A Macroeconomic Framework for Quantifying Systemic Risk*

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Abstract

Systemic risk arises when shocks lead to states where a disruption in financial intermediation adversely affects the economy and feeds back into further disrupting financial intermediation. We present a macroeconomic model with a financial intermediary sector subject to an equity capital constraint. The novel aspect of our analysis is that the model's solution is a stochastic steady state in which only some of the states correspond to systemic risk states. The model allows us to examine the transition from "normal" states to systemic risk states. We calibrate our model and use it to match the systemic risk apparent during the 2007/2008 financial crisis.

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1 Introduction

It is widely understood that a disruption in financial intermediation, triggered by losses on housing-related investments, has played a central role in the recent economic crisis. Figure 1 plots the market value of equity for the financial intermediary sector, along with a credit spread, investment, and a land price index. The equity value, land price index, and investment are in real per-capita terms, and normalized to be one in 2007Q2. The figure illustrates the close relation between reductions in the value of financial intermediary equity, rising spreads, and falling land prices and aggregate investment.

In the wake of the crisis, understanding *systemic risk*, i.e., the risk of a disruption in financial intermediation with adverse effects for the real economy (see, e.g., Bernanke, 2009, Brunnermeier, Gorton and Krishnamurthy, 2010), has been a priority for both academics and policy-makers. The objective of this paper is to develop a macroeconomic model within which systemic risk can be quantified. We embed a financial intermediary sector within a simple real business cycle model. Equity capital constraints in the intermediation sector affect asset prices, real investment, and output. Moreover, since the tightness of constraints depend endogenously on expected future output, there is a two-way feedback between financial intermediation and real activity. These aspects of the model are by now familiar from the macroeconomics literature on financial frictions (see, e.g., Bernanke, Gertler, and Gilchrist, 1999, Kiyotaki and Moore, 1997, Gertler and Kiyotaki, 2010).

The principal innovation of the paper relative to much of the prior literature is that we model an occasionally binding constraint. We think this is a necessary methodological step in order to study systemic risk because systemwide financial disruptions are rare, and in most cases we are interested in understanding the transition of the economy from non-systemic states into systemic states. The model's equilibrium is a stochastic steady state in which "systemic risk" states correspond to only some of the possible realizations of the state variables. Moreover, in any given state, agents understand that shocks may realize that lead to constraints tightening, triggering systemic risk. Behavior in crisis states is substantially different than the behavior in non-crisis states. Particularly in crisis states, agents are concerned that shocks could lead to greater instability, and demand a higher risk premium to own and finance risky investments. Since other papers (e.g.,Bernanke, Gertler, and Gilchrist, 1999, Kiyotaki and Moore, 1997) log-linearize around a steady state where constraints are assumed to bind, they cannot speak meaningfully to these non-linear effects that are central to systemic risk.

We calibrate our model to replicate a systemic crisis, as in 2008. A significant challenge in quantifying the model is that crises are rare so that there is little data on which to calibrate the model. Our approach is to calibrate the model to match data during a downturn in which financial friction effects are present, but are not acute. We then use the non-linear structure imposed by the theoretical model to extrapolate to a more extreme crisis. We simulate an extreme crisis to see how well it matches patterns in the fall of 2008. We find that with plausibly small shocks the model is able to replicate patterns in 2008. That is, the model's equity capital constraint drives a quantitatively significant amplification mechanism. In addition, a key to this result is our modeling of a housing sector whereby the demand for land is affected by the intermediary capital constraint. We assume that land is in fixed supply while physical capital is subject to adjustment costs. When the equity capital constraint tightens, land prices fall sharply, while the price of physical capital only falls slightly. In particular, we find that the amplification mechanism in our model is substantially through the feedback between the value of intermediary equity and land prices.

We also ask how variables determined prior to the crisis (i.e., during a normal period) affect the likelihood of the crisis. Here we find that the odds of a crisis are small, and as result, although beginning from a better initial condition lowers the odds of a crisis, the effects are small.

The papers that are most similar to ours are Mendoza (2010) and Brunnermeier and Sannikov (2010). Both papers develop stochastic and non-linear financial frictions models to study financial crises. Mendoza is interested in modeling and calibrating crises, or sudden stops, in emerging markets. From a technical standpoint, Mendoza relies on numerical techniques to solve his model, while we develop a model with unidimensional state variable whose equilibrium behavior can be fully characterized by a system of ordinary differential equations. Our approach is thus complementary to his. Brunnermeier and Sannikov also take the differential equation approach of our paper. Their model illustrates the non-linearities in crises by showing that behavior deep in crises regions is substantially different than that in normal periods and underscores the importance of solving non-linear models. In particular, their model delivers a steady state distribution in which the economy can have high occupation time in systemic risk states. While our model is somewhat different than theirs, the principal difference relative to their paper is that we aim to quantitatively match the non-linearities in the data, thus providing a model that can be used to quantify systemic risk. Finally, both Mendoza and Brunnermeier-Sannikov study models with an exogenous interest rate, while the interest is endogenous in our model.

The model we employ is closely related to our past work in He and Krishnamurthy (2010, 2011). He and Krishnamurthy (2011) develop a model integrating the intermediary sector into a general equilibrium asset pricing model. The intermediary sector is modeled based on a moral hazard problem, akin to Holmstrom and Tirole (1997), and optimal contracts between intermediaries and households are derived. Asset prices are also derived analytically. He and Krishnamurthy (2010) assume the form of intermediation contracts, matching the derived contract in He and Krishnamurthy (2011), but enrich the model so that it can be realistically calibrated to match asset market phenomena during the mortgage market financial crisis of 2007 to 2009. In the present paper, we also assume the structure of intermediation in reduced form. The main innovation relative to our prior work is that the present model allows for a real investment margin with capital accumulation and lending. Thus the current paper speaks to not only effects on asset prices but also real effects on economic activities.

The paper is laid out as follows. The next section presents empirical evidence for non-linearity in the relationship between macro variables and intermediary equity, which is motivation for the model we propose. Section 3 describes the model. Section 4 goes through the steps of how we solve the model. Section 5 presents our choice of parameters for the calibration. Sections 6, 7, and 8.1 present the results from our model. Figures and an appendix with further details on the model solution are at the end of the paper.

2 Evidence for Nonlinearity

The relation reflected in Figure 1 between the value of financial intermediary equity, spreads, and aggregate investment is particularly strong in periods of financial distress. Figure 2 illustrates this point. We estimate a three variable VAR, with intermediary equity, investment, and a credit risk spread, using quarterly data from 1975Q1 to 2009Q4. The first two of the variables are based on log-differences, while the spread is in levels. The intermediary equity measure is the sum across all financial firms (banks, broker-dealers, insurance and real estate) of their stock price times the number of shares from the CRSP database.[1] The investment measure is the sum of household durable goods purchases, business investment in software, structures, and residential investment, all from NIPA. Both measures are expressed in per-capita terms and adjusted for inflation using the GDP deflator. The credit risk spread is drawn from Gilchrist and Zakrajsek (2010). There is a large literature showing that credit spreads (e.g., the commercial paper to Treasury bill spread) are a leading indicator for economic activity (see Philippon (2010) for a recent contribution). Credit spreads have two components: expected default and an economic risk premium that lenders charge for bearing default risk. In an important recent paper, Gilchrist and Zakrajsek (2010) show that the spread's forecasting power stems primarily from variation in the risk premium component (the "excess bond premium" or EBP). The authors also show that the EBP is closely related to measures of financial intermediary health. As we will explain, our model has predictions for the link between intermediary equity and the risk premium demanded by intermediaries, while being silent on default (there is no default in the equilibrium of the model). Thus, for our purposes, the EBP is the most natural credit risk spread on which to focus.

The VAR is estimated allowing for coefficients to depend on whether or not the economy is in a distress period. Table 1 lists the distress periods. They are constructed by considering the

[1] Muir (2011) shows that this measure is useful for predicting aggregate stock returns as well as economic activity. Moreover, intermediary equity is a priced factor in the cross-section of stock returns.

highest one-third of realizations of the excess bond premium from Gilchrist and Zakrajsek (2010), but requiring that the distress or non-distress periods span at least two contiguous quarters. The distress periods roughly correspond to NBER recession dates, with one exception. We classify a distress period starting in 1985, while the recession starts in 1990. The S&L crisis and falling real estate prices in the late 80s put pressure on banks which appears to result in a high EBP and hence leads us to classify this period as distress.

The VAR is ordered with equity first, then the EBP spread, and finally investment.[2] We plot the cumulative impulse responses to an orthogonalized shock to intermediary equity. Panel A corresponds to the distress periods. The shock increasing equity by 14% causes the EBP four-quarters out to fall by a little over 2% (relative to a mean value in distress periods of 2.6%). The shock increases investment four-quarters out by close to 3%. The effects are statistically different from zero. Panel B corresponds to the non-distress periods. Here the point estimates for a shock of 10% in intermediary equity are much smaller, and the effects are statistically indistinguishable from zero. The confidence band on investment implies that the effect on investment is at most 1.2%.

These empirical results get to the heart of systemic risk: there is a non-linear relation between financial intermediary equity and macroeconomic outcomes. We aim to quantitatively match this non-linearity with our model.

Table 1: Distress Classification

Distress Periods	NBER Recessions
1975Q1 - 1975Q3	11/73 - 3/75
1982Q2 - 1982Q4	7/81 - 11/82
1985Q4 - 1990Q2	7/90 - 3/91
2001Q2 - 2003Q1	3/01 - 11/01
2007Q3 - 2009Q3	12/07 - 6/09

3 Model

Time is continuous and indexed by t. The continuous time framework is attractive because the solution is given by an ordinary differential equation (ODE) that is easy to handle numerically. The economy has two types of capital: productive capital K_t and housing capital H. We assume that housing is in fixed supply and normalize $H \equiv 1$. We denote P_t as the price of a unit of housing, and q_t as the price of a unit of capital. The numeraire is the consumption good. There are three types of agents: equity households, debt households, and bankers.

[2]The effect of equity on EBP or investment is not sensitive to ordering long as equity is placed before the corresponding variable.

We begin by describing the production technology and the household sector. These elements of the model are a slight variant on a standard stochastic growth model. We then describe bankers and intermediaries, which are the non-standard elements of the model. The modeling of intermediation follows He and Krishnamurthy (2010, 2011).

3.1 Production and Households

There is an "AK" production technology that generates per-period output Y_t:

$$Y_t = AK_t, \tag{1}$$

where A is a positive constant. Commonly, RBC models introduce shocks to the productivity parameter A. Introducing shocks to A will add another state variable and complicate solutions to the model. Instead, we assume shocks directly in the evolution of the capital stock ("capital quality shocks"):

$$\frac{dK_t}{K_t} = i_t dt - \delta dt + \sigma dZ_t \tag{2}$$

The term i_t is the amount of new capital installed at date t. Capital depreciates by δdt, where δ is constant. The last term σdZ_t is a capital quality shock, following Gertler and Kiyotaki (2010). For example, K_t can be thought of as the effective quality/efficiency of capital rather than the amount of capital outstanding. The capital quality shock is a simple device to introduce an exogenous source of variation in the value of capital. Note that the price of capital, q_t, is endogenous. Thus, we will be interested in understanding how the exogenous capital quality shock translates into endogenous capital price shocks. Finally, the shock σdZ_t is the only source of uncertainty in the model ($\{Z_t\}$ is standard Brownian motion, while σ is a positive constant).

We assume adjustment costs so that installing $i_t K_t$ new units of capital costs $\Phi(i_t, K_t)$ units of consumption goods where,

$$\Phi(i_t, K_t) = i_t K_t + \frac{\kappa}{2}\left(i_t - \delta\right)^2 K_t.$$

That is, the adjustment costs are assumed to be quadratic in net investment as is standard in the literature. After describing the neoclassical elements of the model, we will introduce financing frictions that affect capital investment.

There is a unit measure of households. Each household enters period t with financial wealth W_t. It consumes out of this wealth and allocates resources to real investment. The household then splits its wealth, allocating $1 - \lambda$ fraction to an "equity household" and retaining the rest with a "debt household." These households individually make financial investment decisions. The investments pay off at period $t + dt$, at which point the members of the household pool their wealth again to give wealth of W_{t+dt}. We describe the financial investment decisions, as well as the reason for the equity/debt labels, in greater detail below. Modeling debt households, i.e., the constant

λ, is useful to calibrate the leverage of the intermediary sector. The modeling device of using the representative family follows Lucas (1990).

The utility of the combined household is,[3]

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t}\left((1-\phi)\ln c_t^y + \phi \ln c_t^h\right) dt\right],$$

where ρ is the discount rate, c_t^y is consumption of the output good, and c_t^h is consumption of housing services. Then, given the preferences, the optimal consumption rule must satisfy:

$$\frac{c_t^y}{c_t^h} = \frac{1-\phi}{\phi} D_t, \tag{3}$$

where D_t is the endogenous rental rate on housing to be determined in equilibrium. In equilibrium, ϕ affects the relative market value of the housing sector to the goods producing sector.

3.2 Bankers and Equity Capital

We assume that all productive capital and housing stock can only be owned directly by "financial intermediaries." There is a continuum of competitive intermediaries. The intermediaries are owned by households, but run by bankers who have the know-how to manage investments. These bankers make all investment decisions of the intermediary. That is, we assume that there is a separation between the ownership and control of the intermediary. At time t, a given banker has "reputation" of ϵ_t. Faced with such a banker, we assume that equity-households are willing to invest up to ϵ_t to own the equity of the intermediary. Any remaining funds raised by the intermediary are in the form of short-term (from t to $t + dt$) debt financing. Equity can only be raised from equity-households, while debt can be raised from either equity or debt households.

Denote the realized profit-rate on the intermediary's investments from t to $t + dt$, net of any debt repayments, as $d\tilde{R}_t$. This is the return on the shareholder's equity of the intermediary. The profit is stochastic and depends on shocks at time $t + dt$. Then, we assume that the reputation of the banker making that intermediary's investment decisions evolves as,

$$\frac{d\epsilon_t}{\epsilon_t} = m d\tilde{R}_t,$$

where $m > 0$ is a constant. Poor investment returns reduce ϵ_t and thus reduce the amount of equity a given intermediary can raise in the next period. For simplicity, we assume this reputation dynamic in reduced form rather than modeling learning on the part of the households. Reputation is one way to think about how past returns may affect household's willingness to invest in intermediaries. But there are other ways, such as moral hazard or adverse selection, in which past

[3] We assume households have log utility to highlight the effects of the intermediary sector's (endogenous) risk tolerance on prices and quantities. However, our modeling approach can handle richer specifications of the household's utility function. At this time, we have not explored alternatives to log, although it may be interesting to do so.

returns of the intermediary reduces net worth and thereby reduces households' willingness to invest in intermediaries.

We assume that a banker makes investment decisions to maximize his future reputation. Bankers do not consume goods (a feature which is convenient when clearing the goods market). A given banker may die at any date at a Poisson rate of η. Thus, a banker makes investment decisions to maximize,

$$\mathbb{E}\left[\int_0^\infty e^{-\eta t}\ln\epsilon_t dt\right].$$

Given the log form objective function, it is easy to show that the time t decision of the banker is chosen to maximize,

$$\mathbb{E}_t[d\tilde{R}_t] - \frac{m}{2}Var_t[d\tilde{R}_t]. \tag{4}$$

The constant m thus parameterizes the "risk aversion" of the banker.

To summarize, a given intermediary can raise at most ϵ of equity capital. If the intermediary's investments perform poorly, then ϵ falls going forward, and the equity capital constraint tightens. The banker in charge of the intermediary chooses the intermediary's investments to maximize the mean excess return on equity of the intermediary minus a penalty for variance multiplied by the "risk aversion" m.

Our modeling of bankers appears exotic at first glance: they make intermediary investment decisions, but only have preferences over their future reputations, thus de-facto ruling out any incentive contracts. We note that the key feature of our modeling is that bankers who do badly raise less equity going forward, and that bankers maximize the mean-minus-variance of the return on equity of the intermediary. In He and Krishnamurthy (2011) we consider a standard setting where bankers have preferences over consumption and households write incentive contracts with bankers to manage intermediaries. In that setting, we find that bankers' equity capital constraint is similarly a function of their investment performance. Moreover, from a macroeconomic standpoint all of the model's dynamics are driven by the equity capital constraint. Thus, our modeling in this paper is a simplification that captures the essence of an equity capital constraint.[4]

3.3 Aggregate Intermediary Capital

Consider now the aggregate intermediary sector. We denote by $\mathcal{E}_t$ the maximum equity capital that can raised by this sector, which is just the aggregate version of individual banker's reputation ϵ. The maximum equity capital $\mathcal{E}_t$ will be the key state variable in our analysis, and its dynamics

[4]The modeling leads to two changes relative to He and Krishnamurthy (2010, 2011). First, we do not have to keep track of the bankers' consumption decisions which simplifies the model's analysis somewhat. More substantively, in our previous work we find that, in crisis states, the interest rate diverges to negative infinity. In the present modeling, the interest rate is determined purely off the household's Euler equation, which leads to a better behaved interest rate.

are given by,

$$\frac{d\mathcal{E}_t}{\mathcal{E}_t} = md\tilde{R}_t - \eta dt + d\psi_t. \tag{5}$$

The first term here reflects that all intermediaries are identical, so that the aggregate stock of intermediary reputation evolves with the return on the intermediaries' equity. The second-term, $-\eta dt$, captures exit of bankers who die at the rate η. Exit is important to include otherwise $\frac{d\mathcal{E}_t}{\mathcal{E}_t}$ has strictly positive drift which makes the model non stationary. In other words, without exit, intermediary capital will grow and the capital constraint will not bind. The last term, $d\psi_t \geq 0$ reflects entry. We describe this term more fully below when describing the boundary conditions for the economy (see Section 4.4). In particular, we will assume that entry occurs when the aggregate intermediary sector has low capital, because the incentives to enter are high in these states.

3.4 Capital Goods Producers

Capital goods producers, owned by households, undertake real investment. We make assumptions so that the financing costs in the intermediary sector indirectly affects the capital goods producers' investment decision, thus capturing a possible credit crunch.

At date t, suppose a producer makes $i_t K_t$ units of capital at cost

$$\Phi(i_t, K_t) = i_t K_t + \frac{\kappa}{2}\left(i_t - \delta\right)^2 K_t$$

units of consumption goods, where κ is a positive constant. We assume that the producer must then sell the capital to the intermediary sector at price q_t. The price q_t is the intermediaries' valuation of capital and reflects the "risk aversion" of the intermediary sector. As we will see, if the intermediary sector has low equity capital, this factor will tend to increase the risk premium demanded by intermediaries and thus reduce q_t.

There are two ways to interpret the investment-intermediation relationship. Most directly, we can think of intermediaries as venture capital/private equity investors. In this case, the household creates a business and raises q_t from the intermediary. If q_t is low, say because intermediaries are capital constrained, the household has less incentive to build the business. Alternatively, we can think of the intermediary as a bank that makes a collateralized loan. Suppose, the household purchases a car, but raises money from the bank to finance the purchase. The bank evaluates the collateral and determines it is willing to lend q_t against it, which then affects the household's car buying decision.

Given q_t, i_t is chosen to solve,

$$\max_{i_t} q_t i_t K_t - \Phi(i_t, K_t) \Rightarrow i_t = \delta + \frac{q_t - 1}{\kappa}. \tag{6}$$

Recall that $\Phi(i_t, K_t)$ reflects a quadratic cost function on investment net of depreciation.

3.5 Household Members and Portfolio Choices

Each household splits its wealth of W_t to allocate $1-\lambda$ fraction to an "equity household" and retaining the rest with a "debt household." We assume that the debt household can only invest in intermediary debt, while the equity household can invest in either debt or equity.

Collectively, equity households invest their allocated wealth of $(1-\lambda)\,W_t$ into the intermediaries subject to the restriction that, given the stock of banker reputations, they do not purchase more than $\mathcal{E}_t$. When $\mathcal{E}_t > W_t(1-\lambda)$ so that the intermediaries reputation is sufficient to absorb the households' maximum equity investment, we say that the capital constraint is not binding. But when $\mathcal{E}_t < W_t(1-\lambda)$ so that the capital constraint is binding, the equity household restricts its equity investment and places any remaining wealth in bonds. In the case where the capital constraint does not bind, it turns out to be optimal – since equity offers a sufficiently high risk-adjusted return – for the equity households to purchase $(1-\lambda)W_t$ of equity in the intermediary sector. We will prove the latter statement when solving the model.

Let,

$$E_t \equiv \min\left(\mathcal{E}_t, W_t(1-\lambda)\right)$$

be the amount of equity capital raised by the intermediary sector. The households' portfolio share in intermediary equity, paying return $d\tilde{R}_t$, is thus,

$$x_t^H = \frac{E_t}{W_t}.$$

The debt household simply invests his portion λW_t into the riskless bond. The household budget constraint implies that the amount of debt purchased by the combined household is equal to $W_t - E_t$.

3.6 Bond Market Interest Rate

Denote the interest rate on the short-term bond as r_t. Given our Brownian setting with continuous sample paths, the short-term debt is riskless.[5] At the margin, if a household cuts its consumption, it can increase its bond market investment and finance more consumption tomorrow. Thus, the standard household Euler equation holds at the equilibrium interest rate r_t:

$$r_t = \rho + \mathbb{E}_t\left[\frac{dc_t^y}{c_t^y}\right] - Var_t\left[\frac{dc_t^y}{c_t^y}\right]. \tag{7}$$

3.7 Intermediary Portfolio Choice

Each intermediary chooses how much debt and equity financing to raise from households, subject to the capital/reputation constraint, and then makes a portfolio choice decision to own housing

[5] Note that we place no restriction on the raising of debt financing by the intermediary. Debt is riskless and is always over-collateralized so that a debt constraint would not make sense in our setting. It is clear in practice that there are times in which debt or margin constraints are also quite important. Our model sheds light on the effects of limited equity capital (e.g., limited bank capital) and its effects on intermediation.

and capital. The return on purchasing one unit of housing is,

$$dR_t^h = \frac{dP_t + D_t dt}{P_t}, \tag{8}$$

where P_t is the pricing of housing, and D_t is the equilibrium rental rate given in (3). For capital, if the intermediary buys one unit of capital at price q_t, the capital is worth q_{t+dt} next period and pays a dividend equal to Adt. However, the capital depreciates at the rate δdt and is subject to the capital quality shocks σdZ_t. Thus, the return on capital investment, accounting for the Ito quadratic variation term, is as follows:

$$dR_t^k = \frac{dq_t + Adt}{q_t} - \delta dt + \sigma dZ_t + \left[\frac{dq_t}{q_t}, \sigma dZ_t\right]. \tag{9}$$

We use the following notation in describing an intermediary's portfolio choice problem. Denote α_t^k (α_t^h) as the fraction of the equity raised by an intermediary that is invested in capital (housing).[6] The intermediary's return on equity is,

$$d\tilde{R}_t = \alpha_t^k dR_t^k + \alpha_t^h dR_t^h + (1 - \alpha_t^k - \alpha_t^h) r_t. \tag{10}$$

A banker solves,

$$\max_{\alpha_t^k, \alpha_t^h} \mathbb{E}_t[d\tilde{R}_t] - \frac{m}{2} Var_t[d\tilde{R}_t]. \tag{11}$$

3.8 Market Clearing and Equilibrium

1. In the goods market, the total output must go towards consumption and real investment (where we use capital C to indicate aggregate consumption)

$$Y_t = C_t^y + \Phi(i_t, K_t). \tag{12}$$

   Note again that bankers do not consume and hence do not enter this market clearing condition.

2. The housing rental market clears so that

$$C_t^h = H \equiv 1. \tag{13}$$

3. The intermediary sector holds the entire capital and housing stock. The intermediary sector raises total equity financing of $E_t = \min(\mathcal{E}_t, W_t(1-\lambda))$. Its portfolio share into capital and housing are α_t^k and α_t^h. The total value of capital in the economy is $q_t K_t$, while the total value of housing is P_t. Thus, market clearing for housing and capital are:

$$\alpha_t^k E_t = K_t q_t \text{ and } \alpha_t^h E_t = P_t. \tag{14}$$

   These expressions pin down the equilibrium values of the portfolio shares, α_t^k and α_t^h.

[6] Here, our convention is that when the sum of αs exceed one, the intermediary is shorting the bond (i.e., raising debt) from households. For example, if $\alpha_t^k = \alpha_t^h = 1$, then an intermediary that has one dollar of equity capital will be borrowing one dollar of debt (i.e. $1 - \alpha_t^k - \alpha_t^h = -1$) to invest one dollar each in housing and capital.

4. The total financial wealth of the household sector is equal to the value of the capital and housing stock:

$$W_t = K_t q_t + P_t.$$

An equilibrium of this economy consists of prices, (P_t, q_t, D_t, r_t), and decisions, $(c_t^y, c_t^h, i_t, \alpha_t^k, \alpha_t^h)$. Given prices, the decisions are optimally chosen, as described by (3), (6), (7) and (11). Given the decisions, the markets clear at these prices.

4 Model Solution

Figure 3 summarizes the key elements of the model. Household financial wealth of W_t goes toward purchasing equity and debt in the intermediary sector. The equity investment is subject to the constraint that it cannot exceed $\mathcal{E}_t$. Bankers who run the intermediaries choose portfolio shares to maximize the mean return on equity of the intermediary minus a penalty for variance of returns. In equilibrium, the intermediary sector owns all of the capital and housing stock. Thus, via market clearing, we arrive at the equilibrium portfolio shares in capital and housing that must be optimally chosen by each banker. Note that when these equilibrium portfolio shares are large, which happens when intermediary equity is low, equilibrium risk premia have to adjust upwards to make it optimal for the bankers to choose these large risky portfolio shares. This is the central factor in determining P_t and q_t. Finally, given intermediary portfolio shares and realized returns, we can compute the actual return on the intermediary portfolio. The dynamics of the equity capital constraint, $\mathcal{E}_t$, depend on these realized returns.

We derive a Markov equilibrium where the state variables are K_t and $\mathcal{E}_t$. That is, we look for an equilibrium where all the price and decision variables can be written as functions of these two state variables. Given homogeneity features of the economy, we can simplify this further. We look for price functions of the form $P_t = p(e_t)K_t$ and $Q_t = q(e_t)$ where

$$e_t \equiv \frac{\mathcal{E}_t}{K_t}.$$

Therefore, K_t scales the economy while e_t describes the equity capital constraint of the intermediary sector. The equity capital constraint, e_t, evolves stochastically. We write the evolution of e_t in equilibrium as

$$de_t = \mu_e dt + \sigma_e dZ_t,$$

The functions μ_e and σ_e are state-dependent drift and volatility to be solved in equilibrium.

The solution strategy is to derive a system of ODEs for $p(e)$ and $q(e)$. Asset returns and volatility are functions of $p(e)$, $q(e)$, and derivatives of these functions. The intermediaries' portfolio demand for housing and capital are a function of the asset return and volatility. In equilibrium, the intermediary sector most hold the entire housing and capital stock. This leads to a pair of equations that $p(e)$ and $q(e)$ must satisfy.

We go through the steps in detail next.

4.1 Asset returns and Intermediary Optimality

Let us write,

$$dR_t^h = (\pi_t^h + r_t)dt + \sigma_t^h dZ_t.$$

Here the expected excess return on housing investment is π_t^h and the volatility of investment in housing is σ_t^h. Likewise, we write,

$$dR_t^k = (\pi_t^k + r_t)dt + \sigma_t^k dZ_t.$$

The intermediary maximizes the mean-variance objective, (11), which we can write as:

$$\max_{\alpha_t^k, \alpha_t^h} \left\{ \alpha_t^k \pi_t^k + \alpha_t^h \pi_t^h - \frac{m}{2} \left(\alpha_t^k \sigma_t^k + \alpha_t^h \sigma_t^h \right)^2 \right\}$$

The optimality conditions are,

$$\frac{\pi_t^k}{\sigma_t^k} = \frac{\pi_t^h}{\sigma_t^h} = m \left(\alpha_t^k \sigma_t^k + \alpha_t^h \sigma_t^h \right). \tag{15}$$

The intermediary chooses portfolio shares so that the Sharpe ratio on each asset is equalized. Moreover, the Sharpe ratio is equal to the riskiness of the intermediary portfolio times the "risk aversion" of m.

We next express the terms in equation (15) in terms of the state variables of the model. First note that the market clearing condition (14) pins down α_t^k and α_t^h. Consider the risk and return terms on each investment. We can use the rental market clearing condition $C_t^h = H = 1$ to solve for the housing rental rate D_t:

$$D_t = \frac{\phi}{1-\phi} C_t^y = \frac{\phi}{1-\phi} K_t (A - i_t - \frac{\kappa}{2}(i_t - \delta)^2),$$

where we have used the goods market clearing condition in the second equality. Note that i_t, as given in (6), is only a function of $q(e_t)$. Thus, D_t can be expressed as a function of K_t and e_t.

Given the conjecture $P_t = p(e_t)K_t$, we use Ito's lemma to write the return on housing as,

$$\begin{aligned} dR_t^h &= \frac{dP_t + D_t dt}{P_t} = \frac{K_t dp_t + p_t dK_t + [dp_t, dK_t] + D_t dt}{p_t K_t} \\ &= \left[\frac{p'(e)(\mu_e + \sigma\sigma_e) + \frac{1}{2}p''(e)\sigma_e^2 + \frac{\phi}{1-\phi}\left(A - i_t - \frac{\kappa}{2} i_t^2\right)}{p(e)} + i_t - \delta \right] dt + \sigma_t^h dZ_t, \end{aligned} \tag{16}$$

where the volatility of housing returns is,

$$\sigma_t^h = \sigma + \sigma_e \frac{p'(e)}{p(e)}$$

The return volatility has two terms: the first term is the exogenous capital quality shock and the second term is the endogenous price volatility due to the dependence of housing prices on the intermediary reputation e (which is equal to equity capital, when the constraint binds). In addition, when e is low, prices are more sensitive to e (i.e. $p'(e)$ is high), which further increases volatility.

Similarly, for capital, we can expand (9):

$$dR_t^k = \left[-\delta + \frac{(\mu_e + \sigma\sigma_e)\, q'(e) + \frac{1}{2}\sigma_e^2 q''(e) + A}{q(e)} \right] dt + \sigma_t^k dZ_t,$$

with the volatility of capital returns,

$$\sigma_t^k = \sigma + \sigma_e \frac{q'(e)}{q(e)}$$

The volatility of capital has the same terms as that of housing. However, when we solve the model, we will see that $q'(e)$ is far smaller than $p'(e)$ which indicates that the endogenous component of volatility is small for capital.

We substitute these volatility terms, and the market clearing portfolio shares, to find, an expression for the volatility of the intermediary's portfolio,

$$\alpha_t^k \sigma_t^k + \alpha_t^h \sigma_t^h = \frac{K_t}{E_t} \left(\sigma_e (q' + p') + \sigma (p + q) \right). \tag{17}$$

When K_t/E_t is high, or when intermediary equity is low, volatility is high. In addition, we have noted earlier that p' is high when E_t is low, which further raises volatility. Returning to the optimality condition (15), we thus see that the intermediary demands a high Sharpe ratio on investments when intermediary equity is low.

We substitute from the volatility expression and the expression for returns back into (15) to find a pair of second-order ODEs: First, capital,

$$(\mu_e + \sigma\sigma_e)\, q' + \frac{1}{2}\sigma_e^2 q'' + A - (\delta + r_t) q = m \left(\sigma q + \sigma_e q' \right) \frac{K_t}{E_t} \left(\sigma_e (q' + p') + \sigma (p + q) \right); \tag{18}$$

And for housing:

$$\begin{aligned} (\mu_e + \sigma\sigma_e)\, p' + \frac{1}{2}\sigma_e^2 p'' \quad &+ \quad \frac{\phi}{1-\phi} \left(A - i_t - \frac{\kappa}{2}(i_t - \delta)^2 \right) - (\delta + r_t - i_t)\, p \\ &= \quad m \left(\sigma p + \sigma_e p' \right) \frac{K_t}{E_t} \left(\sigma_e (q' + p') + \sigma (p + q) \right) \end{aligned} \tag{19}$$

4.2 Dynamics of State Variables

We derive equations for μ_e and σ_e which describe the dynamics of the capital constraint. Applying Ito's lemma to $\mathcal{E}_t = e_t K_t$, and substituting for dK_t from (2), we find:

$$\frac{d\mathcal{E}_t}{\mathcal{E}_t} = \frac{K_t de_t + e_t dK_t + \sigma_e \sigma K dt}{e_t K_t} = \frac{\mu_e + \sigma_e \sigma + e\,(i_t - \delta)}{e} dt + \frac{\sigma_e + e\sigma}{e} dZ_t. \tag{20}$$

We can also write the equity capital dynamics directly in terms of intermediary returns and exit, from (5). When the economy is not at at a boundary (hence $d\psi = 0$), equity dynamics are given by,

$$\begin{aligned} \frac{d\mathcal{E}_t}{\mathcal{E}_t} &= m\alpha_t^k \left(dR_t^k - r_t\right) + m\alpha_t^h \left(dR_t^h - r_t\right) + (mr_t - \eta)dt \\ &= m\alpha_t^k \left(\pi_t^k dt + \sigma_t^k dZ_t\right) + m\alpha_t^h \left(\pi_t^h dt + \sigma_t^h dZ_t\right) + (mr_t - \eta)dt. \end{aligned}$$

We use (15) relating equilibrium expected returns and volatilities to rewrite this expression as,

$$\frac{d\mathcal{E}_t}{\mathcal{E}_t} = m^2 \left(\alpha_t^k \sigma_t^k + \alpha_t^h \sigma_t^h\right)^2 dt + m \left(\alpha_t^k \sigma_t^k + \alpha_t^h \sigma_t^h\right) dZ_t + (mr_t - \eta)dt \tag{21}$$

where the portfolio volatility term is given in (17). We match drift and volatility in both equations (20) and (21), to find expressions for μ_e and σ_e. Matching volatilities, we have,

$$m\frac{K_t}{E_t}\left(\sigma_e(q' + p') + \sigma(p+q)\right) = \frac{\sigma_e}{e} + \sigma$$

while matching drifts, we have,

$$\left(m\frac{K_t}{E_t}\left(\sigma_e(q' + p') + \sigma(p+q)\right)\right)^2 + mr_t - \eta = \frac{\mu_e + \sigma_e\sigma + e\left(i_t - \delta\right)}{e}.$$

These equations can be rewritten to solve for μ_e and σ_e in terms of p, q, K, and E_t.

4.3 Interest Rate

Based on the household consumption Euler equation, we can derive the interest rate r_t. Since

$$C_t^y = Y_t - i_t K_t - \frac{\kappa K_t}{2}(i_t - \delta)^2 = \left(A + \delta - \frac{q_t - 1}{\kappa} - \frac{(q_t - 1)^2}{2\kappa}\right) K_t,$$

we can derive $\mathbb{E}_t\left[dC_t^y / C_t^y\right]$ and $Var_t\left[dC_t^y / C_t^y\right]$ in terms of $q\left(e\right)$ (and its derivatives), along with μ_e and σ_e. Then using (7) it is immediate to derive r_t in these terms as well.

4.4 Boundary Conditions

The equilibrium is fully characterized by an ODE system, which are the ODEs (18) and (19), with substitutions for μ_e, σ_e and r_t. The exact expressions are tedious and are placed in the Appendix. The ODEs are solved numerically subject to two boundary conditions. First, the upper boundary is characterized by the economy with $e \to \infty$ so that the capital constraint never binds. We derive exact pricing expressions for the economy with no capital constraint and impose these as the upper boundary. The Appendix provides details.

The lower boundary condition is as follows. We assume that new bankers enter the market when the Sharpe ratio reaches γ, which is an exogenous parameter in the model. This captures the idea that the value of entry is high when the Sharpe ratio of the economy is high. Entry alters the evolution of the state variable e. In particular, the entry point $\underline{e}$ is endogenous and is a reflecting barrier. We assume that entry increases the aggregate intermediary reputation (and therefore aggregate intermediary equity capital), but requires physical capital. We assume that paying β units of capital increases $\mathcal{E}$ by one unit. Since the entry point is a reflecting barrier it must be that the price of a unit of housing, pK, and the price of a unit of capital, q, have zero derivative with respect to e at the barrier (if not, an investor can make unbounded profits by betting on an almost sure increase/decrease in the asset price). Hence we have that $q'(\underline{e}) = 0$. For the housing price, imposing that pK has zero derivative implies the lower boundary condition $p'(\underline{e}) = \frac{p(\underline{e})\beta}{1+\underline{e}\beta} > 0$. The derivative is positive because K falls at the entry boundary, since entry uses up capital, and hence p must rise in order to keep pK constant. See the Appendix for the exact argument and derivation.

5 Calibration

Table 2: Parameters

	Parameter	Choice	Target
Panel A: Intermediation			
m	Performance sensitivity	2.5	Average Sharpe ratio
λ	Debt ratio	0.5	Average intermediary leverage
η	Banker exit rate	13%	Good model dynamics
γ	Entry barrier	5.5	Highest Sharpe ratio
β	Entry cost	1.9	Land price volatility
Panel B: Technology			
σ	Capital quality shock	5%	Investment volatility
δ	Depreciation rate	10%	Literature
κ	Adjustment cost	2	Literature
A	Productivity	0.14	Investment-to-capital ratio
Panel C: Other			
ρ	Time discount rate	2%	Literature
ϕ	Housing share	0.5	Housing-to-wealth ratio

The parameters, ρ (household time preference), δ (depreciation), and κ (adjustment cost) are relatively standard. We use conventional values for these parameters (see Table 2). Note that since our model is set in continuous time, the values in Table 2 correspond to annual values rather than the typical quarterly values one sees in discrete time DSGE parameterizations.

The most important parameter in the model is σ which governs the exogenous uncertainty in this model. Increasing σ increases the volatility of all quantities and prices in the model. We choose $\sigma = 5\%$ as our baseline, and show how changing σ affects results. The baseline generates volatility of investment growth in the model of 5.51% and volatility of consumption growth of 3.70%. In the data, the volatility of investment growth from 1975 to 2009 is 6.70% while the volatility of consumption growth is 1.47%. We have chosen a σ value that is too low for investment but too high for consumption. Some calibrations of consumption growth that include data from the pre-war period use values as high as 3.3%, so that our consumption growth number is not implausible.

The main intermediation parameters are m and λ. The parameter m governs the "risk aversion" of the banker. As we vary m, the Sharpe ratio in the model changes. The choice of $m = 2.5$ gives an average Sharpe ratio from the model of 38%, which is in line with typical asset pricing calibrations. The parameter λ affects the financial intermediary sector's leverage when the capital constraint does not bind. In the states where the equity capital constraint does not bind the financial leverage is simply λ. He and Krishnamurthy (2010) measure the leverage of the financial sector, broadly defined to includes banks, mutual funds, hedge funds, and insurance companies. In 2004, prior to the crisis and hence plausibly a period with no equity capital constraints, they compute a leverage of 0.52. We thus set $\lambda = 0.5$.

The entry boundary condition (i.e. lower boundary) is determined by γ and β. We set $\gamma = 5.5$, so that new entry occurs when the Sharpe ratio is 550%. Based on movements in credit spreads, as measured by Gilchrist and Zakrajsek (2010)'s excess bond premium, we compute that the EBS during the 2008 crisis was roughly 15 times the average. Since in our simulation the Sharpe ratio is around 38%, we set the highest Sharpe ratio to be 550%. Although high entry threshold is crucial for our model, the exact choice of γ is less important because the probability of reaching the entry boundary is almost zero. Our choice is principally motivated by setting γ sufficiently high that it does not affect the model's dynamics in the main part of the distribution. The value of β is far more important because it determines the slope of the land price function at the entry boundary, and therefore the slope all through the capital constrained region. The volatility of land prices is closely related to the slope of the price function (see equation (17)). We choose $\beta = 2.85$ which produces land price volatility in distress regions of 22.87%,[7], which matches the empirical volatility of land price growth in distress periods (22.79% from 1975 to 2009).[8]

We set η (the bankers' death rate) equal to 13% in our baseline. This value implies that the average banker lives for 7.8 years. It is hard to pin down η based on data on the U.S. economy. Our choice is rather dictated by targeting "good" model dynamics. The choice of η is important for our results because it shifts the center of the steady state distribution of intermediary equity capital. For example, if η is very small, the steady state distribution places little weight on being a crisis region. If η is too large, the model is always in a crisis region. We have chosen η so that the drift of intermediary reputation ($\mathcal{E}$) is near zero but positive in the unconstrained region. By targeting

[7]This choice of β leads to a lower entry boundary $p'(\underline{e}) = 0.32$.

[8]Note that it is tautological within our model that at the entry barrier the household sector is willing to pay exactly βK units of capital to boost wealth (i.e. P and q) by increasing e. That is, the value of β cannot be independently pinned down from this sort of computation.

the drift near zero, we allow the probability of a crisis to be driven primarily by the volatility of the economy rather than a contrived death rate parameter. When we vary parameters we also vary η so as to keep the average value of e across the simulation to be the same.

We set $\phi = 0.5$. The parameter ϕ governs the dividend on housing which in turns drives the total value of the housing assets relative to wealth. From Flow of Funds data, Table B100, the total wealth of the household sector in 2005 is 64tn. Of this wealth, real estate accounts for 25tn, or 39%. In our simulation, the choice of $\phi = 0.5$ yields that the mean ratio $\frac{p}{p+q}$ is 38%.

Finally, we we set $A = 0.14$ to match the average investment to capital ratio in the data. From 1975 to 2009, this average is 11% in the data and 10.20% in the simulation.

6 Price and Policy Functions

Figures 4 and 5 plot the price and policy functions for the baseline parameterization and three variations. Consider the baseline in Figure 4 first. The X-axis in all of the graphs is $e = \frac{\mathcal{E}}{K}$. The equity capital constraint binds for points to the left of 0.646. The lower right panel graphs plots the steady-state distribution of the intermediary equity state variable. Most of the weight is on the part of the state space where the capital constraint does not bind.

The first two panels on the top row are $p(e)$ (housing price divided by capital) and $q(e)$. Both price functions are increasing in equity capital as one would expect. It is worth noting that going from right-to-left, prices fall before entering the capital constrained region. As the economy moves closer to the constraint, the likelihood of falling into the constrained region rises and this affects asset prices immediately.

Comparing the first two panels, the main difference is that the range of variation for q is considerably smaller than that for p. This is because housing is in fixed supply while physical capital is subject to adjustment costs. With the $\kappa = 2$ parameterization, the adjustment costs are sufficiently small that capital prices do not vary much. It may be possible to arrive at higher volatility in q if we consider higher adjustment costs or flow adjustment costs as in Gertler and Kiyotaki (2010). However, the model as it stands is unable to generate meaningful variation in q, which is a failure of the model.

The graph illustrates that the aggregate asset price volatility in the economy is substantially driven by housing volatility. The middle and right panel of the second row are for return volatility of q and p. Housing volatility is much higher than q volatility. Note also that the actual price of housing is equal to p times K, and since K is also volatile, housing prices are more volatile than just p.

The top row, third panel is the Sharpe ratio. The Sharpe ratio is about 0.35 in the unconstrained region and rises rapidly upon entering the constrained region. The interest rate also falls sharply when the economy enters the constrained region. Both effects reflect the endogenous increase in "risk aversion" of the intermediary sector.

The first and second panels in the bottom row graph the investment and consumption policy functions. Investment-to-capital falls as q falls. The resource constraint implies that $C/K + \Phi(I, K)/K = A$. Thus, consumption-to-capital rises as the constraint becomes tighter. Note that aggregate consumption depends on this policy function and the dynamics of capital. In the constrained region, capital falls so that while C/K rises, K falls, and the net effect on aggregate consumption depends on parameters. For our baseline parameters, consumption growth when the capital constraint does not bind averages 0.35% while it is -0.37% when the constraint binds.

Figure 5 plots the baseline plus a variation with lower sigma ($\sigma = 4\%$). The results are intuitive. With lower volatility, Sharpe ratio and risk premia are lower. Thus asset prices are higher and investment is higher.

7 Simulation and Model Non-linearity

We compare the results from simulating the model to quarterly data from 1975 to 2009. We split this data into distress periods and non-distress periods as explained earlier (see Table 1), computing sample moments from each subperiod. In choosing the distress/non-distress classification, we face the tradeoff that if we choose too high a cutoff to define distress (say, worst 10% of observations), then we have too little data on which to compute meaningful statistics. On the other hand, when simulating the model we follow the same procedure and label distress as the worst one-third of the sample realizations. Importantly therefore our definitions are consistent and comparable across both model and data. From Figure 4, points to the left of 4 are classified as distressed.

The data we consider are aggregate consumption, investment, intermediary equity, land prices, and EBP-Sharpe. The consumption and investment data are from NIPA. Consumption is non-housing services and nondurable goods. Investment is household durable goods purchases, business investment in software, structures, and residential investment. The intermediary equity measure is the sum across all financial firms (banks, broker-dealers, insurance and real estate) of their stock price times the number of shares from the CRSP database. Land price data is from the Lincoln Institute (http://www.lincolninst.edu/subcenters/land-values/price-and-quantity.asp), where we use *LAND_PI* series based on Case-Shiller-Weiss. Each of these four series are expressed in per-capita terms and deflated using the GDP price deflator. We sample the data quarterly but compute annual log changes in the series, to get an annual growth rate. We focus on annual growth rates because there are slow adjustment mechanisms in practice (e.g., flow adjustment costs to investment) that our model abstracts from. We thus sample at a frequency where these adjustment mechanisms play out fully.

The EBP-Sharpe is from Gilchrist and Zakrajsek (2010). As discussed earlier, these authors construct the EBP as the expected return on a risky corporate bond investment in excess of

Table 3: Model Simulation and Data

The table presents standard deviations and covariances for intermediary equity growth (Eq), investment growth (I), consumption growth (C), land price growth (PL), and Sharpe ratio (EB). Growth rates are computed as annual changes in log value from t to $t+1$. The Sharpe ratio is the value at $t+1$. The column labeled data are the statistics for the period 1975 to 2009, where the Sharpe ratio is constructed from the excess bond premium, and other variables are standard and defined in the text. The next four columns are from the model, reflecting different parameter choices. Numbers are presented conditional on being in the distress period or non-distress period. For the data, the classification of the periods follows Table 1. For the model simulation, the distress period is defined as the 33% worst realizations of the Sharpe ratio.

	Data	Baseline	$\sigma = 4\%$	$\phi = 0$	$m = 2$
Panel A: Distress Periods					
vol(Eq)	26.71%	26.13	24.92	14.62	25.89
vol(I)	6.47%	5.74	4.15	5.14	5.27
vol(C)	1.93%	3.60	3.67	4.52	4.36
vol(PL)	22.79%	22.87	6.31		6.78
vol(EB)	61.22%	49.65	52.49	9.42	45.67
cov(Eq, I)	1.17%	0.81	0.41	0.53	0.67
cov(Eq, C)	0.29%	0.35	0.01	0.44	0.11
cov(Eq, PL)	4.48%	5.00	1.17		1.20
cov(Eq, EB)	-8.07%	-6.86	-5.49	-0.33	-4.33
Panel B: Non-distress Periods					
vol(Eq)	18.40%	6.84	4.13	5.00	5.09
vol(I)	5.95%	5.40	4.00	5.01	5.00
vol(C)	1.21%	3.92	3.97	4.94	4.96
vol(PL)	9.14%	9.61	4.27		5.18
vol(EB)	13.14%	6.65	3.80	0.03	0.59
cov(Eq, I)	0.17%	0.37	0.16	0.25	0.25
cov(Eq, C)	0.04%	0.26	0.16	0.25	0.25
cov(Eq, PL)	0.01%	0.65	0.17		0.26
cov(Eq, EB)	0.39%	-0.09	-0.00	-0.00	-0.00

the riskless rate, after accounting for the expected default on that bond. For our model, it is more natural to focus on a Sharpe ratio rather than an expected return. Equation (15) says that intermediaries will require the same Sharpe ratio on all of their investments. The required Sharpe ratio depends on the intermediary capital constraint. We construct a measure of the Sharpe ratio by dividing the EBP by the expected default on the bond. One can show that this ratio is roughly proportional to the Sharpe ratio of our model.[9]

[9] Suppose that the yield on a corporate bond is y^c, the yield on the riskless bond is y^r and the default rate on the bond is $E[d]$. The expected return on the bond is $y^c - y^r - E[d]$, which is the counterpart to the excess bond premium of Gilchrist and Zakrajsek (2010). To compute the Sharpe ratio on this investment, we need to divide by the riskiness of the corporate bond investment. Plausibly, the risk is proportional to $E[d]$ (for example, if default is binomial or Poisson, this approximation is exact). Thus the ratio $\frac{y^c - y^r - E[d]}{E[d]}$ is proportional to the Sharpe ratio on the investment.

We simulate the model, quarterly, for 2000 years. To minimize the impact of the initial condition, we first simulate the economy for 2000 years, and then record data from the economy for the next 2000 years. We then compute sample moments and the probability of distress region accordingly. We run the simulation 5000 times and report the sample average.

Table 3 provides numbers from the data and the simulation. When reading these numbers it is important to keep in mind that our calibration has not explicitly targeted the asymmetry across distress and non-distress periods. The calibration targets are neutral. Thus an important criterion for the success of our work is whether the non-linearity imposed by the theoretical structural of the model can match the asymmetry in the data.

Comparing the numbers in the second column across distress and non-distress periods shows the non-linearity in the data (also evident in Figure 2). There is almost no relation between equity and the other variables in the non-distress periods, while the variables are closely related in the distress periods. Volatilities are much higher in the distress periods than the non-distress periods.

Now consider the calibration. In the data, the covariance between equity and investment is 1.17% in distress and 0.17% in non-distress. In the simulation, these numbers are 0.81% and 0.37%. The model also comes close to matching the asymmetry in land price volatility and covariance with land prices and equity. In the data, the volatility numbers are 22.79% and 9.14%. We have chosen parameters to get close to the 22.79% so that it is not surprising that the land price volatility from the model in the distress periods is 22.87%. The result of the model is to match the much lower volatility of 9.61% in the non-distress periods. The land-equity covariances in the data are 4.48% and 0.01%. In the model, they are 5.00% and 0.65%. The model is also quite close in matching the asymmetry patterns in the Sharpe ratio and its covariance with intermediary equity.

The model misses in two dimensions. First equity volatility is too low relative to the data in non-distress periods. Part of the explanation here is that there are likely shocks in the non-distress periods that our one-shock model is missing. The model also produces consumption volatility that is uniformly too high. Moreover, the consumption volatility in the non-distress period is higher than that in the distress periods. This happens in the model because if investment becomes more volatile, holding output volatility fixed, consumption must become less volatile. In the model, output volatility does rise in the constrained region, but not as much as investment volatility. It seems clear that more work needs to be done on the household sector preferences to better match consumption dynamics. For example, introducing endogenous labor supply can help to better match consumption dynamics.

The last three columns in the table consider variations where in different ways we make the economy less volatile. In each of these variations, we ensure that the mean of the steady-state distribution of the state variable is the same as in the baseline. We do this by altering η. Thus, the variations should be thought of as delivering a mean-preserving spread around the baseline.

The variation with σ lowered to 4% from 5% reduces the volatility of all variables considerably. In the non-distress periods, the reduction is roughly 1% for each of the variables. In the distress periods the reduction is much more, which should be expected given the non-linearity in the model. The variation with $m = 2$ also effectively reduces volatility in a similar manner.

The variation with $\phi = 0$ is interesting in that it reveals the workings of the model. When $\phi = 0$, land drops out of the model. From Figure 5 note that land price volatility rises in the constrained region while the volatility of q remains roughly constant. Thus, when land is removed from the economy, the volatility of intermediary equity in the constrained region falls from 26.13% to 14.92%. The intermediary pricing kernel is far less volatile which in turn greatly reduces the non-linearity in the model. Because land is in fixed supply, reduced demand for assets in the constrained region causes land prices to fall. Physical capital is subject to adjustment costs so that reduced demand both reduces quantity and price. This distinction is what drives the high volatility of land relative to physical capital in our baseline. And eliminating land thereby reduces the non-linear effects produced by the model.

8 Systemic Risk

8.1 Measures in Systemic States

Table 4 reports the values of prices and policies in a given state, focusing in on a systemic event when the intermediary capital constraints binds. At the mean Sharpe ratio of the model the capital constraint does not bind. Row (1) provides numbers for the mean Sharpe ratio, the unconditional probability that the Sharpe ratio will exceed the mean, the ratio of intermediary equity to capital, housing values, q, the investment rate, interest rate and consumption growth. These latter numbers are compute at the mean Sharpe ratio. Columns (2), (3), and (4) report the same measures at higher Sharpe ratios. The equity capital constraint binds for $E/K < 0.646$, so that the constraint binds at each of these values. The numbers illustrate how housing prices and investment fall non-linearly as the constraint tightens. The numbers also show that q barely moves, which as we have noted before is a failure of the model.

The last two rows give the interest rate and consumption growth. The real interest rate is negative in the systemic states. If our model had a monetary side, the analysis could bring in zero-lower-bound considerations which have been the subject of a large literature recently (see, e.g., Christiano, Eichenbaum and Rebelo, 2010). The interest rate is negative largely because expected consumption growth is negative in the model's crisis.

Table 4: Systemic Crisis

The table compares values of asset prices and macroeconomic aggregates at different points in a crisis, indexed by the Sharpe ratio. The first column of data are the numbers at the mean Sharpe ratio. The rest of the columns present data at a given multiple of that mean Sharpe ratio.

	(1)	(2)	(3)	(4)
	At mean e	X4	X8	X16
Sharpe Ratio	0.327	1.308	2.616	5.232
Prob Sharpe being larger	71.79%	0.31	0.09	0.03
Equity (E/K)	0.87	0.40	0.26	0.13
Housing (P/K)	0.74	0.24	0.19	0.15
Capital (q)	1.0068	0.9943	0.9936	0.9933
Investment (I/K)	10.34%	9.71	9.68	9.66
Interest rate	2.44%	-2.61	-7.76	-19.98
Consumption growth	0.40%	-4.57	-9.72	-21.68

8.2 Crisis Simulation

We use our model to attempt to replicate the crisis of 2008, as reflected in Figure 1. We assume that the economy in 2007Q2 is at $e = 4$ which is the threshold we have used earlier in classifying distress and non-distress states. In our classification from Table 1, 2007Q3 is the start of the distress period. Note that our classification has the economy in distress even though the intermediary capital constraint is not binding. We think of the latter case as corresponding to a "systemic crisis." Starting from the $e = 4$ state, we impose a sequence of exogenous shocks, $\sigma(Z_t - Z_{t-1})$, to the capital dynamics equation (2). These shock are in units of percentage change in capital. We impose quarterly shocks from 2007Q2 to 2009Q4. The exact shocks are $(-3.7\%, -7.1\%, -6.5\%, -2.8\%, -0.5\%, -3.1\%, -2.3\%, -1.2\%, -0.1\%, -0.8\%)$ which totals -28%. We compute the values of all endogenous variables, intermediary equity, land prices, investment, and the Sharpe ratio, after each shock. The shocks are chosen so that the endogenous value of intermediary equity matches that from data from the crisis as reflected in Figure 1. Figure 6 plots the values of the endogenous variables from the model simulation at each quarter.

Note first that the exogenous shocks total 28% while intermediary equity and land prices fall by about 70% in the trough of the model. Thus, there is clearly an amplification of shocks. The equity capital constraint comes to bind after the first three shocks, totaling 20%, and corresponding to 2008Q2. The Sharpe ratio rises dramatically after 2008Q2. Also, note that from that point on, the shocks are smaller but the response of the endogenous variables is larger, reflecting the non-linearity of the model.

Figure 1 should be compared to Figure 6. It is apparent that the model can replicate the crisis with a sequence of plausible shocks.

Table 5: Probability of Crisis

We simulate the model from the given initial condition and compute the probability of falling into a "crisis" state, as described by the different metrics.

Metric	Initial Condition		
	Median	10% higher E/K	10% lower E/K
Prob>Distress	0.16	0.13	0.185
Prob(Sharpe>2XMean)	0.0014	0.00105	0.00176
Prob(Sharpe>4XMean)	0.00042	0.00026	0.00047

8.3 Probability of Crisis

We also use our model to compute the probability of falling into a systemic crisis. We assume that the economy is at the median value of the state variable e. From this point, we simulate the model for 10 years and compute the fraction of time that the economy transits into a systemic crisis, using different metrics for the systemic cutoff. The computation here is different than that presented in Table 4 because those results are unconditional measures of the economy ever spending time in crisis states. Table 5 presents the results. The most salient feature of these results is that the probabilities of the capital constraint binding are close to zero. The table also illustrates how beginning at a different initial condition as measured by a better or worse capitalized intermediary sector affects the probability of a crisis. The qualitative result is as expected: starting from a better capital position reduces the probability of a crisis.

9 Conclusion

We present a fully stochastic model of a systemic crisis in which the main friction is an equity capital constraint on the intermediary sector. We calibrate the model and find that the model is able to replicate behavior in non-distress periods, distress periods, and extreme systemic crisis. In particular, the model is able to quantitatively match the nonlinearities that distinguish patterns across these states. The same calibration predicts that crises are exceptionally unlikely to occur when starting from the median point of the steady state distribution, indicating the difficulty of predicting crises. We continue to examine the model and its performance.

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10 Appendix

10.1 The System of ODEs

Here we give the expressions of ODEs, expecially write the second-order terms p'' and q'' in terms lower order terms. For simplicity, we ingore the argument for $p(e)$, $q(e)$ and their derivateives. Let

$$w \equiv p + q, F(e) \equiv \frac{w}{e} - m\theta(e)w', \text{ and } G(e) \equiv \left(A - \delta - \widehat{\imath} - \frac{\kappa\widehat{\imath}^2}{2}\right)\kappa F(e) + qq'm(1-\theta(e))w,$$

where

$$\theta(e) \equiv \max\left[\frac{w}{e}, \frac{1}{1-\lambda}\right] \text{ and } \widehat{\imath} \equiv \frac{q-1}{\kappa}.$$

We have

$$\sigma_e = \frac{e(w)\sigma(m\theta(e_t)-1)}{w - e_t m\theta(e_t)w'}.$$

Note that $c^y = A - \delta - \widehat{\imath}(e_t) - \frac{\kappa\widehat{\imath}^2}{2}$. =Define $\left[\begin{array}{c} p'\left(\frac{c^y\kappa}{G(e)}\left(-m(1-\theta(e_t))\left(\frac{\frac{1}{2}Q\sigma_e^2}{c^y\kappa}\right)w + m\theta(e_t)\frac{1}{2}\sigma_e^2\right)\right) \\ +\frac{p}{G(e)}\left(qq'm\theta(e_t)\frac{1}{2}\sigma_e^2 + \frac{F(e)}{2}q\sigma_e^2\right) \end{array}\right]$

$$\begin{aligned}
a_{11} &\equiv p'\left(\frac{c^y\kappa}{G(e)}\left(-m(1-\theta(e_t))\left(\frac{\frac{1}{2}Q\sigma_e^2}{c^y\kappa}\right)w + m\theta(e_t)\frac{1}{2}\sigma_e^2\right)\right) + \frac{p}{G(e)}\left(qq'm\theta(e_t)\frac{1}{2}\sigma_e^2 + \frac{F(e)}{2}q\sigma_e^2\right), \\
a_{12} &\equiv p'\left(\frac{c^y\kappa}{G(e)}m\theta(e_t)\frac{1}{2}\sigma_e^2\right) + \frac{1}{2}\sigma_e^2 + \frac{p}{G(e)}\left(qq'm\theta(e_t)\frac{1}{2}\sigma_e^2\right), \\
a_{21} &\equiv Q'\frac{c^y\kappa}{G(e)}\left(\left[-m(1-\theta(e_t))\frac{\frac{1}{2}Q\sigma_e^2}{c^y\kappa}\right]w + \frac{1}{2}m\theta(e_t)\sigma_e^2\right) + \frac{1}{2}\sigma_e^2 + \frac{Q}{G(e)}\left(qq'm\theta(e_t)\frac{1}{2}\sigma_e^2 + \frac{F(e)}{2}q\sigma_e^2\right) \\
a_{22} &\equiv Q'\frac{c^y\kappa}{G(e)}m\theta(e_t)\frac{1}{2}\sigma_e^2 + \frac{q^2}{G(e)}q'm\theta(e_t)\frac{1}{2}\sigma_e^2,
\end{aligned}$$

and

$$\begin{aligned}
b_1 &\equiv (p\sigma + p'\sigma_e)\sigma m\theta(e_t)\frac{w-ew'}{eF(e)} - p'\left(\frac{c^y\kappa}{G(e)}\left[\begin{array}{c}\left[m(1-\theta(e_t))\left(\rho + \widehat{\imath} - \frac{\frac{1}{2}\left(qq''+(q')^2\right)\sigma_e^2}{c^y\kappa} - \frac{\left[c^y\sigma - \frac{qq'\sigma_e}{\kappa}\right]^2}{c^{y2}}\right) - \widehat{\imath} - \eta \right. \\ +m\theta(e_t)\left[\frac{A-\delta}{\phi} + \left(p_t - \frac{\phi}{1-\phi}\right)\widehat{\imath} - \frac{\phi}{1-\phi}\frac{\kappa}{2}\widehat{\imath}_t^2 + \delta(1-Q(e_t))\right]\end{array}\right.\right. \\
&\quad -\frac{\phi}{1-\phi}c^y - p\widehat{\imath} + \frac{p}{G(e)}\left(\begin{array}{c}\left(\rho+\widehat{\imath}\right)c^y\kappa F(e) - \frac{F(e)}{2}(q')^2\sigma_e^2 - \frac{F(e)\kappa\left[c^y\sigma - \frac{qq'\sigma_e}{\kappa}\right]^2}{c^y} \\ -qq'\left(\left[-\widehat{\imath} - \eta\right]w + m\theta(e_t)\left[\frac{A-\delta}{1-\phi} + \left(p - \frac{\phi}{1-\phi}\right)\widehat{\imath}(e_t) - \frac{\phi}{1-\phi}\frac{\kappa}{2}\widehat{\imath}_t^2 + \delta(1-q)\right]\right)\end{array}\right), \\
b_2 &\equiv (\sigma_e q' + q\sigma)\sigma m\theta(e_t)\frac{w-ew'}{eF(e)} - q'\left(\frac{c^y\kappa}{G(e)}\left(\begin{array}{c}\left[m(1-\theta(e_t))\left(\rho + \widehat{\imath} - \frac{\frac{1}{2}(q')^2\sigma_e^2}{c^y\kappa} - \frac{\left[c^y\sigma - \frac{qq'\sigma_e}{\kappa}\right]^2}{c^{y2}}\right) - \widehat{\imath} - \eta\right]w \\ +m\theta(e_t)\left[\frac{A-\delta}{1-\phi} + \left(p - \frac{\phi}{1-\phi}\right)\widehat{\imath} - \frac{1-\phi}{\phi}\frac{\kappa}{2}\widehat{\imath}_t^2 + \delta(1-Q(e_t))\right]\end{array}\right)\right) \\
&\quad -A + q\delta + \frac{q}{G(e)}\left(\begin{array}{c}\left(\rho+\widehat{\imath}\right)c^y\kappa F(e) - \frac{F(e)}{2}(Q')^2\sigma_e^2 - \frac{F(e)\kappa\left[c^y\sigma - \frac{qq'\sigma_e}{\kappa}\right]^2}{c^y} \\ -qq'\left(\left[-\widehat{\imath} - \eta\right]w + m\theta(e_t)\left[\frac{A-\delta}{1-\phi} + \left(p - \frac{\phi}{1-\phi}\right)\widehat{\imath} - \frac{1-\phi}{\phi}\frac{\kappa}{2}\widehat{\imath}^2 + \delta(1-q)\right]\right)\end{array}\right).
\end{aligned}$$

Then the second-order terms can be solved as

$$\begin{bmatrix} q'' \\ p'' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{bmatrix} a_{22}b_1 - a_{12}b_2 \\ -a_{21}b_1 + a_{11}b_2 \end{bmatrix}.$$

10.2 Boundary Conditions and Numerical Methods

10.2.1 When $e \to \infty$ without capital constraint

When $e \to \infty$, we have q and p as constants. Let $\widehat{i} = \frac{q-1}{\kappa}$, and since

$$C_t^y = \left(A - \delta - \frac{Q_t - 1}{\kappa} - \frac{(Q_t - 1)^2}{2\kappa} \right) K_t = \left(A - \delta - \widehat{i} - \frac{\kappa \widehat{i}^2}{2} \right) K_t,$$

we have $dC^y/C^y = dK/K = \widehat{i}dt + \sigma dZ_t$. As a result, the interest rate is

$$r = \rho + \widehat{i} - \sigma^2,$$

and both assets have the same return volatility $\sigma_R^k = \sigma_R^h = \sigma$. Because the intermediary's portfolio weight $\theta = \frac{1}{1-\lambda}$, the banker's pricing kernel is $\sigma m \theta (e_t) = \frac{m\sigma}{1-\lambda}$. Therefore

$$\frac{\mu_R^k - r}{\sigma_R^k} = \frac{m\sigma}{1-\lambda} \Rightarrow \mu_R^k = \frac{m\sigma^2}{1-\lambda} + \rho + \widehat{i} - \sigma^2 = \rho + \widehat{i} + \frac{m-1+\lambda}{1-\lambda}\sigma^2.$$

Because $\mu_R^k = -\delta + \frac{A}{q}$ by definition, we can solve for

$$q = \frac{A}{\rho + \widehat{i} + \delta + \frac{m-1+\lambda}{1-\lambda}\sigma^2}.$$

Because $\widehat{i} = \frac{Q-1}{\kappa}$, plugging in the above equation we can solve for

$$q = \frac{-\left(\rho + \delta + \frac{m-1+\lambda}{1-\lambda}\sigma^2 - \frac{1}{\kappa}\right) + \sqrt{\left(\rho + \delta + \frac{m-1+\lambda}{1-\lambda}\sigma^2 - \frac{1}{\kappa}\right)^2 + \frac{4A}{\kappa}}}{\frac{2}{\kappa}} \tag{22}$$

which gives the value of q and $\widehat{i}$ when $e = \infty$.

Now we solve for p. Using $\frac{\mu_R^h - r}{\sigma_R^h} = \frac{m\sigma}{1-\lambda}$ we know that $\mu_R^h = \rho + \widehat{i} + \frac{m-1+\lambda}{1-\lambda}\sigma^2$. Since $\frac{\frac{\phi}{1-\phi}\left(A - \delta - \widehat{i} - \frac{\kappa\widehat{i}^2}{2}\right)}{p} + \widehat{i} = \mu_R^h$ by definition, we have

$$p = \frac{\frac{(1-\phi)}{\phi}\left(A - \delta - \widehat{i} - \frac{\kappa\widehat{i}^2}{2}\right)}{\rho + \frac{m-1+\lambda}{1-\lambda}\sigma^2}. \tag{23}$$

Numerically, instead of (22) and (22) we impose the slope conditions $p'(\infty) = q'(\infty) = 0$ which gives more stable solutions.

10.2.2 Lower entry barrier

Consider the boundary condition at $\underline{e}$ which is a reflecting barrier due to linear technology of entry. More specifically, at the entry boundary $\underline{e}$, we have

$$dE = m\theta(e_t)\left[dR_t^{agg} - r_t dt\right] E_t dt + dU_t$$

where dU_t reflects E_t at $\underline{e}K$. Heuristically, suppose that at $\underline{E} = \underline{e}K$, a negative shock ϵ sends E to $\underline{e}K - \epsilon$ which is below $\underline{e}K$. Then immediately there will be βx unit of physical capital to be converted into x units of E, so that the new level $\widehat{E} = \underline{e}K - \epsilon + x = \underline{e}\widehat{K} = \underline{e}(K - \beta x)$. This implies that the amount of capital to be converted to E is $x = \frac{\epsilon}{1+\underline{e}\beta} > 0$, and the new capital is

$$\widehat{K} = K - \beta x = K - \beta\frac{\epsilon}{1+\underline{e}\beta}$$

This result is useful in characterizing $p'(\underline{e})$.

Now we give the boundary conditions for p and q. First, although entry reduces physical capital K, since q is measured as per unit of K, the price should not change during entry. Therefore we must have $q'(\underline{e}) = 0$. The story for scaled housing price p is different. When entry lowers the aggregate physical capital K, equilibrium consumption, as well as equilibrium housing rents, going forward is lower, which translates to a lower P directly. Formally, right after the negative shock described above, the housing price is $p\left(\frac{E}{K}\right)K$, which can be rewritten as

$$p\left(\frac{E}{K}\right)K = p\left(\underline{e} - \frac{\epsilon}{K}\right)K.$$

This must equal the housing price $p(\underline{e})\widehat{K} = p(\underline{e})\left(K - \beta\frac{\epsilon}{1+\underline{e}\beta}\right)$ right after the adjustment, i.e.

$$p(\underline{e})\left(K - \beta\frac{\epsilon}{1+\underline{e}\beta}\right) = p\left(\underline{e} - \frac{\epsilon}{K}\right)K = p(\underline{e})K - p'(\underline{e})\epsilon$$

where we have used the fact that ϵ can be arbitrarily small in the continuous-time limit. As a result, implying that

$$p'(\underline{e}) = \frac{p(\underline{e})\beta}{1+\underline{e}\beta} > 0.$$

Define $\xi \equiv \frac{p(\underline{e})\beta}{1+\underline{e}\beta}$. In numerical solution instead of imposing β, we directly impose the following boundary conditions for equilibrium pricing functions

$$p'(\underline{e}) = \xi \text{ and } q'(\underline{e}) = 0. \tag{24}$$

We will treat ξ as our primitive parameters to match the housing price volatility in crisis time.

10.2.3 Numerical method

Given (24), the following results is useful. We know that at $\underline{e}$ the equilibrium Sharpe ratio is (recall $w(e) = p(e) + q(e)$)

$$\gamma = \sigma m \theta(\underline{e}) \frac{w(\underline{e}) - \underline{e} w'(\underline{e})}{w(\underline{e}) - \underline{e} m \theta(\underline{e}) w'(\underline{e})} = \sigma m \frac{w(\underline{e})}{\underline{e}} \frac{w(\underline{e}) - \underline{e} w' \xi}{w(\underline{e}) - \underline{e} m \frac{w(\underline{e})}{\underline{e}} \xi} = \sigma m \frac{w(\underline{e}) - \underline{e}\xi}{\underline{e}(1 - m\xi)}$$

which implies that

$$p(\underline{e}) + Q(\underline{e}) = w(\underline{e}) = \frac{\gamma \underline{e}(1 - m\xi)}{\sigma m} + \underline{e}\xi. \tag{25}$$

Based on (25) numerically we use the following 2-layer loops to solve the ODE system in (10.1) with endogenous entry boundary $\underline{e}$.

1. In the inner loop, we fix $\underline{e}$. Consider different trials of $q(\underline{e})$; given $q(\underline{e})$, we can get $p(\underline{e}) = \frac{\gamma \underline{e}(1-m\xi)}{\sigma m} + \underline{e}\xi - q(\underline{e})$. Then based on the four boundary conditions

$$p(\underline{e}), q(\underline{e}), p'(\infty) = q'(\infty) = 0,$$

   we can solve this 2-equation ODE system with boundary conditions using the *Matlab* builtin ODE solver bvp4c. We then search for the right $q(\underline{e})$ so that $p'(\underline{e}) - q'(\underline{e}) = \xi$ holds.

2. In the outer loop, we search for appropriate $\underline{e}$. For each trial of $\underline{e}$, we take the inner loop, and keep searching until $q'(\underline{e}) = 0$.

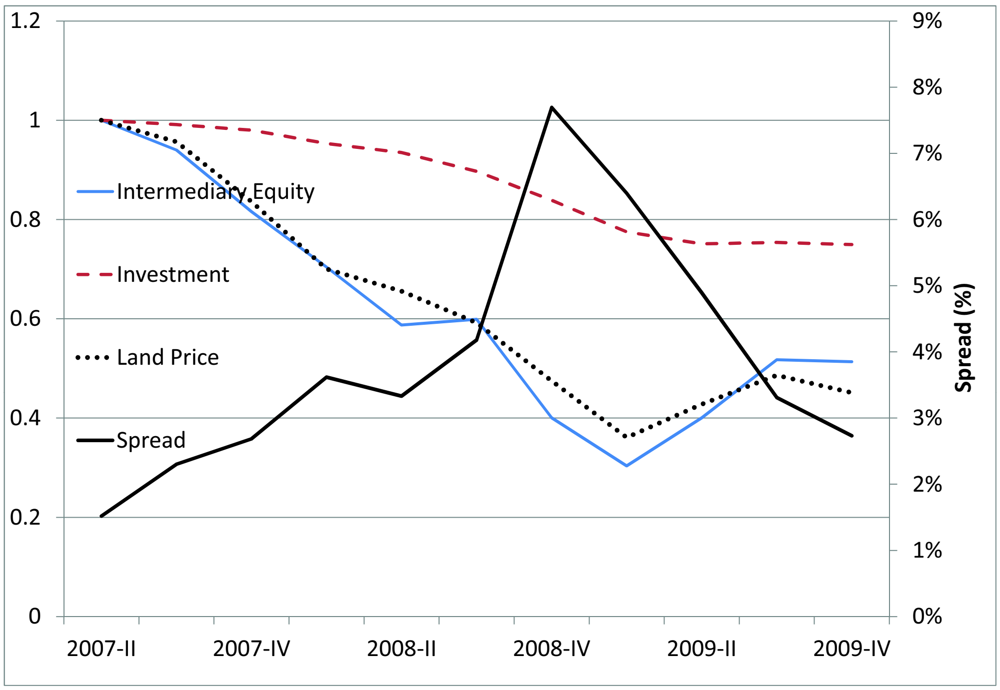
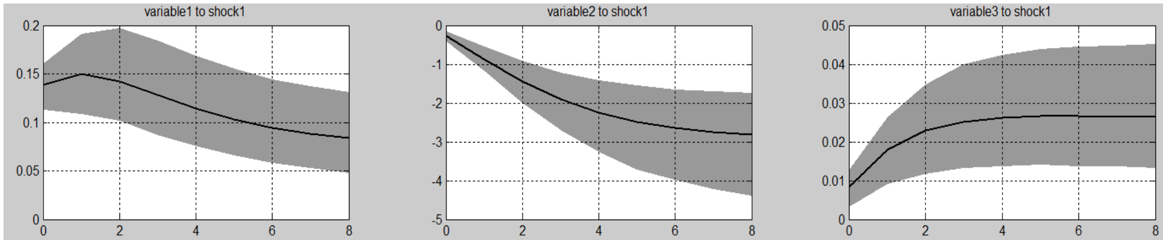
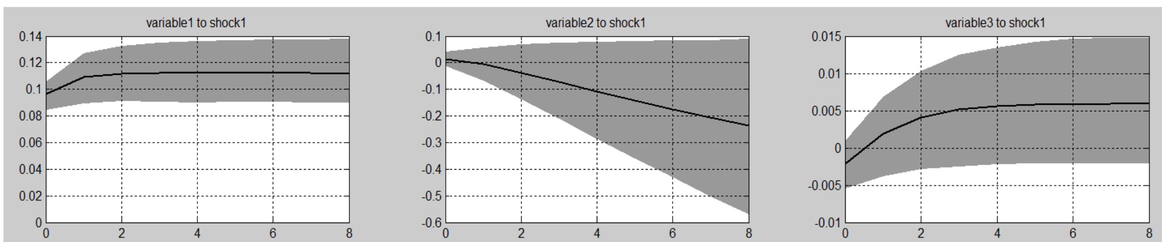


Figure 1: Data from 2007 to 2009. Intermediary equity, investment, land price index are on left-axis. Variables are scaled by their initial values in 2007Q2. Excess bond premium (labeled spread) is on right-axis, with no scaling adjustment.

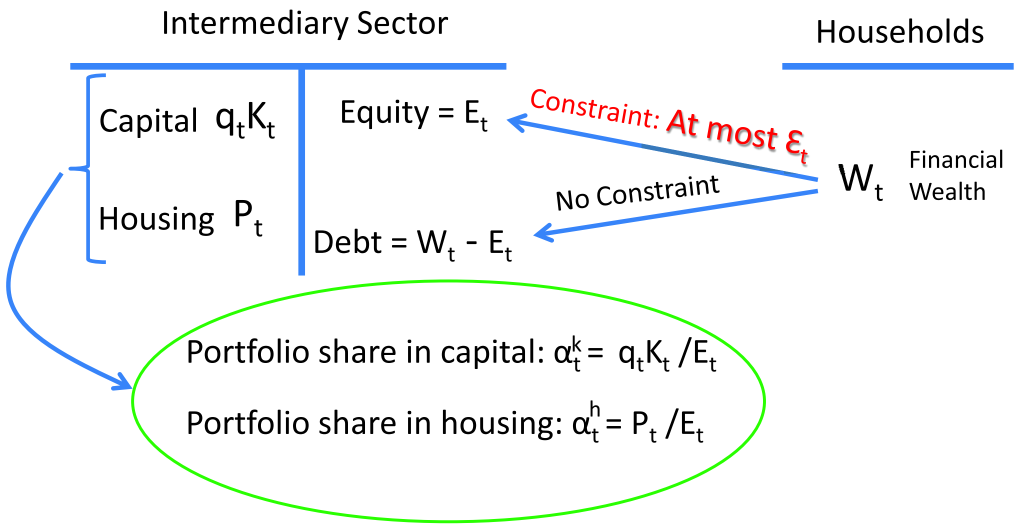


Panel A: Distress Periods



Panel B: Non-distress Periods

Figure 2: Impulse responses to a one-standard deviation shock to intermediary equity growth on intermediary equity growth (first column), EBS-Sharpe (second column), and investment growth (third-column). Responses are cumulated. Time is quarterly. We present results for distress and non-distress periods as classified in Table 1.



Return on equity : $d\tilde{R}_t = \alpha_t^k dR_t^k + \alpha_t^h dR_t^h + (1 - \alpha_t^k - \alpha_t^h) r_t$

Banker solves : $\max_{\alpha_t^k, \alpha_t^h} \mathbb{E}_t[d\tilde{R}_t] - \frac{m}{2} Var_t[d\tilde{R}_t]$

Figure 3: Model Summary

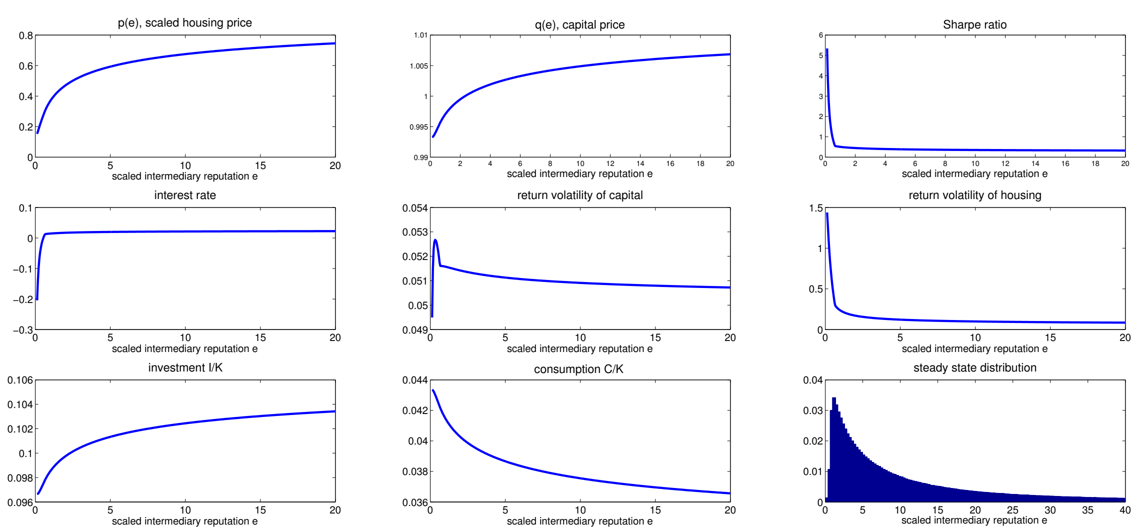


Figure 4: Price and Policy Functions for Baseline Parameters

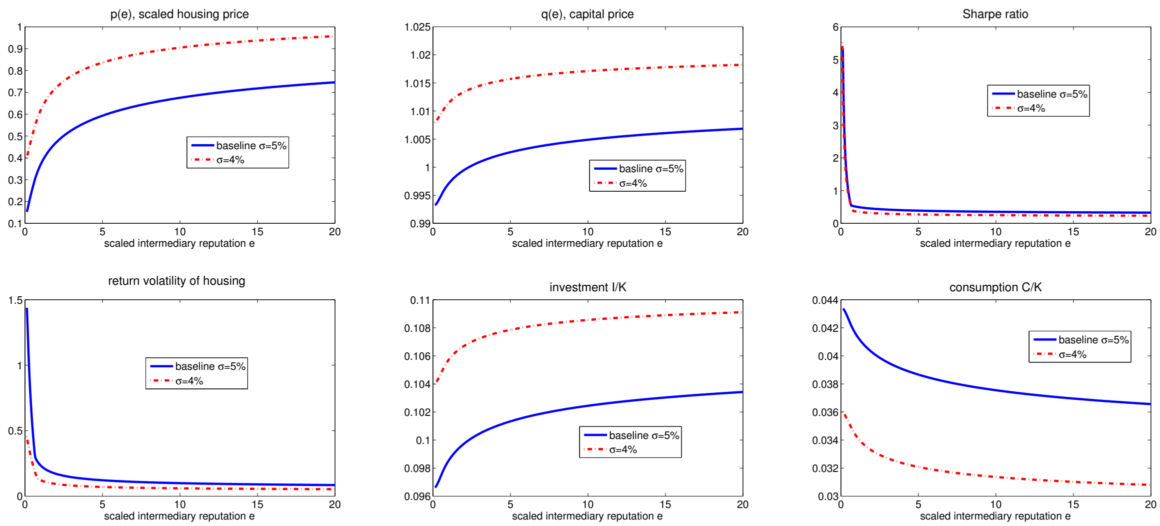


Figure 5: Price and Policy Functions for $\sigma = 4\%$ Case

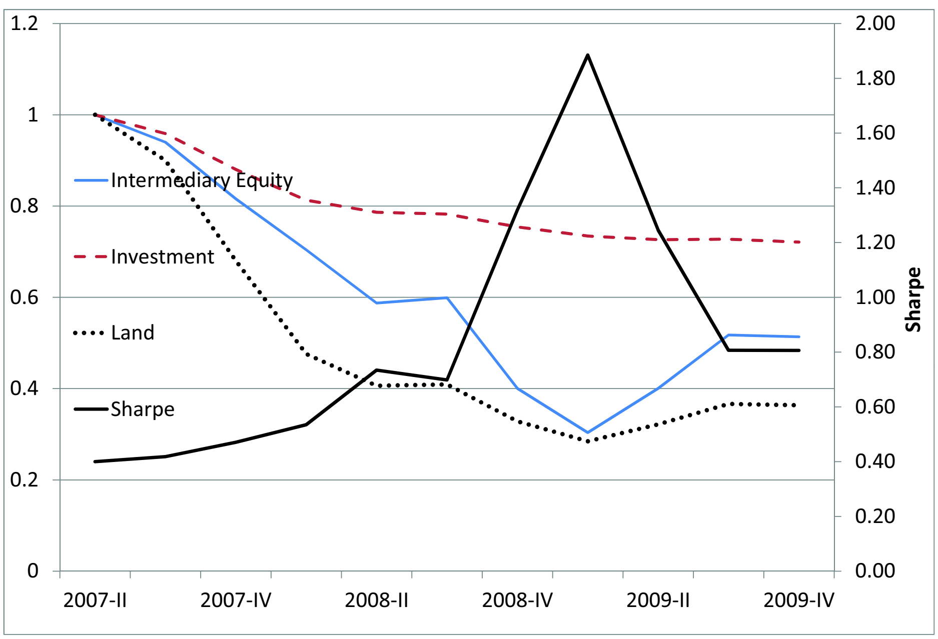


Figure 6: Model simulation matching data from 2007 to 2009. Intermediary equity, investment, land price index are on left-axis. Variables are scaled by their initial values in 2007Q2. Sharpe ratio is on right-axis, with no scaling adjustment.