e. $\overline{K} = \infty$, the system does not reach a steady state; instead, R&D continues forever: $d(t) > 0$, for all t. It has to be shown that only one path is consistent with the transversality condition (4g). Suppose such a path exists. A necessary condition is $\lim_{t\to\infty} e^{-\rho t}\lambda_K = 0$. Rearranging (4e), integrating across the interval $s \in [t, t_0)$ and taking the limit as $t_0 \to \infty$, I obtain

$$\lambda_K(t) = -\int_t^{\infty} e^{-\rho(s-t)} q_B(s) x'(K(s))\, \mathrm{d}s \tag{17}$$

The assumptions on $x(\cdot)$ and (4c) imply that $K \to \infty$, and $\lim_{t\to\infty} x'(K(t)) = 0$. Denoting the quantity of backstop resource consumed at the minimum price by $\overline{q}_B$, $\lambda(t) < \int_t^{\infty} e^{-\rho(s-t)} \overline{q} x'(K(t))\, \mathrm{d}s \to 0$. The assumptions on $c(\cdot)$ then dictate that also $d \to 0$.

A stage-two phase diagram in (K, λ_K)-space is presented in Figure 14a. $\dot{K} >$

0 for all $\lambda_K > 0$, with loci $\dot{K} = 0$ located at $\lambda_K = 0$. The loci of points $\dot{\lambda}_K = 0$ is illustrated; it is decreasing and approaches the K-axis asymptotically. The optimal path has to be sandwiched between the two loci. This path is unique. Suppose it weren't; then there would exist two paths $\lambda_K^1(K)$ and $\lambda_K^2(K)$, both asymptotically converging to the K-axis. Suppose $\lambda_K^1(\tilde{K}) > \lambda_K^2(\tilde{K})$, for some $\tilde{K}$. As K increases, the vertical distance between the two paths would have to decrease. However, at $\tilde{K}$

$$\begin{aligned}\frac{\mathrm{d}(\lambda_K^1 - \lambda_K^2)}{\mathrm{d}K} &= \frac{\dot{\lambda}_K^1}{\dot{K}^1} - \frac{\dot{\lambda}_K^2}{\dot{K}^2} \\ &= \frac{\dot{\lambda}_K^1}{r^1} - \frac{\dot{\lambda}_K^2}{r^2} > 0\end{aligned}$$

as both terms are negative, and decreasing in absolute value with λ_K. Hence the paths would diverge, while converging towards zero—a contradiction. □

Proof of Proposition 2. The backstop will always be used eventually as $u'(0) > x(0)$. Note that this implies that $\lambda_K(0) > 0$; otherwise the costate variable will become negative and the transversality condition (4g) is not met (note that $x'(\cdot) < 0$). For the same reason, $\lambda_K(t) = 0$ is only possible for $t \geq t^{**}$ (in fact, integrating (4e), one confirms that, if t^{**} is finite, then $\lambda(t) = 0$ for $t \geq t^{**}$). But then, due to the assumptions on $c(\cdot)$, research takes place at all times until the attainment of the lower bound (if ever): $d > 0$ for all $t < t^{**}$.

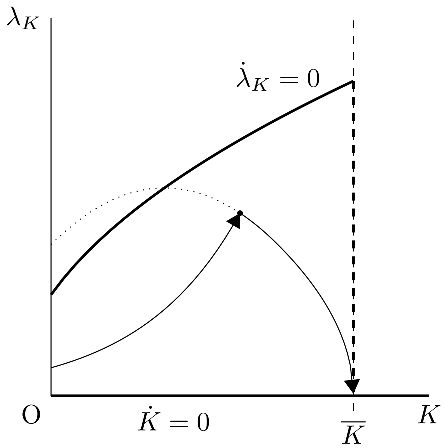
Suppose there is an interval of time of non-zero length such that both resources are used simultaneously. Then, from the first-order conditions, during this period $\lambda_S = x$. Taking time derivatives and using (4d), $0 \leq \rho\lambda_S = x'(K)d < 0$ which is a contradiction. Hence, there cannot exist an interval during which both resources are used.

That the exhaustible resource will be used up entirely is immediately implied (4a) and (4f). Marginal utility of resource consumption increases in stage one; in stage two, as the backstop cost decreases, marginal utility decreases. This yields the monotonicity properties of resource use over time. Prior to the time of switch t^*, $\dot{\lambda}_K > 0$ (as $q_B = 0$). This yields the monotonicity of R&D intensity prior to the switch.

In (K, λ_K)-space, following exhaustion, we have

$$\left.\frac{\mathrm{d}\lambda_K}{\mathrm{d}K}\right|_{q_B>0} = \frac{\dot{\lambda}_K}{\dot{K}} = \frac{\rho\lambda_K + p^{-1}(x(K))x'(K)}{(c')^{-1}(\lambda_K)} \leq \frac{\rho\lambda_K}{(c')^{-1}(\lambda_K)} = \left.\frac{\mathrm{d}\lambda_K}{\mathrm{d}K}\right|_{q_B=0}$$

and so the path will lie below the terminal path in (K, λ_K)-space (Figure 14a). □



(a)

Figure 14: Behaviour of the economy in (K, λ_K)-space; illustrated the case in which the lower bound $\underline{x}$ is attained in finite time. Along the terminal path, the economy approaches $(\overline{K}, 0)$. Before this, the economy has reached the terminal path at some finite date, prior to which it lies on a path below the terminal path. A higher knowledge stock at the switching date $K(t^*)$ implies a lower R&D intensity path before the switch. The knowledge stock at the switching date is determined by the resource constraint.

Proof of Proposition 3. Note first that, for any given K, $\lambda_K > 0$, an increase in ρ increases the slope of the phase arrows in (K, λ_K)-space: $\dot{K}$ remains unchanged, but $\dot{\lambda}_K$ strictly increases (Figure 15a). This further implies that the new terminal path will lie strictly below the old terminal path. Both have to end at $(\overline{K}, 0)$. Suppose the new terminal path would, somewhere, lie (weakly) above the old one. Then it would be impossible for the terminal path to arrive at the required point.

Take optimal paths A and B such that $\rho_A < \rho_B$. Suppose $K_B(t_B^*) \geq K_A(t_A^*)$. Then it is immediate from the phase diagram that $d_B(0) < d_A(0)$, and that $t_A^* < t_B^*$. Now note that the marginal utility of consuming fossil fuels also has to rise at a higher rate, and terminate at $x_B(t_B^*) \leq x_A(t_A^*)$. This implies that the marginal utility will always be lower along B than along A, that is extraction rates have to be always higher; and for a longer time. This will break the resource constraint. Hence, $K_B(t_B^*) < K_A(t_A^*)$.

Suppose $\frac{\mathrm{d}t^*}{\mathrm{d}\rho} < 0$. Then $\lambda_S(0)$ has to fall with ρ; if not, then there will be less resource extraction at all times, for a shorter period of time, and the resource constraint is not satisfied.

Suppose $\frac{\mathrm{d}t^*}{\mathrm{d}\rho} > 0$. Now, from the phase diagram, it is obvious that if $\lambda_K(0)$

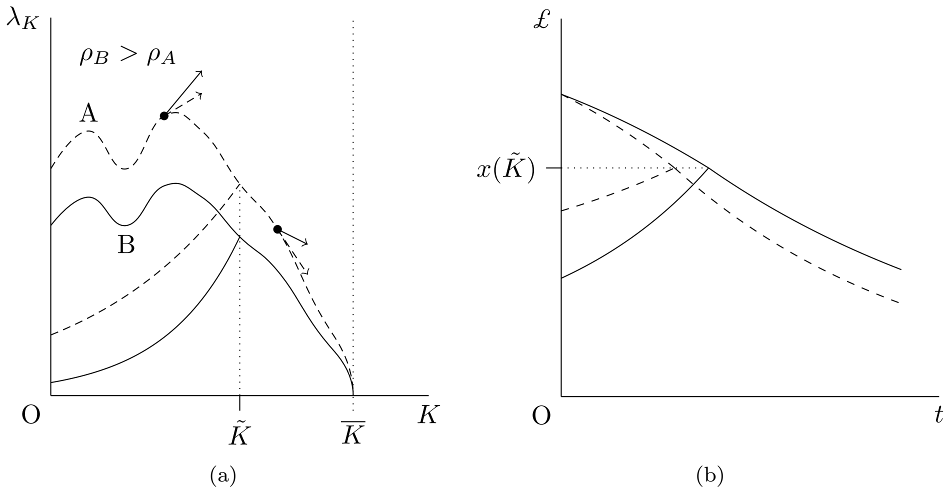


Figure 15: An increase in ρ leads to the terminal path contracting down and the phase arrows all skewing up (15a). A (weakly) lower knowledge stock at switching time would imply higher t^* and higher extraction (lower marginal utility) at all moments, breaking resource constraint (15a).

were to rise with ρ, the terminal path would be hit more quickly—a contradiction.

Thus at least one of the capital stocks must fall in terms of the initial shadow values; in fact, both may do so. This means that either the initial R&D rate or the initial extraction rate (or both) have to fall. □

Proof of Proposition 4. By arguments employed in the proof of Proposition 3, for two equilibria A and B which do not vary in the terminal path (i.e. which have identical discount rates, R&D cost functions and backstop technologies)

$$K_A(t_A^*) \geq K_B(t_B^*) \Leftrightarrow t_A^* \geq t_B^*, \quad q_A(t^*) \geq q_B(t_B^*), d_A(0) < d_B(0)$$

Suppose A and B vary only in terms of initial resource stock: $S_A(0) > S_B(0)$, $d_A(0) \geq d_B(0)$. Then the path $q_A(t) < q_B(t)$ for all $t \in [0, t_A^*]$ and the resource constraint is broken. Hence it must be that $d_A(0) < d_B(0)$ and $q_A(0) > q_B(0)$.

Suppose instead that A and B vary only in terms of the initial knowledge stock: $K_A(0) \geq K_B(0)$. Then if $q_A(0) \leq q_B(0)$, the price of the exhaustible resource will hit the backstop price earlier: $t_A^* < t_B^*$, and again not all of the resource is used up in A. Hence $q_A(0) \geq q_B(0)$. Similar claims are not applicable for the R&D process. □

B Open-loop equilibrium

Proof of Proposition 5. All costate variables are denoted by the same symbols as for the social planner's problem, but now represent the marginal value of the stocks to their respective 'owners'.

Necessary conditions for a solution to the exporter's problem are

$$R'(q_F) = p'(q_F)q_F + p(q_F) \leq \lambda_S, \qquad q_F \geq \max\{0, q^{-1}(x)\}, \quad \text{C.S.} \tag{18a}$$

$$\dot{\lambda}_S = \rho\lambda_S \tag{18b}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_S(t)S(t) = 0 \tag{18c}$$

$$\lambda_S(T) = x(K(T)) \tag{18d}$$

where T denotes the time at which the resource is exhausted. Equation (18a) is just the Hotelling Rule for the monopolist: marginal revenue $R'(q_F)$ has to equal the scarcity rent (which increases at the discount rate). The entire stock has to be exhausted eventually; the optimal date of exhaustion is given by (18d) (following from the condition that the Hamiltonian equal zero at the date at which the resource is used up). The marginal revenue is the discounted price at which the very last unit can be sold, at the end of the limit-pricing stage.

The resulting solution has three stages. For $t \in [0, t^*)$, the price of the resource is strictly below the backstop cost and only the exhaustible resource is consumed. If resource stocks are low, or if resource demand is inelastic for the relevant range, this stage is degenerate (Hoel (1978)). In the second stage, for $t \in [t^*, T)$, the resource price equals the backstop cost but only the exhaustible resource is consumed. The costate trajectory is continuous and so $p(q_F(t^*)) = x(K(t^*))$. Finally, from $t = T$, the exhaustible resource has been used up and only the backstop resource is consumed. The exporter has nothing further to do as the extraction rate is constrained to zero.

Turn now to the importer's problem. The discontinuity in backstop demand, with respect to the knowledge stock ((1) and (2)), poses a problem: namely, that the objective function in (6) is not differentiable with respect to K at $x(K) = p^{-1}(q_F)$. Hence the Maximum Principle must be modified, as by Hartl and Sethi (1984), to obtain necessary conditions:

$$c'(d) \leq \lambda_K, \quad d \geq 0, \quad \text{C.S.} \tag{19a}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_K(t)K(t) = 0 \tag{19b}$$

$$\dot{\lambda}_K \in \begin{cases} \{\rho\lambda_K + q_B x'(K)\} & p(q_F) \neq x(K) \\ [\rho\lambda_K + q_F x'(K), \rho\lambda_K] & p(q_F) = x(K) \end{cases} \tag{19c}$$

In words, when the monopolist is limit pricing, then the time derivative of the costate variable can take any of a number of values. Notice that when there is no backstop use, λ_K rises at the discount rate. When there is only backstop consumption, this rate of increase is reduced by $|q_B x'(K)|$. A path of λ_K implies a path for $x(K)$; a higher $\dot{\lambda}_K$ corresponds to a faster decrease in $\dot{x} = x'(K)\dot{K} = x'(K)(c')^{-1}(\lambda_K)$, i.e. the backstop price slope falling faster. Integrated over some period, this implies the backstop price rising less or falling more steeply.

Once the backstop cost hits the resource price path, i.e. limit pricing begins, any equilibrium path will have to have $\dot{x} = \dot{p}_F \equiv p'(q_F)\dot{q}_F$ for some λ_K satisfying (19) above. This will ensure that limit pricing continues. Suppose q_F is, indeed, such that it is possible to find a λ_K which allows limit pricing to continue, and consider a candidate optimum in which λ_K would be such as to diverge from the limit pricing outcome. A lower λ_K would mean that the backstop price would immediately rise above the resource price; this would accelerate R&D effort again to bring λ_K back to the limit pricing path. The same argument holds for a λ_K inducing faster R&D; as soon as the backstop cost fell infinitesimally below the exhaustible resource price, R&D would slow down and the backstop price would immediately begin tracking the resource price—i.e. limit pricing.

Following the exhaustion of the resource at time T, the exporter ceases to play a role in the game and the importer behaves as the social planner. The final stage can be analysed as for the social optimum. The stages are tied together by continuity of K and λ_K at times t^* and T. □

Proof of Proposition 6. It is well-known that if $\epsilon'(q) \geq 0$, the rate of increase of the resource price is greater than ρ. Suppose that the open-loop equilibrium path hits the terminal curve at a higher K than the socially optimal path: $K_S(t^*_S) < K_{OL}(t^*_{OL})$. Then, by arguments used in the previous proposition, $t^*_S < t^*_{OL}$; further, $q^S_F(t^*_S) < q^{OL}_F(t^*_{OL}) < q^{OL}_F(t^*_S)$ and the extraction path $q^{OL}_F(t)$ lies above $q^S_F(t)$. But the social optimum exhausts the entire stock by t^*_S, in which case the along open-loop trajectory exhaustion occurs before t^*_{OL}. Thus $K_{OL}(t^*_{OL}) < K_S(t^*_S)$, implying $d_{OL}(0) > d_S(0)$.

Suppose $\epsilon'(q) \leq 0$, so that $\dot{q}_F^{OL} < \rho q_F^{OL}$. If $q_F^{OL} \leq q_F^S(0)$, and $q_F^{OL}(t) > q_F^S(t)$ for $t > 0$, and again the resource stock is exhausted before t^*_{OL}. Hence $q_F^{OL}(0) > q_F^S(0)$. □

Proof of Proposition 7. With isoelastic demand $q = p^{-\frac{1}{\sigma}}$, and extending the argument used by Hoel (1978), as

$$q_F = \max\left\{\left(\frac{\lambda_S(t)}{1-\sigma}\right)^{-\frac{1}{\sigma}}, (x(K(t)))^{-\frac{1}{\sigma}}\right\}$$

(where the max operator captures the limit pricing behaviour), and as

$$\lambda_S(t) = e^{-\rho(T-t)} x(K(T)) \tag{20}$$

it follows that $e^{-\rho(T-t^*)} = (1-\sigma)\frac{x(K(t^*))}{x(K(T))}$. Suppose S_0 increases but $q_F(0)$ falls (weakly). Then t^* falls weakly; further, limit pricing begins at a lower $K(t^*)$, with $S(t^*)$ higher and T higher. This implies that the RHS of (20) (less than one) will be greater, and so $T - t^*$ is lower—a contradiction. Hence $q_F(0)$, $K(t^*)$, and $d(0)$ all increase. □

Proof of Proposition 1. The boundary will satisfy, with equality, conditions

$$MR(p^{-1}(x(K)) = e^{-\rho(T-t^*)}$$
$$\int_{t^*}^{T} p^{(}-1)(x(K(t)))\,\mathrm{d}t = S$$

for any t^*. The sign of the slope of the boundary is easily obtained by the implicit function theorem. □

Proof of Proposition 8 (sketch). One only has to confirm that, given that the other player is following the open-loop strategy (R&D according to the terminal path for the importer, limit-pricing for all K for the exporter), the optimal response is indeed that player's own open-loop strategy. □

C Equilibrium with externality and extraction costs

Proof of Proposition 9. TO BE WRITTEN—STRAIGHTFORWARD. □

Proof of Proposition 10. The Hamiltonian for this problem is

$$\mathcal{H} = u(p^{-1}(x)) - x(K)p^{-1}(x) - c(d) + \lambda_K d - (\lambda_S - \lambda_G)p^{-1}(x)$$

assuming limit-pricing begins immediately. The necessary conditions are, from Note 2, Chapter 2.2 in Seierstad and Sydsæter (1987):

$$c'(d) = \lambda_K \tag{21a}$$

$$\dot{\lambda}_K = \rho\lambda_K + x'(K)\left(p^{-1}(x) + (\lambda_S - \lambda_K)(p^{-1})'(x)\right) \tag{21b}$$

$$\dot{\lambda}_S = \rho\lambda_S \tag{21c}$$

$$\dot{\lambda}_G = \rho\lambda_G + Z'(G) \tag{21d}$$

$$\lambda_K(T) = \lambda_K^\infty(K(T)) - \mu x'(K) \tag{21e}$$

$$\lambda_S(T) = \mu C'(S(T)) \tag{21f}$$

$$\lambda_G(T) = -\frac{Z'(G)}{\rho} \tag{21g}$$

$$\mathcal{H}(T) = \rho\left(\pi^\infty(K(T)) - \frac{Z(G)}{\rho}\right) \tag{21h}$$

where μ is a shadow value related to the constraint $C(S(T)) = x(K(T))$. Equation (21h) yields the optimal stopping time T. Note that for the terminal path, this holds for all K with $\lambda_S = \lambda_G = 0$, and $\lambda_K = \lambda_K^\infty(K)$, $d = d^\infty(K)$. Hence

$$\begin{aligned} \lambda_K(T)d(T) - c(d(T)) - (\lambda_K^\infty(K(T))d^\infty(K(T)) - c(d^\infty(K(T)))) \\ = (\lambda_S(T) - \lambda_G(T))\, p^{-1}(x(K(T))) \end{aligned} \tag{22}$$

Using the first-order condition on d, the function $\Phi(d) \equiv c'(d)d - c(d) = \lambda_K d - c(d) =$ is increasing in d. The above statement thus relates the difference between $\Phi(d(T))$ and $\Phi(d^\infty(K(T))$ to the reduction in welfare caused by pollution at the moment of exhaustion. Note that $\lambda_G(T) < 0$.

I will now argue that $d(T) > d^\infty(K(T))$. This follows if $\lambda_S(T) < 0$, so that $\mu > 0$: then $\lambda_K(T) - \lambda_K^\infty(K(T)) = -\mu x'(K) > 0$, which yields the result. Then, from (22), $\lambda_S(T) - \lambda_G(T) > 0$, i.e. $\lambda_G(T)$ is more negative than $\lambda_S(T)$. This makes intuitive sense: the welfare impact of having more of the exhaustible resource lies in the fact that a part of it will eventually become a pollutant—but only part, and only eventually.

To complete the argument, suppose that $\lambda_S(T) \geq 0$, so that $\mu \leq 0$. Then, for sure, the LHS of (22) is positive, so that $d(T) > d^\infty(K(T))$; but from the transversality condition on $\lambda_K(T)$, the opposite must hold—a contradiction.

I will finally establish the property that the optimal trajectory crosses the terminal path once at most; and, so, that there are two distinct phases of R&D, the first (if it exists) with R&D lower, and the latter with R&D higher, than in the reference equilibrium. Note that $\lambda_S - \lambda_G$ has a positive sign. Now, suppose there exists a point in time when the optimal trajectory coincides with the terminal path. Then, along the optimal path, $\dot{\lambda}_K$ must be higher (comparing the equations of motion for λ_K) while clearly the R&D rate, and hence $\dot{K}$, is equal in both cases. Thus the optimal trajectory will cross to above the terminal path and stay there until exhaustion. □

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