Competitive Insurance Markets with Limited Commitment

Simon Board, Moritz Meyer-ter-Vehn

UCLA

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Motivation

Insurance subject to commitment problems

- Agent can cancel policy.
- Firm can exit if unprofitable.

Micro and macro interactions

- Competition aggravates agents' commitment problem.
- We solve for equilibrium in optimal self-enforcing contracts.

Main findings

- Stationary contracts are optimal.
- Eqm contracts determined by "competitive rationing" rule
- Endogenous contract dispersion.

Literature

Optimal single firm contracts

- Harris and Holmstrom (1982)
- Thomas and Worrall (1988)

Endogenizing outside option

- Bulow and Rogoff (1989)
- Sigouin (2004), Rudanko (2009)

Applications

- Health insurance without individual mandates.
- Implicit contracts in labor markets
- Optimal risk-sharing in development
- Financial markets with solvency constraints

Big Picture: Competitive Contracting

Contracting inefficiency

- Employment with moral hazard (Shapiro and Stigilitz, 1983)
- Insurance without commitment (Bulow and Rogoff, 1989)

Competitive rationing

- Propose family of rationing rules
- Without contracting problem, obtain first best
- With contracting problem, obtain contract distribution

Properties of equilibrium

- Endogenous contract dispersion
- Endogenous number of contracts offered

Model

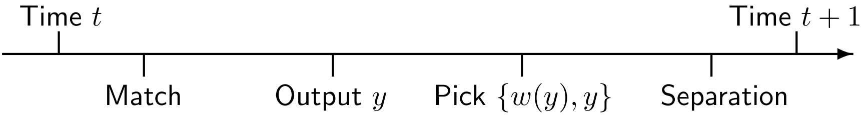
Model Overview

Economy

- Mass 1 identical agents; free entry of n identical firms.
- Time $\{1, 2, \ldots\}$; discount rate $\delta \in (0, 1)$.

Stage game

1. Agents receive outside offers $\{w(y^t)\}_{t=1}^{\infty}$
2. IID output $y_t \in [\underline{y}, \overline{y}]$ is publicly realized
3. Agent has choice of $w(y^t)$ (honor contract) and y_t (renege)
4. Separation with prob. $1 - \alpha$, and if either party terminates



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Stage payoffs

- Utility $u_t = u(w(y^t))$; assume $u'(\underline{y}) = \infty$, $u''(y) < 0$.
- Profit $\pi_t = y_t - w(y^t)$.

Firm's Problem

Firm's Problem

Matching Stage

- $W \sim F^e$ is cont. value of best offer; may have atom at 0.
- Firm fills contract instantly.

Restrictions:

- F^e stationary and anonymous.
- Contract $\langle w(y^t) \rangle$ only depends on history within relationship.

Self-enforcing contracts

- SPNE in pure strategies.
- No voluntary terminations.
- Harshest penal code off equilibrium.

Firm's Problem

Firm's problem is to choose $\langle w(y^t)\rangle$ to maximise Π_1 s.t.

$$u(w(y^t)) + \delta\alpha V_{t+1} + \delta(1-\alpha)V^{\varnothing} \geq u(y_t) + \delta V^{\varnothing} \qquad \text{(IR)}$$

$$\Pi_t \geq \Pi_1 \qquad \text{(FIR)}$$

- Agent's pre- and post-matching value functions

$$V_t = \int \max\{W, W_t\} dF^e(W)$$

$$W_t = E[u(w(y^t))] + \delta\alpha V_{t+1} + \delta(1-\alpha)V^{\varnothing}$$

- Firm's pre-matching profit function

$$\Pi_t = F^e(W_t)E[y_t - w(y^t) + \delta(\alpha\Pi_{t+1} + (1-\alpha)\Pi_1)] + [1 - F^e(W_t)]\Pi_1$$

Stationary Contracts

Stationary contract: Payments $w(y_t)$ independent of history y^{t-1}.

Theorem 1.
For any self-enforcing contract there is a stationary self-enforcing contract with weakly higher initial profit and utility.

Idea

- Thomas and Worrall: Increasing wages $w_t = \underline{w}(\max_{s \leq t} y_s)$
- Profits become negative, so firm terminates contract.

Notation

- Utility $\mu = E[u(w(y))]$ sufficient statistic for contract in matching market.
- Value of contract $V(\mu)$.

Firm Optimal Contract

Firm's problem is to choose $\langle w(y) \rangle$ to maximize

$$\pi = E[y - w(y)]$$

$$\text{s.t.} \quad u(w(y)) + \delta(\alpha V(\mu) + (1-\alpha)V^{\varnothing}) \geq u(y) + \delta V^{\varnothing} \qquad \text{(IR)}$$

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Optimal contract (Δ, w^*) given by,

$$u(w(y)) = \max\{u(w^*), u(y) - \Delta\}$$

$$\frac{1/u'(w^*)}{\alpha\delta V'(\mu)} = \frac{E\left[1/u'(w(y))\right]}{1 + \alpha\delta V'(\mu)}$$

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Value of job

$$V(\mu) = \int_{\underline{\mu}}^{\overline{\mu}} \max\{\mu + \alpha\delta V(\mu); \hat{\mu} + \alpha\delta V(\hat{\mu})\} dF^e(\hat{\mu}) + \delta(1-\alpha)V^{\varnothing}$$

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Value of job

$$V'(\mu) = (1 + \delta\alpha V'(\mu))F^e(\mu) = \frac{F^e(\mu)}{1 - \alpha\delta F^e(\mu)}$$

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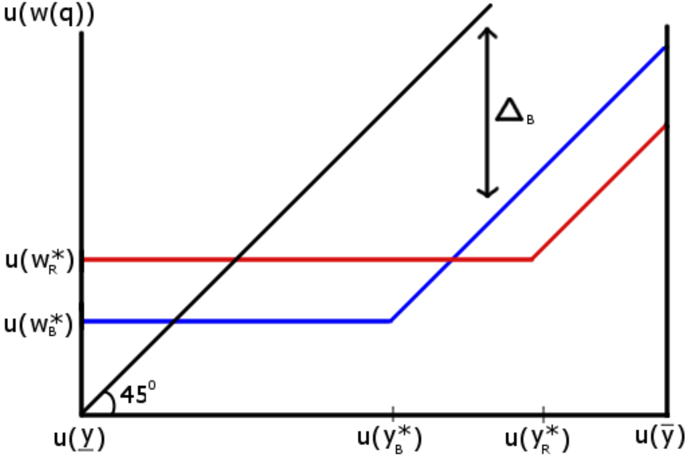
Multiple Optimal Contracts



Competitive Rationing

Competitive Rationing

Economy

- Mass 1 identical agents and $n \leq 1$ identical firms.
- αn filled contracts, $(1-\alpha)n$ vacancies with cdf $F(\mu)$.
- Axioms: Individual rationality, Anonymity, Market clearing.

- Market clearing

$$\underbrace{(1-\alpha n)(1-F^{\varnothing}(\mu))}_{\text{unemployed}} + \underbrace{\alpha n F(\mu)(1-F^{e}(\mu))}_{\text{employed below } \mu} = \underbrace{(1-\alpha)n(1-F(\mu))}_{\text{vacancies above } \mu}$$

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$$\underbrace{(1-\alpha n)(1-\psi(F^e(\mu)))}_{\text{unemployed}} + \underbrace{\alpha n F(\mu)(1-F^e(\mu))}_{\text{employed below } \mu} = \underbrace{(1-\alpha)n(1-F(\mu))}_{\text{vacancies above } \mu}$$

Matching function $\psi : [0,1] \to [0,1]$, weakly increasing.

- Comparative advantage of employed: $F^{\varnothing}(u) = \psi(F^e(\mu))$.

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$$\underbrace{(1-\alpha n)(1-\psi(\beta(q)))}_{\text{unemployed}} + \underbrace{\alpha n q(1-\beta(q))}_{\text{employed below } q} = \underbrace{(1-\alpha)n(1-q)}_{\text{vacancies above } q}$$

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- Comparative advantage of employed: $F^{\varnothing}(u) = \psi(F^e(\mu))$.
- Retention rate $F^e(\mu) = \beta(F(\mu))$ on range of $F(\cdot)$.
- Let $q = F(\mu)$ and expand $\beta(q)$ to all $q \in [0,1]$.

Competitive Rationing

Examples

- Shapiro-Stiglitz: $\psi(z) = 0$.
- Fully anonymous: $\psi(z) = z$.
- Intern matching: $\psi(z) = 1$.

Properties

- We assume unemployed are better searchers: $\psi(z) \leq z$.
- $\psi(\cdot)$ obeys OJS if it is continuous (i.e. $\beta(q)$ strictly inc. in q).
- More OJS under $\tilde{\psi}(\cdot)$ than $\psi(\cdot)$ if $\tilde{\psi}(z) \geq \psi(z)$.

Equilibrium

Equilibrium Definition

An industry equilibrium is mass n of contracts $\langle w_x(y)\rangle_{x\in[0,1]}$ s.t.

(a) Every contract $\langle w_x(y)\rangle$ is firm-optimal w.r.t. F^e and $V^{\varnothing}$.

(b) Every contract yields zero profits, $\pi = 0$.

(c) F^e and $V^{\varnothing}$ derived from rent distribution $F(\mu)$

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The value when without a contract is

$$V^\varnothing = \int \left(\mu + \delta\alpha V(\mu) + \delta(1-\alpha)V^\varnothing\right) dF^\varnothing(\mu) + (1-\theta)[\mu^\varnothing + \delta V^\varnothing]$$

where $1 - F^\varnothing(\varnothing) = \theta = (1-\alpha)n/(1-\alpha n)$ and $\mu^\varnothing = Eu(y)$.

Equilibrium Construction

Equilibrium $\langle \Delta(x), w^*(x), n \rangle_{x \in [0,1]}$ is characterized by:

1. Optimality condition (or marginal IR constraint)

$$\frac{1}{u'(w^*)} = \alpha\delta\beta(x)E\left[\frac{1}{u'(w(y))}\right]$$

First condition on $(\Delta(x), w^*(x))$ given n.

2. Zero Profits:

$$E[w(y) - y] = 0$$

Second condition on $(\Delta(x), w^*(x))$ given n.

3. IR constraint for highest firm

$$\Delta(1) = \alpha\delta[V(\mu(1)) - V^{\varnothing}]$$

Uniquely determines n.

Equilibrium Characterization

Theorem 2.

(a) *Equilibrium exists and is unique.*

(b) *Equilibrium insurance is determined by*

$$\frac{1}{u'(w^*)} = \alpha\delta\beta(x)E\left[\frac{1}{u'(w(y))}\right]$$

(c) *With OJS, $F(u)$ is strictly increasing and continuous.*

(d) *With FA matching, there is mass $n = 1$ of contracts.*

(e) *With less OJS, there are fewer contracts, $n < 1$.*

Example: Fully Anonymous Matching

Fully anonymous matching

- Employed and unemployed receive same offers: $\psi(z) = z$.
- Retention rate: $\beta(q) = (1 - n(1-q))/(1 - \alpha n(1-q))$.

Lowest job

- Retention rate $\beta(0) = 1 - \theta$.
- If $\beta(0) > 0$ then zero-profits and Thm 2 mean $\Delta(0) > 0$.
- But then $V(0) > V^{\varnothing}$ and (IR) does not bind.
- Hence firm is not maximizing profits.

In equilibrium

- Full employment, $n = 1$.
- Lowest contract has $\beta(0) = 0$, so that $\Delta(0) = 0$, $w^*(0) = \underline{y}$.

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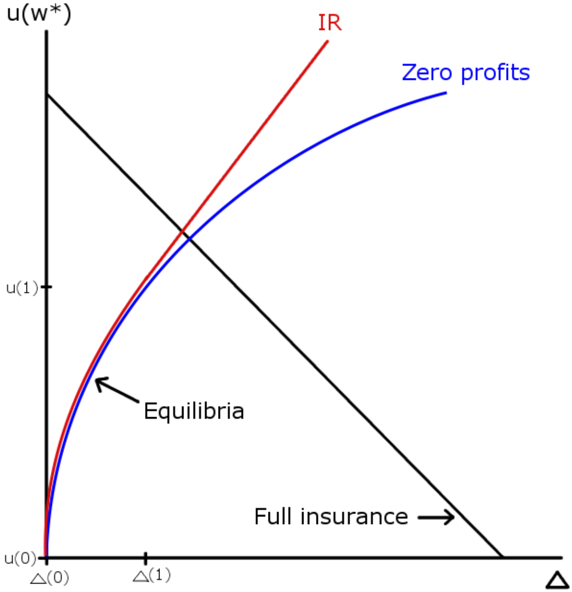


Example: Shapiro Stiglitz Matching

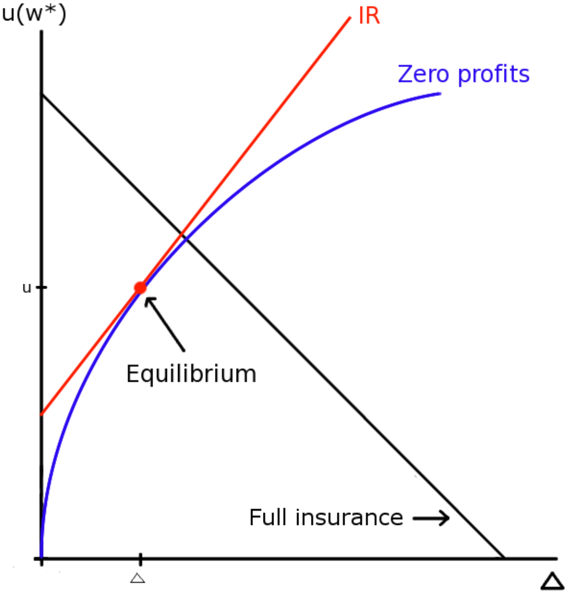
Shapiro-Stiglitz matching

- Only unemployed receive offers: $\psi \equiv 0$ and $\beta \equiv 1$.
- Theorem 2: All firms offer same contract.

Some agents are uninsured

- If take contract then cannot look for another one.
- Opportunity cost means zero-insurance contract not IR.
- Acts like fixed cost of offering contract.

Example: Shapiro-Stiglitz Matching



The End

Summary

Methodology for studying competitive contracts

- Propose family of competitive rationing rules
- Study distribution of contracts as function of the rule

Insurance contracts with limited commitment

- Stationary contracts are optimal.
- Endogenous contract dispersion.
- Under FA matching, everybody is (under)insured.

What next?

- Markovian output
- Competition ex-post instead of ex-ante
- Asymmetric risk aversion