Output contingent securities and efficient investment by firms

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Economic environment

In our paper, we consider a model close to MQ'09 and MQ'10

Here, to simplify the presentation:

- we follow the model and notations of MQR'12
- we assume that labor is supplied inelastically: there are no workers
- we do not separate consumers and capitalists

$$U(m^i, c^i) = m_0^i + \delta \mathbb{E}\left[m_1^i + u(c_1^i)\right]$$

- there are n firms: each firm has
  - two levels of investment $A = \{a_{\mathrm{H}}, a_{\mathrm{L}}\}$
  - two levels of productions $Y = \{y_{\mathrm{L}}, y_{\mathrm{H}}\}$

Output profiles

- An investment profile is denoted by $a^n = (a_1, \dots, a_n)$

$$A^n = \{a_{\text{L}}, a_{\text{H}}\}^n$$

- An output profiles is denoted by $y^n = (y_1, \dots, y_n)$

$$Y^n = \{y_{\text{L}}, y_{\text{H}}\}^n$$

- We let $\Pi(\cdot, a^n) : Y^n \to [0, 1]$ denote the output probability distribution where

$$\Pi(y^n, a^n)$$

denote the probability of output profile y^n under investment profile a^n

Aggregate Output

- Aggregate output is represented by

$$\Sigma = \underbrace{Y + \ldots + Y}_{n\text{-times}}$$

- Output profiles consistent with aggregate output $\sigma \in \Sigma$

$$Y^n(\sigma) = \{y^n \in Y^n \ : \ y_1 + \ldots + y_n = \sigma\}$$

- We let $\pi_\sigma(a^n)$ be the probability of aggregate output σ under investment a^n, i.e.,

$$\pi_\sigma(a^n) = \sum_{y^n \in Y^n(\sigma)} \Pi(y^n, a^n)$$

Primitive states of nature

Assume that "exogenous circumstances" are represented by

$$(\Omega, \mathbb{P})$$

which combined with firms' actions explain production risk through the production functions

$$(\omega, a_k) \longmapsto f_k(\omega, a_k) \in Y$$

such that

$$\Pi(y^n, a^n) = \mathbb{P}\left[\{f^n(\cdot, a^n) = y^n\}\right] = \mathbb{P}\left[\bigcap_{k=1}^{n} \{f_k(\cdot, a_k) = y_k\}\right]$$

Contracts contingent on primitive states-of-nature

- Agents can trade all contracts contingent to primitive states of nature
- Agents' budget set

$$m_0^i + \bar{q}^n \theta^i + \sum_{\omega \in \Omega} \bar{\kappa}(\omega) z^i(\omega) \leqslant e_0^i + \varphi^i[-\bar{a}^n + \bar{q}^n]$$

$$m_1^i(\omega) + \bar{p}(\omega) c^i(\omega) \leqslant e_1^i + \bar{p}(\omega) f^n(\omega, \bar{a}) \theta^i + z^i(\omega)$$

- Market clearing

$$\sum_{i \in I} \bar{c}^i(\omega) = f_1(\omega, \bar{a}_1) + \ldots + f_n(\omega, \bar{a}_n)$$

$$\sum_{i \in I} \bar{m}_0^i + \sum_{k=1}^{n} \bar{a}_k = \sum_{i \in I} e_0^i \quad \text{and} \quad \sum_{i \in I} \bar{m}_1^i(\omega) = \sum_{i \in I} e_1^i$$

Contracts contingent on primitive states-of-nature

- Equivalently, we can write agent i's budget set in its reduced form

$$m_0^i + \sum_{\omega \in \Omega} \bar{\kappa}(\omega) z^i(\omega) \leqslant e_0^i + \varphi^i[-\bar{a}^n + \bar{q}^n]$$

and for every primitive state ω

$$m_1^i(\omega) + \bar{p}(\omega) c^i(\omega) \leqslant e_1^i + z^i(\omega)$$

- Together with the asset pricing equation[1]

$$\bar{q}_k = \sum_{\omega \in \Omega} \bar{\kappa}(\omega) \bar{p}(\omega) f_k(\omega, \bar{a}_k)$$

[1] By quasi-linearity we have $\bar{\kappa}(\omega) = \delta \mathbb{P}(\omega)$

Contracts contingent on primitive states-of-nature

- If each firm k maximizes Net Present Value

$$\bar{a}_k \in \operatorname{argmax} \left\{ -a + \sum_{\omega \in \Omega} \bar{\kappa}(\omega)\bar{p}(\omega) f_k(\omega, a) \ : \ a \in A \right\}$$

  Then we get efficiency

- Behavioral assumption: firm k takes prices as given and expects the market to pay the price $\tilde{q}_k(a)$ for its equity under investment a where

$$\tilde{q}_k(a) = \sum_{\omega \in \Omega} \bar{\kappa}(\omega)\bar{p}(\omega) f_k(\omega, a)$$

  in other words, it maximizes

$$\text{NPV}_k(a) = -a + \sum_{\omega \in \Omega} \bar{\kappa}(\omega)\bar{p}(\omega) f_k(\omega, a)$$

Contracts contingent on observable output

- Markets based on primitive states ω are difficult to operate
  - Writing contracts: ex-ante, it would be "too costly" to specify precisely in a contract all the primitive states of nature
  - Enforcing contracts: ex-post, it is difficult to verify a primitive state of nature
- Modern financial markets trade assets whose payoffs depend on the observable profits of the firms
  - bonds, equities, derivatives and indexes

Assumption

Agents can trade equities and all contracts contingent on aggregate output

Contracts contingent on aggregate output

- Agents' budget set

$$m_0^i + \sum_{\sigma\in\Sigma} \bar{\kappa}_\sigma z_\sigma^i \leqslant e_0^i + \varphi^i[-\bar{a}^n + \bar{q}^n]$$

and for every aggregate output σ

$$m_\sigma^i + \bar{p}_\sigma c_\sigma^i \leqslant e_1^i + z_\sigma^i$$

- Together with the asset pricing equation

$$\bar{q}_k = \sum_{\sigma\in\Sigma} \sum_{y^n\in Y^n(\sigma)} \bar{\kappa}_\sigma \bar{p}_\sigma y_k$$

- By quasi-linearity we have

$$\bar{\kappa}_\sigma = \delta\pi_\sigma(\bar{a}^n) = \delta \sum_{y^n\in Y^n(\sigma)} \Pi(y^n, \bar{a}^n)$$

Contracts contingent on output: which objective?

- What is the “correct” objective that firms should maximize?
- We have incomplete markets, how should firm k value the out-of-equilibrium output stream

$$\omega \mapsto f_k(\omega, a)?$$

- What are the “correct” discount factors?
- Do market prices provide enough information to coordinate agents’ decisions?

The Shareholder Value

- Let $\bar{p}(y^n) = \bar{p}_\sigma$ for $\sigma = y_1 + \ldots + y_n$
- Recall that, at equilibrium, firm k's equity satisfies

$$\bar{q}_k = \delta \sum_{y^n \in Y^n} \Pi(y^n, \bar{a}^n) \bar{p}(y^n) y_k$$

- One may consider the Shareholder Value

$$\mathrm{SV}_k(a_k) = -a_k + \delta \sum_{y^n \in Y^n} \Pi(y^n, a_k, \bar{a}_{-k}) \bar{p}(y^n) y_k$$

- Maximizing this Shareholder Value may not lead to efficiency

What causes inefficiency?

- Market incompleteness (output contingent securities)
- Non-competitive behavior of firm k which
  - takes the good prices $\bar{p}_\sigma$ as given
  - but fully considers the impact of new investments on the probability of y^n and therefore on the probability of aggregate output $\sigma = y_1 + \ldots + y_n$

Asset pricing ... our contribution

$$
\begin{aligned}
\bar{q}_k &= \delta \sum_{y^n \in Y^n} \Pi(y^n, \bar{a}^n) \bar{p}(y^n) y_k \\
&= \delta \sum_{\sigma \in \Sigma} \bar{p}_\sigma \sum_{y^n \in Y^n(\sigma)} \Pi(y^n, \bar{a}^n) y_k \\
&= \sum_{\sigma \in \Sigma} \bar{\kappa}_\sigma \bar{p}_\sigma \sum_{y^n \in Y^n(\sigma)} \frac{\Pi(y^n, \bar{a}^n)}{\pi_\sigma(\bar{a})} y_k \\
&= \sum_{\sigma \in \Sigma} \bar{\kappa}_\sigma \bar{p}_\sigma \sum_{y^n \in Y^n} \overline{\Pi}(y^n | \sigma) y_k \\
&= \sum_{\sigma \in \Sigma} \bar{\kappa}_\sigma \bar{p}_\sigma \sum_{y_k \in Y} \overline{\Pi}_k(y_k | \sigma) y_k
\end{aligned}
$$

$\overline{\Pi}(y^n|\sigma) = \Pi(y^n, \bar{a}^n)/\pi_\sigma(\bar{a}^n)$ if $y_1 + \ldots + y_n = \sigma$ and 0 elsewhere

$\overline{\Pi}_k(y_k|\sigma) \equiv \sum_{y^{-k} \in Y^{-k}} \overline{\Pi}(y^n|\sigma)$

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\bar{q}_k &= \delta \sum_{y^n \in Y^n} \Pi(y^n, \bar{a}^n)\bar{p}(y^n) y_k \\
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&= \sum_{\sigma \in \Sigma} \bar{\kappa}_\sigma \bar{p}_\sigma \sum_{y^n \in Y^n(\sigma)} \frac{\Pi(y^n, \bar{a}^n)}{\pi_\sigma(\bar{a})} y_k \\
&= \sum_{\sigma \in \Sigma} \bar{\kappa}_\sigma \bar{p}_\sigma \sum_{y^n \in Y^n} \overline{\Pi}(y^n|\sigma) y_k \\
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Asset pricing ... our contribution

$$\begin{aligned}
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&= \sum_{\sigma \in \Sigma} \bar{\kappa}_\sigma \bar{p}_\sigma \sum_{y_k \in Y} \overline{\Pi}_k(y_k|\sigma) y_k
\end{aligned}$$

$\overline{\Pi}(y^n|\sigma) = \Pi(y^n, \bar{a}^n)/\pi_\sigma(\bar{a}^n)$ if $y_1 + \ldots + y_n = \sigma$ and 0 elsewhere

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Asset pricing ... our contribution

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\begin{aligned}
\bar{q}_k &= \delta \sum_{y^n \in Y^n} \Pi(y^n, \bar{a}^n) \bar{p}(y^n) y_k \\
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&= \sum_{\sigma \in \Sigma} \bar{\kappa}_\sigma \bar{p}_\sigma \sum_{y^n \in Y^n(\sigma)} \frac{\Pi(y^n, \bar{a}^n)}{\pi_\sigma(\bar{a})} y_k \\
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$\overline{\Pi}(y^n|\sigma) = \Pi(y^n, \bar{a}^n)/\pi_\sigma(\bar{a}^n)$ if $y_1 + \ldots + y_n = \sigma$ and 0 elsewhere

$\overline{\Pi}_k(y_k|\sigma) \equiv \sum_{y^{-k} \in Y^{-k}} \overline{\Pi}(y^n|\sigma)$

Our contribution

- At equilibrium, the market value of firm k can be expressed as follows

$$\overline{V}_k = -\bar{a}_k + \underbrace{\sum_{\sigma \in \Sigma} \bar{\kappa}_\sigma \bar{p}_\sigma \sum_{y_k \in Y} \overline{\Pi}_k(y_k|\sigma) y_k}_{\bar{q}_k}$$

- Every shareholder of firm k would like that firm k chose $\bar{a}_k$ to maximize

$$\widetilde{V}_k(a_k) = -a_k + \widetilde{q}_k(a_k)$$

where $\widetilde{q}_k(a_k)$ is agents' perceptions about the way the new investment decision a_k would affect the price that the "market" will pay for the equity

Standard behavioral assumption: Price taking

- Perceptions should be correct at equilibrium

$$\widetilde{q}_k(\bar{a}_k) = \bar{q}_k = \sum_{\sigma \in \Sigma} \bar{\kappa}_\sigma \bar{p}_\sigma \sum_{y_k \in Y} \overline{\Pi}_k(y_k|\sigma) y_k$$

- Price-taking assumption: agents conceive that the investment decision of a single firm does not affect prices
- This leads to the following expression

$$\widetilde{q}_k(a_k) = \sum_{\sigma \in \Sigma} \bar{\kappa}_\sigma \bar{p}_\sigma \sum_{y_k \in Y} \widetilde{\Pi}_k(y_k, a_k|\sigma) y_k$$

where $\widetilde{\Pi}_k(y_k, a_k|\sigma)$ is the probability (perceived by agents) that firm k's output is y_k when it chooses the investment a_k conditional to the aggregate output σ

Our behavioral assumption: Aggregate output taking

Agents have *competitive beliefs*

They are convinced that a change in firm k's investment will not affect the likelihood of each aggregate output σ, i.e.,

$$\widetilde{\Pi}_k(y_k, a_k|\sigma) = \mathbb{P}(\{f_k(a_k) = y_k\}|\{g(\bar{a}) = \sigma\})$$

where $g(\bar{a}) = f_1(\bar{a}) + \ldots + f_n(\bar{a})$

Remark

In our opinion, this assumption is consistent with the price-taking assumption: agents take prices for aggregate output as given

Competitive market value of the firm

Theorem

If each firm k chooses its investment maximizing the "competitive" market value

$$V_k(a_k) = -a_k + \sum_{\sigma \in \Sigma} \bar{\kappa}_\sigma \bar{p}_\sigma \sum_{y_k \in Y} \widetilde{\Pi}_k(y_k, a_k|\sigma) y_k$$

then the corresponding equilibrium is efficient

Competitive firms

- As the traditional general equilibrium literature, we assume that firms are price takers and do not seek to manipulate prices
- Contrary to MQ'09, MQ'10 and MQR'12, we assume that firms take aggregate production as given: each firm believes that a unilateral deviation has little influence on aggregate output
- Under these two assumptions, we have proved that profit-maximization (when profit is correctly computed) is socially well justified
- Our result shows that the stakeholder theory is not founded by the mere presence of output-contingent contracts
- An additional feature (e.g., a non-competitive production sector) is needed to reject the profit-maximization criterium