High Hopes and Disappointment

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Preference for timing of information arrival: motivations

Ordinary expected utility (von Neumann-Morgenstern preferences): agent cares only about the distribution of consumption itself, not about arrival of information about consumption.

Why might an agent care about arrival of information?

- Primitive preferences: an agent may be less stressed if consumption surprises are learned well in advanced, or an agent may be excited and happy if information comes at the last minute.
- Household production: having information earlier may be valuable because it helps an agent to make better choices. For example, knowing how much consumption expenditure you will make a year from now may make spending on a cruise more efficient because you can get a discount from booking earlier.

Recursive Utility: Kreps-Porteus, Epstein-Zin, Seldon

Kreps and Porteus [1978, 1979] provided an axiomatic formulation for preferences that are similar to von Neumann-Morgenstern preferences but allow for preference for time resolution of uncertainty. Their axioms are similar to axioms used by other people except that there is a time element, and lotteries resolving at different times cannot be substituted. They have an additional assumption that preferences about consumption from some node forward depends only on what happens going forward, and this assumption implies a recursive structure that looks like a nonlinear version of the backwards equation of dynamic programming (which would be linear given von Neumann-Morgenstern preferences).

What is different in our formulation

Our preference assumption is different than recursive utility because we do not assume that preferences are only forward-looking along the subtree from the node where we start. For recursive utility,

- Preferences looking forward do not depend on past consumption. (This rules out models of habit formation such as Constantinides [1990] or Dybvig [1995].)
- Preferences looking forward do not depend on what anticipations the agent had in the path, i.e., on what was expected to happen in the rest of the tree not in the current subtree.

The second part of the assumption is what we are relaxing. Two agents with the same wealth and prospects may feel and behave differently depending on whether this wealth level is better or worse than anticipated.

An Example (not vN-M or Machina)

Scenario 1

$$
\begin{tikzcd}[column sep=small]
 & & & c_{uu} = 4 \\
 & L_u = 3 \arrow[urr] \arrow[drr] & & \\
 & & & c_{ud} = 3 \\
L = 1 \arrow[uur] \arrow[ddr] & & & \\
 & & & c_{du} = 2 \\
 & L_d = 1 \arrow[urr] \arrow[drr] & & \\
 & & & c_{dd} = 1
\end{tikzcd}
$$

Scenario 2

$$
\begin{tikzcd}[column sep=small]
 & & & c_{uu} = 4 \\
 & L_u = 2 \arrow[urr] \arrow[drr] & & \\
 & & & c_{ud} = 2 \\
L = 1 \arrow[uur] \arrow[ddr] & & & \\
 & & & c_{du} = 3 \\
 & L_d = 1 \arrow[urr] \arrow[drr] & & \\
 & & & c_{dd} = 1
\end{tikzcd}
$$

$$(1-p)(U(L) + \frac{1}{2}(U(L_u) + U(L_d))) + \frac{p}{4}(U(c_{uu}) + U(c_{ud}) + U(c_{du}) + U(c_{dd}))$$

$$\pi_{uu} = \pi_{ud} = \pi_{du} = \pi_{dd} = \frac{1}{4}$$

Another Example (not Kreps-Porteus)

Scenario 5

$$\begin{tikzcd}[row sep=tiny]
 & & c_{uu} = 4 \\
 & L_u = 1 \arrow[ur] \arrow[dr] & \\
 & & c_{ud} = 1 \\
L = 1 \arrow[uur] \arrow[ddr] & & \\
 & & c_{du} = 2 \\
 & L_d = 2 \arrow[ur] \arrow[dr] & \\
 & & c_{dd} = 2
\end{tikzcd}$$

Scenario 7

$$\begin{tikzcd}[row sep=tiny]
 & & c_{uu} = 4 \\
 & L_u = 1 \arrow[ur] \arrow[dr] & \\
 & & c_{ud} = 1 \\
L = 1 \arrow[uur] \arrow[ddr] & & \\
 & & c_{du} = 1 \\
 & L_d = 1 \arrow[ur] \arrow[dr] & \\
 & & c_{dd} = 1
\end{tikzcd}$$

Scenario 6

$$\begin{tikzcd}[row sep=tiny]
 & & c_{uu} = 2 \\
 & L_u = 2 \arrow[ur] \arrow[dr] & \\
 & & c_{ud} = 2 \\
L = 2 \arrow[uur] \arrow[ddr] & & \\
 & & c_{du} = 2 \\
 & L_d = 2 \arrow[ur] \arrow[dr] & \\
 & & c_{dd} = 2
\end{tikzcd}$$

Scenario 8

$$\begin{tikzcd}[row sep=tiny]
 & & c_{uu} = 2 \\
 & L_u = 2 \arrow[ur] \arrow[dr] & \\
 & & c_{ud} = 2 \\
L = 1 \arrow[uur] \arrow[ddr] & & \\
 & & c_{du} = 1 \\
 & L_d = 1 \arrow[ur] \arrow[dr] & \\
 & & c_{dd} = 1
\end{tikzcd}$$

$$(1-p)(U(L) + \frac{1}{2}(U(L_u) + U(L_d))) + \frac{p}{4}(U(c_{uu}) + U(c_{ud}) + U(c_{du}) + U(c_{dd}))$$

$$\pi_{uu} = \pi_{ud} = \pi_{du} = \pi_{dd} = \frac{1}{4}$$

The choice problem: simple case

Given W_0 and L_0, choose adapted $\{c_t\}$ and nondecreasing adapted $\{L_t\}$ to maximize
$E \int_0^\infty e^{-\rho t}\{\ (1-p)U(L_t) + pU(c_t)\ \}\, dt$
subject to:
$E[\int_{t=0}^\infty \xi_t c_t] = W_0$
$c_t \geq L_t$.

W_0: initial wealth
L_0: initial anticipation
c_t: rate of consumption at time t
L_t: anticipation at t = lower bound on future consumption
$\rho > 0$: given pure rate of time discount
$p \in [0,1]$: given weight inf preferences for anticipation
U concave increasing: utility function of consumption
ξ_t: state-price density, usual lognormal model

Less absolute and probably more reasonable model

Given W_0 and L_0, choose adapted $\{c_t\}$ and nondecreasing adapted $\{L_t\}$ to
maximize
$E \int_0^\infty e^{-\rho t} \{ \ (1-p)U(L_t) + p[\,U(c_t) - K(U(L_t) - U(c_t))^+\,] \ \} \, dt$
subject to:
$E[\int_{t=0}^\infty \xi_t c_t] = W_0$.

This specializes to the other problem when $K = \infty$.

Obviously, many other specifications are possible. This specification captures the economic ideas we are trying to model without being so complicated we cannot obtain a useful solution to portfolio problems. In particular, this specification adds only one state variable L more than recursive utility models, and is separable across levels of L into simpler problems involving stopping times.

Solution Strategy

Lagrangian:

$$E \int_0^\infty e^{-\rho t} \{ (1-p) U(L_t) + p[U(c_t) - K(U(L_t) - U(c_t))^+] - \lambda e^{\rho t} \xi_t c_t \} \, dt + \lambda w_0.$$

1. Given λ and the process for L, optimize over c_t and substitute the resulting value, a dual function, into the objective function.
2. Given λ optimize by L by integrating by parts and solving a family of very simple stopping problems.
3. Solve for λ using the budget constraint, an option pricing problem.

Solving for c

Terms of the Lagrangian involving c_t are

$$p[\,U(c_t) - K(U(L_t) - U(c_t))^+\,] - \lambda e^{\rho t}\xi_t c_t$$

The value from maximizing these terms over c_t is the dual function of the first term, evaluated at the second term. This can be written as

$$\Phi(\Lambda_t, L_t) = \begin{cases} p\tilde{U}(\Lambda_t) & (\ \Lambda_t < U'(L_t)\ ) \\ p(1+K)\tilde{U}\left(\frac{\Lambda_t}{1+K}\right) - pKU(L_t) & (\ \Lambda_t > (1+K)U'(L_t)\ ) \\ p(U(L_t) - L_t\Lambda_t) & \text{otherwise.} \end{cases}$$

where $\Lambda_t = \xi_t \lambda \exp(\rho t)/p$.

It is also easy to write down the optimal c_t, an option-like payoff.

Solving for L

$$V = E\int_0^\infty e^{-\rho t}\{\,(1-p)U(L_t) + \Phi(\Lambda_t, L_t)\,\}\,dt$$

We can write

$$(1-p)U(L_t) + \Phi(\Lambda_t, L_t) = (1-p)U(L_0) + \Phi(\Lambda_t, L_0) + \int_{L=L_0}^{L_t} ((1-p)U'(L) + \Phi'_L(\Lambda_t, L))dL$$

Substituting this into the value function and reversing the order of integration, we have that V is a constant plus

$$E[\int_{L=L_0}^\infty e^{-\rho\tau_L}\int_{\delta=0}^\infty e^{-\rho\delta}((1-p)U'(L) + \Phi_L(\Lambda_{t+\delta}, L))d\delta]$$

where $\tau_L = \inf\{t|L_t > L\}$

Solving for L: continued

Again, V is a constant plus

$$E[\int_{L=L_0}^{\infty} e^{-\rho\tau_L} \int_{\delta=0}^{\infty} e^{-\rho\delta}((1-p)U'(L) + \Phi_L(\Lambda_{t+\delta}, L))d\delta]$$

where $\tau_L = \inf\{t|L_t > L\}$. For each L onsider the optimal stopping problem

Choose τ_L to maximize

$$E[\int_{L=L_0}^{\infty} e^{-\rho\tau_L} E[\int_{\delta=0}^{\infty} e^{-\rho\delta}((1-p)U'(L) + \Phi_L(\Lambda_{t+\delta}, L))d\delta]]$$

The value multiplying $e^{-\rho\tau_L}$ is equal to $U'(L)$ (a constant) times a function of $\Lambda_t/U'(L)$, suggesting a solution has the simple form

$$\tau_x = \inf\{t : \Lambda_t \leq z_* U'(x)\}$$

Value function

Using mostly standard calculations (including a calculation of the resolvent of the state-price density ξ), the value function can be written as

$$V = \Theta(L_0) + \Lambda_0^2 \int_{\Lambda_0/U'(L_0)}^{\infty} y^{-3}\tilde{U}''(\Lambda_0/y)\bar{f}_1(y)\, dy,$$

where $\Theta(L_0)$ and $\bar{f}_1$ are known functions, with some simplification for power utility.

What are L_t and c_t?

Let

$$\Lambda_t = p^{-1} \lambda e^{\rho t} \xi_t$$

and

$$\eta_t \equiv \left(\frac{\inf_{0 \le s \le t} \Lambda_s}{z_*} \right) \wedge U'(L_0)$$

Then,

$$L_t^* = I(\eta_t)$$

$$c_t^* = \begin{cases} I(\Lambda_t), & \Lambda_t < \eta_t, \\ L_t^*, & \eta_t \le \Lambda_t \le (K+1)\eta_t \\ I\left(\frac{\Lambda_t}{K+1}\right), & (K+1)\eta_t \le \Lambda_t. \end{cases}$$

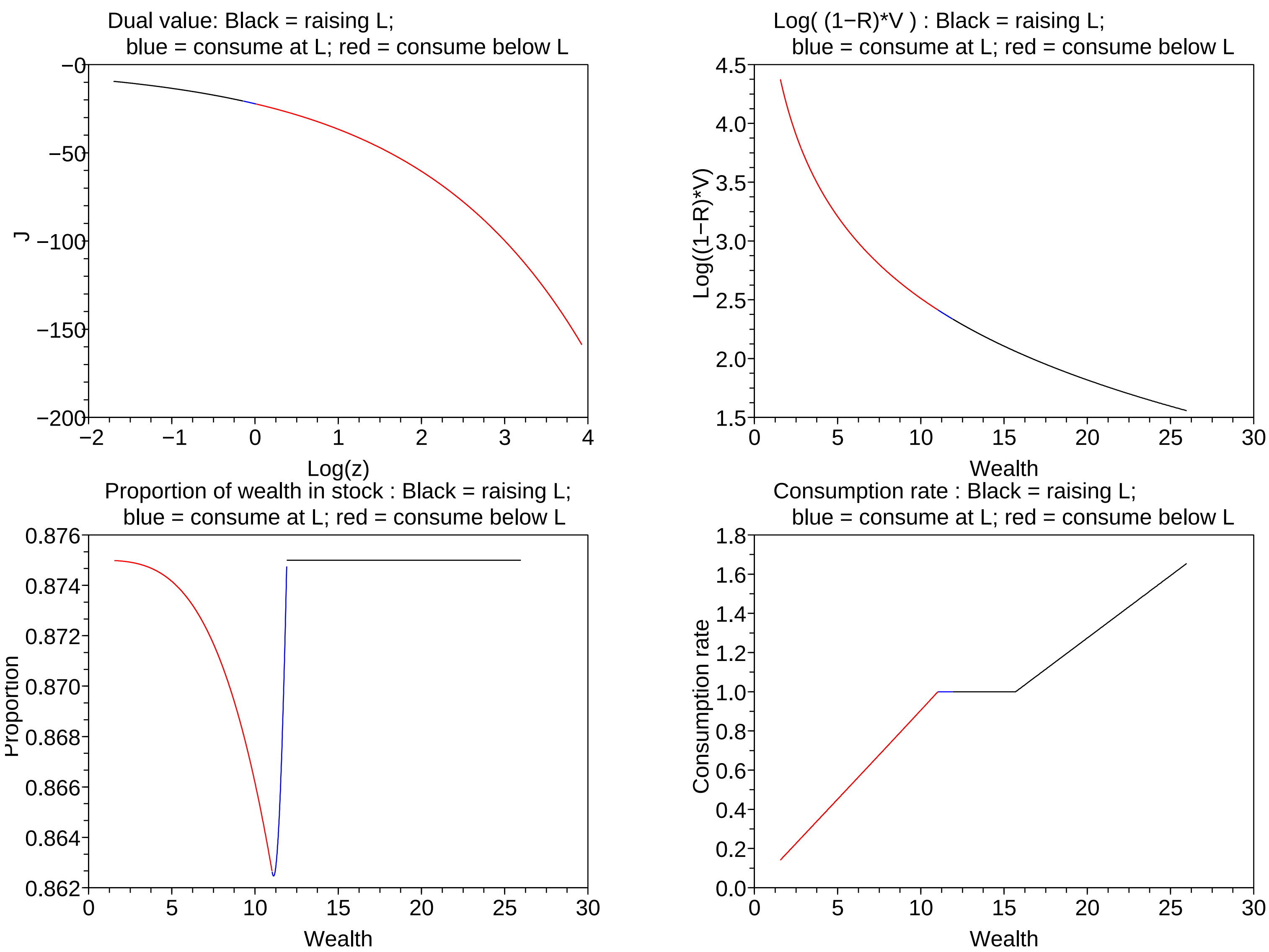
Solving for λ

Putting the explicit form for c_t into the budget constraint we can do a one-dimensional search provided we can solve the "option pricing problem" of valuing consumption to find wealth (and we can numerically). Probably we cannot obtain a closed-form solution because it is a somewhat messy lookback option.

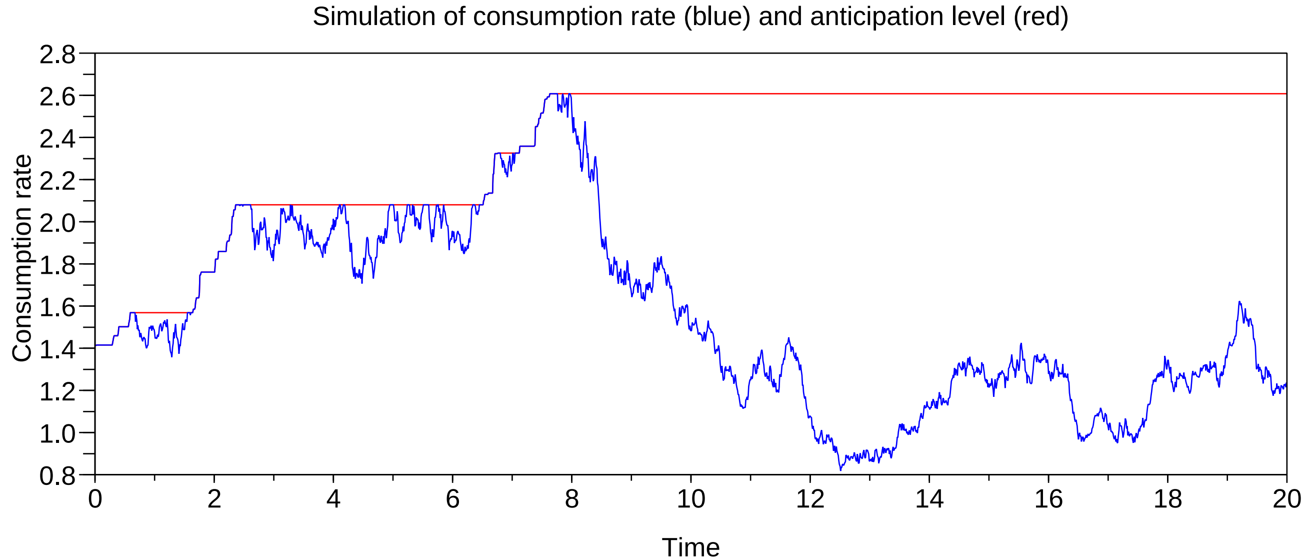
Parameters for the Plots

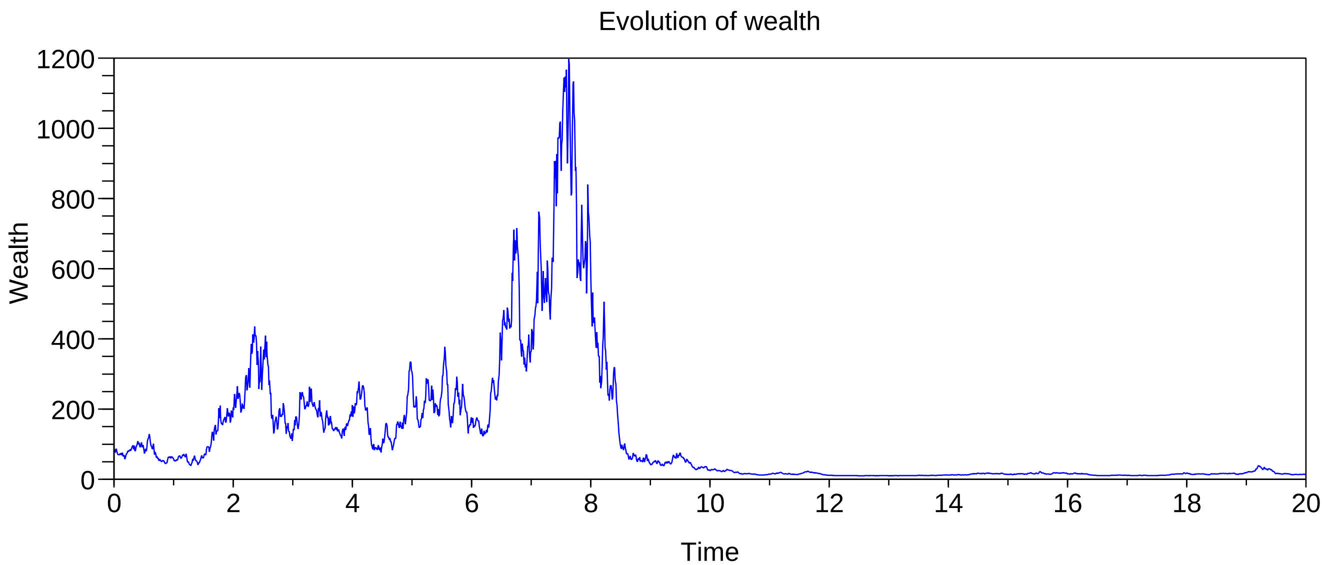
We shall suppose throughout that $\sigma = 0.2$, $\mu = 0.12$, $r = 0.05$, $\rho = 0.1$, $R = 2$, and that $L_0 = 1$. We fix the value of p to be 0.5, and consider what happens for different values of $K = 1.03, 2.03, 3.03, 43$. In the first two examples, the anticipation level is an aspiration, and consumption never exceeds L, whereas in the last two the anticipation level is in general a lower bound for consumption, though it may exceed consumption if things go badly.

Plots for $K = 1.03$

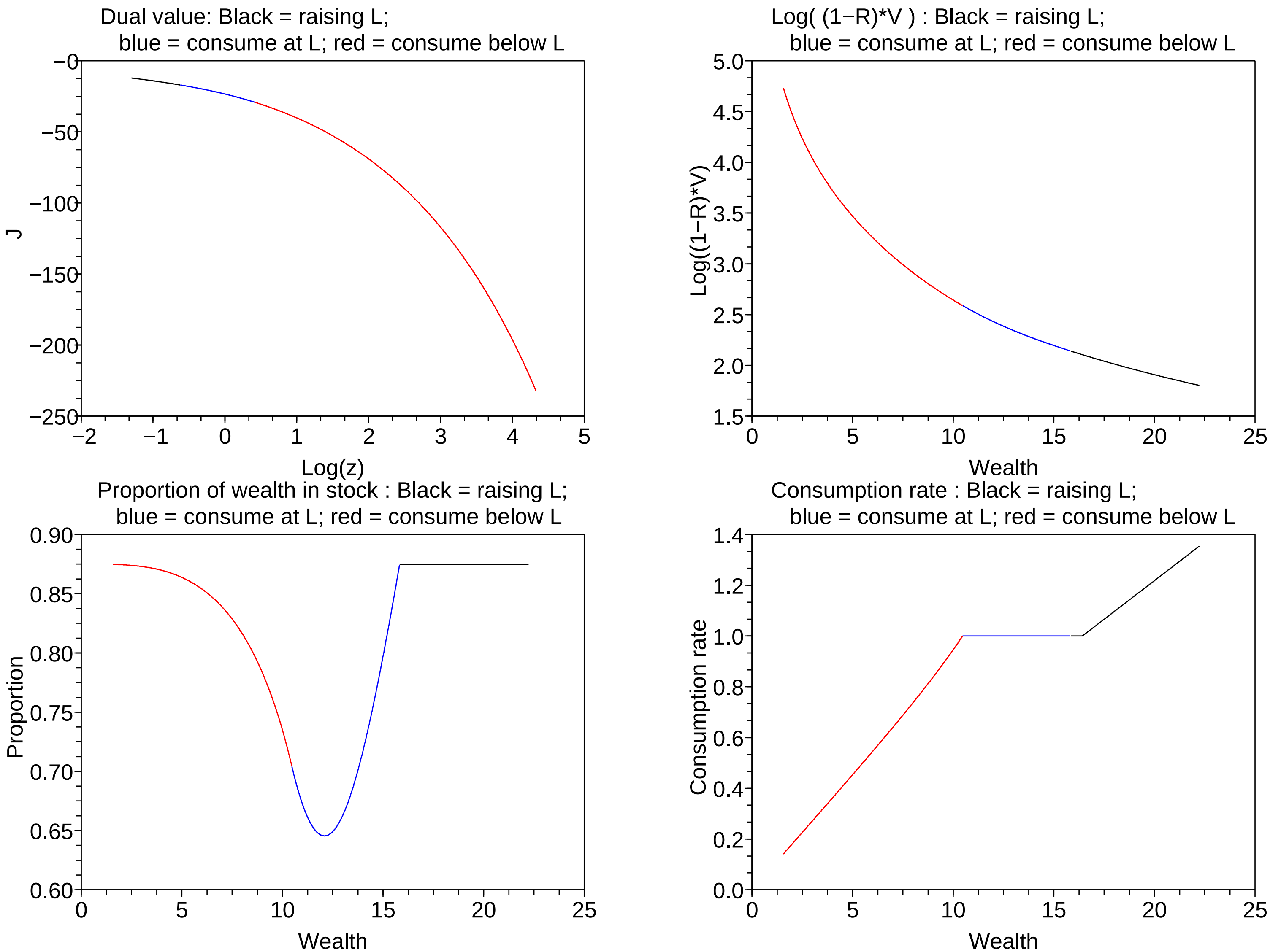


Plots for $K = 1.03$

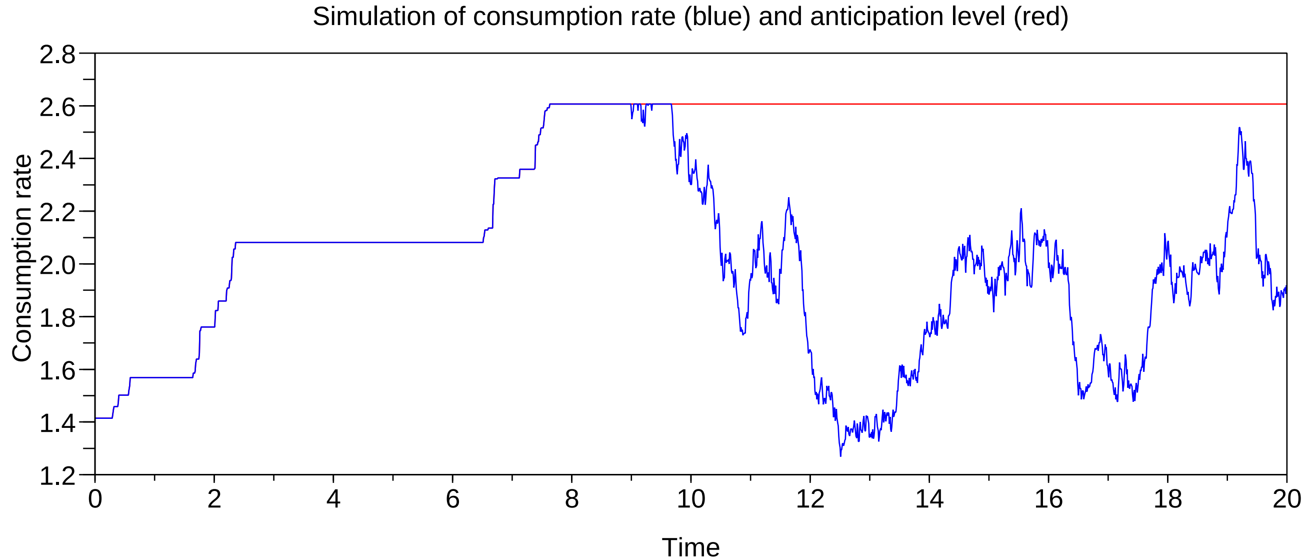


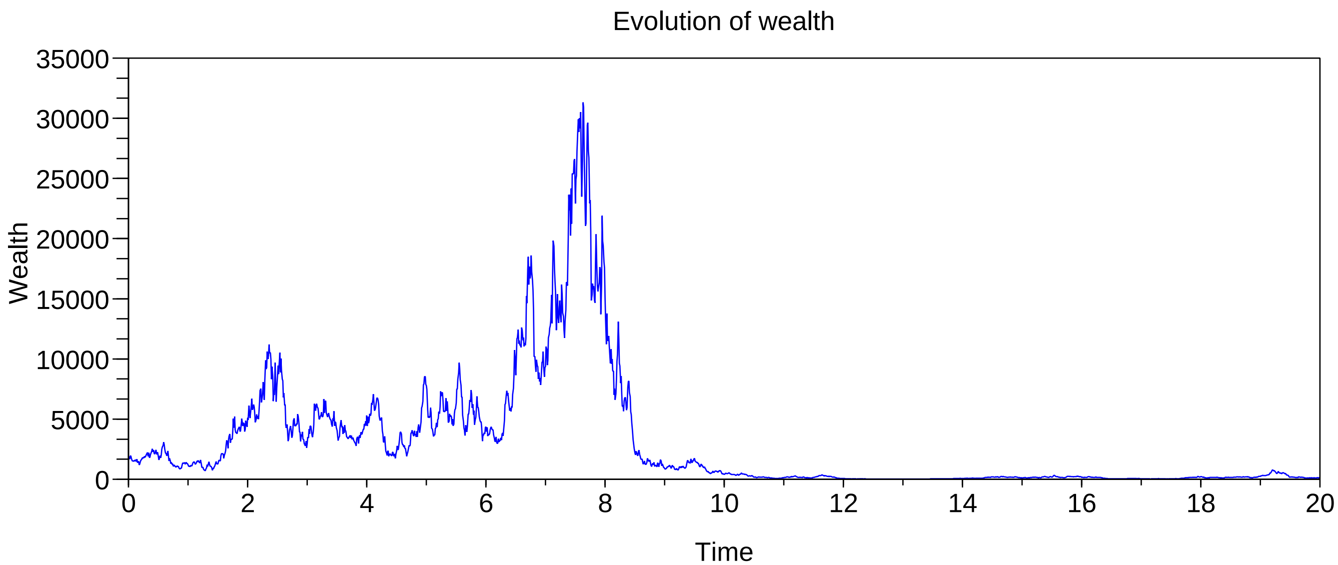


Plots for $K = 2.03$

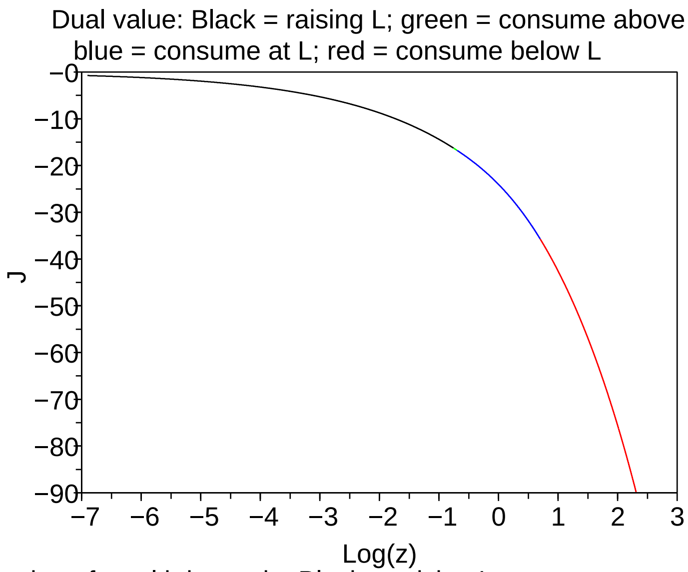


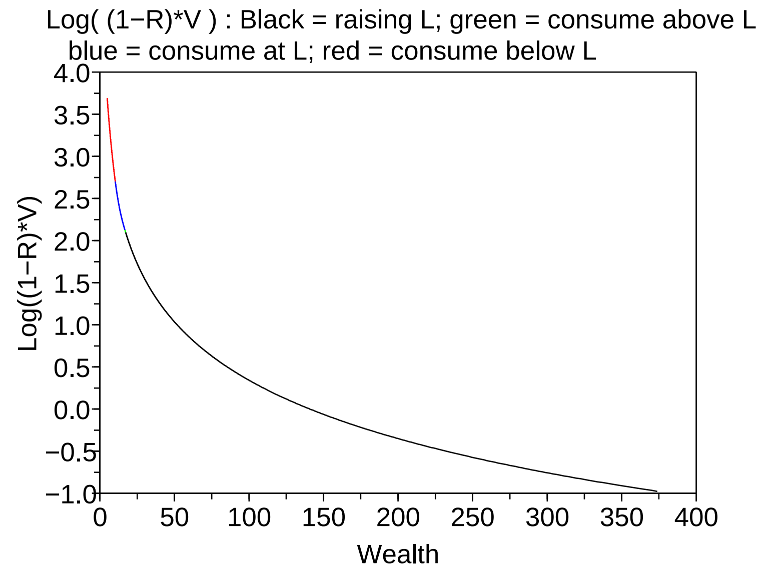
Plots for $K = 2.03$

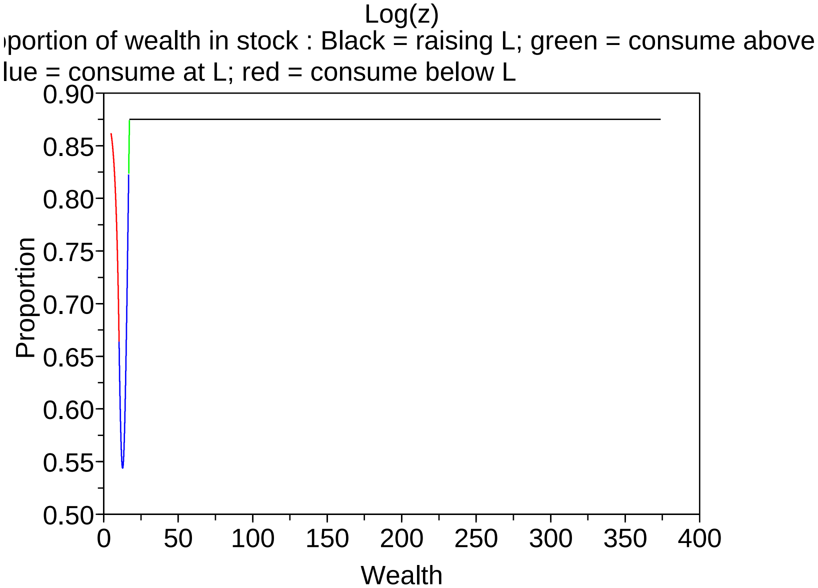


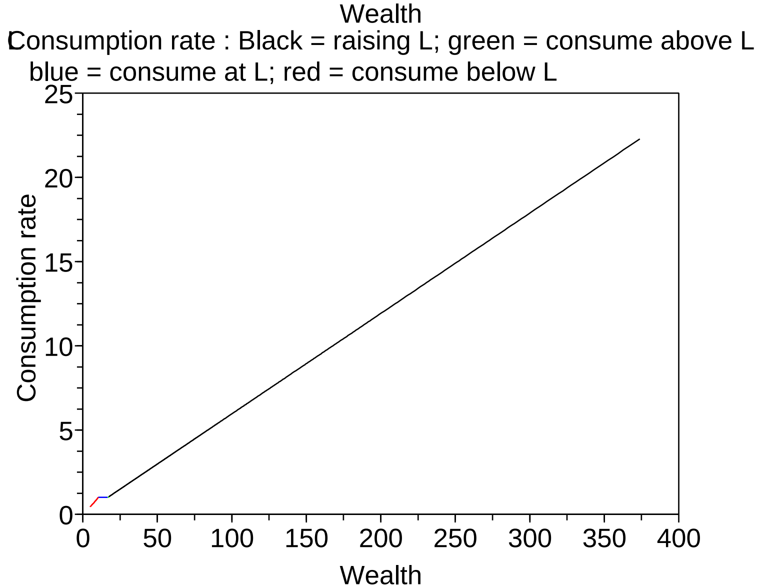


Plots for $K = 3.03$



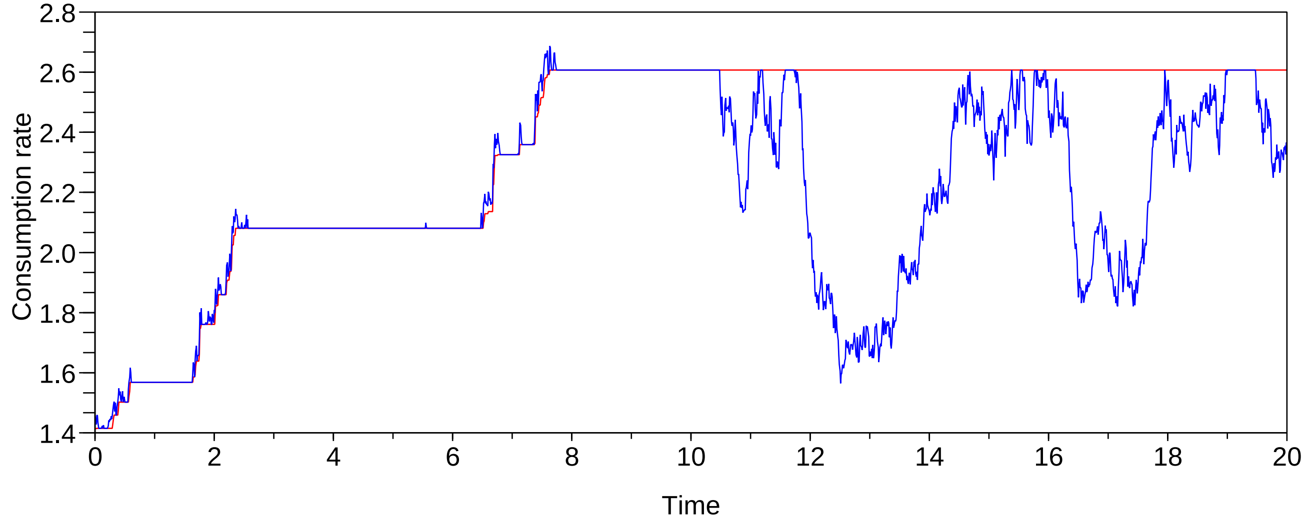




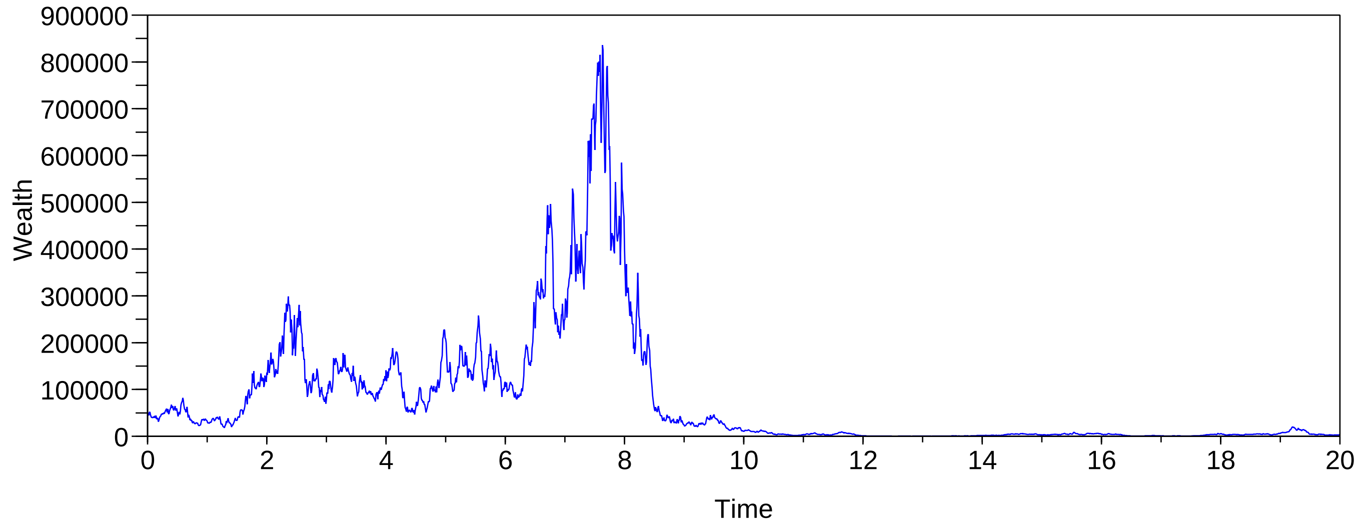


Plots for $K = 3.03$

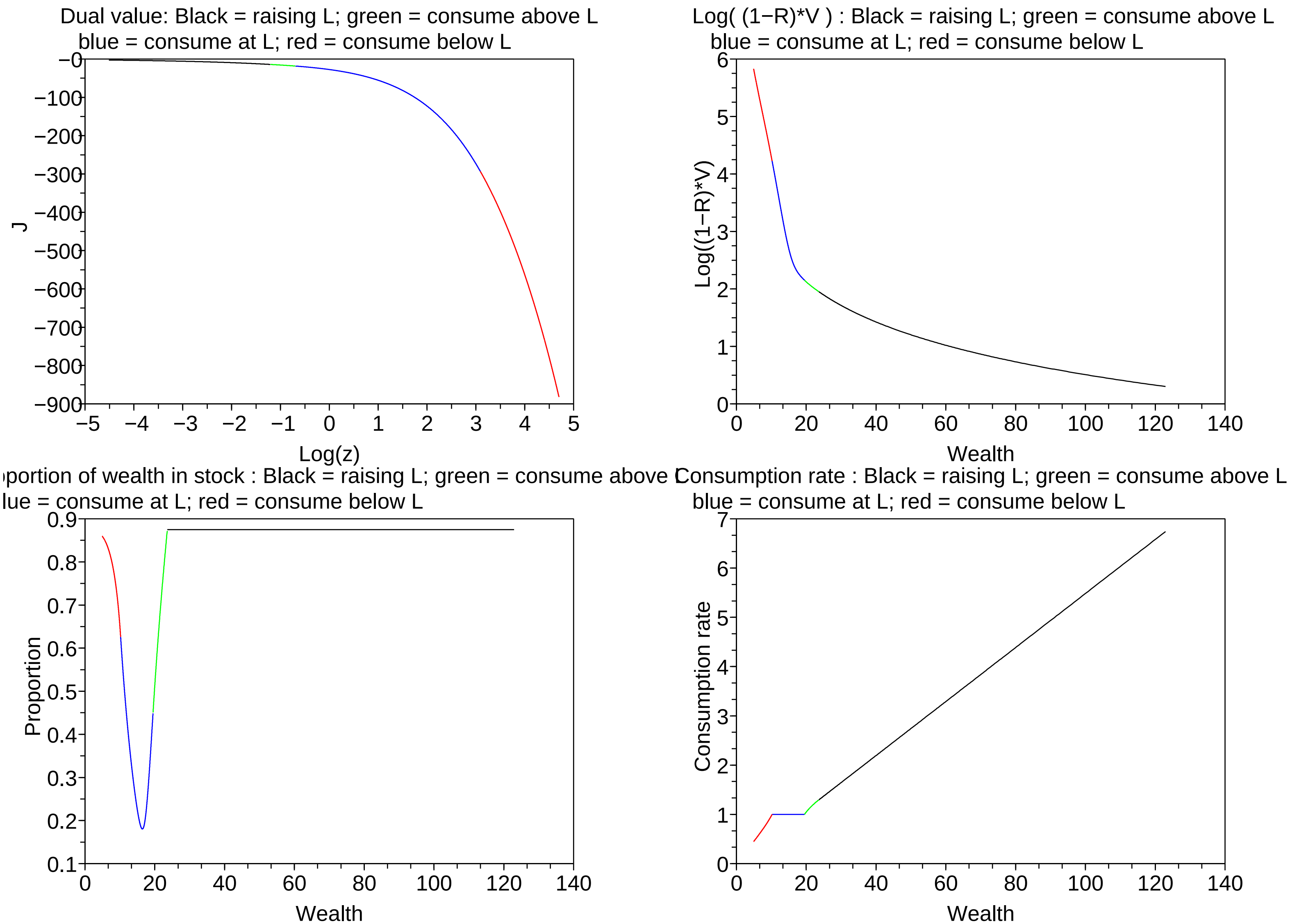
Simulation of consumption rate (blue) and anticipation level (red)



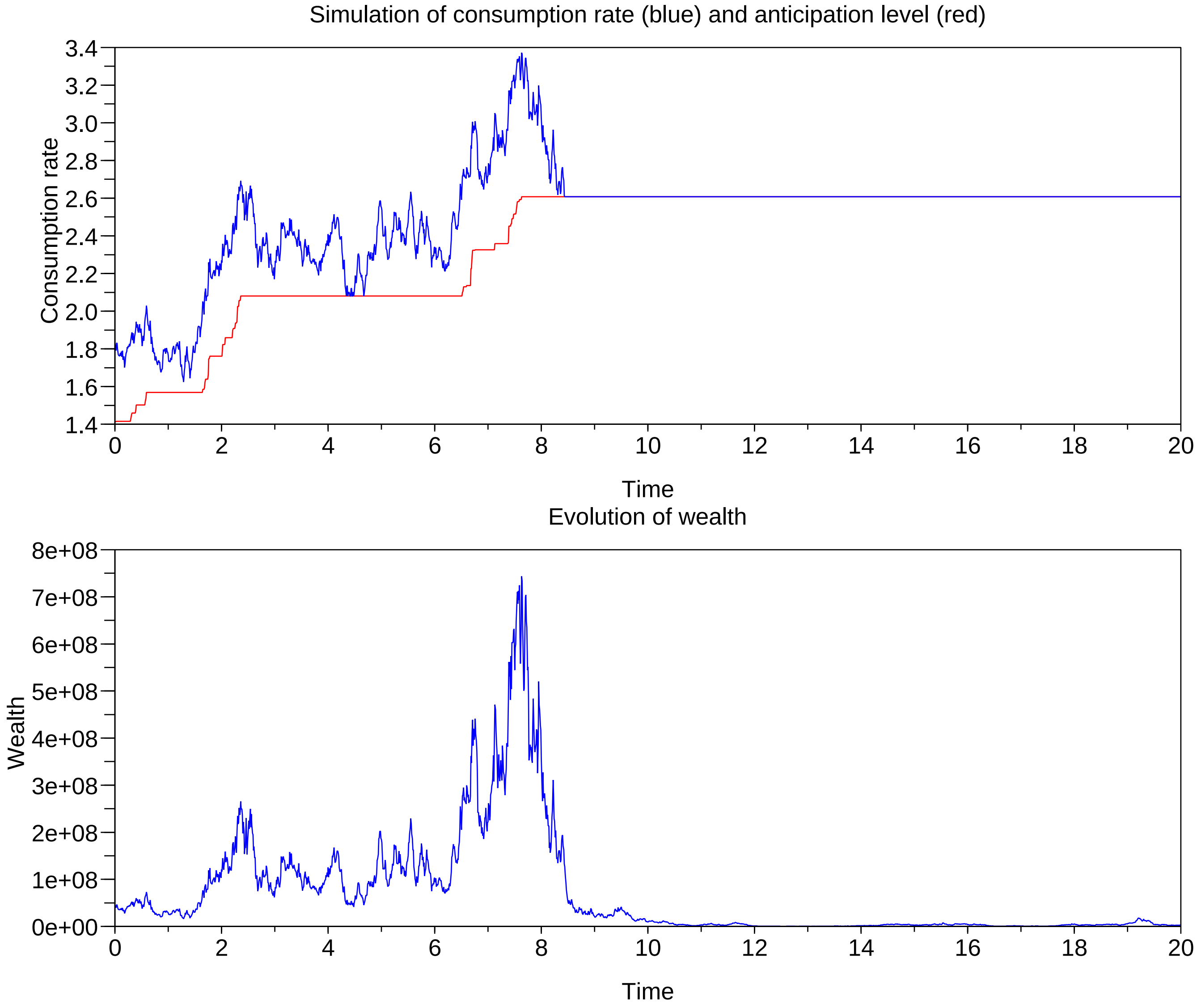
Evolution of wealth



Plots for $K = 43$



Plots for $K = 43$



Summary

Chris and I have proposed and solved investment and consumption withdrawal problems using a new type of preferences that include a preference for time resolution of uncertainty. Unlike the recursive utility, the preferences can depend on what was anticipated but did not happen. In some cases (with power utility), we have a closed-form solution up to one constant that can be solved numerically using a one-dimensional search. For a general functional form for the utility function, the solution involves integrals that can easily be done numerically (just as in the Merton case). A variant of the model (Dybvig, Jang, and Koo) can be used to formulate and solve a model of learning how to consume effectively.

Learning how to consume

This slide talks about the work with Hyeng Keun Koo and Bong-gyu Jang. Given $\underline{L}_0$, $\overline{L}_0$, and W_0, choose $\underline{L}_0$, $\overline{L}_0$, and c_t to maximize

$$E\left[\int_0^\infty e^{-\rho t}\left\{(U(c_t) - \overline{K}(U(c_t) - U(\overline{L}_t))^+ \right.\right.$$
$$\left.\left. -\underline{K}(U(\underline{L}_t) - U(c_t))^+)dt - \overline{K}dU(\overline{L}_t) + \beta\underline{K}dU(\underline{L}_t\right\}\right]$$

subject to the usual budget constraint, nonnegative wealth, $\overline{L}_t$ nondecreasing, and $\underline{L}_t$ nonincreasing. Then the range $[\underline{L}_t, \overline{L}_t]$ is the range of consumptions where the agent knows how to consume optimally at time t. Psychic effort can be spent to increase this range, and the range is set optimally to trade off the psychic effort now and improved competence for consuming in the wider range (and beyond) in the future. This model can be solved (more-or-less exactly for the one-sided problem) using the same general approach as in the paper with Chris I have been presenting.