NBER WORKING PAPER SERIES

ADVERSE SELECTION, REPUTATION AND SUDDEN COLLAPSES IN SECONDARY LOAN MARKETS

V.V. Chari
Ali Shourideh
Ariel Zetlin-Jones

Working Paper 16080
http://www.nber.org/papers/w16080

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
June 2010

We are grateful to Kathy Rolfe for editorial assistance and Hugo Hopenhayn, Roozbeh Hosseini, Larry Jones, Patrick Kehoe, Guido Lorenzoni, Chris Phelan as well as seminar participants at ASU, Kellogg, Yale, the 2009 SED Meeting, New York and Minneapolis Fed, the Conference on Money and Banking at University of Wisconsin, and XII International Workshop in International Economics and Finance in Rio for helpful comments. Chari and Shourideh are grateful to the National Science Foundation for support. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis, or the Federal Reserve System, or the views of the National Bureau of Economic Research.

Adverse Selection, Reputation and Sudden Collapses in Secondary Loan Markets
V.V. Chari, Ali Shourideh, and Ariel Zetlin-Jones
NBER Working Paper No. 16080
June 2010
JEL No. E44,E50,G01,G14,G18,G38

ABSTRACT

Banks and financial intermediaries that originate loans often sell some of these loans or securitize them in secondary loan markets and hold on to others. New issuances in such secondary markets collapse abruptly on occasion, typically when collateral values used to secure the underlying loans fall. These collapses are viewed by policymakers as signs that the market is not functioning efficiently. In this paper, we develop a dynamic adverse selection model in which small reductions in collateral values can generate abrupt inefficient collapses in new issuances in the secondary loan market. In our model, reductions in collateral values worsen the adverse selection problem and induce some potential sellers to hold on to their loans. Reputational incentives induce a large fraction of potential sellers to hold on to their loans rather than sell them in the secondary market. We find that a variety of policies that have been proposed during the recent crisis to remedy market inefficiencies do not help resolve the adverse selection problem.

V.V. Chari
Department of Economics
University of Minnesota
1035 Heller Hall
271 - 19th Avenue South
Minneapolis, MN 55455
and NBER
varadarajanvchari@gmail.com

Ali Shourideh
Department of Economics
University of Minnesota
4-101 Hanson Hall
1925 Fourth Street South
Minneapolis, MN 55455
shour004@umn.edu

Ariel Zetlin-Jones
Department of Economics
University of Minnesota
4-101 Hanson Hall
1925 Fourth Street South
Minneapolis, MN 55455
zetli001@umn.edu

1 Introduction

Following the sharp decline in the volume of new issuances in the U.S. secondary loan market in the fall of 2007, policymakers argued that the market was not functioning normally and proposed and carried out a variety of policy interventions intended to restore the normal functioning of this market. Here we construct a model in which new issuances in the secondary loan market abruptly collapse and this collapse is associated with an increase in inefficiency. We also argue that reductions in the value of the collateral used to secure the underlying loans are particularly likely to trigger sudden collapses associated with increased inefficiency. Since sudden collapses are associated with increased inefficiency, our model is consistent with policymakers' views that the market was functioning poorly. We use this model to analyze proposed and actual policy interventions and argue that these interventions typically do not remedy the inefficiency associated with the market collapse.

In our model, the main economic function of the secondary loan market is to allocate originated loans to institutions that have a *comparative advantage* in holding and managing the loans. This economic function is disrupted by informational frictions. In our model, loan originators differ in their ability to originate high quality loans. The originators are better informed about their ability to generate high quality loans than are potential purchasers. This informational friction creates an *adverse selection* problem. The focus of our analysis is to examine the extent to which reputational considerations ameliorate or intensify the adverse selection problem in these markets. In order to analyze these reputational considerations, we develop a dynamic adverse selection model of the secondary loan market.

Our main finding is that our model has fragile outcomes in which sudden collapses in the volume of new issuances in secondary loan markets are associated with increased inefficiency. We say that outcomes are fragile if the model has multiple equilibria or if a large number of originators change their decisions in response to small changes in aggregate fundamentals.

In terms of fragility as multiplicity, we show that our baseline dynamic adverse selection model with reputation has multiple equilibria for a range of reputation levels. In one of these equilibria, labeled the *positive reputational equilibrium*, high quality loan originators have incentives to sell at a current loss in order to improve their reputations and command higher prices for future loans.

In the other equilibrium, labeled the *negative reputational equilibrium*, loan originators who sell their loans are perceived by future buyers to have low quality loans. These perceptions induce high quality loan originators to hold on to their loans. Since low quality originators always sell their loans, the volume of new issuances is larger in the positive reputational equilibrium than in the negative reputational equilibrium.

To see that the multiplicity of equilibria implies that our model can generate sudden collapses in the volume of new issuances, consider some exogenous event that induces originators and buyers to switch from the positive to the negative reputational equilibrium. If many originators have reputation levels in the multiplicity region, this event induces a sudden collapse in the volume of new issuances. We provide conditions under which the positive reputational equilibrium yields higher welfare than the negative reputational equilibrium (both in the interim and ex-ante sense of efficiency as in Holmstrom and Myerson (1983).) Therefore, our model can generate sudden collapses associated with increased inefficiency.

While the multiplicity of equilibria has the attractive feature that it implies that the model can be consistent with observations of sudden collapses, such multiplicity makes it difficult to conduct policy analysis. We propose a refinement adapted from the coordination games literature (see Carlsson and Van Damme (1993) and Morris and Shin (2003)). Our refinement is also motivated by the idea that sudden collapses in the volume of new issuances in loan markets are associated with falls in the value of the collateral that supports the underlying loans. These considerations lead us to add fluctuations in the collateral value and to assume that the collateral value is observed with an arbitrarily small error.

We show that fluctuations in the collateral value make the outcomes of our model consistent with our second notion of fragility, namely, a large fraction of loan originators choose to change their decisions on whether to sell or hold their loans in response to small changes in collateral values. In this sense, reductions in collateral values can induce sudden collapses in the volume of new issuances for the market as a whole.

Both adverse selection and the dynamics induced by reputation acquisition play central roles in generating sudden collapses from small changes in collateral values. A simple way of seeing the role of adverse selection is to note that the version of our model with symmetrically informed originators and buyers does not produce sudden collapses in new issuances. With asymmetrically informed agents, originators with high reputations receive higher prices for their loans and are therefore more willing to sell their loans. We show that a fall in collateral values makes high quality originators less willing to sell their loans. This result follows because the market price, being a weighted average of the loans sold by low and high quality originators, falls by a larger amount than does the return to a high quality originator to holding a loan. A fall in collateral values tends to induce originators who were close to being indifferent about selling versus holding to hold. Small changes in collateral values can induce a large number of originators to switch to holding from selling only if they are all close to the point of indifference. In a static model, we have no reason to expect that the distribution of originators by reputation levels will be concentrated close to the indifference point.

In a dynamic model with learning by market participants, we argue that originators' reputations are likely to be clustered. The reason is that in models like ours, the reputation levels of high quality originators have an upward trend over time resulting in the reputation levels of many high quality originators tending to become similar in the long run. We show that in an infinitely repeated version of our model, the long run or invariant distribution of reputation levels displays significant clustering. This clustering in turn implies that small changes in fundamentals can lead a large number of originators to change their decisions when the fundamentals are close to the point of indifference. A related result is that small changes in collateral values when these values are far away from the point of indifference do not lead to large changes in the volume of new issuances.

The fragility of equilibrium in our model implies that it is consistent with the observed large fluctuations in the volume of new issuances in the market for asset backed securities. Figure 1 displays the volume of new issuances of asset-backed securities for various categories from the first quarter of 2000 to the first quarter of 2009. The figure shows that the total volume of new issuances of asset-backed securities rose from roughly \$50 billion in the first quarter of 2000 to roughly \$300 billion in the fourth quarter of 2006. The volume of new issuances fell abruptly to roughly \$100 billion in the third quarter of 2007 and then fell again to near zero in roughly the fourth quarter of 2008. The figure also shows similar large fluctuations in the volume of new issuances for each category.

Ivashina and Scharfstein (2008) document a similar pattern for new issues of syndicated loans. Figure 1, Panel-A of their paper shows that syndicated lending rose from roughly \$300 billion in the first quarter of 2000 to roughly \$700 billion in the second quarter of 2007. This lending declined

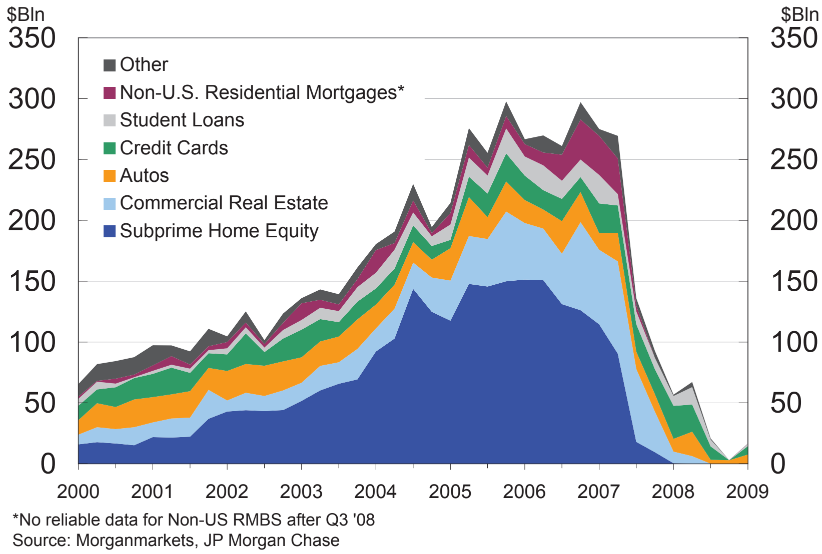

Figure 1: New Issuance of Asset Backed Securities (Source: JP Morgan Chase)

sharply thereafter and fell to roughly $100 billion by the third quarter of 2008.

The reduction in the volume of new issuances in the secondary market roughly coincided with a reduction in collateral values. One way of seeing this coincidence is to consider the Case-Shiller home price index (available at http://www.standardandpoors.com/indices). This index stopped growing in late 2006 and declined through 2007. The coincidence of the reduction in the volume of new issuances and the reduction in collateral values is consistent with our model.

White (2009) has argued that the United States experienced a boom bust cycle in securitization of real estate assets in the 1920's similar to its recent experience. Figure 2 displays the change in the outstanding stock in real estate bonds in the 1920s based on data in Carter and Sutch (2006). Such bonds were issued against single large commercial mortgages or pools of commercial or real estate mortgages and were publicly traded. To make this data comparable to more recent data, we scale the data from the 1920s by nominal GDP in 2009. Specifically, we multiply the change in the nominal stock of outstanding debt in each year by ratio of the nominal GDP in 2009 to that in the relevant year. This figure shows that the changes in the stock rose dramatically from essentially

0 in 1919 to an average of 145 billion dollars in the period from 1925 to 1928. The market then collapsed sharply and changes in the stock fell to roughly 50 billion dollars in 1929. Such large changes in the stock are likely to have been associated with similar large changes in the volume of new issuances.

Figure 2: Change in Stock of Real Estate Bonds 1920-1930

We have argued that our model is consistent with abrupt collapses in secondary loan markets. Our model is also consistent with the widespread view among policymakers that such abrupt collapses were associated with sharp increases in the inefficiency of the operation of such markets. For example, the Treasury Department, in its Fact Sheet dated March 23, 2009 releasing details of a proposed Public-Private Investment Program for Legacy Assets asserts,

> "Secondary markets have become highly illiquid, and are trading at prices below where they would be in normally functioning markets." (Treasury Department 2009)

Similarly, the Federal Reserve Bank of New York, in a White Paper dated March 3, 2009 making the case for the Temporary Asset Loan Facility (TALF) asserts that

> "Nontraditional investors such as hedge funds, which may otherwise be willing to invest in these securities, have been unable to obtain funding from banks and dealers because of a general reluctance to lend." (TALF White Paper 2009)

In the wake of the 2007 collapse of secondary loan markets, policymakers proposed a variety of programs intended to remedy inefficiencies in the market for securitized assets. Some of these programs, such as the proposed Public-Private Partnership for purchasing assets held by distressed financial institutions, were not implemented. Others, such as TALF, were implemented. This program allows participants to purchase securitized assets by borrowing from the Federal Reserve and using the assets as collateral. To the extent that the interest rate charged by the Federal Reserve is below market interest rates, this program is effectively a subsidy for the private purchase of assets in the secondary loan market. To the extent that the interest rate charged by the Federal Reserve is at market interest rates, it is not clear why this program would be effective.

We use our model to evaluate the effects of various policies. One such policy which resembles the Public-Private Partnership and the TALF program is that the government offers to purchase loans at prices at or above existing market values. Another policy, which is intended to capture the effects of the Federal Reserve's monetary policy actions, is to change the time path of interest rates. In terms of purchase policies, we show that if the price is set below that level that prevails in the positive reputational equilibrium, the policy by itself does not change equilibrium outcomes but it does involve transfers to banks and implies that the government makes negative profits. If the purchase price is set at a sufficiently high level, this policy can eliminate the fragility of equilibria. At this high level, the policy also involves transfers to banks and implies that the government makes negative profits.

In terms of policies that change the time path of interest rates, we show that temporary decreases in interest rates worsen the adverse selection problem. Interestingly, anticipated decreases in interest rates in the future can have beneficial current effects by reducing the range of reputations over which the economy has multiple equilibria.

1.1 Related Literature

Our work here is related to an extensive literature on adverse selection in asset markets, including the work of Myers and Majluf (1984), Glosten and Milgrom (1985), Kyle (1985), and Garleanu and Pedersen (2004) as well as to the related securitization literature, specifically, the work of DeMarzo and Duffie (1999) and DeMarzo (2005). We add to this literature by analyzing how reputational incentives affect adverse selection problems.

Our assumption that buyers have less information concerning the loan quality of a bank is in line with a descriptive literature that argues that secondary loan markets feature adverse selection (see, for example, the work of Dewatripont and Tirole (1994), Ashcraft and Schuermann (2008), and Arora et al. (2009)). Also, a growing literature provides data on the presence of adverse selection in asset markets. For example, Downing et al. (2009) find that loans which banks held on their balance sheets yielded more on average relative to similar loans which they securitized and sold. Drucker and Mayer (2008) argue that underwriters of prime mortgage-backed securities are better informed than buyers and present evidence that these underwriters exploit their superior information when trading in the secondary market. Specifically, the tranches that such underwriters avoid bidding on exhibit much worse-than-average ex-post performance than the tranches that they do bid on.

Our work is also related to an extensive literature on reputation. Kreps and Wilson (1982) and Milgrom and Roberts (1982) argue that equilibrium outcomes are better in models with reputational incentives than in models without them. In the banking literature, Diamond (1989) develops this argument. More recently, Mailath and Samuelson (2001) analyze the role of reputational incentives in infinite horizon economies and provide conditions under which they can improve outcomes. In contrast, Ely and Välimäki (2003) and Ely et al. (2008) describe models in which reputational incentives can worsen outcomes. Our work here combines the results in this literature by showing that reputational models can have multiple equilibria. In some of these equilibria, reputational incentives can generate better outcomes; in others, worse. Furthermore, using techniques from the global games literature, we develop a refinement that produces a unique, fragile equilibrium. Perhaps the work most closely related to ours is that of Ordoñez (2008). An important difference between our work and his is that our model has equilibria that are worse than the static equilibrium, so that reputational incentives can lead to outcomes that are ex-post less efficient than in a model without these incentives.

Our analysis of policy is closely related to recent work by Philippon and Skreta (2009) who analyze a variety of policies in a model with adverse selection. The main difference with our work is that we focus on the incentives induced by reputation while they analyze a static model.

2 Reputation in a Secondary Loan Market Model

We develop a finite horizon model of the secondary loan market and use the model to demonstrate how adverse selection and reputation interact to yield abrupt collapses with increased inefficiency.

We begin with a static version of our benchmark model. We use the unique equilibrium of this model to construct equilibria in a repeated finite horizon model. We show that reputational equilibria typically exhibit dynamic coordination problems in the sense that for a wide range of parameters, the repeated model has multiple equilibria. Although reputation is always valued, across the different equilibria loan originators choose different actions based on the different inferences future buyers draw from the current actions of originators.

2.1 Static Model: A Unique Equilibrium

We start with the static model. This model can also be interpreted as describing the last period of a finite horizon model. We show that the static model has a unique equilibrium in which the equilibrium outcomes depend on the informed originator's reputation.

2.1.1 Agents and Timing

The model has three types of agents: a loan originator referred to as a bank, a continuum of buyers, and a continuum of lenders. All agents are risk neutral.

The bank is endowed with a risky loan indexed by π. The loan can also be thought of more generally as an investment opportunity such as a project, a mortgage, or an asset-backed security. Each loan requires q units of inputs, which represents the loan's size. A loan of type π yields a return of $v = \bar{v}$ with probability π and $v = \underline{v}$ with probability $1 - \pi$ at the end of the period. For the analysis in this section, we normalize $\underline{v}$ to 0. Later, when we allow for aggregate shocks and introduce our refinement, we will allow $\underline{v}$ to be a random variable, possibly different from zero. We assume that $\pi \in \{\underline{\pi}, \bar{\pi}\}$ with $\underline{\pi} < \bar{\pi}$. We refer to a bank which has a loan of type $\bar{\pi}$ as a *high quality bank* and one with a loan of type $\underline{\pi}$ as a *low quality bank*. We assume that $\underline{\pi}\bar{v} \geq q$ so that each loan has positive net present value if sold.

The bank can either sell the loan in a secondary market or it can hold the loan. Selling the loan at a price p yields a payoff to the bank of $p - q$. The purchaser of the loan is entitled to the resulting return. If the bank chooses to hold the loan, it must borrow q from lenders to finance the loan and repay $q(1 + r)$ at the end of the period, where r is the within-period interest rate paid to lenders. We allow r to be positive or negative in order to examine the effects of various policy experiments described below. If the bank holds the loan it is entitled to the return from its projects; however, the bank then incurs a cost of holding the loan, c, in addition to the cost of repaying its debt, $q(1 + r)$.

Besides the quality of its loan, the bank is indexed by a cost type, which represents the costs, relative to the marketplace, that the bank incurs when it holds the loan to maturity. We intend the cost of the loan to represent funding liquidity costs, servicing costs, renegotiation costs in the event of a loan default, and costs associated with holding a loan that may be correlated in a particular way with the rest of the bank's portfolio, among other potential factors. We assume that $c \in \{\underline{c}, \bar{c}\}$ with $\underline{c} < -qr < 0 < \bar{c}$. We refer to a bank of type $\bar{c}$ as a *high cost bank* and a bank of type $\underline{c}$ as a *low cost bank*. We normalize the cost of holding and managing the loan for the market to be zero. We assume the quality type and cost types are drawn independently of each other.

Hence, the model has four types of banks: $(\pi, c) \in \{\underline{\pi}, \bar{\pi}\} \times \{\underline{c}, \bar{c}\}$. We refer to the different types of banks, $(\bar{\pi}, \bar{c}), (\bar{\pi}, \underline{c}), (\underline{\pi}, \bar{c}), (\underline{\pi}, \underline{c})$, as, HH, HL, LH, LL banks, respectively.

Timing of the Static Game

We formalize the interactions in this economy as an extensive form game with the following timing.

1. Nature draws the quality and cost types of the bank.

2. Buyers simultaneously offer a price to purchase a loan, p.

3. The bank sells the loan to one of the buyers or holds the loan to maturity.

We assume that, as perceived by buyers and lenders, the bank has quality type $\bar{\pi}$ with probability μ_2 and quality type $\underline{\pi}$ with probability $1 - \mu_2$. (The subscript 2 on the probability is meant to indicate that these are the beliefs of lenders associated with the second period of our two period model described below.) Following the work of Kreps and Wilson (1982) and Milgrom and Roberts (1982), we refer to μ_2 as the bank's reputation. Also, buyers believe that the bank has cost type $\underline{c}$ with probability α and cost type $\bar{c}$ with probability $1 - \alpha$. The cost and quality types are independently drawn.

2.1.2 Strategy and Equilibrium

A strategy for the bank consists of a decision of whether to sell or hold its loan, and which buyer to sell to if the bank chooses to sell. Clearly, the bank will choose the buyer offering the highest price if the bank decides to sell, so we suppress this aspect of the bank's strategy. Let a denote the decision of the bank whether to sell or hold the loan. If the bank chooses to sell, we denote the decision by $a = 1$, and if the bank chooses to hold the loan, we denote the decision by $a = 0$. A strategy for the bank is a function $a(\cdot)$ which maps the highest offered price, p, into a decision of whether to sell or hold the loan. The payoffs to a type (π, c) bank are given by

$$w_2(a|p, \pi, c) = a(p - q) + (1 - a)\left[\pi\bar{v} - q(1 + r) - c\right]$$

A strategy for a buyer consists of the choice of a price to offer a bank for its loan. The payoffs to a buyer with an accepted price p and a strategy $a_2(\cdot|\pi, c)$ for each type of bank is

$$u_2(p|a_2) = E_{\pi,c}[v|a_2(p|\pi, c) = 1] - p.$$

Since buyers move simultaneously, they engage in a form of Bertrand competition, so that the price is equal to the expected return of the loan.

A (pure strategy) Perfect Bayesian Equilibrium is a price p_2 and a strategy for each bank type, $a_2(\cdot|\pi, c)$, such that for all p, each bank type chooses the optimal loan decision and buyers offer the highest price that yields a payoff of 0; i.e., $p_2 \in \max\{p|u_2(p|r, a_2) = 0\}$.

Before characterizing the equilibria of this game, we characterize the outcomes under full information, when the bank's type is known by buyers. When buyers and lenders are informed of the bank's type, (π, c), Bertrand competition among buyers implies that the price in the secondary loan market is $p = \pi\bar{v}$. Consider the decision of whether to sell or hold a loan by a bank of type (π, c). Facing a price p, the bank chooses to sell the loan in the secondary market if and only if

$$p - q \geq \pi\bar{v} - q(1+r) - c.$$

Since Bertrand competition implies that the price $p = \pi\bar{v}$, the bank sells if and only if

$$qr + c \geq 0$$

which can also be written as $c \geq -qr$. Since we have assumed that $\underline{c} < -qr < 0 < \bar{c}$, in equilibrium if the bank has a high cost, it sells its loan while if it has a low cost it holds its loan.

Notice that the equilibrium allocation under full information is ex-post efficient. Low cost banks have a comparative advantage (over the market) in holding loans to maturity while the market has a comparative advantage over high cost banks. The full information equilibrium allocates loans to agents with a comparative advantage in holding and managing the loan. Thus, if the bank has a low cost of holding and managing the loan, it holds its loan, and if the bank has a high cost of holding and managing the loan, it sells its loan.

Next, we characterize the equilibria of the game with private information. For expositional simplicity, we focus on the decisions of the high quality, high cost bank (HH) and restrict the strategy sets of the low cost type banks as well as the low quality, high cost bank. Specifically, we assume that the low cost type banks hold their loans while the LH bank sells its loan. In Proposition 3 below, we show that, if $\underline{c}$ is sufficiently negative, the assumed strategies for these three types of banks are indeed optimal. In terms of the strategy of the HH bank, we show that it can be characterized by a threshold level of μ_2, which we denote by μ_2^*, such that below μ_2^*, the high quality, high cost type bank holds its loan, and above μ_2^*, this type sells its loan.

Consider now the loan decision of the high quality, high cost (HH) bank. The HH bank sells if and only if

$$p - q \geq \bar{\pi}\bar{v} - q(1+r) - \bar{c}. \tag{1}$$

Note that (1) implies that if the HH bank is willing to sell at any price, it is also willing to sell at a higher price. This result implies that, in any equilibrium, Bertrand competition drives buyers profits to zero. In terms of buyers' decisions, note that at any candidate equilibrium price, the HH bank either sells or holds its loan. Consider a candidate price at which the HH bank sells. Then, with probability μ_2, the selling bank is a high quality bank. Since we have assumed that a low quality high cost bank always sells, with probability $(1 - \mu_2)$ the selling bank is low quality. Thus, Bertrand competition among buyers implies that any candidate equilibrium price at which the HH bank sells must satisfy the following equality:

$$\hat{p}(\mu_2) := [\mu_2 \bar{\pi} + (1 - \mu_2)\underline{\pi}]\, \bar{v}. \tag{2}$$

At a candidate price at which the HH bank holds, only the low quality bank sells so that the equilibrium price must satisfy

$$p = \underline{\pi}\bar{v}. \tag{3}$$

When facing the highest possible price, $\hat{p}(\mu_2)$, the HH bank sells if and only if

$$\hat{p}(\mu_2) - q \geq \bar{\pi}\bar{v} - q(1 + r) - \bar{c}$$

or, substiuting from (2),

$$[\mu_2 \bar{\pi} + (1 - \mu_2)\underline{\pi}]\, \bar{v} - q \geq \bar{\pi}\bar{v} - q(1 + r) - \bar{c}. \tag{4}$$

Let μ_2^* be the value of reputation such that the HH bank is indifferent between selling and holding at $\hat{p}(\mu_2)$. Then, at any interior value of reputation, μ_2^* must satisfy

$$[\mu_2^* \bar{\pi} + (1 - \mu_2^*)\underline{\pi}]\, \bar{v} - q = \bar{\pi}\bar{v} - q(1 + r) - \bar{c}$$

or

$$\mu_2^* = 1 - \frac{qr + \bar{c}}{(\bar{\pi} - \underline{\pi})\bar{v}}. \tag{5}$$

Clearly for $\mu_2 \geq \mu_2^*$, our model has an equilibrium in which the HH bank sells its loan at a price

$\hat{p}(\mu_2)$. If $\mu_2 < \mu_2^*$, our model has an equilibrium in which the HH bank holds its loan and buyers offer a price $p = \underline{\pi}\bar{v}$. To see that this equilibrium is unique, note that if $\mu_2 \geq \mu_2^*$, if the offered price is below $\hat{p}(\mu_2)$, one of the buyers can deviate and offer a price just below $\hat{p}(\mu_2)$ and induce the HH bank to sell. This deviation yields strictly positive profits.

We use this characterization of the static equilibrium to calculate the payoffs associated with a given level of reputation μ_2 at the beginning of the period before a bank's cost type is realized. These payoff calculations play a crucial role in our dynamic game. They are given by

$$V_2(\mu_2) = \begin{cases} \bar{\pi}\bar{v} - q(1+r) - Ec, & \mu_2 < \mu_2^* \\ (1-\alpha)\left\{[\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi}]\bar{v} - q\right\} + \alpha[\bar{\pi}\bar{v} - q(1+r) - \underline{c}], & \mu_2 \geq \mu_2^*. \end{cases} \tag{6}$$

Similarly, we can define the value of the equilibrium for a low quality bank type:

$$W_2(\mu_2) = \begin{cases} (1-\alpha)\left[\underline{\pi}\bar{v} - q\right] + \alpha[\underline{\pi}\bar{v} - q(1+r) - \underline{c}], & \mu_2 < \mu_2^* \\ (1-\alpha)\left\{[\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi}]\bar{v} - q\right\} + \alpha[\underline{\pi}\bar{v} - q(1+r) - \underline{c}], & \mu_2 \geq \mu_2^*. \end{cases}$$

It is clear that V_2 is weakly increasing and convex in μ_2. We have proved the following proposition.

Proposition 1 *If $\underline{\pi}\bar{v} > q$ and $qr + \bar{c} > 0$, then for any $\mu \in [0,1]$, the static model has a unique equilibrium. Let μ_2^* be defined by (5). For $\mu_2 < \mu_2^*$, the equilibrium price is $\underline{\pi}\bar{v}$ and the HH bank holds its loan. For $\mu_2 \geq \mu_2^*$, the equilibrium price is $[\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi}]\,\bar{v}$ and the HH bank sells its loan. Furthermore, the payoff to the HH bank given in (6) is weakly increasing and convex in μ_2.*

Note that we have modeled buyers as behaving strategically. This modeling choice plays an important role in ensuring that the static game has a unique equilibrium. Suppose that rather than modeling buyers as behaving strategically, we had instead simply required that market prices satisfy a zero profit condition. One rationale for this requirement is that buyers take prices as given and choose how many loans to buy as in a competitive equilibrium. It is easy to show that with this requirement the economy has multiple equilibria in the static game if $\mu_2 \geq \mu_2^*$. One of these equilibria corresponds to the unique equilibrium of our game. In the other equilibrium, the buyers offer a price of $\underline{\pi}\bar{v}$. At this offered price, the HH bank holds its loan and only the low quality, high cost banks sells its loan. We find multiplicity of this kind unattractive in our model because obvious bilateral gains to trade are not being exploited. Each of the buyers has a strong incentive to offer a price slightly below $[\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi}]\,\bar{v}$. At this offered price, the HH bank strictly prefers to sell, and the buyer making such an offer makes strictly positive profits. In our formulation, with strategic behavior by the buyers, this low price outcome cannot be an equilibrium.

While we prefer our strategic formulation, we emphasize that our results that reputational incentives induce multiplicity do not rely on the static game having a unique equilibrium. We chose a formulation in which the static game has a unique equilibrium in order to argue that reputational incentives by themselves can induce multiplicity.

2.2 Two Period Benchmark Model

Consider now a two-period repetition of our static game in which the bank's quality type is the same in both periods. We assume that the bank's second period payoffs are discounted at rate β. In period 1, a continuum of buyers who are present in the market for only one period choose to offer prices for loans sold in that period. In period 2, a new set of buyers each offer prices for loans sold in that period. This new set of buyers observes whether the bank sold or held its loan in the previous period, and, if the bank sold its loan, buyers observe the realized value of the loan. If the loan is held, we assume that period 2 buyers do not observe the realized value of the loan.

The timing of the game is an extension of that described in the static game. As in that game, at the beginning of period 1, nature draws the bank's quality and cost type. We assume that the bank's quality type is fixed for both periods. At the beginning of period 2, nature draws a new cost type for the bank. In any period, the bank's quality and cost types are unknown to buyers. The timing within each period is the same as in the static game. We also assume that the returns to successful loans, $v = \bar{v}$, and to unsuccessful loans, $v = 0$, are the same in both periods.

In order to define an equilibrium in this repeated game, we must develop language that will allow us to describe how second period buyers update their beliefs about the bank's type based on observations from period 1. To do so, we let the public history at the beginning of period 2 be denoted by θ_1 where $\theta_1 \in \{h, s0, s\bar{v}\}$ where $\theta_1 = h$ denotes that the bank held its loan in period 1, $\theta_1 = s0$ denotes that the bank sold its loan and the loan paid off $v = 0$, and $\theta_1 = s\bar{v}$ denotes that the bank sold its loan and the loan paid off $v = \bar{v}$.

As in the static game, we focus on the strategic incentives of the HH bank and restrict the strategy sets of the low cost type banks as well as the low quality, high cost bank. Specifically, we assume that the low cost type banks must hold their loans while the LH bank must sell its loan. A strategy for the high cost, high quality bank is now given by a pair of functions, $a_1(p_1)$ representing the decision in period 1 and $a_2(p_2, \theta_1)$ representing the loan decision in period 2, if the bank realizes a high cost in period 2, as a function of offered prices.

Consider next how the buyers in the last period update their beliefs about the bank's type. This update depends through Bayes rule on the prior belief of the buyers, the loan decision of the bank and the loan return realization if the bank sold, as well as on the first period strategies chosen by the HH bank and period 1 buyers. From Bayes rule, these posterior probabilities are given by

$$\mu_2(\mu_1, \theta_1 = h, a_1(\cdot), p_1) = \frac{\mu_1\left(\alpha + (1-\alpha)(1-a_1(p_1))\right)}{\mu_1\left(\alpha + (1-\alpha)(1-a_1(p_1))\right) + (1-\mu_1)\alpha} \tag{7}$$

$$\mu_2(\mu_1, \theta_1 = s\bar{v}, a_1(\cdot), p_1) = \frac{\mu_1 a_1(p_1)(1-\alpha)\bar{\pi}}{\mu_1 a_1(p_1)(1-\alpha)\bar{\pi} + (1-\mu_1)(1-\alpha)\underline{\pi}} \tag{8}$$

$$\mu_2(\mu_1, \theta_1 = s0, a_1(\cdot), p_1) = \frac{\mu_1 a_1(p_1)(1-\alpha)(1-\bar{\pi})}{\mu_1 a_1(p_1)(1-\alpha)(1-\bar{\pi}) + (1-\mu_1)(1-\alpha)(1-\underline{\pi})} \tag{9}$$

For notational convenience, we suppress the dependence on strategies and priors and let μ_h denote the posterior associated with the bank holding its loan, and $\mu_{s\bar{v}}$ and μ_{s0} denote the posteriors associated with selling and yielding a high or low return.

Given the updating rules, the period 1 payoffs for the HH bank are given by

$$\begin{aligned} w_1(a|p) = & a\left[p - q + \beta\left(\bar{\pi} V_2(\mu_{s\bar{v}}) + (1-\bar{\pi})V_2(\mu_{s0})\right)\right] \\ & + (1-a)\left[(\bar{\pi}\bar{v} - q(1+r) - \bar{c}) + \beta V_2(\mu_h)\right] \end{aligned}$$

where $\mu_h, \mu_{s\bar{v}}$, and μ_{s0} are given by equations (7), (8), and (9). Buyers' payoffs associated with an accepted price, p, in period t are given by

$$u_t(p|r, a_t, \mu_t) = \frac{\mu_t(1-\alpha)a_t(p)\bar{\pi} + (1-\mu_t)(1-\alpha)\underline{\pi}}{\mu_t(1-\alpha)a_t(p) + (1-\mu_t)(1-\alpha)}\bar{v} - p.$$

A Perfect Bayesian Equilibrium is a first period price, p_1, a first period loan decision for the high quality, high cost bank $a_1(\cdot)$ which maps accepted prices into loan decisions, updating rules

$\mu_h, \mu_{s\bar{v}}, \mu_{s0}$ which map observations on loan decisions into posterior beliefs, a second period price, p_2, which maps second period beliefs into prices, and a second period loan decision $a_2(\cdot)$ which maps accepted prices and histories into loan decisions such that

1. for all p, the HH bank chooses the optimal action in period 1 so that $w_1(a_1(p)|p) \geq \max_{a'} w_1(a|p)$,
2. for all p, the HH bank chooses the optimal action in period 2 so that $w_2(a_1(p)|p) \geq \max_{a'} w_2(a|p)$,
3. the first period price, p_1 satisfies $p_1 \in \max\{p|u_1(p|a_1) = 0\}$,
4. the second period price, p_2 satisfies $p_2 \in \max\{p|u_2(p|a_2) = 0\}$,
5. the updating rules, $\mu_h, \mu_{s\bar{v}}, \mu_{s0}$ satisfy Bayes' Rule, namely, (7), (8), and (9).

Next, we characterize the set of equilibria in the two period game under the following assumption,

Assumption 1 *α and β satisfy $\beta(1-\alpha) \leq 1$.*

Later we provide a partial characterization of the set of equilibria when this assumption is relaxed.

We show that the game has two equilibria for a range of period 1 reputations, μ_1 around the static threshold, μ_2^*. In one equilibrium, the HH bank chooses to sell its loan in period 1. The posteriors associated with selling now depend non-trivially on the realized values of the loan. In particular, when the loan has a high realized value, the bank is rewarded with a higher posterior, and when the loan has a low realized value, the bank's posterior is lower than its prior. The posterior associated with holding the loan is exactly equal to the bank's period 1 reputation. These posteriors provide reputational incentives for the HH bank to sell the loan in order to signal its type and receive a higher period 2 reputation. Notice, for an HH bank with initial reputation above the static threshold, μ_2^*, the bank's equilibrium strategy coincides with repetition of the static perfect Bayesian equilibrium, but for HH banks with reputations below the static threshold, reputational incentives dominate their static incentives.

In the second type of equilibrium, the HH bank chooses to hold its loan. In this equilibrium, uninformed agents believe that the only type of bank that sells its loan is the LH bank. Hence, regardless of the return of the loan, if the bank sells it receives a posterior reputation of 0. Because uninformed agents believe that high quality banks hold their loans, the posterior associated with holding the loan is higher than the prior reputation. These posteriors provide reputational incentives for the bank to hold its loan in order to signal its type. In this equilibrium, the action of HH banks with reputations below the static threshold coincides with repetition of the static perfect Bayesian equilibrium. High quality, high cost banks with reputations above the static threshold now hold their loan because of reputational concerns. In this sense, reputation is harmful as it induces high quality, high cost banks to hold their loans while in a static setting the market place can offer a sufficiently high price to induce these banks to sell their loans.

To see these results, consider first supporting equilibria in which the HH bank chooses to sell its loan in period 1. In this case, the period 1 price is given by equation $\hat{p}(\mu_1)$. Given this price, selling is optimal if the difference in payoffs between selling and holding the loan is non-negative, or if the following incentive constraint is satisfied:

$$(\mu_1\bar{\pi} + (1-\mu_1)\underline{\pi})\,\bar{v} - q + \beta\left(\bar{\pi}V_2(\mu_{s\bar{v}}) + (1-\bar{\pi})V_2(\mu_{s0})\right) \geq \bar{\pi}\bar{v} - q(1+r) - \bar{c} + \beta V_2(\mu_h) \qquad (10)$$

or if

$$\mu_1(\bar{\pi} - \underline{\pi})\bar{v} + \beta\left(\bar{\pi}V_2(\mu_{s\bar{v}}) + (1-\bar{\pi})V_2(\mu_{s0}) - V_2(\mu_h)\right) \geq (\bar{\pi} - \underline{\pi})\bar{v} - (qr + \bar{c})$$

where

$$\mu_h = \mu_1,\ \mu_{s\bar{v}} = \frac{\mu_1\bar{\pi}}{\mu_1\bar{\pi} + (1-\mu_1)\underline{\pi}},\ \text{and } \mu_{s0} = \frac{\mu_1(1-\bar{\pi})}{\mu_1(1-\bar{\pi}) + (1-\mu_1)(1-\underline{\pi})} \qquad (11)$$

We will show that there is some value of μ, denoted $\underline{\mu} < \mu_2^*$ such that for all $\mu_1 \geq \underline{\mu}$, the inequality in (10) holds so that the HH bank sells its loan in period 1. To show this result, the following lemma is useful. This lemma also plays a key role in our proof that in an infinite horizon version of our model, reputation levels tend to cluster.

Lemma 1 *If the HH bank sells its loan in the first period, then* $\bar{\pi}\mu_{s\bar{v}} + (1-\bar{\pi})\mu_{s0} \geq \mu_h$.

Proof. From (11) we have (as an implication of Bayes Rule) that if the HH bank sells its loan in the first period, the reciprocal of the posterior beliefs is a martingale. Formally, we have

$$\frac{\bar{\pi}}{\mu_{s\bar{v}}} + \frac{1-\bar{\pi}}{\mu_{s0}} = \frac{1}{\mu_1} = \frac{1}{\mu_h}$$

Since $1/\mu$ is a convex function, it follows that

$$\bar{\pi}\mu_{s\bar{v}} + (1-\bar{\pi})\mu_{s0} \geq \mu_1 = \mu_h. \tag{12}$$

■

Let the reputational gain be defined as

$$\Delta^g(\mu_1) = \beta\left(\bar{\pi}V_2(\mu_{s\bar{v}}) + (1-\bar{\pi})V_2(\mu_{s0}) - V_2(\mu_h)\right)$$

Recall from Proposition 1 that V_2 is a convex function, so that $\bar{\pi}V_2(\mu_{s\bar{v}}) + (1-\bar{\pi})V_2(\mu_{s0}) \geq V_2(\bar{\pi}\mu_{s\bar{v}} + (1-\bar{\pi})\mu_{s0})$. This convexity together with Lemma 1 implies that at μ_2^*, $\Delta^g(\mu_2^*) > 0$ so that the left side of (10) is strictly greater than the right side. This result implies that, as we show in the Appendix, our model has an equilibrium in which there is some value of μ, denoted $\underline{\mu} < \mu_2^*$ such that at $\underline{\mu}$, (10) holds as an equality and for all $\mu_1 \geq \underline{\mu}$, the inequality in (10) holds.

Now consider the equilibrium in which the HH bank holds its loan in period 1. In this case the equilibrium price is given $\underline{\pi}\bar{v}$. A bank holds its loan if and only if

$$(\mu_1\bar{\pi} + (1-\mu_1)\underline{\pi})\,\bar{v} - q + \beta\left(\bar{\pi}V_2(\mu_{s\bar{v}}) + (1-\bar{\pi})V_2(\mu_{s0})\right) \leq \bar{\pi}\bar{v} - q(1+r) - \bar{c} + \beta V_2(\mu_h) \tag{13}$$

where

$$\mu_h = \frac{\mu_1}{\mu_1 + (1-\mu_1)\alpha}, \text{ and } \mu_{s\bar{v}} = \mu_{s0} = 0$$

If the inequality in (13) is reversed, there is a deviation by buyer to price $\hat{p}(\mu_1)$ that would break down the equilibrium. Analogously to the positive equilibrium, we define the reputational gain as

$$\Delta^b(\mu_1) = \beta(V_2(0) - V_2(\mu_h))$$

Since $\mu_h > \mu_1$,using Proposition 1, $\Delta^b(\mu_2^*) < 0$ so that (13) holds as a strict inequality. This result implies that, as we show in the Appendix, our model has an equilibrium in which there is some value of μ, denoted $\bar{\mu} > \mu_2^*$ such that at $\bar{\mu}$, (13) holds as an equality and for all $\mu_1 \leq \bar{\mu}$, the inequality in (13) holds.

We have proved the following proposition.

Proposition 2 *(Multiplicity of Equilibria) Suppose Assumption (1) is satisfied and* $0 < \mu_2^* < 1$. *Then, there exist* $\underline{\mu}$ *and* $\bar{\mu}$ *with* $\underline{\mu} < \mu_2^* < \bar{\mu}$ *such that*

1. *if* $\mu_1 \in [\underline{\mu}, \bar{\mu})$, *the model has two equilibria: in one the HH bank sells its loan, and in the other the HH bank holds its loan,*
2. *if* $\mu_1 < \underline{\mu}$, *the model has a unique equilibrium in which the HH bank holds its loan in period 1,*
3. *if* $\mu_1 \geq \bar{\mu}$, *the model has a unique equilibrium in which the HH bank sells its loan in period 1.*

Next we provide a partial characterization of the set of equilibria when we relax Assumption (1). We show that even when this assumption is relaxed, the game has a region of multiplicity near μ_2^*. We have also shown that multiplicity can arise for values of μ close to 1. Details are available upon request.

Proposition 3 *(Region of Multiplicity). There exist* $\underline{\mu}$ *and* $\bar{\mu}$ *with* $\underline{\mu} < \mu_2^* < \bar{\mu}$ *such that if* $\mu_1 \in [\underline{\mu}, \bar{\mu})$, *the game has two equilibria: in one the HH bank sells its loan, and in the other the HH bank holds its loan.*

Therefore, we have shown that introducing reputation as a device for mitigating lemons problems results in equilibrium multiplicity, that is, reputation can both be a blessing and a curse. The game has a positive reputational equilibrium in which, encouraged by reputational incentives, banks with a high quality asset sell their asset. In this equilibrium, reputation helps sustain market activity in a market that would be illiquid without reputational incentives. The game also has a negative reputational equilibrium in which reputational incentives discourage selling and banks with a high quality asset hold on to their asset. In this equilibrium, reputation helps depress market activity in a market that would be liquid without reputational incentives.

It is straightforward to extend this two period model to a multi-period model. (Indeed, we extend a version of the model with a refinement that produces a unique equilibrium to a multi-period model below). It is also straightforward to see that a version of our model with three or more periods will feature multiple equilibria. In addition to the multiplicity demonstrated in the two period model, a model with three or more periods will feature multiplicity induced by trigger strategies as in Benoit and Krishna (1985).

A version of our model with three or more periods can generate sudden collapses in the volume of loans sold in secondary markets. To see these sudden collapses, consider a version of our model with three or more periods. The equilibrium outcomes in the last two periods of such a model clearly coincide with the equilibrium outcomes of our two period model. Suppose that in the first period of the three period model, the equilibrium coincides with the analog of the positive reputational equilibrium so that new issue volumes are large. Suppose that in the next to last period, banks and buyers observe a 'sunspot' at the beginning of the period. This sunspot acts as a coordinating device which allows agents to select amongst the equilibria. If the sunspot is such that private agents choose the positive equilibrium, the volume of loans that are sold in secondary markets is high, while if the sunspot is such that private agents choose the negative equilibrium, the volume of loans sold in secondary markets is low. In this sense, a multi-period version of our model generates sudden collapses in the volume of trade.

To draw an analogy to models of reputation as incomplete information, our model nests features of the model in Mailath and Samuelson (2001) and Ordoñez (2008) as well as that of Ely and Välimäki (2003). In Mailath and Samuelson (2001) and Ordoñez (2008), strategic types are good and want to separate from non-strategic types - though in Mailath and Samuelson (2001) reputation generally fails to deliver this type of equilibria. Nevertheless, in their environments, there is no long run reputational loss from good behavior. Ely and Välimäki (2003), share the property that strategic types are good and want to separate, however, structure of learning is such that good behavior never implies long-run positive reputational gains and therefore reputational incentives exacerbate bad behavior in equilibrium.

Recall that thus far, we have restricted the strategies of all bank types except the HH type. Under a sufficient condition that $\underline{c}$ is sufficiently negative, we can show that the assumed strategies are optimal. This sufficient condition is given by

Assumption 2

$$(\bar{\pi} - \underline{\pi})\,\bar{v} + qr + \max_{\mu_1 \in [0,1]} \Delta^g(\mu_1) < -\underline{c} \tag{14}$$

We then have the following Proposition.

Proposition 4 *Suppose Assumption 1 and Assumption (2) hold. Then the unique equilibrium of the static game described in Proposition 1 and the multiple equilibria of the dynamic game described in Proposition 2 are also equilibria of the associated games when all bank types behave strategically.*

2.3 Sudden Collapses and Increased Inefficiency

In this section, we study the efficiency properties of the positive and the negative reputational equilibria. We provide sufficient conditions under which the positive reputational equilibrium Pareto dominates the negative reputational equilibrium in the sense of interim utility (see Holmstrom and Myerson (1983)), and sufficient conditions under which the positive equilibrium dominates the negative equilibrium in the sense of ex-ante utility. In this sense, sudden collapses of trade volume in our model due to switches between equilibria are associated with increased inefficiency.

In order to develop these sufficient conditions, suppose that $\mu_1 \in [\underline{\mu}, \mu_2^*]$ and that in the negative equilibrium, posterior beliefs conditional on future buyers observing a hold decision by a bank in the first period, μ_h^n, are less than the static cutoff, μ_2^*. Consider the difference in utility level of high quality high cost bank in the two equilibria. This difference is given by:

$$\Delta U(\bar{\pi}, \bar{c}) = \hat{p}(\mu_1) - (\bar{\pi}\bar{v} - qr - \bar{c}) + \beta\left[\bar{\pi} V_2(\mu_{s\bar{v}}^p) + (1-\bar{\pi}) V_2(\mu_{s0}^p) - V_2(\mu_h^n)\right]$$

where $\mu_{s\bar{v}}^p$ and μ_{s0}^p are the posterior beliefs in the positive equilibrium. Since, $\mu_h^n, \mu_h^p \leq \mu_2^*$ and $V_2(\cdot)$ is constant for $\mu \leq \mu_2^*$, it follows that $V_2(\mu_h^n) = V_2(\mu_h^p)$. Then, from (10), it is clear that $\Delta U(\bar{\pi}, \bar{c}) \geq 0$. The difference in utility level of a low quality high cost bank is given by

$$\Delta U(\underline{\pi}, \bar{c}) = \hat{p}(\mu_1) - \underline{\pi}\bar{v} + \beta\left[\underline{\pi} W_2(\mu_{s\bar{v}}^p) + (1-\underline{\pi}) W_2(\mu_{s0}^p) - \underline{\pi} W_2(\mu_{s\bar{v}}^n) + (1-\underline{\pi}) W_2(\mu_{s0}^n)\right]$$

Note that $\mu_{s\bar{v}}^n = \mu_{s0}^n = 0$. Therefore, the difference in continuation value is positive. Since the price in the positive equilibrium is higher, we must have $\Delta U(\underline{\pi}, \bar{c}) \geq 0$. Moreover, since $\mu_h^n, \mu_h^p \leq \mu_2^*$, the continuation values for low cost types is the same in the two equilibrium and since they are holding in the first period, their utility levels are the same. Since buyers make zero profits in both equilibria, we have established the following proposition:

Proposition 5 *Suppose that* $(1-\alpha)/(\bar{\pi}\left(\alpha - \frac{\pi}{\bar{\bar{\pi}}}\right)) < \beta(1-\alpha)$ *and suppose* μ_1 *is close to* $\underline{\mu}$*. Then, the utility level for each type of bank and the buyers in the positive equilibrium is at least as large as the utility level for the corresponding type of bank and the buyers in the negative equilibrium.*

In the appendix, we show that $(1-\alpha)/(\bar{\pi}\left(\alpha - \frac{\pi}{\bar{\bar{\pi}}}\right)) \leq \beta(1-\alpha)$ is a sufficient condition for μ_h^n to be less than or equal to μ_2^*.

In the case that $\mu_h^n > \mu_2^*$, one can show that the utility level of the low cost types is lower in the positive reputational equilibrium than in the negative reputational equilibria. Hence, the two equilibria are not comparable in interim utility terms. However, under appropriate sufficient conditions, the positive equilibrium yields a higher ex-ante utility than the negative equilibrium. Consider, the allocations in the two equilibria in the first period. The only difference in allocations is that, in the positive equilibrium the high quality high cost type sells while in the negative equilibrium this type holds. Thus difference in ex-ante utility (or social surplus) in the first period between the two equilibria is given by

$$(1-\alpha)\mu(qr+\bar{c}).$$

Clearly, first period utility is higher in the positive equilibrium than in the negative equilibrium. However, in the second period social surplus is higher in the negative equilibrium than in the positive equilibrium because the high cost types always sell in the negative equilibrium whereas in the positive equilibrium they hold the asset some fraction of the time - when the signal quality is bad in the first period or after a hold decision in the first period. Therefore, the change in social surplus in the second period is given by

$$-\mu(1-\alpha)((1-\alpha)(1-\bar{\pi})+\alpha)(qr+\bar{c})$$

Thus, the overall change in the social surplus is given by

$$\mu(1-\alpha)(1-\beta(1-\bar{\pi}(1-\alpha)))(qr+\bar{c})$$

Clearly, this overall change is positive if and only if $\beta(1-\bar{\pi}(1-\alpha)) < 1$. We have established the following proposition:

Proposition 6 *Suppose that $\beta(1-\bar{\pi}(1-\alpha)) < 1$. Then the ex-ante utility of the bank is higher in the positive reputational equilibrium than in the negative reputational equilibrium and the ex-ante utility of the buyers is the same in the two equilibria.*

3 Adding Aggregate Shocks

Consider a version of our benchmark model with aggregate shocks to collateral values, namely the default value of the loans banks originate. One motivation for adding aggregate shocks is that sudden collapses seem to be associated with reductions in collateral values. Specifically, we will assume that $\underline{v}$ fluctuates randomly. Notice that under our formulation, in the event of no default, the payoff from the loan, $\bar{v}$, is not random while in the event of default, the payoff from the loan, $\underline{v}$ is subject to aggregate shocks. In the event of no default, the payment on a loan such as a mortgage is known in advance, while in the event of default the amount the lender collects depends on the value of the collateral. To the extent that this value is correlated across different types of loans, the payoff in the event of default fluctuates in the same way across a wide variety of loans. The obvious example is that of a mortgage on residential or commercial property. The value of such property is often subject to aggregate shocks.

We begin with a version of our static model. Suppose that $\underline{v}$ is drawn from some distribution $F(\underline{v})$ with finite mean at the beginning of the period. We will show that in the static model, the unique equilibrium is characterized by a cutoff threshold $\mu_2^*(\underline{v})$ such that banks with reputation models above $\mu_2^*(\underline{v})$ sell their loans and banks below this threshold hold their loans and a fall in $\underline{v}$ raises $\mu_2^*(\underline{v})$. In this sense a fall in collateral values worsens the adverse selection problem. To see this result, note that an HH bank sells its loan if and only if

$$\hat{p}(\mu_2) - q \geq \bar{\pi}\bar{v} + (1-\bar{\pi})\underline{v} - q(1+r) - \bar{c}. \tag{15}$$

where

$$\hat{p}(\mu_2) := [\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi}]\,\bar{v} + [\mu_2(1-\bar{\pi}) + (1-\mu_2)(1-\underline{\pi})]\,\underline{v}. \tag{16}$$

Substituting for $\hat{p}(\mu_2)$ from (16) into (15) and simplifying, we obtain

$$[\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi}]\,(\bar{v} - \underline{v}) - q \geq \bar{\pi}(\bar{v} - \underline{v}) - q(1+r) - \bar{c}. \tag{17}$$

From (17), we obtain that $\mu_2^*(\underline{v})$ is given by

$$\mu_2^*(\underline{v}) = 1 - \frac{qr + \bar{c}}{(\bar{\pi} - \underline{\pi})(\bar{v} - \underline{v})}.$$

Clearly $\mu_2^*(\underline{v})$ is decreasing in $\underline{v}$.

The dynamic model with aggregate shocks has multiple equilibria. Consider a version of our dynamic model in which the default value $\underline{v}_t$, $t = 1.2$ is drawn independently over time from a distribution F with finite mean. The argument for multiplicity is essentially the same as in the dynamic model without aggregate shocks. The proof of the following Proposition mirrors the proof of Proposition 2 and is omitted.

Proposition 7 *For every $\underline{v}_1$ such that $0 < \mu_2^*(\underline{v}_1) < 1$, there exists functions $\underline{\mu}(\underline{v}_1)$ and $\bar{\mu}(\underline{v}_1)$ satisfying $\underline{\mu}(\underline{v}_1) < \mu_2^*(\underline{v}_1) < \bar{\mu}(\underline{v}_1)$ such that for all $\mu_1 \in [\underline{\mu}(\underline{v}_1), \bar{\mu}(\underline{v}_1)]$, the model with aggregate shocks has two equilibria. In one equilibrium, the HH bank sells its asset in period 1 and equilibrium price is $p_1^H(\underline{v}_1) = [\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi}]\,\bar{v} + [\mu_2(1-\bar{\pi}) + (1-\mu_2)(1-\underline{\pi})]\,\underline{v}_1$ $(= \hat{p}(\mu_2; \underline{v}_1))$. In the other equilibrium, the HH bank holds the loan period 1 and the equilibrium price is given by $p_1^L(\underline{v}_1) = \underline{\pi}\bar{v} + (1-\underline{\pi})\underline{v}_1$.*

Thus, since the model with aggregate shocks has multiple equilibria, it suffers from the same Dynamic Coordination Problem as the model without aggregate shocks.

4 Aggregate Shocks and Imperfect Observability

In this section, we use a perturbation of the model with aggregate shocks in order to select a unique equilibrium. The method used is in the spirit of the refinement literature on static coordination games (see, for example, Carlsson and Van Damme (1993), Morris and Shin (2003).)

We use a refinement which selects a unique equilibrium for two reasons. One reason is that we would like to understand how outcomes in the model respond to various kinds of policy interventions. As is typically the case in models with multiple equilibria, comparative static exercises are not meaningful. Therefore, we seek a refinement that allows us to select a unique equilibrium. The other reason is that we want to establish a well defined notion of fragility. In many macroeconomic environments with multiple equilibria, small shocks to the environment can cause sudden changes in behavior. Without a selection device, multiplicity leads to a lack of discipline on how equilibrium behavior changes in response to shocks. Techniques adapted from the literature on coordination games, however, enable us to impose such discipline. We demonstrate the precise nature of fragility in our environment using the unique equilibrium selected by our perturbation described below.

We adapt techniques from the literature on cordination games because, as we have noted in Chari et al. (2009), we think of the multiplicity of equilibria in our environment as arising from a coordination problem between future buyers and current banks, and hence refer to it as a *Dynamic Coordination Problem.* To see the sense in which lack of coordination leads to multiplicity, suppose that in period 1, period 2 buyers could commit to buy the asset in period 2 at pre-specified prices contingent on observed realizations of asset quality. Then a bank whose quality type is just below the quality threshold μ_2^* has an incentive to sell the asset in period 1. Such commitment would eliminate the multiplicity of equilibria. The unique equilibrium with commitment is the positive reputational equilibrium. This argument suggests that coordination failure is at the root of the multiplicity result. This interpretation helps us develop a refinement concept similar to that in the coordination games literature.

Consider the following model with aggregate shocks and imperfect observability. In each period $t = 1, 2$, an aggregate shock $\underline{v}_t \sim F(\underline{v}_t)$ with finite mean is drawn. These shocks are drawn independently across periods. Banks and buyers at the beginning of each period observe a noisy signal of $\underline{v}_t$ given by $v_t = v_t + \sigma\varepsilon_t$ where $\varepsilon_t \sim G(\varepsilon_t)$ with $E[\varepsilon_t] = 0$ is i.i.d. across periods and $\sigma > 0$. We assume that F and G have full support over $\mathbb{R}$.

The timing of the game is as follows:

1. At the beginning of each period t, agents observe $\underline{v}_{t-1}$. Buyers do not observe previous period signals v_{t-1} or the market price p_{t-1}. (We believe that our uniqueness result goes through if future buyers receive a noisy signal about previous prices.)

2. The new aggregate state $\underline{v}_t$ is drawn, the bank and current period buyers do not observe the

current state, $\underline{v}_t$, but they do observe the noisy signal, v_t.

3. Buyer offer prices.

4. The bank decides whether to sell or hold.

The payoffs of holding to a high cost high quality bank in period t is

$$\bar{\pi}\bar{v} + (1-\bar{\pi})\underline{v}_t - q(1+r) - \bar{c}$$

Hence, when selling occurs, the payoff from holding to the bank which has observed the signal v_t^d is

$$\bar{\pi}\bar{v} + (1-\bar{\pi})E\left[\underline{v}_t|v_t\right] - q(1+r) - \bar{c}$$

When $\sigma > 0$, the updating rules for the signal are given by

$$\begin{aligned}
\Pr(v_1 &\leq \hat{v}_1|\underline{v}_1) = \Pr(\underline{v}_1 + \sigma\varepsilon_1 \leq \hat{v}) = G\left(\frac{\hat{v}_1 - \underline{v}_1}{\sigma}\right) \\
\Pr(v_1 &\leq \hat{v}_1|\underline{v}_1) = \frac{\int_{-\infty}^{\hat{v}_1} f(v)g\left(\frac{\underline{v}_1 - v}{\sigma}\right)dv}{\int_{-\infty}^{\infty} f(v)g\left(\frac{\underline{v}_1 - v}{\sigma}\right)dv} = H(\hat{v}_1|\underline{v}_1)
\end{aligned}$$

Assumption 3 *(Monotone Likelihood Ratio) The posterior belief function $H(\underline{v}_1|v_1)$ is a decreasing function of v_1*

The assumption implies that when the signal, v_1, about the shock is high, the value of the shock, $\underline{v}_1$, is likely to be high. Straightforward algebra can be used to show that this assumption is satisfied if a monotone likelihood ratio property on g holds, namely that for any $v_1 > v_1'$, $g(v_1 - \underline{v}_1)/g(v_1' - \underline{v}_1)$ is increasing in $\underline{v}_1$.

To develop conditions under which the equilibrium is unique, we begin with the second period.

Proposition 8 *In the second period, given a reputation level μ_2 and a default value signal v_2, there is a unique equilibrium outcome in which bank's decision is given by:*

$$a_2 = \begin{cases} 1 & \text{if } \mu_2 \geq \mu_2^*(v_2) \\ 0 & \text{if } \mu_2 < \mu_2^*(v_2) \end{cases}$$

and the market price is given by

$$p_2 = \begin{cases} [\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi}]\,\bar{v} + [\mu_2(1-\bar{\pi}) + (1-\mu_2)(1-\underline{\pi})]\,v_2 & \text{if } \mu_2 \geq \mu_2^*(v_2) \\ \underline{\pi}\bar{v} + (1-\underline{\pi})\underline{v}_2 & \text{if } \mu_2 < \mu_2^*(v_2) \end{cases}$$

where

$$\mu_2^*(v_2) = \max\left\{\min\left\{1 - \frac{qr + \bar{c}}{(\bar{\pi} - \underline{\pi})(\bar{v} - v_2)}, 1\right\}, 0\right\}$$

The equilibrium in the subgame in the second period is similar to the previous section. The payoff from holding to a HH bank is

$$\bar{\pi}\bar{v} + (1-\bar{\pi})v_2 - q(1+r) - \bar{c}$$

since $E[\underline{v}_2|v_2] = v_2$ and the payoff from selling is $[\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi}]\,\bar{v} + [\mu_2(1-\bar{\pi}) + (1-\mu_2)(1-\underline{\pi})]\,v_2 - q$ $(:= \hat{p}(\mu_2; v_2) - q.\mu_2^*(v_2))$ is defined as the value of reputation that makes the bank indifferent between selling and holding. The equilibrium in the sub-game implies the following value function for the bank:

$$\hat{V}_2(\mu_2, v_2) = \alpha\left[\bar{\pi}\bar{v} - q(1+r) - \underline{c}\right] + (1-\alpha)\max\{\hat{p}(\mu_2; v_2) - q, \bar{\pi}\bar{v} + (1-\bar{\pi})v_2 - q(1+r) - \bar{c}\}$$

and the ex-ante value of period 2 reputation is

$$V_2(\mu_2) = \int\int \hat{V}_2(\mu_2, v_2)dG\left(\frac{v_2 - \underline{v}_2}{\sigma}\right)dF(\underline{v}_2) \tag{18}$$

Proving that the perturbed game has a unique equilibrium is easiest when F is an improper uniform distribution, $U[-\infty, \infty]$. However, an improper uniform implies that the ex-ante value function, $V_2(\mu_2)$, is not well-defined. To ease the exposition, in the next proposition we assume that $\underline{v}_t$ is distributed independently but not identically across periods. In particular, we assume $\underline{v}_1$ is drawn from the improper uniform distribution while $\underline{v}_2$ is drawn from a proper distribution F. In section 4.2, we state our result for the case where $\underline{v}_t$ is i.i.d across periods and F is a proper distribution. Notice that if $\underline{v}_1$ is drawn from an improper uniform distribution,

$$H(v_1|\hat{v}_1) = G\left(\frac{\hat{v}_1 - v_1}{\sigma}\right)$$

The next proposition states our uniqueness result.

Proposition 9 *For each $\sigma > 0$ and $V_2(\mu_2)$ given by (18), the game with uniform improper priors has a unique equilibrium in which in period 1, HH bank's action is characterized by a cutoff $v_1^*(\sigma) \in \mathbb{R}$ and is given by*

$$a_1(v_1) = \begin{cases} 1 & \text{if } v_1 \geq v_1^*(\sigma) \\ 0 & \text{if } v_1 < v_1^*(\sigma) \end{cases}$$

and the market price is given by

$$p_1^*(v_1) = \begin{cases} \hat{p}(\mu_1; v_1) & \text{if } v_1 \geq v_1^*(\sigma) \\ \underline{\pi}\bar{v} + (1 - \underline{\pi})v_1 & \text{if } v_1 < v_1^*(\sigma) \end{cases}$$

We prove this proposition using a similar method to that in (Carlsson and Van Damme (1993). We begin by restricting attention to switching strategies in which the bank sells for all default values above a threshold and holds for all default values below that threshold. We show that the game has a unique equilibrium in switching strategies. We then prove that the equilibrium switching strategy is the only strategy that survives iterated elimination of strictly dominant strategies so that we have a unique equilibrium.

The intuition for the iterated elimination argument is as follows. Note that we can define equilibrium as a strategy for the bank in period 1, and a belief - about the bank's action in period 1 - by period 2 buyers used for Bayesian updating. In equilibrium beliefs have to coincide with strategies. Obviously reputational incentives depend on future buyers' beliefs. When v_1 is very large, independent of future buyers' beliefs, an HH bank sells the asset. Similarly, when v_1 is very low, an HH bank holds onto the asset, independent of future beliefs. This argument establishes two bounds $\hat{v}^1 > \tilde{v}^1$, such that any equilibrium strategy must prescribe a sale for v_1 higher than $\hat{v}^1$ and holding for v_1 lower than $\tilde{v}^1$. This result means that the set of beliefs by future buyers have to satisfy the same property. Limiting the set of beliefs puts tighter upper and lower bounds on reputational incentives, which in turn implies new bounds $\hat{v}^2 > \tilde{v}^2$. We show that iterating in this manner implies that the bounds $\hat{v}^n$ and $\tilde{v}^n$ converge to a common limit.

Here we sketch the steps of the proof and leave the details to the Appendix.

4.1 Outline of Proof with Improper Priors

1. **Unique Equilibrium in Switching Strategies**

   (a) **Switching Strategies**

   Restrict attention to Bank strategies of the following form:

$$d_k(v_1) = \begin{cases} 1 & v_1 \geq k \\ 0 & v_1 < k \end{cases}$$

   where k represents the switching point. We characterize the best response of the HH bank when future buyers use d_k to form their posteriors over the bank's type. To do so, we must define Bayesian updating.

   (b) **Bayesian Updating**

   Consider an arbitrary belief $\hat{a}_1(\cdot)$ by period 2 buyers about the HH bank's period 1 action. Based on the observed history and signal $\underline{v}_1$, Bayesian updating implies the following rules:

$$\begin{aligned}
\mu_{sg}(\underline{v}_1;\hat{a}_1) &= \frac{\mu_1\bar{\pi}\int \hat{a}_1(v_1)dG\left(\frac{v_1-\underline{v}_1}{\sigma}\right)}{\mu_1\bar{\pi}\int \hat{a}_1(v_1)dG\left(\frac{v_1-\underline{v}_1}{\sigma}\right) + (1-\mu_1)\underline{\pi}} \\
\mu_{sd}(\underline{v}_1;\hat{a}_1) &= \frac{\mu_1(1-\bar{\pi})\int \hat{a}_1(v_1)dG\left(\frac{v_1-\underline{v}_1}{\sigma}\right)}{\mu_1(1-\bar{\pi})\int \hat{a}_1(v_1)dG\left(\frac{v_1-\underline{v}_1}{\sigma}\right) + (1-\mu_1)(1-\underline{\pi})} \\
\mu_h(\underline{v}_1;\hat{a}_1) &= \frac{\mu_1\left[(1-\alpha)\int\left[1-\hat{a}_1(v_1)\right]dG\left(\frac{v_1-\underline{v}_1}{\sigma}\right)+\alpha\right]}{\mu_1\left[(1-\alpha)\int\left[1-\hat{a}_1(v_1)\right]dG\left(\frac{v_1-\underline{v}_1}{\sigma}\right)+\alpha\right]+(1-\mu_1)\alpha}
\end{aligned}$$

For switching strategies, these formulas simplify to

$$\mu_{sg}(\underline{v}_1; d_k) = \frac{\mu_1 \bar{\pi}\left[1 - G\left(\frac{k-\underline{v}_1}{\sigma}\right)\right]}{\mu_1 \bar{\pi}\left[1 - G\left(\frac{k-\underline{v}_1}{\sigma}\right)\right] + (1-\mu_1)\underline{\pi}} \tag{19}$$

$$\mu_{sd}(\underline{v}_1; d_k) = \frac{\mu_1 (1-\bar{\pi})\left[1 - G\left(\frac{k-\underline{v}_1}{\sigma}\right)\right]}{\mu_1 (1-\bar{\pi})\left[1 - G\left(\frac{k-\underline{v}_1}{\sigma}\right)\right] + (1-\mu_1)(1-\underline{\pi})}$$

$$\mu_h(\underline{v}_1; d_k) = \frac{\mu_1 \left[(1-\alpha) G\left(\frac{k-\underline{v}_1}{\sigma}\right) + \alpha\right]}{\mu_1 \left[(1-\alpha) G\left(\frac{k-\underline{v}_1}{\sigma}\right) + \alpha\right] + (1-\mu_1)\alpha}$$

(c) **Characterizing the Gain from Reputation**

Given any belief $\hat{a}_1$, we define the gain from reputation as

$$\Delta(v_1; \hat{a}_1) = \beta \int \left[\bar{\pi} V_2(\mu_{sg}(\underline{v}_1; \hat{a}_1)) + (1-\bar{\pi}) V_2(\mu_{sd}(\underline{v}_1; \hat{a}_1)) - V_2(\mu_h(\underline{v}_1; \hat{a}_1))\right] dG\left(\frac{v_1 - \underline{v}_1}{\sigma}\right) \tag{20}$$

In the appendix we prove the following Lemma which characterizes the gain from reputation for general strategies and for switching strategies..

Lemma 2 *The gain from reputation $\Delta(v_1; \hat{a}_1)$ is uniformly bounded and strictly increasing in $\hat{a}_1$ according to a point-wise ordering on beliefs. In particular, if $\hat{a}_1$ is a switching strategy, d_k, then $\Delta(v_1; d_k)$ is strictly decreasing in k. Moreover, when $\hat{a}_1$ is a switching strategy, $\Delta(v_1; \hat{a}_1)$ is strictly increasing in v_1.*

(d) **Equilibrium in Switching Strategies**

Facing a switching strategy belief of future buyers, d_k, clearly, the HH bank sells if and only if

$$\hat{p}(\mu_1; v_1) - q + \Delta(v_1; d_k) \geq \bar{\pi}\bar{v} + (1-\bar{\pi})v_1 - q(1+r) - \bar{c}. \tag{21}$$

Note that the value of selling, given by the left side of (21) is increasing in v_1 and its partial derivative with respect to v_1 is at least the derivative of $\hat{p}(\mu_1; v_1)$, given by $\mu_1(1-\bar{\pi}) + (1-\mu_1)\underline{\pi}$. The value of holding, given by the right side of (21) is increasing in v_1 and its derivative is $1-\bar{\pi}$. Since the derivative for the value of selling is greater than the value of holding, there exists a unique solution, $b(k)$, that solves the equation

$$\hat{p}(\mu_1; b(k)) - q + \Delta(b(k); d_k) = \bar{\pi}\bar{v} + (1-\bar{\pi})b(k) - q(1+r) - \bar{c}.$$

Hence, the best response of the HH bank to a switching strategy belief of future buyers, d_k, is a switching strategy, $d_{b(k)}$ in which the bank sells for all returns above $b(k)$ and holds for all return values below $b(k)$. An equilibrium in switching strategies must be a fixed point of the above equation, so an equilibrium switching point, k^* satisfies

$$\hat{p}(\mu_1; k^*) - q + \Delta(k^*; d_{k^*}) = \bar{\pi}\bar{v} + (1-\bar{\pi})k^* - q(1+r) - \bar{c}.$$

In the Appendix, we prove the following lemma.

Lemma 3 *The best response function* $b(k)$ *has a unique fixed point* k^* *which is globally stable.*

Hence, the game with switching strategies has a unique equilibrium.

2. **Restriction to Switching Strategies is Without Loss of Generality**

   (a) **Limit Dominance Regions**

   We show that regardless of future buyers belief functions, the bank has a dominant strategy for extreme values of default values. Consider two numbers $\hat{v} < \tilde{v}$. We define an *extreme monotone strategy* to be a strategy that calls for selling when $v_1 \geq \tilde{v}$ and holding for $v_1 \leq \hat{v}$. We define $A_{\hat{v},\tilde{v}}$ to be the set of such strategies. Notice that $A_{-\infty,\infty}$ is the set of all strategies. Define the Best Response set operator on a subset of beliefs, A, as

$$BR(A) = \{a_1 | a_1(v_1) = 1 \Leftrightarrow \hat{p}(\mu_1; v_1) - q + \Delta(v_1; \hat{a}_1) \geq \bar{\pi}\bar{v} + (1-\bar{\pi})v_1 - q(1+r) - \bar{c}\}$$

   We show that there exist bounds $\hat{v}^0 < \tilde{v}^0$ such that the HH bank holds for $v_1 \leq \hat{v}^0$ and it sells the asset for $v_1 \geq \tilde{v}^0$, independent of future buyers' belief function $\hat{a}_1$. That is

$$\begin{aligned} \forall \hat{a}_1, v_1 &\geq \tilde{v}^0; \hat{p}(\mu_1; v_1) - q + \Delta(v_1; \hat{a}_1) \geq \bar{\pi}\bar{v} + (1-\bar{\pi})v_1 - q(1+r) - \bar{c} \quad (22) \\ \forall \hat{a}_1, v_1 &\leq \hat{v}^0; \hat{p}(\mu_1; v_1) - q + -q + \Delta(v_1; \hat{a}_1) \leq \bar{\pi}\bar{v} + (1-\bar{\pi})v_1 - q(1+r) - \bar{c} \end{aligned}$$

Using the result from Lemma (2) that $\Delta(v_1; \hat{a}_1)$ is uniformly bounded in (22), it follows that these bounds exist. We have established that any equilibrium strategy must be an extreme monotone strategy with cutoffs $\hat{v}^0 < \tilde{v}^0$. That is,

$$BR(A_{-\infty,\infty}) \subseteq A_{\hat{v}^0,\tilde{v}^0}.$$

Thus, we can restrict attention to extreme monotone strategies without loss of generality.

(b) **Best Response Sets Converge.**

We show that the best response set operator is decreasing in the sense that it induces a best response set which is a strict subset of any arbitrary set of extreme monotone beliefs. Repeatedly applying this operator induces a decreasing sequence of sets which converges to a unique equilibrium.

To show that the best response set operator is decreasing, we show that for any $\hat{v} < \tilde{v}$, $BR(A_{\hat{v},\tilde{v}}) \subseteq A_{b(\hat{v}),b(\tilde{v})} \subset A_{\hat{v},\tilde{v}}$. Since $\Delta(v_1; \hat{a}_1)$ is increasing in $\hat{a}_1$, for all $\hat{a}_1 \in A_{\hat{v},\tilde{v}}$ we have

$$\hat{p}(\mu_1; v_1) - q + \Delta(v_1; d_{\tilde{v}}) \leq \hat{p}(\mu_1; v_1) - q + \Delta(v_1; \hat{a}_1) \leq \hat{p}(\mu_1; v_1) - q + \Delta(v_1; d_{\hat{v}})$$

because $\hat{a}_1$ first order stochastically dominates $d_{\tilde{v}}$ and is dominated by $d_{\hat{v}}$. This result implies that

$$\bar{\pi}\bar{v} + (1-\bar{\pi})v_1 - q(1+r) - \bar{c} \geq \hat{p}(\mu_1; v_1) - q + \Delta(v_1; \hat{a}_1)$$

if

$$\bar{\pi}\bar{v} + (1-\bar{\pi})v_1 - q(1+r) - \bar{c} \geq \hat{p}(\mu_1; v_1) - q + \Delta(v_1; d_{\hat{v}})$$

This result implies that if a_1 is the best response to $\hat{a}_1$, then

$$\forall v_1 < b(\hat{v}), \quad a_1(v_1) = 0$$

Similarly, we can show that the best response to $\hat{a}_1$ must satisfy $a_1(v_1) = 1$ for all $v_1 \geq b(\tilde{v})$. We have proved that $BR(A_{\hat{v},\tilde{v}}) \subseteq A_{b(\hat{v}),b(\tilde{v})}$. Since $b(k)$ is globally stable, $A_{b(\hat{v}),b(\tilde{v})} \subset A_{\hat{v},\tilde{v}}$ so that $BR(A_{\hat{v},\tilde{v}}) \subseteq A_{b(\hat{v}),b(\tilde{v})} \subset A_{\hat{v},\tilde{v}}$. Finally, because $b(k)$ has a unique fixed point, $A^n_{b(\hat{v}),b(\tilde{v})}$ converges to $A_{k^*,k^*} = \{d_{k^*}\}$ so that $BR^n(A_{-\infty,\infty})$ also converges to $\{d_{k^*}\}$.

4.2 Uniqueness Result with Proper Priors

In this section, we provide a characterization of equilibria in the limiting perturbed game with general proper priors. In particular, we prove that in the perturbed game as $\sigma \to 0$, the set of period 1 equilibrium strategies converges to a unique strategy. We use the method of Laplacian beliefs introduced by Frankel et al. (2003) and reviewed by Morris and Shin (2003) to prove our uniqueness result. In fact we show that the game described above is equivalent to a game discussed by Morris and Shin (2003). We then use their result to prove the following theorem. The proof is in the Appendix.

Proposition 10 *Given the value function $V_2(\mu_2)$ given by (18), as $\sigma \to 0$ the set of first period equilibrium strategies in the game with proper priors converges to a unique strategy by the HH bank given by*

$$a_1(v_1) = \begin{cases} 1 & \text{if } v_1 \geq v_1^* \\ 0 & \text{if } v_1 < v_1^* \end{cases}$$

where v_1^* satisfies:

$$\hat{p}(\mu_1; v_1^*) - q + \beta \int_0^1 \left[\bar{\pi} V_2\left(\hat{\mu}_{sg}(l)\right) + (1-\bar{\pi}) V_2\left(\hat{\mu}_{sd}(l)\right) - V_2\left(\hat{\mu}_h(l)\right)\right] dl = \bar{\pi}\bar{v} + (1-\bar{\pi}) v_1^* - q(1+r) - \bar{c}$$

and

$$\hat{\mu}_{sg}(l) = \frac{\mu_1\bar{\pi}l}{\mu_1\bar{\pi}l + (1-\mu_1)\underline{\pi}}$$
$$\hat{\mu}_{sd}(l) = \frac{\mu_1(1-\bar{\pi})l}{\mu_1(1-\bar{\pi})l + (1-\mu_1)(1-\underline{\pi})}$$
$$\hat{\mu}_{h}(l) = \frac{\mu_1\left[(1-\alpha)(1-l)+\alpha\right]}{\mu_1\left[(1-\alpha)(1-l)+\alpha\right] + (1-\mu_1)\alpha}$$

5 The Multi-Period Model

In this section, we extend the model to many (possibly an infinite number of) periods. The qualitative properties of the model are very similar to the model with two periods. In particular, we show that the game with noisy signal has a unique equilibrium in the limit as the observation error converges to zero.

The extension of the model to multi periods is as follows: time is discrete and $t = 1, \cdots, T$. At $t = 0$, the bank draws a quality type $\pi \in \{\underline{\pi}, \bar{\pi}\}$ where $\Pr(\pi = \bar{\pi}) = \mu_0$ is given - μ_0 is the initial reputation level of the bank. In each period t the bank also draws a cost shock $c_t \in \{\underline{c}, \bar{c}\}$ where c_t is i.i.d. over time and $\Pr(c_t = \underline{c}) = 1 - \alpha$. In each period, the bank originates a loan. When the bank's quality type is given by π, the loan yields a return of $\bar{v}$ with probability π and a return of $\underline{v}_t$ with probability $1 - \pi$, where $\underline{v}_t \leq \bar{v}$. The return $\underline{v}_t$ is an i.i.d. stochastic process that is drawn from $F(\underline{v}_t)$ in each period. The economy is also populated by a continuum of buyers who live for one period. The information structure of the game is as in the two period model in section 4. In each period before trading occurs, all agents in the economy observe $v_t = \underline{v}_t + \sigma_t\varepsilon_t$ where ε_t is i.i.d. and distributed according to $G(\varepsilon)$. They do not, however, observe $\underline{v}_t$. Given this information, the agents trade in the market. After the trade, the default value of the return $\underline{v}_t$ becomes public information. Previous prices are not observed by current buyers. Based on observables, agents update their beliefs at the end of period t.

Let $V_T(\mu_T)$ denote the last period's ex-ante value function $V_T(\mu_T)$. Note that, the equilibrium strategy in the last period is a cutoff strategy with cutoff $v_T^*(\mu_T)$ given by

$$v_T^*(\mu_T) = \bar{v} - \frac{qr + \bar{c}}{(1-\mu_T)(\bar{\pi} - \underline{\pi})}$$

and hence,

$$V_T(\mu_T) = (1-\alpha)\int_{-\infty}^{v_T^*(\mu_T)} \{\bar{\pi}\bar{v} + (1-\underline{\pi})\underline{v}_t - q(1+r) - \bar{c}\}\, dF(\underline{v}_t)$$
$$+(1-\alpha)\int_{v_T^*(\mu_T)}^{\infty} \{\hat{p}(\mu_T;\underline{v}_t) - q\}\, dF(\underline{v}_t)$$

From Proposition 10 , as σ_{T-1} converges to zero, the set of equilibrium strategies in period $T-1$ converges to a cutoff strategy with cutoff $v_{T-1}^*(\mu_{T-1})$ given by

$$v_{T-1}^*(\mu_{T-1}) = \bar{v} - \frac{qr + \bar{c} + \beta\int_0^1 [\bar{\pi}V_T(\hat{\mu}_{sg}(l;\mu_{T-1})) + (1-\bar{\pi})V_T(\hat{\mu}_{sg}(l;\mu_{T-1})) - V_T(\hat{\mu}_h(l;\mu_{T-1}))]\, dl}{(1-\mu_{T-1})(\bar{\pi}-\underline{\pi})}$$

Notice that for σ_{T-1} small and given the above cutoff strategy, the value function at period $T-1$, $V_{T-1}(\mu_{T-1};\sigma_{T-1})$ is given by

$$V_{T-1}(\mu_{T-1};\sigma_{T-1}) = (1-\alpha)\int_{\underline{v}_t}\int_{-\infty}^{\frac{v_{T-1}^*(\mu_{T-1})-\underline{v}_t}{\sigma_{T-1}}} \{\bar{\pi}\bar{v} + (1-\underline{\pi})\underline{v}_t - q(1+r) - \bar{c}$$
$$+\beta V_T\left(\hat{\mu}_h\left(1-G\left(\frac{v_{T-1}^*(\mu_{T-1})-\underline{v}_t}{\sigma_{T-1}}\right)\right)\right)\Bigg\} dG(\varepsilon_{T-1})dF(\underline{v}_t) +$$
$$(1-\alpha)\int_{\underline{v}_t}\int_{-\infty}^{\frac{v_{T-1}^*(\mu_{T-1})-\underline{v}_t}{\sigma_{T-1}}} \{\hat{p}(\mu_{T-1};\underline{v}_t) - q$$
$$+\beta\bar{\pi}V_T\left(\hat{\mu}_{sg}\left(1-G\left(\frac{v_{T-1}^*(\mu_{T-1})-\underline{v}_t}{\sigma_{T-1}}\right)\right)\right)$$
$$+\beta(1-\bar{\pi})V_T\left(\hat{\mu}_{sb}\left(1-G\left(\frac{v_{T-1}^*(\mu_{T-1})-\underline{v}_t}{\sigma_{T-1}}\right)\right)\right)\Bigg\} dG(\varepsilon_{T-1})dF(\underline{v}_t)$$
$$+\alpha\int_{\underline{v}_t}\int_{-\infty}^{\infty} \{\bar{\pi}\bar{v} + (1-\underline{\pi})\underline{v}_t - q(1+r) - \bar{c}$$
$$+\beta V_T\left(\hat{\mu}_h\left(1-G\left(\frac{v_{T-1}^*(\mu_{T-1})-\underline{v}_t}{\sigma_{T-1}}\right)\right)\right)\Bigg\} dG(\varepsilon_{T-1})dF(\underline{v}_t)$$

and hence, the above formula becomes the following as $\sigma_{T-1}\to 0$:

$$
\begin{aligned}
V_{T-1}(\mu_{T-1}) \;=\; & (1-\alpha)\int_{-\infty}^{v^*_{T-1}(\mu_{T-1})} \{\bar{\pi}\bar{v} + (1-\bar{\pi})\underline{v}_t - q(1+r) - \bar{c} + \beta V_T(\hat{\mu}_h(0))\}\, dF(\underline{v}_t) \\
& +(1-\alpha)\int_{v^*_{T-1}(\mu_{T-1})}^{\infty} \{\hat{p}(\mu_{T-1};\underline{v}_t) - q + \beta\bar{\pi}V_T(\hat{\mu}_{sg}(1)) + \beta(1-\bar{\pi})V_T(\hat{\mu}_{sd}(1))\}\, dF(\underline{v}_t) \\
& +\alpha\int_{-\infty}^{v^*_{T-1}(\mu_{T-1})} \{\bar{\pi}\bar{v} + (1-\bar{\pi})\underline{v}_t - q(1+r) - \underline{c} + \beta V_T(\hat{\mu}_h(0))\}\, dF(\underline{v}_t) \\
& +\alpha\int_{v^*_{T-1}(\mu_{T-1})}^{\infty} \{\bar{\pi}\bar{v} + (1-\bar{\pi})\underline{v}_t - q(1+r) - \underline{c} + \beta V_T(\hat{\mu}_h(1))\}\, dF(\underline{v}_t)
\end{aligned} \tag{23}
$$

The value functions in each period can be constructed in a similar maneer. If the value function in period t, V_t is increasing the same proof as in Proposition 10 applies. Therefore, we can state the following theorem:

Proposition 11 *If $V_{t+1}(\mu_{t+1})$ is increasing in μ_{t+1}, for each μ_t, there is a unique equilibrium strategy in period t as $\sigma_t \to 0$. The equilibrium strategy in period t is given by a cutoff strategy as follows:*

$$
a_t(v_t;\mu_t) = \begin{cases} 1 & v_t \geq v^*_t(\mu_t) \\ 0 & v_t < v^*_t(\mu_t) \end{cases}
$$

where $v^*_t(\mu_t)$ satisfies the following

$$
\hat{p}(\mu_t;v^*_t) - q + \beta\int_0^1 [\bar{\pi}V_{t+1}(\hat{\mu}_{sg}(l)) + (1-\bar{\pi})V_{t+1}(\hat{\mu}_{sb}(l)) - V_{t+1}(\hat{\mu}_h(l))]\, dl = \bar{\pi}\bar{v} + (1-\bar{\pi})v^*_t - q(1+r) - \bar{c}
$$

and $V_{t+1}(\cdot)$ is defined recursively as in (23).

It is straightforward to show that the part of the value function associated with the type $(\bar{\pi}, \bar{c})$ is increasing by proving that both the value of selling and holding is increasing in μ_t and therefore that part is increasing. However, whether the part associated with the type $(\bar{\pi}, \underline{c})$ is increasing requires further assumptions on the set of parameter values - one assumption is that α is relatively small. For all of our numerical examples, $V_t(\mu_t)$ is an increasing function.

6 Fragility

We think of equilibrium outcomes as *fragile* in two ways. One notion of fragility is simply that the economy has multiple equilibria so that sunspot-like fluctuations can induce changes in outcomes. A second notion of fragility is that small changes in fundamentals induce large changes in aggregate outcomes.

Equilibrium outcomes in our unperturbed game are clearly fragile under the first notion because that game has multiple equilibria. They are also fragile under the second notion if agents in the model coordinate on different equilibria depending on the realization of the fundamentals and if a large mass of agents have reputation levels in the multiplicity region.

Since our perturbed game has a unique equilibrium, it is not fragile under the first notion. We argue that it is fragile under our second notion. In our multi-period model, the history of past outcomes induces dispersion in the reputation levels of different banks. In order for our equilibrium to display fragility under the second notion, we must have that either banks with a wide variety of reputation levels change their actions in the same way in response to aggregate shocks or that the reputation levels of banks cluster close to each other. We conducted a wide variety of numerical exercises and found that the clustering effect is very strong in our model. This clustering effect clearly depends on the details of the history of exogenous shocks. To abstract from these details, we consider the invariant distribution associated with our model and show that this invariant distribution displays clustering. The invariant distribution is that associated with the infinite horizon limit of our multi-period model. We allow for a small probability of replacement in order to ensure that the invariant distribution is not concentrated at a single point.

Figure 3 displays the cutoff values for each reputation type for the ergodic set associated with the invariant distribution.[1] This ergodic set contains reputation levels between roughly 0.25 and 0.85. For collateral values above the cutoffs shown in Figure 3, banks sell their loans and below the cutoffs banks hold their loans. This figure illustrates that as the collateral value falls, the adverse selection problem worsens in the sense that banks with a wider range of reputations hold their loans. For example, at a collateral value of 5, banks with reputation levels below roughly 0.4 hold

[1] The parameters used in this simulation are the following: $\bar{\pi} = 0.8, \underline{\pi} = 0.3, \bar{v} = 7, \bar{c} = 0.5, \underline{c} = -3, \alpha = 0.15, q = .1, r = 0.5, \beta(1-\lambda) = .99, \lambda = .4, \mu_0 = .6$ where λ represents the exogenous probability of replacement and μ_0 is reputation of a newly replaced bank. The distribution of $\underline{v}$ is $N(0, 2)$.

their loans and the banks with higher reputation levels sell their loans. At a collateral value of 4, banks with reputation levels below roughly 0.65 hold their loans and banks with higher reputation levels sell their loans. Thus, a fall in collateral values from 5 to 4 induces banks with reputation levels roughly between 0.4 and 0.65 to switch from selling to holding their loans.

Figure 4 displays the invariant distribution of reputation levels for high quality banks. This figure shows that the invariant distribution displays significant clustering. Roughly 70 per cent of high quality banks have reputation levels between 0.8 and 0.85. Small fluctuations in the default value of loans around the cutoff values for such banks can induce a large mass of banks to alter their behavior.

Figure 5 plots the volume of trade, measured as the fraction of all banks that sell their loans. A decrease in the default value from 1.3 to 1.1 induces a 50 per cent decrease in the volume of trade. In this sense, Figure 5 suggests that equilibrium outcomes in our model are fragile under the second notion.

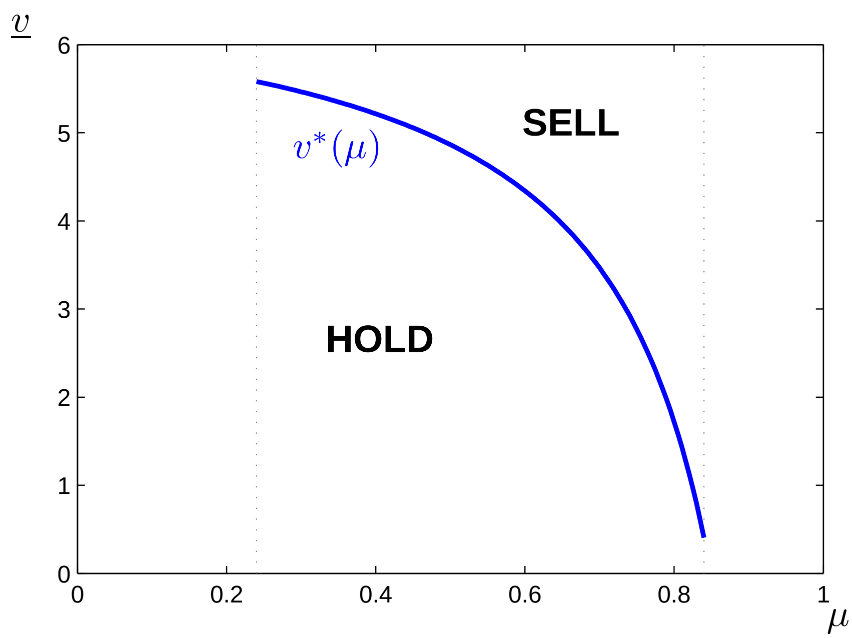

Figure 3: Cutoff thresholds for high quality banks.

Next we analyze the forces that induce clustering in our model. Recall from Lemma 1 that, conditional on a high quality, high cost bank selling, Bayes rule implies that $\frac{1}{\mu_t}$ is a martingale. Since $\frac{1}{\mu_t}$ is a convex function, Jensen's inequality implies that the reputation of a bank, μ_t, is a

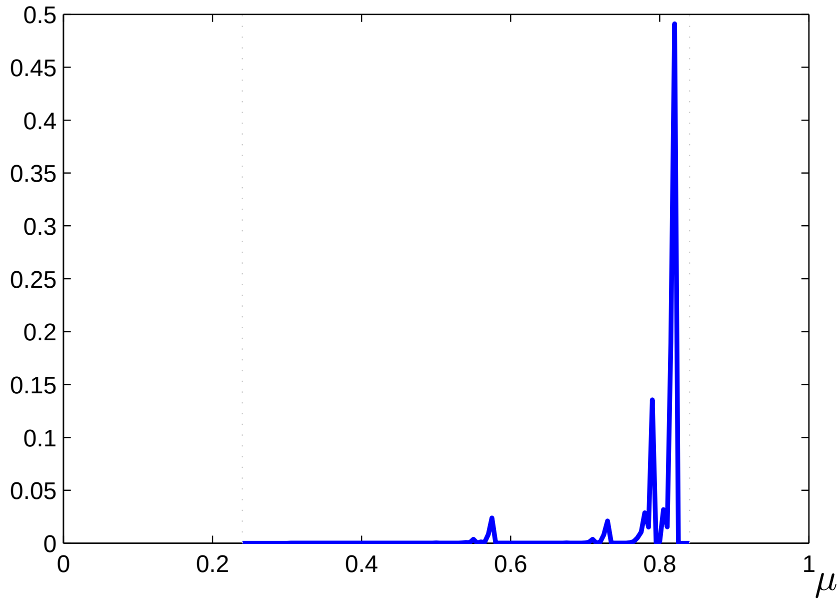

Figure 4: Invariant Distribution of reputations of high quality banks.

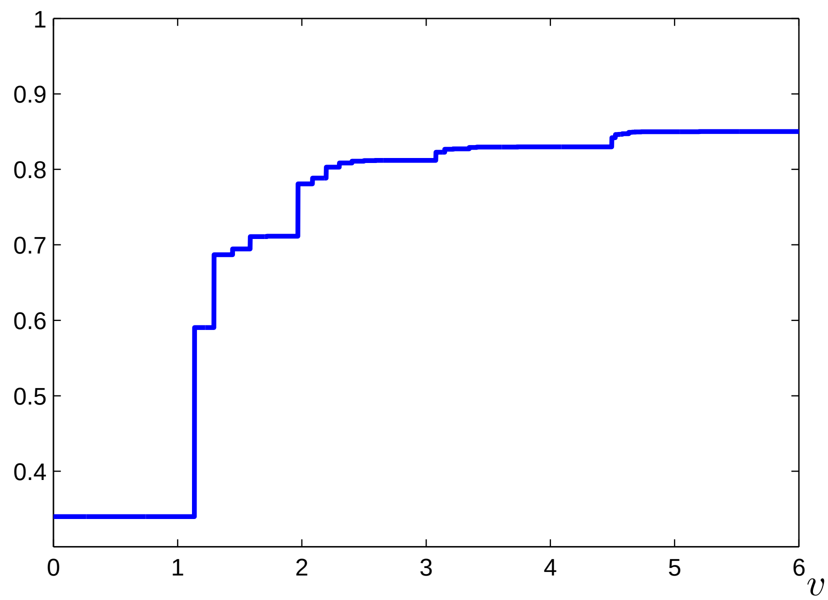

Figure 5: Volume of Trade as a function of shock to default value.

submartingale so that μ_t tends to rise. Conditional on a high quality, high cost bank holding, the analysis of our equilibrium implies that the reputation of such a bank also rises. These forces imply that the reputation of a high quality bank displays an upward trend. This upward trend is dampened by replacement. Since all high quality banks tend to have an upward trend in their reputations, these reputations tend to cluster towards each other.

This reasoning suggests that fragility under the second notion does not depend on the particular equilibrium that we have selected. In both the positive and the negative reputational equilibria, the reputations of high quality banks rises over time and tend eventually to cluster together. This clustering tends to make them react in the same way to fluctuations in the default value of the underlying loans. We conjecture that any continuous selection procedure will produce periods of high volumes of new issuances followed by sudden collapses.

We have analyzed the effect of other aggregate shocks in our model. In particular, we allowed the comparative advantage cost, $\bar{c}$, to be subject to aggregate shocks. In that version of the model, we found that banks with a wide variety of reputations tend to have cutoffs that are very close to each other. That model displays fragility under our second notion because small fluctuations in holding costs around a critical value induce large changes in actions by banks with a wide variety of reputations. (Details are available upon request).

7 Policy Exercises

In this section, we use our model to evaluate the effects of various policies intended to remedy problems credit markets that have been proposed since the 2007 collapse of secondary loan markets in the U.S. We focus on the effects of policies in which the government would purchase asset backed securities at prices above existing market value, such as the Public-Private Partnership plan as well as on policies which decreased the costs of holding loans to maturity, including changes in the Fed Funds target rate, the Term Asset-Backed Securities Loan Facility (TALF), and increased FDIC insurance.

We first consider policies in which the government attempts to purchase so-called toxic assets at above-market values. Consider the following government policy in the limiting version of the perturbed game as $\sigma \to 0$. The government offers to buy the asset at some price p in the first period.

Suppose first, that $p \leq \hat{p}(\mu_1; v_1)$. We claim that the unique equilibrium without government is also the unique equilibrium with this government policy. To see this claim, note that the equilibrium in the second period is the same with and without the government policy so that the reputational gains are the same with and without government policy. Consider the first period and a realization of first period return $v_1 < v_1^*$. In the game without the government, the HH bank found it optimal not to sell at a price $\hat{p}(\mu_1; v_1)$. Since the reputational gains are the same with and without the government policy, in the game with the government, it is also optimal for the HH not to sell at this price. A similar argument implies that the equilibrium strategy of the HH bank is unchanged for $v_1 > v_1^*$. Thus, this government policy has no effect on the equilibrium strategy of the HH bank. Of course, under this policy, the government ends up buying the asset from low quality banks. The only effect of this policy is to make transfers to low quality banks.

Suppose next that the price set by the government, p, is sufficiently larger than $\hat{p}(\mu_1; v_1)$.Then, the HH bank will find it optimal to sell and will enjoy the reputational gain associated with a policy of selling. In this sense, if the government offers a sufficiently high price, it can ensure that reputational incentives work to overcome adverse selection problems. Note however that this policy necessarily implies that the government must earn negative profits.

Consider now a policy which reduces interest rates in period 1 and leaves period 2 interest rates unchanged. We begin the analysis with the unperturbed game. Such a policy increases the static payoff in period 1 from holding loans which worsens the static incentives for the HH bank to sell its loan. Specifically, this policy raises both the threshold $\underline{\mu}$ below which banks find it optimal to hold in the positive reputational equilibrium and the threshold $\bar{\mu}$ below which banks find it optimal to hold their loans in the negative reputational equilibrium. Thus, this policy serves only to aggravate the lemons problem in secondary loans markets.

Consider next a policy under which the government commits to reducing period 2 interest rates but leaves period 1 interest rates unchanged. Obviously, this policy increases incentives for banks to hold their loans in period 2 and thereby increases the threshold below which banks hold their loans, μ_2^*. In this sense, it makes period 2 allocations less efficient. We will show that this policy reduces the region of multiplicity in period 1 and in this sense can improve period 1 allocations. To show the reduction in the region of multiplicity, consider the reputational gain in the positive reputational equilibrium evaluated at $\underline{\mu}$:

$$\beta\left(\bar{\pi}V_2(\mu_{s\bar{v}}) + (1-\bar{\pi})V_2(\mu_{s0}) - V_2(\mu_h)\right)$$

Using 6, it is straightforward to see that an arbitrarily small reduction in interest rates of dr in period 2 reduces $V_2(\mu_{s\bar{v}})$ by $\alpha q dr$ since $\mu_{s\bar{v}} > \mu_2^*$. Moreover, since μ_{s0} and μ_h are strictly less than μ_2^*, $V_2(\mu_{s0})$ and $V_2(\mu_h)$ fall by qdr. As a result, the reputational gain falls by $\beta\bar{\pi}(1-\alpha)qdr$. This decline in reputational gain induces an increase in the threshold $\underline{\mu}$. Similarly, we can show that the policy induces a fall in the threshold $\bar{\mu}$. Thus, the region of multiplicity shrinks and in this sense can improve period 1 allocations. Interestingly, such a policy is time inconsistent because the government has a strong incentive in period 2 not to make period 2 allocations less efficient.

An alternative policy which has not been proposed is to consider forced asset sales in which the government randomly forces banks to sell their loan. Such a policy in our model would mitigate the lemons problem in secondary loan markets by generating a pool of loans in secondary markets consistent with the ex-ante mix of loan types. While this is a standard intervention directed at increasing the price and volume of trade in markets that suffer from adverse selection, in our model such an intervention comes at the cost of misallocating loans to those without comparative advantage. Specifically, some banks with low costs of holding loans will be forced to sell to the marketplace.

It is straightforward to show that a policy under which the government commits to purchase assets in period 2 at prices which are contingent on the realization of the signals can eliminate the multiplicity of equilibria and support the positive reputational equilibrium. While such a policy would be desirable, the feasibility of such a policy can only be analyzed by developing a model in which private agents cannot commit but the government can.

8 Conclusion

This paper is an attempt to make three contributions: a theoretical contribution to the literature on reputation, a substantive contribution to the literature on the behavior of financial markets during crises, and a contribution to analyses of proposed and actual policies during the recent crisis. In terms of the theoretical contribution, we have combined insights from the literature that emphasizes the positive aspects of reputational incentives (see Mailath and Samuelson (2001)) with the literature that emphasizes the negative aspects of reputational incentives (see Ely and Välimäki (2003)) to show that multiplicity of equilibria naturally arise in reputation models like ours. We have also shown how techniques from the coordination games literature can be adapted to develop a refinement method that produces a unique equilibrium. In terms of the literature on the behavior of financial markets during crises, we have argued that sudden collapses in secondary loan market activity are particularly likely when the collateral value of the underlying loan declines. In terms of policy, we have argued that a wide variety of proposed policy responses would not have averted either the sudden collapse or the associated inefficiency. An important avenue for future work is to analyze policies which might in fact remedy the inefficiencies.

9 Appendix

Proposition 2. (Multiplicity of Equilibria) Suppose Assumption (1) is satisfied and $0 < \mu_2^* < 1$. Then, there exist $\underline{\mu}$ and $\bar{\mu}$ with $\underline{\mu} < \mu_2^* < \bar{\mu}$ such that if $\mu_1 \in [\underline{\mu}, \bar{\mu})$, the model has two equilibria: in one the HH bank sells its loan, and in the other the HH bank holds its loan, if $\mu_1 < \underline{\mu}$, the model has a unique equilibrium in which the HH bank holds its loan in period 1, if $\mu_1 \geq \bar{\mu}$, the model has a unique equilibrium in which the HH bank sells its loan in period 1.

Proof: We show that our economy has a positive reputational equilibrium. Using Lemma 1 and the result from Proposition 1 that V_2 is nondecreasing, it follows that $\Delta^g(\mu_1) \geq 0$. Next we show that there is some critical value of μ_1 denoted $\mu_g < \mu_2^*$ such that for all μ_1 in the interval $\mu_g < \mu_1 \leq \mu_1^*$, $\Delta^g(\mu_1)$ is strictly positive and increasing in μ_1 and $\Delta^g(\mu_1) = 0$ for $\mu_1 \leq \mu_g$. To obtain these results, define μ_g implicitly by

$$\mu_2^* = \frac{\mu_g \bar{\pi}}{\mu_g \bar{\pi} + (1 - \mu_g)\underline{\pi}}.$$

That is μ_g denotes that initial reputation level such that if the HH bank sells and receives a good signal, its reputation level would rise to μ_2^*.Since $\bar{\pi} > \underline{\pi}$, $\mu_g < \mu_2^*$. To see that for all $\mu_g < \mu_1 \leq \mu_1^*$, $\Delta^g(\mu_1)$ is strictly positive and increasing in μ_1, rewrite the reputational gain as

$$\Delta^g(\mu_1) = \beta \left(\bar{\pi}(V_2(\mu_{s\bar{v}}) - V_2(\mu_h)) + (1 - \bar{\pi})(V_2(\mu_{s0}) - V_2(\mu_h))\right).$$

Since $\mu_h = \mu_1$ and $\mu_{s0} < \mu_1$, from Proposition 1 it follows that for all $\mu_g < \mu_1 \leq \mu_1^*, V_2(\mu_{s0}) = V_2(\mu_h)$ Since $\mu_{s\bar{v}} > \mu_h = \mu_1$, it follows that $\Delta^g(\mu_1)$is positive and since $\mu_{s\bar{v}}$ is strictly increasing in μ_1 it follows that $\Delta^g(\mu_1)$ is strictly increasing. To see that $\Delta^g(\mu_1) = 0$ for $\mu_1 \leq \mu_g$, note that $\mu_{s\bar{v}} \leq \mu_2^*$ so that $V_2(\mu_{s\bar{v}}) = V_2(\mu_h)$.

Next, rewrite (10) as

$$(\mu_1 \bar{\pi} + (1 - \mu_1)\underline{\pi})\, \bar{v} - q + \Delta^g(\mu_1) \geq \bar{\pi}\bar{v} - q(1 + r) - \bar{c} \tag{24}$$

Consider $\mu_1 \leq \mu_2^*$. Since $\Delta^g(\mu_1)$ is a nondecreasing function of μ_1 in this range and $(\mu_1 \bar{\pi} + (1 - \mu_1)\underline{\pi})\, \bar{v}$ is a strictly increasing function of μ_1, it follows that the left side of (24) is strictly increasing in this range. Since $\Delta^g(\mu_1^*)$ is strictly positive, using (5) the left side of (24) is strictly greater than the right side of this inequality at μ_1^*. Since $\Delta^g(\mu_g) = 0$ and $\mu_g < \mu_2^*$, the left side is strictly less than the right side at μ_g. Thus, there is a unique value of μ at which (24) holds as an equality. For $\mu_1 > \mu_2^*$, $(\mu_1\bar{\pi} + (1-\mu_1)\underline{\pi})\,\bar{v} - q > \bar{\pi}\bar{v} - q(1+r) - \bar{c}$ and $\Delta^g(\mu_1) \geq 0$ so that (24) is satisfied. We have established that our model has an equilibrium in which all HH banks with reputation levels above $\mu_1 \geq \underline{\mu}$ sell.

To obtain the negative reputational equilibrium, define μ_b implicitly by

$$\mu_2^* = \frac{\mu_b}{\mu_b + (1-\mu_b)\alpha}.$$

That is μ_b denotes that initial reputation level such that if the HH bank holds, its reputation level would rise to μ_2^*. Clearly $\mu_b < \mu_2^*$.

Since $\mu_h = \mu_1/(\mu_1 + (1-\mu_1)\alpha)$ is greater than μ_1, it follows that $\Delta^b(\mu_1)$ is negative for $\mu_1 > \mu_b$. If $\mu_1 \in [\mu_b, \mu_2^*]$, selling has a static cost, i.e. $\hat{p}(\mu_2) - q \leq \bar{\pi}\bar{v} - q(1+r) - \bar{c}$ as well as a loss from reputation, i.e. $\Delta^b(\mu_1) < 0$ so that the HH bank prefers to hold the asset. If $\mu_1 \in (\mu_2^*, 1]$, there are benefits from selling the asset, i.e. $\hat{p}(\mu_2) - q \geq \bar{\pi}\bar{v} - q(1+r) - \bar{c}$, while there is a loss from reputation $\Delta^b(\mu_1) < 0$. Assumption (1) ensures that when $\mu_1 = 1$, the static benefit outweighs the loss from reputation, i.e. (13) is reversed at $\mu_1 = 1$. Moreover, Since $\mu_h = \mu_1/(\mu_1 + (1-\mu_1)\alpha)$,it is easy to show that $(\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi})\,\bar{v} - q + \Delta^b(\mu_1)$ is a strictly convex function of μ_1 for $\mu_1 \in [\mu_2^*, 1]$. Since the value of this function is strictly less than $\bar{\pi}\bar{v} - q(1+r) - \bar{c}$ at $\mu_1 = \mu_2^*$ and weakly higher when $\mu_1 = 1$, there exists a unique $\bar{\mu} \in (\mu_2^*, 1]$, at which (13) holds with equality. For $\mu_1 \leq \bar{\mu}$, (13) holds and for $\mu_1 > \bar{\mu}$ (13) is violated. *Q.E.D.*

Proposition 4. Suppose Assumption 1 and Assumption 2 hold. Then the unique equilibrium of the static game described in Proposition 1 and the multiple equilibria of the dynamic game described in Proposition 2 are also equilibria of the associated games when all bank types behave strategically.

Proof: Consider the static game. It is sufficient to show that given the constructed equilibrium and specified strategies for all agents, there is no profitable deviation by any agent. Note that in the proof of Proposition 2 we show that $\Delta^g(\mu_1) \geq 0$ for all $\mu_1 \in [0, 1]$. Hence, Assumption 2 implies that

$$\mu_1\left(\bar{\pi}-\underline{\pi}\right)\bar{v}+qr<-\underline{c}$$

or

$$[\mu_1\bar{\pi}+(1-\mu_1)\underline{\pi}]\bar{v}-q<\underline{\pi}\bar{v}-q(1+r)-\underline{c} \tag{25}$$

Inequality (25) implies that facing break even prices the low cost type bank would like to hold. Moreover a deviation by a buyer must attract these types of bank and (25) implies that buyers must offer a price higher than the actuarially fair price. Hence, there is no deviation by any buyer or a low cost bank type. Moreover, an LH bank wants to sell even at the lowest possible price, $\underline{\pi}\bar{v}$, since $\bar{c}>0$. Thus there are no profitable deviation from the specified strategies in the static game.

Consider the positive equilibrium of the dynamic game. Given future beliefs, the value of selling to a low quality bank adjusted by the future reputational gain from holding is given by

$$[\mu_1\bar{\pi}+(1-\mu_1)\underline{\pi}]\bar{v}-q+\beta\left[\underline{\pi}V_2(\mu^g_{s\bar{v}})+(1-\underline{\pi})V_2(\mu^g_{s0})-V_2(\mu)\right]$$

where $\mu_{s\bar{v}}=\bar{\pi}\mu_1/(\mu_1\bar{\pi}+(1-\mu_1)\underline{\pi})$ and $\mu^g_{s0}=(1-\bar{\pi})\mu_1/((1-\bar{\pi})\mu_1+(1-\underline{\pi})(1-\mu_1)$. The value of selling to a high quality bank is given by

$$[\mu_1\bar{\pi}+(1-\mu_1)\underline{\pi}]\bar{v}-q+\Delta^g(\mu_1)$$

From assumption (14) and $\beta\left[\underline{\pi}V_2(\mu^g_{s\bar{v}})+(1-\underline{\pi})V_2(\mu^g_{s0})-V_2(\mu)\right]=\Delta^g(\mu_1)$, we have

$$\begin{aligned}
[\mu_1\bar{\pi}+(1-\mu_1)\underline{\pi}]\bar{v}-q+\beta\left[\underline{\pi}V_2(\mu^g_{s\bar{v}})+(1-\underline{\pi})V_2(\mu^g_{s0})-V_2(\mu)\right] &\leq \underline{\pi}\bar{v}-q(1+r)-\underline{c}\\
[\mu_1\bar{\pi}+(1-\mu_1)\underline{\pi}]\bar{v}-q+\Delta^g(\mu_1) &\leq \bar{\pi}\bar{v}-q(1+r)-\underline{c}
\end{aligned}$$

Hence, there is no profitable deviation by the low cost types. As for the LH type bank, note that in the positive equilibrium

$$[\mu_1\bar{\pi}+(1-\mu_1)\underline{\pi}]\bar{v}-q+\beta\left[\bar{\pi}V_2(\mu^g_{s\bar{v}})+(1-\bar{\pi})V_2(\mu^g_{s0})-V_2(\mu)\right]\geq\bar{\pi}\bar{v}-q(1+r)-\bar{c} \tag{26}$$

We use the above inequality to show that the LH type bank does not have a profitable deviation.

There are two possible cases: Case 1. $\bar{c}+qr \geq (\bar{\pi}-\underline{\pi})\bar{v}$. In this case, $\mu_2^* = 0$ and $V_2(\mu)$ is a constant function. Therefore, $\Delta^g(\mu_1) = 0$ for all μ_1 and $\beta\left[\underline{\pi}V_2(\mu_{s\bar{v}}^g) + (1-\underline{\pi})V_2(\mu_{s0}^g) - V_2(\mu)\right] = 0$. In this case, we are back to the static game and as we have shown before, the LH bank finds it optimal to sell always. Case 2. $\bar{c} + qr < (\bar{\pi} - \underline{\pi})\bar{v} < \bar{v}$. In this case, we have

$$\begin{aligned}\beta\left[V_2(\mu_{s\bar{v}}) - V_2(\mu_{s0})\right] &\leq \beta(1-\alpha)\left\{[\mu_{s\bar{v}}\bar{\pi} + (1-\mu_{s\bar{v}})\underline{\pi}]\bar{v} - q - \bar{\pi}\bar{v} + q(1+r) + \bar{c}\right\} \\ &= \beta(1-\alpha)\left\{-(1-\mu_{s\bar{v}})(\bar{\pi}-\underline{\pi})\bar{v} + qr + \bar{c}\right\}\end{aligned}$$

The last expression is increasing in μ_1 and therefore maximized at $\mu_1 = 1$. Hence, we must have

$$\beta\left[V_2(\mu_{s\bar{v}}) - V_2(\mu_{s0})\right] \leq \beta(1-\alpha)(qr+\bar{c}) < \bar{v}$$

Therefore,

$$-\beta(\bar{\pi}-\underline{\pi})\left[V_2(\mu_{s\bar{v}}) - V_2(\mu_{s0})\right] > -\bar{v}(\bar{\pi}-\underline{\pi})$$

Adding this inequality to (26) , we get

$$[\mu_1\bar{\pi} + (1-\mu_1)\underline{\pi}]\bar{v} - q + \beta\left[\underline{\pi}V_2(\mu_{s\bar{v}}^g) + (1-\underline{\pi})V_2(\mu_{s0}^g) - V_2(\mu)\right] \geq \bar{\pi}\bar{v} - q(1+r) - \bar{c}$$

which implies that the LH type bank does not have a profitable deviation in the constructed equilibrium.

As for the negative equilibrium, it is clear that a bank with low cost does not want to sell its loan, since selling only punishes the bank. Therefore, it is sufficient to show that the LH bank wants to sell its loan. That is, we need to show that for all $\mu_1 \in [0, \bar{\mu}]$, we have

$$\underline{\pi}\bar{v} - q + \beta[V_2(0) - V_2(\mu_h^b)] \geq \underline{\pi}\bar{v} - q(1+r) - \bar{c} \tag{27}$$

where $\mu_h^b = \mu_1/(\mu_1 + (1-\mu_1)\alpha)$. To do so, we first show that this inequality is satisfied at $\mu_1 = \bar{\mu}$. Now, since $\Delta^b(\mu_1) = \beta[V_2(0) - V_2(\mu_h^b)]$ is decreasing, this implies that (27) holds for all $\mu_1 \in [0, \bar{\mu}]$. By definition, $\bar{\mu}$ satisfies

$$\underline{\pi}\bar{v} - q + \beta[V_2(0) - V_2(\mu_h^b)] = \bar{\pi}\bar{v} - q(1+r) - \bar{c}$$

Obviously, this equality leads to the above inequality. Therefore, we have shown that LH bank still finds it optimal to sell in the negative equilibrium. *Q.E.D.*

Proposition 5. Suppose that $(1-\alpha)/(\bar{\pi}\left(\alpha - \frac{\underline{\pi}}{\bar{\pi}}\right)) \leq \beta(1-\alpha)$. Then, the utility level for each type of bank and the buyers in the positive equilibrium is at least as large as the utility level for the corresponding type of bank and the buyers in the negative equilibrium.

Proof: Here we show that if $(1-\alpha)/(\bar{\pi}\alpha - \underline{\pi}) \leq \beta(1-\alpha)$, then $\mu_h^n \leq \mu_2^*$. The Proposition follows from the discussion in the text. To see this result, notice that if $(1-\alpha)/(\bar{\pi}\alpha - \underline{\pi}) \leq \beta(1-\alpha)$ then

$$\frac{1 - \frac{\underline{\pi}}{\bar{\pi}}}{\alpha - \frac{\underline{\pi}}{\bar{\pi}}} \leq 1 + \beta\bar{\pi}(1-\alpha)$$

Let $\lambda = \frac{1}{1+\beta\bar{\pi}(1-\alpha)}$. Then, $\lambda(1 - \frac{\underline{\pi}}{\bar{\pi}}) \leq \alpha - \frac{\underline{\pi}}{\bar{\pi}}$ or $\lambda + (1-\lambda)\frac{\underline{\pi}}{\bar{\pi}} \leq \alpha$. Now consider the following two linear functions, $f_1(\mu) = \lambda + (1-\lambda)(\mu + (1-\mu)\frac{\underline{\pi}}{\bar{\pi}})$ and $f_2(\mu) = \mu + (1-\mu)\alpha$. The value of the two function coincide at $\mu = 1$. Moreover, at $\mu = 0$, by the above inequalities $f_1(0) \leq f_2(0)$. Hence, we must have for all $\mu \in [0,1]$, $f_1(\mu) \leq f_2(\mu)$. In other words:

$$\frac{\mu}{\lambda + (1-\lambda)(\mu + (1-\mu)\frac{\underline{\pi}}{\bar{\pi}})} \geq \frac{\mu}{\mu + (1-\mu)\alpha}$$

Therefore, by Jensen's inequality - $E\frac{1}{X} \geq \frac{1}{EX}$, we must have that

$$\lambda\mu + (1-\lambda)\frac{\mu}{\mu + (1-\mu)\frac{\underline{\pi}}{\bar{\pi}}} \geq \frac{\mu}{\lambda + (1-\lambda)(\mu + (1-\mu)\frac{\underline{\pi}}{\bar{\pi}})} \tag{28}$$

Note that by definition of $\underline{\mu}$, we must have

$$\hat{p}(\underline{\mu}) + \beta(1-\alpha)\bar{\pi}\left[V_2(\mu_{s\bar{v}}) - V_2(\underline{\mu})\right] = \bar{\pi}\bar{v} - qr - \bar{c}$$

Further simplification of the above implies that

$$\frac{1}{1+\beta\bar{\pi}(1-\alpha)}\underline{\mu} + \frac{\beta\bar{\pi}(1-\alpha)}{1+\beta\bar{\pi}(1-\alpha)}\frac{\underline{\mu}}{\underline{\mu} + (1-\underline{\mu})\frac{\underline{\pi}}{\bar{\pi}}} = \mu_2^*$$

Then, by (28), we must have $\mu_2^* \geq \mu_h^n$. *Q.E.D.*

Lemma 2 *The reputation gain* $\Delta(v_1; \hat{a}_1)$ *has the following properties:*

1. $\Delta(v_1;\hat{a}_1)$ *is continuous in* v_1 *and* $\hat{a}_1$*. Furthermore, if* $\hat{a}_1$ *is point-wise higher than* $\hat{a}_1'$, $\Delta(v_1;\hat{a}_1) \geq \Delta(v_1;\hat{a}_1')$*. Moreover, if* $\hat{a}_1(v_1) \neq \hat{a}_1'(v_1)$ *for a positive measure subset of* v_1*'s,* $\Delta(v_1;\hat{a}_1) > \Delta(v_1;\hat{a}_1')$*. In particular* $\Delta(v_1;d_k)$ *is decreasing in* k*.*

2. *If* $\hat{a}_1$ *is switching strategy,* $\Delta(v_1;\hat{a}_1)$ *is increasing in* v_1*.*

3. $\Delta(v_1;\hat{a}_1)$ *is bounded in the sense that there exists* $\underline{\Delta} < \bar{\Delta}$ *such that for all* $v_1 \in \mathbb{R}$ *and* $\hat{a}_1$*, we have* $\underline{\Delta} \leq \Delta(v_1;\hat{a}_1) \leq \bar{\Delta}$*.*

Proof: 1. Consider the set $A = \{v_1; \hat{a}_1(v_1) > \hat{a}_1'(v_1)\}$. Then

$$\int \hat{a}_1(v_1) dG\left(\frac{v_1 - \bar{v}_1}{\sigma}\right) - \int \hat{a}_1'(v_1^d) dG\left(\frac{v_1 - \bar{v}_1}{\sigma}\right) = \int_A dG\left(\frac{v_1 - \bar{v}_1}{\sigma}\right) \geq 0$$

with equality only if A is measure zero. Given the Bayesian updating formulas, this inequality implies that for any $\bar{v}_1$,

$$\mu_{sg}(\bar{v}_1;\hat{a}_1) \geq \mu_{sg}(\bar{v}_1;\hat{a}_1'), \mu_{sd}(\bar{v}_1;\hat{a}_1) \geq \mu_{sd}(\bar{v}_1;\hat{a}_1'), \mu_h(\bar{v}_1;\hat{a}_1) \leq \mu_h(\bar{v}_1;\hat{a}_1')$$

with strict inequalities only if A is zero measure. Therefore, for each v_1, the integrand in (20) is higher for $\hat{a}_1$and therefore $\Delta(v_1;\hat{a}_1) \geq \Delta(v_1;\hat{a}_1')$ with equality only if A is measure zero.

2. If $\hat{a}_1$ is a switching strategy with switching point k,from (19) it is straightforward to see that $\mu_{sg}(\bar{v}_1;\hat{a}_1), \mu_{sd}(\bar{v}_1;\hat{a}_1)$ are strictly increasing and $\mu_h(\bar{v}_1;\hat{a}_1)$ is strictly decreasing in $\bar{v}_1$. Thus the integrand in (20)c is increasing in $\bar{v}_1$. Since we have assumed that $H(\hat{v}_1|v_1)$ is decreasing in v_1, from first order stochastic dominance, it follows that $\Delta(v_1;\hat{a}_1)$ is strictly increasing.

3. To show boundedness, we first show that for all μ_2, $V_2(\mu_2)$ is well defined and continuous. Since μ_2 lies in a compact set, it follows that $V_2(\mu_2)$ is bounded. To show continuity, note that when $v_2 \geq (\mu^*)^{-1}(\mu_2)$, $V_2(\mu_2, v_2) = \hat{p}(\mu_2; v_2) - q$ and if $v_2 < (\mu^*)^{-1}(\mu_2)$, $V_2(\mu_2, v_2^d) = \bar{\pi}\bar{v} + (1 -$

$\bar{\pi})v_2^d - q(1+r) - \bar{c}$. Therefore,

$$
\begin{aligned}
V_2(\mu_2) &= \int_{-\infty}^{\infty}\left[\int_{\mu^{*-1}(\mu_2)}^{\infty}\{[\hat{p}(\mu_2;v_2)-q\}\,dG\left(\frac{v_2-\bar{v}_2}{\sigma}\right)\right. \qquad (29)\\
&\quad \left.+\int_{-\infty}^{\mu^{*-1}(\mu_2)}\{\bar{\pi}\bar{v}+(1-\bar{\pi})v_2-q(1+r)-\bar{c}\}\,dG\left(\frac{v_2-\bar{v}_2}{\sigma}\right)\right]dF(v_2)\\
&= \{\hat{p}(\mu_2;v_2)-q\}\\
&\quad +\int_{-\infty}^{\infty}\int_{-\infty}^{\mu^{*-1}(\mu_2)}\{(1-\mu_2)(\bar{\pi}-\underline{\pi})(\bar{v}-v_2)-qr-\bar{c}\}\,dG\left(\frac{v_2-\bar{v}_2}{\sigma}\right)dF(\bar{v}_2).
\end{aligned}
$$

Using our assumption that the random variable v_2 has a finite mean with respect to G in (29), it follows that $V_2(\mu_2)$ is bounded. Continuity follows by inspection of (29) noting that so that G and F are continuous functions. Thus, there exist bounds $\underline{\Delta} \leq \bar{\Delta}$ such that for any $v_1, \hat{a}_1$

$$\underline{\Delta} \leq \bar{\pi}V_2(\mu_{s\bar{v}}(\bar{v}_1;\hat{a}_1)) + (1-\bar{\pi})V_2(\mu_{s0}(\bar{v}_1;\hat{a}_1)) - V_2(\mu_h(\bar{v}_1;\hat{a}_1)) \leq \bar{\Delta}.$$

Q.E.D.

Lemma 3 *The best response function $b(k)$ satisfies the following:*

1. *$b(k)$ is continuous and strictly increasing in k.*
2. *There exists a unique v_1^*, such that $b(v_1^*) = v_1^*$.*
3. *For all $k > v_1^*$, $b(k) < k$ and for all $k < v_1^*$, $b(k) > k$.*

Proof: 1.. $b(k)$ satisfies the following

$$\hat{p}(\mu_1;b(k)) - q + \Delta(b(k);d_k) = \bar{\pi}\bar{v} + (1-\bar{\pi})b(k) - q(1+r) - \bar{c} \qquad (30)$$

Since $\Delta(b;d_k)$ is continuous in b and k, it is obvious that $b(k)$ is continuous. An increase in k, causes the function $\Delta(b;d_k)$ to decrease by Lemma 2. Since $\hat{p}(\mu_1;b) - (1-\bar{\pi})b$ is increasing in b, from (30), $b(k)$ must be an increasing function of k.

2. Any fixed point of $b(k)$, v_1^* must satisfy

$$\hat{p}(\mu_1;v_1^*) - q + \Delta(v_1^*;d_{v_1^*}) = \bar{\pi}\bar{v} + (1-\pi)v_1^* - q(1+r) - \bar{c}$$

Now, notice that under $d_{v_1^*}$, from the Bayesian updating rules, the updating rules are functions of only $1 - G\left(\frac{v_1^* - \bar{v}_1}{\sigma}\right)$. Therefore, we can rewrite $\Delta(v_1^*; d_{v_1^*})$ as the following

$$\Delta(v_1^*; d_{v_1^*}) = \beta \int_{-\infty}^{\infty} \left\{ \bar{\pi} V_2 \left(\hat{\mu}_{sg} \left(1 - G \left(\frac{v_1^* - \bar{v}_1}{\sigma} \right) \right) \right) + (1 - \bar{\pi}) V_2 \left(\hat{\mu}_{sd} \left(1 - G \left(\frac{v_1^* - \bar{v}_1}{\sigma} \right) \right) \right) \right.$$
$$\left. - V_2 \left(\hat{\mu}_h \left(1 - G \left(\frac{v_1^* - \bar{v}_1}{\sigma} \right) \right) \right) \right\} dG \left(\frac{v_1^* - \bar{v}_1}{\sigma} \right)$$

Let $l = 1 - G\left(\frac{v_1^* - \bar{v}_1}{\sigma}\right)$. Then the above integral becomes

$$\Delta(v_1^*; d_{v_1^*}) = \beta \int_0^1 \left[\bar{\pi} V_2 \left(\hat{\mu}_{sg}(l) \right) + (1 - \bar{\pi}) V_2 \left(\hat{\mu}_{sd}(l) \right) - V_2 \left(\hat{\mu}_h(l) \right) \right] dl$$

and v_1^* must satisfy

$$-q + \beta \int_0^1 \left[\bar{\pi} V_2 \left(\hat{\mu}_{sg}(l) \right) + (1 - \bar{\pi}) V_2 \left(\hat{\mu}_{sd}(l) \right) - V_2 \left(\hat{\mu}_h(l) \right) \right] dl = \bar{\pi}\bar{v} + (1 - \underline{\pi}) v_1^* - \hat{p}(\mu_1; v_1^*) - q(1+r) - \bar{c}$$

The left side of the above equation does not depend on v_1^* and the right side is strictly decreasing in v_1^*. Since the right side ranges from plus infinity to minus infinity, there exists a unique v_1^* that satisfies the above equation. Now, notice that under $d_{v_1^*}$, from the Bayesian updating rules, the updating rules are functions of only $1 - G\left(\frac{v_1^* - \bar{v}_1}{\sigma}\right)$. Therefore, we can rewrite $\Delta(v_1^*; d_{v_1^*})$ as the following

$$\Delta(v_1^*; d_{v_1^*}) = \beta \int_{-\infty}^{\infty} \left\{ \bar{\pi} V_2 \left(\hat{\mu}_{sg} \left(1 - G \left(\frac{v_1^* - \bar{v}_1}{\sigma} \right) \right) \right) + (1 - \bar{\pi}) V_2 \left(\hat{\mu}_{sd} \left(1 - G \left(\frac{v_1^* - \bar{v}_1}{\sigma} \right) \right) \right) \right.$$
$$\left. - V_2 \left(\hat{\mu}_h \left(1 - G \left(\frac{v_1^* - \bar{v}_1}{\sigma} \right) \right) \right) \right\} dG \left(\frac{v_1^* - \bar{v}_1}{\sigma} \right)$$

and v_1^* must satisfy

$$-q + \beta \int_0^1 \left[\bar{\pi} V_2 \left(\hat{\mu}_{sg}(l) \right) + (1 - \bar{\pi}) V_2 \left(\hat{\mu}_{sd}(l) \right) - V_2 \left(\hat{\mu}_h(l) \right) \right] dl = \bar{\pi}\bar{v} + (1 - \underline{\pi}) v_1^* - \hat{p}(\mu_1; v_1^*) - q(1+r) - \bar{c}.$$

The left side of the above equation does not depend on v_1^* and the right side is strictly decreasing in v_1^*. Since the right side ranges from plus infinity to minus infinity, there exists a unique v_1^* that satisfies the above equation.

3. Suppose $k < v_1^*$ and $b(k) \leq k$. Since $\lim_{k \to -\infty} b(k) = \hat{v}^0 > -\infty$. Then by continuity of $b(\cdot)$, there must exists $k \in (-\infty, k]$ such that $b(\hat{k}) = \hat{k}$. Contradicting part 2. Similarly, we can show that for all $k > v_1^*$, $b(k) < k$. *Q.E.D.*

Proposition 10: Given the value function $V_2(\mu_2)$ given by (18), as $\sigma \to 0$ the set of first period equilibrium strategies in the game with proper priors converges to a unique strategy by the HH bank given by

$$a_1(v_1) = \begin{cases} 1 & \text{if } v_1 \geq v_1^* \\ 0 & \text{if } v_1 < v_1^* \end{cases}$$

where v_1^* satisfies:

$$\hat{p}(\mu_1; v_1^*) - q + \beta \int_0^1 \left[\bar{\pi} V_2\left(\hat{\mu}_{sg}(l)\right) + (1-\bar{\pi}) V_2\left(\hat{\mu}_{sd}(l)\right) - V_2\left(\hat{\mu}_h(l)\right)\right] dl = \bar{\pi}\bar{v} + (1-\bar{\pi}) v_1^* - q(1+r) - \bar{c}$$

and

$$\begin{aligned} \hat{\mu}_{sg}(l) &= \frac{\mu_1 \bar{\pi} l}{\mu_1 \bar{\pi} l + (1-\mu_1)\underline{\pi}} \\ \hat{\mu}_{sd}(l) &= \frac{\mu_1 (1-\bar{\pi}) l}{\mu_1 (1-\bar{\pi}) l + (1-\mu_1)(1-\underline{\pi})} \\ \hat{\mu}_h(l) &= \frac{\mu_1 \left[(1-\alpha)(1-l) + \alpha\right]}{\mu_1 \left[(1-\alpha)(1-l) + \alpha\right] + (1-\mu_1)\alpha} \end{aligned}$$

Proof: We prove Proposition 10 by mapping our environment into that described in Morris and Shin (2003) and show that their requirements for existence of a unique equilibrium in the limit are satisfied.

Given a value function $V_2(\mu_2)$, consider an equilibrium strategy profile in the first period $(a_1(\cdot), \hat{a}_1(\cdot), p_1(\cdot))$. In a game with full information about shocks to returns, when agents in period 2 believe that HH bank sells with probability l in the first period 1, the HH bank's differential gain from selling is given by

$$\hat{\pi}(v_1, l) = \hat{p}(\mu_1; v_1) + qr + \bar{c} - \bar{\pi}\bar{v} - (1-\bar{\pi}) v_1 + \beta \left[\bar{\pi} V_2(\hat{\mu}_{sg}(l)) + (1-\bar{\pi}) V_2(\hat{\mu}_{sd}(l)) - V_2(\hat{\mu}_h(l))\right]$$

Then, in the game with private information, $l = \int \hat{a}_1(v_1) dH(v_1|\bar{v}_1)$ is a random variable. We, then, show that $\hat{\pi}$ satisfies the conditions A1-A3, A4*, A5, and A6 in Morris and Shin (2003).

We then can apply Proposition 2.2 in Morris and Shin (2003) and that completes the proof of our Propoisition. It is easy to see that $\hat{\mu}_{sg}(l)$ and $\hat{\mu}_{sd}(l)$ are increasing in l and $\hat{\mu}_h(l)$ is decreasing in l. Since, $V_2(\mu_2)$ is non-decreasing in μ_2, $\hat{\pi}(v_1, l)$ is non-decreasing in l - condition A1. Obviously $\hat{\pi}(v_1, l)$ is increasing in v_1- condition A2. Since $\hat{\pi}(v_1, l)$ is separable in v_1 and l, and $\hat{\pi}(v_1, l)$ is linearly increasing in v_1, there must exist a unique v_1^* such that $\int \hat{\pi}(v_1^*, l)dl = 0$ - condition A3. Since $V_2(\mu_2)$ is a continuous function over a compact set $[0, 1]$, $\beta\left[\bar{\pi}V_2(\hat{\mu}_{sg}(l)) + (1-\bar{\pi})\, V_2(\hat{\mu}_{sd}(l)) - V_2(\hat{\mu}_h(l))\right]$ is a bounded above and below by $\underline{\Delta}$ and $\bar{\Delta}$, respectively. Now let

$$
\begin{aligned}
0 &= -\hat{p}(\mu_1; \underline{v}_1) - qr + \bar{\pi}\bar{v} + (1-\bar{\pi})\underline{v}_1 - \bar{c} - \bar{\Delta} - \varepsilon \\
0 &= -\hat{p}(\mu_1; \hat{v}_1) - qr + \bar{\pi}\bar{v} + (1-\bar{\pi})\hat{v}_1 - \bar{c} - \underline{\Delta} + \varepsilon
\end{aligned}
$$

Then, if $v_1 \leq \underline{v}_1$, $\hat{\pi}(c_1, l) \leq -\varepsilon$ for all $l \in [0, 1]$. Moreover, if $v_1 \geq \hat{v}_1$, $\hat{\pi}(v_1, l) \geq -\varepsilon$ for all $l \in [0, 1]$ - condition A4*. Continuity of V_2 implies that $\hat{\pi}(v_1, l)$ is a continuous function of v_1 and l. Therefore, $\int_0^1 g(l)\hat{\pi}(v_1, l)dl$ is a continuous function of $g(\cdot)$ and v_1 - condition A5. Moreover, by definition of $F(\cdot)$ and $G(\cdot)$, noisy signal v_1 has a finite expectation, $E[v_1] \in \mathbb{R}$ - condition A6. Therefore, we can rewrite proposition 2.2 in Morris and Shin (2003) for our environment:

Proposition *Let v_1^* satisfy $\int \hat{\pi}(v_1^*, l)dl = 0$. For any $\delta > 0$, there exists a $\bar{\sigma} > 0$ such that for all $\sigma \leq \bar{\sigma}$, if strategy a_1 survives iterated elimination of dominated strategies, then $a_1(v_1) = 1$ for all $v_1 \geq v_1^* + \delta$ and $a_1(v_1) = 0$ for all $v_1 \leq v_1^* - \delta$.*

Q.E.D.

References

Arora, S., B. Boaz, M. Brunnermeier, and R. Ge (2009): "Computational Complexity and Information Asymmetry in Financial Products," Working Paper, Princeton University, Department of Computer Science.

Ashcraft, A. and T. Schuermann (2008): "Understanding the securitization of subprime mortgage credit," 2:3, 191–309.

Benoit, J. and V. Krishna (1985): "Finitely Repeated Games," *Econometrica*, 53, 905–922.

Carlsson, H. and E. Van Damme (1993): "Global games and Equilibrium selection," *Econometrica: Journal of the Econometric Society*, 989–1018.

Carter, S. and R. Sutch (2006): *Historical Statistics of the United States: Earliest Times to the Present*, Cambridge University Press.

Chari, V. V., A. Shourideh, and A. Zetlin-Jones (2009): "Moral Hazard, Reputation, and Fragility in Credit Markets," Working Paper, University of Minnesota.

DeMarzo, P. (2005): "The pooling and tranching of securities: A model of informed intermediation," *Review of Financial Studies*, 18, 1.

DeMarzo, P. and D. Duffie (1999): "A liquidity-based model of security design," *Econometrica*, 67, 65–99.

Dewatripont, M. and J. Tirole (1994): *The Prudential Regulation of Banks*, MIT Press.

Diamond, D. (1989): "Reputation acquisition in debt markets," *The Journal of Political Economy*, 97, 828–862.

Downing, C., D. Jaffee, and N. Wallace (2009): "Is the Market for Mortgage-Backed Securities a Market for Lemons?" *Review of Financial Studies*, 22, 2257.

Drucker, S. and C. Mayer (2008): "Inside Information and Market Making in Secondary Mortgage Markets," Tech. rep., working paper, Renaissance Technologies LL3 and Duke University.

Ely, J., D. Fudenberg, and D. Levine (2008): "When is reputation bad?" *Games and Economic Behavior*, 63, 498–526.

Ely, J. and J. Välimäki (2003): "Bad Reputation*," *Quarterly Journal of Economics*, 118, 785–814.

Frankel, D., S. Morris, and A. Pauzner (2003): "Equilibrium selection in global games with strategic complementarities," *Journal of Economic Theory*, 108, 1–44.

Garleanu, N. and L. Pedersen (2004): "Adverse selection and the required return," *Review of Financial Studies*, 17, 643–665.

Glosten, L. and P. Milgrom (1985): "Bid, ask and transaction prices in a market-maker market with heterogeneously informed traders," *Journal of Financial Economics*, 14, 71–100.

Holmstrom, B. and R. Myerson (1983): "Efficient and Durable Decision Rules with Asymmetric Information," *Econometrica*, 51, 1799–1899.

Ivashina, V. and D. Scharfstein (2008): "Bank Lending During the Financial Crisis of 2008," Tech. rep., Harvard Business School Working Paper.

Kreps, D. and R. Wilson (1982): "Reputation and Imperfect Information," *Journal of Economic Theory*, 27, 253–279.

Kyle, A. (1985): "Continuous auctions and insider trading," *Econometrica*, 53, 1315–1335.

Mailath, G. and L. Samuelson (2001): "Who wants a good reputation?" *The Review of Economic Studies*, 68, 415–441.

Milgrom, P. and J. Roberts (1982): "Predation, Reputation, and Entry Deterrence," *Journal of Economics Theory*, 27, 280–312.

Morris, S. and H. Shin (2003): "Global Games: Theory and Applications," 1, 56–114.

Myers, S. and N. Majluf (1984): "Corporate financing and investment decisions when firms have informationthat investors do not have," .

Ordoñez, G. (2008): "Fragility of Reputation and Clustering in Risk Taking," Working Paper, University of California Los Angeles.

Philippon, T. and V. Skreta (2009): "Efficient Interventions in Markets with Adverse Selection," Working Paper, NYU.

White, E. N. (2009): "Lessons from the Great American Real Estate Boom and Bust of the 1920s," .