

# Sentiments\*

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## Abstract

This paper shows how extrinsic fluctuations, and forces akin to “animal spirits”, can be accommodated in unique-equilibrium, rational-expectations, macroeconomic models like those in the RBC/DSGE paradigm. To this goal, we limit the communication that is embedded in a neoclassical economy by letting trading be random and decentralized. We then show that, as long as this prevents agents from reaching identical equilibrium expectations, these expectations, and macroeconomic outcomes, may vary with a certain type of extrinsic shocks which we call “sentiments”. These shocks are akin to sunspots, but operate in unique-equilibrium economies. We further show how communication may help propagate these shocks in a way that resembles the spread of fads and rumors and that gives rise to “boom-and-bust” cycles. We finally illustrate the quantitative potential of our insights within a variant of the RBC model.

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# 1 Introduction

Macroeconomic fluctuations are tied to shifts in market expectations. Consider, for example, the recent crisis. The earlier boom in housing markets has been attributed to “exuberant” beliefs about future prices; the subsequent bust came with a fast reversal in these beliefs; and the ongoing recovery is said to hinge on how quickly firms and households regain their “confidence” in the economy.

These observations are commonplace; they merely pinpoint to the apparent co-movement of market expectations and market outcomes. The challenge for the macroeconomist is to first formalize and then quantify the “deeper” forces that might be driving this co-movement.

In the standard paradigm, these forces are modeled as shocks to the technological frontier of the economy, the stock of capital, or other payoff-relevant fundamentals.<sup>1</sup> To many economists, this is unsatisfactory: shifts in “market sentiment” and “aggregate demand” often appear to obtain without obvious innovations in people’s tastes and abilities, firms’ know-how, and the like.

Motivated by this conviction, a long tradition in macroeconomics has sought to accommodate extrinsic fluctuations, and forces akin to “animal spirits”, by introducing non-convexities and multiple equilibria.<sup>2</sup> In this paper, we are motivated by the same theme but make a distinct methodological contribution. We show how extrinsic fluctuations can emerge in conventional, unique-equilibrium macroeconomic models, such as those in the dominant RBC/DSGE paradigm, once these models make room for the observed heterogeneity in expectations of economic outcomes.

**Model.** We consider a convex neoclassical economy in which agents are rational, markets are competitive, the equilibrium is unique, and there is no room for randomization devices. To sharpen our results, we also rule out aggregate shocks to preference, technologies, or any other (payoff-relevant) fundamentals. More crucially, we deviate from the standard paradigm by introducing trading frictions, which serve precisely two roles in our model: they introduce idiosyncratic trading risk and they limit the communication that takes place through markets or other means.

The economy is thus split into multiple “islands” (Lucas, 1972), which are heterogeneous in terms of TFP, information, and trading opportunities. Each island specializes in production of a certain good but wishes to consume also the good of at least one other island, which gives rise to trade. Importantly, this trade is decentralized and takes place through random matching: in each period, each island meets and trades with only one other, randomly selected, island. Furthermore, certain employment and production choices are made in anticipation of these trading opportunities, but before the observation of the actual terms of trade. Finally, communication is impeded in the sense that the islands may be unable to talk to one another or otherwise reach the same beliefs about future market outcomes, such as the terms of their trade, prior to their physical meeting.

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<sup>1</sup>To avoid confusion, let us fix some terminology. By “standard paradigm” we refer to the class of micro-founded, unique-equilibrium, rational-expectations, general-equilibrium models that have dominated academic research since the RBC revolution. By “fundamentals” we refer to any payoff-relevant variable, such as preferences, endowments, technologies, and government policies, or news thereof. Finally, by “extrinsic shocks” we refer to any residual, payoff-irrelevant, random variable.

<sup>2</sup>See, *inter alia*, Azariadis (1981), Benhabib and Farmer (1994), Cass and Shell (1983), Diamond (1982), Cooper and John (1988), and Guesnerie and Woodford (1992).

**Results.** As with any other rational-expectations framework, the equilibrium of our economy is defined as the fixed point between market outcomes (actual allocations and prices) and market expectations (expectations of allocations and prices). Furthermore, any variation in these endogenous variables must ultimately be driven by some sort of exogenous shocks. The question of interest for us, as for the literature on self-fulfilling fluctuations, is whether the equilibrium variation in market expectations is spanned by the variation in exogenous payoff-relevant variables and beliefs thereof, or whether there is also some residual, extrinsic variation.

Theorem 1 establishes that the aforementioned fixed point exists and is unique, which rules out the usual formalization of self-fulfilling fluctuations. Theorem 2 establishes that extrinsic variation in market expectations is nevertheless possible as long as these expectations remain imperfectly aligned across different agents—which, in turn, can be true as long as communication is imperfect.

To understand this result, take any two islands  $i$  and  $j$  that are about to meet and trade. Next, note that the output of each island is pinned down by the local preferences and technologies, and the local belief about the upcoming terms of trade: other things equal, an island produces more if it expects its terms of trade to improve. Finally, consider the following question: can there exist states of Nature in which both islands expect their terms of trade to improve?

Clearly, this cannot be the case if communication is perfect: if island  $i$  expects its terms to improve, and if both islands share the same beliefs about market outcomes, then island  $j$  must expect its own terms to deteriorate. As we show in Theorem 1, this logic guarantees that, whenever equilibrium expectations are homogeneous across agents, actual macroeconomic outcomes are pinned down by the underlying fundamentals, even if the latter are not per se known.

Now consider the case where communication is imperfect, so that the two islands are holding heterogeneous beliefs about the terms of their trade. This means that there can exist states of Nature in which they both expect their terms to improve, as well as states of Nature in which they both expect their terms to deteriorate. What is more, these events can be correlated in the cross-section of the economy, giving rise to aggregate fluctuations.

During a boom, each island produces more because it expects its trading partner to produce more, and hence the demand for its own product to increase. During a recession, each island expects its demand to be low, and acts in a way that drives down the demand for other islands. These fluctuations therefore have the same flavor, and the same empirical content, as the self-fulfilling fluctuations that obtain in models with multiple equilibria.

What drives these fluctuations is a particular kind of aggregate shocks, which we call “sentiment shocks”. These shocks impact the information that is available to each island, without however affecting the latter’s belief about either the aggregate fundamentals (which are fixed) or the idiosyncratic fundamentals of its trading partner (which is random). In this sense, these shocks are extrinsic. These shocks nevertheless impact *equilibrium* expectations, because they alter, in effect, the belief that each island holds about the beliefs and choices of other islands. One can thus think of, say, a positive sentiment shock as a shock that rationalizes the optimism of one island by making that island receive news (signals) that other islands are themselves optimistic.

These shocks can thus also be understood as shocks to higher-order beliefs. By imposing that the *aggregate* fundamentals are fixed and common knowledge, we rule out the particular type of higher-order uncertainty that has been the focus of previous work (e.g., Morris and Shin, 2002, 2003, Woodford, 2003). Nevertheless, by introducing trading frictions and imperfect communication, we open the door to higher-order uncertainty at the *micro* level: when two islands are matched together, they are uncertain, not only about each other’s productivities, but also about each other’s beliefs of their productivities, each other’s beliefs of their beliefs of their productivities, and so on. The fluctuations we document reflect correlated variation in this kind of higher-order beliefs.

That being said, we prefer to interpret our “sentiment shocks”, not as shocks to higher-order beliefs of exogenous fundamentals, but rather as shocks to *first-order* beliefs of *endogenous* economic outcomes. This is both for theoretical and for empirical reasons. In a rational-expectations setting like ours, firms and households do not *need* to form any of the aforementioned higher-order beliefs. Rather, as emphasized in Lucas (1972), they need only to form the right (rational) first-order beliefs regarding the relevant endogenous economic outcomes. Furthermore, what is observed in survey evidence is only the latter kind of first-order beliefs. Finally, there are multiple specifications of the belief hierarchy that are consistent with the same joint distribution for equilibrium outcomes and equilibrium expectations, which means that the former cannot be uniquely identified by data on the latter. By contrast, what can be identified is the extrinsic variation in first-order beliefs of economic outcomes—this is what we are after in this paper.

Complementing this perspective, we argue that correlation in higher-order beliefs may emerge endogenously as agents learn from realized market outcomes or otherwise exchange their forecasts of economic activity. In this sense, the information structures we use in this paper are only convenient proxies for the complex ways through which agents communicate with one another through markets, surveys, macro statistics, and many other ways. We next proceed to show that such communication may also serve as a powerful propagation mechanism—leading to contagion effects akin to the spread of fads and rumors, and giving rise to “boom-and-bust cycles” like those experienced in recent years.

Moving beyond our model, the broader contribution is to show how the notions of “animal spirits” and “self-fulfilling beliefs” can be accommodated in the RBC/DSGE paradigm without abandoning the discipline of rational expectations and equilibrium uniqueness. Relatedly, our approach permits us to capture “news” about economic activity without news about technology or other fundamentals.

To illustrate the quantitative potential of this contribution, we embed a certain variant of our “sentiment shocks” in the neoclassical growth model. We then use this to show that our theory appears to have no serious difficulty in matching key business-cycle facts such as the co-movement of employment, output, consumption, and investment.

**Layout.** The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 characterizes the equilibrium. Section 4 contains our main results regarding the possibility of extrinsic fluctuations. Section 5 shows how communication helps generate fad dynamics and boom-and-bust cycles. Section 6 explores the quantitative potential. Section 7 concludes. Appendices A and B contain, respectively, all the proofs and a detailed analysis of the model of Section 6.

## 2 Model

The economy consists of a continuum of islands, indexed by  $i \in \mathcal{I} = [0, 1]$ . Each island is populated by a representative household and a representative, locally-owned firm. All agents are price-takers. Each island produces a single good, which can either be consumed at “home” or be traded for a good produced “abroad” (by some other island). Production exhibits constant returns to scale with respect to local labor, which is supplied elastically by the local household, and local land, which is in fixed supply. Time is discrete, indexed by  $t \in \{0, 1, \dots\}$ , and each period contains two stages. Employment and production are set in stage 1, while trading and consumption occur in stage 2. Finally, and importantly, trading takes place through random pair-wise matching.

**Firms and technologies.** Consider the firm of island  $i$ . Its technology is given by

$$y_{it} = A_i(n_{it})^\theta(k_{it})^{1-\theta}, \quad (1)$$

where  $y_{it}$  is the quantity produced,  $A_i$  is the local total factor productivity (TFP),  $n_{it}$  is the labor input,  $k_{it}$  is the land input, and  $\theta \in (0, 1)$  parameterizes the income share of labor. The profit of this firm is  $\pi_{it} = p_{it}y_{it} - w_{it}n_{it} - r_{it}k_{it}$ , where  $p_{it}$  denotes the local price of the local good,  $w_{it}$  denotes the local wage, and  $r_{it}$  the local rental rate of land.

TFP varies across islands but not over time, thus ruling out both aggregate and idiosyncratic shocks. The cross-sectional distribution of TFP is described by a p.d.f.  $\mathcal{F}_A : \mathcal{A} \rightarrow (0, 1)$ , where  $\mathcal{A}$  is a compact subset of  $\mathbb{R}_+$ . This distribution is invariant over time and common knowledge—and so is the exact mapping from the identity  $i$  of a particular island to its idiosyncratic productivity  $A_i$ .

**Households and preferences.** Preferences on island  $i$  are given by

$$\mathcal{U}_i = \sum_{t=0}^{\infty} \beta^t [U(c_{it}, c_{it}^*) - V(\ell_{it})]$$

where  $\beta \in (0, 1)$  is the discount factor,  $c_{it} \in \mathbb{R}_+$  and  $c_{it}^* \in \mathbb{R}_+$  are the consumptions of, respectively, the “home” and the “foreign” good,<sup>3</sup>  $U(c_{it}, c_{it}^*)$  is the utility flow from these two forms of consumption,  $\ell_{it} \in \mathbb{R}_+$  is labor supply, and  $V(\ell_{it})$  is the implied disutility.  $U$  and  $V$  are given by

$$U(c, c^*) = \left(\frac{c}{1-\eta}\right)^{1-\eta} \left(\frac{c^*}{\eta}\right)^\eta \quad \text{and} \quad V(\ell) = \frac{\ell^\epsilon}{\epsilon},$$

where  $\eta \in (0, 1)$  parameterizes the extent to which there is specialization and trade (the fraction of “home” expenditure that is spent on the “foreign” good), while  $\epsilon > 1$  parameterizes the Frisch elasticity of labor supply. Finally, the period- $t$  budget constraint is given by

$$p_{it}c_{it} + p_{it}^*c_{it}^* \leq w_{it}\ell_{it} + \pi_{it} \quad (2)$$

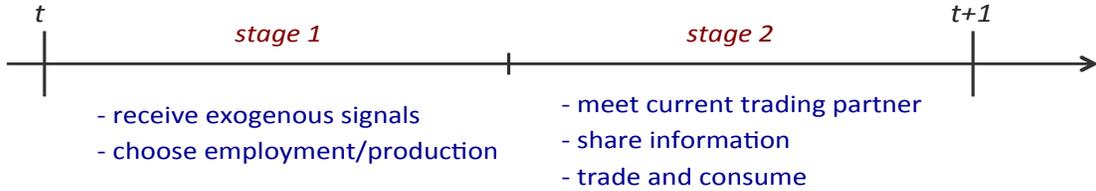
where  $p_{it}$  and  $p_{it}^*$  denote the local prices of, respectively, the “home” and the “foreign” good.

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<sup>3</sup>To have well-defined preferences over the entire commodity space, we can think of the home agents as either being indifferent among the goods of all other islands, or as liking only the good of their current random match.

**Matching, timing, and information.** To simplify, the matching is assumed to be uniform and i.i.d. over time: each island has an equal probability of being matched with any other island. Nature draws all the matches at the beginning of time, but does not reveal who is matched with whom and when. Thus fix a period  $t$  and a pair of islands that have been matched together in that period. In stage 2, the two islands meet, figure out that they were in the same match, and trade. The two islands, however, choose their employment and production levels in stage 1, *before* observing either their identities or the terms of their trade. Key economic decisions are thus made in anticipation of future trading opportunities, and with incomplete information about these opportunities.<sup>4</sup>

Our results do not depend on the precise details of how we model the information structure. To be concrete, however, we will assume (i) that exogenous information arrives only in stage 1 of each period and (ii) that every island shares its information with its trading partner once the two meet in stage 2. The flow of information and the timing of choices are thus as in the following figure.



More formally, for each  $t$ , we fix a compact set  $\mathcal{X}_t \subset \mathbb{R}^n$  and let  $x_{it}$  be a random variable drawn from  $\mathcal{X}_t$ . This variable represents the signal(s) that island  $i$  receives in stage 1 of that period and can be quite arbitrary. For instance, it may contain information, not only about the TFP of  $i$ 's trading partner, but also about the information that the latter has acquired either by Nature or by past trades. We will consider specific examples in due course. For now, we only impose a certain form of symmetry: the signal received by a particular island does not depend per se on either its own "name" or the precise identities of its trading partners. It follows that all the relevant information that is available to an island in stages 1 and 2 of period  $t$  can be summarized in, respectively, the variables  $\omega_{it} \in \Omega_t$  and  $z_{it} \in \mathcal{Z}_t$ , which are defined recursively as follows: for all  $t \geq 0$ ,  $\omega_{i,t} = (z_{i,t-1}, x_{i,t})$  and  $z_{it} = (\omega_{it}, \omega_{m_t(i),t})$ , where  $m_t(i)$  henceforth denotes  $i$ 's match in period  $t$  and where  $z_{i,-1} \equiv A_i$ . That is, information sets (or "types", or "local states") are updated either by the arrival of exogenous signals in stage 1 or by the endogenous information exchange during stage 2.<sup>5</sup>

**Sentiment shocks.** The joint distribution of the signals  $x_{it}$  in the population of islands is allowed to depend on an exogenous random variable  $\xi_t$  drawn from a compact set  $\Xi \subset \mathbb{R}^n$ . This variable is akin to a sunspot in the sense that it affects information sets without affecting either the true aggregate fundamentals or any agent's beliefs about these fundamentals (for the latter are fixed and common knowledge). As will become clear in due course, we can further refine the notion that this variable is extrinsic by imposing that variation in  $\xi_t$  does not cause variation in any island's

<sup>4</sup>This stylizes a simple fact. When firms decide employment and investment, they are uncertain about consumer demand; when consumers decide spending, they are uncertain about employment opportunities and income.

<sup>5</sup>Accordingly, the sets  $\Omega_t$  and  $\mathcal{Z}_t$  are compact and constructed recursively by letting  $\mathcal{Z}_{-1} = \mathcal{A}$  and  $\Omega_t = \mathcal{Z}_{t-1} \times \mathcal{X}_t$  and  $\mathcal{Z}_t = \Omega_t \times \Omega_t$  for any  $t \geq 0$ .

belief about the TFP level of either its own current and future trading partners, or of any other match in the economy. This variable will thus permit us to introduce aggregate variation in beliefs of equilibrium outcomes without any variation in beliefs of fundamentals. To fix language, we refer to  $\xi_t$  as a “sentiment shock”. The history of this shock is denoted by  $\xi^t \equiv (\xi_1, \dots, \xi_t)$ .

**Market clearing and trade balance.** The local labor and rental markets clear if and only if, respectively,  $n_{it} = \ell_{it}$  and  $k_{it} = K$ . The market for the consumption good clears if and only if  $c_{it} + c_{jt}^* = y_{it}$ , where  $j$  stands for  $i$ 's period- $t$  match. Finally, since the islands cannot trade financial claims, the goods trade must be balanced:  $p_{it}^* c_{it}^* = p_{it}(y_{it} - c_{it})$ .

**Technicalities and equilibrium definition.** Note that the underlying probability space is quite rich, as it involves the realizations of all matches and signals in the population. For our purposes, however, it suffices to focus on the joint distribution of the history  $\xi^t$  of the sentiment shock and of the pair of information sets  $(\omega_{it}, \omega_{jt})$  of an arbitrary match  $(i, j)$ . We assume that this distribution is represented by a continuous probability density function, which we henceforth denote by  $\mathcal{P}_t(\omega_{it}, \omega_{jt}, \xi^t)$ . Next, note that any allocation and price system can be represented with a collection of functions  $\{n_t, k_t, y_t, \ell_t, w_t, r_t, p_t, p_t^*, c_t, c_t^*\}_{t=0}^\infty$  such that, for all islands, dates, and possible states,  $n_{it} = n_t(\omega_{it})$ ,  $k_{it} = k_t(\omega_{it})$ ,  $y_{it} = y_t(\omega_{it})$ ,  $\ell_{it} = \ell_t(\omega_{it})$ ,  $w_{it} = w_t(\omega_{it})$ ,  $r_{it} = r_t(\omega_{it})$ ,  $p_{it} = p_t(z_{it})$ ,  $p_{it}^* = p_t^*(z_{it})$ ,  $c_{it} = c_t(z_{it})$ , and  $c_{it}^* = c_t^*(z_{it})$ , with  $z_{it} = (\omega_{it}, \omega_{jt})$  and  $j = m_t(i)$ .<sup>6</sup> We require that these functions be continuous and bounded, which permits us to apply the contraction mapping theorem to prove existence and uniqueness of the equilibrium. Modulo these qualifications, a competitive equilibrium is defined in an otherwise conventional manner.

**Definition 1.** *An equilibrium is a collection of continuous and bounded allocation and price functions such that (i) given current prices and expectations of future prices, the associated allocations are optimal for households and firms; (ii) prices clear all markets; and (iii) expectations are rational.*

Finally, we define aggregate output,  $Y_t$ , as the logarithmic average of local output in the cross-section of islands:  $\log Y_t(\xi^t) \equiv \int_{\Omega_t} \log y_t(\omega) \mathcal{P}_t(\omega | \xi^t)$ . The question of interest for us is then to understand under what conditions the equilibrium value of  $Y_t$  varies with the extrinsic shocks in  $\xi^t$ .

### 3 Equilibrium characterization

We now characterize the equilibrium. Consider first the consumption decisions of the household of island  $i$  during stage 2 of period  $t$ . Let  $\lambda_{it}$  denote the Lagrange multiplier on its budget and normalize the local nominal prices so that  $\lambda_{it} = 1$ . Optimal consumption choices satisfy

$$U_c(c_{it}, c_{it}^*) = p_{it} \quad \text{and} \quad U_{c^*}(c_{it}, c_{it}^*) = p_{it}^*. \quad (3)$$

By trade balance,  $p_{it}^* c_{it}^* = p_{it}(y_{it} - c_{it})$ . By market clearing,  $c_{it} + c_{jt}^* = y_{it}$ . Combining these conditions with the corresponding ones for  $i$ 's trading partner (denoted here by  $j$ ), and using the

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<sup>6</sup>Note that the price functions  $p_t$  and  $p_t^*$  must satisfy  $p_t(\omega, \omega')/p_t^*(\omega, \omega') = p_t^*(\omega', \omega)/p_t(\omega', \omega)$  for all  $\omega, \omega' \in \Omega_t$ . This simply means that any two islands that trade face, of course, the same the terms of trade.

Cobb-Douglas specification of  $U$ , we obtain the following:

$$c_{it} = (1 - \eta)y_{it}, \quad c_{it}^* = \eta y_{jt}, \quad \text{and} \quad p_{it} = y_{it}^{-\eta} y_{jt}^{\eta}. \quad (4)$$

The interpretation of these results should be familiar from international trade theory: a fraction  $1 - \eta$  of the good of each island is consumed at “home”, while the rest is “exported”; and the terms of trade increase with the “foreign” supply relative to the “home” one.<sup>7</sup>

Consider now the labor-supply and labor-demand decisions that the local household and the local firm take during stage 1 of period  $t$ . These are given by the following first-order conditions:

$$V'(\ell_{it}) = w_{it} \quad \text{and} \quad w_{it} = \mathbb{E}_{it}[p_{it}] \theta \frac{y_{it}}{n_{it}}, \quad (5)$$

where  $\mathbb{E}_{it}[\cdot]$  is a short-cut for the rational expectation conditional on  $\omega_{it}$ . In words, workers equate the wage with the expected marginal disutility of effort, while firms equate the wage with the expected marginal revenue product of labor. It follows that the local marginal disutility of labor is equated with expected local marginal revenue product of labor.

This last finding means that we can understand the *local* equilibrium of any given island as the solution to the problem of a (benevolent) local planner that takes as given the local beliefs of terms of trade. The *general* equilibrium is then pinned down by requiring that these beliefs are consistent with the local equilibrium behavior of each island, that is, by requiring  $p_{it}$  to satisfy (4).

**Proposition 1.** *The equilibrium production levels and the equilibrium terms of trade solve the following fixed-point problem:*

$$y_t(\omega) = \left( \theta^\vartheta A_t(\omega) K^{1-\theta} \right)^{\frac{1}{1-\vartheta}} \left( \int_{\Omega_t} p_t(\omega, \omega') \mathcal{P}_t(\omega'|\omega) d\omega' \right)^{\frac{\vartheta}{1-\vartheta}} \quad (6)$$

$$p_t(\omega, \omega') = y_t(\omega)^{-\eta} y_t(\omega')^{\eta} \quad (7)$$

where  $\vartheta \equiv \frac{\theta}{\epsilon} \in (0, 1)$ ,  $A_t(\omega)$  identifies the productivity of an island of type  $\omega \in \Omega_t$ , and  $\mathcal{P}_t(\omega'|\omega)$  is the probability that this island attaches to meeting an island of type  $\omega' \in \Omega_t$ .

Proposition 1 is an example of the fixed-point relation between equilibrium outcomes and equilibrium expectations that is endemic to any rational-expectations economy. This fixed point is particularly simple here, and is essentially static because of the absence of capital. However, as illustrated by the dynamic variant that we study in Section 6, our insights apply more generally.

Interestingly, this fixed point can also be understood as the perfect Bayesian equilibrium of a *fictitious* game. To see this, substitute (7) into (6) to get the following:

$$\log y_{it} = (1 - \alpha) f_i + \alpha \mathbf{E}_{it}[\log y_{jt}], \quad (8)$$

where  $f_i \equiv \frac{1}{1-\vartheta} \log(\theta^\vartheta A_i K^{1-\theta})$  summarizes  $i$ 's fundamentals,  $\alpha \equiv \frac{\eta}{\eta + (1-\vartheta)/\vartheta} \in (0, 1)$  is a scalar that is pinned down by preference and technology parameters, and  $\mathbf{E}_{it}$  is an adjusted expectation

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<sup>7</sup>Island  $i$ 's terms of trade are given by  $R_{it} \equiv \frac{p_{it}^*}{p_{it}^*}$  (the ratio of “export” to “import” prices). Note then that  $R_{it} = \frac{y_{jt}}{y_{it}} = p_{it}^{1/\eta}$ . Since this is an increasing function of  $p_{it}$ , we henceforth interpret  $p_{it}$  also as the terms of trade.

operator defined by  $\mathbf{E}_{it}[X] \equiv H^{-1}(\mathbb{E}_{it}[H(X)])$ , with  $H(X) \equiv \exp(\eta X)$ . It follows that we can represent our economy as a game in which the players are the islands (or their local planners), their choices are their output levels, their best responses are described by (8), and the coefficient  $\alpha$  is, in effect, the degree of strategic complementarity.

This game-theoretic interpretation reveals an important connection between our micro-founded business-cycle economy and the class of more abstract coordination games studied by Morris and Shin (2002) and Angeletos and Pavan (2007): it is *as if* the islands are trying to coordinate their production choices. We will revisit this connection in Sections 4.3 and 4.4. For now, we note that competitive general-equilibrium effects are the sole origin of what looks like strategic interaction in our economy: our model is a Walrasian economy, not a game; the actual agents (firms and households) are infinitesimal price-takers, not strategic players; and the interdependence of allocations across islands is a by-product of the dependence of equilibrium prices on these allocations, not a symptom of production externalities and the like.<sup>8</sup>

Putting aside these interpretations, we can show that conditions (6) and (7), or equivalently condition (8), define a contraction mapping over the set of (bounded and continuous) functions that map the local state of an island to its equilibrium output. The following is then immediate.

**Theorem 1.** *The equilibrium exists and is unique.*

The proof of this result rests on the assumption that  $\Omega_t$  is compact. Without this, we cannot generally guarantee existence. Nevertheless, as long as an equilibrium exists, it has to be unique, because a contraction mapping admits at most one fixed point. In the examples we consider in Section 4.2 and on,  $\Omega_t$  is not compact, but the equilibrium can be obtained by guessing and verifying.

## 4 Extrinsic Fluctuations

We now proceed to study whether the equilibrium can exhibit extrinsic fluctuations, that is, whether economic outcomes can vary with the sentiment shock  $\xi_t$ . As we show below, answering this question does not require one to know the precise details of how agents collect information and communicate with one another—objects that are most likely beyond the hope of measurement and quantification. Rather, it suffices to inspect the forecasts that agents end up forming about endogenous economic outcomes (allocations and prices) along the equilibrium.

With this in mind, we bypass the details of the information structure and index the extent of communication by whether equilibrium beliefs are the same or different across agents.

**Definition 2.** *We say that the economy exhibits “perfect communication” if and only if the following property holds along its unique equilibrium: for any period  $t$ , any match  $(i, j)$ , and any local states  $(\omega_{it}, \omega_{jt})$ , the two islands share in stage 1 the same equilibrium belief about either their output levels (allocations) or their terms of trade (prices).*

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<sup>8</sup>Note that  $\alpha$  is positive only because employment increases with terms of trade, which depends on substitution effects dominating income effects. Extrinsic fluctuations, however, would be possible even if  $\alpha$  were negative.

We can then state our key result as follows.

**Theorem 2.** *Along the unique equilibrium, aggregate economic activity can vary with the extrinsic shock  $\xi_t$  if and only if communication is imperfect.*

In the remainder of this section, we prove this theorem in two steps: subsection 4.1 establishes the “only if” part, while subsection 4.2 proves the “if” part with a specific example. Subsection 4.3 then discuss the broader insights behind our results and their empirical content.

#### 4.1 Perfect communication

If the islands share the same beliefs about their output levels, they must also share the same belief about the terms of their trade, for the latter are pinned down by their output levels. To prove the “only if” part of Theorem 1, it therefore suffices to show that the latter property rules out extrinsic fluctuations. Thus fix a period  $t$  and a match  $(i, j)$ , and suppose that  $i$  and  $j$  share the same belief about  $p_{it}$ . As we show in the appendix, this implies that

$$\log \mathbb{E}_{it} p_{it} = -\log \mathbb{E}_{jt} p_{jt}.$$

Intuitively, whenever  $i$  expects its terms of trade to move in one direction,  $j$  necessarily expects its own terms to move in the opposite direction. Combining this property with condition (6), and with the corresponding condition for  $j$ , we infer that the joint output of the two islands satisfies

$$\log y_{it} + \log y_{jt} = f_i + f_j + \frac{\vartheta}{1-\vartheta} \{\log \mathbb{E}_{it} p_{it} + \log \mathbb{E}_{jt} p_{jt}\} = f_i + f_j,$$

where, recall,  $f_i \equiv \frac{1}{1-\vartheta} \log (\theta^\vartheta A_i K^{1-\theta})$  and similarly for  $f_j$ . Intuitively, any potential movement in expected terms of trade has a perfectly offsetting effect on the behavior of the two islands, so that their joint output is necessarily pinned down by their local TFP levels alone.

Aggregating this finding across all matches, we reach the following result.

**Proposition 2.** *When communication is perfect, the equilibrium level of aggregate output is invariant to  $\xi_t$  and is given by*

$$\log Y_t = \frac{1}{1-\vartheta} \overline{\log A} + \frac{\vartheta}{1-\vartheta} \log K \tag{9}$$

where  $\overline{\log A} \equiv \int \log A \mathcal{F}_A(A) dA$  measures aggregate TFP.

Note that this result requires only that the islands reach the same equilibrium beliefs about allocations and/or prices, which may actually happen even without the islands sharing the same beliefs about their idiosyncratic fundamentals. Furthermore, it is straightforward to show that the above result extends to the case of aggregate TFP shocks, irrespectively of the information that the islands might have about these shocks.<sup>9</sup> These facts underscore that information frictions matter only in so far they induce heterogeneity in first-order beliefs of the relevant equilibrium outcomes—beliefs of the underling fundamentals do not matter *per se*.

<sup>9</sup>In particular, if we let  $\mathcal{F}_A$  be time-varying and relax the assumption that the latter is common knowledge, Proposition 2 continues to hold as soon as we replace the constant  $\overline{\log A}$  with the corresponding time-varying  $\overline{\log A}_t$ .

## 4.2 Imperfect communication: an example

We now prove that heterogeneity in equilibrium beliefs of allocations/prices opens the door to extrinsic fluctuations. For this purpose it suffices to illustrate the possibility of extrinsic fluctuations with a specific example; a discussion of the broader insights follows in the subsequent subsections.

The land endowment is  $K_i = 1$  for all  $i$ . The cross-sectional distribution of TFP is log-normal:  $\log A_i \sim \mathcal{N}(0, \sigma_A^2)$ ,  $\sigma_A > 0$ . The extrinsic shock is i.i.d Normal over time:  $\xi_t \sim \mathcal{N}(0, \sigma_\xi)$ ,  $\sigma_\xi > 0$ . Finally, the exogenous signal received by  $i$  is given by the pair  $x_{it} = (x_{it}^1, x_{it}^2)$ , where

$$x_{it}^1 = \log A_j + u_{it}^1 \quad \text{and} \quad x_{it}^2 = x_{jt}^1 + \xi_t + u_{it}^2,$$

where  $j = m(i, t)$  is  $i$ 's trading partner, and where  $u_{it}^1 \sim \mathcal{N}(0, \sigma_{u1}^2)$  and  $u_{it}^2 \sim \mathcal{N}(0, \sigma_{u2}^2)$  are idiosyncratic noises, with  $\sigma_{u1}, \sigma_{u2} > 0$ . Note that  $x_{it}^1$  represents a private signal that  $i$  receives about  $j$ 's TFP, while  $x_{it}^2$  represents a private signal that  $i$  receives about  $j$ 's information about its own TFP. The shock  $\xi_t$  then introduces an aggregate noise component in the second type of signals.

Note that the posterior belief of island  $i$  about the TFP of its trading partner is pinned down by the signal  $x_{it}^1$  alone, which is itself invariant to the sentiment shock  $\xi_t$ . It follows that  $\xi_t$  does not affect beliefs of either aggregate or idiosyncratic fundamentals. Yet, as we verify below,  $\xi_t$  triggers aggregate fluctuations.

**Proposition 3.** *Consider the equilibrium of the economy described above and let  $F_t^a$  and  $F_t^b$  denote the logarithmic cross-sectional averages of  $\mathbb{E}_{it} y_{jt}$  and  $\mathbb{E}_{it} Y_t$ .*

(i)  $\log Y_t$ ,  $\log F_t^a$  and  $\log F_t^b$  are increasing linear functions of  $\xi_t$ .

(ii) There exist scalars  $\phi_0, \psi_0 \in \mathbb{R}$  and  $\phi_a, \phi_1, \phi_2, \psi_a, \psi_1, \psi_2 \in \mathbb{R}_+$  such that, for all islands, dates, and states of nature,

$$\begin{aligned} \log y_{it} &= \phi_0 + \phi_a \log A_i + \phi_1 x_{it}^1 + \phi_2 x_{it}^2 \\ \mathbb{E}_{it} \log p_{it} &= \psi_0 - \psi_a \log A_i + \psi_1 x_{it}^1 + \psi_2 x_{it}^2 \end{aligned}$$

Part (i) characterizes the aggregate behavior of the economy: variation in  $\xi_t$  triggers positive co-movement in aggregate economic activity, as measured by  $Y_t$ , and in forecasts of economic activity, as measured by either  $F_t^a$  or  $F_t^b$ . Part (ii) reveals the micro-level behavior that rests beneath these fluctuations: an increase in either  $x_{it}^1$  or  $x_{it}^2$  leads island  $i$  to expect an improvement in its terms of trade, which explains why  $y_{it}$  increases with either of these signals, and thereby also with  $\xi_t$ .

To build intuition for this result, suppose for a moment that the output of island  $i$  depended only on local TFP. It would then be optimal for island  $j$  to condition its own output, not only on its own TFP, but also on  $x_{jt}^1$ : a higher  $x_{jt}^1$  signals that  $i$ 's output is likely to be higher and hence that the demand for  $j$ 's product is also likely to be higher (equivalently, that its terms of trade will improve). But then it would become optimal for island  $i$  to raise its own production when it observes either a higher  $x_{it}^1$  or a higher  $x_{it}^2$ , for either observation would now signal that island  $j$  is likely to produce more and hence that the demand of  $i$ 's product is likely to be higher. This explains why an island's expected terms of trade and its output increase with either signal.

The above intuition is based on recursive reasoning—equivalently, on iterating the contraction mapping behind Proposition 1. While illuminating, this is not strictly needed. A simpler intuition emerges once one focuses directly on the fixed point. In equilibrium, *either* of the two signals serves as a signal of the likely level of “foreign” demand. The fact that one signal is intrinsic while the other is extrinsic is irrelevant to the decisions of firms and households. Rather, all that matters for them is simply that either signal contains “news” about the level of economic activity in other islands, and hence about the likely level of demand for the local product. Whenever a positive innovation occurs in  $\xi_t$ , *all* islands receive “good news” of the extrinsic type. For firms, this means an increase in expected marginal returns, which motivates them to expand their production and raise their demand for labor and land. In equilibrium, this stimulates employment and output, while also raising the wage and the rental rate (and thereby land prices). All in all, the economy ends up experiencing a boom that may appear self-fulfilling in the eyes of an outside observer.<sup>10</sup>

The insight that emerges above is more general than the specific example we have used to illustrate it. Whenever a boom obtains in our economy, it *necessarily* reflects optimistic beliefs about the level of economic activity in other islands. How exactly these equilibrium beliefs are rationalized, or engineered, by one information structure or another is not per se relevant.

### 4.3 Imperfect communication: higher-order uncertainty and sentiments

To elaborate on the generality of the preceding insight, we first explain how our fluctuations can be understood as a symptom of higher-order uncertainty. This underscores that they are a robust feature of economies with informational frictions. We then discuss the theoretical and empirical reasons that motivate us to side-step this game-theoretic interpretation and, instead, favor a rational-expectations one: our sentiment shocks are meant to capture extrinsic movements in first-order beliefs of economic activity, not per se shocks to higher-order beliefs of exogenous fundamentals.

To elaborate on the role of higher-order uncertainty, consider the following generalization of the example we studied in the previous subsection. Fix a finite  $H > 1$  and suppose that the signal  $x_{it}$  is now given by  $x_{it} = (x_{it}^1, x_{it}^2, \dots, x_{it}^H)$ , where

$$x_{it}^1 = \log A_{jt} + \varepsilon_{it}^1 \quad \text{and} \quad x_{it}^h = x_{jt}^{h-1} + \varepsilon_{it}^h \quad \forall h \geq 2.$$

That is, islands get signals of the signals... of the signals of others. Suppose further that the error terms  $\varepsilon_{it}^h$  have both idiosyncratic and aggregate components:  $\varepsilon_{it}^h = \xi_t^h + u_{it}^h$ , where  $\xi_t^h$  is the aggregate component and  $u_{it}^h$  is the idiosyncratic one. These components are uncorrelated across  $h$  and  $t$ , as well as with one another, and are drawn from Normal distributions with zero means and variances  $(\sigma_\xi^h)^2$  and  $(\sigma_u^h)^2$ , respectively. Finally, to contrast our sentiment shocks to conventional technology shocks, we let  $\log A_{it} = a_i + \bar{a}_t$ , where  $a_i$  is an island-specific fixed effect and  $\bar{a}_t$  is the period- $t$  aggregate TFP shock. The former is Normally distributed in the cross-section of islands, while the latter is common knowledge, Normal, and i.i.d. over time.

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<sup>10</sup>Note here how our theory formalizes news of economic activity in terms of extrinsic forces rather than news about fundamentals. We revisit this point, which connects to the recent work on “news shocks”, in Section 6.

Consider now the implied hierarchy of beliefs about the fundamentals within a particular match (i.e.,  $i$ 's belief of  $A_j$ ,  $i$ 's belief of  $j$ 's belief of  $A_i$ , and so on). It is easy to check that variation in  $\xi_t^h$  causes variation in beliefs of order  $h$  and above, but not in beliefs of order lower than  $h$ . Each of these shocks has thus a *distinct* effect on the hierarchy of beliefs about the fundamentals. Nevertheless, as shown in the next proposition, these shocks are completely indistinguishable when it comes to equilibrium behavior: macroeconomic outcomes depend only on a single composite of all these shocks, which we denote below by  $\bar{\xi}_t$ . It is then only this composite shock that we wish to think of as the proper measure of what a “sentiment shock” is.

**Proposition 4.** *Consider the equilibrium of the economy described above. There exist scalars  $\Phi, \Psi, \Lambda > 0$  such that*

$$\begin{aligned}\log Y_t &= \Phi \bar{a}_t + \bar{\xi}_t \\ \log F_t^a &= \Phi \bar{a}_t + \Psi \bar{\xi}_t \\ \log F_t^b &= \Phi \bar{a}_t + \Lambda \bar{\xi}_t + v_t\end{aligned}$$

where  $\bar{\xi}_t$  is a linear combination of  $(\xi_t^1, \dots, \xi_t^h)$ , and where  $v_t$  is a random variable that is orthogonal to both  $\bar{a}_t$  and  $\bar{\xi}_t$ .

To understand this result, recall from condition (8) that the equilibrium output of each island depends *only* on its *first-order* beliefs of the level of output in other islands—not on the details of the information structure upon which these beliefs are formed. It follows that the entire extrinsic variation in aggregate output,  $Y_t$ , can be captured in a single random variable  $\bar{\xi}_t$ , which also summarizes the impact of all the exogenous  $\xi_t^h$  shocks on the average of the aforementioned beliefs,  $F_t^a$ . One can thus think of  $\bar{\xi}_t$  as *the* sentiment shock. The alternate forecast measure  $F_t^b$ , which might be easier to observe in survey data, can then be thought of as a noisy proxy of the aforementioned composite sentiment shock.

This result clarifies two points. First, there are multiple ways to shock the information structure so as to obtain the type of extrinsic fluctuations we are interested in: *any* of the  $\xi_t^h$  variables can serve our goals. And second, what matters for the observables of the model is only the variation in *first-order* beliefs of *endogenous* economic outcomes. The details of how these beliefs are ultimately rationalized by certain signals or by shocks to, say, tenth-order beliefs of fundamentals is not relevant for the model’s predictions—and is also of no particular interest to us.

Suppose now that an “econometrician” views the available data on aggregate employment and output, along with surveys of economic forecasts, through the lens of our model. This data may well permit the econometrician to identify separately the composite extrinsic shock  $\bar{\xi}_t$  from the technology shock  $\bar{a}_t$ . For example, the technology shock  $\bar{a}_t$  can first be identified by the Solow residual, and the composite sentiment shock  $\bar{\xi}_t$  can then be identified by the remaining variation in  $Y_t$ ; the average forecasts  $F_t^a$  and  $F_t^b$  can then be used either as alternative sources of identification, or as over-identifying restrictions. By contrast, the information structure and the hierarchy of beliefs are not uniquely identified: there are multiple specifications of these objects that give rise to *exactly* the same joint distribution for equilibrium beliefs and equilibrium outcomes.

These observations explain how our “sentiment shocks” are meant to capture extrinsic movements in first-order beliefs of equilibrium outcomes rather than shocks to higher-order beliefs of exogenous fundamentals—an interpretation that squares well with the rational-expectations tradition, which only requires that agents form rational first-order beliefs about equilibrium outcomes.

Consistent with this interpretation, note that our fluctuations hinge only on the existence of correlated movements in these beliefs, not on the precise details of where this correlation originates from. In our preceding examples, this correlation has been hard-wired in the exogenous information structure. More naturally, however, such correlation may emerge as the by-product of how agents communicate through, say, the markets or the media—communication *means* correlation. We illustrate this idea in Section 5: an exogenous sentiment shock hits only a few islands, but spreads in the rest of the economy as these islands trade and communicate with other islands.

#### 4.4 Trading frictions

We conclude this section with a few remarks on the role of trading frictions. As anticipated in the Introduction, trading frictions (random matching) serve two functions in our model: they impede communication; and they introduce idiosyncratic trading risk. These two ingredients permit us to sustain aggregate volatility in equilibrium outcomes without any aggregate shocks to preferences and technologies, thus sharpening the notion that the fluctuations we document are extrinsic.

If we allow for aggregate shocks to fundamentals, we can engineer additional extrinsic volatility from higher-order uncertainty about these shocks. This would bring our exercise closer to Morris and Shin (2002), Lorenzoni (2010), and Angeletos and La’O (2009). Note, however, that these prior works fail to obtain extrinsic fluctuations because they consider environments in which all the available signals impact first-order beliefs of fundamentals. Furthermore, the entire volatility of aggregate economic outcomes in these papers is bounded by the variance of the aggregate fundamentals: as the latter vanishes, the former also vanishes. This is because *all* higher-order uncertainty in these papers originates from shocks to aggregate fundamentals. By contrast, our model ties the level of higher-order uncertainty—and hence the size of extrinsic fluctuations—to the level of idiosyncratic trading risk, which can be large even if the aggregate fundamentals are fixed. This adds flexibility for applied/quantitative purposes.<sup>11</sup>

No matter whether the aggregate fundamentals are fixed or uncertain, trading frictions are necessary for the existence of our extrinsic fluctuations. This is a direct implication of Grossman (1981), which establishes that complete and centralized markets implement first-best outcomes even if the exogenous information is incomplete. By ruling out aggregate shocks to fundamentals and focusing exclusively on trading frictions, we isolate the minimal ingredient that is needed for our results. We also build a certain bridge to Diamond (1982). Diamond used trading frictions to formalize the Keynesian notion of “coordination failure” and to accommodate self-fulfilling phenomena. But whereas he achieves these objectives only by letting trading frictions introduce non-convexities and multiple equilibria, we achieve the same objectives by letting trading frictions impede communication.

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<sup>11</sup>See the *Online Supplement* for a more detailed discussion of these points.

## 5 Contagion and Boom-and-Bust Cycles

The preceding analysis has concentrated on documenting the possibility of extrinsic fluctuations. We now shift focus to studying the dynamic patterns that these fluctuations may exhibit. More specifically, we demonstrate that communication helps propagate our extrinsic shocks in a manner that resembles contagion effects, or the spread of fads and rumors. We thus illustrate how our theory can accommodate persistent waves of optimism and pessimism, and boom-and-bust cycles similar to those experienced in asset markets and macroeconomic activity during the recent years.

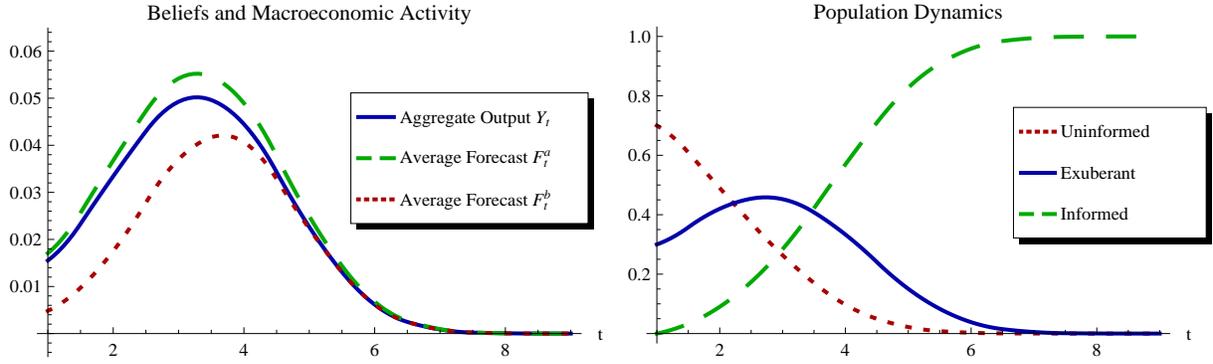
Consider the following variant of our model. At  $t = 0$ , the islands are split into two equally-sized groups. TFP is the same within a group but differs across groups. Think of these groups as “North” and “South”, let  $A_N$  and  $A_S$  be the respective TFP levels, and assume that these are i.i.d. draws from a log-Normal distribution. Each of these two groups is then split into two subgroups. Islands in the first subgroup observe nothing more than their own productivities; we refer to them as “uninformed”. Islands in the second subgroup, which we refer to as “partially informed”, get to see two additional signals. Similarly as in Section 4.2, these signals are given by  $x_N^1 = \log A_S + \varepsilon_N$  and  $x_N^2 = x_S^1 + \xi$  for the North, and  $x_S^1 = \log A_N + \varepsilon_S$  and  $x_S^2 = x_N^1 + \xi$  for the South, where  $\varepsilon_N, \varepsilon_S$  and  $\xi$  are all Normally distributed, independent of one another, and independent of the TFP draws. The initial fraction of partially informed islands is given by  $\chi \in (0, 1/2)$ ; the rest are uninformed.

The exogenous aggregate state is summarized in  $\tilde{s} = (A_N, A_S, \varepsilon_N, \varepsilon_S, \xi)$ . Once Nature draws  $\tilde{s}$  at  $t = 0$ , no other aggregate shock ever hits the economy, and no further exogenous information ever arrives— islands learn only in an endogenous manner, as they meet and “talk” to one another. The entire dynamics we document below are thus the sole product of this kind of communication.

To obtain a closed-form solution of the equilibrium, the random matching is assumed to take the following form. First, an uninformed island can meet either a similarly uninformed island from its own productivity group, in which case it learns nothing, or a partially informed one from its own productivity group, in which case it learns the latter’s information and hence turns into a partially informed island next period. Second, a partially informed island can meet either an uninformed one from its own productivity group, in which case it learns nothing itself, or a partially informed one from the *other* productivity group, in which case they both learn the entire state  $\tilde{s}$  and turn into a third category, which we call “fully informed”. Third, a fully informed island can only meet with a fully informed from its own productivity group. And finally, each island knows beforehand (in stage 1) whether it is matched with an island of the same or different information category.

This structure defines an “information ladder”, with the uninformed islands at the bottom, the partially informed in the middle, and the fully informed at the top. In each period, an island ascends at most one step in this ladder. Eventually, all islands reach the top, but this takes time. The dynamics we document below are a manifestation of how the population ascends this ladder.

It is easy to check that the only islands whose employment and production choices are sensitive to  $\xi$  are partially informed islands that expect to be matched with other partially informed islands. These islands behave in essentially the same way as in the example of Section 4.2. But, whereas in this earlier example all the islands behave in this fashion, here only a fraction does. Furthermore,



**Figure 1: Contagion and boom-and-bust cycle.** The left panel illustrates the response of aggregate output to a positive sentiment shock (solid line), along with that of the average of the beliefs that each island holds about either the output of its trading partner (dashed line) or aggregate output (dotted line). The right panel illustrates the underlying population dynamics.

this fraction evolves over time, due to the communication that takes place as islands meet and trade. To fix language, we henceforth focus on positive realizations for  $\xi$ , we refer to these islands as “exuberant”, and we let  $\lambda_t$  be the fraction of such islands in the population.

**Proposition 5.** (i) *The economy experiences a “fad”: the fraction of “exuberant” islands,  $\lambda_t$ , initially increases, but later on falls and eventually converges to zero.*

(ii) *There exists a scalar  $\Phi > 0$  such that the dynamic response of aggregate output to the initial sentiment shock is given by*

$$\frac{\partial \log Y_t}{\partial \xi} = \Phi \lambda_t, \quad \forall t.$$

These results are illustrated in Figure 1. The left panel documents the dynamic response of aggregate output, and of the islands’ forecasts of economics, to the initial positive sentiment shock. The right panel reveals the underlying population dynamics (i.e., the evolution of the distribution of islands along the aforementioned information ladder). It is evident that the dynamics of actual and expected output track the dynamics of the fraction of “exuberant” islands, which is first increasing and then decreasing. A similar result holds for asset (land) prices, which are, in effect, forecasts of future economic activity. The economy thus experiences a “wave of optimism” that builds up force for a while, only to fade away after enough time—there is a boom followed by a bust.

During the boom phase, more and more islands receive “good news” about the level of economic activity in other islands, and hence about their terms of trade. For those islands that were born exuberant at  $t = 0$ , this news arrives exogenously, from Nature. For those islands that become exuberant in any subsequent period, these news arrive endogenously, as these islands meet islands that were already exuberant. Finally, as time passes, more and more islands become fully informed. The bust phase is thus associated with a “correction” in previously exuberant beliefs. Communication causes the fraction of exuberant islands first to increase and then to fall.

The contagion effects behind these population dynamics are reminiscent of those discussed, *inter alia*, in Shiller (2005) and Akerlof and Shiller (2009): “irrational exuberance” is said to spread in the economy as one agent hears “stories” from other agents. In fact, our dynamics are very similar to those found in Burnside, Eichenbaum, and Rebelo (2011), in a study of the recent boom-and-bust in housing prices. But whereas these authors model the contagion between different agents as the product of behavioral (irrational) heuristics, here we show that it may be merely the symptom of the (imperfect) communication that takes place via the market mechanism and other social interactions. Exuberance then spreads *because* of rationality.

Putting aside any interpretations, three additional remarks are worth making regarding the mechanics of our theory, as illustrated in the above example. First, although our theory (like any other theory) requires an exogenous initial trigger for our fluctuations to kick off, this trigger may rest in a small fraction of the population and nevertheless give rise to a pervasive wave of optimism or pessimism in the entire economy. Second, as long as communication is imperfect, more communication may actually amplify our fluctuations: markets, macroeconomic statistics, the media, and the blogosphere may serve as channels of contagion. Finally, to the extent that communication gets finer and finer with time, equilibrium beliefs must eventually converge, which guarantees that the impact of any given extrinsic shock eventually vanishes. The fluctuations we formalize in this paper therefore embed, not only a natural propagation mechanism, but also a natural mean-reverting mechanism: booms must be followed by busts, recessions by recoveries.

## 6 A Quantitative Exploration

Although the contribution of this paper is primarily theoretical/methodological, we also wish to illustrate the quantitative potential of our insights. Towards this goal, we consider an RBC-like variant of our model that allows for investment and variable capital utilization; this permits us to study the predictions of our theory for the co-movement of key macroeconomic variables (employment, output, labor productivity, consumption, investment).

**Set up.** We first reinterpret the specialized goods as intermediate inputs rather than consumption goods. Within each trading pair, trade takes place in terms of these specialized intermediate inputs only.<sup>12</sup> These specialized goods are then used as intermediate inputs to produce a final good, which in turn can be used either for consumption or for investment.

The production function of the final-good producers is given by

$$y_{it} = \frac{1}{\zeta} (h_{it})^{1-\eta} (h_{it}^*)^\eta$$

where  $y_{it}$  is the final-good output,  $h_{it}$  and  $h_{it}^*$  are the “home” and “foreign” intermediate inputs, and  $\zeta \equiv (1 - \eta)^{(1-\eta)}\eta^\eta$  is a constant. This final good is then used for either consumption or investment purposes, so that the market-clearing condition of the local final good (equivalently, the local resource constraint) is  $c_{it} + i_{it} = y_{it}$ . Firm profits are given by  $\pi_{it}^y = y_{it} - p_{it}h_{it} - p_{it}^*h_{it}^*$ .

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<sup>12</sup>Allowing trades of the final good or of financial securities complicates the equilibrium characterization but does not remove our fluctuations. .

Turning to the intermediate goods, we incorporate variable capital utilization as in King and Rebelo (2000). The production function of the specialized firm on island  $i$  is given by

$$q_{it} = A_i(e_{it}k_{it})^{1-\theta}(n_{it})^\vartheta,$$

where  $k_{it}$  is the installed capital stock and  $e_{it}$  is the rate of capital utilization. Capital is accumulated within the firm, according to the following law of motion:

$$k_{i,t+1} = (1 - \Delta(e_{it}))k_{it} + i_{it}$$

where  $i_{it}$  is gross investment and  $\Delta(e_{it})$  is the depreciation rate, with  $\Delta(e) = \frac{\delta}{\mu}e^\mu$ ,  $\delta > 0$ , and  $\mu > 1$ . Market clearing for the intermediate good imposes  $h_{it} + h_{jt}^* = q_{it}$ , while firm profits are given by  $\pi_{it}^q = p_{it}q_{it} - w_{it}n_{it} - (r_{it} + \Delta(e_{it}))k_{it}$ , where  $r_{it}$  is the net rental rate of capital.

Finally, the households have standard preferences:

$$\mathcal{U}_i = \sum_{t=0}^{\infty} \beta^t [U(c_{it}) - V(n_{it})].$$

where  $U(c) = \frac{1}{1-\gamma}$ ,  $V(n) = \frac{1}{\epsilon}n^\epsilon$ ,  $\gamma > 0$ , and  $\epsilon > 1$ . As for their budget constraint, this is given by

$$c_{it} + k_{it+1} = \pi_{it}^y + \pi_{it}^q + w_{it}n_{it} + (1 + r_{it})k_{it}.$$

**Characterization.** A detailed characterization of the equilibrium is delegated to Appendix B. Here we simply note that the equilibrium allocation solves the following system:

$$V'(n_{it}) = \theta \zeta \mathbb{E}_{it} \left[ U'(c_{it}) \frac{y_{it}}{n_{it}} \middle| \omega_{it} \right] \quad (10)$$

$$\Delta'(e_{it}) e_{it} = (1 - \theta) \zeta \mathbb{E}_{it} \left[ U'(c_{it}) \frac{y_{it}}{k_{it}} \middle| \omega_{it} \right] \quad (11)$$

$$U'(c_{it}) = \beta \mathbb{E}_{it} \left[ U'(c_{i,t+1}) \left( 1 + (1 - \theta) \zeta \frac{\mu}{1+\mu} \frac{y_{it+1}}{k_{it+1}} \right) \middle| z_{it} \right] \quad (12)$$

$$c_{it} + k_{it+1} = y_{it} + (1 - \Delta(e_{it}))k_{it}. \quad (13)$$

$$y_{it} = q_{it}^{1-\eta} q_{jt}^\eta \quad (14)$$

$$q_{it} = A_{it}(e_{it}k_{it})^{1-\theta}(n_{it})^\theta \quad (15)$$

The top four conditions should be familiar: they are the optimality conditions for labor and capital utilization, the Euler condition, and the resource constraint. The remaining two conditions specify the production levels of the various goods. Compared to the RBC model, the only essential novelties are therefore (i) that the income of each island depends on the production choices of another island, through the relevant terms-of-trade effect; and (ii) that expectations are heterogeneous.

The key mechanism thus remains the same as in our baseline model: booms and recessions are driven by extrinsic shocks to beliefs about “demand” (about the output of other islands). Interestingly, however, these fluctuations now manifest, not only in employment, but also in investment and capital utilization. What is more, as all these decisions are infinitely forward-looking, economic

activity in one period may respond to extrinsic belief shifts about economic activity far in the future—it is *as if* the islands are playing a dynamic game in which an island’s optimal employment, consumption and investment choices during one period depend on the expected output of its likely trading partner, not only in the current period, but also in all future periods.

**Priors and sentiments.** While the characterization of the equilibrium is conceptually straightforward, an exact solution is no more possible because of the introduction of capital as an endogenous state variable. Furthermore, a numerical solution remains computationally challenging unless we make heroic assumptions about the information structure.<sup>13</sup>

We thus propose a heterogeneous-prior variant of the information structure that permits us to capture persistent fluctuations without a sacrifice in tractability.<sup>14</sup> In this variant, each island receives a single signal about its trading partner. This signal is given by

$$x_{it} = \log A_{jt} + \varepsilon_{it},$$

where  $\varepsilon_{it}$  is an error term. Differently from our previous analysis, however, the islands do not share a common prior about these error terms. Rather, each island believes (i) that its own error is unbiased, drawn from a Normal distribution with zero mean and variance  $\sigma_\varepsilon^2 > 0$ , and (ii) that the errors of all other islands are biased, drawn from a Normal distribution with the same variance but a mean equal to  $\xi_t$ , where  $\xi_t$  is itself a random variable.

The variable  $\xi_t$ , which can now be interpreted as the bias that each island perceives in the information of *others*, plays exactly the same modeling role as before: a positive innovation in  $\xi_t$  causes each island to become optimistic about the economic activity of other islands, and thereby about the demand for its own product. The main benefit is on the computational side: we can let  $\xi_t$  be known to all agents, which guarantees a low-dimensionality for the equilibrium dynamics. In particular, if we assume that  $\xi_t$  follows a Markov process, then (see Appendix B) the log-linearized dynamics of the economy can be summarized in a linear policy rule  $\Gamma : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$  such that

$$\tilde{K}_{t+1} = \Gamma(\xi_t, \tilde{K}_t).$$

where the tilde denotes log-deviation from steady state. This is akin to the policy rule of the standard RBC model, except that the familiar TFP shock is now replaced by our sentiment shock (and, of course, the precise form of  $\Gamma$  is different).<sup>15</sup>

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<sup>13</sup>The familiar trick of truncating the relevant state by assuming that all shocks become common knowledge after a finite time (e.g., Townsend, 1983, Lorenzoni, 2010) works fine in our baseline model, but not in the dynamic extension of this section, because the introduction of capital (an endogenous state variable) appears to add infinite history.

<sup>14</sup>This is not a free lunch: the present variant is in tension with the strong version of the rational-expectations axiom, which insists on all agents sharing a common prior that coincides with the “objective truth”. That been said, note that agents remain rational in the Bayesian sense; they simply start with different priors. Furthermore, this is only meant to be a convenient modeling short-cut.

<sup>15</sup>This suggests suggests that the heterogeneous-prior variant we have introduced here may prove a convenient short-cut for embedding our extrinsic fluctuations in richer DSGE models as well. A more “purist” approach, however, may opt to avoid this short-cut and, instead, explicitly model the learning dynamics. We took such a route in Section 5.

	The Model		U.S. Data	
	<i>std. dev.</i>	<i>corr(X,Y)</i>	<i>std. dev.</i>	<i>corr(X,Y)</i>
output $Y$	1.73	1.00	1.74	1.00
employment $N$	1.47	1.00	1.34	0.87
consumption $C$	1.26	0.98	1.19	0.79
investment $I$	4.30	0.96	4.98	0.76
labor productivity $Y/N$	0.27	0.99	0.87	0.66
labor wedge $LW$	5.14	-1.00	4.47	-0.82

**Table 1:** This table documents the business-cycle statistics of our model along with those of the US economy. All quantities are in quarterly frequency and HP-filtered. See Appendix B for details.

**Numerical results.** We work at quarterly frequency and set  $\beta = .99$ ,  $\gamma = 2$ ,  $\theta = .65$ ,  $\epsilon = 2$ , and  $\mu = 2$ ; these values are consistent with King and Rebelo (2000). Next, we assume that  $\xi_t$  follows an AR(1) process:  $\xi_t = \rho\xi_{t-1} + \nu_t$ , where  $\rho \in (0, 1)$  and  $\nu_t$  is i.i.d. Normal with mean zero and variance  $\sigma_\xi^2$ . We set  $\rho = .98$ , which builds strong persistence in our fluctuations. The remaining parameters ( $\eta$ ,  $\sigma_A$ ,  $\sigma_\varepsilon$  and  $\sigma_\xi$ ) then matter for aggregate dynamics only through a single composite coefficient, which itself scales up and down all aggregate outcomes. Exploiting this property, we fix  $\eta = 1$  and  $\sigma_a = \sigma_\varepsilon = \sigma_\xi = \sigma$ , and then set  $\sigma = 0.038$ , which induces the variance of the HP-filtered aggregate output in our model to match the corresponding moment in the US data.<sup>16</sup> We then simulate the dynamics of the economy and report the model’s HP-filtered business-cycle statistics in Table 1, along with the corresponding statistics for the US economy.

Given that the volatility of output is matched by design, the question of interest is whether our model also matches the relative volatility and the co-movement of all the other macroeconomic variables. As evident in Table 1, our model is quite successful in this respect. Sentiment shocks cause employment, consumption, investment, and labor productivity to co-move with output, as in actual business cycles. Furthermore, the quantitative effects are in the ballpark of the actual data.

Relative to the standard RBC model, we do worse in that we do not generate enough procyclicality in labor productivity. This is simply because we do not allow for technology shocks to drive the business cycle.<sup>17</sup> But we also do better in that we generate a counter-cyclical labor wedge,<sup>18</sup> which is an important feature of the data (Chari, Kehoe and McGrattan, 2007; Shimer, 2009). To understand this last property, consider the stripped-down version of our model where capital and utilization are both fixed. As a negative sentiment shock causes firms to turn pessimistic about their profitability, labor demand and employment fall. As this happens, the average labor

<sup>16</sup>Our calibration of  $\rho$  and  $\sigma$  is consistent with standard DSGE practice, where the various shocks are estimated so that the model matches the data. As anticipated in Section 4.3, additional discipline can be found in the theory’s predictions about forecasts of economic activity, but we will not explore this route here.

<sup>17</sup>If capital utilization were fixed, labor productivity would have been countercyclical, due to diminishing returns; employment, consumption and investment would, however, remain procyclical. Furthermore, the procyclicality of labor productivity in the data has actually vanished during the last two decades (Gali and van Rens, 2010).

<sup>18</sup>The latter is defined, in logs, by  $LW_t \equiv \log \left[ \frac{U'(C_t)}{V'(N_t)} \theta \frac{Y_t}{N_t} \right]$ , so that a positive wedge maps to a tax on labor.

productivity actually goes up. Standard business-cycle accounting will thus register the resulting fluctuation as an increase in the implicit tax on labor.

It is also worth contrasting the cyclical properties of our theory with those of the literature on “news shocks”. Bound by conventional DSGE practice, this literature formalizes news of economic activity as news of future technology. In so doing, it faces a significant difficulty in generating the observed joint procyclicality in employment, consumption, and investment (Beaudry and Portier, 2006). The usual fixes involve exotic preferences (Jaimovich and Rebelo, 2009), suboptimal monetary policy (Lorenzoni, 2010), or both (Christiano, Ilut, Motto, Rostagno, 2008). By contrast, our theory permits one to formalize news of economic activity as news about extrinsic forces that induce an increase in both current and future firm profitability. This helps stimulate employment, consumption and investment in a similar fashion as a conventional technology shock, which explains why our theory has no difficulty in capturing the observed co-movement in these variables.

## 7 Concluding remarks

Are business cycles driven by changes in preferences and technologies? Or are they driven by “animal spirits”, “market psychology”, and self-reinforcing waves of optimism and pessimism?

This question is not just an empirical matter. To address it, one must first propose a precise theory that formalizes the aforementioned popular but vague notions of “animal spirits” and the like; to paraphrase Lucas (2001), one needs equations that explain what these words mean.

This paper makes a contribution in precisely this direction: we develop a formalization of the aforementioned notions that can be embedded in the type of unique-equilibrium, micro-founded, rational-expectations models that populate the modern RBC/DSGE paradigm.

To achieve this, we relax the conventional assumption that all agents share the same expectations about economic activity, and we introduce a certain type of shocks that we call “sentiments”. These shocks capture aggregate movements in the aforementioned expectations that obtain without any innovations in the underlying technologies or other payoff-relevant fundamentals. They are akin to sunspots, but operate in unique-equilibrium models.

To outside observers, the resulting fluctuations might look as “self-fulfilling”, or as the product of mysterious “demand shocks” that are disconnected from preferences and technologies. In this respect, they have a genuinely Keynesian flavor. They are nevertheless consistent with the neoclassical paradigm, resting merely on the heterogeneity, or misalignment, of equilibrium expectations.

The combination of these points underscores what, in our view, is the relative strength of our theory. Not only is it capable of matching key business-cycle facts, as illustrated in the previous section; it also helps accommodate a set of popular notions about the “real” workings of the economy that have so far been hard to reconcile with the dominant business-cycle paradigm.

Introducing “sentiment shocks” in richer DSGE models, and estimating their contribution to observed business cycles, is a natural direction for future research. Studying their welfare properties and translating our insights in the context of asset markets are two other possible directions.

## Appendix A: Proofs

**Proof of Proposition 1.** From the the optimality condition for labor (5), we get

$$n_{it} = (\mathbb{E}_{it}[p_{jt}]\theta y_{it})^{\frac{1}{\epsilon}}$$

Substituting the above into the production function yields

$$y_{it} = A_i K^{1-\theta} (\mathbb{E}_{it}[p_{jt}]\theta y_{it})^{\frac{\theta}{\epsilon}}$$

Finally, solving the above for  $y_{it}$ , and letting  $\vartheta \equiv \theta/\epsilon$ , we obtain

$$y_{it} = \left( \theta^\vartheta A_i K^{1-\theta} \right)^{\frac{1}{1-\vartheta}} (\mathbb{E}_{it}[p_{it}])^{\frac{\vartheta}{1-\vartheta}}$$

which gives the first condition in the proposition. The second condition follows directly from condition (4). *QED*

**Proof of Theorem 1.** Substituting (7) into (6) and rearranging, we get

$$y_t(\omega)^{1+\eta\frac{\vartheta}{1-\vartheta}} = \left( \theta^\vartheta A_t(\omega) K^{1-\theta} \right)^{\frac{1}{1-\vartheta}} \left( \int_{\Omega_t} y_t(\omega')^\eta \mathcal{P}_t(\omega'|\omega) d\omega' \right)^{\frac{\vartheta}{1-\vartheta}}$$

Taking logs, we reach the following condition

$$\log y_t(\omega) = \frac{\frac{1}{1-\vartheta}}{1 + \eta\frac{\vartheta}{1-\vartheta}} \log \left( A_t(\omega) K^{1-\theta} \right) + \frac{\frac{\vartheta}{1-\vartheta}}{1 + \eta\frac{\vartheta}{1-\vartheta}} \log \left( \int_{\Omega_t} y_t(\omega')^\eta \mathcal{P}_t(\omega'|\omega) d\omega' \right)$$

This reduces to condition (8) in the main text once we let  $\alpha \equiv \frac{\eta}{\eta+(1-\vartheta)/\vartheta} \in (0, 1)$  and  $H(x) \equiv \eta \exp(x)$ . It also means that we can recast the equilibrium allocations in period  $t$  as the solution to the above fixed point problem.

In particular, for each  $t$ , let  $\mathcal{Y}_t$  be the set of real, bounded, and continuous functions with domain  $\Omega_t$ , and endow this set with the sup-norm to obtain a complete metric space. Next, define the operator  $\mathcal{T}_t : \mathcal{Y}_t \rightarrow \mathcal{Y}_t$  as follows: for any  $f \in \mathcal{Y}_t$  and any  $\omega \in \Omega_t$ ,

$$\mathcal{T}_t f(\omega) = (1 - \alpha) \left\{ \frac{\log A_t(\omega) + (1 - \theta) \log K}{1 - \vartheta} \right\} + \alpha \left\{ H^{-1} \left( \int_{\Omega_t} H(f(\omega')) \mathcal{P}_t(\omega'|\omega) d\omega' \right) \right\} \quad (16)$$

where, recall,  $A_t(\omega)$  identifies the productivity of an island of type  $\omega \in \Omega_t$  and  $\mathcal{P}_t(\omega'|\omega)$  is the probability density with which this island meets an island of type  $\omega' \in \Omega_t$ .<sup>19</sup> Now, take any equilibrium and let  $y_t \in \mathcal{Y}_t$  be the equilibrium output function in period  $t$ , for any  $t$ . Then, and only then,  $\log y_t$  is a fixed point of  $\mathcal{T}_t$ .

Existence and uniqueness of the equilibrium then follows from the fact that the operator  $\mathcal{T}_t$  is a contraction with modulus equal to  $\alpha \in (0, 1)$ —a property that we verify below by showing that  $\mathcal{T}_t$  satisfies Blackwell's sufficiency conditions.

<sup>19</sup>The functions  $A_t$  and  $\mathcal{P}_t$  are pinned down by the primitives of the economy:  $A_t$  is simply the function that, for any  $\omega \in \Omega_t$ , returns the first element of  $\omega$ , while  $\mathcal{P}_t$  follows from the exogenous stochastic structure of the economy.

(i) Monotonicity. Suppose  $f, g \in \mathcal{Y}_t$  and  $f(\omega) \geq g(\omega)$  for all  $\omega \in \Omega_t$ . First, note that

$$\mathcal{T}_t f(\omega) - \mathcal{T}_t g(\omega) = \alpha \left\{ H^{-1} \left( \int_{\Omega_t} H(f(\omega')) \mathcal{P}_t(\omega'|\omega) d\omega' \right) - H^{-1} \left( \int_{\Omega_t} H(g(\omega')) \mathcal{P}_t(\omega'|\omega) d\omega' \right) \right\}$$

Note that  $\alpha > 0$  and that  $H^{-1}(x) = \log(x/\eta)$ , which is a monotonically increasing function. We infer that  $\mathcal{T}_t f(\omega) - \mathcal{T}_t g(\omega) \geq 0$  if and only if

$$\int_{\Omega_t} \eta \exp(f(\omega')) \mathcal{P}_t(\omega'|\omega) d\omega' \geq \int_{\Omega_t} \eta \exp(g(\omega')) \mathcal{P}_t(\omega'|\omega) d\omega'. \quad (17)$$

Now, note that  $f(\omega) \geq g(\omega)$  for all  $\omega \in \Omega$  implies that  $\eta \exp(f(\omega')) \geq \eta \exp(g(\omega'))$  for all  $\omega \in \Omega_t$ . This immediately implies that condition (17) is always satisfied. Therefore,  $f \geq g$  implies  $\mathcal{T}_t f \geq \mathcal{T}_t g$ , which proves that  $\mathcal{T}_t$  is monotonic.

(ii) Discounting. Let  $a \geq 0$  be a constant. Then, using the fact that  $H$  is an exponential function, we have:

$$\begin{aligned} \mathcal{T}_t [f(\omega) + a] &= (1 - \alpha) \left\{ \frac{1}{1-\vartheta} \log A_t(\omega) \right\} + \alpha \left\{ H^{-1} \left( \int_{\Omega_t} H(f(\omega') + a) \mathcal{P}_t(\omega'|\omega) d\omega' \right) \right\} \\ &= (1 - \alpha) \left\{ \frac{1}{1-\vartheta} \log A_t(\omega) \right\} + \alpha \left\{ H^{-1} \left( \int_{\Omega_t} H(f(\omega')) \mathcal{P}_t(\omega'|\omega) d\omega' \right) \right\} + \alpha a \end{aligned}$$

Therefore,  $\mathcal{T}_t [f(\omega) + a] = \mathcal{T}_t f(\omega) + \alpha a$ , where  $\alpha \in (0, 1)$ , which proves that  $\mathcal{T}_t$  satisfies discounting.

As both the monotonicity and the discounting conditions of Blackwell's theorem are satisfied, we conclude that the operator  $\mathcal{T}_t$  is indeed a contraction, which in turn proves that the equilibrium exists and is unique. *QED*

**Proof of Theorem 2.** This follows from Propositions 2 and 3.

**Proof of Proposition 2.** By the assumption that that the two islands share the same belief (i.e., the same probability distribution) about  $p_{it}$ , we have that

$$\mathbf{E}_{it} \log p_{it} = \mathbf{E}_{jt} \log p_{it},$$

where  $\mathbf{E}_{it}$  and  $\mathbf{E}_{jt}$  are the adjusted expectation operators defined in the previous section. Replacing  $p_{it} = y_{it}^{-\eta} y_{jt}^{\eta}$  in the above, we infer that

$$\eta(\mathbf{E}_{it} \log y_{jt} - \log y_{it}) = -\eta(\mathbf{E}_{jt} \log y_{it} - \log y_{jt})$$

Note that the left-hand side of the above equals  $\log \mathbb{E}_{it} p_{it}$ , while the right-hand side equals  $-\log \mathbb{E}_{jt} p_{jt}$ . It follows that

$$\log \mathbb{E}_{it} p_{it} = -\log \mathbb{E}_{jt} p_{jt}.$$

The result then follows from the discussion in the main text.

**Proof of Proposition 3.** In the proposed equilibrium, the period- $t$  output of island  $j$  is log-normally distributed conditional on the information of island  $i$ , for any  $i, j$ , and  $t$ . Furthermore, the conditional variance  $Var(\log y_{jt}|\omega_{it})$  is invariant to  $\omega_{it}$ :

$$Var(\log y_{jt}|\omega_{it}) = \sigma_y^2 \equiv \phi_a^2 \sigma_a^2 + \phi_1^2 \sigma_{u1}^2 + \phi_2^2 (\sigma_{u2}^2 + \sigma_\xi^2)$$

It follows that  $\mathbf{E}_{it} \log y_{jt} = \mathbb{E}_{it} \log y_{jt} + \frac{1}{2} \eta^2 \sigma_y^2$ . The fixed-point condition (8) thus reduces to

$$\log y(\omega_i) = const + (1 - \alpha) \frac{1}{1 - \vartheta} a_i + \alpha \mathbb{E}_{it} [\log y(\omega_{jt})] \quad (18)$$

where  $a_i \equiv \log A_i$  and where  $const$  is a scalar that is invariant with  $\omega_{it}$  and that we henceforth ignore without any loss of generality.

We guess and verify a log-linear equilibrium under the log-normal specification for the shock and information structure. Suppose the equilibrium production strategy of the island of type  $\omega_{jt}$  takes a log-linear form given by  $\log y_t(\omega_{jt}) = \phi_a a_j + \phi_1 x_{jt}^1 + \phi_2 x_{jt}^2$ , for some coefficients  $(\phi_a, \phi_1, \phi_2)$ . It follows that  $\log y_t(\omega_{jt})$  is indeed log-normal, with

$$\mathbb{E} [\log y_t(\omega_{jt})|\omega_{it}] = \phi_a \mathbb{E} [a_j|\omega_{it}] + \phi_1 (a_i + \mathbb{E} [u_{jt}^1|\omega_{it}]) + \phi_2 (x_{it}^1 + \mathbb{E} [\xi_t|\omega_{it}] + \mathbb{E} [u_{jt}^2|\omega_{it}]) \quad (19)$$

Let  $\gamma_1 \equiv \sigma_{u1}/\sigma_A$ ,  $\gamma_2 \equiv \sigma_{u2}/\sigma_A$ ,  $\gamma_\xi \equiv \sigma_\xi/\sigma_A$  denote the relative noise ratios. Then

$$\begin{bmatrix} \mathbb{E} [a_j|\omega_{it}] \\ \mathbb{E} [u_{jt}^1|\omega_{it}] \\ \mathbb{E} [\xi_t|\omega_{it}] \\ \mathbb{E} [u_{jt}^2|\omega_{it}] \end{bmatrix} = \begin{bmatrix} \frac{1}{1+\gamma_1^2} x_{it}^1 \\ \frac{\gamma_1^2}{\gamma_1^2 + \gamma_2^2 + \gamma_\xi^2} (x_{it}^2 - a_i) \\ \frac{\gamma_\xi^2}{\gamma_1^2 + \gamma_2^2 + \gamma_\xi^2} (x_{it}^2 - a_i) \\ 0 \end{bmatrix}$$

Substituting these expressions into (18) gives us

$$\log y(\omega_{it}) = (1 - \alpha) \frac{1}{1 - \vartheta} a_i + \alpha \left[ \phi_a \frac{1}{1 + \gamma_1^2} x_{it}^1 + \phi_1 \left( a_i + \frac{\gamma_1^2}{\gamma_1^2 + \gamma_2^2 + \gamma_\xi^2} (x_{it}^2 - a_i) \right) + \phi_2 \left( x_{it} + \frac{\gamma_\xi^2}{\gamma_1^2 + \gamma_2^2 + \gamma_\xi^2} (x_{it}^2 - a_i) \right) \right]$$

By symmetry, equilibrium output for type  $\omega_{it}$  must satisfy  $\log y(\omega_{it}) = \phi_a a_i + \phi_1 x_{it}^1 + \phi_2 x_{it}^2$ . For this to coincide with the above condition for every  $z$ , it is necessary and sufficient that the coefficients  $(\phi_a, \phi_1, \phi_2)$  solve the following system:

$$\begin{aligned} \phi_a &= (1 - \alpha) \frac{1}{1 - \vartheta} + \alpha \phi_1 - \phi_2 \\ \phi_1 &= \alpha \left( \phi_a \frac{1}{1 + \gamma_1^2} + \phi_2 \right) \\ \phi_2 &= \alpha \left( \phi_1 \frac{\gamma_1^2}{\gamma_1^2 + \gamma_2^2 + \gamma_\xi^2} + \phi_2 \frac{\gamma_\xi^2}{\gamma_1^2 + \gamma_2^2 + \gamma_\xi^2} \right) \end{aligned}$$

The unique solution to this system gives us the following equilibrium coefficients.

$$\phi_a = \frac{(1 - \alpha)(1 + \gamma_1^2)((1 - \alpha^2)\gamma_1^2 + \gamma_2^2 + (1 - \alpha)\gamma_\xi^2)}{(1 - \vartheta)((1 - \alpha^2)(\gamma_1^4 + \gamma_2^2 + (1 - \alpha)\gamma_\xi^2) + \gamma_1^2(1 - \alpha^2 + \gamma_2^2 + (1 - \alpha)\gamma_\xi^2))} > 0 \quad (20)$$

$$\phi_1 = \frac{(1 - \alpha)\alpha(\gamma_1^2 + \gamma_2^2 + (1 - \alpha)\gamma_\xi^2)}{(1 - \vartheta)((1 - \alpha^2)(\gamma_1^4 + \gamma_2^2 + (1 - \alpha)\gamma_\xi^2) + \gamma_1^2(1 - \alpha^2 + \gamma_2^2 + (1 - \alpha)\gamma_\xi^2))} > 0 \quad (21)$$

$$\phi_2 = \frac{(1 - \alpha)\alpha^2\gamma_1^2}{(1 - \vartheta)((1 - \alpha^2)(\gamma_1^4 + \gamma_2^2 + (1 - \alpha)\gamma_\xi^2) + \gamma_1^2(1 - \alpha^2 + \gamma_2^2 + (1 - \alpha)\gamma_\xi^2))} > 0 \quad (22)$$

Furthermore, the expected equilibrium price must satisfy

$$E_{it} \log p_{it} = \frac{1 - \vartheta}{\vartheta} \log y(\omega_{it}) - \frac{1}{\vartheta} a_i$$

Using the above results, we have that the expected equilibrium price is given by  $E_{it} \log p_{it} = -\psi_a a_i + \psi_1 x_{it}^1 + \psi_2 x_{it}^2$ . with

$$\psi_a = -\left(\frac{1 - \vartheta}{\vartheta} \phi_a - \frac{1}{\vartheta}\right), \quad \psi_1 = \frac{1 - \vartheta}{\vartheta} \phi_1 > 0, \quad \text{and} \quad \psi_2 = \frac{1 - \vartheta}{\vartheta} \phi_2 > 0.$$

To sign the coefficient  $\psi_a$ , it is straightforward to check the following: (i)  $\psi_a$  is strictly decreasing in  $\gamma_2$ , and (ii)  $\lim_{\gamma_2 \rightarrow 0} \psi_a > \lim_{\gamma_2 \rightarrow \infty} \psi_a > 0$ . Together, this implies that  $\psi_a$  is everywhere positive.

Given the log-linear structure of equilibrium output and the log-normal specification for productivity and the noises, we find that aggregate output is given by  $\log Y_t = \Phi_0 + \Phi_\xi \xi_t$ , where  $\Phi_0 \equiv \phi_0 + \frac{1}{2} \left[ (\phi_a + \phi_1 + \phi_2)^2 + (\phi_1 + \phi_2)^2 \gamma_1 \right]$  and  $\Phi_\xi = \phi_2$ .

Next, due to the log-normal shock and information structure, we can infer that (i)  $\log \mathbb{E}_{it} y_{jt} = \mathbb{E}_{it} \log y_{jt} + \text{const}^a$ , and (ii)  $\log \mathbb{E}_{it} Y_t = \mathbb{E}_{it} [\log Y_t] + \text{const}^b$ , where  $\text{const}^a$  and  $\text{const}^b$  are simply constants. Second, note that  $\mathbb{E}_{it} [\xi_t] = \frac{\gamma_\xi^2}{\gamma_1^2 + \gamma_2^2 + \gamma_\xi^2} (x_{it}^2 - a_i)$ . It follows that the average belief of  $\xi_t$  equals  $\frac{\gamma_\xi^2}{\gamma_1^2 + \gamma_2^2 + \gamma_\xi^2} \xi_t$ , which in turn gives us that the average belief of aggregate output is given by

$$\log F_t^b \equiv \int \log \mathbb{E}_{it} Y_t di = \text{const}^b + \int \mathbb{E}_{it} [\log Y_t] di = \Delta_0 + \Delta_\xi \xi_t,$$

with  $\Delta_0 = \Phi_0 + \text{const}^b$ ,  $\Delta_\xi = \frac{\gamma_\xi^2}{\gamma_1^2 + \gamma_2^2 + \gamma_\xi^2} \Phi_\xi$ . Furthermore, from (18) we have that island  $i$ 's belief  $j$ 's log output must satisfy

$$\mathbb{E}_{it} [\log y_{jt}] = \frac{1}{\alpha} \left[ \log y(\omega_i) - (1 - \alpha) \frac{1}{1 - \vartheta} a_i \right]$$

This implies that the average belief of trading partner's output is given by

$$\log F_t^a \equiv \int \log \mathbb{E}_{it} y_{jt} di = \text{const}^a + \int \mathbb{E}_{it} [\log y_{jt}] di = \text{const}^a + \frac{1}{\alpha} \log Y_t$$

This establishes that both  $\log F_t^a$  and  $\log F_t^b$  are linear functions of  $\xi_t$ . *QED*

**Proof of Proposition 4.** Let  $\mathbf{x}_{it} \equiv (a_i, x_{it}^1, x_{it}^2, \dots, x_{it}^h)'$  and note that

$$\mathbf{x}_{it} = \mathbf{M} \xi_t + \mathbf{m}_1 \mathbf{u}_{it} + \mathbf{m}_2 \mathbf{u}_{jt} + \mathbf{m}_a \mathbf{a}_{ijt}$$

where  $\xi_t \equiv (\xi_t^1, \dots, \xi_t^h)'$ ,  $\mathbf{u}_{it} = (u_{it}^1, \dots, u_{it}^h)'$ ,  $\mathbf{u}_{jt} = (u_{jt}^1, \dots, u_{jt}^h)'$ , and  $\mathbf{a}_{ij} = (a_i, a_j)'$ , and where  $\mathbf{M}$ ,  $\mathbf{m}_1$ ,  $\mathbf{m}_2$ ,  $\mathbf{m}_a$  are some fixed matrices full of zeros and ones.

We guess and verify a log-linear equilibrium under the log-normal specification for the shock and information structure. Suppose the equilibrium production strategy of the island of type  $\omega_{jt}$  takes a log-linear form given by

$$\log y_t(\omega_{jt}) = \chi \bar{a}_t + \phi \mathbf{x}_{jt}$$

for some coefficients  $\chi \in \mathbb{R}$  and  $\phi = (\phi_a, \phi_1, \phi_2, \dots, \phi_h) \in \mathbb{R}_+^{H+1}$ . It follows that  $\log y_t(\omega_{jt})$  is indeed log-normal, with

$$\mathbb{E}[\log y_t(\omega_{jt})|\omega_{it}] = \chi \bar{a}_t + \phi \mathbb{E}[\mathbf{x}_{jt}|\omega_{it}] \quad (23)$$

Furthermore,  $i$ 's conditional expectation of  $\mathbf{x}_{jt}$  is simply the projection of  $\mathbf{x}_{jt}$  on  $\mathbf{x}_{it}$ :

$$\mathbb{E}[\mathbf{x}_{jt}|\omega_{it}] = \mathbf{H}\mathbf{x}_{it}$$

where  $\mathbf{H}$  is the relevant projection matrix. Substituting these expressions into (18) gives us

$$\log y(\omega_{it}) = (1 - \alpha) \frac{1}{1 - \vartheta} (a_i + \bar{a}_t) + \alpha [\chi \bar{a}_t + \phi \mathbf{H}\mathbf{x}_{it}]$$

For this to coincide with  $\log y(\omega_{it}) = \chi \bar{a}_t + \phi \mathbf{x}_{it}$  for every  $\omega_{it}$ , it is necessary and sufficient that the coefficients  $\chi$  and  $\phi$  are given the solution to the following system:

$$\begin{aligned} \chi &= (1 - \alpha) \frac{1}{1 - \vartheta} + \alpha \chi \\ \phi &= (1 - \alpha) \frac{1}{1 - \vartheta} \mathbf{e}_1 + \alpha (\phi \mathbf{H})' \end{aligned}$$

where  $\mathbf{e}_j$  is a column vector of length  $h + 1$  composed of zeros except for a unit in the  $j$ th position. Finally, noting that  $\int \mathbf{x}_{it} di = \mathbf{M}\xi_t$ , we find that aggregate output is given by  $\log Y_t = \chi \bar{a}_t + \phi \mathbf{M}\xi_t$ . Equivalently,  $\log Y_t = \Phi \bar{a}_t + \bar{\xi}_t$ , where  $\bar{\xi}_t \equiv \phi \mathbf{M}\xi_t$ .

Furthermore, the average belief of the trading partners output must satisfy

$$\mathbb{E}_{it}[\log y(\omega_{jt})] = \frac{1}{\alpha} \left[ \log y(\omega_i) - (1 - \alpha) \frac{1}{1 - \vartheta} (a_i + \bar{a}_t) \right]$$

Which implies that

$$\begin{aligned} \log F_t^a &= \int \mathbb{E}_{it}[\log y_{jt}] di = \frac{1}{\alpha} \left[ \log Y_t - (1 - \alpha) \frac{1}{1 - \vartheta} \bar{a}_t \right] \\ &= \frac{1}{\alpha} \left( \chi - (1 - \alpha) \frac{1}{1 - \vartheta} \right) \bar{a}_t + \frac{1}{\alpha} \phi \mathbf{M}\xi_t \end{aligned}$$

where we abstract from the constant (see proof of Proposition 3). Using the definitions of  $\bar{\xi}_t$  and  $\Phi$  along with the equilibrium value for  $\chi$ , and letting  $\Psi \equiv 1/\alpha$ , we get  $\log F_t^a = \chi \bar{a}_t + \frac{1}{\alpha} \bar{\xi}_t = \Phi \bar{a}_t + \Psi \bar{\xi}_t$ .

Finally, by projecting  $\xi_t$  on  $\mathbf{x}_{it}$ , we get  $\mathbb{E}[\xi_t|\omega_{it}] = \mathbf{B}\mathbf{x}_{it}$  for some matrix  $\mathbf{B}$ . It follows that

$$\mathbf{E}_{it}[\log Y_t] = \Phi \bar{a}_t + \phi \mathbf{M}\mathbf{B}\mathbf{x}_{it}$$

and therefore the corresponding average belief is given by

$$\log F_t^b = \int \mathbb{E}_{it}[\log Y_t] di = \Phi \bar{a}_t + \phi \mathbf{M} \left( \int \mathbf{x}_{it} di \right) = \Phi \bar{a}_t + \phi \mathbf{M}\mathbf{B}\mathbf{M}\xi_t$$

where again we abstract from the constant. Since  $\bar{\xi}_t \equiv \phi \mathbf{M}\xi_t$  and  $\phi \mathbf{M}\mathbf{B}\mathbf{M}\xi_t$  are both functions of  $\xi_t$ , and the latter is orthogonal to  $\bar{a}_t$ , we can regress  $\phi \mathbf{M}\mathbf{B}\mathbf{M}\xi_t$  on  $\bar{\xi}_t$  to obtain

$$\phi \mathbf{M}\mathbf{B}\mathbf{M}\xi_t = \Lambda \bar{\xi}_t + v_t,$$

and therefore  $\log F_t^b = \Phi \bar{a}_t + \Lambda \bar{\xi}_t + v_t$ , where  $\Lambda \equiv Cov(\phi \mathbf{M}\mathbf{B}\mathbf{M}\xi_t, \phi \mathbf{M}\xi_t) / Var(\phi \mathbf{M}\xi_t)$  and where  $v_t$  is a linear function of  $\xi_t$  that is orthogonal to both  $\bar{\xi}_t$  and  $\bar{a}_t$ .

**Proof of Proposition 5.** *Part (i).* For any period and any history up to that point, the type of an island belongs to the following set:

$$\bar{\Omega} \equiv \{\omega_U^N, \omega_{U+}^N, \omega_P^N, \omega_{P+}^N, \omega_F^N; \omega_U^S, \omega_{U+}^S, \omega_P^S, \omega_{P+}^S, \omega_F^S\},$$

where, for each group  $g \in \{N, S\}$ ,  $\omega_U^g$  are uninformed islands that are matched with a uninformed island from their group,  $\omega_{U+}^g$  are uninformed islands that are matched with a partially informed island,  $\omega_P^g$  are partially informed islands that are matched with an uninformed island;  $\omega_{P+}^g$  are partially informed islands that are matched with a partially informed island from the *other* group; and  $\omega_F^g$  are fully informed that are matched with a fully informed island from their group.

The period- $t$  cross-sectional distribution of types is thus summarized in a vector  $\mathbf{m}_t \in \Delta(\bar{\Omega})$ , with the  $n$ -th element of this vector giving the fraction of islands whose types is the  $n$ -th element of  $\bar{\Omega}$ . The dynamics of  $\mathbf{m}_t$  follows directly from the presumed matching technology.

Clearly,  $\omega_F^N$  and  $\omega_F^S$  are absorbing states for, respectively, the North and the South. Along with the fact that  $\lambda_0 > 0$ , this proves that  $\lambda_t$  must eventually decrease and must converge to zero as  $t \rightarrow \infty$ . Finally, the fact that  $\lambda_t$  must initially increase follows from the assumption  $\chi < 1/2$ .

*Part (ii).* To understand the determination of equilibrium output, consider first all the matches between islands of types  $\omega_{P+}^N$  and  $\omega_{P+}^S$ . These matches are, in effect, identical to those featured in Section (4.2). The equilibrium output for these types must therefore satisfy  $\log y(\omega_{P+}^N) = \phi_a a_N + \phi_1 x_N^1 + \phi_2 x_N^2$  and  $\log y(\omega_{P+}^S) = \phi_a a_S + \phi_1 x_S^1 + \phi_2 x_S^2$ , where the coefficients  $(\phi_a, \phi_1, \phi_2)$  are given in (20)-(22). For all other matches, on the other hand, it is straightforward to check that output is given either by  $\phi_a a_N$  (for the Northern islands) or  $\phi_a a_S$  (for the Southern islands), where  $\phi_a = \frac{1}{1-\vartheta}$ . We thus infer that local output is given as follows:

$$\log y_{it} = \begin{cases} \phi_a a_N + \phi_1 x_N^1 + \phi_2 x_N^2 & \text{if } \omega_{it} = \omega_{P+}^N, \\ \phi_a a_S + \phi_1 x_S^1 + \phi_2 x_S^2 & \text{if } \omega_{it} = \omega_{P+}^S, \\ \phi_a a_i & \text{otherwise} \end{cases} \quad (24)$$

Aggregating this across all islands, we obtain

$$\log Y_t = \phi_a \bar{a} + \lambda_t [\phi_1 \bar{\varepsilon} + \phi_2 \bar{\xi}]$$

where  $\bar{a} \equiv \frac{1}{2}(a_N + a_S)$  and  $\bar{\varepsilon} \equiv \frac{1}{2}(\varepsilon_1 + \varepsilon_2)$ , and where  $\lambda_t$  is the fraction of islands with types either  $\omega_{P+}^N$  or  $\omega_{P+}^S$ . The result then follows by letting  $\Phi \equiv \phi_2$ . *QED*

## Appendix B: RBC Variant

*Characterization.* By combining the optimality conditions for the final-good firms with market clearing (trade balance), we get

$$z_{it} = (1 - \eta)q_{it}, \quad z_{it}^* = \eta q_{jt}, \quad p_{it} = q_{it}^{-\eta} q_{jt}^{\eta}, \quad \text{and} \quad p_{it}^* = q_{it}^{1-\eta} q_{jt}^{\eta-1}$$

This is similar to the baseline model; we only have to re-interpret the consumption goods as the intermediate inputs.

Consider now the behavior of the intermediate-good firms. The first-order conditions with respect to labor, the capital stock, and the rate of capital utilization are, respectively, as follows:

$$\begin{aligned}\mathbb{E}_{it} [\lambda_{it} w_{it}] &= \mathbb{E}_{it} [\lambda_{it} p_{it}] \theta \frac{q_{it}}{n_{it}} \\ \mathbb{E}_{it} [\lambda_{it} (r_{it} + \Delta(e_{it}))] &= \mathbb{E}_{it} [\lambda_{it} p_{it}] (1 - \theta) \frac{q_{it}}{k_{it}} \\ \mathbb{E}_{it} [\lambda_{it}] \Delta'(e_{it}) k_{it} &= (1 - \theta) \mathbb{E}_{it} [\lambda_{it} p_{it}] \frac{q_{it}}{e_{it}}\end{aligned}$$

where  $\lambda_{it}$  is the marginal value of wealth on island  $i$ . These conditions simply state that the expected marginal costs of labor, capital, and capital utilization are equated with their respective expected marginal revenue products, which in turn depend on the island's expected terms of trade.

Next, on the household's side, the Envelope condition, the optimality condition for labor, and the Euler condition give the following:

$$\begin{aligned}\lambda_{it} &= U'(c_{it}) \\ V'(n_{it}) &= \mathbb{E}_{it} [\lambda_{it} w_{it}] \\ U'(c_{it}) &= \mathbb{E}_{it}^2 [\beta U'(c_{i,t+1}) (1 + r_{i,t+1})]\end{aligned}$$

where, recall,  $\mathbb{E}_{it}^2$  denotes the expectation conditional on stage-2 information.

Combining the aforementioned conditions, using  $p_{it} q_{it} = q_{it}^{1-\eta} q_{jt}^\eta = \zeta y_{it}$  where  $\zeta = (1 - \eta)^{1-\eta} \eta^\eta$ , and adding the local resource constraint, we get the system (10)-(15) in the main text.

*Log-linearization and numerical solution.* To characterize the equilibrium dynamics, we first log-linearize conditions (10)-(15) to get the following linear dynamic system:

$$\epsilon \tilde{n}_{it} = \mathbb{E}_{it}^1 [\tilde{y}_{it} - \gamma \tilde{c}_{it}] \quad (25)$$

$$(1 + \mu) \tilde{e}_{it} = \mathbb{E}_{it}^1 [\tilde{y}_{it} - \tilde{k}_{it}] \quad (26)$$

$$\tilde{c}_{it} = \mathbb{E}_{it}^2 \left[ \tilde{c}_{i,t+1} - \frac{(1-\beta)}{\gamma} (\tilde{y}_{it+1} - \tilde{k}_{i,t+1}) \right] \quad (27)$$

$$\bar{c} \tilde{c}_{it} + \bar{k} \tilde{k}_{i,t+1} = \bar{y} \tilde{y}_{it} + \left( 1 - \frac{1-\beta}{\beta\mu} \right) \bar{k} \tilde{k}_{it} - (1 + \mu) \frac{1-\beta}{\beta\mu} \bar{k} \tilde{e}_{it} \quad (28)$$

$$\tilde{y}_{it} = (1 - \eta) \tilde{q}_{it} + \eta \tilde{q}_{jt} \quad (29)$$

$$\tilde{q}_{it} = a_{it} + \theta \tilde{n}_{it} + (1 - \theta) (\tilde{e}_{it} + \tilde{k}_{it}) \quad (30)$$

where the bars denote steady-state values and the tildes denote log-deviations from steady state.

Let  $\tilde{\rho}_{it}$  denote the resources available in stage 2, in terms of log-deviation from steady state:

$$\tilde{\rho}_{it} = \rho \left( \tilde{y}_{it}, \tilde{k}_{it}, \tilde{e}_{it} \right) \equiv \bar{y} \tilde{y}_{it} + \left( 1 - \frac{1-\beta}{\beta\mu} \right) \bar{k} \tilde{k}_{it} - (1 + \mu) \frac{1-\beta}{\beta\mu} \bar{k} \tilde{e}_{it} \quad (31)$$

We conjecture the following island-level policy rules, along with a rule for aggregate capital:

$$(\tilde{e}_{it}, \tilde{n}_{it}, \tilde{q}_{it}) = f \left( a_{it}, x_{it}, \xi_t; \tilde{k}_{it}, \tilde{K}_t \right)$$

$$(\tilde{c}_{it}, k_{i,t+1}) = g \left( \tilde{\rho}_{it}; \xi_t, \tilde{K}_t \right)$$

$$\tilde{K}_{t+1} = \Gamma \left( \xi_t, \tilde{K}_t \right)$$

where the functions  $f, g$ , and  $\Gamma$  are linear. This guess is justified by the following considerations. First, an island’s employment, utilization, and production choices during stage 1 depend on its own productivity, its current signal of the productivity of its trading partner, and on the perceived bias in the latter’s signal for essentially the same reasons that it does in our baseline model; but now it also depends on its own capital stock, and on the aggregate capital stock, because the former enters local production while the latter is  $i$ ’s best forecast of the capital stock of its trading partner.<sup>20</sup> Second, an island consumption and investment during stage 2 are pinned down by realized resources, for the usual reasons, and by the aggregate state of the economy, for the latter determines  $i$ ’s beliefs of its future terms of trade, local income, and local prices. Finally, the aggregate policy rule for capital obtains from aggregating the corresponding individual policy rules and noting that the cross-sectional average of resources is ultimately pinned down by the current sentiment shock  $\xi_t$  and the current aggregate capital  $\tilde{K}_t$ .

We then solve the equilibrium by the method of undetermined coefficients: we write the policy rules in terms of arbitrary coefficients; we next plug these rules in the aforementioned log-linearized system (25)-(30) along with the definition of  $\rho_{it}$  in (31) and the aggregation consistency between  $g$  and  $G$ ; we then arrive to a system of equations in the aforementioned coefficients, which can be solved for the equilibrium. This procedure is, in effect, quite similar to the way one solves the log-linearized version of the RBC model, except for the extra complication that our log-linearized system embeds also a fixed point between island-specific and aggregate policy rules.

Once we have the policy rules, we create 500 random time series for the sentiment shock, each of length 250 periods (which is approximately as many quarters as in our data). For each of these series, we first compute the equilibrium time series of all the key macroeconomic variables, we next apply an HP-filter with the conventional weight (1600), and we finally compute the relevant business-cycle statistics on the HP-filtered series. We then take averages of these statistics across all the 500 series, and report these averages in the left two columns of Table 1 in the main text.

Finally, to obtain the empirical counterparts of these statistics, we use the actual U.S. time series data in Smets and Wouters (2007). This data covers the period 1947-2004, are at quarterly frequency, and are seasonally adjusted. Output, consumption, and investment are measured by, respectively, GDP, Personal Consumption Expenditures, and Fixed Private Investment; these variable as taken from the BEA, are deflated by the BEA’s GDP Price Deflator, and are normalized in per-capita terms, using the Civilian Noninstitutional Population aged 16 and over (the latter taken from the BLS). Employment is measured by Nonfarm Hours, as taken from the U.S. Department of Labor. The same HP-filter is applied to this data as with the model’s simulated data, and the corresponding statistics are finally reported in the right two columns of Table 1.

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<sup>20</sup>To understand why the aggregate capital stock  $K_t$  is  $i$ ’s best forecast of  $j$ ’s capital stock, recall that we have assumed that the idiosyncratic productivity of an island is i.i.d. over time and across islands. Learning about  $j$ ’s current productivity therefore gives no information about  $j$ ’s history of past productivity shocks and hence also about its capital stock. If, instead, productivity were persistent, then  $i$ ’s best forecasts of  $j$  would be a linear combination of the aggregate capital stock and  $i$ ’s signals about  $j$ ’s productivity. This would complicate a bit the solution, but is unlikely to affect the results.

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