Arbitrageurs, bubbles and credit conditions

Julien Hugonnier (SFI @ EPFL) and Rodolfo Prieto (BU)

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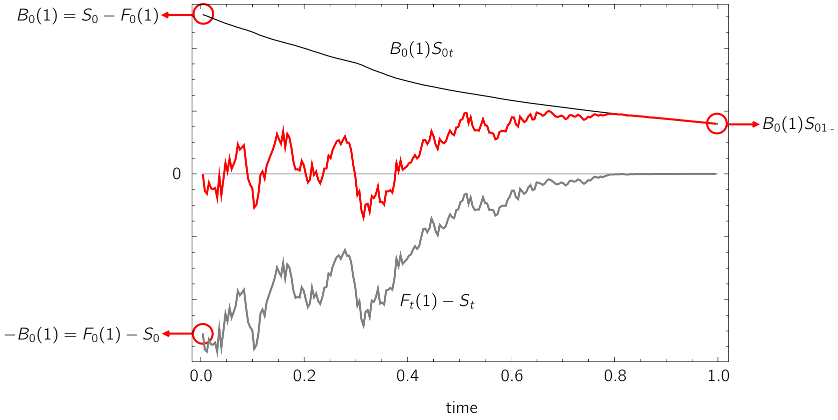
Motivation

- Loewenstein and Willard (2000): Bubbles may exist on zero net supply securities in frictionless finite horizon continuous-trade economies, but not on positive net supply securities (stocks).
- Hugonnier (2011): Portfolio constraints may give rise to bubbles on stocks even if there are unconstrained investors (subject to a std. wealth constraint).
  - Unconstrained agents face an implicit liquidity provision constraint due to the presence of constrained agents.
  - Bubbles arise to incite agents to hold positions that are compatible with market clearing (portfolio imbalances/asset supply shortages) ($W_2 > F(c_2)$).
  - For example, financial assets in the limited participation model with log agents of Basak and Cuoco (1998) contain bubbles.

Motivation (cont'd)

- A (rational) bubble is defined in the traditional way: the difference between the market price of a security and the lowest cost (self-financing) portfolio that produces the same cash-flows with pathwise nonnegative wealth.
- An arbitrage strategy that exploits a bubble usually involves short selling the higher cost security and buying the lower cost replicating portfolio.
- This type of strategy requires no initial wealth and provides positive payoffs, but may not be feasible at all scales due to wealth constraints.

Limited arbitrage opportunity



This paper

- A stylized exchange economy:
  - Three types of agents with homogeneous (log) preferences and beliefs.
  - One group faces portfolio constraints.
  - Key departure: One group (arbitrageurs) faces better credit conditions.
- Equilibrium models with bubbles provide a good laboratory to study questions such as:
  - How credit conditions (wealth constraints) map into prices?.
  - What is the impact of risky arbitrage activity on prices?.
  - Who bears the costs of deflating bubbles?.

Main findings

1. Bubble size depends (negatively) on credit conditions (arbitrageurs bring prices closer to fundamentals).
2. Risky arbitrage trading amplifies fundamental shocks.
3. Price effects with log agents: P/D ratio goes down when volatility goes up (leverage effect) $(S(\delta, s), \sigma(s))$.

Outline

- Related literature
- Model
- Equilibrium with arbitrage activity
- Analysis

Related literature

- Equilibrium models with portfolio constraints (nonnegative wealth constraint):
  - Homogeneous agents (log preferences→flat vol): Detemple and Murthy (1997), Basak and Cuoco (1998), Basak and Croitoru (2000), Pavlova and Rigobon (2007).
  - Heterogeneous agents: Kogan and Uppal (2001), Shapiro (2002), Wu (2004), Kogan, Makarov and Uppal (2007), Gallmeyer and Hollifield (2008), Gârleanu and Pedersen (2011), Chabakauri (2011).

Related literature

- Bubbles:
  - Santos and Woodford (1997): fundamental value as the lowest cost replicating portfolio.
  - Loewenstein and Willard (2000a, 2000b), Heston et al. (2007): bubbles in zero net supply assets, multiple solutions for the fundamental PDE.
  - Jarrow et al. (2010, 2011, 2012): partial equilibrium results.
  - Hugonnier (2011), Prieto (2011): constrained and unconstrained agents.
- Role of arbitrageurs:
  - Gromb and Vayanos (2002): segmented mkts., arbitrageur may impact positively constrained agents.
  - Basak and Croitoru (2000, 2006): all agents constrained, bounded riskless arbitrage, impact on interest rates only.

1. Riskless asset is in zero net supply

$$S_{0t} = 1 + \int_0^t S_{0u} r_u du.$$

2. Stock is in positive supply (one share) and evolves according to

$$S_t = S_0 + \int_0^t S_u \left((r_u + \sigma_u \theta_u) du + \sigma_u dZ_u \right) - \int_0^t \delta_u du,$$

where exogenous dividends follow a geometric Brownian motion

$$\delta_t = \delta_0 + \int_0^t \delta_u \left(\mu_\delta du + \sigma_\delta dZ_u \right).$$

and $\delta_0, \mu_\delta, \sigma_\delta > 0$.

- Initial stock price S_0, interest rate r, volatility σ and market price of risk θ are endogenously determined.

Agents

- Preferences of agent $k = 1, 2, 3$

$$U_k(c) \equiv E\left[\int_0^\infty e^{-\rho t}\log(c_{kt})dt\right].$$

- Finite horizon model gives similar results.
- Portfolio strategies

$$\begin{aligned} W_{kt} &= \underbrace{\phi_{kt}}_{\text{money market}} + \underbrace{\pi_{kt}}_{\text{stock}} \\ &= w_k + \int_0^t (W_{ku}r_u - c_{ku})\,du + \int_0^t \pi_{ku}\sigma_u(dZ_u + \theta_u du). \end{aligned}$$

Agents (cont'd)

k	Agent	Initial wealth w_k	Wealth constraint	Portfolio constraint
1	Unconstrained	$(1-\alpha)S_0 - \beta$	$W_{1t} \geq 0$	No
2	Constrained	$\alpha S_0 + \beta$	$W_{2t} \geq 0$	$\lvert\pi_2 \sigma_t\rvert \leq (1-\varepsilon)\sigma_\delta W_{2t}$
3	Arbitrageurs	0	$W_{3t} \geq -\psi S_t$	No

- Parameter $\varepsilon \in [0, 1]$ controls the severity of the portfolio constraint.
- Credit conditions are controlled by the parameter $\psi \geq 0$.
- Credit improves in good times (when stock is high) and dries up in bad times.
- The model with a single arbitrageur of type ψ is equivalent to a model with N arbitrageurs of type ψ_n, with $\psi = \sum_{n=1}^{N} \psi_n$.

Price components

- Consider the process

$$\xi_t = \frac{1}{S_{0t}} \exp\left[-\frac{1}{2}\int_0^t \theta_u^2 du - \int_0^t \theta_u dZ_u\right].$$

- ξ_t can be used as a pricing kernel by agents 1 and 3, even though there may not be a 'risk neutral probability measure' ($\xi_t S_{0t}$ may not be a density).
- Given a security with cash process $c \geq 0$ over $[0, \tau]$,

$$F_{ct}(\tau) \equiv E_t\left[\int_t^\tau \frac{\xi_u}{\xi_t} c_u du\right]$$

is the minimal amount that unconstrained agents need to hold at time t to replicate the cash flows of this security while maintaining nonnegative wealth.

Price components (cont'd)

Cash flows	Fundamental value	Bubble
Stock over $[t, \infty)$	$F_t \equiv E_t \left[\int_t^{\infty} \frac{\xi_u}{\xi_t} \delta_u du \right]$	$B_t \equiv S_t - F_t$
Stock over $[t, \tau]$	$F_t(\tau) \equiv E_t \left[\int_t^{\tau} \frac{\xi_u}{\xi_t} \delta_u du + \frac{\xi_\tau}{\xi_t} S_\tau \right]$	$B_t(\tau) \equiv S_t - F_t(\tau)$
Money mkt. over $[t, \tau]$	$F_{0t}(\tau) \equiv E_t \left[\frac{\xi_\tau}{\xi_t} S_{0\tau} \right]$	$B_{0t}(\tau) \equiv S_{0t} - F_{0t}(\tau)$

Why bubbles persist in equilibrium

- Assume the stock has a bubble.
- Consider the following arbitrage strategy $W_t^{\text{arbitrage}}$ in $[0, 1]$.
  - Sell short 1 unit of the stock, $+S_0$,
  - Buy the replicating portfolio, $-F_0(1)$,
  - Invest $S_0 - F_0(1) = B_0(1) > 0$, in the money market account.

Why bubbles persist in equilibrium (cont'd)

- The wealth process associated with this arbitrage strategy is given by

$$
\begin{aligned}
W_0^{\text{arbitrage}} &= 0, \\
W_t^{\text{arbitrage}} &= F_t(1) - S_t + B_0(1)S_{0t} = -B_t(1) + B_0(1)S_{0t}, \\
W_1^{\text{arbitrage}} &= B_0(1)S_{01}.
\end{aligned}
$$

- This strategy is not admissible for agent 1. But:
  - It can be implemented with a sufficiently large 'collateral' position.
  - For example,

    $$S_t + W_t^{\text{arbitrage}}$$

    is nonnegative at all times (→the stock is a dominated asset).
- $W_t^{\text{arbitrage}}$ might be admissible for arbitrageurs ($\psi \geq 1$).

Feasible consumption plans

- (Static budget constraint) A consumption plan c is feasible iff:

  Unconstrained (1):

$$E\left[\int_0^\infty \xi_t c_{1t} dt\right] \le w_1 \equiv (1-\alpha)S_0 - \beta.$$

  Constrained (2):

$$E\left[\int_0^\infty \xi_{2t} c_{2t} dt\right] \le w_2 \equiv \alpha S_0 + \beta.$$

  Arbitrageur (3):

$$E\left[\int_0^\infty \xi_t c_{3t} dt\right] \le \underbrace{\psi(S_0 - F_0)}_{\text{PV arbitrage profits given wealth constraint}}.$$

Equilibrium

- An equilibrium is a pair of security price processes (S_0, S) and a set $\{c_k, (\phi_k, \pi_k)\}_{k=1}^3$ of consumption plans and trading strategies such that:
  - Given (S_0, S) the consumption plan c_k maximizes U_k over the feasible set of agent k and is financed by the trading strategy (ϕ_k, π_k).
  - Markets clear:

$$\begin{aligned} &\text{Money mkt.} \quad && \phi_1 + \phi_2 + \phi_3 = 0, \\ &\text{Stock mkt.} \quad && \pi_1 + \pi_2 + \pi_3 = S, \\ &\text{Good mkt.} \quad && c_1 + c_2 + c_3 = \delta. \end{aligned}$$

- An equilibrium is said to have arbitrage activity if the consumption plan of the arbitrageur is not identically zero.
- Arbitrage activity $\Leftrightarrow$ bubble.

Individual optimality

- Assume $B \neq 0$.
- Consumption plans:
$$c_{kt} = \rho \left(W_{kt} + 1_{\{k=3\}} \psi B_t \right).$$
- Trading strategies:
$$\pi_{1t} = (\theta_t / \sigma_t) W_{1t},$$
$$\pi_{2t} = (\theta_t / \sigma_t) \kappa_t W_{2t},$$
$$\pi_{3t} = (\theta_t / \sigma_t)(W_{3t} + \psi B_t) - \psi(\Sigma_t^B / \sigma_t),$$

where
$$\kappa_t = \min \left(1; \frac{(1-\varepsilon)\sigma_\delta}{|\theta_t|} \right) \quad \in [0, 1],$$
and Σ^B denotes the diffusion coefficient of the process B.

Individual optimality (cont'd)

- The arbitrageur (3) holds:
  - A mean-variance efficient position over his effective wealth, $W_{3t} + \psi B_t$.
  - A position proportional to the credit conditions, whose sign depends on the correlation of the stock price with B_t.
- The arbitrageur's wealth is given by

$$W_{3t} = \underbrace{E_t \left[\int_t^{\infty} \frac{\xi_u}{\xi_t} c_{3u} du \right]}_{\frac{c_{3t}}{\rho}} - \psi B_t.$$

- Source of price level effects.

Representative agent construction

- The utility function of the representative agent is defined by

$$u(c, \gamma, \lambda_t) \equiv \max_{c_1+c_2+c_3=c} \{\log(c_1) + \lambda_t \log(c_2) + \gamma \log(c_3)\}.$$

- $\lambda_t = c_{2t}/c_{1t}$, encapsulates the differences across investment opportunity sets.
- $\gamma = c_{3t}/c_{1t}$, determines the relative weight of arbitrageurs in the economy.
- The state price density of unconstrained agents is thus given by

$$\xi_t = e^{-\rho t} \frac{u_c(\delta_t, \gamma, \lambda_t)}{u_c(\delta_0, \gamma, \lambda_0)} = e^{-\rho t} \frac{\delta_0(1 + \gamma + \lambda_t)}{\delta_t(1 + \gamma + \lambda_0)}.$$

Consumption plans

- Consumption plans

$$
\begin{aligned}
c_{1t} &= \frac{1}{1+\gamma}(1-s_t)\delta_t, \\
c_{2t} &= s_t\delta_t, \\
c_{3t} &= \frac{\gamma}{1+\gamma}(1-s_t)\delta_t.
\end{aligned}
$$

- The consumption share of the constrained agent is given by

$$
s_t \equiv \frac{c_{2t}}{\delta_t} = \frac{\lambda_t}{1+\gamma+\lambda_t} \in (0,1).
$$

- It evolves according to

$$
\begin{aligned}
ds_t &= -s_t\varepsilon\sigma_\delta\left(dZ_t + \frac{s_t}{1-s_t}\varepsilon\sigma_\delta dt\right), \\
s_0 &= \rho(\alpha S_0+\beta)/\delta_0.
\end{aligned}
$$

Market price of risk and interest rate

- In equilibrium:
- The riskless rate of interest and the market price of risk are given by

$$r_t = \rho + \mu_\delta - \sigma_\delta^2 \left(1 + \frac{\varepsilon s_t}{1 - s_t}\right),$$

$$\theta_t = \sigma_\delta \left(1 + \frac{\varepsilon s_t}{1 - s_t}\right).$$

- Note:
  - $\varepsilon = 0 \rightarrow$ unconstrained equilibrium.
  - $\varepsilon = 1 \rightarrow$Basak and Cuoco (1998)+arbitrageurs.

Prices

- There exists a unique equilibrium (a solution for s_0, γ), iff

$$-\frac{\alpha\delta_0}{\rho} < \beta < \frac{\delta_0(1-\alpha)}{\rho(1+\psi)}.$$

- Prices are given by

$$S_t = \frac{\delta_t}{\rho}\left(1 - \frac{\psi}{1+\psi} s_t^{\eta}\right),$$

$$S_{0t} = e^{\left(\rho - \sigma_\delta^2/2(1-\varepsilon)\right)t} \frac{\delta_t s_t^{1/\varepsilon}}{\delta_0 s_0^{1/\varepsilon}},$$

where $\eta \equiv \frac{1}{2}\left(1 + \sqrt{1 + 8\rho/(\varepsilon\sigma_\delta)^2}\right)$.

- Both prices contain bubbles.

Properties

- The stock price is a convex combination of its fundamental value and the stock price in the absence of the arbitrageur,

$$S_t = \frac{1}{1+\psi}\frac{\delta_t}{\rho} + \frac{\psi}{1+\psi}F_t.$$

  - As $\psi \to 0$, the stock price converges to the price in an economy where only agents 1 and 2 trade.
  - As $\psi \to \infty$, the stock price converges to its fundamental value.
- Collateral services provided by the stock need not be as high in the presence of arbitrageurs.
- The volatility of the stock is given by

$$\sigma_t = \sigma_\delta \left(1 + \frac{\psi \eta \varepsilon s_t^{\eta}}{1+\psi(1-s_t^{\eta})}\right).$$

- Price effects with log agents!.

Bubbles

Cash flows	Fundamental value	Bubble
Stock over $[t, \infty)$	$F_t = \frac{\delta_t}{\rho}\left(1 - s_t^\eta\right)$	$B_t = \frac{\delta_t}{\rho}\frac{1}{1+\psi}s_t^\eta$
Stock over $[t, \tau]$	$F_t(\tau) = S_t - B_t(\tau)$	$B_t(\tau) = \frac{\delta_t}{\rho(1+\psi)}H\left(\tau - t, s_t; 2\eta - 1\right)$
Money mkt. over $[t, \tau]$	$F_{0t}(\tau) = S_{0t} - B_{0t}(\tau)$	$B_{0t}(\tau) = S_{0t}s_t^{-1/\varepsilon}H\left(\tau - t, s_t; 2/\varepsilon - 1\right)$

- With:

$$H(\tau, s; \alpha) \equiv s^{\frac{1+\alpha}{2}}\Phi(d_+(\tau, s; \alpha)) + s^{\frac{1-\alpha}{2}}\Phi(d_-(\tau, s; \alpha)),$$

$$d_\pm(\tau, s; \alpha) \equiv \frac{1}{\varepsilon\sigma_\delta\sqrt{\tau}}\log(s) \pm \frac{\alpha}{2}\varepsilon\sigma_\delta\sqrt{\tau}.$$

Properties

- Bubbles: compensation to the unconstrained agents for the implicit liquidity provision constraint due to the presence of constrained agents

$$\begin{aligned} B_t = S_t - F_t &= \frac{1}{1+\psi}\left(W_{2t} - F_t(c_2)\right) \\ &= \frac{\delta_t}{1+\psi}\frac{1}{1+\gamma+\lambda_t}\int_t^\infty e^{-\rho(u-t)}\underbrace{\left(\lambda_t - E_t\left[\lambda_u\right]\right)}_{>0} du, \end{aligned}$$

  where

$$\lambda_t = \lambda_0 - \int_0^t \lambda_u(\theta_u - \kappa_u\theta_u)dZ_u.$$

- The riskless asset bubble is larger than the stock's bubble,

$$\frac{B_{0t}(\tau)}{S_{0t}} \geq \frac{B_t(\tau)}{S_t}, \qquad \text{for } \tau \geq t.$$

Portfolios

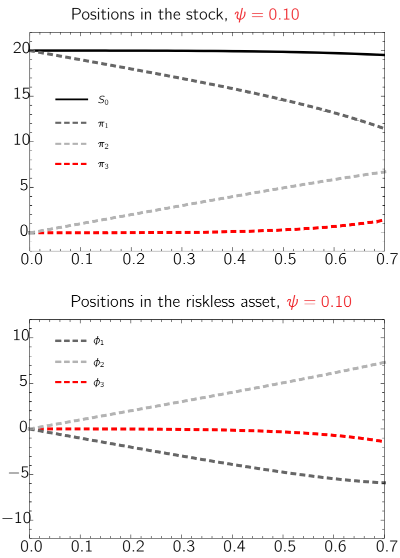
- The unconstrained agent (1) is levered in the stock

$$\pi_{1t}/W_{1t} = \frac{1+\varepsilon\frac{s_{2t}}{1-s_t}}{1+\frac{\psi\eta\varepsilon s_t^{\eta}}{1+\psi(1-s_t^{\eta})}} > 1.$$

- The constrained agent (2) invests in the riskless asset due to the portfolio constraint

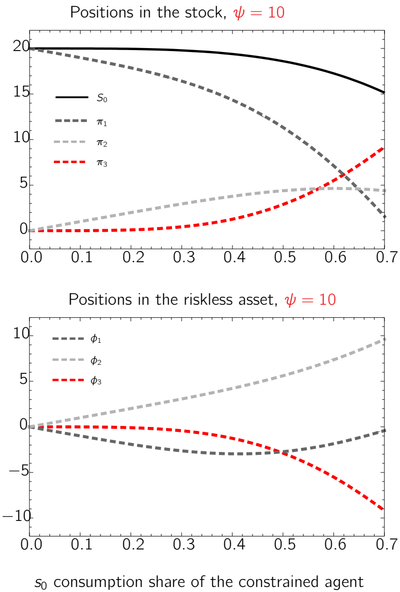
$$\pi_{2t}/W_{2t} = \frac{1-\varepsilon}{1+\frac{\psi\eta\varepsilon s_t^{\eta}}{1+\psi(1-s_t^{\eta})}} \in [0,1].$$

Portfolios $\psi = 0.10$

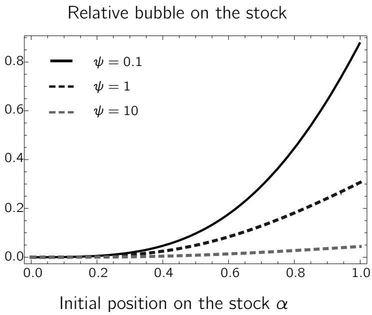


s_0 consumption share of the constrained agent

Portfolios $\psi = 10$



Bubbles decrease in size as $\psi \uparrow$



Understanding agent 1's strategy

- Agent 1 holds positions in the stock and in the riskless asset's bubble

$$W_{1t} = \phi_{1t}^{S}(\tau) + \phi_{1t}^{B_0}(\tau),$$

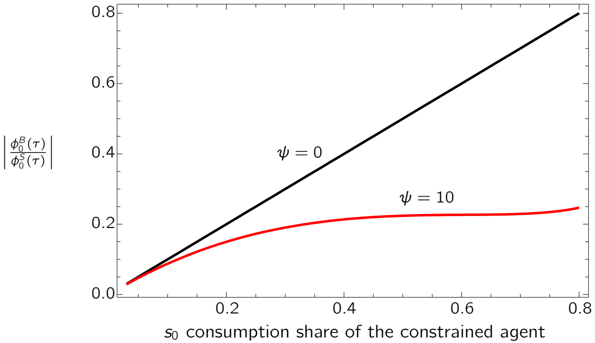
where

$$\phi_{1t}^{S}(\tau) = \frac{\delta_t(1+\psi)\left(\left(\sigma_\delta - \Sigma_{t,\tau}^{0}\right)(1-s_t) + \varepsilon\sigma_\delta s_t\right)}{\rho(\sigma_t - \Sigma_{t,\tau}^{0})(1+\gamma)} > 0,$$

$$\phi_{1t}^{B_0}(\tau) = -\frac{\delta_t\left((\sigma_\delta - \sigma_t)(1-s_t) + \varepsilon\sigma_\delta s_t\right)}{\rho(\sigma_t - \Sigma_{t,\tau}^{0})(1+\gamma)} < 0,$$

and the process $\Sigma_{t,\tau}^{0} < 0$ is the diffusion coefficient of $\log B_{0t}(\tau)$.

Agent 1 positions as $\psi \uparrow$



Welfare

- Risky arbitrage trading may be detrimental to both agents 1 and 2.
- Expected utility:

$$U_1(\psi) = U_0 - \rho^{-1}\log(1+\gamma) + E\left[\int_0^\infty e^{-\rho t}\log(1-s_t)dt\right],$$

$$U_1'(\psi) >< 0.$$

$$U_2(\psi) = U_0 + E\left[\int_0^\infty e^{-\rho t}\log(s_t)dt\right],$$

$$U_2'(\psi) < 0.$$

$$U_3(\psi) = U_0 + \rho^{-1}\log\left(\frac{\gamma}{1+\gamma}\right) + E\left[\int_0^\infty e^{-\rho t}\log(1-s_t)dt\right],$$

$$U_3'(\psi) > 0.$$

Bubble deflating policies - shifting (ε, ψ)

- Relaxation of collateral requirements $\psi \uparrow$
  - Constrained agents (2) are worse off.
  - Higher volatility in the stock market.
- Higher participation in the stock market $\varepsilon \downarrow$
  - Unconstrained agents (1) and arbitrageurs (3) are worse off.
  - Lower volatility in the stock market.

Concluding remarks

- A stylized pure exchange economy with log agents and one friction.
- Main findings:
  - Risky arbitrage trading amplifies fundamental shocks.
  - The leverage effect (with log agents): P/D ratio goes down when volatility goes up.
  - The bubble size depends (negatively) on credit conditions.
- What if we introduce liquidity shocks (jumps) in ψ or δ? (bailouts).

- For an arbitrary consumption and investment plan,

$$\xi_t W_{3t} + \int_0^t \xi_u c_{3u} du = \int_0^t \xi_s \left(\pi_{3u}\sigma_u - W_{3u}\theta_u\right) dZ_u.$$

- The deflated stock and riskless asset price processes satisfy

$$\xi_t S_t + \int_0^t \xi_u \delta_u du = S_0 + \int_0^t \xi_u \left(S_u\sigma_u - S_u\theta_u\right) dZ_u,$$

$$\xi_t S_{0t} = 1 - \int_0^t \xi_u S_{0s}\theta_u dZ_u.$$

- Let N_t be defined by

$$N_t = \xi_t W_{3t} + \psi \xi_t S_t + \ell \xi_t S_{0t} + \int_0^t \xi_u (c_{3u} + \psi \delta_u) du$$
$$= \psi S_0 + \ell + \int_0^t \xi_u \left((\pi_{3u} + \psi S_u)\sigma_u - (\psi S_u + W_{3u} + \ell S_{0u})\theta_u \right) dZ_u$$

with $\psi \geq 0$ and $\ell \geq 0$.

- N_t is a nonnegative local martingale for positive consumption plans, and hence a supermartingale. This implies that

$$E\left[\int_0^T \xi_t (c_{3t} + \psi \delta_t) dt + \ell \xi_T S_{0T}\right] \leq \psi S_0 + \ell.$$