Output contingent securities and efficient investment by firms

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Economic environment

In our paper, we consider a model close to MQ’09 and MQ’10

Here, to simplify the presentation:

- we follow the model and notations of MQR’12
- we assume that labor is supplied inelastically: there are no workers
- we do not separate consumers and capitalists

\[ U(m^i, c^i) = m_0^i + \delta E [m_1^i + u(c_1^i)] \]

- there are \( n \) firms: each firm has
  - two levels of investment \( A = \{a_H, a_L\} \)
  - two levels of productions \( Y = \{y_L, y_H\} \)
Output profiles

- An investment profile is denoted by $a^n = (a_1, \ldots, a_n)$

\[ A^n = \{a_L, a_H\}^n \]

- An output profiles is denoted by $y^n = (y_1, \ldots, y_n)$

\[ Y^n = \{y_L, y_H\}^n \]

- We let $\Pi(\cdot, a^n) : Y^n \to [0, 1]$ denote the output probability distribution where

\[ \Pi(y^n, a^n) \]

 denote the probability of output profile $y^n$ under investment profile $a^n$. 
Aggregate Output

- Aggregate output is represented by

\[ \Sigma = Y + \ldots + Y \]

\( n \)-times

- Output profiles consistent with aggregate output \( \sigma \in \Sigma \)

\[ Y^n(\sigma) = \{ y^n \in Y^n : y_1 + \ldots + y_n = \sigma \} \]

- We let \( \pi_\sigma(a^n) \) be the probability of aggregate output \( \sigma \) under investment \( a^n \), i.e.,

\[ \pi_\sigma(a^n) = \sum_{y^n \in Y^n(\sigma)} \Pi(y^n, a^n) \]
Primitive states of nature

Assume that “exogenous circumstances” are represented by

$$(\Omega, \mathbb{P})$$

which combined with firms’ actions explain production risk through the production functions

$$(\omega, a_k) \mapsto f_k(\omega, a_k) \in Y$$

such that

$$\Pi(y^n, a^n) = \mathbb{P}[\{f^n(\cdot, a^n) = y^n\}] = \mathbb{P}\left[\bigcap_{k=1}^n \{f_k(\cdot, a_k) = y_k\}\right]$$
Contracts contingent on primitive states-of-nature

- Agents can trade all contracts contingent to primitive states of nature
- Agents’ budget set

\[ m^i_0 + \bar{q}^n \theta^i + \sum_{\omega \in \Omega} \bar{k}(\omega) z^i(\omega) \leq e^i_0 + \varphi^i [-\bar{a}^n + \bar{q}^n] \]

\[ m^i_1(\omega) + \bar{p}(\omega) c^i(\omega) \leq e^i_1 + \bar{p}(\omega) f^n(\omega, \bar{a}) \theta^i + z^i(\omega) \]

- Market clearing

\[ \sum_{i \in I} \bar{c}^i(\omega) = f_1(\omega, \bar{a}_1) + \ldots + f_n(\omega, \bar{a}_n) \]

\[ \sum_{i \in I} \bar{m}^i_0 + \sum_{k=1}^{n} \bar{a}_k = \sum_{i \in I} e^i_0 \quad \text{and} \quad \sum_{i \in I} \bar{m}^i_1(\omega) = \sum_{i \in I} e^i_1 \]
Contracts contingent on primitive states-of-nature

- Equivalently, we can write agent $i$’s budget set in its reduced form

\[
m_0^i + \sum_{\omega \in \Omega} \kappa(\omega) z^i(\omega) \leq e^i_0 + \varphi^i [-\bar{a}^n + \bar{q}^n]
\]

and for every primitive state $\omega$

\[
m^i_1(\omega) + \bar{p}(\omega) c^i(\omega) \leq e^i_1 + z^i(\omega)
\]

- Together with the asset pricing equation\(^1\)

\[
\bar{q}_k = \sum_{\omega \in \Omega} \kappa(\omega) \bar{p}(\omega) f_k(\omega, \bar{a}_k)
\]

\(^1\)By quasi-linearity we have $\kappa(\omega) = \delta \mathbb{P}(\omega)$
Contracts contingent on primitive states-of-nature

- If each firm $k$ maximizes Net Present Value

\[ \tilde{a}_k \in \arg\max \left\{ -a + \sum_{\omega \in \Omega} \bar{k}(\omega) \bar{p}(\omega) f_k(\omega, a) : a \in A \right\} \]

Then we get efficiency

- Behavioral assumption: firm $k$ takes prices as given and expects the market to pay the price $\tilde{q}_k(a)$ for its equity under investment $a$ where

\[ \tilde{q}_k(a) = \sum_{\omega \in \Omega} \bar{k}(\omega) \bar{p}(\omega) f_k(\omega, a) \]

in other words, it maximizes

\[ \text{NPV}_k(a) = -a + \sum_{\omega \in \Omega} \bar{k}(\omega) \bar{p}(\omega) f_k(\omega, a) \]
Contracts contingent on observable output

- Markets based on primitive states $\omega$ are difficult to operate
  - Writing contracts: ex-ante, it would be “too costly” to specify precisely in a contract all the primitive states of nature
  - Enforcing contracts: ex-post, it is difficult to verify a primitive state of nature
- Modern financial markets trade assets whose payoffs depend on the observable profits of the firms
  - bonds, equities, derivatives and indexes

Assumption

Agents can trade equities and all contracts contingent on aggregate output
Contracts contingent on aggregate output

- Agents’ budget set

\[ m_i^0 + \sum_{\sigma \in \Sigma} \kappa_\sigma z_\sigma^i \leq e_i^0 + \phi_i^i [-\bar{a}^n + \bar{q}^n] \]

and for every aggregate output \( \sigma \)

\[ m_i^\sigma + \bar{p}_\sigma c_\sigma^i \leq e_1^i + z_\sigma^i \]

- Together with the asset pricing equation

\[ \bar{q}_k = \sum_{\sigma \in \Sigma} \sum_{y^n \in Y^n(\sigma)} \kappa_\sigma \bar{p}_\sigma y_k \]

- By quasi-linearity we have

\[ \kappa_\sigma = \delta \pi_\sigma(\bar{a}^n) = \delta \sum_{y^n \in Y^n(\sigma)} \Pi(y^n, \bar{a}^n) \]
Contracts contingent on output: which objective?

- What is the “correct” objective that firms should maximize?
- We have incomplete markets, how should firm $k$ value the out-of-equilibrium output stream $\omega \mapsto f_k(\omega, a)$?
- What are the “correct” discount factors?
- Do market prices provide enough information to coordinate agents’ decisions?
The Shareholder Value

- Let $\bar{p}(y^n) = \bar{p}_\sigma$ for $\sigma = y_1 + \ldots + y_n$
- Recall that, at equilibrium, firm $k$’s equity satisfies
  \[
  \bar{q}_k = \delta \sum_{y^n \in Y^n} \Pi(y^n, \bar{a}^n)\bar{p}(y^n)y_k
  \]
- One may consider the Shareholder Value
  \[
  SV_k(a_k) = -a_k + \delta \sum_{y^n \in Y^n} \Pi(y^n, a_k, \bar{a}_k)\bar{p}(y^n)y_k
  \]
- Maximizing this Shareholder Value may not lead to efficiency

What causes inefficiency?

- Market incompleteness (output contingent securities)
- Non-competitive behavior of firm $k$ which
  - takes the good prices $\bar{p}_\sigma$ as given
  - but fully considers the impact of new investments on the probability of $y^n$ and therefore on the probability of aggregate output $\sigma = y_1 + \ldots + y_n$
Asset pricing ... our contribution

\[ \bar{q}_k = \delta \sum_{y^n \in Y^n} \Pi(y^n, \bar{a}^n) \bar{p}(y^n) y_k \]

\[ = \delta \sum_{\sigma \in \Sigma} \bar{p}_\sigma \sum_{y^n \in Y^n(\sigma)} \Pi(y^n, \bar{a}^n) y_k \]

\[ = \sum_{\sigma \in \Sigma} \bar{\kappa}_\sigma \bar{p}_\sigma \sum_{y^n \in Y^n(\sigma)} \frac{\Pi(y^n, \bar{a}^n)}{\pi_\sigma(\bar{a})} y_k \]

\[ = \sum_{\sigma \in \Sigma} \bar{\kappa}_\sigma \bar{p}_\sigma \sum_{y^n \in Y^n} \Pi(y^n|\sigma) y_k \]

\[ = \sum_{\sigma \in \Sigma} \bar{\kappa}_\sigma \bar{p}_\sigma \sum_{y_k \in Y} \Pi_k(y_k|\sigma) y_k \]

\[ \Pi(y^n|\sigma) = \frac{\Pi(y^n, \bar{a}^n)}{\pi_\sigma(\bar{a}^n)} \text{ if } y_1 + \ldots + y_n = \sigma \text{ and } 0 \text{ elsewhere} \]

\[ \Pi_k(y_k|\sigma) = \sum_{y^{-k} \in Y^{-k}} \Pi(y^n|\sigma) \]
Asset pricing ... our contribution

\[ \bar{q}_k = \delta \sum_{y^n \in Y^n} \Pi(y^n, \bar{a}^n) \bar{p}(y^n)y_k \]

\[ = \delta \sum_{\sigma \in \Sigma} \bar{p}_\sigma \sum_{y^n \in Y^n(\sigma)} \Pi(y^n, \bar{a}^n)y_k \]

\[ = \sum_{\sigma \in \Sigma} \bar{\kappa}_\sigma \bar{p}_\sigma \sum_{y^n \in Y^n(\sigma)} \frac{\Pi(y^n, \bar{a}^n)}{\pi_\sigma(\bar{a})}y_k \]

\[ = \sum_{\sigma \in \Sigma} \bar{\kappa}_\sigma \bar{p}_\sigma \sum_{y^n \in Y^n} \bar{\Pi}(y^n | \sigma)y_k \]

\[ = \sum_{\sigma \in \Sigma} \bar{\kappa}_\sigma \bar{p}_\sigma \sum_{y_k \in Y} \bar{\Pi}_k(y_k | \sigma)y_k \]

\[ \bar{\Pi}(y^n | \sigma) = \frac{\Pi(y^n, \bar{a}^n)}{\pi_\sigma(\bar{a}^n)} \text{ if } y_1 + \ldots + y_n = \sigma \text{ and } 0 \text{ elsewhere} \]

\[ \bar{\Pi}_k(y_k | \sigma) = \sum_{y^{n-k} \in Y^{n-k}} \bar{\Pi}(y^n | \sigma) \]
Asset pricing ... our contribution

\[
\bar{q}_k = \delta \sum_{y^n \in Y^n} \Pi(y^n, \bar{a}^n) \bar{p}(y^n) y_k \\
= \delta \sum_{\sigma \in \Sigma} \bar{p}_{\sigma} \sum_{y^n \in Y^n(\sigma)} \Pi(y^n, \bar{a}^n) y_k \\
= \sum_{\sigma \in \Sigma} \bar{\kappa}_{\sigma} \bar{p}_{\sigma} \sum_{y^n \in Y^n(\sigma)} \frac{\Pi(y^n, \bar{a}^n)}{\pi_{\sigma}(\bar{a})} y_k \\
= \sum_{\sigma \in \Sigma} \bar{\kappa}_{\sigma} \bar{p}_{\sigma} \sum_{y^n \in Y^n} \bar{\Pi}(y^n | \sigma) y_k \\
= \sum_{\sigma \in \Sigma} \bar{\kappa}_{\sigma} \bar{p}_{\sigma} \sum_{y_k \in Y} \bar{\Pi}_k(y_k | \sigma) y_k
\]

\[
\bar{\Pi}(y^n | \sigma) = \Pi(y^n, \bar{a}^n) / \pi_{\sigma}(\bar{a}^n) \text{ if } y_1 + \ldots + y_n = \sigma \text{ and } 0 \text{ elsewhere} \\
\bar{\Pi}_k(y_k | \sigma) \equiv \sum_{y^{\neg k} \in Y^{\neg k}} \bar{\Pi}(y^n | \sigma)
\]
Asset pricing ... our contribution

\[
\bar{q}_k = \delta \sum_{y^n \in Y^n} \Pi(y^n, \bar{a}^n) \bar{p}(y^n) y_k = \delta \sum_{\sigma \in \Sigma} \bar{p}_\sigma \sum_{y^n \in Y^n(\sigma)} \Pi(y^n, \bar{a}^n) y_k
\]

\[
= \sum_{\sigma \in \Sigma} \bar{K}_\sigma \bar{p}_\sigma \sum_{y^n \in Y^n(\sigma)} \frac{\Pi(y^n, \bar{a}^n)}{\pi_\sigma(\bar{a})} y_k
\]

\[
= \sum_{\sigma \in \Sigma} \bar{K}_\sigma \bar{p}_\sigma \sum_{y^n \in Y^n} \bar{\Pi}(y^n | \sigma) y_k
\]

\[
= \sum_{\sigma \in \Sigma} \bar{K}_\sigma \bar{p}_\sigma \sum_{y_k \in Y} \bar{\Pi}_k(y_k | \sigma) y_k
\]

\[
\bar{\Pi}(y^n | \sigma) = \Pi(y^n, \bar{a}^n)/\pi_\sigma(\bar{a}^n) \text{ if } y_1 + \ldots + y_n = \sigma \text{ and } 0 \text{ elsewhere}
\]

\[
\bar{\Pi}_k(y_k | \sigma) = \sum_{y^{-k} \in Y^{-k}} \bar{\Pi}(y^n | \sigma)
\]
Asset pricing ... our contribution

\[
\bar{q}_k = \delta \sum_{y^n \in Y^n} \Pi(y^n, \bar{a}^n) \bar{p}(y^n) y_k \\
= \delta \sum_{\sigma \in \Sigma} \bar{p}_\sigma \sum_{y^n \in Y^n(\sigma)} \Pi(y^n, \bar{a}^n) y_k \\
= \sum_{\sigma \in \Sigma} \bar{\kappa}_\sigma \bar{p}_\sigma \sum_{y^n \in Y^n(\sigma)} \Pi(y^n, \bar{a}^n) \frac{y_k}{\pi_\sigma(\bar{a})} \\
= \sum_{\sigma \in \Sigma} \bar{\kappa}_\sigma \bar{p}_\sigma \sum_{y^n \in Y^n} \Pi(y^n | \sigma) y_k \\
= \sum_{\sigma \in \Sigma} \bar{\kappa}_\sigma \bar{p}_\sigma \sum_{y_k \in Y} \Pi_k(y_k | \sigma) y_k
\]

\[
\Pi(y^n | \sigma) = \Pi(y^n, \bar{a}^n)/\pi_\sigma(\bar{a}^n) \text{ if } y_1 + \ldots + y_n = \sigma \text{ and 0 elsewhere} \\
\Pi_k(y_k | \sigma) \equiv \sum_{y^{\neg k} \in Y^{\neg k}} \Pi(y^n | \sigma)
\]
Our contribution

- At equilibrium, the market value of firm $k$ can be expressed as follows

$$
\bar{V}_k = -\bar{a}_k + \sum_{\sigma \in \Sigma} \bar{\kappa}_{\sigma} \bar{p}_{\sigma} \sum_{y_k \in Y} \Pi_k(y_k|\sigma)y_k
$$

- Every shareholder of firm $k$ would like that firm $k$ chose $\bar{a}_k$ to maximize

$$
\tilde{V}_k(a_k) = -a_k + \tilde{q}_k(a_k)
$$

where $\tilde{q}_k(a_k)$ is agents’ perceptions about the way the new investment decision $a_k$ would affect the price that the “market” will pay for the equity
Standard behavioral assumption: Price-taking

- Perceptions should be correct at equilibrium

\[ \tilde{q}_k(\bar{a}_k) = \bar{q}_k = \sum_{\sigma \in \Sigma} \bar{\kappa}_\sigma \bar{p}_\sigma \sum_{y_k \in Y} \bar{\Pi}_k(y_k|\sigma) y_k \]

- Price-taking assumption: agents conceive that the investment decision of a single firm does not affect prices

- This leads to the following expression

\[ \tilde{q}_k(a_k) = \sum_{\sigma \in \Sigma} \bar{\kappa}_\sigma \bar{p}_\sigma \sum_{y_k \in Y} \bar{\tilde{\Pi}}_k(y_k, a_k|\sigma) y_k \]

where \( \bar{\tilde{\Pi}}_k(y_k, a_k|\sigma) \) is the probability (perceived by agents) that firm \( k \)'s output is \( y_k \) when it chooses the investment \( a_k \) conditional to the aggregate output \( \sigma \)
Our behavioral assumption: Aggregate output taking

Agents have *competitive beliefs*

They are convinced that a change in firm $k$’s investment will not affect the likelihood of each aggregate output $\sigma$, i.e.,

$$\tilde{\Pi}_k(y_k, a_k | \sigma) = \mathbb{P}(\{f_k(a_k) = y_k\} | \{g(\bar{a}) = \sigma\})$$

where $g(\bar{a}) = f_1(\bar{a}) + \ldots + f_n(\bar{a})$

**Remark**

In our opinion, this assumption is consistent with the price-taking assumption: agents take prices for aggregate output as given
Theorem

If each firm $k$ chooses its investment maximizing the “competitive” market value

$$V_k(a_k) = -a_k + \sum_{\sigma \in \Sigma} \bar{k}_\sigma \bar{p}_\sigma \sum_{y_k \in Y} \tilde{\Pi}_k(y_k, a_k | \sigma) y_k$$

then the corresponding equilibrium is efficient
Competitive firms

- As the traditional general equilibrium literature, we assume that firms are price takers and do not seek to manipulate prices.
- Contrary to MQ’09, MQ’10 and MQR’12, we assume that firms take aggregate production as given: each firm believes that a unilateral deviation has little influence on aggregate output.
- Under these two assumptions, we have proved that profit-maximization (when profit is correctly computed) is socially well justified.
- Our result shows that the stakeholder theory is not founded by the mere presence of output-contingent contracts.
- An additional feature (e.g., a non-competitive production sector) is needed to reject the profit-maximization criterium.