Competitive Insurance Markets with Limited Commitment

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Motivation

Insurance subject to commitment problems

- Agent can cancel policy.
- Firm can exit if unprofitable.

Micro and macro interactions

- Competition aggravates agents’ commitment problem.
- We solve for equilibrium in optimal self-enforcing contracts.

Main findings

- Stationary contracts are optimal.
- Eqm contracts determined by “competitive rationing” rule
- Endogenous contract dispersion.
Literature

Optimal single firm contracts

- Harris and Holmstrom (1982)
- Thomas and Worrall (1988)

Endogenizing outside option

- Bulow and Rogoff (1989)
- Sigouin (2004), Rudanko (2009)

Applications

- Health insurance without individual mandates.
- Implicit contracts in labor markets
- Optimal risk-sharing in development
- Financial markets with solvency constraints
Big Picture: Competitive Contracting

Contracting inefficiency

- Employment with moral hazard (Shapiro and Stiglitz, 1983)
- Insurance without commitment (Bulow and Rogoff, 1989)

Competitive rationing

- Propose family of rationing rules
- Without contracting problem, obtain first best
- With contracting problem, obtain contract distribution

Properties of equilibrium

- Endogenous contract dispersion
- Endogenous number of contracts offered
Model
Model Overview

Economy
▶ Mass 1 identical agents; free entry of $n$ identical firms.
▶ Time $\{1, 2, \ldots\}$; discount rate $\delta \in (0, 1)$.

Stage game
1. Agents receive outside offers $\{w(y^t)\}_{t=1}^{\infty}$
2. IID output $y_t \in [\underline{y}, \overline{y}]$ is publicly realized
3. Agent has choice of $w(y^t)$ (honor contract) and $y_t$ (renege)
4. Separation with prob. $1 - \alpha$, and if either party terminates
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Stage payoffs

- Utility $u_t = u(w(y^t))$; assume $u'(y) = \infty$, $u''(y) < 0$.
- Profit $\pi_t = y_t - w(y^t)$. 
FIRM’S PROBLEM
Firm’s Problem

Matching Stage

- $W \sim F^e$ is cont. value of best offer; may have atom at 0.
- Firm fills contract instantly.

Restrictions:

- $F^e$ stationary and anonymous.
- Contract $\langle w(y^t) \rangle$ only depends on history within relationship.

Self-enforcing contracts

- SPNE in pure strategies.
- No voluntary terminations.
- Harshest penal code off equilibrium.
Firm’s Problem

Firm’s problem is to choose \( \langle w(y^t) \rangle \) to maximise \( \Pi_1 \) s.t.

\[
\begin{align*}
  u(w(y^t)) + \delta \alpha V_{t+1} + \delta (1 - \alpha) V^\emptyset & \geq u(y_t) + \delta V^\emptyset \\
  \Pi_t & \geq \Pi_1
\end{align*}
\]

(IR) (FIR)

- Agent’s pre- and post-matching value functions

\[
\begin{align*}
  V_t &= \int \max\{W, W_t\} dF^e(W) \\
  W_t &= E[u(w(y^t))] + \delta \alpha V_{t+1} + \delta (1 - \alpha) V^\emptyset
\end{align*}
\]

- Firm’s pre-matching profit function

\[
\Pi_t = F^e(W_t) E[y_t - w(y^t) + \delta (\alpha \Pi_{t+1} + (1 - \alpha) \Pi_1)] + [1 - F^e(W_t)] \Pi_1
\]
Stationary Contracts

Stationary contract: Payments $w(y_t)$ independent of history $y^{t-1}$.

**Theorem 1.**
*For any self-enforcing contract there is a stationary self-enforcing contract with weakly higher initial profit and utility.*

Idea

▶ Thomas and Worrall: Increasing wages $w_t = w(\max_{s \leq t} y_s)$
▶ Profits become negative, so firm terminates contract.

Notation

▶ Utility $\mu = E[u(w(y))]$ sufficient statistic for contract in matching market.
▶ Value of contract $V(\mu)$. 
Firm Optimal Contract

Firm's problem is to choose $\langle w(y) \rangle$ to maximize

$$\pi = E[y - w(y)]$$

s.t. $$u(w(y)) + \delta(\alpha V(\mu) + (1 - \alpha)V^\emptyset) \geq u(y) + \delta V^\emptyset \quad \text{(IR)}$$
Firm Optimal Contract

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Optimal contract \((\Delta, w^*)\) given by,

\[
u(w(y)) = \max\{u(w^*), u(y) - \Delta\}
\]

\[
\frac{1}{u'(w^*)} = \frac{E[1/u'(w(y))]}{1 + \alpha \delta V'(\mu)}
\]
**Firm Optimal Contract**

**Firm's problem** is to choose \( \langle w(y) \rangle \) to maximize

\[
\pi = E[y - w(y)]
\]

subject to

\[
u(w(y)) + \alpha \delta V(\mu) \geq u(y) + \alpha \delta V^\emptyset \quad \text{(IR)}
\]

Optimal contract \((\Delta, w^*)\) given by,

\[
u(w(y)) = \max \{u(w^*), u(y) - \Delta\}
\]

\[
\frac{1}{u'(w^*)} = \frac{E[1/u'(w(y))]}{1 + \alpha \delta V'(\mu)}
\]

Value of job

\[
V(\mu) = \int_{\mu}^{\bar{\mu}} \max \{\mu + \alpha \delta V(\mu); \hat{\mu} + \alpha \delta V(\hat{\mu})\} dF^e(\hat{\mu}) + \delta (1 - \alpha) V^\emptyset
\]
Firm Optimal Contract

Firm's problem is to choose \( \langle w(y) \rangle \) to maximize

\[
\pi = E[y - w(y)]
\]

s.t. \( u(w(y)) + \alpha \delta V(\mu) \geq u(y) + \alpha \delta V^\emptyset \) (IR)

Optimal contract \((\Delta, w^*)\) given by,

\[
\begin{align*}
  u(w(y)) &= \max\{u(w^*), u(y) - \Delta\} \\
  \frac{1}{u'(w^*)} &= \frac{\alpha \delta V'(\mu)}{E[1/u'(w(y))]} \\
  \frac{\alpha \delta V'(\mu)}{\alpha \delta V'(\mu)} &= \frac{E[1/u'(w(y))]}{1 + \alpha \delta V'(\mu)}
\end{align*}
\]

Value of job

\[
V'(\mu) = (1 + \delta \alpha V'(\mu))F^e(\mu) = \frac{F^e(\mu)}{1 - \alpha \delta F^e(\mu)}
\]
Firm Optimal Contract

Firm’s problem is to choose $\langle w(y) \rangle$ to maximize

$$
\pi = E[y - w(y)]
$$

s.t. $u(w(y)) + \alpha \delta V(\mu) \geq u(y) + \alpha \delta V^\emptyset \quad (IR)

Optimal contract $(\Delta, w^*)$ given by,

$$
u(w(y)) = \max\{u(w^*), u(y) - \Delta\}
$$

$$
\frac{1}{u'(w^*)} = \frac{E[1/u'(w(y))]}{\alpha \delta F^e(\mu)}
$$

$$
\frac{1}{u'(w^*)} = \frac{E[1/u'(w(y))]}{1}
$$
Multiple Optimal Contracts
Competitive Rationing
Competitive Rationing

Economy

- Mass 1 identical agents and \( n \leq 1 \) identical firms.
- \( \alpha n \) filled contracts, \( (1 - \alpha) n \) vacancies with cdf \( F(\mu) \).

- Market clearing

\[
(1 - \alpha n)(1 - F^\emptyset(\mu)) + \alpha n F(\mu)(1 - F^e(\mu)) = (1 - \alpha) n (1 - F(\mu))
\]

unemployed \hspace{2cm} employed below \( \mu \) \hspace{2cm} vacancies above \( \mu \)
Competitive Rationing

Economy

- Mass 1 identical agents and \( n \leq 1 \) identical firms.
- \( \alpha n \) filled contracts, \( (1 - \alpha)n \) vacancies with cdf \( F(\mu) \).

- Market clearing

\[
(1 - \alpha n)(1 - \psi(F_e(\mu))) + \alpha n F(\mu)(1 - F_e(\mu)) = (1 - \alpha)n(1 - F(\mu))
\]

\( \psi \) is the matching function: \( \psi : [0, 1] \rightarrow [0, 1] \), weakly increasing.

- Comparative advantage of employed: \( F^\varnothing(u) = \psi(F_e(\mu)) \).
### Competitive Rationing

**Economy**

- Mass 1 identical agents and $n \leq 1$ identical firms.
- $\alpha n$ filled contracts, $(1 - \alpha)n$ vacancies with cdf $F(\mu)$.

**Market clearing**

\[
(1 - \alpha n)(1 - \psi(\beta(F(\mu)))) + \alpha n F(\mu)(1 - \beta(F(\mu))) = (1 - \alpha)n(1 - F(\mu))
\]

Matching function $\psi : [0, 1] \rightarrow [0, 1]$, weakly increasing.

- Comparative advantage of employed: $F^\varnothing(u) = \psi(F^e(\mu))$.
- Retention rate $F^e(\mu) = \beta(F(\mu))$ on range of $F(\cdot)$. 
Competitive Rationing

Economy

- Mass \(1\) identical agents and \(n \leq 1\) identical firms.
- \(\alpha n\) filled contracts, \((1 - \alpha)n\) vacancies with cdf \(F(\mu)\).

- Market clearing

\[
(1 - \alpha n)(1 - \psi(\beta(q))) + \alpha n q(1 - \beta(q)) = (1 - \alpha)n(1 - q)
\]

Matching function \(\psi : [0, 1] \rightarrow [0, 1]\), weakly increasing.

- Comparative advantage of employed: \(F^\emptyset(u) = \psi(F^e(\mu))\).
- Retention rate \(F^e(\mu) = \beta(F(\mu))\) on range of \(F(\cdot)\).
- Let \(q = F(\mu)\) and expand \(\beta(q)\) to all \(q \in [0, 1]\).
Competitive Rationing

Examples

- Shapiro-Stiglitz: $\psi(z) = 0$.
- Fully anonymous: $\psi(z) = z$.
- Intern matching: $\psi(z) = 1$.

Properties

- We assume unemployed are better searchers: $\psi(z) \leq z$.
- $\psi(\cdot)$ obeys OJS if it is continuous (i.e. $\beta(q)$ strictly inc. in $q$).
- More OJS under $\tilde{\psi}(\cdot)$ than $\psi(\cdot)$ if $\tilde{\psi}(z) \geq \psi(z)$.
Equilibrium
An **industry equilibrium** is mass $n$ of contracts $\langle w_x(y) \rangle_{x \in [0,1]}$ s.t.

(a) Every contract $\langle w_x(y) \rangle$ is firm-optimal w.r.t. $F^e$ and $V^{\emptyset}$.

(b) Every contract yields zero profits, $\pi = 0$.

(c) $F^e$ and $V^{\emptyset}$ derived from rent distribution $F(\mu)$.
Equilibrium Definition

An industry equilibrium is mass \( n \) of contracts \( \langle w_x(y) \rangle_{x \in [0,1]} \) s.t.

(a) Every contract \( \langle w_x(y) \rangle \) is firm-optimal w.r.t. \( F^e \) and \( V^\emptyset \).
(b) Every contract yields zero profits, \( \pi = 0 \).
(c) \( F^e \) and \( V^\emptyset \) derived from rent distribution \( F(\mu) \)

The value when without a contract is

\[
V^\emptyset = \int (\mu + \delta \alpha V(\mu) + \delta (1 - \alpha) V^\emptyset) \, dF^\emptyset(\mu) + (1 - \theta)[\mu^\emptyset + \delta V^\emptyset]
\]

where \( 1 - F^\emptyset(\emptyset) = \theta = (1 - \alpha)n/(1 - \alpha n) \) and \( \mu^\emptyset = Eu(y) \).
Equilibrium Construction

Equilibrium $\langle \Delta(x), w^*(x), n \rangle_{x \in [0,1]}$ is characterized by:

1. **Optimality condition (or marginal IR constraint)**

   $$\frac{1}{u'(w^*)} = \alpha \delta \beta(x) E \left[ \frac{1}{u'(w(y))} \right]$$

   First condition on $(\Delta(x), w^*(x))$ given $n$.

2. **Zero Profits:**

   $$E[w(y) - y] = 0$$

   Second condition on $(\Delta(x), w^*(x))$ given $n$.

3. **IR constraint for highest firm**

   $$\Delta(1) = \alpha \delta [V(\mu(1)) - V^\varnothing]$$

   Uniquely determines $n$. 
Theorem 2.

(a) Equilibrium exists and is unique.
(b) Equilibrium insurance is determined by

\[ \frac{1}{w'(w^*)} = \alpha \delta \beta(x) E \left[ \frac{1}{w'(w(y))} \right] \]

(c) With OJS, \( F(u) \) is strictly increasing and continuous.
(d) With FA matching, there is mass \( n = 1 \) of contracts.
(e) With less OJS, there are fewer contracts, \( n < 1 \).
Example: Fully Anonymous Matching

Fully anonymous matching

- Employed and unemployed receive same offers: $\psi(z) = z$.
- Retention rate: $\beta(q) = (1 - n(1 - q))/(1 - \alpha n(1 - q))$.

Lowest job

- Retention rate $\beta(0) = 1 - \theta$.
- If $\beta(0) > 0$ then zero-profits and Thm 2 mean $\Delta(0) > 0$.
- But then $V(0) > V^\varnothing$ and (IR) does not bind.
- Hence firm is not maximizing profits.

In equilibrium

- Full employment, $n = 1$.
- Lowest contract has $\beta(0) = 0$, so that $\Delta(0) = 0$, $w^*(0) = y$. 
Example: Fully Anonymous Matching
Example: Shapiro Stiglitz Matching

Shapiro-Stiglitz matching

- Only unemployed receive offers: $\psi \equiv 0$ and $\beta \equiv 1$.
- Theorem 2: All firms offer same contract.

Some agents are uninsured

- If take contract then cannot look for another one.
- Opportunity cost means zero-insurance contract not IR.
- Acts like fixed cost of offering contract.
Example: Shapiro-Stiglitz Matching
The End
Summary

Methodology for studying competitive contracts

- Propose family of competitive rationing rules
- Study distribution of contracts as function of the rule

Insurance contracts with limited commitment

- Stationary contracts are optimal.
- Endogenous contract dispersion.
- Under FA matching, everybody is (under)insured.

What next?

- Markovian output
- Competition ex-post instead of ex-ante
- Asymmetric risk aversion