A Macroeconomic Framework for Quantifying Systemic Risk

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April 2012

Systemic Risk and Nonlinearity

- The economy occasionally can arrive in a state where
  - financial intermediation is disrupted
  - small fundamental shocks can have quantitatively large effects on macro economy
- This is **systemic risk**; for example, as in the recent crisis

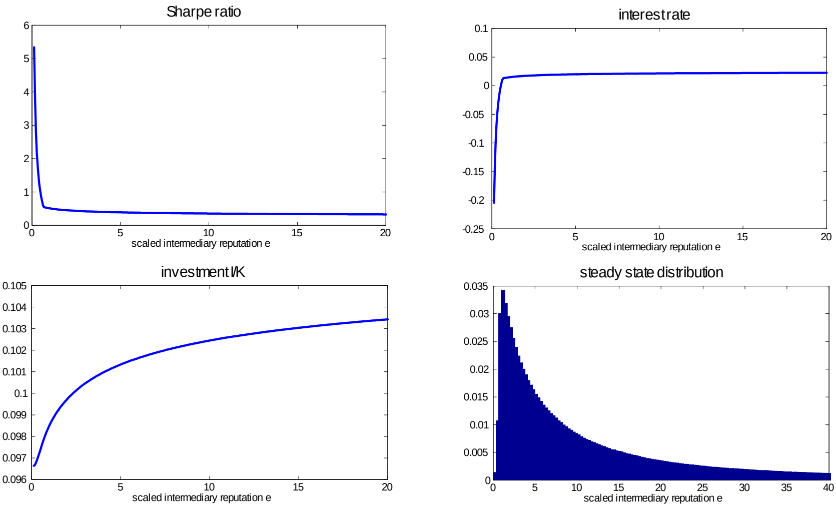
Systemic Risk and Nonlinearity

- The economy occasionally can arrive in a state where
  - financial intermediation is disrupted
  - small fundamental shocks can have quantitatively large effects on macro economy
- This is **systemic risk**; for example, as in the recent crisis
- Goal: Write down a calibratable DSGE model with nonlinearity, in which
  - much of the time the link between financial intermediation and macro economy is small
  - but at (crisis) times the effects are greatly amplified
- Quantitatively evaluate the model predictions, including the transition between normal and systemic states

Innovation Relative to Much of Literature

- We study a model with occasionally binding financial constraint
- Typical models (e.g., Kiyotaki-Moore (1997),...) linearize around steady state where constraint binds.
  - Cannot talk about 1) likelihood that intermediation is disrupted (its always disrupted...) and 2) how severe it is disrupted
- Our model solution has stochastic steady state, with fully solved equilibrium prices and policies
  - Main drawback: need to reduce state variables
  - Have to leave out some common DSGE elements
- Similar methodology to Mendoza (2010) and Brunnermeier-Sannikov (2011)
- Model elements adopted from He-Krishnamurthy (2012), with real investment and housing

Preview of model result



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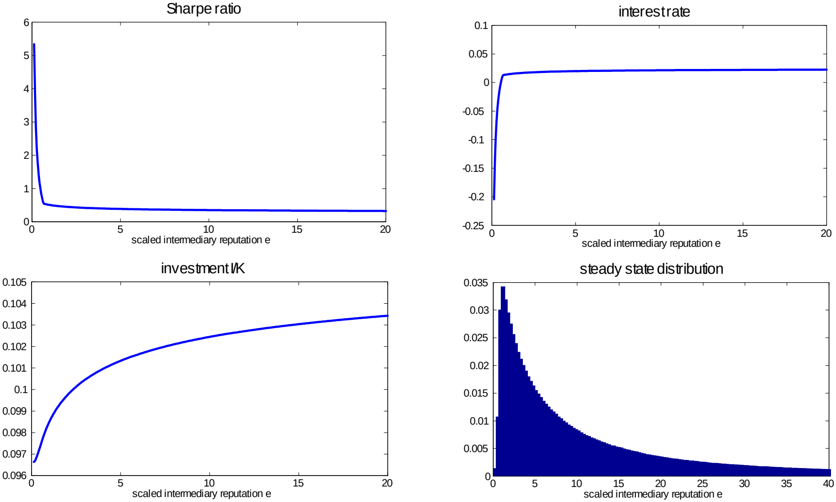


- **Crisis:** $e_{crisis} = 0.65$, binding capital constraint
- **Distress:** $e_{distress} = 4$ so that $\Pr\left(e \leq e_{distress}\right) = 33\%$ as in data

Evidence of Non-Linearity

- **Excess bond premium** (EBP): the risk premium part of credit spread (removing default part), Gilchrist and Zakrajsek (2010). Correlates with measures of intermediary health.
- Use EBP to classify distress periods (33%) and non-distress periods (the rest)

Distress Periods	NBER Recessions
1975Q1 - 1975Q3	11/73 - 3/75
1982Q2 - 1982Q4	7/81 - 11/82
1985Q4 - 1987Q3	
1988Q4 - 1990Q1	7/90 - 3/91
1992Q4 - 1993Q2	
2001Q2 - 2003Q1	3/01 - 11/01
2007Q3 - 2009Q3	12/07 - 6/09

- Caveat: distress periods are much "milder" relative to 07/08 crisis

State-Dependent Covariances (1)

- Equity = Total market value of equity of finance, insurance and real estate sectors. (works as well if only include banks + broker/dealers)
- All variables are growth, except Sharpe ratio constructed from EBP

	Distress		Non Distress	
	Cov	Corr	Cov	Corr
Equity, Investment	1.31%	51.48	0.07	5.79
Equity, Consumption	0.25%	45.85	0.03	14.74
Equity, Sharpe	-6.81%	-35.96	-0.14	-0.06
Equity, Landprice	4.06%	60.65	0.12	0.07

State-Dependent Covariances (2)

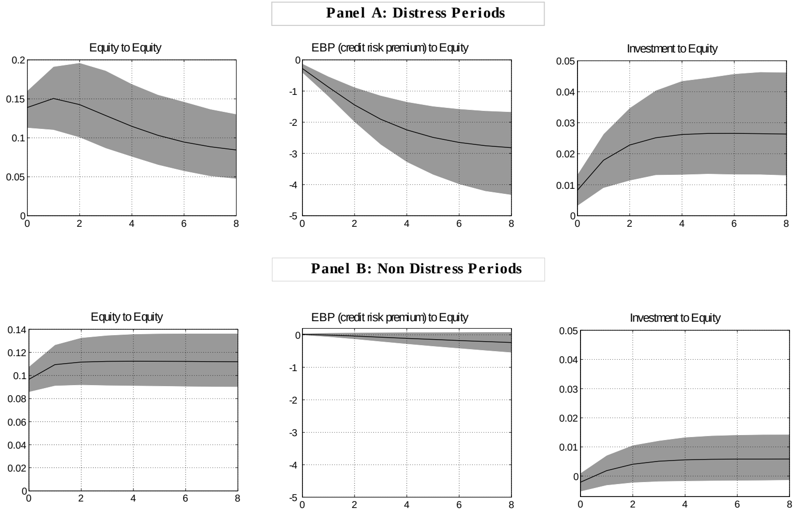
All variables are growth, except Sharpe ratio constructed from EBP

	Distress		Non Distress	
	NBER+2	Excl-Crisis	NBER+2	Excl-Crisis
Equity, *Investment*	31.78%	22.10	17.11	17.26
Equity, *Consumption*	0.13%	0.04	0.01	0.03
Equity, *Sharpe*	-7.57%	-2.12	-0.78	-0.19
Equity, *Landprice*	4.39%	-0.63	-0.31	-0.01

Note: Similar numbers if only use NBER dates, but sample is only 20% of observations.

VAR Evidence of Non-Linearity (3)

VAR order: [intermediary equity, EBP, investment]. Coefficients depend on distress state



Road Map of the Rest of Talk

- Model, mechanism, and solution
- Calibration
  - Baseline parameters
  - Prices and polices, comparative statics
- Matching data on distress and non-distress (i.e. mild crises)
- Systemic crisis
  - Extrapolate to crisis state
  - Uncover fundamental shocks in the recent crisis
  - How likely are crises?
- Advantage of structural models: we have essentially one data point with deep systemic crisis

Agents and Technology

- Two classes of agents: households and bankers
  - Households own the entire economy, but subject to frictions related to bankers who control intermediaries (next slide)
- Two types of capital: productive capital K_t and housing capital H. Fixed supply of housing $H \equiv 1$
  - Price of capital q_t and price of housing P_t determined in equilibrium

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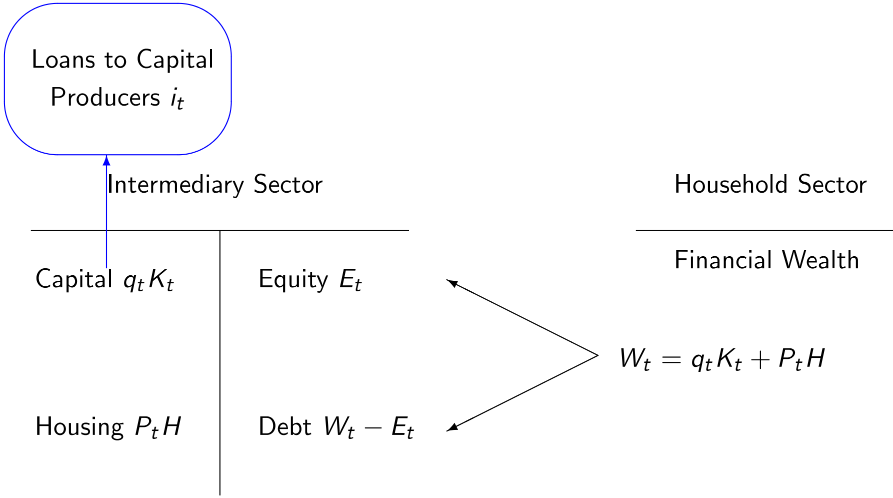
- Two types of capital: productive capital K_t and housing capital H. Fixed supply of housing $H \equiv 1$
  - Price of capital q_t and price of housing P_t determined in equilibrium
- Production $Y = AK_t$, with A being constant
- Fundamental shocks: stochastic capital quality shock dZ_t

$$\frac{dK_t}{K_t} = i_t dt - \delta dt + \sigma dZ_t$$

- Investment/Capital i_t, quadratic adjustment cost

$$\Phi(i_t, K_t) = i_t K_t + \frac{\kappa}{2} (i_t - \delta)^2 K_t$$

Aggregate Balance Sheet



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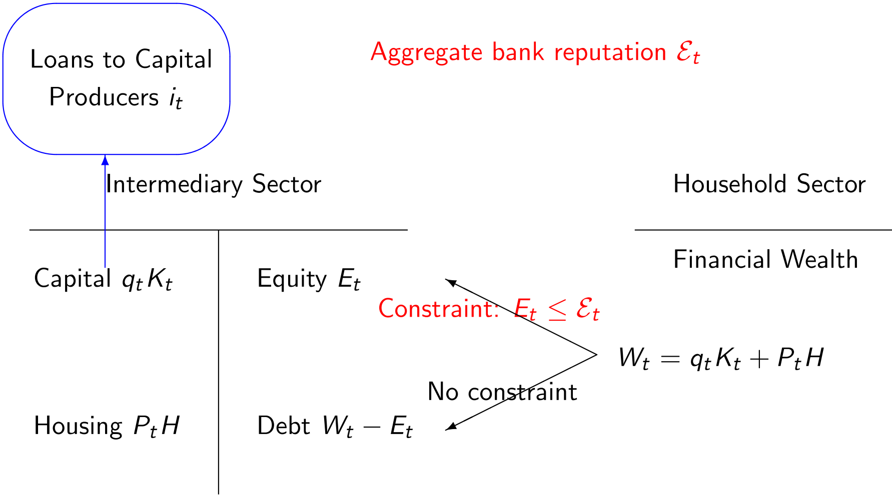

Single Bank/Banker

Capital $q_t k_t$	Equity e_t
Housing $P_t h_t$	Debt d_t

Portfolio share in capital: $\alpha_t^k = \frac{q_t k_t}{e_t}$

Portfolio share in housing : $\alpha_t^h = \frac{P_t h_t}{e_t}$

Borrowing (no constraint): $d_t = q_t k_t + P_t h_t - e_t = (\alpha_t^k + \alpha_t^h - 1) e_t$

Return on bank equity: $d\tilde{R}_t = \alpha_t^k dR_t^k + \alpha_t^h dR_t^h - (\alpha_t^k + \alpha_t^h - 1) r_t dt$

Banker (log preference) solves: $\max_{\alpha_t^k, \alpha_t^h} E[d\tilde{R}_t - r_t dt] - \frac{m}{2} Var_t[d\tilde{R}_t]$

Single Bank/Banker

Assets	Liabilities
Capital $q_t k_t$	Equity e_t
Housing $P_t h_t$	Debt d_t

Properties

- (k, h) scale up with e
- (k, h) increasing in $E_t[dR - r]$
- (k, h) decreasing in $Var[dR]$

Portfolio share in capital: $\alpha_t^k = \frac{q_t k_t}{e_t}$

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General Equilibrium (1)

Intermediary Sector			Household Sector
Capital $q_t K_t$	Equity E_t	Constraint: $E_t \leq \mathcal{E}_t$	Financial Wealth
Housing $p_t H$	Debt $W_t - E_t$		$W_t = q_t K_t + p_t H$

Portfolio share in capital: $\alpha_t^k = \frac{q_t K_t}{E_t}$

Portfolio share in housing: $\alpha_t^h = \frac{P_t H}{E_t}$

- Given a particular state $(K_t, \mathcal{E}_t)$, the portfolio shares are pinned down by GE
- Portfolio shares must also be optimally chosen by banks

$$\max_{\alpha_t^k, \alpha_t^h} \ \mathbb{E}_t[d\tilde{R}_t - r_t dt] - \frac{m}{2} Var_t[d\tilde{R}_t]$$

General Equilibrium (2)

Intermediary Sector

Capital $q_t K_t$	Equity E_t
Housing $p_t H$	Debt $W_t - E_t$

Constraint: $E_t \leq \mathcal{E}_t$

Household Sector

Financial Wealth
$W_t = q_t K_t + p_t H$

Portfolio share in capital: $\alpha_t^k = \frac{q_t K_t}{E_t}$

Portfolio share in housing: $\alpha_t^h = \frac{P_t}{E_t}$

- Prices (returns) have to adjust for optimality:
  - $\mathbb{E}_t[dR_t^h - r_t dt], \mathbb{E}_t[dR_t^k - r_t dt] \Rightarrow$ equations for $\mathbb{E}_t[dP_t], \mathbb{E}_t[dq_t]$
- Rewrite to get ODEs for $P(K, \mathcal{E})$ and $q(K, \mathcal{E})$
- Scale invariance: Define $e \equiv \mathcal{E}/K$; then $P = Kp(e)$ and $q(e)$

Reputations and Capital Constraint

- Single bank has reputation ϵ_t linked to intermediary performance (constant m)

$$\frac{d\epsilon_t}{\epsilon_t} = m\tilde{R}_t.$$

  - Poor returns reduce reputation!
- Households invest a maximum of ϵ_t dollars of equity capital with this banker

Reputations and Capital Constraint

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  - Poor returns reduce reputation!
- Households invest a maximum of ϵ_t dollars of equity capital with this banker
- Death rate η, and entry $d\psi_t > 0$ of new bankers in extreme states (modeled later)
- $\mathcal{E}_t$: aggregate reputation. Identical banks, aggregate dynamics of $\mathcal{E}_t$

$$\frac{d\mathcal{E}_t}{\mathcal{E}_t} = md\tilde{R}_t - \eta dt + d\psi_t$$

Bankers' Problem

- Banker may die at a Poisson rate of η which is effective discount rate
- Banker maximizes his expected reputation: $\mathbb{E}\left[\int_0^\infty e^{-\eta t} \ln \epsilon_t dt\right]$
  - Bankers do not consume goods – preserves some "representative household" features.
- With log preferences, banker makes portfolio decision (for intermediary) to

$$\max_{\alpha_t^k, \alpha_t^h} \quad \mathbb{E}_t[d\tilde{R}_t - r_t dt] - \frac{m}{2} Var_t[d\tilde{R}_t]$$

Here, m parameterizes banker's "risk aversion"

Capital Producers and Investment

- Capital goods producers (owned by households) undertake real investment
- Producers must sell the capital stock to intermediaries at price q_t
  - Risk averse intermediaries bear aggregate fundamental shocks
  - Real investment is affected by financial condition of intermediaries to capture "credit crunch"
- Possible interpretations:
  - Entrepreneurs raise capital from VC/PE at the price of q_t
  - Commercial banks makes collateralized loans
- Investment decision

$$\max_{i_t} \; q_t i_t K_t - \Phi(i_t, K_t) \;\Rightarrow\; i_t = \delta + \frac{q_t - 1}{\kappa}$$

Households' Problem (1)

- Choose consumption c_t^y and housing c_t^h to maximize

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t}\left((1-\phi)\ln c_t^y + \phi \ln c_t^h\right) dt\right]$$

- Equilibrium rental price D_t (housing asset dividend), FOC $\frac{c_t^h D_t}{\phi} = \frac{c_t^y}{1-\phi}$. In equilibrium ($C_t^h = H = 1$)

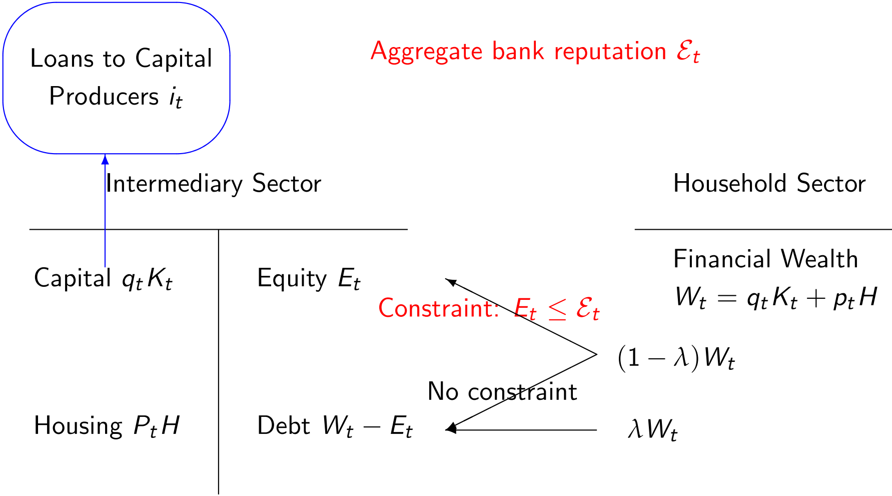
$$D_t = \frac{\phi}{1-\phi} C_t^y$$

  - ϕ: expenditure share in housing, or the relative size of housing sector
- Interest rate $r_t = \rho + \mathbb{E}_t\left[dC_t^y \,/\, C_t^y\right] - Var_t\left[dC_t^y \,/\, C_t^y\right]$

Households' Problem (2)

- Representative household enters time t with financial wealth W_t
- The household splits wealth: $(1-\lambda) W_t$ to "equity households," λW_t to "bond households"
  - Equity households invest their portion of wealth as equity of intermediaries, **subject to capital frictions**
  - Bond households invest in riskless bonds
- Once returns are realized, both members pool their wealth again (as in Lucas 1990)
- The only role of bond households (i.e. parameter λ) is to introduce intermediary's leverage in normal time

Model Scheme



Equity Capital Constraint

- Equity households wish to buy equity of at most $(1-\lambda)W_t$
- Intermediary equity capital E is given by

$$E_t = \min\left[\mathcal{E}_t, (1-\lambda)W_t\right]$$

- If $\mathcal{E}_t > (1-\lambda)W_t$ then equity capital constraint is slack

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- If $\mathcal{E}_t > (1-\lambda)W_t$ then equity capital constraint is slack
- How can capital constraint come to bind, if now $\mathcal{E}_t > (1-\lambda)W_t$?
- Suppose a -10% shock to real estate and price of capital, so that $W_t \downarrow 10\%$ (Household wealth $=$ aggregate wealth)
- Reputation follows $\frac{d\mathcal{E}_t}{\mathcal{E}_t} = m d\tilde{R}_t + ...$ Two forces make $\mathcal{E}_t \downarrow$ more than 10%:
  - If $m > 1$
  - $d\tilde{R}_t < -10\%$: $d\tilde{R}_t$ is a levered claim on assets (bond households)

Boundary Conditions

- When $e = \infty$, $\mathcal{E}_t > (1 - \lambda)\, W_t$ frictionless economy
  - We solve for $p(\infty), q(\infty)$ analytically
- As $e \to 0$, intermediaries' portfolio volatility, i.e. Sharpe ratio, rises
- New bankers enter if $e = \underline{e}$ (Sharpe ratio hits γ, exogenous constant)
  - Entry increases aggregate $\mathcal{E}$ but requires physical capital K at conversion rate of β
  - $\underline{e}$ is a reflecting boundary
- Boundary conditions at the entry point $\underline{e}$

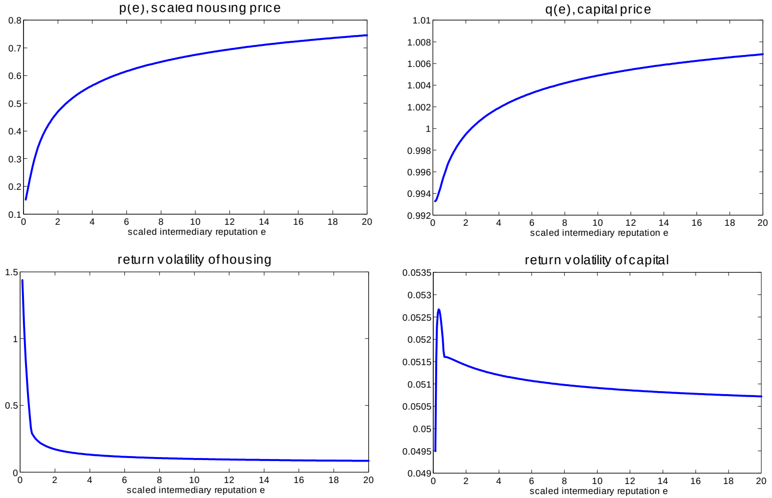
$$q'\left(\underline{e}\right) = 0,\ p'\left(\underline{e}\right) = \frac{p\left(\underline{e}\right)\beta}{1 + \underline{e}\beta},\ \text{and}\ Sharpe\_Ratio\left(\underline{e}\right) = \gamma$$

Calibration: Baseline Parameters

	Parameter	Choice	Target
Panel A: Intermediation			
m	Performance sensitivity	2.5	Average Sharpe ratio (38%)
λ	Debt ratio	0.5	Average intermediary leverage
η	Banker exit rate	13%	Good model dynamics
γ	Entry trigger	5.5	Highest Sharpe ratio
β	Entry cost	1.9	Land price volatility
Panel B: Technology			
σ	Capital quality shock	5%	Investment and Consumption volatilities
δ	Depreciation rate	10%	Literature
κ	Adjustment cost	2	Literature
A	Productivity	0.14	Investment-to-capital ratio
Panel C: Others			
ρ	Time discount rate	2%	Literature
ϕ	Housing share	0.5	Housing-to-wealth ratio

Equilibrium Prices and Policies (1)

- $e_{crisis} = 0.65$: binding capital constraint
- $e_{distress} = 4$ so that $\Pr\left(e \leq e_{distress}\right) = 33\%$ as in data



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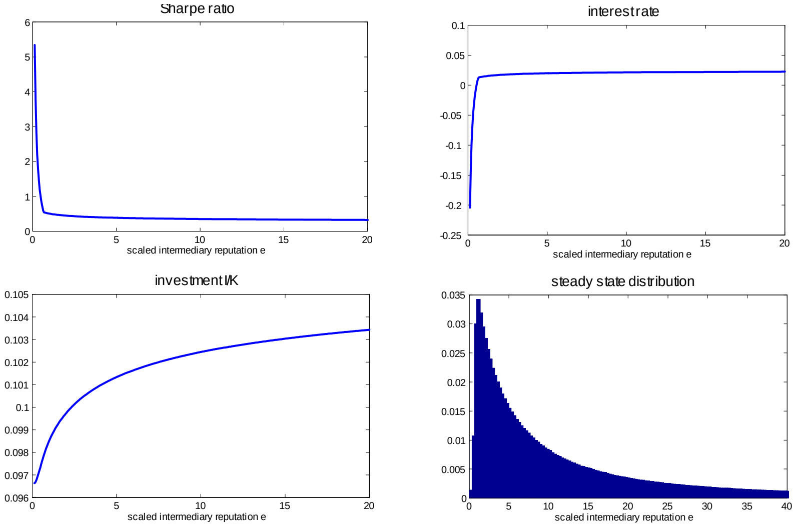


Matching State-Dependent Covariances: Baseline

	Distress		Non Distress	
	Data	Baseline	Data	Baseline
vol (*Eq*)	31.48%	26.13	17.54	6.84
vol (*I*)	8.05%	**5.74**	6.61	**5.40**
vol (*C*)	1.71%	**3.60**	1.28	**3.92**
vol (*LP*)	21.24%	**22.87**	9.79	9.61
vol (*EB*)	60.14%	49.65	12.72	6.65
cov (*Eq*, *I*)	1.31%	0.81	0.07	0.37
cov (*Eq*, *C*)	0.25%	0.35	0.03	0.26
cov (*Eq*, *LP*)	4.06%	5.00	0.12	0.65
cov (*Eq*, *EB*)	-6.81%	-6.86	-0.14	-0.09

Matching State-Dependent Covariances: lower σ

	Distress			Non Distress		
	Data	Baseline	$\sigma = 4\%$	Data	Baseline	$\sigma = 4\%$
$vol\,(Eq)$	31.48%	26.13	20.78	17.54	6.84	5.18
$vol\,(I)$	8.05%	5.74	4.42	6.61	5.40	4.19
$vol\,(C)$	1.71%	3.60	2.71	1.28	3.92	3.32
$vol\,(LP)$	21.24%	22.87	14.05	9.79	9.61	6.65
$vol\,(EB)$	60.14%	49.65	37.77	12.72	6.65	4.49
$cov\,(Eq, I)$	1.31%	0.81	0.47	0.07	0.37	0.22
$cov\,(Eq, C)$	0.25%	0.35	0.22	0.03	0.26	0.17
$cov\,(Eq, LP)$	4.06%	5.00	2.27	0.12	0.65	0.34
$cov\,(Eq, EB)$	-6.81%	-6.86	-3.82	-0.14	-0.09	-0.04

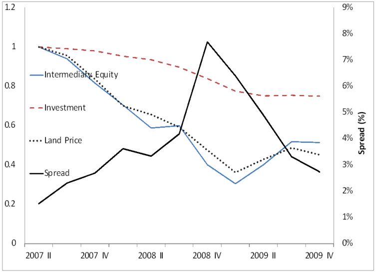
Matching State-Dependent Covariances: No Housing

	Distress			Non Distress		
	Data	Baseline	$\phi = 0$	Data	Baseline	$\phi = 0$
$vol\,(Eq)$	31.48%	26.13	14.62	17.54	6.84	5.00
$vol\,(I)$	8.05%	5.74	5.14	6.61	5.40	5.01
$vol\,(C)$	1.71%	3.60	4.52	1.28	3.92	4.94
$vol\,(LP)$	21.24%	22.87		9.79	9.61	
$vol\,(EB)$	60.14%	49.65	9.42	12.72	6.65	0.03
$cov\,(Eq, I)$	1.31%	0.81	0.53	0.07	0.37	0.25
$cov\,(Eq, C)$	0.25%	0.35	0.44	0.03	0.26	0.25
$cov\,(Eq, LP)$	4.06%	5.00		0.12	0.65	
$cov\,(Eq, EB)$	-6.81%	-6.86	-0.33	-0.14	-0.09	-0.00

Uncovering Shocks in the Recent Crisis

Data

Model



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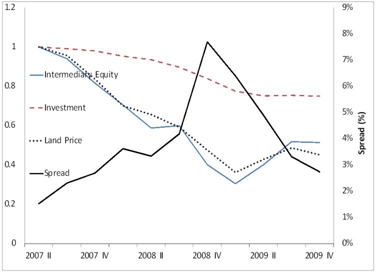

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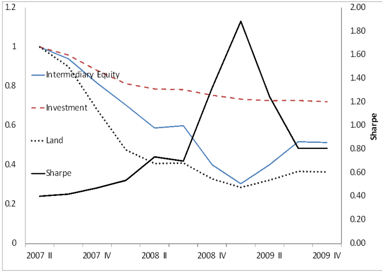


- Based on realized equity return we uncover fundamental shocks to K

07QIII	07QIV	08QI	08QII	08QIII	08QIV	09QI	09QII	09QIII	09QIV
-3.69%	-7.06	-6.46	-2.78	-0.52	-3.05	-2.34	-1.26	-0.14	-0.80

- Total -25%. Capital constraint binds after 08QII—systemic crisis
  - In the model (data), land price fall by 71% (55%)

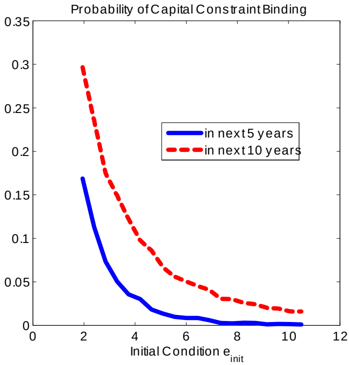
Systemic Crisis and Consequences

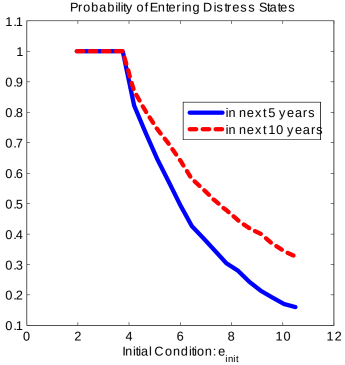
- Economic variables in systemic crisis. Recall $e_{crisis} = 0.65$

	At mean e	X4	X8	X16
Sharpe Ratio	0.327	1.308	2.616	5.232
Prob Sharpe being higher	71.79%	0.31	0.09	0.03
Equity (E/K)	0.87	0.40	0.26	0.13
Housing (P/K)	0.74	0.24	0.19	0.15
Capital (q)	1.0068	0.9943	0.9936	0.9933
Investment (I/K)	10.34%	9.71	9.68	9.66
Interest rate	2.44%	-2.61	-7.76	-19.98
Consumption growth	0.40%	-4.57	-9.72	-21.68

Probability of Crisis

- Conditional probability of hitting crisis (left) or distress (right)





- 2007Q2, Prob(crisis occurs in the next 2 years)=0.13%

Conclusion

- We develop a fully stochastic model of systemic crisis, with two major frictions:
  - Equity capital constraint on intermediary sector
  - Intermediaries have substantial holdings in real assets (physical capital or housing)
- We find that the model
  - not only qualitatively delivers the nonlinearity observed in the data
  - but also quantitatively matches the differential comovements in distress and non-distress periods
- Recent 07/08 crisis requires a cumulative negative shock around -25%
- Things we are working on: more on model-based measure of systemic risk