Environmental Policy and Directed Technical Change in a Global Economy: Is There a Case for Carbon Tariffs?

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Abstract

In an open economy, can unilateral environmental policies undertaken by a group of committed countries ensure sustainable growth? This paper addresses this question in a dynamic model with directed technical change. There are two countries and two tradeable goods. One of the two goods (the polluting good) is produced with a clean input and a dirty input, which causes a global externality. Innovation can be targeted at both sectors and, within the polluting sector, at clean or dirty technologies. For most of the analysis, innovation is local. I show that carbon taxes in a single country are generally unable to ensure sustainable growth, that is, to prevent environmental quality from falling below some critical threshold. A temporary combination of clean research subsidies and a tariff in a single country can ensure sustainable growth for sufficiently large initial quality of the environment - in some cases, clean research subsidies alone may not do so. I characterize the first best policy, the world optimal policy under the constraint that one country must be in laissez-faire, and the optimal policy from the point of view of a single country. Calibrated numerical simulations show that, relative to autarky, trade accelerates environmental degradation, but that when one country undertakes the appropriate policies, trade helps reducing environmental degradation. Finally, I add knowledge spillovers and show that carbon taxes in a single country are still generally unable to ensure sustainable growth.

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1 Introduction

Countries not subject to any binding constraints under the Kyoto protocol account for an increasing fraction of CO_2 emissions; their share in world emissions has risen from a third in 1990 to more than a half in 2008. In the meantime, climate negotiations have stalled, and no global agreement seems to be in sight. In response, several countries either have undertaken unilateral actions or are considering doing so, and more and more, these policies harbor some protectionist aspects. For instance, the American Clean Energy and Security Act, which was supposed to set up a cap-and-trade system in the US, planned to implement trade barriers with countries that did not have a similar system in the absence of an international agreement by 2018.[1] This raises two questions: First, can unilateral policies be enough to ensure sustainable growth? Second, how necessary is protectionism to achieve this goal?

Fundamentally, these questions are about the long-run behavior of the economy, and addressing them requires understanding the dynamic evolution of comparative advantage and its interaction with environmental policies. This paper tries to shed light on the latter by integrating directed technical change in a trade model with a global pollution externality. It first shows that unilateral environmental policies that have no protectionist component (like a carbon tax) typically fail at ensuring sustainable growth, as they generally cannot reduce emissions in the long-run. This occurs because production of the polluting good gets reallocated from intervening to non-intervening countries through the well-known "pollution haven effect." The innovation response amplifies this pollution haven effect and these policies are likely to accelerate environmental degradation. Second, the paper shows that intervening countries can achieve sustainable growth without cooperation from the rest of the world by implementing a temporary industrial policy which combines clean research subsidies and a tariff. Such a policy develops clean technologies in the polluting sector in the intervening countries, so that emissions can be reduced while the intervening countries acquire a comparative advantage in the polluting sector. As a consequence, production and innovation in non-intervening countries can be redirected away from the polluting sector, ensuring a decrease in emissions in the non-intervening countries as well.

More formally, I consider a dynamic version of a two country (North and South), two sector and two factor Heckscher-Ohlin model - with another factor, scientists, used for innovation. The North represents countries willing to implement an environmental policy (the intervening countries), while the South represents countries that do not undertake any policy. One sector

[1] In this case, the trade barrier was an international reserve allowance. The bill passed the House in 2009 but was rejected by the Senate. Trade barriers have also been discussed for the European Union Emissions Trading System (EU ETS), but have not been imposed yet. However, the EU ETS is going to be extended to air transport from January 2012, and should concern all airlines, making it the first attempt at taxing foreign firms for pollution.

never pollutes, while the other sector can be more or less polluting depending on the balance between dirty and clean technologies in the country. In practice, the polluting sector includes the manufacturing of chemicals and chemical products, of non-metallic mineral products and of basic metals; clean technologies might correspond to renewable energy or bioplastics. Innovation can be directed at the non-polluting or the polluting sector. Within the polluting sector, it can be targeted at the clean or the dirty technologies. The allocation of innovation between the two sectors depends on the relative size of both sectors in the country as measured by their revenue share (Acemoglu (1998)). Since the exporting sector has a relatively larger market size than the importing sector, innovation is tilted towards the exporting sector in comparison to the other country, which creates a force towards the amplification of comparative advantage over time. Within the polluting sector, the allocation of innovation between the clean and dirty technologies is tilted towards the most advanced of the two: there is path dependence in innovation. For the main part of the analysis, innovation is assumed to be completely local.

In laissez-faire, if clean technologies are initially less advanced than dirty ones in both countries, innovation is continually directed mostly towards the dirty technologies. Emissions keep increasing and the economy eventually reaches an "environmental disaster" as the quality of the environment falls below a critical threshold. In other words, economic growth is not sustainable. Carbon taxes or taxes on dirty research in the North (which have no protectionist aspects) increase the costs of producing the polluting good in the North; therefore, they can only reinforce a potential initial comparative advantage of the South in the polluting sector. In this case, such policies cannot reverse the process of increased specialization of the South in the polluting sector over time, and thus they are unable to prevent an environmental disaster. As the reallocation of production goes hand in hand with a reallocation of innovation towards the polluting sector in the South, and as innovation in the polluting sector in the South will be mostly directed at dirty technologies, such non-protectionist policies are likely to accelerate environmental degradation. In contrast, temporary clean research subsidies in the North can redirect innovation from both dirty technologies and the non-polluting sector towards clean technologies in the North. A temporary policy that combines clean research subsidies with a tariff allows the North to develop a comparative advantage in the polluting sector at the same time as this sector is becoming cleaner. Once clean technologies in the North are sufficiently advanced, and the initial comparative advantage is reversed, market forces that were previously driving the economy into a disaster now work towards averting it: emissions decrease both in the North (as innovation keeps being directed to clean technologies) and in the South (as the South specializes over time in the non-polluting sector). If the initial environmental quality is sufficiently large, an environmental disaster will be averted. One may think that such a reversal of comparative advantage could be ensured with clean research subsidies only, but this is not always true. Under free trade, the South may fully specialize, in which case all of its innovation will be directed towards the polluting sector, such that the North will not be able to acquire a comparative advantage in the polluting sector. A tariff can prevent such an outcome. Overall, to avert a disaster without the cooperation from the South, the North must enact policies that affect the evolution of comparative advantage in a way that the South ends up specializing in the non-polluting sector. Directed technical change is essential for this result: with exogenous technical change, policies in the North only may fail at preventing a disaster - no matter how large the initial quality of the environment is.

I characterize the world first best policy and second best policy under the constraint that no intervention can occur in the South. This second best policy can be decentralized through a carbon tax and research subsidies in the North, along with a trade tax on the polluting good, which typically takes the initial form of a tariff and then of an export subsidy.[2] The expression for the optimal trade tax reflects two objectives for the social planner: reducing emissions in the South and redirecting Southern innovation towards the non-polluting sector. I also derive the optimal policy for a Northern social planner and show that the same instruments are used, but in this case, the tariff also reflects terms of trade considerations.

I carry out a simple calibration exercise which illustrates the main results of the paper. It shows that for reasonable parameter values, the welfare costs of not being able to implement any policy in the South are very large. It highlights the double-edged nature of both trade and directed technical change: both accelerate environmental degradation under laissez-faire, but help reduce environmental degradation when one country intervenes.

Finally, I relax the assumption that knowledge is purely local. First, I look at the impact of technological diffusion through spillovers by letting the more backward country catch up partially. In this case, a combination of clean research subsidies in the North and a tariff can enhance clean technologies in the South to a sufficient extent that a switch towards clean innovation occurs there too. Therefore, an environmental disaster can be prevented without a reversal of comparative advantage when the South initially exports the polluting good. I show that the previous results are generalizable: a carbon tax may fail at preventing an environmental disaster and clean research subsidies alone might fail too, but a combination of clean research subsidies and a carbon tariff can prevent it. Second, I look at the case where innovating firms are global, and can (partially or fully) use an innovation developed in one country in the other country. I show that if innovation is sufficiently transferrable across countries, clean research subsidies in the North only - without a tariff - can induce a switch to clean innovation in the South, thereby preventing a disaster.

In its mechanism, this paper bears some resemblance to papers defending the infant industry

[2] Production relies on the presence of monopolistically produced intermediates, so there is also a subsidy to correct for the monopoly distortion.

argument that trade can be detrimental to growth by leading countries in the South to specialize in sectors with poor development prospects (Young (1991), Matsuyama (1992) and Galor and Mountford (2008)). Here as well, a country risks specializing in the "wrong" sector, not because that sector offers poor growth prospects, but because this country cannot prevent the environmental externality associated with the production in this sector. Chapter 8 of Grossman and Helpman (1991) builds a two country, two sector model where one sector is differentiated and features productivity growth with local knowledge. The authors show that a country with a comparative advantage in that sector will keep that comparative advantage forever in laissez-faire but that temporary research subsidies can allow the other country to catch up and eventually take the lead. Temporary research subsidies here play a similar role in reversing comparative advantage.[3]

The economic literature on trade and the environment has long recognized that in an open world, the effectiveness of unilateral policies in reducing world pollution can be hampered by the pollution haven effect - see for instance Pethig (1976), and empirical evidence can be found in Copeland and Taylor (2004). Markusen (1975) and Hoel (1996) show that the optimal instrument to address the pollution haven effect is a tariff. In the specific context of global warming, where the pollutant (CO_2) enters differently at several stages of the production process, several papers using computable general equilibrium models have attempted to track carbon through the global economy in order to determine the pattern of trade and compute the carbon leakage rate (that is the rate at which emissions abroad increase following a domestic reduction). It is generally agreed that the carbon contents of exports from developing countries to developed countries largely exceeds the carbon content of their imports.[4] Elliott et al. (2010) compute a carbon leakage rate of 20% from a reduction in Annex I countries (the countries with binding constraints under the Kyoto protocol), and show that border tax adjustments eliminate half of it.[5] The paper also relates to the literature on the impact of trade on the environment (see Copeland and Taylor (1995)): here in the absence of global cooperation, trade is necessary to avert an environmental disaster, but needs to be managed in order to deliver

[3] Models featuring trade and directed technical change also include Acemoglu (2003), who studies the impact of trade on the skill bias of technological change, and Gancia and Bonfiglioli (2008), who show that trade amplifies international wage differences.

[4] For instance, Atkinson et al. (2011) find that the net US imports of carbon from China in 2004 were of 244 millions of tons of CO_2, which represents 0.9% of total world emissions that year.

[5] Other studies delivering similar results include: Babiker and Rutherford (2005), Schenker and Bucher (2010), Böhringer, Fischer and Rosendahl (2010) or Böhringer, Carbon and Rutherford (2011). Introducing imperfect competition, Babiker (2005) finds a leakage rate greater than 100%, while, introducing the possibility for energy-saving innovation and international knowledge spillovers Gerlagh and Kuik (2007) find a negative rate. There are, however, few real empirical studies: Aichele and Felbermayr (2010) use a gravity model of trade and find that committing to the Kyoto protocol increases the carbon content of imports from not-committed countries by 10%, while Aichele and Felbermayr (2012) find that when committing to the Kyoto protoc countries reduce domestic CO_2 emissions on average by about 7% but that their overal CO_2 consumption does not change.

the right outcome.[6] Since this literature has focused on static models, it has fully ignored the evolution of comparative advantage over time and the dynamic consequences of policies. Therefore it could not study the long-run behavior of the economy and the sustainability of economic growth.

In fact, a growing literature has shown the importance of taking into account directed technical change when designing policies against global warming. On the empirical side, Popp (2002) shows that an increase in energy prices leads to more energy-saving innovation and Newell, Jaffe and Stavins (1999) find similar results, focusing on air conditioners. Aghion et al. (2011) focus on the car industry and establish both that an increase in fuel prices leads to clean innovation at the expense of dirty innovation, and that there is path dependence in clean versus dirty innovation (which is one of the features of my model). This empirical evidence has led to several theoretical papers integrating directed technical change in the study of climate change issues; here, I build specifically on the model developed by Acemoglu et al. (2011) (henceforth AABH).[7] The final good in AABH and the polluting sector in my paper are both produced with a clean and a dirty input, which are substitutes for each other, and, because of knowledge externalities of "the building on the shoulder of giants" type, there is path dependence in the direction of innovation (clean or dirty). AABH focus primarily on the single country case but the working paper already presents an international version of the model. Trade is between the clean and dirty inputs (which are substitutes), while in my current paper, it is between two sectors which are complements or Cobb-Douglas in consumption. In my model, innovation can reduce the emission rate in the polluting sector, so that a country can build a comparative advantage in the polluting sector while reducing its emissions; this is impossible in AABH. Moreover, AABH rule out innovation in the South. Maria and Smulders (2004) and Maria and van der Werf (2008) have also tackled the issue of modeling the interaction between directed technical change and international trade, but neither allow for innovation within one of the traded goods. In other words, these models study the allocation of innovation between an energy intensive sector and a non-energy intensive sector, but ignore that within the energy intensive sector innovations could be pollution-enhancing or saving.[8] I further discuss the differences between this approach and mine at the end of section 2.

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[6] Empirical studies (Antweiler, Copeland and Taylor (2001) or Frankel and Rose (2005)) point towards a positive effect of trade on the environment for local pollutents but the effect for CO_2 emissions is not clear.

[7] Earlier work on the environment and directed technical change include Bovenberg and Smulders (1995), Bovenberg and Smulders (1996), Goulder and Schneider (1999), van der Zwaan et al. (2002), Popp (2004), Grimaud and Rouge (2008) and Aghion and Howitt (2009).

[8] In Maria and Smulders (2004) the North develops technologies that the South imitates, and opening up to trade leads to a reallocation of innovation towards the sector that the North exports. Carbon leakage is reduced when the goods are substitutes and amplified otherwise. In Maria and van der Werf (2008) both countries innovate and carbon leakage is always reduced by the innovation response to a cut in emissions in a single country. Golombek and Hoel (2004) study the interaction between environmental policy and innovation in an open world in a static model.

This paper is structured as follows: Section 2 presents the model, section 3 studies the laissez-faire equilibrium and which policies are able to ensure sustainable growth, section 4 solves for the world optimal policy, the world second best policy when the South is in laissez-faire, and for the optimal policy for the North, section 5 presents a stylized calibration, and finally, section 6 discusses how the main results are generalized when knowledge flows across countries. Appendix A presents some direct extension of the model, Appendix B contains the main proofs, Appendices C and D are available online.

2 Model

I consider an infinite-horizon version of a 2 country (North, N, and South, S), 2 sector (G and H), 2 + 1 factor (capital and labor plus scientists) Heckscher-Ohlin-Ricardo model, where sector G is similar to the economy of AABH. Time is discrete. Each country is endowed with a fixed amount of labor and capital: L_N, K_N and L_S, K_S, and there is a fixed mass one of scientists in both countries.

Preferences. In a given period the flow of utility for consumers in country X can be represented by an instantaneous utility function $u(C_{Xt}, S_t)$,where C_{Xt} is the quantity of final consumption in country $X \in \{N, S\}$ and S_t is the quality of the environment (identical in the North and in the South - the externality is global). Final consumption is a CES aggregate of the consumption of two goods denoted G and H:

$$C_{Xt} = \left(\nu C_{XGt}^{\frac{\sigma-1}{\sigma}} + (1-\nu)\, C_{XHt}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where C_{XYt} represents the quantity of good $Y \in \{G, H\}$ consumed in country $X \in \{N, S\}$. σ is the elasticity of substitution between goods G and H. I restrict attention to the case where the two goods are either complements ($\sigma < 1$) or where final consumption is Cobb-Douglas ($\sigma = 1$), so that both goods are essential for final consumption. Goods G and H will be the only goods traded internationally. Good G represents the traded goods responsible for greenhouse gases emissions (in particular, energy intensive goods), and good H, the traded goods which do not generate emissions. In the calibration section, good G will be identified with manufacturing of chemicals and chemical products (ISIC code 24), of other non-metallic mineral products (26) and of basic metals (27), and good H will be the rest of manufacturing.

Production. Good H in country X is produced competitively according to

$$Y_{XHt} = \left(\int_0^1 A_{XHit} x_{XHit}^{\gamma} di\right) \left(K_{Xht}^{\beta} L_{Xht}^{1-\beta}\right)^{1-\gamma}, \tag{2}$$

where K_{Xht} (respectively L_{Xht}) is the capital (respectively labor) hired in the assembly of good H in country X, x_{XHit} is the quantity of intermediates i hired in sector H, and A_{XHit}

is the productivity of intermediate i, specific to the country and the sector. γ represents the factor share of intermediates. Intermediates are produced monopolistically according to

$$x_{XHit} = \psi K_{XHit}^{\beta} L_{XHit}^{1-\beta}, \tag{3}$$

where K_{XHit} (respectively L_{XHit}) is the capital (respectively labor) hired in the production of intermediate i for good H in country X. Intermediates cannot be traded internationally. As the same factor share is used in the production of intermediates and in the final assembly of the good, $\beta \in (0,1)$ is the overall factor share of capital in sector H. I define total employment of capital in sector H (the sum of capital hired to produce good H and the intermediates specific to good H) as:

$$K_{XHt} \equiv K_{Xht} + \int_0^1 K_{XHit} di, \tag{4}$$

similarly L_{XHt} is total employment of labor in sector H in country X.

Good G is produced competitively with a "clean" input Y_{Xct} and a "dirty" input Y_{Xdt} according to:

$$Y_{XGt} = \left(Y_{Xct}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{Xdt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{5}$$

where $\varepsilon > 1$ is the elasticity of substitution between the clean and the dirty input (I will also consider the case $\varepsilon = \infty$, when the two inputs are perfect substitutes). The dirty input is the source of pollution. Both inputs are produced competitively according to:

$$Y_{Xzt} = \left(\int_0^1 A_{Xzit} x_{Xzit}^{\gamma} di \right) \left(K_{Xzt}^{\alpha} L_{Xzt}^{1-\alpha} \right)^{1-\gamma} \text{ for } z \in \{c, d\}, \tag{6}$$

where K_{Xzt} (respectively L_{Xzt}) is the capital (respectively labor) hired in the assembly of input z in country X, x_{Xzit} is the quantity of intermediates i hired in sector z, and A_{Xzit} is the productivity of intermediate i. Both clean and dirty intermediates are produced monopolistically according to:

$$x_{Xzit} = \psi K_{Xzit}^{\alpha} L_{Xzit}^{1-\alpha}, \tag{7}$$

so that $\alpha \in (0,1)$ represents the total factor share of capital in sector G. The share of intermediates γ is the same for the clean input, the dirty input and sector H (so that the monopoly distortion only has a scale effect and does not affect the pattern of comparative advantage). I assume throughout that $\alpha > \beta$, which is true empirically: the most polluting sectors tend to be more capital intensive. This is without loss of generality, everything is identical when $\alpha < \beta$, and the analysis can be extended to a pure Ricardian model with $\alpha = \beta$.[9] I also define K_{XGt} as the total employment of capital in sector G:

$$K_{XGt} \equiv K_{Xct} + K_{Xdt} + \int_0^1 K_{Xcit} di + \int_0^1 K_{Xdit} di, \tag{8}$$

[9] The only issue is that when the initial difference in comparative advantages is too small, it will be impossible to rule out multiple equilibria with different patterns of comparative advantages.

and define similarly L_{XGt} the total employment of labor in sector G in country X. I could have alternatively assumed that good G was produced with capital, labor and a CES aggregate of clean and dirty intermediates without affecting the results qualitatively for a sufficiently large elasticity of substitution ε. In practice the clean input models non-polluting inputs that could substitute for polluting inputs, for instance renewable energies to replace fossil fuel energy, or bioplastics to replace traditional petroleum products (the functional form is discussed in greater extent in AABH).

Market clearing for each factor in each country requires that:

$$K_{XGt} + K_{XHt} \leq K_X \text{ and } L_{XGt} + L_{XHt} \leq L_X, \tag{9}$$

and market clearing for each good requires that:

$$C_{NGt} + C_{SGt} \leq Y_{NGt} + Y_{SGt} \text{ and } C_{NHt} + C_{SHt} \leq Y_{NHt} + Y_{SHt}. \tag{10}$$

Environment. The quality of the environment S_t evolves according to

$$S_t = (1 + \Delta) S_{t-1} - \xi (Y_{dNt} + Y_{dSt}), \tag{11}$$

whenever the right hand side belongs to $(0, \overline{S})$, it is otherwise bounded above by $\overline{S}$, the pristine level of environmental quality, and below by 0. ξ measures the rate of environmental degradation from the production of dirty input, and Δ is the regeneration rate of the environment. Without loss of generality, I assume that $S_0 = \overline{S}$. Although some results require environmental regeneration, the exact law of motion is not crucial (the only important assumption is that if total emissions $\xi (Y_{dNt} + Y_{dSt})$ become arbitrarily large, environmental quality goes to 0). Note that 0 is an absorbing state for the quality of the environment: if the economy reaches that point in finite time, it will stay there forever. I refer to such an event as "an environmental disaster," or as "non-sustainable" economic growth.

Innovation. At the beginning of every period, entrepreneurs are allocated monopoly rights on the production of intermediates such that each entrepreneur holds monopoly rights on the production of a finite number of intermediates for one period only. Moreover, entrepreneurs can hire scientists to increase the productivity of their variety. By hiring s_{Xzit} scientists, the entrepreneur holding monopoly rights on variety i in sector $z = H$ or subsectors $z \in \{c, d\}$ can increase the initial productivity $A_{Xi(t-1)}$ of his intermediate to:

$$A_{Xzit} = \left(1 + \kappa s_{Xzit}^{\iota} \left(\frac{A_{Xz(t-1)}}{A_{Xzi(t-1)}}\right)^{\frac{1}{1-\gamma}}\right)^{1-\gamma} A_{Xzi(t-1)}, \text{ for } z \in \{c, d, H\}, \tag{12}$$

where $0 < \iota < 1$, so that the innovation function κs^{ι} is increasing, concave and satisfies the Inada conditions (the analysis could be generalized to innovation functions of the form

$\kappa \left((s_{Xzit} + \Upsilon)^{\iota} - \Upsilon^{\iota} \right) \left(\frac{A_{Xz(t-1)}}{A_{Xzi(t-1)}} \right)^{\frac{1}{1-\gamma}}$, with $\Upsilon > 0$, which do not satisfy Inada condition, such generalization will be useful in section 6). The concavity of the innovation function represents decreasing return to scale in innovation during a single period (the more scientists innovate on a particular technology during one period, the more they may reproduce the same type of innovation). A_{Xzt} is the average productivity of (sub)sector $z \in \{c, d, H\}$, at time t, defined as:

$$A_{Xzt} \equiv \left(\int_0^1 A_{Xzit}^{\frac{1}{1-\gamma}} di \right)^{1-\gamma} \text{ for } z \in \{c, d, H\}. \tag{13}$$

The factor $A_{Xzi(t-1)}^{-\frac{1}{1-\gamma}}$ represents decreasing return to scale in innovation (the more advanced is a technology the more difficult it is to further innovate), $A_{Xz(t-1)}^{\frac{1}{1-\gamma}}$ represent knowledge spillovers from all the other intermediates in the same sector in the same country. These two last effects exactly compensate each other and do not create any inefficiency in the economy. This formulation ensures that the innovation decision remains symmetric across varieties and that aggregate productivity grows exponentially for a given mass of scientists working in the (sub)sector. In both countries, there is a mass 1 of scientists, and market clearing requires that:

$$\int_0^1 (s_{XHit} + s_{Xcit} + s_{Xdit})\, di \leq 1. \tag{14}$$

Since entrepreneurs have monopoly rights for one period only, they will hire scientists so as to maximize their current profits instead of the entire flow of profits generated by their innovation. The allocation of scientists across (subsectors) is therefore "short-sighted". One period monopoly rights are the only inefficiency in innovation[10] and allow to model in the simplest way the "building on the shoulder of giants" externality. The existence of this externality has long been recognized by the endogenous growth literature. In the specific context of climate change it plays a crucial role in explaining why clean technologies have failed to really take off so far, and why direct research incentives on top of carbon taxes are welfare improving (this is the point made by AABH). As will become clearer below, one period monopoly rights also help ensuring that the equilibrium is unique. With permanent monopoly rights and no environmental externality, the efficient innovation allocation would be an equilibrium but not the unique one (unless the discount rate is very large).

Further, note that there are no technological spillovers between countries and that technologies are country specific. Section 6 analyzes alternative scenarios. A fixed mass of scientists in both countries allows me to focus on the direction of technical change only, and an equal mass in both countries ensures that one country does not become arbitrarily large relative to the other one (this assumption is relaxed in Appendix A).

[10] When the environmental externality is corrected for, the allocation of innovation maximizes the welfare of an infinitely impatient agent every period.

Finally, it is important that innovation can occur in all three (sub)sectors. If innovation were limited to clean and dirty technologies within the polluting sector, the North could not build a comparative advantage in a specific sector which will be crucial here. With clean innovation in the polluting sector only, the model would ignore all innovations that are directed at increasing productivity without decreasing emissions - to some extent this is what Maria and Smulders (2004) and Maria and van der Werf (2008) do - while, in fact, this is still the bulk of innovation in manufacturing: Aghion et al. (2011) show that in the car industry there are more "dirty" innovations than "clean" ones. If, on the contrary, only dirty innovations were available in the polluting sector, it would not be possible to generate innovation to "replace" existing polluting technologies, as the polluting sector and the non-polluting sector are complements ($\sigma \leq 1$). In fact, the working paper version of AABH deals with such a case, where the two sectors are substitutes ($\sigma > 1$), but this may not be the best representation of inter-industry trade between basic metals and machinery, for instance.

Policy tools. Section 4 solves an optimal allocation problem under some constraints and does not restrict the range of instruments that a social planner can use. In the following section, however, I restrict attention to some specific policy instruments (which will later be part of the optimal solution). These policy instruments also turn out to be the types of policies that countries are implementing (or consider doing so). More specifically, I will introduce add-valorem taxes on the dirty input (τ_{Xt}), which are the equivalents of a carbon tax, and sector specific research subsidies or taxes on scientists (as add-valorem subsidies or taxes on the wage of scientists working in a specific sector).[11] In addition, when I look at unilateral policies for the North, I allow for an add-valorem trade tax on the polluting good G (because of Lerner symmetry this is without loss of generality, the trade tax could also be on the other good). Therefore, prices in the South are always equal to international prices: $p_{SGt} = p_{Gt}$ and $p_{SHt} = p_{Ht}$, while in the North the price of good H is also equal to the international price $p_{Ht} = p_{NHt}$, but the price of good G is given by $p_{NGt} = p_{Gt}(1 + b_t)$, where b_t is the tariff (or export subsidy). When the North is the only country intervening, I assume that trade balance must be maintained, where trade balance writes as:

$$p_{Gt}(Y_{SGt} - C_{SGt}) + p_{Ht}(Y_{SHt} - C_{SHt}) = 0. \tag{15}$$

Note that the tariff is not explicitly related to the carbon content of imports. When the South does not undertake any policy, relating it explicitly to the average carbon content of imports from a given country and in a given sector, would not change anything (each Southern firm being atomistic, its impact on average emission is infinitesimal and so its behavior will

[11] A country may also implement a subsidy to the use of all intermediates identical across every subsector to correct for the monopoly distortion. This subsdiy can differ across time and countries without affecting the following results, as it has a pure scale effect.

not affect the tariff it pays). To affect the behavior of Southern firms, the North would either need to know the carbon content of its imports individually for each exporting firm - which seems implausible - or the South would have to implement a policy in response to the North tariff. I come back to these issues in subsection **??**.

Overall, a policy is characterized by a sequence of add-valorem taxes on the dirty input τ_{Xt} in each country, a sequence of taxes and subsidies on scientists in every subsector, and a sequence of trade taxes b_t on the polluting good. All subsidies and taxes are financed (or rebated) through lump-sum taxation at the country-level.

3 Equilibrium

In this section I consider policy as a given.

Definition 1 *A feasible allocation is a sequence of demands for capital ($K_{Xht,}$ K_{XHit}, K_{Xct}, K_{Xcit}, K_{Xdt}, K_{Xdit}), demands for labor ($L_{Xht,}$ L_{XHit}, L_{Xct}, L_{Xcit}, L_{Xdt}, L_{Xdit}), demands for intermediates (x_{Xzit} for $z \in \{c, d\}, H$), demands for inputs (Y_{Xct}, Y_{Xdt}), goods production (Y_{XGt}, Y_{XHt}), demands for goods (C_{XGt}, C_{XHt}), research allocations (s_{Xzit} for $z \in \{c, d\}, H$) and quality of the environment S_t, such that in each period t and in each country $X \in \{N, S\}$, factor and good markets clear ((9), (14), and (10) hold).*

Definition 2 *For a given policy, an equilibrium is given by a feasible allocation and sequences of wages of workers (w_{Xt}), returns to capital (r_{Xt}), wages of scientists (υ_{Xt}), (consumer) prices for intermediates (φ_{Xzit} for $z \in \{c, d\}, H$), (producer) prices for clean and dirty inputs (p_{Xct}, p_{Xdt}), international prices of goods (p_{Gt}, p_{Ht}) for $X \in \{N, S\}$, such that (i) ($\varphi_{Xzit}, x_{Xzit}, s_{Xzit}, K_{Xzit}, L_{Xzit}$) maximizes profits by the producer of intermediate i in sector $z \in \{c, d, H\}$ in country X, (ii) L_{Xzt}, K_{Xzt} maximize the profits of the producer of good $z \in \{c, d, H\}$, (iii) Y_{Xct} and Y_{Xdt} maximize the profits of producer of good G, (iv) C_{XGt} and C_{XHt} maximize consumers' utility under the trade balance constraint, (v) (15) is satisfied.*

In the first subsection, I detail the behavior of the economy under laissez-faire; in particular, I explain the pattern of trade and the allocation of innovation across sectors. In subsection 3.2, I analyze how sustainable growth can be achieved with policy in both countries, and subsection 3.3 explains why taxing the polluting sector in the North only can fail at preventing a disaster. Subsection 3.4 explains how a disaster can be avoided with unilateral policies in the North, and subsection 3.5 discusses the results.

3.1 Laissez-faire

Trade pattern. This subsection analyzes the laissez-faire equilibrium, the full equilibrium pattern is derived in Appendix B.1 (and generalized to positive carbon taxes). First, in each country aggregate production in each sector is written as:

$$Y_{XGt} = \zeta A_{XGt} K_{XGt}^{\alpha} L_{XGt}^{1-\alpha} \text{ and } Y_{XHt} = \zeta A_{XHt} K_{XHt}^{\beta} L_{XHt}^{1-\beta}, \tag{16}$$

with $\zeta \equiv \frac{\gamma^{2\gamma}(1-\gamma)^{1-\gamma}}{(1-\gamma+\gamma^2)\psi^\gamma}$ and $A_{XGt} \equiv \left(A_{Xct}^{\varepsilon-1} + A_{Xdt}^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}$ is the average productivity of sector G when there is no carbon tax. This formulation highlights that, in a given period, the model collapses into a Heckscher-Ohlin model with varying productivity across countries. The South has the comparative advantage in the polluting good G and exports it if and only if

$$\left(\frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_S}{L_S} > \left(\frac{A_{NGt}}{A_{NHt}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_N}{L_N}. \tag{17}$$

Trade results both from Ricardian forces (relative productivity) and Heckscher-Ohlin forces (relative endowment). As long as the difference in comparative advantage is not too large, both countries will produce both goods, but once it becomes sufficiently large one country fully specializes and, if the difference in comparative advantage grow even further, both countries fully specialize. Emissions are given by: $E_{Xt} = \xi \left(\frac{A_{Xdt}}{A_{XGt}}\right)^{\varepsilon} Y_{XGt}$, so that the emission rate in the polluting sector increases in the relative productivity of the dirty technology A_{Xdt}/A_{Xct}. Over time, innovation will modify comparative advantage and the emission rate. I now describe how innovation gets allocated across the different subsectors in laissez-faire.

Allocation of innovation. The innovation decision results from two forces: path dependence in clean versus dirty technologies and amplification of comparative advantage. Entrepreneurs face a two stages problem. In the second stage, they choose prices in order to maximize their profits given their productivity. Post-innovation profits in sector $z \in \{c, d, H\}$ are given by (see Appendix B.2):

$$\pi_{Xzit} = (1-\gamma)\gamma \left(\frac{A_{Xzit}}{A_{Xzt}}\right)^{\frac{1}{1-\gamma}} p_{Xzt} Y_{Xzt}. \tag{18}$$

These profits are directly proportional to the revenues of their (sub)sector and increase with the productivity of their intermediate A_{Xzit}. In the first stage, entrepreneurs hire scientists to increase the productivity of their intermediate. Thanks to the knowledge spillovers (see 12), all monopolists in a given (sub)sector hire the same number of scientists, so that average productivity evolves according to:

$$A_{Xzt} = \left(1 + \kappa s_{Xzt}^{\iota}\right)^{1-\gamma} A_{Xz(t-1)} \text{ for } z \in \{c, d, H\}.$$

Path dependence in clean versus dirty technologies. First, I look at the allocation of innovation within sector G (in the case where some good G is produced in the country, otherwise $s_{Xct} = s_{Xdt} = 0$). Combining the first order conditions with respect to the number of scientists in the clean and dirty subsector (and assuming that some production takes place in sector G in country X), the allocation of scientists within sector G obeys:

$$\frac{s_{Xct}^{1-\iota}\left(1+\kappa s_{Xct}^{\iota}\right)}{s_{Xdt}^{1-\iota}\left(1+\kappa s_{Xdt}^{\iota}\right)}=\frac{p_{Xct}Y_{Xct}}{p_{Xdt}Y_{Xdt}}=\frac{A_{Xct}^{\varepsilon-1}}{A_{Xdt}^{\varepsilon-1}}. \tag{19}$$

The second equality arises from the demand equation for both inputs in sector G and the fact that their production technologies differ only by their level of productivity. The ratio of revenues in the clean over the dirty sector increases with the ratio of clean over dirty technologies. A larger average productivity leads to a larger market share for the input (so if $A_{Xdt} > A_{Xct}$, $Y_{Xdt} > Y_{Xct}$), it also makes the input more expensive ($p_{Xdt} < p_{Xct}$), but when the inputs are substitutes, this price effect is dominated by the market size effect. Thus, more scientists are allocated to the dirty subsector than to the clean subsector if and only if $A_{Xd(t-1)} > A_{Xc(t-1)}$, as long as the size of innovation κ is sufficiently small (otherwise there may be multiple equilibria when $A_{Xd(t-1)}$ and $A_{Xc(t-1)}$ are close to each other, see Appendix B.2). Hence, in the polluting sector G, in the absence of government intervention, innovation tends to be allocated to the sector already the most advanced: there is path-dependence.

Amplification of comparative advantage. Second, I look at the allocation of innovation across sectors (assuming that production holds in both sectors, otherwise the problem is trivial). Combining the first order condition with respect to the number of scientists in sector H and in subsectors c and d, I get:

$$\frac{s_{Xct}^{1-\iota}\left(1+\kappa s_{Xct}^{\iota}\right)+s_{Xdt}^{1-\iota}\left(1+\kappa s_{Xdt}^{\iota}\right)}{s_{XHt}^{1-\iota}\left(1+\kappa s_{XHt}^{\iota}\right)}=\frac{p_{XGt}Y_{XGt}}{p_{XHt}Y_{XHt}}. \tag{20}$$

Therefore, for given ratio of initial productivities within sector G ($A_{Xd(t-1)}/A_{Xc(t-1)}$ given), the larger the revenues in sector G relative to sector H, the more scientists will be allocated to sector G.

In autarky, using consumer demand:

$$\frac{p_{XHt}Y_{XHt}}{p_{XGt}Y_{XGt}}=\frac{1-\nu}{\nu}\left(\frac{Y_{XG}}{Y_{XH}}\right)^{\frac{1-\sigma}{\sigma}}, \tag{21}$$

so that if $\sigma = 1$, innovation remains balanced between the two sectors as the right-hand side term is a constant, while for $\sigma < 1$, innovation tends to occur in the smallest sector – that is the sector with relatively lower productivity – and therefore becomes balanced between the two sectors after a few periods where the laggard sector catches up. Since the two sectors are complements, innovation will not disappear in one sector over time (as it does in the case of clean versus dirty innovation).

Under free-trade, prices are equalized in the North and the South, so each country tends to innovate relatively more in the sector it exports (and does so at equal ratio of initial productivities within sector G). As more innovation in a sector leads to a larger comparative advantage in that sector, which typically prompts more innovation in the same sector in the first place, multiple equilibria could arise. With sufficiently small size of innovation κ, however, changes in productivities in the current period remain sufficiently small, that the innovation allocation problem will have a single solution.[12] Therefore:

Lemma 1 *If κ is small enough, $\varepsilon < \infty$, $\iota \geq 1/2$ and $\alpha \neq \beta$ the equilibrium is unique.*

Proof. See Appendix D.1 ■

From now on, I assume that κ is sufficiently small and $\iota \geq 1/2$[13] to ensure that the equilibrium is unique (uniqueness of the equilibrium extends beyond laissez-faire to any policy where the trade tax is null).[14] I can then derive:

Lemma 2 *In laissez-faire, if either the South initially has a weak comparative advantage in sector G $\left(\frac{A_{SG0}}{A_{SH0}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_S}{L_S} \geq \left(\frac{A_{NG0}}{A_{NH0}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_N}{L_N}$ and $\min\left(A_{Sc0}/A_{Sd0}, A_{Sd0}/A_{Sc0}\right) < \min\left(A_{Nc0}/A_{Nd0}, A_{Nd0}/A_{Nc0}\right)$, or if it has a strong comparative advantage $\left(\frac{A_{SG0}}{A_{SH0}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_S}{L_S} > \left(\frac{A_{NG0}}{A_{NH0}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_N}{L_N}$ and $\min\left(A_{Sc0}/A_{Sd0}, A_{Sd0}/A_{Sc0}\right)$ is sufficiently small, then every period more scientists are hired in sector G in the South than in the North: $s_{SGt} > s_{NGt}$; and, the South fully specializes in producing good G and the North in producing good H in finite time. If $\left(\frac{A_{SG0}}{A_{SH0}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_S}{L_S} = \left(\frac{A_{NG0}}{A_{NH0}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_N}{L_N}$ and $\min\left(A_{Sc0}/A_{Sd0}, A_{Sd0}/A_{Sc0}\right) = \min\left(A_{Nc0}/A_{Nd0}, A_{Nd0}/A_{Nc0}\right)$, there is no trade.*

Proof. See Appendix B.2 ■

Because countries tend to innovate more in the sector they export, the difference in the allocation of research across sectors builds up over time and the relative productivities of both sectors become so different that the two countries eventually fully specialize. Conditions on $\min\left(A_{Xc0}/A_{Xd0}, A_{Xd0}/A_{Xc0}\right)$ are necessary for the following reason: The incentive to innovate in sector G and the growth rate of the average productivity of sector G for a given mass of scientists in that sector depend on the initial relative productivities in clean and dirty technologies $A_{Xc(t-1)}/A_{Xd(t-1)}$ (both decrease when $A_{Xc(t-1)}/A_{Xd(t-1)}$ is close to 1); with the assumption of the lemma, this aspect does not prevent innovation from going to the sector of initial comparative advantage. Such an assumption can be dispensed with when the inputs are perfect substitute ($\varepsilon = \infty$). This increase in comparative advantage effect is where

[12] If monopoly rights were permanent, a large discount rate would also be necessary to ensure that the equilibrium is unique. Otherwise, since innovation would tend to be allocated towards the sector where the economy would eventually specialize in, anticipations of the long-run pattern of specialization would be self fulfilling. Similarly, within the polluting sector, if entrepreneurs expect the economy to move towards a clean path in the future, they will start innovating in clean.

[13] Without this condition there may be situations where an equilibrium wihtout full specialization coexists with an equilibrium with full specialization. However, this would not affect the results of this section, the results of section 4 also hold as long as the interior equilibrium is chosen whenever it is possible.

[14] For $\varepsilon = \infty$, there is unicity except if $(1+\kappa(1))^{-(1-\gamma)} \leq A_{Xc0}/A_{Xd0} \leq (1+\kappa(1))^{(1-\gamma)}$.

the mechanics of the model bear some resemblance with the infant industry argument, as initial comparative advantage affects the path that the economy undertakes. Here, however, increasing specialization is not detrimental to the long-run growth of private consumption in any of the two countries. In fact, growth is maximized when there is full specialization, since in this case there is no overlap between innovations across countries. Instead, it will be detrimental to welfare if production of the polluting good ends up taking place in a country unwilling to implement any environmental policy.

3.2 Avoiding a disaster with policy in both countries

If the dirty subsector is more advanced than the clean one in both countries, then under laissez-faire, innovation in sector G is continually directed primarily towards dirty intermediates. As innovation necessarily keeps occurring in both sectors,[15] the production of good G and emissions grow unboundedly. At some point the regeneration capacity of the environment becomes overwhelmed and the economy reaches a disaster.

Using clean research subsidies, taxes on dirty research or even a carbon tax, a global government could redirect innovation from the dirty towards the clean subsector in both countries. Once clean intermediates will have acquired a sufficient lead over dirty intermediates, market forces will ensure that research is continually directed mostly towards the clean subsector (now the most advanced). Eventually, the emission rate of the polluting good goes towards 0, and a disaster can be avoided if the initial environmental quality is large enough. This analysis is similar to AABH and is presented here to provide contrast with the unilateral policy case. It can be summarized in the following remark:

Remark 1 No matter how large $\overline{S}$ is a disaster occurs in the laissez-faire equilibrium if clean technologies are less developed than dirty ones ($A_{Nc0} \leq A_{Nd0}$ and $A_{Sc0} \leq A_{Sd0}$). For sufficiently large $\overline{S}$, temporary clean research subsidies, taxes on dirty research, or carbon taxes in both countries can prevent a disaster.

Proof. See Appendix D.2 ■

3.3 Taxes on the polluting good in the North only

Assume now that only the North can implement some policy. Can it avoid an environmental disaster alone? First note that in autarky and without knowledge spillovers, no policy in the North could prevent a disaster, as emissions in the South alone would grow unboundedly

[15] The exporting country innovates more in the polluting good than it would under autarky and, since goods G and H are either complements or Cobb-Douglas, innovation does not disappear within the polluting good under autarky.

regardless of what the North does: trade is necessary to avoid an environmental disaster without international cooperation. Now one can show:

Lemma 3 *If clean technologies are less developed than dirty ones in the South $A_{Sc0}/A_{Sd0} \leq 1$, then, to prevent a disaster, all factors in the South must be asymptotically allocated to the non-polluting sector H, and the North must export the polluting good G in the long-run.*

Proof. See Appendix B.3 ■

In other words, the key to avoid an environmental disaster with policies in the North only is to ensure that the South economy asymptotically fully specialize in the non-polluting sector. If the South were to keep allocating a positive share of labor and capital to the polluting sector, the mass of scientists allocated to the polluting sector in the South will never go to 0 either (since the two goods are complements). The production of polluting good in the South would become unbounded, and without an environmental policy in the South, so will emissions.

In this subsection, I focus on taxes on the polluting good in the North (a carbon tax or a tax on dirty research), which are non-protectionist policies. Both can reduce emissions in the North and prompt clean innovation there, and could prevent an environmental disaster if the North were alone or if the South were to undertake the same policy, but such policies may be incompatible with a South specializing in the non-polluting sector and therefore may be unable to prevent an environmental disaster.

Proposition 1 *No matter how large $\overline{S}$ is, no combination of a carbon tax and a tax on dirty research can prevent a disaster if clean technologies are less developed than dirty ones in the North $A_{Nc0}/A_{Nd0} \leq 1$, clean technologies are sufficiently less developed than dirty ones in the South (A_{Sc0}/A_{Sd0} is sufficiently small and in particular $A_{Nc0}/A_{Nd0} \geq A_{Sc0}/A_{Sd0}$), and the South has a weak initial comparative advantage in the polluting sector $(A_{SG0}/A_{SH0})^{\frac{1}{\alpha-\beta}} K_S/L_S \geq (A_{NG0}/A_{NH0})^{\frac{1}{\alpha-\beta}} K_N/L_N$.*

Proof. See Appendix B.4 ■

This result follows lemmas 2 and 3. Under laissez-faire and with the assumptions of the proposition, the South would keep the comparative advantage in the polluting sector or (in the knife edge case) there would never be trade. Using a tax on dirty research or a carbon tax, the North government cannot reverse this pattern. On the contrary: a tax on dirty innovation drives scientists away from the polluting sector G towards the non-polluting sector H, and, within the polluting sector it allocates innovation towards the initially backward clean subsector, further reducing the growth rate of the average productivity A_{NGt}. A carbon tax has the same effect on innovation and also directly reduces the productivity of the polluting sector in the North. As both instruments increase the costs of producing the polluting good in the North, they lead to an increase in its world relative price. This induces an increase in the production of the polluting good G in the South and therefore more emissions: this is the classic pollution haven effect. As the relative revenues of the polluting sector increase in the South, more innovation there takes place in the polluting sector, where it is mostly directed at the dirty technologies.

North domestic taxes on the polluting good can therefore only accelerate the specialization of the South in the polluting sector. Because the growth rate of the polluting sector is maximized when the South fully specializes in it, it is likely that such policies lead to faster environmental degradation (in addition, since the gap between clean and dirty technologies in the South grows faster, the emission rate grows faster as well). Of particular interest is the knife edge case where the South has no comparative advantage $((A_{SG0}/A_{SH0})^{\frac{1}{\alpha-\beta}} K_S/L_S = (A_{NG0}/A_{NH0})^{\frac{1}{\alpha-\beta}} K_N/L_N$ and $A_{Nc0}/A_{Nd0} = A_{Sc0}/A_{Sd0})$. In this case, there would be no trade in laissez-faire, but because of the pollution haven effect, the policy intervention tips the balance towards a comparative advantage for the South in sector G, which then builds on itself over time.[16]

In the proposition, the condition that A_{Sc0}/A_{Sd0} must be sufficiently small (and not simply smaller than 1) is necessary for the same reasons as in lemma 2: with the ratio of clean over dirty technologies further from 1 in the North than in the South, more innovation in the polluting sector might take place in the North than in the South even if the South exports the polluting good.[17] Importantly, the value of $\overline{S}$ has no bearing on how small the initial ratio A_{Sc0}/A_{Sd0} needs to be. Such conditions can be dispensed with if the initial comparative advantage is sufficiently large or when $\varepsilon = \infty$ (in which case, $A_{Sc0}/A_{Sd0} < (1+\kappa)^{-(1-\gamma)}$ would be enough). In the pure Ricardian case $\alpha = \beta$, the proposition holds provided that the initial comparative advantage is sufficiently large (otherwise there may be multiple equilibria and in some of them the North may specialize in the polluting sector). Finally, if the North has the initial comparative advantage in the polluting sector G, the pollution haven effect still pushes the South towards specializing in the polluting sector, but the amplification of initial comparative advantage effect now pushes in the other direction.

[16] It is already possible in a static model that an increase in the carbon tax in the North (τ_{Nt}) leads to an increase in world emissions if the South uses relatively dirtier technologies than the North. Here, the dynamic aspects amplify the static pollution haven effect, so that in the long-run non-protectionist policies cannot prevent a disaster.

[17] More specifically, the incentive to innovate in sector G is lower when the revenues in the clean and dirty subsectors are close to each other, that is when $A_{Xc(t-1)}^{\varepsilon-1}$ and $(1+\tau_{Xt})^{-\varepsilon} A_{Xd(t-1)}^{\varepsilon-1}$ are close to each other. With a tax on dirty research only, it is possible to get $A_{Ndt}/A_{Nct} < A_{Sct}/A_{Sdt}$ at some point once clean technologies in the North have caught up with dirty ones, by preventing any research in dirty technologies. A sufficiently large carbon tax can make the dirty subsector arbitrarily less profitable than the clean one in the North, regardless of relative technologies. The assumption on A_{Sc0}/A_{Sd0} ensures that the difference in comparative advantages becomes sufficiently large should the revenues ratio in clean over dirty be further than one in the North than in the South.

3.4 Introducing clean research subsidies and tariffs

The previous policies failed a preventing an environmental disaster when the South had the initial comparative advantage in the polluting sector because they could not reverse the pattern of trade. Clean research subsidies can reallocate innovation from the non-polluting sector H towards clean technologies in the North and therefore they may be able to reverse the pattern of trade. Surprisingly, I can show:

Proposition 2 *If final consumption is Cobb-Douglas in the polluting and non-polluting goods ($\sigma = 1$), there exist initial factor endowments and technologies such that no matter how large $\overline{S}$ is, no combination of a carbon tax, a tax on dirty research and a subsidy on clean research can prevent a disaster.*

Proof. See Appendix B.5.1 ■

Therefore, clean research subsidies may still fall short in preventing an environmental disaster if enacted alone - even though clean research subsidies have a protectionist aspect since they are an indirect subsidy to the polluting sector. This occurs when the South fully specializes in the polluting sector and clean technologies in the South are sufficiently less advanced than dirty ones. In this case all South scientists are allocated to the polluting sector and asymptotically all of them to dirty technologies. Therefore, even if the North were to allocate all its scientists to clean technologies, A_{SGt} would grow as fast as A_{NGt}. Such a situation is irreversible in the Cobb-Douglas case, and an environmental disaster cannot be avoided. When the two goods are strict complements ($\sigma < 1$), on the contrary, when both countries innovate in the polluting sector only, the demand for the non-polluting good becomes so large that the South cannot stay fully specialized.[18] Full specialization in the South will happen in the first place if its initial comparative advantage in the polluting sector is sufficiently large or if clean technologies are sufficiently backward in the North (as the average productivity of the polluting sector in the North A_{NGt} grows slowly during the period where clean technologies are catching up with dirty ones there).

Finally, I consider that the North can implement a tariff:

Proposition 3 *A combination of a temporary tariff and a temporary clean research subsidy in the North can prevent a disaster provided that the initial environmental quality $\overline{S}$ is sufficiently large.*

Proof. See Appendix B.5.2 ■

[18] Note, however, that in the strict complement case $\sigma < 1$, averting the disaster with clean research subsidies when there is full specialization in both countries is possible only if the North government can still allocate research to the clean subsector even though no clean machines are produced. See Appendix D.3.

While clean research subsidies allow to redirect innovation in the North towards clean technologies, the tariff affects innovation in the South by preventing the South from specializing "too much" in the polluting sector.[19] As long as the initial environmental quality is sufficiently large, a combination of these two policies can prevent a disaster. For instance, the following two phases approach works. In a first phase, the social planner implements a tariff large enough to shut down trade, and sufficiently large clean research subsidies that all scientists in the North innovate in the clean subsector. Innovation in the South must then be balanced between the polluting and non-polluting sectors. As a consequence, not only do clean technologies become more advanced than dirty ones in the North, but the North builds a comparative advantage in the polluting sector G, since it innovates relatively more in the polluting sector than the South does. Once $A_{Nc(t-1)}/A_{Nd(t-1)}$ is sufficiently large, the social planner can stop all policies: following lemma 2, sector G production ends up fully moving to the North (where it has become clean), and a disaster can be avoided.

Good-based carbon content tariff. As discussed in section 2, without policy in the South, the only way for the North to directly affect Southern firms' emission rates is to know the carbon contents of each imports at the firm level, which, without any cooperation from the South, seems impossible. Nevertheless, it is interesting to mention this case as a benchmark (and it is more relevant when there are only few exporters in the South). In this case, when the South has the comparative advantage in the polluting good, the dirty input will taxed for the exports market. If clean technologies are not too backward in the South, and the export market is relatively large (which is possible if the South is small and has a large comparative advantage), a switch to clean technologies in the South is possible, and a reversal of comparative advantage may not be necessary.[20] When the South is willing to undertake some form of policy on the other hand, a realistic approach would be to relate the carbon tariff with average emissions from a given country in a given sector. In this case, the South government would internalize that the tariff depends on its policy and could be incentivized to undertake some environmental policy on its own. Such an analysis would be very interesting but is beyond the scope of this paper. Appendix A presents other direct extensions of the model.

3.5 Discussion

The following lessons can be derived from the previous analysis. First, the pollution haven effect becomes worse in a dynamic setting. Taxes on the polluting sector in the North risk

[19] In the presence of a tariff, the equilibrium may not be unique when $\sigma < \frac{1}{2}$, but this does not affect the analysis, since a sufficiently large tariff can always prevent the South from fully specializing in sector G.

[20] Note that such a trade tax makes sense as a tariff only, not as an export subsidy. A combination of clean research subsidies and a good-based carbon content tariff can always prevent a disaster for sufficiently large environmental quality, provided that it is possible to direct scientists towards clean research in the North even when the polluting sector there is not active.

leading the economy to a path where the South has the comparative advantage in the polluting sector. As comparative advantage tends to get reinforced over time, the bulk of production of the polluting sector ends up occurring in the South, dramatically hampering the effect of the initial intervention on worldwide emissions. To ensure sustainable growth without cooperation from the South, the North must undertake a temporary industrial policy in order not only to make the polluting sector cleaner but also to get the comparative advantage in the polluting sector.

Second, trade acts as a double-edged sword. In laissez-faire, trade leads to specialization, which maximizes long-run growth and therefore leads to faster environmental degradation. Moreover, because of the pollution haven effect, trade makes non-protectionist policies less efficient. However, if well managed, trade is key to avoid a disaster in a non cooperative world, since once the North has developed its clean technologies sufficiently, trade forces ensure that pollution will decrease in the South too.

Third, directed technical change also acts as a double-edged sword. Relative to a model where technological levels grow exogenously at an equal rate, directed technical change accelerates economic growth and environmental degradation, and reduces the benefits from non-protectionist unilateral policies through the South innovation response. On the other hand, avoiding a disaster with unilateral policies only may be impossible without directed technical change regardless of the initial environmental quality: When clean technologies are too backward relative to dirty technologies, even if the North produces only the clean input and exports all its production (which can be achieved with sufficiently large export subsidies), the South may still have to produce some of the polluting good itself to satisfy its own consumption, in which case emissions in the South grow unboundedly. For some parameter values averting a disaster with policies in the North only and without directed technical change is possible, but requires a permanent growing export subsidy, and an unbounded carbon tax. I come back to the role of trade and directed technical change in the calibration section.

4 Optimal policy

I now analyze the optimal policy. I consider a social planner who cares only about world consumption and the quality of the environment, and seeks to maximize the function:

$$U = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \frac{\left(v\left(S_t\right)\left(C_{Nt}+C_{St}\right)\right)^{1-\eta}}{1-\eta}. \tag{22}$$

where $\rho > 0$ is the discount rate, and $\eta \geq 0$ is the inverse elasticity of intertemporal substitution. I further assume $\lim_{S\to 0} v(S) = 0$ and $v'\left(\overline{S}\right) = 0$, which implies, first, that a disaster is as detrimental to welfare as zero consumption and, second, that the marginal damage of the first unit of pollution is zero. The social planner tries to correct for the environmental and knowledge externalities, as well as for the monopoly distortion.

This objective function is a Bergson-Samuelson social welfare function if the economic agents live for one period in the North and in the South. However, except when $\eta = 0$, it is not a Bergson-Samuelson social welfare function if the economic agents live infinitely, since it cannot be written as a function of the utility of one representative agent in the North and one in the South. On the contrary, the social welfare function given by:

$$U = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \frac{v(S_t)^{1-\eta}}{1-\eta} \left(C_{Nt}^{1-\eta} + C_{St}^{1-\eta} \right), \tag{23}$$

is a Bergson-Samuelson social welfare function with agents living infinitely. The (well-known) problem in this case is that the social planner would want to redistribute from the richest to the poorest country, as a positive η also measures an aversion to inequality. For the main analysis, I abstract from redistribution issues, and therefore focus on the formulation given by (22), but I discuss the alternative set-up (23) in the text.

In the first subsection I briefly characterize the first best policy, which generalizes the analysis of AABH. In the second one, I study the second best case where the social planner cannot intervene in the South (the main focus of this paper), and in the third, I look at the optimal policy from the viewpoint of a social planner who cares only about the North.

4.1 First best

In the first best, the social planner maximizes (22) under the production function equations (1), (2), (3), (5), (6), (7), the factor market clearing equations (9), (14), the good market clearing equations (10), the environmental degradation equation (11) and the accumulation of knowledge equations (12). I can then show:

Proposition 4 *The first best policy needs to be decentralized through a combination of a carbon tax in the North and in the South (with the same price for carbon), research subsidies/taxes in the North and the South in both sectors, and a subsidy to the use of all intermediates.*

Proof. See Appendix B.6 ■

Each instrument allows the social planner to correct for one distortion. First, the under-utilization of intermediates due to monopoly pricing is corrected by a subsidy $1-\gamma$ to all intermediates. Second, the environmental externality is corrected by a carbon tax in both countries which equalizes the marginal cost of the tax in terms of lower consumption at the time, with the marginal benefit of higher environmental quality in all subsequent periods from avoiding one unit of pollution (carbon taxes in the North and the South differ in add-valorem form across countries but are identical as a per-unit of CO_2 tax). Finally the social planner corrects for the myopia of monopolists in their innovation decisions. Instead of allocating scientists across sectors depending on the revenues generated by their effort in their sector in the first period, the social planner allocates them according to the discounted value of the entire stream of additional revenues generated by their innovation. More specifically, instead of (19) and (20), scientists are now allocated across the dirty, clean and H (sub)sectors according to:

$$\frac{s_{XHt}^{\iota-1}}{1+\kappa s_{XHt}^{\iota}}\sum_{s=t}^{\infty}B_{s,t}\widehat{p}_{Hs}Y_{Hs}=\frac{s_{Xct}^{\iota-1}}{1+\kappa s_{Xct}^{\iota}}\sum_{s=t}^{\infty}B_{s,t}\widehat{p}_{Xcs}Y_{cs}=\frac{s_{Xdt}^{\iota-1}}{1+\kappa s_{Xdt}^{\iota}}\sum_{s=t}^{\infty}B_{s,t}\widehat{p}_{Xds}Y_{ds}, \tag{24}$$

where $\widehat{p}_{Xcs}$ and $\widehat{p}_{Xds}$ denote the shadow price of the clean and dirty inputs in country X, $\widehat{p}_{Hs}$ the shadow price of good H and $B_{s,t}=\frac{1}{(1+\rho)^{s-t}}\frac{\frac{\partial u}{\partial C}(C_{Ns}+C_{Ss},S_s)}{\frac{\partial u}{\partial C}(C_{Nt}+C_{St},S_t)}$ is the effective discount factor between period s and t.

Since the utility flow is minimal during a disaster and the social planner can always reduce world emissions, the optimal policy always avoid a disaster. The following remarks further characterizes the optimal policy and establishes conditions under which a switch to clean innovation occurs in one country - as in remark 1.

Remark 2 The social planner avoids a disaster. If the discount rate ρ is sufficiently small and the inverse elasticity of intertemporal substitution $\eta \leq 1$, innovation in sector G switches to mostly clean innovation, both countries reach full specialization in finite time, and the carbon tax is temporary.

Proof. See Appendix B.7 ■

The intuition is the following. With the inverse elasticity of intertemporal substitution $\eta \leq 1$ and a sufficiently small discount rate, the optimal policy maximizes the long-run growth rate while avoiding an environmental disaster. A switch towards clean innovation allows the polluting sector to keep growing at a positive rate while avoiding a disaster (the alternative would be to keep relying on dirty technologies, but with a permanent large carbon tax). Moreover, long-run growth is maximized if each country innovates only in its own sector (as innovations in one country and the other do not add up), and in this case, the difference in comparative advantage becomes so large that both countries end-up fully specializing. Since the dirty input becomes a negligible part of the production process, emissions vanish, and the quality of the environment goes back to $\overline{S}$, ensuring that the carbon tax itself reaches zero in finite time.

Interestingly, the country exporting the polluting good is not necessarily the one where consumption is reduced the most due to environmental policy. Indeed, the reduction in the production of the polluting good creates a terms of trade effect beneficial to the polluting country. When final consumption is Cobb-Douglas ($\sigma = 1$), and the policy intervention does not affect the pattern of specialization, long-run consumption is reduced proportionally in both countries relative to laissez-faire (or to a case without the environmental externality). Once countries are fully specialized, the country exporting good G has a fixed income share of ν. When the goods are strict complements ($\sigma < 1$), the country exporting good G in the long run actually ends up having a larger share of world consumption as the terms of trade effect is stronger.

Finally, under the alternative specification (23), the only difference is that the social planner will use a monetary transfer to ensure that in every period $C_{Nt} = C_{St}$: production and emissions follow the same pattern.

4.2 Second best

I now turn to the case where the social planner cannot implement any policy in the South (the economy there must behave as in laissez-faire),[21] and cannot transfer income from one country to another (so that trade balance must be maintained at every point in time: there is no international lending). The second best policy is defined by the social planner maximizing (22) under the following constraints: (1) for the North and the South, the constraints ((2), (3), (5), (6), (7), (9), (14), (12)) for the North only; the environmental degradation constraint (11), market goods clearing in both countries, which are now written as:

$$C_{NYt} = Y_{NYt} + M_{Yt} \text{ and } C_{SYt} = Y_{SYt} - M_{Yt}, \text{ for } Y \in \{G, H\}, \tag{25}$$

where M_{Yt} denotes net imports of the North of good Y, the trade balance constraint

$$p_t M_{Gt} + M_{Ht} = 0, \tag{26}$$

where $p_t \equiv p_{Gt}/p_{Ht}$ is the international price ratio, and constraints describing the laissez-faire economy in the South. These constraints (detailed in Appendix B.8) are given by a consumer demand equation:

$$\frac{\frac{\partial C_S}{\partial C_{SG}}}{\frac{\partial C_S}{\partial C_{SH}}} = \frac{\nu C_{SHt}^{\frac{1}{\sigma}}}{(1-\nu)\, C_{SGt}^{\frac{1}{\sigma}}} = p_t, \tag{27}$$

offer equations in the South of the type:

$$Y_{SGt} = y_{SG}\left(p_t, A_{SGt}, A_{SHt}\right) \text{ and } Y_{SHt} = y_{SH}\left(p_t, A_{SGt}, A_{SHt}\right), \tag{28}$$

an emissions equation $Y_{Sdt} = \left(A_{Sdt}/A_{SGt}\right)^{\varepsilon} Y_{SGt}$, an equation giving the total mass of scientists allocated to sector G,

$$s_{SGt} = s_{SG}\left(p_t, A_{Sdt}, A_{Sct}, A_{SHt}\right), \tag{29}$$

[21] To simplify the exposition, however, I assume that the optimal subsidy to the use of all intermediates is implemented in the South: this is without any loss of generality, since the subsidy only has a scale effect. Moreover I consider that the equilibrium in the South is unique given the innovation allocation in the North - which generally requires κ sufficiently small and $\sigma \geq 1/2$.

and the resulting law of motion of aggregate productivity in the South - where the allocation between clean and dirty innovation is uniquely determined given the total mass s_{SGt} and the ratio $A_{Sc(t-1)}/A_{Sd(t-1)}$-:

$$A_{SHt} = \left(1+\kappa\left(1-s_{SGt}\right)^{\iota}\right)^{1-\gamma} A_{SH(t-1)}, \tag{30}$$

$$A_{Szt} = \left(1+\kappa\left(s_{Szt}\left(s_{SGt},\left(\frac{A_{Sc(t-1)}}{A_{Sd(t-1)}}\right)^{\varepsilon-1}\right)\right)^{\iota}\right)^{1-\gamma} A_{Sz(t-1)}, \text{ for } z\in\{c,d\} \tag{31}$$

First, note that when the inverse elasticity of intertemporal substitution $\eta \geq 1$, a disaster brings a utility of $-\infty$, the social planner will then prevent it whenever it is feasible. As shown in Appendix B.9, the social planner also prevents a disaster whenever it is feasible if $\eta < 1$ and the discount rate ρ is sufficiently small.[22]

Proposition 5 *The second best policy needs to be decentralized through a carbon tax in the North, research subsidies/taxes in the North, a subsidy to the use of all intermediates and a trade tax.*

Proof. See Appendix B.8 ■

In this second best scenario, the social planner uses the same instruments as before to address the inefficiencies of the economy in the North: a subsidy to the use of all intermediates (for the monopoly distortion), a carbon tax, and research subsidies in order to allocate scientists as in (24). The trade tax is the optimal way to affect prices in the South, which is the only channel through which the social planner can intervene on the South economy; it is therefore the only instrument that he uses to influence the South economy. In Appendix B.8, I derive an implicit equation for the value of the optimal trade tax (95), which takes the following form:

$$\begin{aligned} & f_t\left(b_t\right) \\ = \; & c_{1t}\xi\left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon}\sum_{s=t}^{\infty}\left(\frac{1+\Delta}{1+\rho}\right)^{s-t} I_{S_t,..,S_s<\overline{S}}\frac{\partial u}{\partial S}\left(C_{Ns}+C_{Ss},S_s\right) \\ & +c_{2t}\left(\frac{s_{SHt}^{\iota-1}}{1+\kappa s_{SHt}^{\iota}}\mu_{SHt+1}A_{SHt+1}-\frac{s_{Sdt}^{\iota-1}}{1+\kappa s_{Sdt}^{\iota}}\frac{\partial s_{Sdt}}{\partial s_{SGt}}\mu_{Sdt+1}A_{Sdt+1}-\frac{s_{Sct}^{\iota-1}}{1+\kappa s_{Sct}^{\iota}}\frac{\partial s_{Sct}}{\partial s_{SGt}}\mu_{Sct+1}A_{Sct+1}\right) \\ & +term_{3t} \end{aligned} \tag{32}$$

where $f_t\left(b_t\right)$ is an expression that has the sign of b_t, c_{1t} and c_{2t} are positive coefficients when the South is not specialized and null otherwise (when the South is at a corner of specialization,[23] the coefficients can be positive or null), $\mu_{Sz(t+1)}$ denotes the social value of a productivity unit

[22] In this case a social planner always favor a path where the utility flow is unbounded , which is not possible when a disaster occurs.

[23] That is when the South is fully specialized in one sector but a firm could produce an infinitesimal amount of the other good without making losses.

in the South in (sub)sector $z \in \{c, d, H\}$. This formula highlights the two motives behind the trade tax. The first term is always positive and represents the environmental motive: in the South, pollution through dirty input production escapes taxation, and a positive trade tax on the polluting good G (that is a tariff or an export subsidy) reduces the relative price of good G in the South, which reduces its production and therefore emissions in the South. The second term is a correction for the myopia of innovators in the South; it represents the difference between the social value of an additional scientist in the non-polluting sector H and one in the polluting sector G on welfare for all subsequent periods. In principle, it could be of either sign, but in practice it ends up being positive, also pushing towards a positive tariff or export subsidy. According to lemma 3, the South must at least asymptotically fully specialize in the non-polluting sector to prevent an environmental disaster. Therefore, current innovations in the polluting sector are of little (or no) use in the future. A positive trade tax on the polluting good G tilts innovation in the South away from that sector. The third term ($term_{3t}$) has an ambiguous sign but goes to zero as the gap between clean and dirty technologies in the South increase.[24] Overall, the trade tax is generally positive, taking the form of a tariff or an export subsidy when the North is the exporter of the polluting good.

To better relate my analysis with the literature, I derive in Appendix D.4 the following condition for the social optimum:

$$\frac{p_t + M_{Gt}\frac{\partial p_t}{\partial M_G}}{1 + M_{Gt}\frac{\partial p_t}{\partial M_H}} = \frac{\frac{\partial C_{Nt}}{\partial C_{NGt}} + \left(\frac{\phi_t}{\lambda_t}\frac{\partial s_{SGt}}{\partial p_t} - \frac{\omega_t}{\lambda_t}\frac{\partial Y_{SGt}}{\partial p_t}\right)\frac{\partial p_t}{\partial M_{Gt}} - \frac{\partial C_{St}}{\partial C_{SGt}}}{\frac{\partial C_{Nt}}{\partial C_{NHt}} + \left(\frac{\phi_t}{\lambda_t}\frac{\partial s_{SGt}}{\partial p_t} - \frac{\omega_t}{\lambda_t}\frac{\partial Y_{SGt}}{\partial p_t}\right)\frac{\partial p_t}{\partial M_{Ht}} - \frac{\partial C_{St}}{\partial C_{SHt}}} \tag{33}$$

where λ_t is the social value of an additional unit of world consumption at time t and ϕ_t is the social value of moving an infinitesimal mass of scientists in the South to sector G. $M_{Gt}\frac{\partial p_t}{\partial M_G}$ and $M_{Gt}\frac{\partial p_t}{\partial M_H}$ measure the terms of trade effect of an additional unit of imports in sector G and H respectively. This equality stipulates that the ratio of the cost of imports for the North (prices plus terms of trade effects) must be equal to the ratio of marginal social benefit, which include more consumption of the imported good in the North, less consumption in the South, environmental damage, and the impact on innovation. Under the alternative specification (23), the only difference is that the trade tax is also used in order to favor the poorest country: When the South is the poorest, it will push towards a smaller tariff - when the South exports the polluting good G - but then towards a larger export subsidy.

The following proposition further characterizes the second best policy by deriving conditions under which the South fully specializes in finite time in sector H and a switch to clean

[24] This term is fairly technical. It reflects that if more Southern scientists are allocated to sector G today, then for a given number of scientists allocated to sector G tomorrow, more will be allocated to dirty than to clean technologies. Since $A_{Sct}/A_{Sdt} \to 0$, nearly all innovation in sector G ends up occuring in the dirty subsector very fast, so this term goes to zero. It could be positive or negative, as the higher consumption value for dirty innovation is tempered by the environmental externality.

innovation occurs in the North.

Proposition 6 *Assume that clean technologies are less developed than dirty ones in the South, $A_{Sc0} \leq A_{Sd0}$, that there is no intervention in the South, but that avoiding a disaster with policy in the North only is feasible. If the time discount rate ρ is sufficiently small, and either the polluting and non-polluting goods are strict complements ($\sigma < 1$), or the inverse elasticity of intertemporal substitution $\eta \leq 1$, then, under the second best policy, both countries fully specialize in finite time (the South in the non-polluting sector H and the North in the polluting sector G), innovation in sector G switches to being mostly in clean technologies, and the optimal trade tax and carbon tax are temporary.*

Proof. See Appendix B.9 ■

The intuition behind this proposition is similar to the intuition behind remark 2, with the caveat that now the South must not specialize in the polluting sector G. When the inverse of the elasticity of intertemporal substitution $\eta \leq 1$, the logic is identical: long-run growth (and therefore welfare for sufficiently small discount rate) is maximized when both countries fully specialize and the country producing the polluting good switches to clean innovation. The optimal trade tax must equal zero once the South is fully specialized (and not at a corner of full specialization), and once the environment has recovered, the optimal carbon tax also becomes equal to zero. The analysis can be extended to the case $\eta > 1$ if goods G and H are strict complements ($\sigma < 1$). When $\eta > 1$, for sufficiently low discount rate, the optimal path must feature a utility flow growing unboundedly. When the two goods are complements ($\sigma < 1$), this translates into production growing unboundedly in both sectors. Production of the polluting good can grow unboundedly only in the North and if a switch to clean innovation occurs. One can then show that if production of good G grows unboundedly in the North but remains bounded in the South, both countries eventually specialize.

4.3 Optimal policy for the North

I now briefly discuss the case of a North social planner, whose objective is to maximize $U = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \frac{v(S_t)^{1-\eta}}{1-\eta} C_{Nt}^{1-\eta}$, under the same constraints as in the second best case above (a full resolution and a more detailed analysis than below are provided in Appendix D.5). Proposition 5 still holds and only the trade tax is modified. (33) is replaced by:

$$\frac{p_t + M_{Gt}\frac{\partial p_t}{\partial M_G}}{1 + M_{Gt}\frac{\partial p_t}{\partial M_H}} = \frac{\frac{\partial C_{Nt}}{\partial C_{NGt}} + \left(\frac{\phi_t}{\lambda_t}\frac{\partial s_{SGt}}{\partial p_t} - \frac{\omega_t}{\lambda_t}\frac{\partial Y_{SGt}}{\partial p_t}\right)\frac{\partial p_t}{\partial M_{Gt}}}{\frac{\partial C_{Nt}}{\partial C_{NHt}} + \left(\frac{\phi_t}{\lambda_t}\frac{\partial s_{SGt}}{\partial p_t} - \frac{\omega_t}{\lambda_t}\frac{\partial Y_{SGt}}{\partial p_t}\right)\frac{\partial p_t}{\partial M_{Ht}}},$$

as the social planner no longer takes into account South consumption. Terms of trade concerns now matter for the sign of the optimal trade tax: they will push towards a tariff when the

North imports but then towards an export tax, while environmental concerns push towards a tariff initially and then an export subsidy. One can then initially expect a positive tariff, and eventually, as the environment recovers and the North exports the polluting good, an export tax. Further, the optimal North policy can be characterized by:

Proposition 7 *Assume that clean technologies are less developed than dirty ones in the South $A_{Sc0} \leq A_{Sd0}$, and that avoiding a disaster with policies in the North only is feasible, then under the optimal North policy: (i) The South must asymptotically be at the corner of specialization in the non-polluting sector, and the trade tax is permanent. (ii) If the time discount rate ρ is sufficiently small, and either the polluting and non-polluting goods are strict complements ($\sigma < 1$), or the inverse elasticity of intertemporal substitution $\eta \leq 1$, a positive mass of scientists is asymptotically allocated to clean technologies in the North (asymptotically all of them when $\eta \leq 1$) and the share of scientists allocated to the dirty subsector tends to zero.*

Proof. See Appendix D.6 ■

As in the case of the world social planner, the optimal North policy involves avoiding a disaster and positive growth so that a switch to clean technologies in the North must happen (but the North does not necessarily fully specialize). Following lemma 3, the South must move towards full specialization; however, here, the North uses a (permanent) export tax in order to maintain the South approaching the corner of full specialization or just at the corner of full specialization. The reason is the following. If the South is fully specialized in good H (not at the corner), an export tax does not affect its production. Since the South does not produce the polluting good, and since both goods are complements, the demand for the polluting good is so large that the South is ready to give up nearly all its production of good H to consume a bit of good G. Therefore, the North increases the price of good G in order to receive more and more of good H, up to the corner point where an increase starts affecting the production pattern in the South.

5 Stylized Calibration

In this section I carry a simple calibration exercise in order to illustrate the previous results. This exercise should not be taken as a careful quantitative assessment; the level of aggregation of the model is too high for that. In particular, I show how both trade and directed technical change act as "double-edge swords": they tend to accelerate environmental degradation in laissez-faire and when the North undertakes only non-protectionist policies, but they help prevent a disaster when the North undertakes the appropriate policies.

5.1 Parameter choices

I provide a brief description of the calibration, further details are given in Appendix C. I define 1 period as 5 years and the initial values are based on the world economy in 2003-2007, assuming laissez-faire in both countries. For simplicity, I assume that the polluting and non-polluting goods enter in final consumption in a Cobb-Douglas way ($\sigma = 1$), and that the elasticity of intertemporal substitution is equal to 1 ($\eta = 1$) - as in Golosov et al. (2011). The annual time discount rate is 0.015 as in Nordhaus (2008). I identify the North with the countries in Annex I of the Kyoto protocol (the countries submitted to binding constraints on their emissions) and the South as the rest of the world. Data availability restricts the number of countries studied but the sample includes the most important countries (the North comprises 33 countries and the South 18). To identify the polluting and non-polluting sectors, I rely on the IEA data on sectoral emissions of CO2 from fossil fuel combustion across the world (IEA (2010*a*)) and on the UNIDO data on sectoral value added (UNIDO (2011)). I restrict attention to manufacturing and compute at the available aggregation level the world rate of emissions per dollar of value added in each sector. The sectors with the highest rate are identified with sector G and the other ones with sector H (and, according to the model, I ignore the emissions coming from sector H). In practice, the polluting sector corresponds to manufacture of chemicals and chemical products (ISIC code 24), of other non-metallic mineral products (26) and of basic metals (27).[25] I find that the South production is tilted towards sector G relative to the North production ($Y_{NG0}/Y_{SG0} \times Y_{SH0}/Y_{NH0} = 0.77$), which in the framework of the model corresponds to the South having a small initial comparative advantage in the polluting sector G. From world production in sectors G and H, I compute the consumption share of good G ($\nu = 0.257$) when the economy consists only of sectors G and H.

I compute the capital factor share from the ratio of capital compensation over labor compensation in both sectors in the US with the EU KLEMS dataset (Timmer, O'Mahony and van Ark (2008)), and find a capital share $\alpha = .5$ in the polluting sector, and $\beta = .3$ in the non-polluting one. A unit of a good is defined as the quantity with a value added worth one billion of dollars in 2000. Factor shares and initial production values are enough to determine the initial productivity adjusted endowments, which, with the initial ratio A_{Xc0}/A_{Xd0}, are all that matter for the economy when knowledge is purely local.[26] I fix $\gamma = 1/3$, which is a common value in endogenous growth models. For the elasticity of substitution between the clean and the dirty input ε, I choose $\varepsilon = 5$, the medium value used in the working paper version

[25] Sector H corresponds to the other sectors in manufacturing except 23, 25, 33, 36 and 37 for which data are not available.

[26] Nevertheless, I do not assign arbitrary values for endowments, but choose L_X as total employment in sector G and H in country X, and K_X as total capital formation in both sectors in country X (from the UNIDO database).

of AABH.[27] For the innovation function I choose $\iota = 1/4$ (numerically this parameter has a limited impact) and κ is adjusted so that the long-run growth rate of the economy is 2% a year.

The quality of the environment S_t is linearly negatively related to the atmospheric concentration of CO_2 (the assumption that $S_0 = \overline{S}$ is relaxed and the initial environmental quality S_0 is set such that it corresponds to the current atmospheric concentration of 379 ppm). To better relate the paper with the rest of the numerical literature, the law of motion (11) is replaced by:

$$\overline{S} - S_t = (1 - \Delta) \left(\overline{S} - S_{t-1} \right) + \xi \left(Y_{Ndt} + Y_{Sdt} \right), \tag{34}$$

(with bounds at 0 and $\overline{S}$). This law of motion is used by Golosov et al. (2011) amongst others. Δ is calibrated such that at current levels around half of CO_2 emissions are absorbed and do not add to atmospheric concentrations. Under this alternative specification, the environment never fully recovers; therefore the optimal carbon tax is asymptotically null instead of being null in finite time.[28] I compute the emission rate in the North and in the South in sector G, the South's rate is nearly four times as large as the North's. Such a large difference cannot be accounted for by the model if $A_{Nd0} > A_{Nc0}$ and the emission rate per unit of dirty input (ξ) is identical in both countries. Since $A_{Nc0} > A_{Nd0}$ would be very unrealistic, I relax the assumption that the emission rates per unit of dirty input are the same in both countries. To derive a proxy for A_{Xc0}/A_{Xd0}, I use IEA data (IEA (2010b)) and identify the ratio Y_{Xc0}/Y_{Xd0} with the ratio of the production of nonfossil fuel energy over fossil fuel energy in country X's primary energy supply. This ratio is 25% larger for the North than for the South. The rest of the difference between the initial emission rates per unit of good G in the North and in the South is made up by the difference between ξ_N and ξ_S ($\xi_S > \xi_N$ so dirty inputs in the South are more polluting than in the North).[29] Changes in CO_2 atmospheric concentrations are then mapped with changes in temperature, and $S = 0$ is chosen to correspond with a "disaster" temperature level of 6°C. The function $\nu(S_t)$ is the same as in AABH, and mimics the cost function of Nordhaus (2008) for increases up to 3°C.

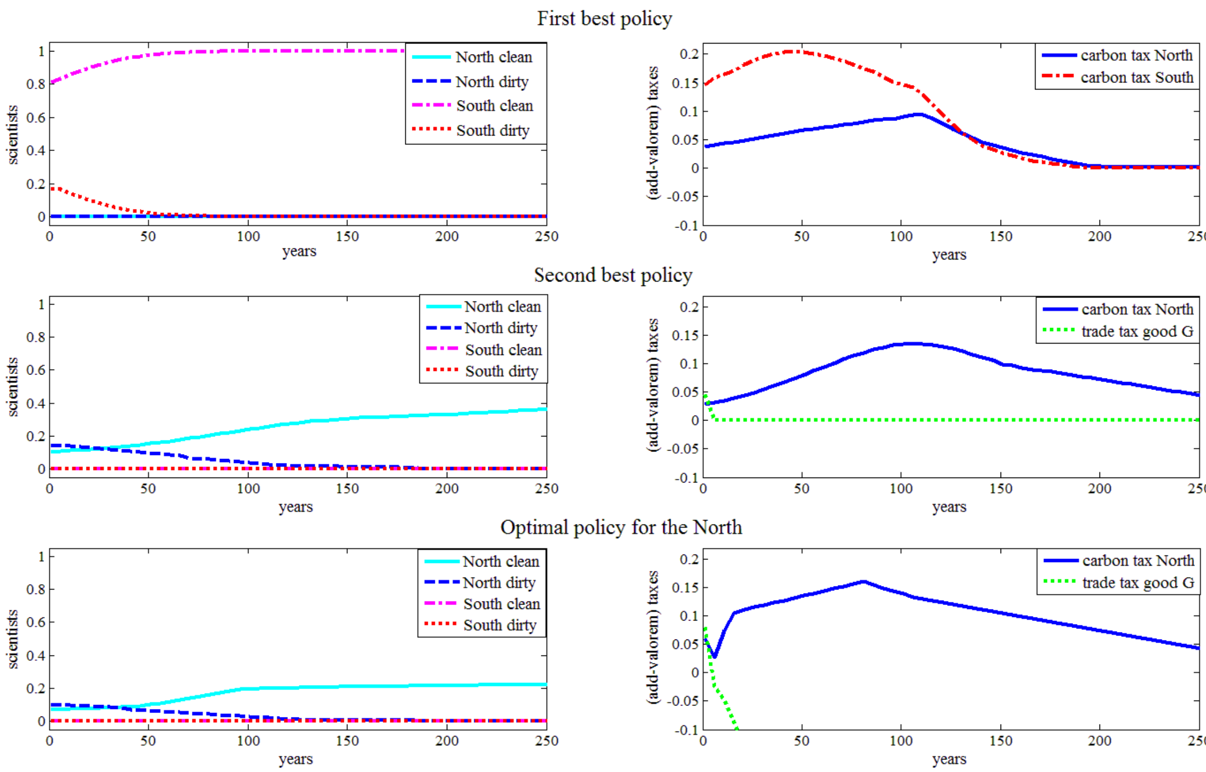

Figure 1: Figure 1: First best, second best and North optimal policies

5.2 Alternative policies and their welfare costs

Figure 1 displays the allocation of innovation, carbon tax and trade tax in the first best case, the second best case and in the optimum for the North case. Figure 1.A shows the allocation of innovation in the polluting sector G in the first best case: nearly all sector G innovation is carried out in the South, and mostly in clean technologies from the first period. Following remark 2; specialization in both countries occurs in finite time, in fact here, it occurs very rapidly, which is why there is nearly no innovation in sector G in the North (and nearly no innovation in sector H in the South). Figure 1.B shows the ad valorem carbon taxes in the North and the South - recall that the per-unit of CO_2 taxes are identical though. The carbon tax in the South initially increases (as temperature increases) and then decreases following the fall in temperature and the development of clean technologies in the South. Figure 1.C shows the allocation of innovation in the second best case. Contrary to the first best case, the North must now export the polluting good G eventually. In this calibration, a large trade tax on good G (see figure 1.D) ensures that right from the first period the South specializes in the non-polluting good H.[30] The switch towards more clean innovation than dirty in the North occurs rapidly but not immediately for two reasons: first, as the South's emission rate is larger than the North's, the temperature increase is initially lower than in the first best case, so the North can afford working on dirty technologies a bit longer; second, keeping some innovation in dirty technologies helps the North build a large comparative advantage in the polluting sector. The amount of clean innovation increases over time and, beyond the time frame of the simulation, eventually reaches one when the North fully specializes in the polluting sector (following proposition 6). The carbon tax in figure 1.D follows a pattern similar to the first best case, however, since clean technologies are developed slower, emissions decrease less fast (see figure 2.A) and the carbon tax goes to zero slower. Finally, figure 1.E shows the allocation of innovation in the North optimal policy. The pattern is similar to the second best case, but less North scientists are allocated to the polluting sector. Since the North becomes the exporter of good G, allocating less scientists to its export sector positively affects its terms of trade. In this calibration, the South is also fully specialized from the first period; however,

[27] There is no good empirical estimation of this parameter yet. Its important role is the focus of the numerical exercise in AABH, it does not strongly affect the comparative statics exercise carried out here.

[28] If one assumes that the law of motion (34) holds only for $S_{t-1} > 0$ and that $S = 0$ is still an absorbing state. In this case remark 2, proposition 6 and proposition 7 are still satisfied (except for the finite carbon tax). If $S = 0$ is not an absorbing state, however, then for $\eta < 1$, temporarily reaching $S_t = 0$ can be part of the solution even for arbitrarily low discount rate, so that the proofs of the previous remarks and propositions are not valid any more (even for $\sigma < 1$). Here $\eta = 1$, so this does not matter.

[29] Finally, ξ_N and ξ_S are adjusted upwards so that emisssions in sector G in the North and in the South correspond to total world emissions.

[30] A sufficiently large tariff on the polluting good G leads to autarky, if the trade tax increases further, it is an export subsidy and it can reverse the pattern of trade. This is what happens in this calibration in the first period.

according to proposition 7, it must remain just at the corner of specialization. To maintain this situation the trade tax turns negative from the second period, and becomes an export tax on the polluting good for the North (for readability the figure is cut, but as the comparative advantage increases, the trade tax tends towards -1, that is a 100% tax on exports). To cope with the smaller innovation in the polluting sector in the first period in the North, the carbon tariff is larger in the first period. The fast specialization in at least one country that occurs in this calibration can be explained by a relatively large growth rate (2% a year) combined with a small difference in capital share in both sector ($\alpha - \beta = .2$). Taking into account imperfect mobility of factors or both cross-sector and cross-country knowledge spillovers would slow down the specialization process.

Table 1: disaster and welfare cost

	time until disaster	welfare cost
Miracle	never	0%
Laissez-faire	80 years	100%
First best	never	5.64%
Second best	never	14.08%
Third best	never	14.22%
Non protect.	80 years	100%

Table 1 shows whether a disaster can be avoided under different scenarios and what the welfare costs of climate change are. The welfare cost is computed as the equivalent percentage loss of world consumption every period relative to the first best in a "miracle" scenario where the dirty input would cease to pollute (that is from the first period $\xi_N = \xi_S = 0$). Under laissez-faire a disaster occurs after 80 years (the welfare cost is 100% with log utility). There is no disaster in the first best case (which is always true) and since initial environmental quality is sufficiently large, there is no disaster either in the second best case. Not being able to intervene in the South, and therefore having to reverse the pattern of trade, sharply increases the welfare costs of climate change policy (they are nearly three times as large).[31] Table 1 also presents the case of a "third" best where the North can implement a carbon tax and research subsidies/taxes, but no trade tax, consumption or production taxes. With the calibrated parameter values, a tariff happens to be not necessary to avoid a disaster and the welfare costs of dispensing with it are relatively small: This is because the initial comparative advantage of the South in the polluting sector is small. No combination of a carbon tax and a tax on dirty research in the North can prevent an environmental disaster, and in table 1, I compute the "non-protectionist policy" that minimizes the amount of CO_2 accumulated. This

[31] Note that this increase in cost is almost entirely due to the environmental externality. In the miracle case, there would also be some welfare costs from not being able to intervene in the South - as innovation there would not be allocated optimally- but these costs would be very small: 0.15%.

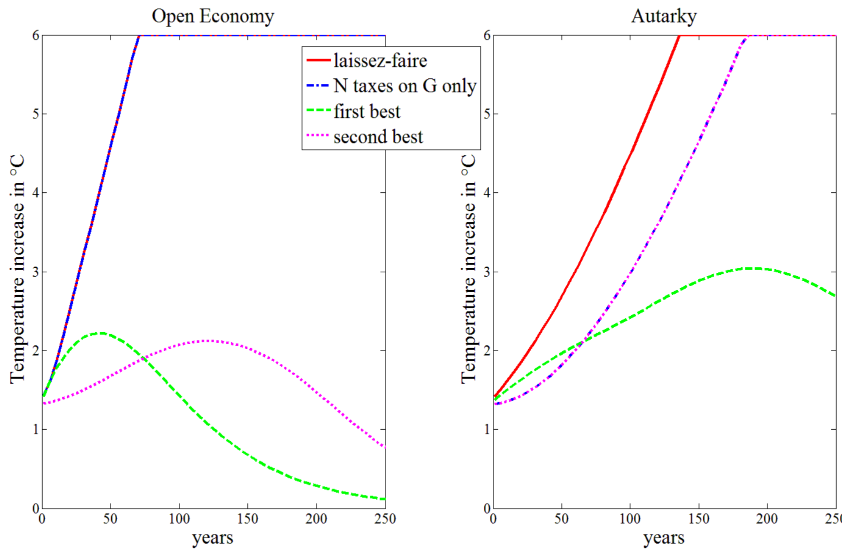

Figure 2: Figure 2: Increases in temperature with trade and in autarky

policy cannot postpone the disaster, and as shown in figure 2 below, its effect on temperature is extremely small relative to laissez-faire.

5.3 Trade, a double-edged sword

Figure 2 shows the increases in temperature in laissez-faire, with the non-protectionist policy that minimizes the amount of CO_2 emissions, in the second best case and in the first best case, and then compares them with the increases in temperature with the same policies in autarky. In line with table 1, non-protectionist policies cannot slow down the disaster; in fact, the difference in temperature is so small that the two curves are indistinguishable. In the first best case the increase in temperature is initially larger than in the second best case, the reason is that in the latter case production of the polluting good moves to the North, where the emissions rate is lower. The temperature starts decreasing earlier in the first best case than in the second best case because (nearly) all Southern scientists innovate in clean technologies in the first best policy, but not all scientists in the North do so in the second best one (see figures 1.A and 1.C). In autarky, the disaster is postponed, as the polluting sector grows slower (a mass of around 0.2 of scientists innovate in the polluting sector in autarky, instead of all scientists of one country in free trade). Non-protectionist policies in the North (a carbon tax and a tax on dirty research) can now postpone a disaster as the pollution haven effect does not exist, but whether clean research subsidies are allowed for or not does not really affect the increase in temperature. In figure 2.B, in the "second best" case (which now refers to the combination of carbon taxes and clean research subsidies in the North that minimizes CO_2 emissions), the temperature

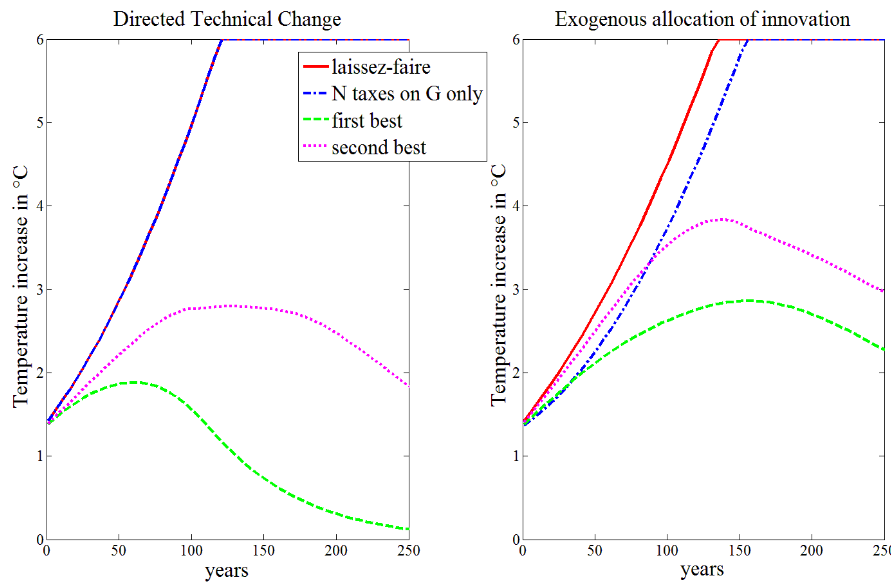

Figure 3: Figure 3: Increases in Temperature with and without Directed Technical Change

increase cannot be distinguished from the temperature increase resulting from taxes on the polluting sector in the North only. Even in the first best case temperature increases more as the growth rate of clean technologies is smaller than in the open economy scenario. Overall, figure 2 illustrates the double-edged sword role of trade: without trade, unilateral policies cannot prevent a disaster, but opening up to trade accelerates environmental degradation if the North does not undertake the appropriate policy.

5.4 Directed technical change, a double-edged sword

Directed technical change (DTC) plays a similar role. With the calibrated values, however, non-protectionist policies cannot delay a disaster either when there is no DTC and in both countries $s_c = s_d = s_H = 1/3$. To better illustrate the impact of DTC, I carry the same exercise but assuming that $\alpha = 0.7$ and $\beta = 0.1$ (a larger difference in capital limits the pollution haven effect in a static model and therefore better illustrates how it is amplified by the innovation response). The resulting increase in temperature across the different policies and under the two scenarios are given in figure 3 (Appendix C.2 carries the exercise with the original values). DTC accelerates the disaster under laissez-faire since it accelerates the growth rate of the production sector. Without DTC, non-protectionist policies are still unable to prevent a disaster, but they can delay it for 20 years. With a permanent and large trade tax, unilateral policies can still avert an environmental disaster with these parameters but the increase in temperature is much larger (despite a much lower growth rate), and even in the first best case temperature increases for a longer time.

6 Extensions

I now relax the assumption that productivity improvements are entirely country specific. The amplification of comparative advantage effect has been one of the dragging forces behind the previous results. In reality, some productivity improvements are likely to at least partly cross borders, so that some economic forces will work against this effect.[32] This raises the issue of the robustness of the previous analysis. I consider in turn technology diffusion (subsection 6.1) and innovation by global firms (subsection 6.2), and show that the main messages of section 3 still hold, even though the underlying intuitions are somewhat different. Note that the assumption that $\alpha \neq \beta$ must be imposed now as a pure Ricardian model would not be suited to study these questions.

6.1 Technology Diffusion

Far from the technological frontier, a country is likely to benefit from the diffusion of innovations produced in other countries. To illustrate this in the simplest way, I assume that at the beginning of every period the country with the least advanced average productivity in a given sector can exogenously progressively catch up with the other country. More specifically, before any innovation happens, the producer of intermediate i in sector $z \in \{c, d, H\}$ gets access to the technology:

$$\overline{A_{Xzit}} = \max\left(\left(\frac{A_{(-X)z(t-1)}}{A_{Xz(t-1)}}\right)^{\delta}, 1\right) A_{Xzi(t-1)},$$

where $\delta \in [0, 1]$ measures the strength of the technological diffusion. This formulation delivers in return the law of motion for aggregate productivity

$$A_{Xzt} = (1 + \kappa s^{\iota}_{Xzt})^{1-\gamma} \max\left(\left(\frac{A_{(-X)z(t-1)}}{A_{Xz(t-1)}}\right)^{\delta}, 1\right) A_{Xz(t-1)},$$

for $z \in \{c, d, H\}$. With this formulation, the ratio of the technological levels across countries cannot diverge: when one country gets a sufficiently strong advantage over the other one, the catching up process ensures that regardless of the pattern of innovation, this difference is reduced next period.

In particular, policies in the North which increase the amount of clean innovation in the North will now also increase the productivity of clean technologies in the South. In fact, they may even put the South on a clean innovation track: if at one point in time pre-innovation

[32] Dechezleprêtre et al. (2010) suggest that clean technologies transfer between developing and developed countries exist but are quite limited.

clean technologies are above dirty ones in the South $\overline{A}_{Sct} > \overline{A}_{Sdt}$, market forces in the South will induce more clean than dirty innovations. The key to preventing a disaster is no longer to push the South towards specializing in the non-polluting sector, but rather to ensure a switch towards clean innovation in the South. Such transition will occur as soon as more scientists are allocated to clean technologies in the North than to dirty technologies in the South (since in the long-run, South clean productivity A_{Sct} grows like the North one A_{Nct}). Whether this is the case or not directly depends on the policies that the North allows for and on the pattern of comparative advantage, similarly to the analysis in section 3. Therefore, the intuitions developed before still apply and, surprisingly, the broad results are not as different as one could have expected. In particular, I can show:

Proposition 8 *Assume that initially (i) technologies are sufficiently close to each other across countries, that κ is sufficiently small and the spillovers δ are sufficiently strong, (ii) that the South is relatively well endowed in capital $\frac{K_S}{L_S} > \frac{K_N}{L_N}$, and that (iii) clean technologies are sufficiently less advanced than dirty ones (A_{Sc0}/A_{Sd0} sufficiently small), then no combination of a carbon tax and a tax on dirty research in the North can prevent a disaster, no matter how large $\overline{S}$ is.*

Proof. See Appendix D.7 ■

This proposition mirrors proposition 1. The assumptions (i) ensure that technological levels remain sufficiently close to each other across countries, which combined with the assumption (ii) ensures that the South keeps a comparative advantage in the polluting sector - assumption (iii) plays the same role as in proposition 1 and ensures that when the South has the comparative advantage in the polluting sector, it does innovate there more than the North; this assumption can be dispensed with when $\varepsilon = \infty$. In this case, the South keeps the comparative advantage in the polluting sector, and since a carbon tax in the North can only reinforce this comparative advantage; more scientists are innovating in dirty technologies in the South than in clean in the North: South clean productivity $\overline{A}_{Sct}$ never catches up, and a switch to clean innovation in the South never occurs.

As before, a temporary combination of clean research subsidies and a tariff can prevent a disaster for sufficiently large initial environmental quality (proposition 3 still holds): clean research subsidies can reallocate innovation in the North to clean technologies while a tariff can limit innovation in dirty technologies in the South, so that $\overline{A}_{Sct}$ grows faster than $\overline{A}_{Sdt}$ and a switch towards clean innovation eventually occurs in the South. As it has been stated, proposition 2 is not robust when the clean and the dirty inputs are not perfect substitutes ($\varepsilon < \infty$). In that case, however, I can show:

Remark 3 Assume that final consumption is Cobb-Douglas in the polluting and non-polluting goods ($\sigma = 1$) and that clean and dirty inputs are perfect substitutes ($\varepsilon = \infty$). First, i) if initial relative endowments are sufficiently close to each other and the initial environmental quality is sufficiently large, then temporary clean research subsidies in the North alone can prevent a disaster. Second, ii) if the South has a sufficiently large capital labor ratio ($\frac{\alpha^{\beta}(1-\alpha)^{1-\beta}}{\beta^{\beta}(1-\beta)^{(1-\beta)}} K_N^{\beta} L_N^{1-\beta} > (1+\kappa)^{\frac{1-\gamma}{\delta}} \frac{1-\nu}{\nu} K_S^{\beta} L_S^{1-\beta}$ and $(1+\kappa)^{\frac{1-\gamma}{\delta}} K_N^{\alpha} L_N^{1-\alpha} < \frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}} \frac{1-\nu}{\nu} K_S^{\alpha} L_S^{1-\alpha}$), technologies are sufficiently close across countries ($A_{Nz0}/A_{Sz0} \in \left((1+\kappa)^{-\frac{1-\gamma}{\delta}}, (1+\kappa)^{\frac{1-\gamma}{\delta}}\right)$ for $z \in \{c, d, H\}$), and clean technologies are less advanced than dirty ones in the South ($A_{Sc0} < A_{Sd0}\,(1+\kappa)^{-(1-\gamma)\left(1+\frac{1}{\delta}\right)}$), then, clean research subsidies alone can never prevent a disaster.

Proof. See Appendix D.7 ■

The introduction of knowledge spillovers ensures that even in the Cobb-Douglas ($\sigma = 1$) case, clean research subsidies can now prevent a disaster without a tariff if initial endowment ratios are not too far apart.[33] The reason is that thanks to the knowledge spillovers, close initial endowment ratios guarantee that the South will not be fully specialized in the long-run. Some scientists in the South must be allocated to the non-polluting sector, so that clean technologies in the North grow faster than dirty technologies in the South with sufficiently large clean research subsidies. On the other hand, if endowment ratios are sufficiently far apart and technologies are close to start with, full specialization in both countries will be maintained indefinitely. As a result, all scientists would be allocated to the polluting sector in the South. If the clean and dirty inputs are perfect substitutes ($\varepsilon = \infty$), they will necessarily all be allocated to the dirty subsector: South clean productivity A_{Sct} never catches up preventing a switch to clean innovation in the South.

If $\varepsilon < \infty$, since clean technologies in the South grow at the same rate as in the North, the ratio of clean over dirty technologies in the South cannot tend towards zero, and the mass of scientists allocated to clean technologies remain bounded away from zero when the South specializes in the polluting sector. Therefore, eventually, pre-innovation clean productivity in the South becomes larger than dirty productivity, $\overline{A}_{Sct} > \overline{A}_{Sdt}$, and a switch towards clean innovation must occur. Note, however, that this result relies on the possibly unrealistic assumption that the North can innovate in clean technologies even though it specializes in the non-polluting sector and therefore does not produce any clean intermediates. If the North could not innovate in an inactive sector, remark 3 would still hold for $\varepsilon < \infty$. Second, this result also relies on the innovation function κs^{ι} satisfying the Inada condition. If instead, the innovation function were $\kappa\left((s+\Upsilon)^{\iota} - \Upsilon^{\iota}\right)$ with $\Upsilon > 0$, then, for clean technologies initially

[33] $\frac{1-\alpha}{\alpha}\frac{K_S}{L_S} < \frac{1-(\alpha\nu+\beta(1-\nu))}{\alpha\nu+\beta(1-\nu)} \frac{(1+\kappa)^{-\frac{1-\gamma}{\delta}} K_N + K_S}{(1+\kappa)^{\frac{1-\gamma}{\delta}} L_N + L_S} < (1+\kappa)^{-2\frac{1-\gamma}{\delta}} \frac{1-\beta}{\beta}\frac{K_N}{L_N}$ is a sufficient condition.

sufficiently less advanced than dirty ones (A_{Sc0}/A_{Sd0} being sufficiently small), all innovation in the South could go towards dirty technologies when the South is fully specialized in the polluting sector and remark 3 would hold.[34]

The structure of the optimal policy (with or without the no intervention in the South constraint) is broadly similar, but subsidies to research and the possible trade tax will have to take into account the presence of the knowledge spillovers. Moreover, the second best policy does not necessarily feature a reversal of comparative advantage any more when the South initially has the comparative advantage in the polluting good.

Note that to some extent technological diffusion itself is a parameter that can be affected by policy: laxer intellectual property rights, direct financing of projects abroad, or migrations of skilled workers could all contribute to the faster diffusion of technology. This analysis therefore suggests that the diffusion of clean technologies in the South makes the need for a tariff less pressing. In fact, since the high social welfare cost from unilateral intervention without knowledge spillovers came from the necessary reversal of comparative advantage, diffusing technologies in order to prevent a disaster without such a reversal in comparative advantage could reduce significantly the costs of the intervention.

6.2 Worldwide entrepreneurs

So far I have assumed that innovation in the North and in the South only responded to local conditions. However many innovative firms are global and make their innovation decision based on the entire world market. I now study this case and, for simplicity, focus on the case where final consumption is Cobb-Douglas ($\sigma = 1$). I show that clean research subsidies alone can now prevent a disaster but that carbon taxes may still fail to do so. The conditions under which they would now refer to the relative size of the polluting sector in the South and the North rather than simply the pattern of comparative advantage.

More specifically, I consider that the producer of intermediate ι in sector $z \in \{c, d, H\}$ is the same in the North and in the South and his intermediate has the same productivity in both countries; however, intermediates are still not tradeable.[35] By hiring s_{Nzit} scientists in the North and s_{Szit} in the South, the entrepreneur for variety i in sector $z \in \{c, d, H\}$ with

[34] In Appendix D.7, I also show that when $\sigma < 1$, clean research subsidies can prevent a disaster for sufficiently close endowment ratios, whether the government in the North can allocate research to the clean subsector when no clean intermediates are produced or not.

[35] In this case trade balance typically does not hold since there will be income transfers across countries. Under free-trade this does not affect the pattern of production and emissions.

initial productivities $A_{zi(t-1)}$ can increases it to:

$$A_{zit} = \left(1+\kappa\left(s_{Nzit}^{\iota}+s_{Szit}^{\iota}\right)\frac{A_{z(t-1)}^{\frac{1}{1-\gamma}}}{A_{zi(t-1)}^{\frac{1}{1-\gamma}}}\right)^{1-\gamma} A_{zi(t-1)}.$$

The results extend to the case where the innovation function is $\kappa\left(s_{Nzit}+s_{Szit}\right)^{\iota}$. As before, since profits are proportional to $\left(A_{zit}/A_{zt}\right)^{\frac{1}{1-\gamma}}$, for $z \in \{c, d, H\}$, every entrepreneur hires the same number of scientists, and the law of motion of aggregate productivity can be written as:

$$A_{zt} = \left(1+\kappa\left(s_{Xzt}^{\iota}+s_{(-X)zt}^{\iota}\right)\right)^{1-\gamma} A_{z(t-1)}.$$

In this subsection, the key to preventing a disaster will be to ensure that A_{ct} grows faster than A_{dt}, since this will bring the emission rate in the polluting sector down to zero even in the South. The next remark shows that it is still the case that a carbon tax can fail at preventing an environmental disaster:

Proposition 9 *Assume that $\sigma = 1$, that clean technologies are sufficiently less advanced than dirty ones (A_{c0}/A_{d0} is sufficiently small), and that the South originally has a weakly larger market share than the North in the polluting good ($Y_{SG0} \geq Y_{NG0}$), then no carbon tax in the North can prevent a disaster, no matter how large the initial environmental quality $\overline{S}$ is.*

Proof. Appendix D.8 ∎

Without direct research incentives, the innovation allocation is identical across countries. Within the polluting sector, the allocation of innovation across the clean and dirty subsectors favors the one with the largest revenues. In the North, a large carbon tax can ensure that the clean input subsector has nearly the same size as the entire North polluting sector, while in the South, if clean technologies are sufficiently less advanced (A_{c0}/A_{d0} is small), the size of the dirty subsector is close to the size of the South polluting sector ($A_{c0} < (1+2\kappa)^{-(1-\gamma)} A_{d0}$ would be sufficient when $\varepsilon = \infty$). Worldwide, the size of the clean subsector is then close to the size of the polluting sector in the North, while the size of the dirty subsector is close to the size of the polluting sector in the South. If the South has a larger market share in the polluting sector, there would be more dirty than clean innovations, and A_{ct} would never catch up. When final consumption is Cobb-Douglas, the relative size of both countries in the production of the polluting good does not change with technologies, and, since a carbon tax can only increase the relative size of the South, an initially larger market share ensures that the South remains the biggest country in the polluting sector.[36]

[36] If the North uses a tax on dirty research on top of the carbon tax, the remark stays true but for Y_{NG0}/Y_{SG0} sufficiently small. In this case, the Northern social planner, can ensure that no scientists in the North innovate in dirty technologies. Yet, if Y_{NG0}/Y_{SG0} is small, most innovation in the North will occur in the non-polluting sector anyway, so most sector G innovation will occur in the South and will be determined by the South market, favoring dirty innovation. See Appendix D.8

If subsidies to clean research in the North are sufficiently strong, however, Northern scientists will nearly all innovate in clean intermediates, thereby also improving the productivity of clean intermediates in the South. In the meantime, innovation in the South will be prevented from moving fully towards sector G and dirty intermediates: even if the South fully specializes in sector G, entrepreneurs having monopoly rights in sector H will hire some scientists from the South to improve the productivity of the (active) variety they own in the North. Therefore, a tariff (which is still part of the optimal policy) is not necessary to prevent a disaster, regardless of initial endowments.

Proposition 10 *Clean research subsidies in the North alone can prevent a disaster if the initial environmental quality is sufficiently large.*

In Appendix D.9, I extend this result to the case where $\sigma < 1$, and when innovation is only partially global.[37] As technological diffusion, the internationalization of the R&D process makes it possible for policy in the North to induce a switch to clean innovation in the South, so that the second best policy needs not feature a reversal of the pattern of comparative advantage. However, even with a very internationalized process, the North needs to undertake a very proactive policy towards the development of clean technologies; otherwise, the direction of innovation within sector G will be dictated by the economic conditions of the South instead of the North.

7 Conclusion

This paper develops a dynamic model of trade and the environment with directed technical change in a two country world, in order to study what type of unilateral policies could achieve sustainable growth. I also characterize the best unilateral policy for the world and for the intervening country. When knowledge is local, a combination of temporary clean research subsidies and a carbon tariff can prevent an environmental disaster, while unilateral taxes on the polluting sector are unlikely to do so, particularly when the South has initially the comparative advantage in the polluting sector. The second best policy which maximizes world welfare under the constraint that no intervention can be undertaken in the South can be decentralized through research subsidies, a carbon tax and a trade tax. The trade tax takes

[37] I look at the case where entrepreneurs have different technologies across countries, and where innovation takes the form: $A_{Xzit} = \left(1 + \kappa \left(s^{\iota}_{Xzit} + \delta s^{\iota}_{(-X)zit}\right) \frac{A_{Xz(t-1)}^{\frac{1}{1-\gamma}}}{A_{Xzi(t-1)}^{\frac{1}{1-\gamma}}}\right)^{1-\gamma} A_{Xzi(t-1)}$, where δ measures how easy it is to transfer innovation from one country to another. I show that as long as $\delta\left(1 + \left(\frac{\delta}{1+\kappa}\frac{\nu}{1-\nu}\right)^{\frac{1}{1-\iota}}\right)^{\iota} > 1$ clean research subsidies alone can prevent a disaster for sufficiently large environmental quality.

the form of a carbon tariff and then of an export subsidy, reflecting the double objective of reducing emissions in the South and of redirecting innovation there. Under some assumptions on the preferences of the social planner, the second best policy features a switch to clean innovation in the North, full specialization in both countries, and a temporary trade tax. The optimal policy for the North can be decentralized using the same instruments, but in this case, the trade tax may turn into an export tax and must be maintained permanently. In the presence of knowledge spillovers, or with international innovating firms, a switch to clean innovation in the South can be achieved with policies in the North, so that a disaster can be avoided without the South having to specialize in the non-polluting sector. In both cases unilateral taxes on the polluting sector may still fail at preventing a disaster.

In response to the question "is there a case for carbon tariffs?", the analysis shows that some form of protectionism is necessary to ensure sustainable growth with unilateral policies. A trade tax is the optimal tool to reduce pollution in countries that do not intervene, but, clean research subsidies alone can sometimes ensure sustainable growth particularly in the presence of knowledge spillovers.

In practice, taken at face value, a full revival of industries like metallurgy in developed countries, which is what the basic model argues for, may look unrealistic. More practically, the paper argues for an industrial policy in the North that aims at cleaning the polluting sectors, without losing to much competitiveness to the South, in order to slow down the move of polluting industries there. In particular, the paper shows the shortcomings of a "carbon tax"-only policy (or equivalent policies like a cap-and-trade) in the North in the presence of imperfect knowledge markets and a non-cooperative South. However, this aggressive protectionist unilateral policy could phase out once clean technologies diffuse to the South, or once a global agreement is found.

This analysis is very much a first step and could be enriched in several directions. First, I have only considered a global social planner faced with the constraint of no intervention in the South or a Northern social planner. A better understanding of climate negotiations would require modelling two competing social planners in a Nash equilibrium. As mentioned in the text, the design of the trade tax (whether it is directly related to average carbon content in the country or not) will then affect the behavior of the South planner. Second, to analyze more carefully the actual implementation of carbon tariffs, one would have to take into account WTO constraints and the possibility for firms or countries to hide the true level of their emissions or to manipulate environmental policies for their own advantage. Third, I have not investigated the issue of intellectual property rights (IPR), which may play a major role in the development of clean technologies. On one hand, laxer IPR could lead to a faster diffusion of clean technologies to the South, which facilitates a switch towards a clean path there. On the other hand, they may reduce the incentive to develop clean technologies in the North in the first place. Finally, the paper suggests that directed technical change makes emissions in the South much more responsive to policies in the North in the long-run. This calls into question the common estimates of the carbon leakage rate which are obtained in static models. Therefore, integrating directed technical change into a full numerical model of the world economy would be very useful in order to reevaluate the impact of carbon taxes and carbon tariffs.

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8 Appendix A

In this appendix, I mention a few direct extensions of section 3.

Clean input production subsidy. Section 3 focused on the role of research subsidies, carbon taxes and trade taxes, since they are the instruments used to decentralize the optimal policy. In practice, other instruments could be used in a unilateral policy (especially if tariffs were forbidden), and it is interesting to know whether a disaster could be avoided with them. For instance, a temporary direct production subsidy to the clean input could avoid an environmental disaster alone (provided that the initial environmental quality is large enough). In a first phase the North could implement a subsidy sufficiently large that the North essentially produces the clean input and clean intermediates only (this is the same as an infinite tax on any other type of production), which guarantees that the North has the comparative advantage in the polluting sector. As the North innovates in clean technologies only, while the South innovates in both sectors, the North can acquire a comparative advantage in the polluting sector even without the subsidy. At which point a disaster can be avoided following the same logic as in proposition 3.[38]

Different mass of scientists in the North and the South. If the mass of scientists in the North was much smaller than in the South, the North would eventually become a small economy relative to the South, and the South's economy will behave as if it were in autarky: regardless of the policies undertaken by the North, a disaster would be unavoidable. On the contrary, if the mass of scientists in the North was much larger than in the South, a disaster could be avoided using clean research subsidies without the need for a tariff: even when the South fully specializes in sector G, A_{NGt} would grow faster than A_{SGt}, so the North could build a comparative advantage in the polluting sector. In fact, depending on parameters, a disaster may also be avoided using taxes on dirty research or a carbon tax under the assumptions of proposition 1.[39]

Limited policy in the South. I now discuss the case where intervention in the South is possible but limited by a low fiscal capacity (the South does not have the necessary infrastructure to enforce full collection of taxes beyond some threshold). More specifically, the size of government revenues in the South cannot exceed some faction g of the size of the econ-

[38] Another possibility is that the North could only implement a consumption tax on the polluting good instead of a tariff. Such a tax can prevent the South from specializing in the polluting good (by reducing its relative price), but may induce the North to (temporarily) specialize in the non-polluting sector, in which case clean intermediates are no longer produced in the North for some periods. Assuming that entrepreneurs can still hire scientists even when they do not produce any intermediates, a combination of the consumption tax and clean research subsidies can avoid a disaster for sufficiently large initial environmental quality.

[39] For instance, in the Cobb-Douglas case $\sigma = 1$, if the consumption share of the polluting good (ν) is close to 1, $\frac{A_{NGt}}{A_{NHt}}$ can grow faster than $\frac{A_{SGt}}{A_{SHt}}$, leading to a reversal of comparative advantage, provided that there is a sufficiently large mass of scientists in the North; if $\nu = 1/2$, on the contrary, the previous analysis carries on.

omy (measured by $p_{Gt}Y_{SGt} + p_{Ht}Y_{SHt}$). To ensure a switch towards clean technologies in the polluting sector, the South needs to maintain tax on dirty innovation such that slightly more innovation occurs in clean than in dirty technologies as long as clean technologies remain less advanced than the dirty ones. From (18 and 19), at the tipping point ($s_{Sct} = s_{Sdt} = \frac{s_{SGt}}{2}$), the size of the intervention (T_t), relative to the size of the economy would be equal to:

$$\frac{T_t}{p_{Gt}Y_{SGt} + p_{Ht}Y_{SHt}} = \frac{\gamma(1-\gamma)\iota\kappa\left(\frac{s_{SGt}}{2}\right)^{\iota}}{\left(1+\kappa\left(\frac{s_{SGt}}{2}\right)^{\iota}\right)^{1-(\varepsilon-1)(1-\gamma)}} \frac{A_{Sdt-1}^{\varepsilon-1} - A_{Sct-1}^{\varepsilon-1}}{A_{Sct}^{\varepsilon-1} + A_{Sdt}^{\varepsilon-1}} \frac{p_{Gt}Y_{GSt}}{p_{Gt}Y_{SGt} + p_{Ht}Y_{SHt}} \quad (35)$$

For the switch to occur this ratio must remain smaller than g, otherwise, the gap between clean and dirty technologies in the South keeps increasing. The more the South specializes in the polluting sector G, the higher this ratio is as $\frac{p_{Gt}Y_{GXt}}{p_{Gt}Y_{SGt}+p_{Ht}Y_{SHt}}$ and s_{SGt} increase. Now, a tariff implemented by the North on the polluting good G can reduce the degree of specialization of the South in this sector, and therefore increases the chance that an intervention in the South becomes possible. An environmental disaster can then be avoided without the South specializing in the non-polluting sector.[40]

9 Appendix B: Main proofs of the paper

9.1 Appendix B.1 Characterization of the equilibrium in a given period

For this subsection I consider the economy at a given time, after innovation has occurred (I drop the subscript t for simplicity). To avoid repetition with the social optimum analysis, I do not impose that the economy is in laissez-faire: I let each country impose a subsidy $\widetilde{q}$ on all intermediates (it can differ across time or across countries without affecting anything as it has a pure scale effect) and a carbon tax τ_X. First I derive the aggregate production functions in each sector given prices, second I solve for prices and third I characterize the pattern of specialization in free trade. Finally, I derive the allocation of innovation.

9.1.1 B.1.1 Deriving aggregate production function.

First, note that if good G is produced, then both subsectors c and d are active. Assume that good G is produced in country X, the maximization problem for producers in subsector

[40] It is important to distinguish this case from one where the desire to participate or not in the South arises from a local government maximizing its own welfare. In that case, intervention in the South may be limited relative to the first best, both because the South social planner does not internalize the damage on the welfare of North citizens, and because of the leakage effect to the North which diminishes the efficiency of a South policy. However, being an exporter of good G pushes the South towards a relatively larger intervention to benefit from better terms of trade. A tariff on the polluting good would reduce these terms of trade effect and therefore potentially the willingness of the South to switch to clean technologies (see Copeland and Taylor (2005)). It would then be even more important to ensure that the tariff is directly related to emissions in the exporting country as mentioned in the previous paragraph.

$z \in \{c, d\}$ leads to the demand function for capital and labor in assembly of good z:

$$r_X K_{Xz} = (1-\gamma)\,\alpha p_{Xz} Y_{Xz} \text{ and } w_X L_{Xz} = (1-\gamma)(1-\alpha)\, p_{Xz} Y_{Xz} \tag{36}$$

and the demand for intermediates:

$$\varphi_{Xzi} = \gamma p_{Xz} A_{Xzi} x_{zi}^{\gamma-1} \left(K_{Xz}^{\alpha} L_{Xz}^{1-\alpha}\right)^{1-\gamma}, \tag{37}$$

with φ_{Xzi} the consumer price of intermediate i. From 7, the cost of producing one unit of intermediate is given by $\psi \left(\frac{r_X}{\alpha}\right)^{\alpha} \left(\frac{w_X}{1-\alpha}\right)^{1-\alpha}$. Monopolists maximizes profits by imposing a mark-up $1/\gamma$ on their costs, so that producer prices are given by:

$$\frac{\varphi_{Xzi}}{1-\widetilde{q}} = \frac{\psi}{\gamma} \left(\frac{r_X}{\alpha}\right)^{\alpha} \left(\frac{w_X}{1-\alpha}\right)^{1-\alpha}. \tag{38}$$

The production of intermediates is then given by:

$$x_{Xzi} = \left(\frac{p_{Xz}\gamma^2}{(1-\widetilde{q})\,\psi} \left(\frac{\alpha}{r_X}\right)^{\alpha} \left(\frac{1-\alpha}{w_X}\right)^{1-\alpha}\right)^{\frac{1}{1-\gamma}} A_{Xzi}^{\frac{1}{1-\gamma}} K_{Xz}^{\alpha} L_{Xz}^{1-\alpha}, \tag{39}$$

and factor demands in the production of intermediate i in sector z follows:

$$K_{Xzi} = \left(\frac{\alpha}{r_X}\frac{w_X}{1-\alpha}\right)^{1-\alpha} \psi x_{Xzi} \text{ and } L_{Xzi} = \left(\frac{r_X}{\alpha}\frac{1-\alpha}{w_X}\right)^{\alpha} \psi x_{Xzi}. \tag{40}$$

Plugging in (36) and (39) into (6), I get the price of good z as:

$$p_{Xz} = \frac{1}{A_{Xz}} \frac{(1-\widetilde{q})^{\gamma}\,\psi^{\gamma}}{(1-\gamma)^{1-\gamma}\,\gamma^{2\gamma}} \left(\frac{r_X}{\alpha}\right)^{\alpha} \left(\frac{w_X}{1-\alpha}\right)^{1-\alpha}, \tag{41}$$

where. Now, profit maximization by producers of good G leads to the demand function:

$$\frac{Y_{Xc}}{Y_{Xd}} = \left(\frac{p_{Xc}}{(1+\tau_X)\,p_{Xd}}\right)^{-\varepsilon}, \tag{42}$$

and the price of good G is given by $p_G = \left(p_{Xc}^{1-\varepsilon} + (1+\tau_X)^{1-\varepsilon} p_{Xd}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$, which, using (41), translates into:

$$p_{XG} = \frac{(1-\gamma)^{\gamma-1}\,\gamma^{-2\gamma}\,(1-\widetilde{q})^{\gamma}\,\psi}{\left(A_{Xz}^{\varepsilon-1} + \left((1+\tau_X)^{-1} A_{Xd}\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}} \left(\frac{r_X}{\alpha}\right)^{\alpha} \left(\frac{w_X}{1-\alpha}\right)^{1-\alpha}. \tag{43}$$

This relationship holds if country X produces good G, if country X does not produce good G, the equality is replaced by: $p_{XG} \leq \frac{(1-\gamma)^{\gamma-1}\gamma^{-2\gamma}(1-\widetilde{q})^{\gamma}\psi}{\left(A_{Xz}^{\varepsilon-1}+\left((1+\tau_X)^{-1}A_{Xd}\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}} \left(\frac{r_X}{\alpha}\right)^{\alpha} \left(\frac{w_X}{1-\alpha}\right)^{1-\alpha}$. Similarly in sector H, $p_{XH} \leq \frac{(1-\gamma)^{\gamma-1}\gamma^{-2\gamma}(1-\widetilde{q})^{\gamma}\psi}{A_{XH}}\left(\frac{r_X}{\alpha}\right)^{\beta}\left(\frac{w_X}{1-\alpha}\right)^{1-\beta}$, with equality if good H is produced in country X.

Note that (42) gives:

$$Y_{Xd} = \left(\frac{(1+\tau_X)^{-1} A_{Xd}}{\left(A_{Xc}^{\varepsilon-1} + \left((1+\tau_X)^{-1} A_{Xd}\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}}\right)^{\varepsilon} Y_{XG}, \tag{44}$$

which directly leads to the expression for the emission rate. Combining (36), (39), (40), (42), and (43), one gets that total factor employment in sector G satisfies:

$$K_{XG} = \left(\frac{\alpha}{r_X}\frac{w_X}{1-\alpha}\right)^{1-\alpha}\frac{1}{\zeta}\frac{A_{Xc}^{\varepsilon-1} + (1+\tau_X)^{-\varepsilon} A_{Xd}^{\varepsilon-1}}{\left(A_{Xc}^{\varepsilon-1} + \left((1+\tau_X)^{-1} A_{Xd}\right)^{\varepsilon-1}\right)^{\frac{\varepsilon}{\varepsilon-1}}} Y_{XG}, \tag{45}$$

$$L_{XG} = \left(\frac{r_X}{\alpha}\frac{1-\alpha}{w_X}\right)^{\alpha}\frac{1}{\zeta}\frac{A_{Xc}^{\varepsilon-1} + (1+\tau_X)^{-\varepsilon} A_{Xd}^{\varepsilon-1}}{\left(A_{Xc}^{\varepsilon-1} + \left((1+\tau_X)^{-1} A_{Xd}\right)^{\varepsilon-1}\right)^{\frac{\varepsilon}{\varepsilon-1}}} Y_{XG}, \tag{46}$$

with $\zeta \equiv \frac{\gamma^{2\gamma}(1-\gamma)^{1-\gamma}(1-\widetilde{q})^{1-\gamma}}{((1-\gamma)(1-\widetilde{q})+\gamma^2)\psi^{\gamma}}$. Combining these two expressions and following the same strategy in sector H, one gets:

$$Y_{XGt} = \zeta\frac{\left(A_{Xct}^{\varepsilon-1} + \left((1+\tau_{Xt})^{-1} A_{Xdt}\right)^{\varepsilon-1}\right)^{\frac{\varepsilon}{\varepsilon-1}}}{A_{Xct}^{\varepsilon-1} + (1+\tau_{Xt})^{-\varepsilon} A_{Xdt}^{\varepsilon-1}} K_{XGt}^{\alpha} L_{XGt}^{1-\alpha} \text{ and } Y_{XHt} = \zeta A_{XHt} K_{XHt}^{\beta} L_{XHt}^{1-\beta}. \tag{47}$$

This equation translates into (16) when there is no carbon tax.

When both sectors are active, taking the ratio of (43) and the equivalent expression for p_{XH}, one can express the capital rent to wage ratio as

$$\frac{r_X}{w_X} = \left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\beta^{\beta}(1-\beta)^{1-\beta}}\right)^{\frac{1}{\alpha-\beta}}\left(\frac{\left(A_{Xc}^{\varepsilon-1} + \left((1+\tau_X)^{-1} A_{Xd}\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}}{A_{XH}}\right)^{\frac{1}{\alpha-\beta}}\left(\frac{p_{XG}}{p_{XH}}\right)^{\frac{1}{\alpha-\beta}}.$$

Plugging this expression into (45) and (46) and the equivalent equations in sector H, and using factor market clearing (9), one gets a system of two equations with two unknowns (Y_{XG},Y_{XH}) that can be solved as:

$$\begin{aligned} Y_{XG} \quad &= \quad \frac{\zeta}{(\alpha-\beta)}\left(\frac{\beta^{\beta\alpha}(1-\beta)^{(1-\beta)\alpha}}{\alpha^{\beta\alpha}(1-\alpha)^{(1-\alpha)\beta}}\right)^{\frac{1}{\alpha-\beta}}\frac{A_{XG}(\tau_X)}{1-\delta_X(\tau_X)} \\ &\times\left(\left(\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\beta)\left(\frac{p_{XG}}{p_{XH}}\frac{A_{XG}(\tau_X)}{A_{XH}}\right)^{\frac{1-\alpha}{\alpha-\beta}} K_X - \beta L_X\left(\frac{p_{XG}}{p_{XH}}\frac{A_{XG}(\tau_X)}{A_{XH}}\right)^{\frac{-\alpha}{\alpha-\beta}}\right) \end{aligned} \tag{48}$$

$$Y_{XH} = \frac{\zeta}{(\alpha-\beta)}\left(\frac{\beta^{\beta\alpha}(1-\beta)^{(1-\beta)\alpha}}{\alpha^{\beta\alpha}(1-\alpha)^{(1-\alpha)\beta}}\right)^{\frac{1}{\alpha-\beta}} A_{XH} \quad (49)$$
$$\times\left(\alpha\left(\frac{p_{XG}}{p_{XH}}\frac{A_{XG}(\tau_X)}{A_{XH}}\right)^{\frac{-\beta}{\alpha-\beta}} L_X - \left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\alpha)\left(\frac{p_{XG}}{p_{XH}}\frac{A_{XG}(\tau_X)}{A_{XH}}\right)^{\frac{1-\beta}{\alpha-\beta}} K_X\right).$$

where $A_{XG}(\tau_X) \equiv \left(A_{Xc}^{\varepsilon-1} + \left((1+\tau_X)^{-1}A_{Xd}\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}$ is a measure of average productivity of sector G in country X, and $\delta_X(\tau_X) \equiv \frac{\tau_X A_{Xd}^{\varepsilon-1}(1+\tau_X)^{-\varepsilon}}{A_{Xc}^{\varepsilon-1}+(1+\tau_X)^{-(\varepsilon-1)}A_{Xd}^{\varepsilon-1}} \in [0,1)$ is a correction term (measuring the difference between imposing a tax and a decrease in productivity of the dirty input).

9.1.2 B.1.2 Equilibrium price

Consumer maximization leads to $\frac{p_{XG}}{p_{XH}} = \frac{\nu}{1-\nu}\left(\frac{C_{XH}}{C_{XG}}\right)^{\frac{1}{\sigma}}$. In autarky this translates into: $\frac{Y_{XG}}{Y_{XH}} = \left(\frac{\nu}{1-\nu}\right)^{\sigma}\left(\frac{p_{XG}}{p_{XH}}\right)^{-\sigma}$, which combined with (48) and (49), defines the equilibrium autarky price uniquely (given technologies) since $\frac{Y_{XG}}{Y_{XH}}$ is increasing in $\frac{p_{XG}}{p_{XH}}$, and the right-hand side decreases. More specifically, one gets that the autarky price must satisfy:

$$\left(\frac{p_{XG}}{p_{XH}}\right)^{\sigma}\frac{A_{XG}(\tau_X)}{1-\delta_X(\tau_X)}\left(\begin{array}{c}\left(\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\beta)\left(\frac{p_{XG}}{p_{XH}}\frac{A_{XG}(\tau_X)}{A_{XH}}\right)^{\frac{1-\alpha}{\alpha-\beta}}K_X\\ -\beta L_X\left(\frac{p_{XG}}{p_{XH}}\frac{A_{XG}(\tau_X)}{A_{XH}}\right)^{\frac{-\alpha}{\alpha-\beta}}\end{array}\right) \quad (50)$$
$$= \left(\frac{\nu}{1-\nu}\right)^{\sigma} A_{XH}\left(\begin{array}{c}\alpha\left(\frac{p_{XG}}{p_{XH}}\frac{A_{XG}(\tau_X)}{A_{XH}}\right)^{\frac{-\beta}{\alpha-\beta}}L_X\\ -\left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\alpha)\left(\frac{p_{XG}}{p_{XH}}\frac{A_{XG}(\tau_X)}{A_{XH}}\right)^{\frac{1-\beta}{\alpha-\beta}}K_X\end{array}\right)$$

If $\varepsilon A_{Xc}^{\varepsilon-1}+(1-(\varepsilon-1)\tau_X)(1+\tau_X)^{-\varepsilon}A_{Xd}^{\varepsilon-1} > 0$, $\frac{A_{XG}(\tau_X)}{1-\delta_X(\tau_X)}$ increases in A_{Xc} and always increase in A_{Xd} and decrease in τ_X. Therefore $\frac{p_{XG}}{p_{XH}}$ decreases in A_{Xc} (when $\varepsilon A_{Xc}^{\varepsilon-1}+(1-(\varepsilon-1)\tau_X)(1+\tau_X)^{-\varepsilon}A_{Xd}^{\varepsilon-1} > 0$), decreases in A_{Xd} and K_X and increases in A_{XH} and L_X. It is direct to check that in the absence of any tax, the relative autarky price of good G over good H is higher in the North than in the South if and only if $(A_{SG}/A_{SH})^{\frac{1}{\alpha-\beta}}K_S/L_S > (A_{NG}/A_{NH})^{\frac{1}{\alpha-\beta}}K_N/L_N$ (so that under free-trade the North imports good G in this case).

Under free-trade, the equilibrium price ratio is the same in both countries and satisfies

$$\frac{p_G}{p_H} = \frac{\nu}{1-\nu}\left(\frac{C_{XH}}{C_{XG}}\right)^{\frac{1}{\sigma}} = \frac{\nu}{1-\nu}\left(\frac{Y_{NH}+Y_{SH}}{Y_{NG}+Y_{SG}}\right)^{\frac{1}{\sigma}}, \quad (51)$$

which similarly defines uniquely the price ratio given technologies (as Y_{XH} is decreasing in the price ratio and Y_{XG} increasing). When both countries produce both goods, one can use (48) and (49) to get:

$$\left(\frac{p_G}{p_H}\right)^{\sigma}\left(\begin{array}{c}\frac{A_{NG}(\tau_N)}{1-\delta_N(\tau_N)}\left(\left(\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\beta)\left(\frac{p_G}{p_H}\frac{A_{NG}(\tau_N)}{A_{NH}}\right)^{\frac{1-\alpha}{\alpha-\beta}}K_N-\beta\left(\frac{p_G}{p_H}\frac{A_{NG}(\tau_N)}{A_{NH}}\right)^{\frac{-\alpha}{\alpha-\beta}}L_N\right)\\+\frac{A_{SG}(\tau_S)}{1-\delta_S(\tau S)}\left(\left(\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\beta)\left(\frac{p_G}{p_H}\frac{A_{SG}(\tau_S)}{A_{SH}}\right)^{\frac{1-\alpha}{\alpha-\beta}}K_S-\beta\left(\frac{p_G}{p_H}\frac{A_{SG}(\tau_S)}{A_{SH}}\right)^{\frac{-\alpha}{\alpha-\beta}}L_S\right)\end{array}\right) \quad (52)$$

$$=\left(\frac{\nu}{1-\nu}\right)^{\sigma}\left(\begin{array}{c}A_{NH}\left(\alpha\left(\frac{p_G}{p_H}\frac{A_{NG}(\tau_N)}{A_{NH}}\right)^{\frac{-\beta}{\alpha-\beta}}L_N-\left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\alpha)\left(\frac{p_G}{p_H}\frac{A_{NG}(\tau_N)}{A_{NH}}\right)^{\frac{1-\beta}{\alpha-\beta}}K_N\right)\\+A_{SH}\left(\alpha\left(\frac{p_G}{p_H}\frac{A_{SG}(\tau_S)}{A_{SH}}\right)^{\frac{-\beta}{\alpha-\beta}}L_S-\left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\alpha)\left(\frac{p_G}{p_H}\frac{A_{SG}(\tau_S)}{A_{SH}}\right)^{\frac{1-\beta}{\alpha-\beta}}K_S\right)\end{array}\right)$$

9.1.3 B.1.3 Pattern of specialization in free trade

I now derive the full pattern of specialization in free trade. To simplify expression I introduce the notations $\widetilde{K_X} \equiv \left(\frac{A_{XG}(\tau_X)^{1-\beta}}{A_{XH}^{1-\alpha}}\right)^{\frac{1}{\alpha-\beta}} K_X$ and $\widetilde{L_X} \equiv \left(\frac{A_{XH}^{\alpha}}{A_{XG}(\tau_X)^{\beta}}\right)^{\frac{1}{\alpha-\beta}} L_X$, which represent "effective endowments". Using (48), (49) and (52), assuming that both countries produce both goods, the condition $Y_{XG} > 0$ translates into

$$\frac{\widetilde{K_X}}{\widetilde{L_X}} > \frac{\beta}{1-\beta}\frac{\left(\frac{\nu}{1-\nu}\right)^{\sigma}\left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\left(\frac{\widetilde{L_X}}{\widetilde{K_X}}\right)^{\alpha-\beta}\right)^{1-\sigma}(1-\alpha)\left(\widetilde{K_N}+\widetilde{K_S}\right)+(1-\beta)\left(\frac{\widetilde{K_N}}{1-\delta_N}+\frac{\widetilde{K_S}}{1-\delta_S}\right)}{\left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\left(\frac{\widetilde{L_X}}{\widetilde{K_X}}\right)^{\alpha-\beta}\right)^{1-\sigma}\alpha\left(\frac{\nu}{1-\nu}\right)^{\sigma}\left(\widetilde{L_N}+\widetilde{L_S}\right)+\beta\left(\frac{\widetilde{L_N}}{1-\delta_N}+\frac{\widetilde{L_S}}{1-\delta_S}\right)}, \quad (53)$$

and $Y_{XH} > 0$ into:

$$\frac{\widetilde{K_X}}{\widetilde{L_X}} < \frac{\alpha}{1-\alpha}\frac{\left(\frac{\nu}{1-\nu}\right)^{\sigma}\left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\left(\frac{\widetilde{L_X}}{\widetilde{K_X}}\right)^{\alpha-\beta}\right)^{1-\sigma}(1-\alpha)\left(\widetilde{K_N}+\widetilde{K_S}\right)+(1-\beta)\left(\frac{\widetilde{K_N}}{1-\delta_N}+\frac{\widetilde{K_S}}{1-\delta_S}\right)}{\left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\left(\frac{\widetilde{L_X}}{\widetilde{K_X}}\right)^{\alpha-\beta}\right)^{1-\sigma}\alpha\left(\frac{\nu}{1-\nu}\right)^{\sigma}\left(\widetilde{L_N}+\widetilde{L_S}\right)+\beta\left(\frac{\widetilde{L_N}}{1-\delta_N}+\frac{\widetilde{L_S}}{1-\delta_S}\right)}, \quad (54)$$

Therefore, conditions (53) and (54) define the set of endowments, productivity and taxes for which there is incomplete specialization in both countries.

Assume now that country X fully specializes in sector G, but country $-X$. does not fully specializes. Using (47), production of good G in X is given by: $Y_{XG} = \frac{\zeta}{1-\delta_X}\widetilde{K_X}^{\alpha}\widetilde{L_X}^{1-\alpha}$. Combining this expression with (51) and (48) and (49) for country $-X$, delivers an implicit equation for the price ratio, which can be used to that the condition $Y_{(-X)G} > 0$ is equivalent to:

$$\left(\frac{\widetilde{K_{-X}}}{\widetilde{L_{-X}}}\right)^{(\alpha-\beta)\sigma}\widetilde{K_{-X}}^{\beta}\widetilde{L_{-X}}^{1-\beta} > \left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)^{\sigma}\left(\frac{1-\nu}{\nu}\right)^{\sigma}\frac{\widetilde{K_X}^{\alpha}\widetilde{L_X}^{1-\alpha}}{1-\delta_X}. \quad (55)$$

This case, therefore corresponds to the opposite of (53) and (55).

Similarly if country X specializes in sector H, one gets $Y_{XH} = \zeta \widetilde{K_X}^{\beta} \widetilde{L_X}^{1-\beta}$ and the condition $Y_{H(-X)} > 0$ writes as:

$$\left(\frac{\alpha^{\beta}(1-\alpha)^{1-\beta}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\sigma} \widetilde{K_X}^{\beta} \widetilde{L_X}^{1-\beta} < \left(\frac{1-\nu}{\nu}\right)^{\sigma} \left(\frac{\widetilde{K_{-X}}}{\widetilde{L_{-X}}}\right)^{(\alpha-\beta)(1-\sigma)} \frac{\widetilde{K_{-X}}^{\beta} \widetilde{L_{-X}}^{1-\beta}}{1-\delta_{-X}}. \tag{56}$$

This case corresponds to the opposite of (54) and (56).

Finally the case where country X fully specializes in G while country $-X$ fully specializes in H corresponds to the opposite of (56).and the opposite of (55), for future use, it is convenient to express these two conditions with the actual endowments and productivities as:

$$\left(\frac{\alpha^{\beta}(1-\alpha)^{1-\beta}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\sigma} A_{(-X)H} K_{-X}^{\beta} L_{-X}^{1-\beta} \geq \left(\frac{1-\nu}{\nu}\right)^{\sigma} \left(\frac{K_X}{L_X}\right)^{(\alpha-\beta)(1-\sigma)} \frac{K_X^{\beta} L_X^{1-\beta}}{1-\delta_X} A_{XH}^{\sigma} \left(A_{XG}(\tau_X)\right)^{1-\sigma} \tag{57}$$

$$A_{(-X)H}^{1-\sigma} \left(A_{(-X)G}\left(\tau_{(-X)}\right)\right)^{\sigma} \left(\frac{L_{-X}}{K_{-X}}\right)^{(\alpha-\beta)(1-\sigma)} K_{-X}^{\alpha} L_{-X}^{1-\alpha} \leq \left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}} \frac{1-\nu}{\nu}\right)^{\sigma} \frac{A_{XG}(\tau_X) K_X^{\alpha} L_X^{1-\alpha}}{1-\delta_X}. \tag{58}$$

One can show that these endowment sets have no overlap. Moreover, in each case scenario the relative price of good G over good H is uniquely defined, Therefore in free trade and for given technologies, the equilibrium is unique.

9.1.4 B.1.4 Equilibrium profits and innovation decision

Using (6), (39), I can express intermediates production in sector $z \in \{c, d\}$ as: $x_{Xzi} = \frac{p_{Xz}\gamma^2}{(1-\widetilde{q})\psi} \left(\frac{\alpha}{r_X}\right)^{\alpha} \left(\frac{1-\alpha}{w_X}\right)^{1-\alpha} \left(\frac{A_{Xzi}}{A_{Xz}}\right)^{\frac{1}{1-\gamma}} Y_{Xzt}$,combining this with (41) and (38) gives

$$\pi_{Xzit} = \frac{(1-\gamma)\gamma}{(1-\widetilde{q})} \left(\frac{A_{Xzit}}{A_{Xzt}}\right)^{\frac{1}{1-\gamma}} p_{Xzt} Y_{Xzt}, \tag{59}$$

or (18) when $\widetilde{q} = 0$. Using (42), this translates into:

$$\pi_{Xcit} = \frac{\gamma(1-\gamma)}{(1-\widetilde{q})} \left(\frac{A_{Xcit}}{A_{Xct}}\right)^{\frac{1}{1-\gamma}} \frac{A_{Xct}^{\varepsilon-1}}{A_{cXt}^{\varepsilon-1} + \left((1+\tau_{Xt})^{-1} A_{dXt}\right)^{\varepsilon-1}} p_{Gt} Y_{GXt}, \tag{60}$$

$$\pi_{Xdit} = \frac{\gamma(1-\gamma)}{(1-\widetilde{q})} \left(\frac{A_{Xdit}}{A_{Xdt}}\right)^{\frac{1}{1-\gamma}} \frac{(1+\tau_{Xt})^{-\varepsilon} A_{Xdt}^{\varepsilon-1}}{A_{cXt}^{\varepsilon-1} + \left((1+\tau_{Xt})^{-1} A_{dXt}\right)^{\varepsilon-1}} p_{Gt} Y_{GXt}. \tag{61}$$

The same reasoning in sector H gives:

$$\pi_{XHit} = \frac{\gamma(1-\gamma)}{(1-\widetilde{q})} \left(\frac{A_{XHit}}{A_{XHt}}\right)^{\frac{1}{1-\gamma}} p_{XHt} Y_{XHt}. \tag{62}$$

To avoid repetition, I let both countries implement a tax q_{Xt} on the wages of scientists in the dirty subsector. Combining the first order conditions with respect to the number of scientists in the clean and dirty subsector (and assuming that some production takes place in sector G in country X) delivers the allocation of scientists within sector G as:

$$\frac{s_{Xct}^{1-\iota}\left(1+\kappa s_{Xct}^{\iota}\right)}{s_{Xdt}^{1-\iota}\left(1+\kappa s_{Xdt}^{\iota}\right)}=\frac{p_{Xct}Y_{Xct}}{p_{Xdt}Y_{Xdt}}=\frac{\left(1-q_{Xt}\right)\left(1+\tau_{Xt}\right)^{\varepsilon}A_{Xct}^{\varepsilon-1}}{A_{Xdt}^{\varepsilon-1}}, \tag{63}$$

where the second equality arises from (42) and (44). Similarly, combining the first order condition with respect to the number of scientists in sector H and subsector d, I get:

$$\frac{s_{Xdt}^{1-\iota}\left(1+\kappa s_{Xdt}^{\iota}\right)}{s_{XHt}^{1-\iota}\left(1+\kappa s_{XHt}^{\iota}\right)}=\frac{\left(1+\tau_{Xt}\right)^{-\varepsilon}A_{Xdt}^{\varepsilon-1}}{A_{Xct}^{\varepsilon-1}+\left(\left(1+\tau_{Xt}\right)^{-1}A_{Xdt}\right)^{\varepsilon-1}}\frac{p_{XGt}Y_{XGt}}{p_{XHt}Y_{XHt}}. \tag{64}$$

With $\tau_{Xt}=0$, these two last equations write as (19) and (20).

9.2 Appendix B.2 Proofs of path dependence and of lemma 2

This proof requires several steps. First, I solve for the allocation of innovation within the polluting sector for a given mass of scientists. Second, I derive the comparative static of the growth rate of A_{XGt} and the incentive to innovate in sector G with the ratio of initial clean and dirty productivities for a given mass of scientists. Third, I show that under the hypothesis of the lemma, A_{SGt} grows faster than A_{NGt} and A_{NHt} grow faster than A_{SHt}. Forth I prove lemma 2.

9.2.1 Allocation of innovation within the polluting sector

To simplify notation, I introduce the function $\widetilde{\kappa}\left(s\right)\equiv\kappa s^{\iota}$, and the notation

$$a_{Xt}\equiv\min\left(\left(\frac{A_{Xc(t-1)}}{A_{Xd(t-1)}}\right)^{\varepsilon-1},\left(\frac{A_{Xd(t-1)}}{A_{Xc(t-1)}}\right)^{\varepsilon-1}\right)\leq 1.$$

In laissez-faire (19) leads to the following equality

$$\begin{aligned}&\widetilde{\kappa}'\left(s_A\left(a_{Xt},s_{XGt}\right)\right)\left(1+\widetilde{\kappa}\left(s_A\left(a_{Xt},s_{XGt}\right)\right)\right)^{(\varepsilon-1)(1-\gamma)-1}\\ ' \quad = \quad &\widetilde{\kappa}'\left(s_a\left(a_{Xt},s_{XGt}\right)\right)\left(1+\widetilde{\kappa}\left(s_a\left(a_{Xt},s_{XGt}\right)\right)\right)^{(\varepsilon-1)(1-\gamma)-1}a_{Xt}\end{aligned}$$

where $s_A\left(a,s_G\right)$ is the allocation of scientists to the subsector amongst clean and dirty with the highest productivity level at $t-1$ and $s_a\left(a_X,s_{XGt}\right)$

9.2.2 Effect of relative productivity of clean and dirty on sector G growth and incentive to innovate

Denote $a_X \equiv \min\left(\left(\frac{A_{Xc}}{A_{Xd}}\right)^{\varepsilon-1}, \left(\frac{A_{Xd}}{A_{Xc}}\right)^{\varepsilon-1}\right) \leq 1$, and define

$$f(a, s_G) \equiv \frac{1+\widetilde{\kappa}(1-s_G)}{\widetilde{\kappa}'(1-s_G)}\frac{1}{2}\left(\frac{\widetilde{\kappa}'(s_a)(1+\widetilde{\kappa}(s_a))^{(\varepsilon-1)(1-\gamma)-1}a+\widetilde{\kappa}'(s_A)(1+\widetilde{\kappa}(s_A))^{(\varepsilon-1)(1-\gamma)-1}}{(1+\widetilde{\kappa}(s_a))^{(\varepsilon-1)(1-\gamma)}a+(1+\widetilde{\kappa}(s_A))^{(\varepsilon-1)(1-\gamma)}}\right),$$

where $s_A(a, s_G)$ and $s_a(a, s_G)$ are defined through and $s_A(a, s_G) + s_a(a, s_G) = s_G$, (that is s_A denotes the investments in the sector the most advanced between clean and dirty and s_a in the other sector). f represents for a given overall share of scientists in sector G, the ratio between the marginal benefit of an additional scientist in sector H divided by sector H revenues over the marginal benefit of an additional scientist in sector G divided by sector G's revenues. Note that with κ sufficiently small, $\widetilde{\kappa}'(s)(1+\widetilde{\kappa}(s))^{(\varepsilon-1)(1-\gamma)-1}$ is decreasing so that $s_a < s_A$. Next note that f is decreasing in s_G since $s_A(s_G)$ and $s_a(s_G)$ are both increasing in s_G. Rewriting $f(a, s_G) = \frac{(1+\widetilde{\kappa}(1-s_G))}{\widetilde{\kappa}'(1-s_G)}\frac{1}{\frac{1+\widetilde{\kappa}(s_a)}{\widetilde{\kappa}'(s_a)}+\frac{(1+\widetilde{\kappa}(s_A))}{\widetilde{\kappa}'(s_A)}}$, I get $\frac{\partial f}{\partial a} = \frac{(1-\iota)(1+\widetilde{\kappa}(1-s_G))}{\widetilde{\kappa}'(1-s_G)\iota\left(\frac{1+\widetilde{\kappa}(s_a)}{\widetilde{\kappa}'(s_a)}+\frac{(1+\widetilde{\kappa}(s_A))}{\widetilde{\kappa}'(s_A)}\right)^2}\left(\frac{1}{s_A^\iota}-\frac{1}{s_a^\iota}\right)\frac{\partial s_A}{\partial a} < 0$,so that f is decreasing in a, since $s_A > s_a$.

I define similarly the growth rate of sector G for a given number of scientists as: $g(a, s_G) \equiv \left(\frac{(1+\widetilde{\kappa}(s_a))^{(\varepsilon-1)(1-\gamma)}a+(1+\widetilde{\kappa}(s_A))^{(\varepsilon-1)(1-\gamma)}}{a+1}\right)$. g is increasing in s_G, and $\frac{\partial g}{\partial a} = \frac{(1+\widetilde{\kappa}(s_a))^{(\varepsilon-1)(1-\gamma)}-(1+\widetilde{\kappa}(s_A))^{(\varepsilon-1)(1-\gamma)}}{(a+1)^2} <$ 0,since $s_A > s_a$. Therefore for a given amount of scientists in sector G, the average productivity grows faster when the productivities of the two sectors are far from each other.

9.2.3 Growth rate of relative productivity

Here I prove the following lemma:

Lemma 4 *Assume that $\tau_{Xt} = 0$ for $X = N, S$ at all time, that country X initially has the comparative advantage in sector G $\left(\frac{A_{XG0}}{A_{XH0}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_X}{L_X} \geq \left(\frac{A_{(-X)G0}}{A_{(-X)H0}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_{(-X)}}{L_{(-X)}}$ and that (i) a_{X0} and $a_{(-X)0}$ are sufficiently small and the previous inequality is strict or (ii) $a_{X0} \leq a_{(-X)0}$; then at all points in time: $s_{XGt} \geq s_{(-X)Gt}$ (with a strict inequality if one of the previous equalities is strict), and, if one of the previous inequality is strict, A_{SGt}/A_{NGt} and A_{NHt}/A_{SHt} tend to infinity.*

Proof. To fix ideas, I assume that country X is the South, and that in both countries $A_{Xd0} > A_{Xc0}$. Assume that at time $t \geq 1$, I have $\left(\frac{A_{SG(t-1)}}{A_{SH(t-1)}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_S}{L_S} \geq \left(\frac{A_{NG(t-1)}}{A_{NH(t-1)}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_N}{L_N}$, and either $a_{N(t-1)}, a_{S(t-1)}$ are both negligible (with a strict inequality) or $a_{S(t-1)} \leq a_{N(t-1)}$. Solving for entrepreneurs maximization (63) and (64), I get that in both countries,

$$\widetilde{\kappa}'(s_{Xct})(1+\widetilde{\kappa}(s_{Xct}))^{(\varepsilon-1)(1-\gamma)-1}a_{X(t-1)} = \widetilde{\kappa}'(s_{Xdt})(1+\widetilde{\kappa}(s_{Xdt}))^{(\varepsilon-1)(1-\gamma)-1},$$

$$f\left(a_{(t-1)}, s_{XGt}\right) = \frac{1+\widetilde{\kappa}\left(s_{XHt}\right)}{\widetilde{\kappa}'\left(s_{XHt}\right)} \frac{1}{2} \frac{\widetilde{\kappa}'\left(s_{Xct}\right)\left(1+\widetilde{\kappa}\left(s_{Xct}\right)\right)^{(\varepsilon-1)(1-\gamma)-1} a_{Xt} + \widetilde{\kappa}'\left(s_{Xdt}\right)\left(1+\widetilde{\kappa}\left(s_{Xdt}\right)\right)^{(\varepsilon-1)(1-\gamma)-1}}{\left(1+\widetilde{\kappa}\left(s_{Xct}\right)\right)^{(\varepsilon-1)(1-\gamma)} a_{Xt} + \left(1+\widetilde{\kappa}\left(s_{Xdt}\right)\right)^{(\varepsilon-1)(1-\gamma)}}$$
$$= \frac{p_{Ht} Y_{XHt}}{p_{Gt} Y_{XGt}}.$$

Using the expressions for (48) and (49), I get that -when there is not full specialization in any country- the equilibrium can be summarized by three equations:

$$f\left(a_{X(t-1)}, s_{XGt}\right) = \frac{\alpha-(1-\alpha)\left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\beta^{\beta}(1-\beta)^{(1-\beta)}} \frac{p_{Gt}}{p_{Ht}} \frac{\left((1+\widetilde{\kappa}(s_{Xct}))^{(\varepsilon-1)(1-\gamma)} A_{Xc(t-1)}^{\varepsilon-1}+(1+\widetilde{\kappa}(s_{Xdt}))^{(\varepsilon-1)(1-\gamma)} A_{Xd(t-1)}^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}}{(1+\widetilde{\kappa}(s_{XHt}))^{1-\gamma} A_{XH(t-1)}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_X}{L_X}}{(1-\beta)\left(\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\beta^{\beta}(1-\beta)^{(1-\beta)}} \frac{p_{Gt}}{p_{Ht}} \frac{\left((1+\widetilde{\kappa}(s_{Xct}))^{(\varepsilon-1)(1-\gamma)} A_{Xc(t-1)}^{\varepsilon-1}+(1+\widetilde{\kappa}(s_{Xdt}))^{(\varepsilon-1)(1-\gamma)} A_{Xd(t-1)}^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}}{(1+\widetilde{\kappa}(s_{XHt}))^{1-\gamma} A_{XH(t-1)}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_X}{L_X}-\beta},$$

for $X \in \{N, S\}$ and the equation determining the price ratio $\frac{p_{Ht}}{p_{Gt}}$. ■

Now if $a_{N(t-1)} \geq a_{S(t-1)}$, then, at given s_{XGt}, the LHS of the previous equation is lower for the North than for the South. Similarly $\left(\frac{\left((1+\widetilde{\kappa}(s_{Xct}))^{(\varepsilon-1)(1-\gamma)} A_{Xc(t-1)}^{\varepsilon-1}+(1+\widetilde{\kappa}(s_{Xdt}))^{(\varepsilon-1)(1-\gamma)} A_{Xd(t-1)}^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}}{(1+\widetilde{\kappa}(s_{XHt}))^{1-\gamma} A_{XH(t-1)}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_X}{L_X}$ can be rewritten as $\left(\frac{\left((1+\widetilde{\kappa}(s_{Xct}))^{(\varepsilon-1)(1-\gamma)} a_{X(t-1)}+(1+\widetilde{\kappa}(s_{Xdt}))^{(\varepsilon-1)(1-\gamma)}\right)^{\frac{1}{\varepsilon-1}}}{\left(a_{X(t-1)}+1\right)(1+\widetilde{\kappa}(s_{XHt}))^{1-\gamma}}\right)^{\frac{1}{(\alpha-\beta)}} \left(\frac{A_{XG(t-1)}}{A_{XH(t-1)}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_X}{L_X}$, which at given s_{XGt} is (weakly) higher for the South than for the North, since $\left(\frac{A_{SG(t-1)}}{A_{SH(t-1)}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_S}{L_S} \geq \left(\frac{A_{NG(t-1)}}{A_{NH(t-1)}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_N}{L_N}$ and since g is decreasing in a, therefore at given s_{XGt}, the RHS of the previous equation is higher for the North than for the South. For given prices, both the LHS and the RHS are decreasing in s_{XGt}, but for sufficiently small κ, the LHS decreases faster, therefore $s_{SGt} \geq s_{NGt}$, with a strict inequality if either $a_{N(t-1)} > a_{S(t-1)}$ or if $\left(\frac{A_{SG(t-1)}}{A_{SH(t-1)}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_S}{L_S} > \left(\frac{A_{NG(t-1)}}{A_{NH(t-1)}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_N}{L_N}$.

Similarly if both $a_{N(t-1)}$ and $a_{S(t-1)}$ are negligible (relative to the difference in comparative advantage), I get that $s_{Xdt} \simeq s_{XGt}$, $f\left(a_{(t-1)}, s_{XGt}\right) \simeq \frac{1+\widetilde{\kappa}(s_{XHt})}{\widetilde{\kappa}'(s_{XHt})} \frac{\widetilde{\kappa}'(s_{XGt})}{1+\widetilde{\kappa}(s_{XGt})}$ and

$$\frac{\left(\left(1+\widetilde{\kappa}\left(s_{Xct}\right)\right)^{(\varepsilon-1)(1-\gamma)} A_{Xc(t-1)}^{\varepsilon-1}+\left(1+\widetilde{\kappa}\left(s_{Xdt}\right)\right)^{(\varepsilon-1)(1-\gamma)} A_{Xd(t-1)}^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}}{\left(1+\widetilde{\kappa}\left(s_{XHt}\right)\right)^{1-\gamma} A_{XH(t-1)}}$$
$$\simeq \frac{\left(1+\widetilde{\kappa}\left(s_{XGt}\right)\right)^{(1-\gamma)} A_{XG(t-1)}}{\left(1+\widetilde{\kappa}\left(s_{XHt}\right)\right)^{1-\gamma} A_{XH(t-1)}},$$

so that, following a similar reasoning, $\left(\frac{A_{SG(t-1)}}{A_{SH(t-1)}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_S}{L_S} > \left(\frac{A_{NG(t-1)}}{A_{NH(t-1)}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_N}{L_N}$ leads to $s_{SGt} > s_{NGt}$.

Therefore,in both cases $A_{SGt}/A_{NGt} > A_{SG(t-1)}/A_{NG(t-1)}$ and $A_{NHt}/A_{SHt} > A_{NH(t-1)}/A_{SH(t-1)}$ (expect in the case where $\left(\frac{A_{SG(t-1)}}{A_{SH(t-1)}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_S}{L_S} = \left(\frac{A_{NG(t-1)}}{A_{NH(t-1)}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_N}{L_N}$ and $a_{N(t-1)} = a_{S(t-1)}$, where the strict inequalities are replaced by equalities). Note that $a_{Xt} < a_{X(t-1)}$, so if both $a_{N(t-1)}$ an $a_{S(t-1)}$ are negligible, a_{Nt} and a_{St} will be negligible too. Moreover, $\frac{1+\widetilde{\kappa}(s_A(a,s_G))}{1+\widetilde{\kappa}(s_a(a,s_G))}$ is increasing in s_G and decreasing in a, so if $a_{N(t-1)} \geq a_{S(t-1)}$ and $s_{SGt} \geq s_{NGt}$ then $a_{Nt} \geq a_{St}$.

The analysis extends directly to the case where one country specializes. By induction, this is enough to show that $s_{SGt} \geq s_{NGt}$ and, that A_{SGt}/A_{NGt} and A_{NHt}/A_{SHt} are increasing (with a strict inequality and strictly increasing if either $\left(\frac{A_{SG0}}{A_{SH0}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_S}{L_S} > \left(\frac{A_{NG0}}{A_{NH0}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_N}{L_N}$ or $a_{N0} > a_{S0}$).

Nevertheless, having $s_{NGt} < s_{SGt}$ every period is not enough to conclude that A_{SGt}/A_{NGt} and A_{NHt}/A_{SHt} tend to infinity:s_{NGt} and s_{SGt} could converge towards each other. However this would require that either $\left(\frac{A_{SG(t-1)}}{A_{SH(t-1)}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_S}{L_S}$ and $\left(\frac{A_{NG(t-1)}}{A_{NH(t-1)}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_N}{L_N}$ also converge towards each other (which is clearly ruled out), or that both s_{NGt} and s_{SGt} tend towards the same corner solution. Therefore, it should be the case that in both countries $\frac{p_{Ht}}{p_{Gt}} \frac{Y_{XHt}}{Y_{XGt}}$ either tends towards 0 or towards infinity, which is impossible too: in the Cobb-Douglas case $\frac{p_{Ht}}{p_{Gt}} \frac{Y_{NHt}+Y_{SHt}}{Y_{NGt}+Y_{SGt}} = \frac{1-\nu}{\nu}$, and when $\sigma < 1$ case, innovation overall favors the most backward sector preventing all scientists from innovating in the same sector in both countries asymptotically.

9.2.4 Reaching full specialization in finite time

Under the assumptions of lemma 2, using lemma 4, I get that at every period $s_{SGt} > s_{NGt}$ and that A_{SGt}/A_{NGt} and A_{NHt}/A_{SHt} grow unboundedly. Using the expressions (48) and (49), avoiding full specialization asymptotically requires that $\frac{p_{Ht}}{p_{Gt}} \frac{A_{NHt}}{A_{NGt}}$ remains bounded (from $Y_{NGt} \geq 0$) and similarly $\frac{p_{Gt}}{p_{Ht}} \frac{A_{SGt}}{A_{SHt}}$ remain bounded. Taking the product of the two, this leads towards $\frac{A_{NHt}}{A_{NGt}} \frac{A_{SGt}}{A_{SHt}}$ bounded which is a contradiction. Therefore at least one country fully specializes. For the sake of the argument assumes that the South fully specializes in sector G. If this is the case, note that asymptotically A_{SGt} must grow at the rate $(1+\kappa)^{1-\gamma} - 1$ (since eventually all scientists are in the dirty sector there). Then to avoid full specialization in the North in finite time, one must keep (from (55)):

$$\left(\frac{A_{NHt}}{A_{SGt}}\right)^{1-\sigma} K_N^{\alpha} L_N^{1-\alpha} > \left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)^{\sigma} \left(\frac{1-\nu}{\nu}\right)^{\sigma} \left(\frac{A_{SGt}}{A_{NGt}}\right)^{\sigma} K_S^{\alpha} L_S^{1-\alpha},$$

where $0 < \sigma \leq 1$. Now A_{SGt}/A_{NGt} grows exponentially, while A_{NHt}/A_{SGt} cannot grow asymptotically since A_{SGt} asymptotically grows at the fastest rate. Therefore, keeping that inequality is impossible and the North must also fully specialize. Similarly if I had assumed that the North specialized first I would get that the South must also specialize in finite time. Therefore I have proved that I reach full specialization in finite time.

9.3 Appendix B.3 Proof of lemma 3

Since emission per-unit of the polluting good increases in the South, avoiding a disaster requires that the production of the polluting good stays bounded. If the South fully specializes in sector G avoiding a disaster is impossible, while if the South fully specializes in sector H, then in finite time all factors are allocated to the non-polluting sector by definition and the North exports the polluting good.

Assume now that there is not full specialization in the South, that is there are an infinite number of periods where the South produces both goods (and in the following I restrict attention to those periods). Rewriting (48) with $\tau_{St} = 0$ and $p_t = p_{SGt}/p_{Ht}$, production in sector G is given by

$$\begin{aligned} Y_{SGt} &= \frac{\zeta A_{SGt}}{(\alpha-\beta)} \left(\frac{\beta^{\beta\alpha}(1-\beta)^{(1-\beta)\alpha}}{\alpha^{\beta\alpha}(1-\alpha)^{(1-\alpha)\beta}}\right)^{\frac{1}{\alpha-\beta}} \left(p_t \frac{A_{SGt}}{A_{SHt}}\right)^{\frac{-\alpha}{\alpha-\beta}} \\ &\times \left(\left(\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}} (1-\beta)\left(p_t \frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}} K_S - \beta L_S\right). \end{aligned} \tag{65}$$

Therefore to keep Y_{SGt} bounded it must be the case that $\left(p_t \frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}}$ is bounded, with either $\lim \left(p_t \frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}} = \left(\frac{\beta^{\beta}(1-\beta)^{(1-\beta)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)^{\frac{1}{\alpha-\beta}} \frac{\beta L_S}{(1-\beta)K_S}$ or with A_{SGt} bounded. Using (49), Y_{SHt} can be rewritten as:

$$\begin{aligned} Y_{SHt} &= \frac{\zeta A_{SHt}}{(\alpha-\beta)} \left(\frac{\beta^{\beta\alpha}(1-\beta)^{(1-\beta)\alpha}}{\alpha^{\beta\alpha}(1-\alpha)^{(1-\alpha)\beta}}\right)^{\frac{1}{\alpha-\beta}} \left(p_t \frac{A_{SGt}}{A_{SHt}}\right)^{\frac{-\beta}{\alpha-\beta}} \\ &\times \left(\alpha L_S - \left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}} (1-\alpha)\left(p_t \frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}} K_S\right). \end{aligned} \tag{66}$$

Combining these two expressions with (64) implies that the allocation of innovation in the South must satisfy

$$\frac{\kappa'(s_{Sdt})}{(1+\kappa(s_{Sdt}))} \frac{1+\kappa(1-s_{Sdt})}{\kappa'(1-s_{Sdt})} \frac{A_{Sdt}^{\varepsilon-1}}{A_{Sc0}^{\varepsilon-1} + A_{Sdt}^{\varepsilon-1}} = \frac{\alpha L_S - \left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}} (1-\alpha)\left(p_t \frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}} K_S}{\left(\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}} (1-\beta)\left(p_t \frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}} K_S - \beta L_S}. \tag{67}$$

If $\lim \left(p_t \frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}} \neq \left(\frac{\beta^{\beta}(1-\beta)^{(1-\beta)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)^{\frac{1}{\alpha-\beta}} \frac{\beta L_S}{(1-\beta)K_S}$, s_{Sdt} cannot tend towards 0 in which case A_{SGt} would become unbounded, as demonstrated above this would lead to a disaster. Therefore it must be the case that $\lim \left(p_t \frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}} = \left(\frac{\beta^{\beta}(1-\beta)^{(1-\beta)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)^{\frac{1}{\alpha-\beta}} \frac{\beta L_S}{(1-\beta)K_S}$ and so asymptotically all factors in the South (scientists, capital, labor) must be allocated to sector H.

Denoting M_{Gt} and M_{Ht} net imports from the North, (51) leads to:

$$\frac{p_{SGt}Y_{SGt}}{p_{SHt}Y_{SHt}} = \frac{\nu}{1-\nu}\left(\frac{Y_{SGt}-M_{Gt}}{Y_{SHt}-M_{Ht}}\right)^{-\frac{1}{\sigma}}\frac{Y_{SGt}}{Y_{SHt}}.$$

The right-hand side if greater than $\frac{\nu}{1-\nu}\left(\frac{Y_{SHt}}{Y_{SGt}}\right)^{\frac{1-\sigma}{\sigma}}$ if $M_{Gt} \geq 0$. But avoiding a disaster requires that the left-hand side tends towards 0, while $\frac{Y_{SHt}}{Y_{SGt}}$ becomes unbounded: this yields a contradiction, so the North must exports the polluting good G.

9.4 Appendix B.4 Proof of proposition 1

The proof is similar to the proof of the previous lemma: I show that as soon as a tax on dirty research or a carbon tax is implemented, $s_{NGt} < s_{SGt}$ and that this eventually leads to full specialization. Note that in the presence of a carbon tax and a tax on dirty research, the allocation of research in the North now obeys:

$$\begin{aligned}&\frac{(1+\widetilde{\kappa}(s_{NHt}))\left(\begin{array}{c}\widetilde{\kappa}'(s_{Nct})(1+\widetilde{\kappa}(s_{Nct}))^{(\varepsilon-1)(1-\gamma)-1}A_{Nc(t-1)}^{\varepsilon-1}\\+\widetilde{\kappa}'(s_{Ndt})(1-q_t)(1+\tau_{Nt})^{-\varepsilon}(1+\widetilde{\kappa}(s_{Ndt}))^{(\varepsilon-1)(1-\gamma)-1}A_{Nd(t-1)}^{\varepsilon-1}\end{array}\right)}{\widetilde{\kappa}'(s_{NHt})2\left((1+\widetilde{\kappa}(s_{Nct}))^{(\varepsilon-1)(1-\gamma)}A_{Nc(t-1)}^{\varepsilon-1}+(1+\tau_{Nt})^{1-\varepsilon}(1+\widetilde{\kappa}(s_{Ndt}))^{(\varepsilon-1)(1-\gamma)}A_{Nd(t-1)}^{\varepsilon-1}\right)} \\ =\ & \frac{p_{Ht}Y_{NHt}}{p_{Gt}Y_{NGt}},\end{aligned} \tag{68}$$

with:

$$\widetilde{\kappa}'(s_{Nct})(1+\widetilde{\kappa}(s_{Nct}))^{(\varepsilon-1)(1-\gamma)-1}A_{Nc(t-1)} = \widetilde{\kappa}'(s_{Ndt})(1+\widetilde{\kappa}(s_{Ndt}))^{(\varepsilon-1)(1-\gamma)-1}(1-q_t)(1+\tau_{Nt})^{1-\varepsilon}A_{Nd(t-1)}$$

and, following (48) and (49), $\frac{p_{Ht}Y_{NHt}}{p_{Gt}Y_{NGt}}$ is increasing in τ at given prices and technological levels. Without loss of generality, assume that from the first period a tax on dirty research or a carbon tax is implemented.

First I explain how the presence the tax on dirty research affects the incentive to innovate in sector G, second I show that in the first period ($t=1$), it is necessarily the case that $s_{NG1} < s_{SG1}$, third I show that in all following periods the same logic applies and finally I show that full specialization is reached.

9.4.1 Tax on dirty research and incentive to innovate

Define

$$\begin{aligned}& f\left(a_{N(t-1)}, s_{NGt}, 1-q_t\right) \\ \equiv\ & \frac{1+\widetilde{\kappa}(s_{NHt})}{\widetilde{\kappa}'(s_{NHt})}\frac{1}{2}\frac{\widetilde{\kappa}'(s_{Nct})(1+\widetilde{\kappa}(s_{Nct}))^{(\varepsilon-1)(1-\gamma)-1}a_{N(t-1)}+(1-q_t)\widetilde{\kappa}'(s_{Ndt})(1+\widetilde{\kappa}(s_{Ndt}))^{(\varepsilon-1)(1-\gamma)-1}}{(1+\widetilde{\kappa}(s_{Nct}))^{(\varepsilon-1)(1-\gamma)}a_{N(t-1)}+(1+\widetilde{\kappa}(s_{Ndt}))^{(\varepsilon-1)(1-\gamma)}}\end{aligned}$$

with s_{Nct} and s_{Ndt} defined by

$$\widetilde{\kappa}'(s_{Nct})(1+\widetilde{\kappa}(s_{Nct}))^{(\varepsilon-1)(1-\gamma)-1} a_{Nct} = (1-q_t)\widetilde{\kappa}'(s_{Ndt})(1+\widetilde{\kappa}(s_{Ndt}))^{(\varepsilon-1)(1-\gamma)-1}.$$

(Note I keep the same notation as above but here $a_{N(t-1)} = \frac{(1+\tau_{Nt})A_{Nc(t-1)}}{A_{Nd(t-1)}}$ and can be greater or smaller than 1).

It is direct to show that $\widetilde{\kappa}'(s_{Nct})(1+\widetilde{\kappa}(s_{Nct}))^{(\varepsilon-1)(1-\gamma)-1} a_{N(t-1)}+(1-q_t)\widetilde{\kappa}'(s_{Ndt})(1+\widetilde{\kappa}(s_{Ndt}))^{(\varepsilon-1)(1-\gamma)-1}$ decreases with q_t when κ is sufficiently small to ensure that $\widetilde{\kappa}'(s)(1+\widetilde{\kappa}(s))^{(\varepsilon-1)(1-\gamma)-1}$ is decreasing, which has been assumed so far. The denominator is decreasing with q_t since for a given mass scientists in sector G, $q_t = 0$, maximizes the growth rate of average productivity. However for sufficiently small κ, the variations in the denominator are negligible, and for $q_t > 0$, $f\left(a_{N(t-1)}, s_{NGt}, 1-q_t\right) < f\left(a_{N(t-1)}, s_{NGt}, 1\right) = f\left(a_{N(t-1)}, s_{NGt}\right)$.

9.4.2 Showing that $s_{NG1} < s_{SG1}$

The equilibrium allocation of innovation in the North (68) can then be rewritten as

$$f\left((1+\tau_{N1})\frac{A_{Nc0}}{A_{Nd0}}, s_{NG1}, \frac{1-q_1}{1+\tau_{N1}}\right) = \frac{p_{H1}Y_{NH1}}{p_{G1}Y_{NG1}}.$$

Since $f\left((1+\tau_{N1})\frac{A_{Nc0}}{A_{Nd0}}, s_{NG1}, \frac{1-q_1}{1+\tau_{N1}}\right) < f\left((1+\tau_{N1})\frac{A_{Nc0}}{A_{Nd0}}, s_{NG1}\right)$, and that the tax increases $\frac{p_{Ht}Y_{NHt}}{p_{Gt}Y_{NGt}}$, the logic of the proof of lemma 2 would fully apply provided that $\min\left((1+\tau_{N1})\frac{A_{Nc0}}{A_{Nd0}}, (1+\tau_{N1})^{-1}\frac{A_{Nd0}}{A_{Nc0}}\right)$ $\frac{A_{Sc0}}{A_{Sd0}}$, since $\frac{A_{Nc0}}{A_{Nd0}} \geq \frac{A_{Sc0}}{A_{Sd0}}$, this is necessarily satisfies unless $(1+\tau_{N1}) \geq \frac{A_{Nd0}}{A_{Nc0}}\frac{A_{Sd0}}{A_{Sc0}}$. However, for $\frac{A_{Sc0}}{A_{Sd0}}$ sufficiently small, the corresponding tax will be very large, so that the difference in initial comparative advantage between the North and the South would become large, and following the logic of lemma 2 - but with a_{Sc0} negligible relative to the difference in comparative advantage-, in that case too $s_{NG1} < s_{SG1}$. The presence of both the tax on dirty research and the carbon tax distorts innovation such that A_{NG1} grows less than without the tax. As a consequence, $\left(\frac{A_{SG1}}{A_{SH1}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_S}{L_S} \geq \left(\frac{A_{NG1}}{A_{NH1}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_N}{L_N}$.

9.4.3 Showing that $s_{NGt} < s_{SGt}$

As long as $A_{Nd(t-1)} \geq A_{Nc(t-1)}$, the same logic exactly applies: every period $\frac{A_{Nc(t-1)}}{A_{Nd(t-1)}} \geq \frac{A_{Sc(t-1)}}{A_{Sd(t-1)}}$, since less scientists are allocated to sector G in the North and the allocation is tilted towards the clean subsector. Once $A_{Nc(t-1)} \geq A_{Nd(t-1)}$, then $\frac{A_{Nd(t-1)}}{A_{Nc(t-1)}} = \min\left(\frac{A_{Nc(t-1)}}{A_{Nd(t-1)}}, \frac{A_{Nd(t-1)}}{A_{Nc(t-1)}}\right)$ could in principle decrease faster than $\frac{A_{Sc(t-1)}}{A_{Sd(t-1)}}$ since all scientists in sector G can be allocated to the clean subsector in the North whereas they are shared between the two sectors in the South; with $\frac{A_{Sc0}}{A_{Sd0}}$ sufficiently small, and $s_{NG(t-1)} < s_{SG(t-1)}$, nearly all scientists in the South can be allocated to the dirty subsector so that this cannot happen and $\frac{A_{Sc(t-1)}}{A_{Sd(t-1)}}$ decreases faster than $\frac{A_{Nd(t-1)}}{A_{Nc(t-1)}}$. It is still possible to get $(1+\tau_{Nt})^{-1}\frac{A_{Nd(t-1)}}{A_{Nc(t-1)}} \leq \frac{A_{Sc(t-1)}}{A_{Sd(t-1)}}$, with a sufficient large tax (the required tax being increasing over time since $\frac{A_{Sc(t-1)}}{A_{Sd(t-1)}}$ decreases faster than $\frac{A_{Nd(t-1)}}{A_{Nc(t-1)}}$), this tax would then again lead to a large difference in comparative advantage, unless $A_{Nd(t-1)}$ has become sufficiently small relative to $A_{Nc(t-1)}$, but this will not happen before a significant number of periods, by which the difference in comparative advantage - which will have built up with innovation - will be large. Therefore $s_{SGt} > s_{NGt}$ every period.

9.4.4 Reaching full specialization

Therefore here as well, $\frac{A_{SGt}}{A_{NGt}}$ and $\frac{A_{NHt}}{A_{SHt}}$ grow unboundedly. From (48) and (49), this necessarily to specialization in at least one country. Assume that there is full specialization in sector G in the South, so that asymptotically A_{SGt} must grow at the rate $(1+\kappa)^{1-\gamma}-1$. Then to avoid full specialization in the North in finite time, I need to keep (from (58)):

$$A_{NHt}^{1-\sigma}\left(A_{Nct}^{\varepsilon-1}+\left((1+\tau_{Nt})^{-1}A_{Ndt}\right)^{\varepsilon-1}\right)^{\frac{\sigma}{\varepsilon-1}}K_N^{\alpha}L_N^{1-\alpha} \geq \left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)^{\sigma}\left(\frac{1-\nu}{\nu}\right)^{\sigma}A_{SGt}K_X^{\alpha}L_X^{1-\alpha},$$

which here again is impossible. Similarly if the North fully specializes in sector H, avoiding specialization in the South would also be impossible. Therefore both countries fully specialize, the emissions in the South necessarily grow unbounded and there is a disaster.

9.5 Appendix B.5 Proofs of subsection 3.4

9.5.1 B.5.1 Proof of proposition 2

Assume that at some period $(t-1)$:

$$\left(\frac{\alpha^{\beta}(1-\alpha)^{1-\beta}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)A_{NH(t-1)}K_N^{\beta}L_N^{1-\beta} > (1+m)\left(\frac{1-\nu}{\nu}\right)K_S^{\beta}L_S^{1-\beta}A_{SH(t-1)}, \tag{69}$$

$$A_{NG(t-1)}K_N^{\alpha}L_N^{1-\alpha}(1+m) < \left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)\left(\frac{1-\nu}{\nu}\right)A_{SG(t-1)}K_S^{\alpha}L_S^{1-\alpha}, \tag{70}$$

where $m>0$, is sufficiently large that regardless of the allocation of research in the following period, (57) and (58) are satisfied (with $\tau_S=0$ and $\sigma=1$). Then the North fully specializes in H and the South in G, therefore, all scientists in the South are allocated to sector G. Now, as innovation in the South occurs in the polluting sector A_{Sct}/A_{Sdt} becomes arbitrarily small. Taking a first order approximation, the number of scientists allocated to the clean sector in the South will be given by: $s_c = \left(\frac{A_{ct-1}^{\varepsilon-1}}{A_{dt-1}^{\varepsilon-1}}\right)^{\frac{1}{1-\iota}}$. Therefore, the growth rate of A_{Sdt} is $(1+\kappa)^{1-\gamma}\left(1-\frac{(1-\gamma)\iota}{1+\kappa}\left(\frac{A_{ct-1}^{\varepsilon-1}}{A_{dt-1}^{\varepsilon-1}}\right)^{\frac{1}{1-\iota}}\right)-1$, while with all scientists allocated to clean technologies in the North, the growth rate of A_{Nct} is $(1+\kappa)^{1-\gamma}-1$. Since the series $\sum -\frac{(1-\gamma)\iota}{1+\kappa}\left(\frac{A_{ct-1}^{\varepsilon-1}}{A_{dt-1}^{\varepsilon-1}}\right)^{\frac{1}{1-\iota}}$ is converging, then $\prod_t (1-\kappa)^{1-\gamma}\left(1-\frac{(1-\gamma)\iota}{1+\kappa}\left(\frac{A_{ct-1}^{\varepsilon-1}}{A_{dt-1}^{\varepsilon-1}}\right)^{\frac{1}{1-\iota}}\right)$ is not dominated by $\prod_t (1+\kappa)^{1-\gamma}$, in other words, A_{Sdt} can remain larger than MA_{NGt} where M is a constant. In other words, (69) and (70) remain satisfied in the next period.

9.5.2 B.5.2 Proof of proposition 3

If the North implements a tariff that prohibits any trade, innovation in the South will be balanced between dirty intermediates and sector H. Using sufficiently large clean research subsidies, the social planner can allocate nearly all innovation in the North towards clean intermediates. Therefore, after a finite number of period, I will have: $\left(\frac{A_{NG(t-1)}}{A_{NH(t-1)}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_N}{L_N} > \left(\frac{A_{SG(t-1)}}{A_{SH(t-1)}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_S}{L_S}$ and $a_{N(t-1)} = \frac{A_{Nd(t-1)}}{A_{Nc(t-1)}} < \frac{A_{Sc(t-1)}}{A_{Sd(t-1)}}$. Using lemma 2, the North will specialize in sector G while the South will specialize in H. Worldwide emissions are bounded, so for a sufficiently large $\overline{S}$, a disaster is avoided.

9.6 Appendix B.6 Proof of Proposition 4

I denote the Lagrange parameters (with the corresponding constraints in parentheses): λ_{Xt} (1), λ_{XHt} (2), λ_{XGt} (5), λ_{Xzt} (6), φ_{Xzit} (7), φ_{XHit} (3), η_{XKt} (9) for capital, η_{XLt} (9) for labor, θ_{Gt} (10) in sector G, θ_{Ht} (10) in sector H, ω_t (11), μ_{Xzit} (12), υ_{Xt} (14), in addition the social planner faces the constraints: $0 \le Y_{XGt}$ and $0 \le Y_{XHt}$, with Lagrange parameters: ι_{XGt}, ι_{XHt}. Taking the first order condition with respect to Y_{XHt} and Y_{XGt} gives:

$$\lambda_{XHt} = \theta_{Ht} + \iota_{XHt} \text{ and } \lambda_{XGt} = \theta_{Gt} + \iota_{XGt}.$$

First order conditions with respect to C_{Nt} and C_{St} lead to:

$$\frac{1}{(1+\rho)^t}\frac{\partial u}{\partial C}(C_{Nt}+C_{St},S_t) = \frac{\nu (S_t)^{1-\eta}}{(1+\rho)^t}(C_{Nt}+C_{St})^{-\eta} = \lambda_{Xt} \equiv \lambda_t.$$

First order conditions with respect to C_{XGt} and C_{XHt} give:

$$\lambda_t \nu C_{XGt}^{\frac{-1}{\sigma}}\left(\nu C_{XGt}^{\frac{\sigma-1}{\sigma}} + (1-\nu) C_{XHt}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} = \theta_{Gt}, \tag{71}$$

$$\lambda_t (1-\nu) C_{XHt}^{\frac{-1}{\sigma}}\left(\nu C_{XGt}^{\frac{\sigma-1}{\sigma}} + (1-\nu) C_{XHt}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} = \theta_{Ht}. \tag{72}$$

θ_{Gt}/λ_t and θ_{Ht}/λ_t can be interpreted as consumer prices in terms of units of welfare. To emphasize this interpretation, I denote $\widehat{p}_{Gt} = \theta_{Gt}/\lambda_t$ and $\widehat{p}_{Ht} = \theta_{Gt}/\lambda_t$. I then get:

$$\frac{\widehat{p}_{Gt}}{\widehat{p}_{Ht}} = \frac{\nu}{1-\nu}\left(\frac{C_{XHt}}{C_{XGt}}\right)^{\frac{1}{\sigma}} = \frac{\nu}{1-\nu}\left(\frac{C_{NHt}+C_{SHt}}{C_{NGt}+C_{SGt}}\right)^{\frac{1}{\sigma}},$$

which is the equivalent to the equilibrium condition (51). Taking the first order condition with respect to with respect to Y_{XHt} and Y_{XGt} gives:

$$\lambda_{XHt} = \theta_{Ht} + \iota_{XHt} \text{ and } \lambda_{XGt} = \theta_{Gt} + \iota_{XGt},$$

so that when production of good $Y \in \{G, H\}$ takes place: $\lambda_{XYt} = \theta_{Yt}$. Defining $\widehat{\varphi}_{Xzit} \equiv \frac{\varphi_{Xzit}}{\lambda_t}$ and $\widehat{p}_{Xzt} \equiv \frac{\lambda_{Xzt}}{\lambda_t}$, which can be interpreted as the price of intermediate x_{Xzi} and of input Y_{Xz}, first order condition with respect to x_{Xzit} gives:

$$\widehat{\varphi}_{Xzit} = \gamma \widehat{p}_{Xzt} A_{Xzit} x_{zit}^{\gamma-1} \left(K_{Xzt}^{\alpha} L_{Xzt}^{1-\alpha}\right)^{1-\gamma},$$

which is the same as (37). Combining the first order conditions with respect to K_{Xzit} and L_{Xzit} further gives

$$\widehat{\varphi}_{Xzit} = \frac{\psi \widehat{r}_{XHt}^{\alpha} \widehat{w}_{XLt}^{1-\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha}},$$

where $\widehat{r}_{XHt} \equiv \frac{\eta_{XKt}}{\lambda_t}$ and $\widehat{w}_{XLt} \equiv \frac{\eta_{XLt}}{\lambda_t}$ are the prices of capital and labor in country X. This last equation is identical to (38) when the optimal subsidy $\widetilde{q} = 1 - \gamma$ is used. Recovering the equations equivalent to (40) is direct. First order conditions with respect to K_{Xzt} and L_{Xzt} allow to recover the equations equivalent to (36). Now taking the first order condition with respect to Y_{Xdt} and Y_{Xct}, one gets (when $Y_{XGt} \neq 0$):

$$\begin{aligned} \widehat{p}_{Gt} Y_{Xdt}^{-\frac{1}{\varepsilon}} \left(Y_{Xct}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{Xdt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}} &= \widehat{p}_{Xdt} + \xi \frac{\omega_t}{\lambda_t}, \\ \widehat{p}_{Gt} Y_{Xct}^{-\frac{1}{\varepsilon}} \left(Y_{Xct}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{Xdt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}} &= \widehat{p}_{Xct} \end{aligned}$$

this is equivalent to (42) with a tax $\tau_t = \xi \frac{\omega_t}{\lambda_{Xdt}}$. Therefore:

$$\lambda_{XG} = \frac{\psi^{\gamma} \eta_{XKt}^{\alpha} \eta_{XLt}^{1-\alpha}}{\left(A_{Xz}^{\varepsilon-1} + \left((1+\tau_X)^{-1} A_{Xd}\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}} (1-\gamma)^{1-\gamma} \gamma^{\gamma} \alpha^{\alpha} (1-\alpha)^{1-\alpha}},$$

so that, as in equilibrium, country X specializes in good H if

$$\widehat{p}_{XG} < \frac{\psi^{\gamma} \widehat{r}_{Xt}^{\alpha} \widehat{w}_{Xt}^{1-\alpha}}{\left(A_{Xz}^{\varepsilon-1} + \left((1+\tau_X)^{-1} A_{Xd}\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}} (1-\gamma)^{1-\gamma} \gamma^{\gamma} \alpha^{\alpha} (1-\alpha)^{1-\alpha}}.$$

The analysis of sector H is identical except that there is of course no tax there. Now the first order condition with respect to S_t gives

$$\omega_t = \frac{1}{(1+\rho)^t} \frac{\partial u}{\partial S} (C_{Ns} + C_{Ss}, S_s) + (1+\Delta) I_{S_{t+1} < \overline{S}} \omega_{t+1}, \tag{73}$$

where I is the index function, so that

$$\omega_t = \sum_{s=t}^{\infty} \frac{(1+\Delta)^{s-t}}{(1+\rho)^s} I_{S_t,..,S_s<\overline{S}} \frac{\partial u}{\partial S} \left(C_{Ns} + C_{Ss}, S_s\right). \tag{74}$$

This delivers the expression for the optimal tax:

$$\tau_{Xt} = \frac{\xi}{\widehat{p}_{Xdt} \frac{\partial u}{\partial C} \left(C_{Nt} + C_{St}, S_t\right)} \sum_{s=t}^{\infty} \left(\frac{1+\Delta}{1+\rho}\right)^{s-t} I_{S_t,...S_s<\overline{S}} \frac{\partial u}{\partial S} \left(C_{Ns} + C_{Ss}, S_s\right). \tag{75}$$

I now turn to the optimal solution for the innovation part. First as in the equilibrium case, only the average level of technologies (defined in (13)) matter, since the law of motion can be written as

$$A_{Xzit}^{\frac{1}{1-\gamma}} = A_{Xzi(t-1)}^{\frac{1}{1-\gamma}} + \widetilde{\kappa}\left(s_{Xzit}\right) A_{Xz(t-1)}^{\frac{1}{1-\gamma}}, \text{ for } z \in \{c, d, H\}, \tag{76}$$

the solution is also symmetric: $s_{Xzit} = s_{Xzt}$ for $z \in \{c, d, H\}$.

Now taking the first order condition with respect to A_{Xzt}, gives:

$$\begin{aligned}
& \mu_{Xzit} \\
= \; & \lambda_{XHt} x_{Xzit}^{\gamma} \left(K_{Xzt}^{\overline{\alpha\beta}} L_{Xzt}^{1-\overline{\alpha\beta}}\right)^{1-\gamma} \\
& + \mu_{Nzi(t+1)} \left(\left(1 + \widetilde{\kappa}\left(s_{Xzt}\right) \left(\frac{A_{Xzt}}{A_{Xzit}}\right)^{\frac{1}{1-\gamma}}\right)^{1-\gamma} - \widetilde{\kappa}\left(s_{Xzt}\right) \left(\frac{A_{Xzt}}{A_{Xzit}}\right)^{\frac{1}{1-\gamma}} \left(1 + \widetilde{\kappa}\left(s_{Xzt}\right) \left(\frac{A_{Xzt}}{A_{Xzit}}\right)^{\frac{1}{1-\gamma}}\right)^{-\gamma} \right) \\
& + \int_0^1 \widetilde{\kappa}\left(s_{Xzt}\right) \frac{A_{Xzit}^{\frac{1}{1-\gamma}-1}}{A_{Xzjt}^{\frac{1}{1-\gamma}}} \left(1 + \widetilde{\kappa}\left(s_{Xzt}\right) \left(\frac{A_{Xzt}}{A_{Xzjt}}\right)^{\frac{1}{1-\gamma}}\right)^{-\gamma} A_{Xzjt} \mu_{Xzj(t+1)} dj,
\end{aligned}$$

(with $\overline{\alpha\beta} = \alpha$ if $z \in \{c, d\}$ and $\overline{\alpha\beta} = \beta$ if $z = H$), multiplying both sides by $A_{Xzit}^{\frac{-\gamma}{1-\gamma}}$, one gets:

$$\mu_{Xzit} A_{Xzit}^{\frac{-\gamma}{1-\gamma}} = \lambda_{Xzt} x_{Xzit}^{\gamma} A_{Xzit}^{\frac{-\gamma}{1-\gamma}} \left(K_{Xzt}^{\beta} L_{Xzt}^{1-\beta}\right)^{1-\gamma} + \mu_{Nzi(t+1)} A_{Xzit+1}^{\frac{-\gamma}{1-\gamma}} + \widetilde{\kappa}\left(s_{Xzt}\right) \int A_{Xzjt+1}^{\frac{-\gamma}{1-\gamma}} \mu_{Nzj(t+1)} dj,$$

since the equivalent of (39) also holds for sector H, $\lambda_{Xzt} x_{Xzit}^{\gamma} A_{Xzit}^{\frac{-\gamma}{1-\gamma}} \left(K_{Xzt}^{\overline{\alpha\beta}} L_{Xzt}^{1-\overline{\alpha\beta}}\right)^{1-\gamma}$ is a constant across varieties i. Therefore, $\mu_{Xzit} A_{Xzit}^{\frac{-\gamma}{1-\gamma}}$ is constant across varieties and one can define:

$$\mu_{Xzt} \equiv \left(\frac{A_{Xzt}}{A_{Xzit}}\right)^{\frac{\gamma}{1-\gamma}} \mu_{Xzit},$$

which represents the shadow value of one unit of average productivity in sector z, in country X at time t. I can then show that μ_{Xzt} follows the law of motion:

$$\mu_{Xzt} A_{Xzt} = \lambda_{Xzt} Y_{Xzt} + \mu_{Xz(t+1)} A_{Xzt+1}. \tag{77}$$

Now taking the first order condition with respect to s_{Xzit} one gets:

$$\nu_{Xt} = \mu_{Xzit} (1-\gamma) \widetilde{\kappa}' (s_{Xzit}) \left(\frac{A_{Xz(t-1)}}{A_{Xzi(t-1)}} \right)^{\frac{1}{1-\gamma}} \left(1 + \widetilde{\kappa} (s_{Xzit}) \left(\frac{A_{Xz(t-1)}}{A_{Xzi(t-1)}} \right)^{\frac{1}{1-\gamma}} \right)^{-\gamma} A_{Xzi(t-1)},$$

which can then be rewritten as:

$$\nu_{Xt} = \frac{(1-\gamma) \widetilde{\kappa}' (s_{Xzt})}{1 + \widetilde{\kappa} (s_{Xzt})} \mu_{Xzt} A_{Xzt}.$$

Or defining $\widehat{\nu}_{Xt} = \nu_{Xt}/\lambda_{Xt}$, the wage of scientists in terms of utility units, I can rewrite the last equality as

$$\widehat{\nu}_{Xt} = \frac{(1-\gamma) \widetilde{\kappa}' (s_{Xzt})}{1 + \widetilde{\kappa} (s_{Xzt})} \sum_{s=t}^{\infty} \frac{\lambda_s}{\lambda_t} \widehat{p}_{zs} Y_{zs}. \tag{78}$$

Using (41), (42), (43), (44) gives:

$$\frac{p_{Xct} Y_{Xct}}{p_{XGt} Y_{XGt}} = \frac{A_{Xct}^{\varepsilon-1}}{A_{Xct}^{\varepsilon-1} + \left((1+\tau_{Xt})^{-1} A_{Xdt} \right)^{\varepsilon-1}}, \quad \frac{p_{Xdt} Y_{Xdt}}{p_{XGt} Y_{XGt}} = \frac{(1+\tau_{Xt})^{-\varepsilon} A_{Xct}^{\varepsilon-1}}{A_{Xct}^{\varepsilon-1} + \left((1+\tau_{Xt})^{-1} A_{Xdt} \right)^{\varepsilon-1}}.$$

Combining this last two equations with (78) and (77), I get (24).

9.7 Appendix B.7 Proof of remark 2

First, I show that the social planner always chooses to avoid a disaster. Avoiding a disaster is always feasible since the social planner could simply stop production of dirty inputs in both countries. If $\eta \geq 1$ a disaster leads to $U = -\infty$, so the social planner would avoid it. If $\eta < 1$, a disaster leads to a utility flow equal to 0 in the current and all the following periods, whereas by reducing the production of dirty input enough, the social planner can achieve a positive utility flow, without any cost on previous periods.

The rest of the proof has three steps: first I show that when there is no environmental damage, and $\eta \leq 1$, the social planner maximizes long-run growth, second I derive the long-run growth properties of the switch and full specialization in finite time case, third I conclude.

9.7.1 Maximizing long-run growth

Here, I establish the following lemma:

Lemma 5 *Assume $\eta \leq 1$. Consider an allocation (C_{Wt}, S_t) such that there is a t_1, a $A > 0$ and a $g > 0$, such that for $t > t_1$, $S_t = \overline{S}$ and $\lim C_{Wt}/(1+g)^t = A > 0$. Then for ρ sufficiently small the social planner would rather choose the path (C_{Wt}, S_t) to any other (C'_{Wt}, S'_t) if there is a $M > 1$, such that for $t > t_2$, $C'_{Wt} < \frac{1}{M} C_{Wt}$.*

Then there is a t_3 such that for all $t > t_3$, $C'_{Wt} < \frac{1}{M} C_{Wt}$, $S_t = \overline{S}$ and there is a ε such that $C_{Wt} > (A - \varepsilon)(1+g)^t$.

First I consider the case where $\eta = 1$:

$$\begin{aligned}
& U - U' \\
= \; & \sum_{t=0}^{t_3-1} \frac{\log (C_{Wt}) - \log (C'_{Wt}) + \log \nu (S_t) - \log \nu (S'_t)}{(1+\rho)^t} + \sum_{t=t_3}^{\infty} \frac{\log (C_{Wt}) - \log (C'_{Wt})}{(1+\rho)^t} + \sum_{t=t_3}^{\infty} \frac{\log \nu \left(\overline{S}\right) - \log \nu (S'_t)}{(1+\rho)^t} \\
> \; & \sum_{t=0}^{t_3-1} \frac{\log (C_{Wt}) - \log (C'_{Wt})}{(1+\rho)^t} + \frac{\log M}{(1+\rho)^{t_3-1}} \frac{1}{\rho} + \sum_{t=t_3}^{\infty} \frac{\log \nu \left(\overline{S}\right) - \log \nu (S'_t)}{(1+\rho)^t}
\end{aligned}$$

the first term is bounded, the second one tends to infinity as $\rho \to 0$ and the third term is positive, so for ρ sufficiently small, $U - U'$ is positive.

Similarly, when $\eta < 1$,

$$\begin{aligned}
& U - U' \\
= \; & \sum_{t=0}^{t_3-1} \frac{\left(\nu (S_t) C_{Wt}^{1-\eta}\right)^{1-\eta} - \left(\nu (S'_t) C'_{Wt}\right)^{1-\eta}}{(1-\eta)(1+\rho)^t} + \sum_{t=t_3}^{\infty} \frac{\left(\nu \left(\overline{S}\right) C_{Wt}^{1-\eta}\right)^{1-\eta} - \left(\nu (S'_t) C'_{Wt}\right)^{1-\eta}}{(1+\rho)^t (1-\eta)} \\
> \; & \sum_{t=0}^{t_3-1} \frac{\left(\nu (S_t) C_{Wt}^{1-\eta}\right)^{1-\eta} - \left(\nu (S'_t) C'_{Wt}\right)^{1-\eta}}{(1-\eta)(1+\rho)^t} + \frac{(A-\varepsilon)^{1-\eta} \nu \left(\overline{S}\right)^{1-\eta} (1+\rho)}{(1-\eta)\left(1+\rho-(1+g)^{(1-\eta)}\right)} \left(1 - \frac{1}{M^{1-\eta}}\right) \left(\frac{(1+g)^{(1-\eta)}}{1+\rho}\right)^{t_3}
\end{aligned}$$

where the first term is bounded, the second tends to infinity when $\rho \to (1+g)^{(1-\eta)} - 1$, therefore for ρ sufficiently small $U - U'$ is positive.

9.7.2 Long-run growth rate in the switch and full specialization case

Consider the case where after a finite number of periods, country X fully specializes in sector G with more innovation in clean than in dirty (so that asymptotically all innovation occurs in clean), while country $(-X)$ specializes and innovates in H only. If this is the case first note that after another finite number of periods, I get $S_t = \overline{S}$, (see Appendix D.2: dirty input production goes to 0), second that A_{XGt} grows like A_{Xct} at a rate $(1+\kappa)^{1-\gamma} - 1$ and that $A_{(-X)Ht}$ grows at a rate $(1+\kappa)^{1-\gamma} - 1$. World consumption then follows:

$$\begin{aligned}
C_{Wt} &= \left(\nu (Y_{NGt} + Y_{SGt})^{\frac{\sigma-1}{\sigma}} + (1-\nu)(Y_{NHt} + Y_{SHt})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\
&= \zeta \left(\nu \left(A_{XGt} K_X^{\alpha} L_X^{1-\alpha}\right)^{\frac{\sigma-1}{\sigma}} + (1-\nu)\left(A_{(-X)Ht} K_{(-X)}^{\beta} L_{(-X)}^{1-\beta}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},
\end{aligned}$$

which grows at a rate $(1+\kappa)^{1-\gamma} - 1$.

9.7.3 Allocation in a non full specialization case

Consider any allocation (C'_{Wt}, S'_t). First note that $(1+\kappa)^{1-\gamma}-1$ is the largest possible growth rate for C_{Wt}, since asymptotically $Y_{NGt}+Y_{SGt}$ cannot grow faster than $\max(A_{NGt}, A_{SGt})$ and $Y_{NHt}+Y_{SHt}$ cannot grow faster than $\max(A_{NHt}, A_{SHt})$, and the fastest asymptotic growth rate for A_{XGt}, A_{XHt} is $(1+\kappa)^{1-\gamma}-1$. Now if either Y_{WGt} or Y_{WHt} does not grow at $(1+\kappa)^{1-\gamma}-1$ asymptotically, then an allocation featuring full specialization and full switch in finite time will necessarily be preferred by the social planner in view of lemma 5 for sufficiently small discount rate ρ.

Now, to get Y_{WGt} growing at a rate $(1+\kappa)^{1-\gamma}-1$, it is necessary that in one country production of good G grows at a rate $(1+\kappa)^{1-\gamma}-1$. Without loss of generality, assume that Y_{NGt} grows at $(1+\kappa)^{1-\gamma}-1$. As Y_{Ndt} must remain bounded to avoid a disaster, Y_{Nct} must grow at a rate $(1+\kappa)^{1-\gamma}$, which requires that A_{Nct} grows at a rate $(1+\kappa)^{1-\gamma}-1$. Similarly to get $Y_{NHt}+Y_{SHt}$ growing at $(1+\kappa)^{1-\gamma}-1$, it is necessary that either A_{NHt} or A_{SHt} also grow at the rate $(1+\kappa)^{1-\gamma}-1$. Since asymptotically all scientists must be allocated to the clean subsector in the North, A_{NHt} cannot grow exponentially and A_{SHt} must be growing at $(1+\kappa)^{1-\gamma}-1$ (and in return, A_{SGt} does grow exponentially). (57) and (58) must then necessarily be satisfied in finite time (since $\frac{\left(A_{Nct}^{\varepsilon-1}+\left((1+\tau_{Nt})^{-1}A_{Ndt}\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}}{1-\delta_{Nt}}$ asymptotically behaves like A_{Nct}), and full specialization is reached.

Therefore the optimal solution for sufficiently small ρ must feature full specialization in finite time with a switch to clean innovation in the country specializing in sector G. Moreover, once the clean technology is sufficiently advanced in the country specialized in sector G, emissions will be negligible so that the quality of the environment will go back to $\overline{S}$ and the input tax goes to 0.

9.8 Appendix B.8: proof of proposition 5

This proof has two steps, first I specify the equilibrium constraints for the South, second I derive the social optimum.

9.8.1 Step 1: Laissez-faire constraints in the South

First I recall the explicit equations for the constraints (28) and (29). Y_{SGt} and Y_{SHt} are given by (65) and (66) if $\left(p_t\frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}} \in \left(\left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)^{\frac{1}{\alpha-\beta}}\frac{L_S}{K_S}, \left(\frac{\beta^{\beta}(1-\beta)^{(1-\beta)}}{\alpha^{\beta}(1-\alpha)^{1-\beta}}\right)^{\frac{1}{\alpha-\beta}}\frac{L_S}{K_S}\right)$,

$$Y_{SGt}=0 \text{ and } Y_{SHt}=\zeta A_{SHt}K_S^{\beta}L_S^{1-\beta}$$

if $\left(p_t \frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}} \leq \left(\frac{\beta^\alpha(1-\beta)^{(1-\alpha)}}{\alpha^\alpha(1-\alpha)^{(1-\alpha)}}\right)^{\frac{1}{\alpha-\beta}} \frac{L_S}{K_S}$ and

$$Y_{SGt} = \zeta A_{SGt} K_S^\alpha L_S^{1-\alpha} \tag{79}$$

if $\left(p_t \frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}} \geq \left(\frac{\beta^\beta(1-\beta)^{(1-\beta)}}{\alpha^\beta(1-\alpha)^{1-\beta}}\right)^{\frac{1}{\alpha-\beta}} \frac{L_S}{K_S}$. This overall delivers the constraint (28) with the function y_{SG} increasing in p_t (weakly), and A_{SGt} and decreasing in A_{SHt} (weakly), and the function y_{SH} decreasing in p_t (weakly) and A_{SGt} (weakly) but increasing in A_{SHt}. y_{SG} and y_{SH} are only piecewise smooth (at the corner of full specialization, the functions are not differentiable). Note that since the South economy maximizes GDP:

$$p_t \frac{\partial y_{SG}}{\partial p} + \frac{\partial y_{SH}}{\partial p} = 0. \tag{80}$$

When $\left(p_t \frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}} > \left(\frac{\beta^\beta(1-\beta)^{(1-\beta)}}{\alpha^\beta(1-\alpha)^{1-\beta}}\right)^{\frac{1}{\alpha-\beta}} \frac{L_S}{K_S}$, the allocation of scientists is trivially given by $s_{dt} = 1$ and when $\left(p_t \frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}} \leq \left(\frac{\beta^\alpha(1-\beta)^{(1-\alpha)}}{\alpha^\alpha(1-\alpha)^{(1-\alpha)}}\right)^{\frac{1}{\alpha-\beta}} \frac{L_S}{K_S}$, by $s_{dt} = 0$. When $\left(p_t \frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}} \in \left(\left(\frac{\beta^\alpha(1-\beta)^{(1-\alpha)}}{\alpha^\alpha(1-\alpha)^{(1-\alpha)}}\right)^{\frac{1}{\alpha-\beta}} \frac{L_S}{K_S}, \left(\frac{\beta^\beta(1-\beta)^{(1-\beta)}}{\alpha^\beta(1-\alpha)^{1-\beta}}\right)^{\frac{1}{\alpha-\beta}} \frac{L_S}{K_S}\right)$, the allocation of scientists is given by (67), that is:

$$\begin{aligned} &\frac{1+\widetilde{\kappa}\left(1-s_{SGt}\right)}{\widetilde{\kappa}'\left(1-s_{SGt}\right)} \frac{\widetilde{\kappa}'\left(\widetilde{s_{Sdt}}\left(s_{SGt}, \frac{A_{Sdt}}{A_{Sct}}\right)\right)}{\left(1+\widetilde{\kappa}\left(\widetilde{s_{Sdt}}\left(s_{SGt}, \frac{A_{Sdt}}{A_{Sct}}\right)\right)\right)} \left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon-1} \\ = &\frac{\alpha\left(\frac{A_{SHt}^\alpha}{A_{SGt}^\beta}\right)^{\frac{1}{\alpha-\beta}} L_S - (1-\alpha)\left(\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{\beta^\beta(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}} \left(\frac{A_{SGt}^{1-\beta}}{A_{SHt}^{1-\alpha}}\right)^{\frac{1}{\alpha-\beta}} p_t^{\frac{1}{\alpha-\beta}} K_S}{(1-\beta)\left(\frac{\alpha^\alpha(1-\alpha)^{(1-\alpha)}}{\beta^\beta(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}} p_t^{\frac{1}{\alpha-\beta}} \left(\frac{A_{SGt}^{1-\beta}}{A_{SHt}^{1-\alpha}}\right)^{\frac{1}{\alpha-\beta}} K_S - \beta\left(\frac{A_{SHt}^\alpha}{A_{SGt}^\beta}\right)^{\frac{1}{\alpha-\beta}} L_S}. \end{aligned} \tag{81}$$

where $\widetilde{s_{Sdt}}$ is itself define through:

$$\frac{\widetilde{\kappa}'\left(s_{SGt} - \widetilde{s_{Sdt}}\left(s_{SGt}, \frac{A_{Sdt}}{A_{Sct}}\right)\right)}{1+\widetilde{\kappa}\left(s_{SGt} - \widetilde{s_{Sdt}}\left(s_{SGt}, \frac{A_{Sdt}}{A_{Sct}}\right)\right)} = \frac{\widetilde{\kappa}'\left(\widetilde{s_{Sdt}}\left(s_{SGt}, \frac{A_{Sdt}}{A_{Sct}}\right)\right)}{1+\widetilde{\kappa}\left(\widetilde{s_{Sdt}}\left(s_{SGt}, \frac{A_{Sdt}}{A_{Sct}}\right)\right)} \left(\frac{A_{Sdt}}{A_{Sct}}\right)^{\varepsilon-1}. \tag{82}$$

This corresponds to the constraint (29).Note that I defined $\widetilde{s_{Sdt}}$ as a function of s_{SGt} and $\frac{A_{Sdt}}{A_{Sct}}$ not of s_{SGt} and $\frac{A_{Sd(t-1)}}{A_{Sc(t-1)}}$ as I did in Appendix B.3, this allows to express s_{SGt} as a function of the current productivity levels, which simplifies considerably the expression of the optimal tariff. I use the tilde to ensure that the difference between the two functions is explicit (however, 81 also implicitly define s_{SGt} as a unique function of p_t and the previous period technology levels). Note that the function s_{SG} (weakly) increases in p_t, and (weakly) decrease in A_{SHt}. It is possible to show that the function s_{SGt} is continuously differentiable. Moreover, (81) can be rewritten as:

$$\left(p_t\frac{\partial y_{SG}}{\partial A_{SHt}}+\frac{\partial y_{SH}}{\partial A_{SHt}}\right)\frac{\widetilde{\kappa}'\left(s_{SHt}\right)}{\left(1+\widetilde{\kappa}\left(s_{SHt}\right)\right)}A_{SHt} \tag{83}$$

$$= \frac{\widetilde{\kappa}'\left(\widetilde{s_{Sdt}}\left(s_{SGt},\frac{A_{Sdt}}{A_{Sct}}\right)\right)}{\left(1+\widetilde{\kappa}\left(\widetilde{s_{Sdt}}\left(s_{SGt},\frac{A_{Sdt}}{A_{Sct}}\right)\right)\right)}\left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon-1}A_{SGt}\left(p_t\frac{\partial y_{SG}}{\partial A_{SGt}}+\frac{\partial y_{SH}}{\partial A_{SGt}}\right), \tag{84}$$

which stipulates that for given prices, innovation in the South maximizes current GDP $p_tY_{SGt}+Y_{SHt}$.

9.8.2 Step 2: Deriving the social optimum

To simplify a bit the exposition, I combine (11) and the emission equation for the South $Y_{Sdt}=(A_{Sdt}/A_{SGt})^{\varepsilon}Y_{SGt}$ into:

$$S_t=\max\left(\min\left(\left(1+\Delta\right)S_{t-1}-\xi\left(Y_{Ndt}+\left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon}Y_{SGt}\right),\overline{S}\right),0\right), \tag{85}$$

I then use the following notations for the Lagrange parameters (the corresponding constraints are in parentheses): λ_{Xt} for (1) - both in North and South -; for the North only: λ_{NHt} (2), λ_{NGt} (5), λ_{Nzt} (6), φ_{Nzit} (7), φ_{NHit} (3), η_{NKt} (9) for capital, η_{NLt} (9) for labor, μ_{Nzit} (12), υ_{Nt} (14); ω_t (85), θ_{NGt}, θ_{NHt}, θ_{SGt} and θ_{SHt} -with obvious subscripts- for the equations in (25), χ_t (26), κ_t (27), λ_{SGt} and λ_{SHt} (28), ϕ_t (29), μ_{SHt}, μ_{Sdt} and μ_{Sct} (30), in addition, the social planner faces the constraints: $0\leq Y_{NGt}$, $0\leq Y_{NHt}$, with Lagrange parameters: ι_{NGt}, ι_{NHt}.

As specified above, the functions y_{SG} and y_{SH} are not everywhere differentiable, in the following I therefore use generalized Karush Kuhn Tucker conditions: at point of non differentiability the notation $\frac{\partial y_{SG}}{\partial p_t}$, $\frac{\partial y_{SG}}{\partial A_{SGt}}$, $\frac{\partial y_{SG}}{\partial A_{SHt}}$ refers to elements of a vector $\left(\frac{\partial y_{SG}}{\partial p_t},\frac{\partial y_{SG}}{\partial A_{SGt}},\frac{\partial y_{SG}}{\partial A_{SHt}}\right)$ belonging to the Clarke generalized gradient of y_{SG}. Therefore it is still the case at these points that $\frac{\partial y_{SG}}{\partial p_t}\geq 0$, $\frac{\partial y_{SG}}{\partial A_{SGt}}>0$, $\frac{\partial y_{SG}}{\partial A_{SHt}}\leq 0$.

First order conditions with respect to all the "North" variables, and S_t allow us to recover exactly the same equations as in the first best for the North part of the economy (up to replacing θ_{Gt} and θ_{Ht} by θ_{NGt} and and θ_{NHt}). This shows that the economy in the North is similar to the first best case (with a carbon tax, subsidy to the use of intermediates, and research taxes/subsidies that which can be used to decentralize the equilibrium). I am now going to derive that the social planner creates a wedge between relative prices in the North and in the South. To assess the generality of the results, I will not immediately use the functional form. Moreover taking first order condition with respect to C_{St}, I get:

$$\frac{1}{\left(1+\rho\right)^t}\frac{\partial u}{\partial C}\left(C_{Nt}+C_{St},S_t\right)=\frac{\nu\left(S_t\right)^{1-\eta}}{\left(1+\rho\right)^t}\left(C_{Nt}+C_{St}\right)^{-\eta}=\lambda_{St}=\lambda_{Nt}\equiv\lambda_t.$$

Taking the first order condition with respect to C_{SHt}, I get:

$$\theta_{SHt} + \kappa_t \frac{\partial}{\partial C_{SHt}} \frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}} = \lambda_t \frac{\partial C_S}{\partial C_{SHt}} = \lambda_t (1-\nu) C_{SHt}^{\frac{-1}{\sigma}} \left(\nu C_{SGt}^{\frac{\sigma-1}{\sigma}} + (1-\nu) C_{SHt}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}}, \quad (86)$$

and with respect to C_{SGt}:

$$\theta_{SGt} + \kappa_t \frac{\partial}{\partial C_{SGt}} \frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}} = \lambda_t \frac{\partial C_S}{\partial C_{SGt}} = \lambda_t \nu C_{SGt}^{\frac{-1}{\sigma}} \left(\nu C_{SGt}^{\frac{\sigma-1}{\sigma}} + (1-\nu) C_{SHt}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}}. \quad (87)$$

Therefore combining the two:

$$\frac{\theta_{SHt} + \kappa_t \frac{\partial}{\partial C_{SHt}} \frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}}{\theta_{SGt} + \kappa_t \frac{\partial}{\partial C_{SGt}} \frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}} = \frac{\frac{\partial C_S}{\partial C_{SHt}}}{\frac{\partial C_S}{\partial C_{SGt}}} = \frac{1}{p_t},$$

so that:

$$\kappa_t = \frac{\frac{\theta_{SGt}}{p_t} - \theta_{SHt}}{\frac{\partial}{\partial C_{SHt}} \frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}} - \frac{1}{p_t} \frac{\partial}{\partial C_{SGt}} \frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}}. \quad (88)$$

First order conditions with respect to Y_{SHt} and Y_{SGt} give:

$$\lambda_{SGt} = \theta_{SGt} - \omega_t \xi \left(\frac{A_{Sdt}}{A_{SGt}} \right)^{\varepsilon} \text{ and } \lambda_{SHt} = \theta_{SHt}, \quad (89)$$

First order conditions with respect to M_{Ht} and M_{Gt} give:

$$p_t \chi_t = \theta_{NGt} - \theta_{SGt} \text{ and } \chi_t = \theta_{NHt} - \theta_{SHt}, \quad (90)$$

so that

$$\frac{\theta_{SGt}}{p_t} - \theta_{SHt} = \frac{\theta_{NGt}}{p_t} - \theta_{NHt}. \quad (91)$$

Finally the first order condition with respect to p_t gives:

$$M_{Gt} \chi_t = \lambda_{SGt} \frac{\partial y_{SG}}{\partial p_t} + \lambda_{SHt} \frac{\partial y_{SH}}{\partial p_t} + \kappa_t + \phi_t \frac{\partial s_{Sdt}}{\partial p_t} \quad (92)$$

Let us denote by $(1 + b_t)$ an add valorem tariff (export subsidy) on good G, using (71) and (72) in the North one gets,

$$\frac{\frac{\partial C_N}{\partial C_{NG}}}{\frac{\partial C_N}{\partial C_{NH}}} = \frac{\nu C_{NHt}^{\frac{1}{\sigma}}}{(1-\nu) C_{NGt}^{\frac{1}{\sigma}}} = \frac{\theta_{NGt}}{\theta_{NHt}} = \frac{\widehat{p}_{NGt}}{\widehat{p}_{Ht}} = p_t (1 + b_t). \quad (93)$$

Now plugging (89), (88), (90) and (91) in (92), I get:

$$M_{Gt}\frac{(\theta_{NGt}-\theta_{SGt})}{p_t}$$
$$= \left(\theta_{SGt}-\omega_t\xi\left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon}\right)\frac{\partial y_{SG}}{\partial p_t}+\theta_{SHt}\frac{\partial y_{SH}}{\partial p_t}+\frac{\frac{\theta_{SGt}}{p_t}-\theta_{SHt}}{\frac{\partial}{\partial C_{SHt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}-\frac{1}{p_t}\frac{\partial}{\partial C_{SGt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}}+\phi_t\frac{\partial s_{SGt}}{\partial p_t}.$$

Further, using (80), (71) and (72) for the North - replacing θ_{Gt} by θ_{NGt}-, (87), (86), (88) and (93):

$$\frac{M_{Gt}}{p_t}\left(\frac{\partial C_N}{\partial C_{NG}}-\frac{\partial C_S}{\partial C_{SGt}}\right) \tag{94}$$
$$= -\frac{\omega_t\xi}{\lambda_t}\left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon}\frac{\partial y_{SG}}{\partial p_t}+\frac{\phi_t}{\lambda_t}\frac{\partial s_{SGt}}{\partial p_t}+\frac{\partial C_N}{\partial C_{NH}}b_t\left(p_t\frac{\partial y_{SG}}{\partial p_t}+\frac{\left(1-\frac{M_{Gt}}{p_t}\frac{\partial}{\partial C_{SGt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}\right)}{\frac{\partial}{\partial C_{SHt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}-\frac{1}{p_t}\frac{\partial}{\partial C_{SGt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}}\right).$$

This expression shows that the social planner imposes a wedge between relative prices in the North and in the South. With homothetic preferences, this wedge is entirely created by two terms: $-\omega_t\xi\left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon}\frac{\partial y_{SG}}{\partial p_t}$ (which is going to be the environmental motive) and $\phi_t\frac{\partial s_{SGt}}{\partial p_t}$ (which is going to be the innovation motive), indeed if $\omega_t=0$ and $\phi_t=0$, the solution to that equation would be $b_t=0$ -as long as preferences are homothetic, so that $\frac{\partial C_N}{\partial C_{NG}}=\frac{\partial C_S}{\partial C_{SGt}}$ at equal relative price). There is no terms of trade motives for the tariff since the social planner cares equally about consumption in the North and consumption in the South. From here, I use the specific functional forms to get a more explicit formula for the tariff to get (see Appendix D.4.2):

$$
\begin{aligned}
& \frac{\left(\nu^\sigma + (1-\nu)^\sigma \left(p_t\left(1+b_t\right)\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}}{1+b_t} b_t p_t \frac{\partial y_S}{\partial p_t} \\
& + \left(\begin{array}{c} \frac{\left(\nu^\sigma+(1-\nu)^\sigma(p_t(1+b_t))^{\sigma-1}\right)^{\frac{1}{\sigma-1}}\left((1+b_t)\nu^\sigma+(1+\sigma b_t)(1-\nu)^\sigma p_t^{\sigma-1}\right)}{(1+b_t)\left(\nu^\sigma+(1-\nu)^\sigma p_t^{\sigma-1}\right)} \\ -\left(\nu^\sigma+(1-\nu)^\sigma p_t^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \end{array} \right) \frac{\nu^\sigma Y_{SHt}}{p_t\left(\nu^\sigma+(1-\nu)^\sigma p_t^{\sigma-1}\right)} \\
& + \left(\begin{array}{c} \left(\nu^\sigma+(1-\nu)^\sigma p_t^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \\ -\frac{\left(\nu^\sigma+(1-\nu)^\sigma(p_t(1+b_t))^{\sigma-1}\right)^{\frac{1}{\sigma-1}}}{(1+b_t)} \frac{\nu^\sigma(1-b_t(\sigma-1))+(1-\nu)^\sigma p_t^{\sigma-1}}{\left(\nu^\sigma+(1-\nu)^\sigma p_t^{\sigma-1}\right)} \end{array} \right) \frac{(1-\nu)^\sigma p_t^\sigma Y_{SGt}}{p_t\left(\nu^\sigma+(1-\nu)^\sigma p_t^{\sigma-1}\right)} \\
& + \frac{(1-\gamma)\, p_t \left(\nu^\sigma+(1-\nu)^\sigma\left(p_t\left(1+b_t\right)\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}} b_t}{(1-\gamma)\, D_t\left(1+b_t\right)} \frac{\partial s_{SGt}}{\partial p_t} \left(\begin{array}{c} A_{SGt}\frac{\partial y_{SG}}{\partial A_{SGt}}\frac{A_{Sdt}^{\varepsilon-1}}{A_{SGt}^{\varepsilon-1}}\frac{\widetilde{\kappa}'(s_{Sdt})}{(1+\widetilde{\kappa}(s_{Sdt}))} \\ -\frac{\partial y_{SG}}{\partial A_{SHt}}\frac{\widetilde{\kappa}'(s_{SHt})}{(1+\widetilde{\kappa}(s_{SHt}))}A_{SHt} \end{array} \right) \\
= \;& \frac{p_t}{\lambda_t}\omega_t\xi\left(\frac{A_{Sdt}}{A_{SGt}}\right)^\varepsilon \left(\frac{\partial y_{SG}}{\partial p_t} + \frac{\partial s_{SGt}}{\partial p_t}\frac{1}{D_t} \left(\begin{array}{c} A_{SGt}\frac{\partial y_{SG}}{\partial A_{SGt}}\frac{A_{Sdt}^{\varepsilon-1}}{A_{SGt}^{\varepsilon-1}}\frac{\widetilde{\kappa}'(s_{Sdt})}{(1+\widetilde{\kappa}(s_{Sdt}))} - \frac{\partial y_{SG}}{\partial A_{SHt}}\frac{\widetilde{\kappa}'(s_{SHt})}{(1+\widetilde{\kappa}(s_{SHt}))}A_{SHt} \\ +\varepsilon\left(\frac{A_{Sct}}{A_{SGt}}\right)^{\varepsilon-1}\left(\frac{\widetilde{\kappa}'(s_{Sdt})}{(1+\widetilde{\kappa}(s_{Sdt}))}\frac{\partial s_{Sdt}}{\partial s_{SGt}} - \frac{\widetilde{\kappa}'(s_{Sct})}{(1+\widetilde{\kappa}(s_{Sct}))}\frac{\partial s_{Sct}}{\partial s_{SGt}}\right)Y_{SGt} \end{array} \right)\right) \\
& + \frac{\partial s_{SGt}}{\partial p_t}\frac{p_t}{\lambda_t D_t} \left(\begin{array}{c} \mu_{SHt+1}\frac{\widetilde{\kappa}'(s_{SHt})}{(1+\widetilde{\kappa}(s_{SHt}))}A_{SHt+1} \\ -\left(\mu_{Sdt+1}\frac{\widetilde{\kappa}'(s_{Sdt})}{(1+\widetilde{\kappa}(s_{Sdt}))}A_{Sdt+1}\frac{\partial s_{Sdt}}{\partial s_{SGt}} + \mu_{Sct+1}\frac{\widetilde{\kappa}'(s_{Sct})}{(1+\widetilde{\kappa}(s_{Sct}))}A_{Sct+1}\frac{\partial s_{Sct}}{\partial s_{SGt}}\right) \end{array} \right) \\
& + \frac{\partial s_{SGt}}{\partial p_t}\frac{p_t(1-\gamma)(\varepsilon-1)}{\lambda_t D_t} \left(\begin{array}{c} \frac{\widetilde{\kappa}'(s_{Sdt})}{1+\widetilde{\kappa}(s_{Sdt})}\frac{\partial s_{Sdt}}{\partial s_{SGt}} \\ -\frac{\widetilde{\kappa}'(s_{Sct})}{1+\widetilde{\kappa}(s_{Sct})}\frac{\partial s_{Sct}}{\partial s_{SGt}} \end{array} \right) \left(\begin{array}{c} \mu_{Sc(t+1)}A_{Sc(t+1)}\frac{\widetilde{\kappa}'\left(s_{Sc(t+1)}\right)}{1+\widetilde{\kappa}\left(s_{Sc(t+1)}\right)} \\ -\mu_{Sd(t+1)}A_{Sd(t+1)}\frac{\widetilde{\kappa}'\left(s_{Sd(t+1)}\right)}{1+\widetilde{\kappa}\left(s_{Sd(t+1)}\right)} \end{array} \right) \frac{A_{Sct}^{\varepsilon-1}}{A_{Sdt}^{\varepsilon-1}}\left(-\frac{\partial s_{Sd(t+1)}}{\partial a_t}\right).
\end{aligned} \tag{95}
$$

D_t given by

$$
\begin{aligned}
D_t \;\equiv\;& (1-\gamma)^{-1} + \frac{\partial s_{SGt}}{\partial A_{SHt}}\frac{\widetilde{\kappa}'\left(s_{SHt}\right)}{\left(1+\widetilde{\kappa}\left(s_{SHt}\right)\right)}A_{SHt} \\
& -\frac{\widetilde{\kappa}'\left(s_{Sdt}\right)}{\left(1+\widetilde{\kappa}\left(s_{Sdt}\right)\right)}A_{Sdt}\frac{\partial s_{Sdt}}{\partial s_{SGt}}\frac{\partial s_{SGt}}{\partial A_{Sdt}} - \frac{\widetilde{\kappa}'\left(s_{Sct}\right)}{\left(1+\widetilde{\kappa}\left(s_{Sct}\right)\right)}A_{Sct}\frac{\partial s_{Sct}}{\partial s_{SGt}}\frac{\partial s_{SGt}}{\partial A_{Sct}},
\end{aligned} \tag{96}
$$

is strictly positive. On the left-hand side, the first term has the sign of b_t since $p_t\frac{\partial y_S}{\partial p_t} \geq 0$, the second term has the sign of b_t except possibly for b_t sufficiently small (close to -1) when $\sigma < 1$, and the third term has the sign of b_t except possibly for b_t sufficiently large when $\sigma < 1$. The sum of the two terms into brackets, however, always has the sign of b_t and when b_t gets sufficiently small, the South exports good G (so that $\nu^\sigma Y_{SHt} < (1-\nu)^\sigma p_t^\sigma Y_{SGt}$) and imports good G when b_t gets sufficiently large, for all purposes the sum of the second and third term will have the sign of b_t. The forth term also has the sign of b_t. Therefore b_t will have the sign of the right-hand side (RHS). On the RHS, the first term in bracket is weakly positive since $\frac{\partial y_{SG}}{\partial p_t} \geq 0$, $\frac{\partial s_{SGt}}{\partial p_t} \geq 0$, $\frac{\partial y_{SG}}{\partial A_{SGt}} \geq 0$, $\frac{\partial y_{SG}}{\partial A_{SHt}} \leq 0$, $\left(\frac{\widetilde{\kappa}'(s_{Sdt})}{(1+\widetilde{\kappa}(s_{Sdt}))}\frac{\partial s_{Sdt}}{\partial s_{SGt}} - \frac{\widetilde{\kappa}'(s_{Sct})}{(1+\widetilde{\kappa}(s_{Sct}))}\frac{\partial s_{Sct}}{\partial s_{SGt}}\right) \geq 0$ and the denominator is positive. This first term therefore pushes towards a positive tariff (this is the environmental term) - with ω_t still given by (74)-, note that the first part of this term represents the effect of the tariff at given technology, while the second term represents the environmental benefits from reducing innovation in sector G. The second term has the sign of

$$\left(1+\widetilde{\kappa}\left(s_{SHt+1}\right)\right)^{1-\gamma}\mu_{SHt+1}\frac{\widetilde{\kappa}'\left(s_{SHt}\right)}{\left(1+\widetilde{\kappa}\left(s_{SHt}\right)\right)}A_{SHt}$$
$$-\left(\left(1+\widetilde{\kappa}\left(s_{Sdt+1}\right)\right)^{1-\gamma}\mu_{Sdt+1}\frac{\widetilde{\kappa}'\left(s_{Sdt}\right)}{\left(1+\widetilde{\kappa}\left(s_{Sdt}\right)\right)}A_{Sdt}\frac{\partial s_{Sdt}}{\partial s_{SGt}}+\left(1+\widetilde{\kappa}\left(s_{Sct+1}\right)\right)^{1-\gamma}\mu_{Sct+1}\frac{\widetilde{\kappa}'\left(s_{Sct}\right)}{\left(1+\widetilde{\kappa}\left(s_{Sct}\right)\right)}A_{Sct}\frac{\partial s_{Sct}}{\partial s_{SGt}}\right)$$

which is the difference between the social value of allocating one scientist in sector H instead of sector G, for all the future periods (that is not including the current one). The last term, reflects how the allocation between clean and dirty innovation in the future period is affected by the current number of scientists allocated to sector G, and has the sign of $\mu_{Sc(t+1)}A_{Sc(t+1)}\frac{\widetilde{\kappa}'\left(s_{Sc(t+1)}\right)}{1+\widetilde{\kappa}\left(s_{Sc(t+1)}\right)}-\mu_{Sd(t+1)}A_{Sd(t+1)}\frac{\widetilde{\kappa}'\left(s_{Sd(t+1)}\right)}{1+\widetilde{\kappa}\left(s_{Sd(t+1)}\right)}$, this could be positive or negative: on one hand, since the South is not going to switch to clean technologies, developing clean technologies in the South has little value for consumption's sake but on the other hand, dirty technologies polluted. However this term vanishes as $A_{Sc(t-1)}/A_{Sd(t-1)}$ goes to 0.

Finally note that when the South is fully specialized- and not at the threshold of specialization ($\left(p_t\frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}}\notin\left(\left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)^{\frac{1}{\alpha-\beta}}\frac{L_S}{K_S},\left(\frac{\beta^{\beta}(1-\beta)^{(1-\beta)}}{\alpha^{\beta}(1-\alpha)^{1-\beta}}\right)^{\frac{1}{\alpha-\beta}}\frac{L_S}{K_S}\right)$), $\frac{\partial y_{SG}}{\partial p_t}=0$ and $\frac{\partial s_{SGt}}{\partial p_t}=0$, so that the optimal tariff turns out to be $b_t=0$.

9.9 Appendix B.9: proof of proposition 6

I first study the case $\eta\leq 1$ and then the case $\sigma<1$ and $\eta>1$.

9.9.1 Case $\eta\leq 1$.

The proof follows closely Appendix B.7.

First note that when $\eta=1$, a disaster leads to a utility flow of $-\infty$, so whenever it is possible the social planner will avoid it. When $\eta<1$, a disaster leads to a permanent utility flow of 0, but as intervention in the South is prohibited, it may not be possible to avoid it without affecting previous period utilities. However, one can follow part 1 in that case - which is still true-, and as for sufficiently low discount rate positive growth of the utility flow is preferred, a disaster is avoided. In Part 2, I must specify that it is the North that specializes in G, such an allocation is feasible as explained in proposition 1 iv)), and since the South cannot impose a tax or switch to clean technologies, the case where the specialization would be reversed could not feature a sector G growing at $(1+\kappa)^{1-\gamma}-1$ and avoid an environmental disaster. Part 3 must be slightly amended. I still have that there must be a switch towards clean in the North, but the argumentation for full specialization is a bit more complicated.

As before (57) and (58) will necessarily be satisfied in finite time, so that absent any trade tax, there would be full specialization. A trade tax that leaves the South specialized is never optimal: this is direct from the expression for the optimal tariff when the relative price of good G is below the price that just leads to specialization, in the corner case, the trade tax does not affect production in the South, cannot affect pollution if clean technologies in the North are sufficiently advanced, and is therefore just distorting for world consumption (since the carbon tax in the North becomes negligible, free trade maximizes world consumption and technological level in the North can be maintained at the same level with appropriate policies). Now let us assume that there is a non null trade tax implemented at a time t (sufficiently large) that does lead to some production of good G in the South (although asymptotically resources devoted to that sector must go to zero), and let us consider the alternative case where everything research subsidies in the North are adjusted so that technological levels remain the same and free trade is implemented in every following periods. At given technological levels, a non null trade tax leads to lower world consumption than free trade. Now, when removing the trade tax, technological levels in the South are affected: A_{SGt} decreases and A_{SHt} increases, but in free-trade this leads to even larger level of world consumption. Therefore welfare is higher under that alternative set-up: no trade tax is implemented and full specialization is reached in finite time. All research is allocated to sector H in the South in finite time, and asymptotically, all research is allocated to clean intermediates in the North.

9.9.2 Case $\eta > 1$ and $\sigma < 1$

First note that in this case a disaster leads to a utility flow of $-\infty$, so whenever it is possible the social planner will avoid it.

The proof is done in 5 steps: first I show that in the optimum C_{Wt} must grow without bound, second I show that in the South, Y_{SGt} is bounded and that asymptotically all scientists must be allocated to sector H, third I show that A_{Nct} must grow without bound and that there is a full switch towards clean innovation in the North, forth I show that I must then have exponential growth in A_{Nct} in the North, fifth I conclude that full specialization must be reached in finite time.

Part 1: the optimal solution must feature positive growth I want to show the following lemma:

Lemma 6 *Assume $\eta > 1$ and $\sigma < 1$. Let us consider an allocation (C_{Wt}, S_t) such that there is a t_1, such that for $t > t_1$, $S_t = \overline{S}$ and $\lim C_{Wt} = \infty$. Then for ρ sufficiently small the social planner would rather choose the path (C_{Wt}, S_t) to any other (C'_{Wt}, S'_t) if C'_{Wt} is bounded.*

Proof. Then there is a t_2 and a $M > 0$ such that for all $t > t_2$, $C'_{Wt} < M < \frac{1}{2}C_{Wt}$, $S_t = \overline{S}$. I then get:

$$\begin{aligned} U - U' &= \sum_{t=0}^{t_2-1} \frac{1}{(1+\rho)^t} \left(\frac{(\nu(S_t) C_{Wt})^{1-\eta} - (\nu(S'_t) C'_{Wt})^{1-\eta}}{1-\eta} \right) + \sum_{t=t_2}^{\infty} \frac{1}{(1+\rho)^t} \frac{\left(\nu\left(\overline{S}\right) C_{Wt}\right)^{1-\eta} - (\nu(S'_t) C'_{Wt})^{1-\eta}}{1-\eta} \\ &> \sum_{t=0}^{t_2-1} \frac{1}{(1+\rho)^t} \left(\frac{C_{Wt}^{1-\eta} - (C'_{Wt})^{1-\eta}}{1-\eta} + \nu(S_t) - \nu(S'_t) \right) + \frac{\nu\left(\overline{S}\right)^{1-\eta}}{\eta - 1} \left(1 - \frac{1}{2^{\eta-1}}\right) \frac{1}{(1+\rho)^{t_2-1}} \frac{1}{\rho} \end{aligned}$$

where the first term is bounded, the second tends to infinity when $\rho \to 0$, therefore for ρ sufficiently small $U - U'$ is positive. ■

An allocation leading in finite time to full specialization in sector H in the South, and to full specialization in the North in G is feasible and satisfies the assumption of the lemma. Therefore the optimal solution must feature positive long-run growth in both sectors.

Part 2: scientists must all be allocated to sector H in the South asymptotically This is a direct consequence of lemma 3: A_{SHt} must grow at the rate $(1+\kappa)^{1-\gamma} - 1$ (technically, A_{SHt} may be dominated by $(1+\kappa)^{(1-\gamma)t}$, but grows faster than $M\left((1+\kappa)^{1-\gamma} - x\right)^t$, no matter how small x is).

Part 3: A_{Nct} must grow and switch to clean in the North Note that

$$C_{Wt} = C_{Nt} + C_{St} < 2\left(\nu\left(Y_{NGt} + Y_{SGt}\right)^{\frac{\sigma-1}{\sigma}} + (1-\nu)\left(Y_{NHt} + Y_{SHt}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},$$

so that with $\sigma < 1$, to get C_{Wt} unbounded it is necessary that $Y_{NGt} + Y_{SGt}$ is unbounded, since Y_{SGt} is bounded, I must have Y_{NGt} unbounded. Y_{Ndt} must remain bounded however, so that using (44), $A_{Nct}/\left((1+\tau_{Nt})^{-1} A_{Ndt}\right)$ must tend towards infinity. Using (24), innovation in sector G in the North must switch to being mostly in clean technologies, so that A_{Nct} grows unboundedly, while A_{Ndt} grows infinitely less, and emissions in the North become negligible. A_{Ndt}/A_{Nct} becomes negligible and s_{Ndt} is negligible relative to s_{Nct}.

Part 4: there must be exponential growth in A_{Nct} Assume that this is not the case, then in the North all scientists must be allocated to sector H asymptotically and A_{NHt} must grow faster than $\left((1+\kappa)^{1-\gamma} - x\right)^t$, no matter how small x is. If the North fully specializes in sector G this is clearly not optimal. For the North not to fully specialize while the South does so in H, it is necessary to prevent (57) from holding with $X = N$, since when the South is fully specialized (not at the corner), $b_t = 0$. Note that this is impossible if A_{SHt} and A_{NHt} both grow faster than $\left((1+\kappa)^{1-\gamma} - x\right)^t$ for all $x > 0$, and A_{Nct} does not grow exponentially, in fact if A_{Nct} does not grow exponentially, this case will never be possible after a finite number of periods. Therefore, after a finite number of periods the South does not specialize.

Lemma 3 already demonstrates that the North cannot import the polluting good, therefore the only possibility left is that the South does not specialize and the North alternates periods with non-specialization or full-specialization in G, and $M_{Gt} < 0$. To ensure that A_{Nct} does not grow exponentially, it is necessary that in periods where the North does not specialize, $\liminf s_{Nct} = 0$.

Now I show that if $\frac{p_{NGt}Y_{NGt}}{p_{Ht}Y_{NHt}}$ is bounded below, $\liminf s_{Nct} \neq 0$. Indeed, in the North innovation switches to clean in sector G in finite time, so that in periods where the North does not specialize:

$$\frac{\widetilde{\kappa}'(s_{NHt})}{(1+\widetilde{\kappa}(s_{NHt}))}\frac{1+\widetilde{\kappa}(s_{NGt})}{\widetilde{\kappa}'(s_{NGt})} = \frac{\mu_{Nct}A_{Nct}}{\mu_{NHt}A_{NHt}}$$

and to ensure that $\liminf s_{Nct} = 0$, I must have $\liminf \frac{\mu_{Nct}A_{Nct}}{\mu_{NHt}A_{NHt}} = 0$. Now:

$$\frac{\lambda_{Nct}Y_{Nct}}{\lambda_{NHt}Y_{NHt}} = \frac{p_{Nct}Y_{Nct}}{p_{NHt}Y_{NHt}} = \frac{p_{NGt}Y_{NGt}}{p_{NHt}Y_{NHt}}\frac{p_{Nct}Y_{Nct}}{p_{NGt}Y_{NGt}},$$

where since $A_{Nct}/\left((1+\tau_{Nt})^{-1}A_{Ndt}\right)$ tends towards infinity $\frac{p_{Nct}Y_{Nct}}{p_{NGt}Y_{NGt}}$ tends toward 1. Now if $\frac{p_{NGt}Y_{NGt}}{p_{NHt}Y_{NHt}}$ is bounded below, then necessarily $\frac{\lambda_{Nct}Y_{Nct}}{\lambda_{NHt}Y_{NHt}}$ is also bounded below. Denote such a bound by m, using (77), I can write that at a time t where the North does not specialize:

$$\frac{\mu_{Nct}A_{Nct}}{\mu_{NHt}A_{NHt}} = \frac{\sum_{t'=t}^{\infty}\lambda_{Nct}Y_{Nct}}{\sum_{t'=t}^{\infty}\lambda_{NHt}Y_{NHt}},$$

since $\mu_{NH0}A_{NH0}$ is finite, $\mu_{NHt}A_{NHt}$ must converge, so that there is a t_2 large enough so that for the t that I am looking at $\mu_{NHt_2}A_{NHt_2} = \sum_{t=t_2}^{\infty}\lambda_{NHt}Y_{NHt} \leq \lambda_{NHt}Y_{NHt} \leq \sum_{t'=t}^{t=t_2-1}\lambda_{NHt}Y_{NHt}$, and

$$\frac{\mu_{Nct}A_{Nct}}{\mu_{NHt}A_{NHt}} = \frac{\sum_{t'=t}^{\infty}\lambda_{Nct}Y_{Nct}}{\sum_{t'=t}^{\infty}\lambda_{NHt}Y_{NHt}} > \frac{\sum_{t'=t}^{t_2}\lambda_{Nct}Y_{Nct}}{2\sum_{t'=t}^{t_2}\lambda_{NHt}Y_{NHt}} > \frac{\sum_{t'=t}^{t_2}m\lambda_{NHt}Y_{NHt}}{2\sum_{t'=t}^{t_2}\lambda_{NHt}Y_{NHt}} = \frac{m}{2},$$

so that $\frac{\mu_{Nct}A_{Nct}}{\mu_{NHt}A_{NHt}}$ must be bounded below, and $\liminf s_{Nct} \neq 0$.

Now I am going to show that indeed, when $M_{Gt} \leq 0$, $\frac{p_{NGt}Y_{NGt}}{p_{NHt}Y_{NHt}}$ must be bounded below.

First, I can write, using (48) and (49),

$$\frac{p_{NGt}Y_{NGt}}{p_{NHt}Y_{NHt}} = \left(\frac{A_{Nct}^{\varepsilon-1}+\left((1+\tau_{Nt})^{-1}A_{Ndt}\right)^{\varepsilon-1}}{\left(A_{Nct}^{\varepsilon-1}+(1+\tau_{Nt})^{-\varepsilon}A_{Ndt}^{\varepsilon-1}\right)}\right)^{\frac{1}{\varepsilon-1}}$$

$$\times\frac{\left(\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\beta)\left(\frac{p_t(1+b_t)\left(A_{Nct}^{\varepsilon-1}+\left((1+\tau_{Nt})^{-1}A_{Ndt}\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}}{A_{NHt}}\right)^{\frac{1}{\alpha-\beta}}K_N-\beta L_N}{\alpha L_N-\left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\alpha)\left(\frac{p_t(1+b_t)\left(A_{Nct}^{\varepsilon-1}+\left((1+\tau_{Nt})^{-1}A_{Ndt}\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}}{A_{NHt}}\right)^{\frac{1}{\alpha-\beta}}K_N}.$$

Now assume that $\frac{p_{NGt}Y_{NGt}}{p_{NHt}Y_{NHt}}$ is not bounded below, then no matter how small m is, there must be a t such that $\frac{p_{NGt}Y_{NGt}}{p_{NHt}Y_{NHt}} < m$. Since $\lim \frac{A_{Nct}^{\varepsilon-1}+\left((1+\tau_{Nt})^{-1}A_{Ndt}\right)^{\varepsilon-1}}{\left(A_{Nct}^{\varepsilon-1}+(1+\tau_{Nt})^{-\varepsilon}A_{Ndt}^{\varepsilon-1}\right)} = 1$, I must get that the numerator must be smaller than some constant times m. This implies that $\left(\frac{p_{NGt}}{p_{NHt}}\frac{A_{NGt}(\tau_{Nt})}{A_{NHt}}\right)^{\frac{1-\alpha}{\alpha-\beta}}$ is bounded above and using (48) and (49) that Y_{NHt}/Y_{NGt} is greater than some constant times the inverse of m. Now, I can also write (for $M_{Gt}\leq 0$ which I have assumed),

$$\frac{p_{NGt}Y_{NGt}}{p_{NHt}Y_{NHt}} = \frac{\nu}{1-\nu}\left(\frac{Y_{NGt}+M_{Gt}}{Y_{NHt}+M_{Ht}}\right)^{-\frac{1}{\sigma}}\frac{Y_{NGt}}{Y_{NHt}} \geq \frac{\nu}{1-\nu}\left(\frac{Y_{NHt}}{Y_{NGt}}\right)^{\frac{1-\sigma}{\sigma}},$$

which must then be greater than some constant times the inverse of $m^{\frac{1-\sigma}{\sigma}}$, clearly that contradicts the assumption that $\frac{p_{NGt}Y_{NGt}}{p_{NHt}Y_{NHt}}$ is not bounded below.

Therefore $\frac{p_{NGt}Y_{NGt}}{p_{NHt}Y_{NHt}}$ is bounded below, I get a contradiction in all cases and A_{Nct} must grow exponentially.

Part 5 full specialization must be reached in finite time As A_{Nct} grows exponentially (a positive mass of scientists are asymptotically allocated to clean intermediates in the North), then it must be the case that A_{NHt}/A_{SHt} tends towards zero. Therefore, $\frac{\widetilde{K_S}}{\widetilde{L_S}}\frac{\widetilde{L_N}}{\widetilde{K_N}}$ must tend towards 0, and (since $\delta_S = 0$ and $\delta_N \to 0$), (53) -with $X = S$- cannot be satisfied, so that asymptotically, one country at least has to specialize and the North has the comparative advantage in producing good G. (57) must be satisfied in finite time (with $X = N$, as $\tau_S = 0$ and $A_{Nct}/A_{Ndt} \to 0$ and $\delta_{Nt} \to 0$), if at some point condition (58) becomes satisfied, than without a trade tax the economy would feature full specialization, and I can apply the same logic as in case $\eta \leq 1$, to conclude that a non null tariff would necessarily be welfare reducing.

Let us then assume that (58) is never satisfied (which requires that A_{Nct} does not grow faster than $A_{NHt}^{1-\sigma}$). This corresponds to a case where without the trade tax, the North would fully specialize in G, while the South would produce both goods. Now if indeed the North were to permanently specialize in G, all innovation in the North would be directed to clean innovation and (58) will end up being satisfied. To ensure that (58) is not satisfied, a trade tax must be imposed in an infinite number of periods. A trade tax is not optimal when the South is specialized and not at the corner of specialization, so that $p_t \frac{A_{SGt}}{A_{SHt}}$ must be bounded below, and as shown in part 2, above too. Moreover, since A_{SGt} does not grow exponentially, p_t must grow at the same rate as A_{SHt}. Knowing that A_{NGt} grows exponentially, to avoid that $p_t (1 + b_t) \frac{A_{NGt}}{A_{NHt}}$ becomes unbounded and that the North specializes in sector G at one point, the trade tax b_t must be negative. I now show that such a tariff cannot be optimal. First note that a negative tariff increases p_t relative to the free trade case, and therefore cannot decrease pollution in the current period (if t is sufficiently large that the North's emissions are negligible). Moreover, world consumption with a trade tax is necessarily smaller than without at given technological levels, that is:

$$C_{Wt|b \neq 0} < \left(\nu \left(\zeta A_{NGt} K_N^{\alpha} L_N^{1-\alpha} + \zeta A_{SGt} K_{SGt}^{\alpha} L_{SGt}^{1-\alpha} \right)^{\frac{\sigma-1}{\sigma}} + (1 - \nu) \left(\zeta A_{SHt} K_{SHt}^{\beta} L_{SHt}^{1-\beta} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} .$$

Now consider the alternative allocation where there is free trade and all research in the North is allocated to clean intermediates. Note that a higher A_{NGt} (and lower A_{NHt}) reduces p_t further (and p_t is already reduced by the removal of a negative b_t), so that environmental quality is reduced (since the North does not pollute any more), A_{SHt} increases and A_{SGt} decreases. Now since A_{NGt} grows exponentially while A_{SGt} does not, the increase in A_{NGt} more then compensate for the decrease in A_{SGt}. Since A_{SGt} is lower, p_t is lower and A_{SHt} is larger emissions in the South are always lower in all subsequent periods. Therefore world consumption is larger every period under the alternative path and the quality of the environment weakly higher, as a consequence this case with a negative b_t is never optimal and (58) must be satisfied in finite time.

The economy must then move towards full specialization, so that the tariff becomes useless, environmental quality recovers as emissions are negligible in both countries and the tax becomes null.