

Counterparty risk externality: Centralized versus over-the-counter markets*

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Abstract

The opacity of over-the-counter (OTC) markets – in which a large number of financial products including credit derivatives trade – appears to have played a central role in the financial crisis in 2007–09. We model such opacity of OTC markets in a general equilibrium setup where agents share risks, but have incentives to default and their financial positions are not mutually observable. We show that in this setting, there is excess “leverage” in that parties in OTC contracts take on short positions that lead to levels of default risk that are higher than Pareto-efficient ones. In particular, OTC markets feature a *counterparty risk externality* that we show can lead to ex-ante productive inefficiency. This externality is absent when trading is organized via a centralized clearing mechanism that provides transparency of trade positions, or a centralized counterparty (such as an exchange) that observes all trades and sets prices.

J.E.L.: G14, G2, G33, D52, D53, D62

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“To restrain private people, it may be said, from receiving in payment the promissory notes of a banker for any sum, whether great or small, when they themselves are willing to receive them; or, to restrain a banker from issuing such notes, when all his neighbours are willing to accept of them, is a manifest violation of that natural liberty, which it is the proper business of law not to infringe, but to support. Such regulations may, no doubt, be considered as in some respects a violation of natural liberty. But those exertions of the natural liberty of a few individuals, which might endanger the security of the whole society, are, and ought to be, restrained by the laws of all governments; of the most free, as well as of the most despotical. The obligation of building party walls, in order to prevent the communication of fire, is a violation of natural liberty, exactly of the same kind with the regulations of the banking trade which are here proposed.” – Adam Smith, *The Wealth of Nations*, 1776.¹

1 Introduction and motivation

An important risk that needs to be evaluated at the time of financial contracting is the risk that a counterparty will not fulfill its future obligations. This counterparty risk is difficult to evaluate because the exposure of the counterparty to various risks is generally not public information. Contractual terms such as prices and collateral that affect a trade can be tailored to mitigate counterparty risk, but the extent to which this can be achieved, and how efficiently so, depends in general on how contracts are traded.

One possible trading infrastructure is an over-the-counter (OTC) market in which each party trades with another, subject to a bankruptcy code that determines how counterparty defaults will be resolved.² A key feature of OTC markets is their opacity. In particular, even within a set of specific contracts, for example, credit default swaps (CDS), no trading party has full knowledge of positions of others. We show theoretically in this paper that such opacity of exposures in OTC markets leads to an important risk

¹The paragraph cited appears on Vol. 1, p. 289, of the J.M. Dent & Sons Publisher edition, London, 1910.

²The bankruptcy code may be specified in the contract or adhere to a uniformly applicable corporate bankruptcy code.

spillover – a *counterparty risk externality*³ – that leads to excessive “leverage” in the form of short positions that collect premium upfront but default ex post. Such excessive leverage results in inefficient levels of risk-sharing and/or deadweight costs of bankruptcy.

Counterparty risk externality is the effect that the default risk on one contract will be increased if the counterparty agrees to the same contract with another agent because the second contract increases the probability that the counterparty will be unable to perform on the first one. Put simply, the default risk on one deal depends on what else is done. The intuition for our result concerning the inefficiency of OTC markets is that in OTC markets it is not at all transparent what else is being done. Hence, counterparties cannot charge price schedules that effectively penalize the creation of counterparty risk. This makes it likely that excessively large short positions will be built by some institutions without the full knowledge of other market participants.

For example, in September 2008, it became known that A.I.G.’s liquidity position was inadequate given that it had written credit default swaps (bespoke CDS) for many investors guaranteeing protection against default on mortgage-backed products. Each investor realized that the value of A.I.G.’s protection was dramatically reduced on its individual guarantee. Investors demanded increased collateral – essentially posting of extra cash – which A.I.G. was unable to provide and the Treasury had to take over A.I.G. The counterparty risks were so widespread globally that a default would probably have spurred many other defaults, generating a downward spiral. The A.I.G. example illustrates the cost that large OTC exposures can impose on the system when a large institution defaults on its obligations. But, more importantly, it also raises the question of whether A.I.G.’s true risk as a counterparty was subject to adequate risk controls in protections they sold. We argue that the opacity of the OTC markets in which these credit derivatives traded was primarily responsible for allowing the build-up of such large exposures in the first place.

While a number of financial innovations in fixed income and credit markets have traded until now in OTC markets, many products linked to commodity and equities have traded successfully on centralized trading platforms such as exchanges. A distinguishing feature of an exchange relative to OTC trading is that even though individual agents still do not see each others’

³The term “counterparty risk externality” is as employed by Acharya and Engle (2009). A part of the discussion below, especially related to A.I.G. is also based on that article.

trades, there is a centralized counterparty – the exchange – that sees all trades (at least on all products traded on that particular exchange). Crucially, this enables the exchange to offer individual parties pricing schedules for trades (in practice, collateral arrangements and exposure limits) that are contingent not just on observable or public characteristics (e.g., credit ratings) but also on its own knowledge of other trades (e.g., net positions in futures contracts). However, exchanges are often viewed as detrimental to ease of search facilitated by bilateral OTC markets, especially for customized or non-standardized financial products. Hence, as an alternative to intermediating trades on a centralized platform or through a centralized counterparty, a centralized clearing mechanism has been proposed that registers and provides transparency of trades in OTC markets.

We show formally that when trading is organized in the form of a centralized clearing mechanism, then transparency can enable market participants to condition contract terms for each counterparty based on its overall positions. Such conditioning is sufficient to get that party to internalize the counterparty risk externality of its trades. In other words, the moral hazard that a party wants to take on excessive leverage through short positions – collect premiums today and default tomorrow – is counteracted by the fact that they face a steeper price schedule by so doing. Effectively, a centralized clearing mechanism with transparency is sufficient to achieve the efficient risk-sharing outcome. We show that a competitive centralized exchange or a centralized counterparty also would induce efficient risk-sharing, but in practice, this would be at the cost of restricting all trades, including those involving non-standardized financial assets, through a single intermediary.

1.1 Model and results

We derive these results in a competitive general equilibrium (GE) model with two periods but allowing for the possibility of default (Geanakoplos, 1997, Geanakoplos and Zame, 1998, Dubey, Geanakoplos and Shubik, 2005). There is a single financial asset, which can be interpreted as a contingent claim on future states of the world, and agents can take long or short positions in the asset. Trades are collateralized by agents' endowments. When an agent has short positions that cannot be met by the pledgeable fraction of endowment, there is default. Default results in deadweight costs which are borne by the short position and are increasing in the size of short positions, e.g., due to a greater number of parties to deal with in a bankruptcy proceeding.

Such costs may arise also due to loss of customers or franchise value in fully dynamic setups. We do not model the structure of bankruptcy costs but simply postulate their pecuniary equivalent in reduced form.

The possibility of default (the option to exercise limited liability, to be precise) implies that long and short positions do not necessarily yield the same payoff and indeed that there is counterparty risk in trading. We assume a natural bankruptcy rule that illustrates why counterparty risk potentially arises in such a setting. In particular, in any given state of the world, the payoff to long positions is determined pro-rata based on delivery from short positions. This rationing of payments implies that each trade imposes a payoff externality on other trades. This spillover is precisely what we refer to as a *counterparty risk externality*.

In this setup, we consider various trading structures and ask whether a counterparty risk externality leads to inefficient risk-sharing. One structure is a centralized clearing mechanism with transparency guarantees that all trades are observable and agents can set pricing schedules that are conditional on this knowledge. Another structure is a centralized exchange is a centralized counterparty that observes all trades and can set pricing schedules based on this knowledge. In contrast, in economies with OTC market structure, trades are not mutually observed and thus pricing schedules faced by agents are not conditional on their other trades (even though they might be conditioned on public information about their type, e.g., their level of endowment).

Our first result is that *competitive equilibria in economies with a centralized clearing mechanism or a centralized exchange are constrained Pareto efficient*. This is true even allowing for market incompleteness so that the result is not simply a consequence of welfare theorems in case of complete markets. Our second result is that *competitive equilibria in economies with OTC markets are robustly constrained inefficient*. We study two different cases, one in which OTC markets operate with a *bilateral netting* mechanism and one without bilateral netting. We show that OTC markets are robustly constrained inefficient in both cases.⁴ This makes it precise that it is the *opacity* or lack of transparency of positions in the OTC markets (rather than differences with centralized trading in how bankruptcy is resolved) that leads to ex-ante inefficiency.

⁴In other words, the counterparty risk externality is orthogonal to what could be called the “netting externality” – that default decision of one party depends on the default decision of its counterparties.

The inefficiency in the OTC setting manifests as excessively large short positions as counterparties taking long positions do not internalize the default risk they impose on other long positions. Intuitively, as long as there is a “risk premium” on the underlying contract (e.g., because the risk being insured in the contract is aggregate in nature) and the costs of defaulting are not excessively large, the short position (the insurer) perceives a benefit from building up short positions, collecting premiums upfront and defaulting ex post. We interpret this outcome as characterizing excessive “leverage”.⁵ Formally, we capture the resulting inefficiency in the form of deadweight costs of bankruptcy. More generally, the inefficiency could manifest as excessive systemic risk due to spillover on to other counterparties.

As an extension, we allow agents to alter their production schedules. The ability to hedge the production risk through financial contracts creates additional demand for long positions. Our third result is that in this case, the incentives to build excessive leverage through short positions in OTC markets translates into a production inefficiency. This result clarifies that the inefficiency of OTC markets extends beyond just inefficient risk-sharing. As an example, suppose that there is insurance being provided on economy-wide mortgage defaults. This would carry a significant hedging premium due to demand from mortgage lenders, giving rise to perverse insurer incentives to default. Thus, in equilibrium, the insurer would take on large and inadequately-collateralized short-selling (of protection) on pools of mortgages and the insured lenders would feed the excessive creation of the housing stock backing such mortgages.⁶

In our economy, transparency and centralized clearing guarantee efficiency. In fact, trading mechanisms that require a centralized clearing for all trades to guarantee the necessary transparency are quite demanding in practice. OTC markets might represent an effective outlay to trade non-standardized financial products. In this case, our analysis suggests that to address the excessive leverage and counterparty risk externality in OTC markets, an efficient regulatory alternative is to specify a bankruptcy rule for seniority of centrally cleared positions over OTC positions. Such subordination of OTC positions relative to centrally cleared ones may suffice to

⁵Interestingly, this implies a lower unit cost of insurance since the realized insurance payoff is smaller when the insurer is more likely to default.

⁶This may be a partial explanation of the role played by credit default swaps, sold in large quantities by A.I.G. on corporate loan and mortgage pools, in fueling the credit boom preceding the crisis of 2007-09.

guarantee efficiency, as junior OTC positions would not dilute the senior centrally cleared positions, for which counterparties would face appropriate incentives and risk controls.

The remainder of the paper is structured as follows. Section 2 provides a simple example of the counterparty risk externality in OTC markets. Section 3 presents the general model, the various trading structures (centralized clearing and OTC), and the welfare analysis of competitive equilibrium under these structures. Section 4 discusses extensions of the model. Section 5 discusses the relationship between the competitive equilibrium in our model with the market microstructure of OTC and centralized trading structures in practice. This section also considers the policy implications of our model for OTC versus centralized clearing. Section 6 relates our work to existing literature. Section 7 concludes.

2 Example of counterparty risk externality in OTC markets

Consider a two-period ($t = 0, 1$) economy with three types of agents ($i = 1, 2, 3$).⁷ There are two states of the world at $t = 1$, denoted by Good (G) and Bad (B). The probabilities of these states are p and $(1 - p)$, respectively. Agents' endowments in the two states are denoted as $w^i(s)$, $i = 1, 2, 3$, and $s = G, B$. Their initial endowments are denoted w_0^i . We assume throughout that initial endowments are large enough that there are no default considerations at $t = 0$. For simplicity, we also assume that

$$w^1(G) > w^2(G) > w^3(G) = 0,$$

and

$$w^1(B) = w^2(B) = 0 < w^3(B).$$

In other words, agents of type 1 and type 2 have endowment in the good state of the economy, but none in the bad state, whereas agents of type 3 are endowed in the bad state but not in the good state.

⁷Since the model is competitive, there is a continuum of agents of each type. Hence, it would suffice as such to simply consider in this example economy two types of agents. Nevertheless, for sake of clearer exposition of counterparty risk externality, we consider three types of agents, two of which will take long positions in a financial contract and the third will take short positions.

Agents of each type have a mean-variance utility function:

$$E[u(x_0, x(s))] = x_0 + E(x(s)) - \frac{\gamma}{2} \text{var}(x(s)),$$

where x_0 is the residual endowment at $t = 0$, and $x(s)$ is the realized endowment at $t = 1$, both taking account of trades that are structured at $t = 0$ and materialize at $t = 1$.

We assume that the only traded contract is an “insurance” (or a credit default protection) which resembles a put option on the bad state of the economy. The contractual payoff of the contract is $R(G) = 0$ and $R(B) > 0$. For simplicity, we will refer to $R(B)$ simply as R . Importantly, the economy will allow for default so that the actual payoff on the contract in the bad state need not coincide with R . The insurance contract must be paid for at $t = 0$ and we denote its price as q .

To highlight our main point, we consider agents 1 and 2 purchasing insurance contract from agents 3. We denote the long positions of agents 1 and 2 as $z^i \geq 0$, $i = 1, 2$, and the short position of agents 3 as $z^3 \geq 0$. Note that the only agents that can default given our assumptions are agents 3. We assume that in case they default, they suffer a linear non-pecuniary penalty as a function of the positions defaulted upon, whose pecuniary equivalent in the bad state is given by ϵz^3 . Broadly speaking, this penalty can be interpreted as loss of continuation of franchise value in a multi-period setting. Hence, the equilibrium cash flow of agents 3 will be negative in the bad state if they default, reflecting this deadweight cost.

2.1 OTC markets

Consider the case of over-the-counter (OTC) trading: agents do not observe the size of the trades put on by other agents and hence prices cannot be conditioned on these. In other words, all agents take the price per unit of insurance as a given constant (and not a schedule depending on total insurance sold by agents 3 in the economy). Agents are fully rational, however, and anticipate correctly the likelihood of default, and its consequent effect on the realized payoff on the insurance contract (R^+) relative to the promised payoff (R). Suppose that $R^+ \leq R$. Then, the $t = 0$ payoffs to the three agents are

$$(x_0^1, x_0^2, x_0^3) = (w_0^1 - z^1 q, w_0^2 - z^2 q, w_0^3 + z^3 q),$$

and $t = 1$ payoffs in good and bad states are given respectively as

$$[x^1(G), x^2(G), x^3(G)] = [w^1(G), w^2(G), w^3(G)], \text{ and}$$

$$[x^1(B), x^2(B), x^3(B)] = [R^+ z^1, R^+ z^2, w^3(B) - R^+ z^3 - \epsilon z^3 1_D],$$

where 1_D is an indicator variable which takes on the value of one if there is default ($R^+ < R$) and zero otherwise. Equivalently, we could have written $x^3(B) = \max(w^3(B) - R z^3, -\epsilon z^3)$.

Then, equilibrium in the economy is characterized by the trading positions, the payoff on the insurance contract (involving the possibility of default) and the cost of insurance: (z^1, z^2, z^3, R^+, q) , such that:

1. Each agent maximizes its expected utility by choosing its trade positions (as we describe below);
2. Market for insurance clears: $z^3 = z^1 + z^2$; and,
3. In case of default, (we assume that) there is pro-rata sharing of agents 3's total endowment between the long positions of agents 1 and 2:

$$R^+ = \begin{cases} \frac{w^3(B)}{z^1 + z^2} & \text{if } 1_D = 1 \\ R & \text{else} \end{cases}$$

Now, consider agent 1's maximization problem:

$$\max_{z^1} w_0^1 - z^1 q + p w^1(G) + (1 - p) R^+ z^1 - \frac{\gamma}{2} \text{var}(x^1(s)),$$

where

$$\text{var}(x^1(s)) = p(1 - p)[w^1(G) - R^+ z^1]^2.$$

Then, the first-order condition for agent 1 implies that:

$$z^1(R^+, q) = \frac{1}{R^+} \left[w^1(G) - \frac{(q - (1 - p)R^+)}{\gamma p(1 - p)R^+} \right]. \quad (1)$$

Similarly, we obtain for agent 2's long position that:

$$z^2(R^+, q) = \frac{1}{R^+} \left[w^2(G) - \frac{(q - (1 - p)R^+)}{\gamma p(1 - p)R^+} \right]. \quad (2)$$

In other words, all else equal, agents 1 and 2 purchase more insurance if they have greater endowment in the good state and less so if the cost of insurance rises. The crucial observation is that even though the payoff R^+ is affected by each agent's long position in equilibrium, agents are competitive and do not internalize this effect. This is the source of counterparty risk externality in the model.

Next, we will show that agents of type 3 have incentives to default in state B whenever deadweight cost of default ε is not too high. To clarify agent 3's choice with regard to default, consider first the case in which it cannot default. In this case, its problem is

$$\max_{z^3} w_0^3 + z^3 q + (1-p)[w^3(B) - Rz^3] - \frac{\gamma}{2} p(1-p)[w^3(B) - Rz^3]^2,$$

which yields

$$z_{ND}^3 = \frac{1}{R} \left[w^3(B) + \frac{(q - (1-p)R)}{\gamma p(1-p)R} \right]. \quad (3)$$

In the limit where there are no default costs, that is, $\varepsilon = 0$, agent 3 with position z_{ND}^3 will not default in equilibrium only if

$$w^3(B) \geq Rz_{ND}^3,$$

which turns out to be equivalent to requiring that $q \leq (1-p)R$. This condition has the intuitive interpretation that the insurer has incentives not to default ex post only if the price of insurance is smaller than or equal to the expected payoff on the insurance, or in other words, that there is no "risk premium" in the insurance price. This will, however, not hold in equilibrium in general, whenever the insurance is against a risk that is aggregate in nature and cannot be fully diversified away, e.g., if $w^1(G) + w^2(G) > w^3(B)$.⁸

More generally, consider then the problem of agent 3, the insurer, when we explicitly allow for default at proportional cost of default $\varepsilon > 0$:

$$\max_{z^3} w_0^3 + z^3 q - (1-p)\varepsilon z^3 - \frac{\gamma}{2} p(1-p)(\varepsilon z^3)^2.$$

⁸A large component of default risk is driven by macroeconomic risks. This explains why there is the moral hazard of default on part of insurers selling credit default swaps (CDS): CDS effectively insure at least some portion of aggregate risk contained in the default risk of the underlying entity. In contrast, there is less risk of such a moral hazard on the part of insurers selling traditional insurance products such as policies on death, accidents, etc. These risks are easily diversified away across agents in the economy, so that insurers simply earn the actuarially fair premium and do not earn a significant risk premium.

Clearly, the insurer pledges the entire endowment in the bad state at $t = 1$ in order to collect as much insurance premium as possible at $t = 0$.⁹ Thus, from the first-order condition, we obtain that

$$z^3 = \frac{q - (1 - p)\epsilon}{\gamma p(1 - p)\epsilon^2}. \quad (4)$$

Thus, the lower the cost of default ϵ and greater the price of insurance q , the greater is the quantity of insurance supplied by the insurers.

Substituting for (z^1, z^2, z^3) in the market-clearing and bankruptcy conditions of the equilibrium yields two equations in the realized insurance payoff R^+ and insurance price q which can be solved to characterize the equilibrium:

$$R^+(q) = \frac{w^3(B)\gamma p(1 - p)\epsilon^2}{q - (1 - p)\epsilon}, \quad (5)$$

$$w^3(B) = w^1(G) + w^2(G) + \frac{2}{\gamma p} - \frac{2q}{\gamma p(1 - p)R^+}. \quad (6)$$

To get intuition while solving this system of equations, we define as “risk premium”:

$$\Delta p = \frac{q}{R^+} - (1 - p), \quad (7)$$

that is, the difference between the “risk-neutral” probability of state B and its actual or statistical probability. Then, solving the system in Δp and R^+ yields as the solution:

$$\Delta p = \frac{1}{2}\gamma p(1 - p) [w^1(G) + w^2(G) - w^3(B)], \quad (8)$$

implying there is a risk premium whenever agents are risk-averse ($\gamma < 0$), there is risk ($0 < p < 1$), and this risk is aggregate in nature ($w^1(G) + w^2(G) > w^3(B)$), and

$$R^+ = \frac{(1 - p)\epsilon + \sqrt{(1 - p)^2\epsilon^2 + 4w^3(B)\gamma p(1 - p) [\Delta p + (1 - p)]}}{2[\Delta p + (1 - p)]} \quad (9)$$

⁹Note that the no-default condition now takes the form:

$$w^3(B) \geq (R - \epsilon)z^3.$$

which is increasing in ϵ . In other words, the higher the bankruptcy costs, the lower is the equilibrium default rate on the contract. It follows then that the contract price $q = [\Delta p + (1 - p)] R^+$ is also increasing in ϵ . In turn, there is default in equilibrium ($R^+ < R$) if and only if bankruptcy costs are sufficiently small (ϵ smaller than a threshold $\bar{\epsilon}$).

2.2 Numerical example

We parametrize the above economy with $w^1(G) = 10$, $w^2(G) = 5$, and $w^3(B) = 10$ so that state B is aggregate risky in nature. We set $\gamma = 1$, $p = 0.9$ and vary ϵ in the range $[0.1, 1.0]$ (a subset of the entire possible range $\epsilon > 0$). Figures 1, 2 and 3 plot respectively the equilibrium quantity of insurance sold (z^3), its realized payoff (R^+), and its price (q), all as a function of ϵ , the proportional deadweight cost of default.

There is a critical value of ϵ below which defaults take place and this value is around 0.548. Above this value, there is no default. Interestingly, for all ϵ smaller than this threshold value, the equilibrium is effectively the same as far as risk-sharing is concerned. In particular, agents of type 3 transfer all their endowment in the bad state at $t = 1$ to agents 1 and 2.

To be precise, the equilibrium utilities (relative to $t = 0$ endowments) are $(U^1, U^2, U^3) = (-1.97, -0.84, 1.35)$ regardless of ϵ in the default range. However, this is not true of the equilibrium quantity of insurance contracts sold and the unit price of insurance.

For example, when $\epsilon = 0.5$, the quantities traded are $(z^1, z^2) = (8.22, 2.74)$ with $z^3 = z^1 + z^2$; there is 9% default on the contract ($R^+ = 0.91$); and, insurance price is $q = 0.30$. In turn, the risk premium Δp equals 0.23.

In contrast, with $\epsilon = 0.01$, the quantities traded become much larger: $(z^1, z^2) = (410.95, 136.98)$; there is 98% default on the contract ($R^+ = 0.02$); and, insurance price is $q = 0.0067$.

In other words, as the default incentives for agents of type 3 become stronger, there is greater quantity of insurance sold, greater default and greater deadweight costs suffered by these agents. In turn, the equilibrium insurance price is smaller too. Default by the insurer lowers the price of insurance since the payoff on the contract is rationally anticipated by those purchasing insurance to be smaller: the quality of insurance has gone down given the insurer's default risk.

2.3 Inefficiency of OTC markets

The inefficiency of the equilibrium in the example above when $\epsilon < 0.548$ stems from excessive deadweight costs of agent 3's bankruptcy. This can be seen in Figure 4 which plots the sum of utilities of all three agents and also separately of agents of type 3. In effect, regardless of the value of ϵ , we obtain full transfer of endowment in the bad state from agents of type 3 to the other two agents. However, when ϵ is small, this occurs in equilibrium with agents of type 3 selling quantities of insurance that lead to their default. Hence, agents 1 and 2 enjoy the same equilibrium utility as ϵ varies; in contrast, for $\epsilon < 0.548$, default leads to deadweight costs borne by agents of type 3 and their equilibrium utility is substantially lower compared to the case where $\epsilon \geq 0.548$. In other words, even though the counterparty risk externality arises due to agents with long positions ignoring the effect of their demand for insurance on the insurance payoff, they rationally anticipate the equilibrium payoff and pay for it accordingly. The result of their ignoring the counterparty risk externality is that there is too much demand for insurance in equilibrium, which gives insurers the incentive to default ex post, for which they pay ex ante as expected deadweight costs of default and therefore reduction in expected utility.

It is clear in this case that the planner can improve upon the OTC case when ϵ is smaller than 0.548. Essentially, the planner needs to enforce a "position limit" that restricts agents of type 3 from selling a quantity of insurance z^3 that is beyond their endowment in the bad state $w^3(B)$. One way in which this position limit can be implemented is through a non-linear pricing schedule: $q(z^3) = 0$ if $z^3 > w^3(B)$, and $q(z^3)$ determined by the markets otherwise. While in our specific example, it is efficient for insurance to be fully collateralized so that any default is ruled out in equilibrium, this is in general not true. What is however true, and we show below, is that the OTC markets always feature greater likelihood of default in equilibrium compared to its (Pareto) efficient level.

3 The general model

We now build on the above example to construct a general model of an OTC market with default risk. In particular, we allow for an arbitrary number of agent types, with arbitrary structure of endowments, and the possibility

of each agent taking long and short positions with each other agent (requiring us to also introduce some additional notation). Without much further complication, we also allow only a part of each agent's endowment to be pledgeable in honoring its short positions. We continue to restrict attention for sake of simplicity to a single financial contract. After completing the analysis of OTC markets, we consider financial markets with a centralized clearing mechanism and compare its equilibrium outcome with that under the OTC markets.

Formally, we extend the two-period General Equilibrium (GE) exchange economy with default (Geanakoplos, 1997, Geanakoplos and Zame, 1998, Dubey, Geanakoplos and Shubik, 2005) to allow for different mechanisms for financial market trading. In the interest of pedagogical clarity, we state the optimization programs in each setting fully even though some parts are common across different programs.

Agents and endowments The economy is populated by $i = 1, \dots, I$ types of agents. Let x_0^i be consumption of agent i at time 0. Let $s = 1, \dots, S$ denote the states of uncertainty in the economy, which are realized at time 1. State s occurs with probability p_s , and $\sum_s p_s = 1$. Let x_1^i be agent i 's consumption at time 1, a random variable over the state space S : $x_1^i(s)$, for $s \in S$. Let w_0^i be the endowment of agent i at time 0; and $w_1^i(s)$ her endowment at time 1 in state s . The utility of agent i over consumption in state s is denoted as $u^i(x_0^i, x_1^i(s))$ and belongs to the von-Neumann Morgenstern class of expected utility functions.

Financial markets and default We assume, for simplicity, that only one financial asset is traded in this economy, an asset whose payoff is an exogenous non-negative S -dimensional vector R . We can imagine it representing a derivative contract, e.g., a credit default swap.

Agents selling the asset might default on their required payments. In particular, agent i 's short positions are collateralized by the pledgeable fraction α of her endowment at time 1. In other words, in the event of default, creditors (counterparties holding long positions on the asset with the defaulting party) have recourse only to a fraction $\alpha \in [0, 1]$ of the debtor's endowment $w_1^i(s)$. Only a fraction of the debtor's endowment can be pledged as collateral, for instance, because part of the endowment is agent-specific. Other than the defaulting agent simply losing her collateral to counterparties, de-

fault is assumed to have a direct deadweight cost that is proportional to the size of the position defaulted upon. Deadweight costs of default will serve the convenient purpose of providing a bound to short positions on the asset.

Agents trade bilaterally in financial markets. Even though one single asset is traded ex ante, the asset pay-off ex post depends on the type of the agent shorting it, as default decisions in turn do. Let z_+^{ij} be long positions of agents of type i sold by agents of type j . Let $\mathbf{z}_+^i = (z_+^{ij})_{j \in I}$ denote the long portfolio vector of agents of type i (with $z_+^{ii} = 0$, by construction). Let z_-^i be the short position of agents of type i . As we will explain shortly, all short positions are symmetric for the agents shorting the asset, independently of the counterparty, so that there is no need to index short positions of an agent by the counterparty. Then, in case of its default, agent i suffers a deadweight cost of default whose pecuniary equivalent is assumed to be εz_-^i , with $\varepsilon > 0$.

3.1 OTC markets

Consider first the case in which trading is intermediated in over-the-counter (OTC) markets. We model OTC markets as standard competitive markets with no centralized clearing or centralized counterparty (such as an exchange). We assume that no creditor has privileged recourse to a debtor's collateral in case of default. Nonetheless, a bankruptcy mechanism operates to distribute the cash flow delivered on the short positions (full cash flow or endowment recovered in case of default) pro-rata amongst the long positions. To be precise, consider an agent of type i shorting the asset. At equilibrium, the total of the repayment cash flow to an agent of type i is distributed pro-rata among the holders of long positions against counterparty i .¹⁰

The default condition An agent of type i with (long, short) portfolio position (\mathbf{z}_+^i, z_-^i) will default in period 1 in state s iff her income after assets pay off is smaller than the non-pledgeable fraction of her endowment. Let $R^j(s)$ denote the payoff in state s of her long asset portfolio with counterparty $j \in I \setminus \{i\}$. The payoff $R^j(s)$ is taken as given by each agent, though it is endogenously determined, depending on the equilibrium default rate of agents of type j in the economy (as shown later).

¹⁰Given the competitive nature of the model, the bankruptcy mechanism pools all repayments of *all* agents of type i and redistributes them pro-rata to all their counterparties. This is without loss of generality, as we concentrate on symmetric equilibria.

Consider an agent i with a net short position $z_-^i > 0$. She will default on her short position in state s iff:

$$w_1^i(s) + \sum_j R^j(s) z_+^{ij} - R(s) z_-^i < (1 - \alpha) w_1^i(s) - \varepsilon z_-^i . \quad (10)$$

Note that we allow for an agent to maintain at the same time both short and long positions on the asset: z_-^i and $z_+^{ij} > 0$, for some j . In other words, we assume that the clearing mechanism provided by OTC markets does not include *bilateral netting*. We shall study netting later on in the section. Let $I^d(\mathbf{z}_+, z_-; i, s)$ be an indicator variable taking on value 1 if agent i with position (\mathbf{z}_+, z_-) will default at equilibrium in state s , and zero otherwise. Finally, let $I^{nd}(\mathbf{z}_+, z_-; i, s) = 1 - I^d(\mathbf{z}_+, z_-; i, s)$. Clearly, $I^d(\mathbf{z}_+, 0; i, s) = 0$.

Equilibrium payoffs on long and short positions: Since all long positions share pro-rata the payments from defaulting and non-defaulting short positions, the equilibrium payoff of the asset shorted by agent j , denoted $R^j(\mathbf{z}_+, z_-^j; s)$, is given by

$$R^j(\mathbf{z}_+, z_-^j; s) = \begin{cases} \frac{\alpha w_1^j(s)}{z_-^j} & \text{if } I^d(\mathbf{z}_+, z_-^j; j, s) = 1 \\ R(s) & \text{otherwise} \end{cases} \quad (11)$$

where (\mathbf{z}_+, z_-^j) is the portfolio of agents of type j at equilibrium.

Opacity In OTC markets, there is no centralized clearing and disclosure, nor any centralized counterparty that sees all trades. Thus, the trades of each agent i , (\mathbf{z}_+, z_-^i) , are *not* observed in OTC markets by other agents.

Prices and budget constraints Long and short bilateral positions will in general be traded at a price q^j , where the apex j denotes the type of the agent in the short position. Importantly though, the price does *not* depend on her portfolio, since it is not observed.

The budget constraints of agent i in the OTC market are thus given by:

$$\begin{aligned} x_0^i + \sum_j q^j z_+^{ij} - q^i z_-^i &= w_0^i, \\ x_1^i(s) &= \max \left\{ w_1^i(s) + \sum_j R^j(s) z_+^{ij} - R(s) z_-^i, (1 - \alpha) w_1^i(s) - \varepsilon z_-^i \right\} \end{aligned} \quad (12)$$

where $z_+^{ij}, z_-^i \geq 0$, for any j .

The unitary price of a bilateral asset trade z_+^{ij} depends on the short agent's type j , as the type determines the agent's endowment which is public knowledge and affects her probability of default. The fact that this price is *not* conditioned on agent j 's trades is the primary distinction between OTC and markets with a centralized clearing mechanism: contract terms, in particular, prices (and more generally, outside of the model, collateral requirements and exposure limits) are not conditioned on agents' trades in the case of OTC markets whereas they will be in case of a centralized clearing mechanism (Section 3.3).

Competitive equilibrium In equilibrium, financial markets clear:

$$\sum_i z_+^{ij} - z_-^j = 0, \text{ for any } j. \quad (13)$$

Furthermore, the equilibrium payoffs $R^j(s)$ satisfy the condition:

$$R^j(s) = R^j(\mathbf{z}_+^j, z_-^j; s) = \begin{cases} \frac{\alpha w_1^j(s)}{z_-^j} \text{ if } I^d(\mathbf{z}_+^j, z_-^j; j, s) = 1 \\ R(s) \text{ otherwise} \end{cases} \quad (14)$$

Let

$$m^i(s) = MRS^i(s) \equiv p_s \frac{\frac{\partial u^i}{\partial x_1^i}(x_0^i, x_1^i(s))}{\frac{\partial u^i}{\partial x_0^i}(x_0^i, x_1^i(s))} \quad (15)$$

denote the marginal rate of substitution between date 0 and state s at date 1 for agents of type i at equilibrium; that is, the *stochastic discount factor* of agents of type i . The equilibrium price of an asset is then simply equal to the discounted value of asset payoffs, where the discount rate is adjusted for risk according to the stochastic discount factor of any agent with a long position in the asset. More precisely, agents with a long position in the asset are those who have the highest marginal valuation for the asset's return, and hence at equilibrium, prices q^j satisfy:

$$q^j = \max_i E(m^i R^j), \text{ for any } j. \quad (16)$$

3.2 OTC markets with netting

In the OTC markets modeled in the previous section, an agent i is allowed to go both short and long on the asset, and in equilibrium it might be that

$z_-^i > 0$ and, at the same time, $z_+^{ij} > 0$ with some counterparty j . In this context, an ex-post mechanism for state-by-state bilateral netting might have welfare consequences. Hence, we also define an economy with *OTC markets and netting* so as to be able to better distinguish the welfare effects of various distinct components of OTC and centralized market clearing mechanisms.

We model *bilateral netting* by requiring that agents are (without loss of generality) on only one side of the market, that is, for an agent of type i :

$$z_+^{ij} z_-^i = 0, \text{ for any } j. \quad (17)$$

As a consequence, an agent of type i with a short position $z_-^i > 0$ will default in state s iff:

$$w_1^i(s) - R(s)z_-^i < (1 - \alpha) w_1^i(s) - \varepsilon z_-^i. \quad (18)$$

With bilateral netting, therefore, the default decision of any agent i is independent of \mathbf{z}_+^i , which is constrained to be equal to 0 whenever $z_-^i > 0$. Let the default indicator of agents of type i be now denoted as $I^d(z_-; i, s)$, taking on value 1 if agent i with short position z_- will default at equilibrium in state s , and zero otherwise. Agent j 's short position payoffs are now written as

$$R^j(z_-^j; s) = \begin{cases} \frac{\alpha w_1^j(s)}{z_-^j} \text{ if } I^d(z_-^j; j, s) = 1 \\ R(s) \text{ otherwise} \end{cases} \quad (19)$$

and budget constraints of agents i are restricted by $z_+^{ij} z_-^i = 0$, for any j . Finally, at a competitive equilibrium in an economy with OTC markets and netting, financial markets clear, the consistency condition $R^j(s) = R^j(z_-^j; s)$ is satisfied, and equilibrium prices satisfy

$$q^j = \max_i E(m^i R^j), \text{ for any } j. \quad (20)$$

3.3 Centralized clearing

In the previous section, we formalized the competitive equilibrium of an economy in which financial market trades are intermediated by an OTC market. In this section we model instead the operation of a centralized clearing mechanism. We model centralized clearing mechanisms as being composed of two fundamental functions: *bilateral netting* and *transparency*. Transparency is

obtained because a centralized clearing mechanism aggregates all the information about trades and disseminates it to market participants. Two points are in order before we proceed. One, in the model, transparency provided by centralized clearing mechanism is coincident with submission and execution of trades. Our equilibrium setup cannot deal with the timing or market micro-structure issues associated with *when* trades are submitted and made transparent. We discuss this issue in some detail in Section 5.1. Second, we stress that a centralized clearing mechanism need not centrally intermedicate the trades as, for instance, a centralized exchange would do. We study centralized counterparties or exchanges in Section 4.1.

Regarding bankruptcy resolution, we continue to assume that no creditor has direct privileged recourse to a debtor's collateral in case of default; and that, at equilibrium, the sum total of cash flows received by the debtor is distributed pro-rata among the holders of long positions against the debtor. Because of *bilateral netting*, an agent i with a short position $z_-^i > 0$ will default in state s iff

$$w_1^i(s) - R(s)z_-^i < (1 - \alpha) w_1^i(s) - \varepsilon z_-^i; \quad (21)$$

and the equilibrium payoff of the asset shorted by agent j is thus given by

$$R^j(z_-^j; s) = \begin{cases} \frac{\alpha w_1^j(s)}{z_-^j} & \text{if } I^d(z_-^j; j, s) = 1 \\ R(s) & \text{otherwise} \end{cases}. \quad (22)$$

Because of *transparency*, each agent in the economy has access to detailed information about all trades and can condition contract terms on this information. We assume that prices are set in a competitive manner. Specifically, agents are price-takers. However, the payoff on the short position of agent j depends on the position itself, z_-^j , and prices will in general reflect such dependence. Different agents will face different prices, reflecting the probability of default implied by their characteristics: their type (e.g., level of endowment) as well as their trading positions. This requires us to modify the price-taking assumption for short positions in an important manner (that is similar in spirit to modifications in Acharya and Bisin, 2008, and Bisin, Gottardi and Ruta, 2009).

Specifically, an agent of type j with short position z_-^j will face an ask price map

$$q_-^j(z_-^j) = \max_i E(m^i R_-^j(z_-^j)). \quad (23)$$

In other words, an agent of type j understands that the price it will face for a short position depends on the total short positions it sells, z_-^j . Furthermore, an agent of type j understands that the price it will face for a short position will reflect a risk adjustment according to the stochastic discount factor of the agents who would hold such a short position, that is, of those agents who share the highest marginal valuation for the payoff associated to its position, $R_-^j(z_-^j)$. Price taking is then represented by the fact that agents take the vector of stochastic discount factors $(m^1, \dots, m^i, \dots, m^J)$ as given.¹¹ On the other hand, regarding long positions, the payoff $R^j(s)$ is taken as given by each agent, and so is the price q^j .¹²

The budget constraints of agent i are thus given by:

$$\begin{aligned} x_0^i + \sum_j q^j z_+^{ij} - q_-^i(z_-^i) z_-^i &= w_0^i, \\ x_1^i(s) &= \max \left\{ w_1^i(s) + \sum_j R^j(s) z_+^{ij} - R(s) z_-^i, (1 - \alpha) w_1^i(s) - \varepsilon z_-^i \right\} \end{aligned} \quad (24)$$

where $z_+^{ij}, z_-^i \geq 0, z_+^{ij} z_-^i = 0$, for any j .

At competitive equilibrium, all markets clear:

$$\sum_i z_+^{ij} - z_-^j = 0, \text{ for any } j, \quad (25)$$

and the price maps and returns are rationally anticipated by agents:

$$q^j = q_-^j(z_-^j) = \max_i E(m^i R_-^j(z_-^j)), \quad (26)$$

$$R^j(s) = \begin{cases} \frac{\alpha w_1^j(s)}{z_-^j} & \text{if } I^d(z_-^j; j, s) = 1 \\ R(s) & \text{otherwise} \end{cases}, \text{ for any } j. \quad (27)$$

¹¹Our definition of competitive price maps can be thought of as capturing the same consistency condition required by Perfect Nash equilibrium in strategic environments: every agent understands that the ask price she will face for any (possibly out of equilibrium) short position z_-^j will depend on the willingness to pay of agents on the long side of the market. In a competitive equilibrium, however, all deviations from equilibrium are necessarily "small," and hence such willingness to pay coincides with the highest marginal valuation at equilibrium.

¹²We stress that competitive prices, as in any Walrasian model, should be taken as a reduced-form allocation mechanism representing a specific (set of) market microstructure(s). We discuss in Section 5 in some detail how this pricing mechanism can be implemented in actual markets.

3.4 Welfare

How does the competitive equilibrium under OTC markets compare in terms of efficiency properties to the competitive equilibrium under centralized clearing with transparency? To answer this question, we write down the constrained Pareto efficient outcome as the solution to the following problem:

$$\begin{aligned}
 & \max_{(x_0^i, x_1^i, z_+^{ij}, z_-^i)_{i,j}} \sum_i \lambda^i E(u^i(x_0^i, x_1^i)) \\
 & \quad \text{s.t.} \\
 & \sum_i x_0^i - w_0^i = 0, \\
 & \sum_i x_1^i(s) - w_1^i(s) = 0, \text{ for any } s \\
 & x_1^i(s) = \max \left\{ w_1^i(s) + \sum_j R^j(s) z_+^{ij} - R(s) z_-^i, (1 - \alpha) w_1^i(s) - \varepsilon z_-^i \right\},
 \end{aligned} \tag{28}$$

$$R^j(z_-^j; s) = \begin{cases} \frac{\alpha w_1^j(s)}{z_-^j} & \text{if } I^d(\mathbf{z}_+, z_-^j; j, s) = 1 \\ R(s) & \text{otherwise} \end{cases} \tag{29}$$

and where $z_+^{ij}, z_-^i \geq 0$, $z_+^{ij} z_-^i = 0$, for any j , and λ^i is the Pareto weight associated to agents of type i .

This is the standard constrained efficiency problem for a GE economy once it is assumed that default is not controlled by the planner. The constraint (28) serves two purposes:

(i) it restricts the planner's allocations to those that can be achieved with the limited financial instruments available in the economy; and

(ii) it accounts for the fact that each agent can choose to default or not, in each state s : consumption in default state s is $(1 - \alpha) w_1^i(s) - \varepsilon z_-^i$, the non-pledgeable fraction of endowment net of the deadweight costs.¹³

¹³Formally, the constraint includes the incentive compatibility constraint for each agent's choice of default:

$$u^i(x_0^i, x_1^i(s)) \geq u^i(x_0^i, (1 - \alpha) w_1^i(s) - \varepsilon z_-^i). \tag{30}$$

3.5 Results

We can derive the following results on the constrained efficiency of the economy with centralized clearing and the (generic) constrained inefficiency of the economy with OTC markets.

Proposition 1. *Any competitive equilibrium of an economy with a centralized clearing mechanism is constrained Pareto optimal.*

The intuition for efficiency of the centralized exchange economy is that each agent i that is short on the asset faces a price $q_-^j(z_-^j) = \max_i E(m^i R_-^j(z_-^j))$ that is conditioned on her positions. Consequently, she internalizes the effect of her default on the payoff of long positions on the asset $R^j(s)$. The observability of all trades allows for such conditioning of prices and internalization of any externality that trading and default choices impose on other agents. Importantly, note that an *economy with a centralized clearing mechanism*, as we have defined it, is characterized by both a *bilateral netting mechanism* and *transparency*. Both these components are needed for efficiency.

We show that the opacity of OTC markets induces inefficiencies through the counterparty risk externality, independently of the netting mechanism in place. On the other hand, it can be shown that the lack of a bilateral netting mechanism is *per se* associated with an externality that induces inefficiencies in equilibrium even when transparency is guaranteed.

We consider then equilibria with OTC markets and show that they are not in general constrained Pareto efficient. First of all, consider an economy with OTC markets *without bilateral netting*. As we noted, in this case, an agent of type i with a short position $z_-^i > 0$ will default in state s iff:

$$x_1^i(s) = \max \left\{ w_1^i(s) + \sum_j R^j(s) z_+^{ij} - R(s) z_-^i, (1 - \alpha) w_1^i(s) - \varepsilon z_-^i \right\}. \quad (31)$$

The default decision of an agent of type i , therefore, depends on $R^j(s)$, that is, on agents j 's default decisions, which in turn depend on i 's default decisions, introducing a *netting externality* at equilibrium. Then, it is the case that:

Competitive equilibria of economies with OTC markets without netting are robustly not Pareto efficient.

The proof of this statement, however, requires some complex differential computations and is omitted. It is an adaptation of that in Bisin, Geanakoplos, Gottardi, Minelli, and Polemarchakis (2001).

On the other hand, in OTC markets with bilateral netting, an agent i with a short position $z_-^i > 0$ will default in state s iff:

$$w_1^i(s) - R(s)z_-^i < (1 - \alpha)w_1^i(s) - \varepsilon z_-^i . \quad (32)$$

No netting externality arises in this case. Nevertheless, we shall show that equilibria of an economy with *OTC markets and netting* are also typically constrained inefficient. In other words, the transparency provided by centralized clearing mechanism (but not by OTC market economies, with or without netting) is required for constrained efficiency.

Proposition 2. *Competitive equilibria of economies with a centralized clearing mechanism cannot be robustly supported as equilibria in economies with OTC markets, with or without netting.*¹⁴ *More specifically, any competitive equilibrium of the economy with centralized clearing mechanism in which default occurs with positive probability cannot be supported in the economy with OTC markets, with or without netting.*

The intuition is that in OTC markets, with or without netting, each agent j that is short on the asset faces a price q^j that is *not* conditioned on her position z_-^j . Consequently, she does not internalize the effect of her default on the payoff of long positions on the asset R^j . This is a *counterparty risk externality* in addition to the *netting externality*. More generally, it is also the case that:

*Competitive equilibria of economies with OTC markets and netting are robustly not constrained Pareto efficient.*¹⁵

Finally, let the leverage of agent j , L^j , be defined as the value of her short positions' contractual payoff (promised debt payment) divided by the value of her endowment (asset value).

$$L^j \equiv \frac{E(m^j R z_-^j)}{E(m^j w_1^j)} . \quad (33)$$

¹⁴Formally, by robustly we mean: *for an open set of economies parametrized by agents' endowments and preferences.*

¹⁵Once again, we omit the proof of this statement to avoid some complex differential computations.

Then,

Proposition 3. *For deadweight costs ε that are small enough, competitive equilibria of economies with OTC markets, with or without netting, are characterized by weakly greater (and robustly by strictly greater) leverage and default compared to equilibria of the same economy with a centralized clearing mechanism.*

Since ask prices in economies with OTC markets, with or without netting, do not penalize the short positions for their own incentives to default, agents have incentives to exceed the Pareto efficient short positions. Indeed, the proof of these main propositions in the Appendix shows that as long as (i) the underlying asset has some aggregate risk, its price will robustly carry a risk premium that is positive (as explained in the example economy of Section 2), and (ii) bankruptcy costs are not too high (ε is small), then agents with endowments in the aggregate risky states have an incentive to go excessively short, collect premiums upfront, and default ex post. This increases the equilibrium default rate and leads to inefficient risk-sharing.¹⁶ For efficient risk-sharing, it is in general necessary to be able to commit to future payoffs on financial assets, but in OTC markets, such commitment is not enforced through prices and incentives to go excessively short and default dilute the claims of shorting agent's counterparties.

Opacity and counterparty risk externality When combined together, Propositions 1, 2, and 3 imply that a centralized clearing mechanism is an efficient response to the moral hazard that in the absence of perfect observability of trades, agents have incentives to take on short positions that allow them to consume today and default tomorrow. Our analysis, especially in Propositions 2 and 3, makes it precise that it is the *opacity* or lack of transparency of the OTC markets that leads to ex ante inefficiency in terms of excessively large short positions or leverage. In equilibrium, agents anticipate the lowering of payoff on long positions due to counterparty risk and the price of insurance falls. However, this is not sufficient to preclude the insurers

¹⁶If $\varepsilon = 0$, z_-^i is unbounded and, strictly speaking, the economy has no equilibrium. This is just an extreme case, which is of interest to identify the “force” towards borrowing and default built into our model of OTC markets. Positive deadweight costs, $\varepsilon > 0$, guarantee the existence of equilibrium.

from selling large quantities of insurance and defaulting ex post, resulting in inefficiently large deadweight costs of bankruptcy.

4 Extensions

4.1 Centralized exchange economy

In this section we study an economy in which all asset trades are operated by a *competitive centralized exchange*. In essence, the exchange is a centralized counterparty that observes all trades and conditions contract terms for individual agents on these trades. In practice, this could be thought of as capturing a setting with a specialist that sees all trades and sets price schedules, or an exchange that sees all trades and imposes collateral requirements and exposure limits on traders.

We continue to assume that no creditor has direct privileged recourse to a debtor's collateral, in case of default. The centralized exchange, on the other hand, has full recourse to the debtors' pledgeable collateral. Furthermore, the exchange operates as a bankruptcy mechanism, by distributing the cash flow of the short positions pro-rata with respect to the long positions.

An agent i , taking a long position $z_+^{ij} > 0$ with short counterparty j , takes the return R^j as well as the price q^j as given. The equilibrium payoff of the asset shorted by agent j , denoted $R^j(z_-^j; s)$, continues to be given by

$$R^j(z_-^i; s) = \begin{cases} \frac{\alpha w_+^i(s)}{z_-^i} & \text{if } I^d(\mathbf{z}_+, z_-^i; i, s) = 1 \\ R(s) & \text{otherwise} \end{cases} \quad (34)$$

Consequently, an agent of type j with short position z_-^j faces an ask price map

$$q_-^j(z_-^j) = \max_i E(m^i R_-^j(z_-^j)). \quad (35)$$

Price taking is represented by the fact that agents take the pricing kernels $(m^1, \dots, m^i, \dots, m^I)$ as given.

We turn next to the decision problem of the competitive centralized exchange, which controls the supply of the asset to agents. Let the supply offered by the exchange to agent i for long and short positions be denoted (θ_+^i, θ_-^i) , where $\theta_+^i = (\theta_+^{ij})_{j \in I}$ and $\theta_+^{ij} \theta_-^i = 0$ for any j .¹⁷

¹⁷By construction, $\theta_+^{ii} = 0$.

Then, given the supplies, the exchange can compute the cash flow of the short positions of agents:

$$R^j(\theta_-^j; s) = \begin{cases} \frac{\alpha w_1^j(s)}{\theta_-^j} & \text{if } I^d(\theta_-^j; j, s) = 1 \\ R(s) & \text{otherwise} \end{cases} . \quad (36)$$

The exchange prices a unitary short position as $\max_i E(m^i R^j(\theta_-^j))$, taking as given the stochastic discount factor of the agent i who values it the most at the margin, that is the agent who would acquire it if offered.

To summarize, a competitive exchange takes as given the stochastic discount factors $(m^1, \dots, m^i, \dots, m^I)$. Crucially, the exchange anticipates the compositional effects on default risk of portfolios of different agent types, that is, it recognizes how each agent i 's incentives to default are affected by her positions (θ_+^i, θ_-^i) . Thus, the exchange solves the following problem:

$$\max_{(\theta_+^i, \theta_-^i)_i} \left[\sum_j \max_k E(m^k R^j(\theta_-^j)) (\theta_+^{ij} - \theta_-^j) \right] \quad (37)$$

s.t.

$$\sum_i \theta_+^{ij} - \theta_-^j = 0, \text{ for any } j. \quad (38)$$

At competitive equilibrium, the portfolios demanded by the agents are offered by the competitive exchange and markets clear:

$$\theta_+^{ij} = z_+^{ij}, \theta_-^i = z_-^i, \forall i, j, \quad (39)$$

and the price maps and returns anticipated by agents are consistent with those perceived by the exchange:

$$q_-^j(z_-^j) = \max_i E(m^i R^j(z_-^j)), \quad (40)$$

$$q_-^j = q_-^j(z_-^j) = \max_i E(m^i R^j(z_-^j)), \text{ and} \quad (41)$$

$$R^j(s) = R^j(z_-^j; s). \quad (42)$$

It is straightforward to show (proof available upon request) that the competitive equilibrium allocations of economies with such a centralized exchange coincide with those of economies with a centralized clearing mechanism. Therefore, by Proposition 1, competitive equilibrium allocations of

economies with a centralized exchange are constrained efficient. Note however that a centralized exchange which intermediates all financial market trades is a much more invasive institution than a centralized clearing mechanism which allows trading to remain decentralized but requires all trades be reported and made transparent to market participants.

4.2 Production risk

In our entire analysis, the aggregate endowment of the economy, $\sum_{i \in I} w_0^i$ at time 0 and $\sum_{i \in I} w_s^i$ in each state $s \in S$, has been kept constant. We showed that with regard to assets that insure bad aggregate states, the presence of an equilibrium risk premium in their price creates incentives for insurers to take on excessively short positions and default ex post. This effect can in fact arise even in the absence of aggregate risk in endowments if we allow for production in the economy and consider assets that help insure the risk of production. The “hedging premium” on such assets then serves the same purpose as the risk premium on assets insuring aggregate risk of exogenously given endowments.

Suppose each agent is endowed with a production function f which transforms consumption goods at time 0 into consumption goods at time 1. More

precisely, consider the following technology. Let $K = \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_A \end{pmatrix}$ denote a capi-

tal allocation vector over A activities (e.g., projects), so that $k_1 + k_2 + \dots + k_A = k$. The production function can then be defined as the output in state s given capital allocation K : $f(s, K), \forall s$.¹⁸ Note that, by allowing for multiple technological activities ($A \geq 2$), this formulation allows for some control of the agents over the distribution of capital across activities and hence over the probability distribution of outcomes, that is, over production risk.¹⁹

In this extension, the equilibrium analysis of centralized clearing and OTC economies can be extended to production. For instance, the budget

¹⁸We assume f is continuously differentiable, strongly increasing, and strictly quasi-concave.

¹⁹While we restrict to an economy with “backyard production” on the part of agents, for simplicity, the analysis directly extends to a firm-level production economy.

constraints in the economy with a centralized clearing mechanism become:

$$x_0^i + \sum_j q^j z_+^{ij} - q_-^i(z_-^i)z_-^i = w_0^i - k^i$$

$$x_1^i(s) = \max \left\{ w_1^i(s) + \sum_j R^j z_+^{ij} - R(s)z_-^i + f(s, K^i), (1 - \alpha) w_1^i(s) - \varepsilon z_-^i \right\} \quad (43)$$

and agent i chooses a non-negative portfolio (z_+^i, z_-^i) , s.t. $z_+^{ij}z_-^i = 0$ for any j , as well as a non-negative capital allocation K^i . Budget constraints in the OTC economies are similarly formulated.

It is easy to show that, in the production economy,

1. A centralized clearing mechanism continues to decentralize constrained Pareto efficient allocations; and
2. The generic inefficiency of OTC markets manifests itself as excessive selling of assets that help hedge production risk, which in equilibrium leads to deadweight default costs of providers of these hedges.

An example of this inefficient risk-taking is the possible effect of credit default swaps sold by A.I.G. to a large number of financial firms in the United States and the Europe, effectively insuring tail risk of corporate bond and loan portfolios and mortgage-backed securities. This is tantamount to selling insurance on assets “produced” by the banking sector. Our model implies that in an OTC setting for selling such insurance, the insurer would take on large and inadequately-collateralized selling of protection on underlying assets and eventually default.²⁰

To see this in a transparent manner, we revisit our example economy of Section 2. We suppress agent 2 so that with $w^1(G) = w^2(B)$, there is no aggregate risk in starting endowments. But we suppose that agents 1 have access to a production technology, e.g., making mortgages, that yields per unit of investment $f(G)$ in the good state and $f(B)$ in the bad state, where the investment contains aggregate risk in that $f(G) > f(B)$. We define average cash flow produced per unit of investment as $\bar{f} = pf(G) + (1-p)f(B)$. Then denoting the level of investment as k , and the cost of investment as $c(k)$, where $c'(k) > 0$ and $c''(k) > 0$, agent 1’s maximization problem is:

$$\max_{z_+^1, k} w_0^1 - c(k) - z_+^1 q + p [w^1(G) + f(G)k] + (1-p) [R^+ z_+^1 + f(B)k] - \frac{\gamma}{2} \text{var}(x^1(s)),$$

²⁰If we cleared the market for the produced assets, e.g., housing stock, then our model could potentially generate a (housing) “bubble.”

where

$$\text{var}(x^1(s)) = p(1-p)[w^1(G) + f(G)k - R^+z^1 - f(B)k]^2.$$

Then, for a given level of insurance z^1 , the first-order condition for agent 1's investment decision implies that

$$c'(k) = \bar{f} - \gamma p(1-p)[f(G) - f(B)][w^1(G) + f(G)k - R^+z^1 - f(B)k] .$$

Intuitively, the investment is more attractive the greater is its average return (\bar{f}), the lower is its risk ($p(1-p)[f(G) - f(B)]$), and greater is the extent of insurance and its quality (R^+z^1) as insurance lowers the attendant risk of the investment.

In turn, the demand for the asset insuring state B reflects a hedging motive:

$$z^1 = \frac{1}{R^+} \left[w^1(G) + [f(G) - f(B)]k - \frac{\Delta p}{\gamma p(1-p)} \right] ,$$

where Δp is now the hedging premium in the price of the asset, defined as in Section 2 as $[\frac{q}{R^+} - (1-p)]$.

Now, assuming the investment is indeed net present value, the availability of insurance for the producers is desirable up to an extent. Hedging with such insurance reduces risk of the producer (agent 1) and facilitates greater investment in the economy. However, as is clear from the above maximization problem, agent 1 does not take account of the fact that in case insurance is associated with default of the insurer (agent 3), there may be deadweight costs (ϵz^3 whenever $z^3 > w^3(B)$) to facilitating production in the economy. On the other side, the insurer is unable to commit not to default given the attraction of collecting hedging premium upfront and defaulting on the insurance ex post.

If we let $c(k) = \frac{1}{2}\mu k^2$, then the equilibrium investment is

$$k^* = \frac{\bar{f}}{[\mu + \gamma p(1-p)[f(G) - f(B)]]},$$

and the hedging premium is

$$\Delta p = \frac{\gamma p(1-p)\bar{f}[f(G) - f(B)]}{\mu + \gamma(1-p)[f(G) - f(B)]^2} .$$

Interestingly, equilibrium R^+ satisfies the same expression as in the solution of the example economy in Section 2 (given Δp). And unsurprisingly, we again obtain that there is inefficiency due to counterparty risk externality and deadweight default costs whenever ε is sufficiently small.

As explained in the last remarks of Section 2, default could be restricted (and if it be Pareto efficient, even eliminated) by requiring that insurers suitably (possibly fully) collateralize insurance sold. Doing so would restrict leverage in the economy and ensure productive efficiency in that investments take account of deadweight costs of risk transfers involved in managing the risk of investments. It is clear then that centralized clearing with transparency can in fact achieve the outcome that the planner would produce (even in decentralized markets).

5 Discussion

A central feature of our modeling technology is the strategic nature of default – sell insurance today and default tomorrow – and the ex-post costs associated with such a default. It is useful to interpret this feature in view of the practical settings in which financial firms trade. Again, the example with A.I.G. as the protagonist is useful.

A.I.G. had traders (specifically, at A.I.G. Financial Products) who were engaged in the business of selling insurance through synthetic credit default swaps on portfolios of mortgages and corporate loans. It seems with the benefit of hindsight that their incentive to sell a large quantity of such swaps was (as in the model) to collect premiums upfront and get paid salaries and bonuses based on these premiums. The result was a highly levered bet of A.I.G. on the tail risk of the economy, that is, the likelihood of default of A.I.G. in the case where aggregate risk materialized was rather large. Indeed, A.I.G. as an enterprise itself suffered the substantial costs of the resulting default on these swaps. These costs can be interpreted as the parameter ϵ in the model. Due to limited liability, these costs were not borne by traders who sold the swaps but by the rest of A.I.G.'s businesses (such as life and property insurance) whose franchise value was at least in part being deployed to pay off A.I.G.'s (non bailed-out) positions.

Next, we discuss two issues relating to how our competitive equilibrium could be implemented in actual market settings and the implications of our results for proposed reforms to the OTC markets.

5.1 Implementing centralized clearing

The crucial aspect of centralized clearing, in our economy, is that agents condition the terms of the contracts they trade on the total financial position of the counterparty and not just on bilateral positions. While this is a natural reduced-form trading mechanism in the context of competitive equilibrium modeling, the issue of its implementation in actual financial markets remains open, especially considering that financial positions are contracted sequentially in a dynamic setting.

What is required to implement competitive pricing and centralized clearing is a trading mechanism that allows prices and other contractual terms to adjust continuously with each agent's total position. In fact, such a trading mechanism would look much like a margin or collateral arrangement, a mechanism which is commonplace in financial markets. Such arrangements typically require counterparties to post cash (or treasury) collateral based on mark-to-market valuation of their positions and also based on an overall assessment of counterparty risk (e.g., through a credit rating). While such arrangements are not exactly equivalent to continuously observing each agent's total position and conditioning price on that information, they serve as a way of dynamically responding to knowledge about such information. Further, such arrangements may in fact preclude an agent from positions beyond a certain size due to natural collateral constraints, in effect implementing non-linear pricing schedules - or "position limits" - as also explicitly employed on exchanges.

Trading mechanisms of this kind can be informationally demanding in that they require centralized clearing for all trades to guarantee the necessary transparency. In environments characterized by a stationary behavior of economic agents over time, post-trade transparency - in which trades are done, say during the day, but reconciled and registered with a centralized clearing agency at the end of the day and transparency provided to market participants on these trades thereafter - is enough for efficiency. However, in the absence of such stationarity, our results suggest that even pre-trade transparency may be necessary because in absence of information about trades an agent plans to do, it is not possible to charge the right pricing schedule. More generally, this suggests that agents and financial institutions will in fact have ex post incentives to maintain positions which are not centrally cleared. Indeed, they might have incentives to create OTC "clones" of centrally cleared products. Our analysis implies that if trades outside of a centralized clearing

mechanism were to be allowed, it would leave room for excessive leverage and a counterparty risk externality to be built up through OTC markets.

However, this would only be a literal interpretation of our results. An alternative regulatory solution would be to specify a bankruptcy rule whereby centrally cleared positions have seniority over OTC positions. In that case, OTC positions do not dilute or impose any counterparty risk externality on centrally cleared positions. Thus, subordinating OTC positions relative to centrally cleared ones may suffice to guarantee efficiency, as counterparties would face the appropriate incentives and risk controls with respect to centrally cleared positions with other agents.

Finally, some recent changes in OTC markets, especially in contract terms of standardized credit default swaps (the so-called “Big Bang” protocol laid out in April 2009), require counterparties to exchange a large part of their exchanged risk in pre-funded terms. This effectively amounts to requiring a high upfront or initial margin from the short position, or in other words, to reducing the leverage that can be built through a short position. Such direct leverage restrictions may be desirable if there are limitations to implementing centralized clearing, transparency or central counterparties (though such restrictions may be harmful in terms of limiting risk-sharing if they are designed to be too strict).

5.2 Proposals to reform the OTC markets

We now consider the implications of our analysis for the debates raised by the financial crisis of 2007-09 on the desirability of over-the-counter versus centralized trading. In particular, the study of the role played by OTC markets in the ongoing financial crisis has led to several reform proposals.²¹ Our theoretical analysis can help provide a normative framework for evaluation of these proposals.

For example, Acharya, Engle, Figlewski, Lynch and Subrahmanyam (2009) divide the proposals into requiring a (i) centralized registry with no disclosure to market participants; (ii) centralized clearing with disclosure of aggregate trade information to market participants; and (iii) exchange with full public disclosure of prices and volumes. Our theoretical analysis makes it clear that a centralized registry by itself is *not* sufficient as it only gives regulators

²¹See Stulz (2009) for a summary of the dimensions along which OTC markets for credit derivatives likely contributed, and did not contribute, to the crisis.

ex-post access to trade-level information but does not counteract the ex-ante moral hazard of institutions wanting to take on excessive leverage. Both centralized clearing and exchange improve on this ground but it is transparency that is crucial; in other words, it is sufficient that centralized clearing disseminates trade positions to market participants and they themselves set price schedules and risk controls conditional on that information. In particular, requiring all trades to take place through a centralized exchange is not necessary though in that case there would be no need to disclose information on all trades to individual agents.

Regulatory reforms announced in March 2009, and since then under debate in the United States (and similarly in the U.K. and the Europe), involve significant changes to the trading infrastructure of OTC markets, with the objective of reducing systemic risk in the financial sector. Under the proposed reforms, mature and standardized credit derivatives such as the single-name and index credit default swaps (CDS) will be traded through a centralized counterparty; there is no proposal yet to mandate that these be traded on an exchange. Regulators will gain unfettered access to information on prices, volumes and exposures from the centralized counterparties, but the proposals do not require that such information be made public.²² While some aggregate information will be disseminated to all market participants, such as the recent data published by the Depository Trust and Clearing Corporation (DTCC) on all live positions in credit derivatives, full transparency is being required only for regulatory usage.

Our results suggest that these proposed changes are unlikely to be fully adequate, though they take some steps in the right direction. Importantly, not all products are being required, or are in fact amenable, to trade on exchanges. In particular, many financial products such as the customized or “bespoke” collateralized debt and loan obligations (CDOs and CLOs) will remain OTC. Our results suggest that these products should also be subject to trade-level transparency among market participants. And if such transparency requirements should be seen as too restrictive, perhaps because they would limit generation of proprietary information which is costly to acquire, then our analysis suggests a possibly crucial role for a seniority rule

²²In fact, there are arguments linked to political economy for limiting rather than extending the information to which regulators have access to, for the fear that such information may be exploited eventually for political goals, e.g., the pursuit of inefficient fiscal policies achieved through regulatory forbearance towards the financial sector; see Bisn and Rampini (2006b).

in bankruptcy in favor of centrally cleared positions over OTC positions. This will be necessary to limit the counterparty risk externality of opaque OTC positions.

It should be noted that while we have focused on centralized clearing and exchange as mechanisms to eliminate the counterparty risk externality, another possible remedy is to directly address the issue of default moral hazard. This would require the regulator to increase the bankruptcy costs suffered by defaulting financial firms, e.g., by imposing state-contingent penalties. To the extent that such penalties are restricted by limited liability, our analysis highlights that improving trading infrastructure of markets can serve as an important part of regulatory design to contain systemic risk and leverage.

6 Related Literature

The bilateral nature of contracts in the OTC markets has been stressed in the recent literature on the subject. Duffie, Garleanu and Pedersen (2005, 2007) focus on search frictions, dynamic bargaining and valuation in OTC markets; Caballero and Simsek (2009) analyze the role of complexity introduced by bilateral connections and their role in causing financial panics and crises; and, Golosov, Lorenzoni, and Tsyvinski (2009) examine what kind of bilateral contracts will get formed when agents have private information about their endowment shocks.

In the context of insurance provision through financial contracts, the literature (e.g., Duffee and Zhou, 2001, Acharya and Johnson, 2007, Parlour and Winton, 2008) has largely focused on moral hazard on part of the insured due to the presence of information frictions, rather than moral hazard on part of the insurer, the latter being the focus of our paper. On this issue, our paper is more closely related to Allen and Carletti (2006), Thompson (2009) and Zawadowski (2009).

Allen and Carletti (2006) consider contagion from the insurance sector to the financial sector when there is credit risk transfer, but they do not consider agency-theoretic issues. In contrast, we allow for default incentives of the insurer and model credit risk transfer more generally as risk-sharing through financial contracts in a GE setting. Zawadowski (2009) analyzes counterparty risk in entangled financial systems. The system is “entangled” because banks hedge risks using bilateral OTC contracts but do not internalize the cost of their own failure on other banks (through counterparty risk exposures). As a

result of this network externality, banks purchase less insurance against low probability events. Thus, there is less insurance in his model whereas due to moral hazard on part of the insurer, there is in fact excessive insurance in our setup but it is of low quality and entails default by the insurer. Thompson (2009) considers the moral hazard of default on part of the insurer when there is credit risk transfer in the financial sector. His focus is on analyzing how this moral hazard provides incentives to the insured parties to reveal information about their type, so that the two agency problems interact and reduce each others' adversity.

More specifically on the benefits of OTC versus centralized markets, our analysis did not consider practical issues relating to the *extent* of netting of positions that is possible under different market structures. Duffie and Zhu (2009) explain that for a centralized exchange for credit default swaps to reduce counterparty risk more than in the OTC setting, it would require netting not just across CDS but also across other products such as interest-rate swaps. In our model, the primary role of the centralized clearing mechanism or the exchange is not necessarily to reduce or eliminate counterparty risk but to improve its price by aggregating information on trades. We conjecture that if there were a centralized registry of positions (centralized or OTC) that is observed by different clearing platforms or exchanges and disseminated to market participants, then the pricing (or collateral) arrangements would be efficient *ex ante*, and so would be the levels of *ex-post* default risk. This is related to Leitner (2009)'s result that a clearinghouse-style mechanism, allowing each party to declare its trades and revealing publicly those that hit pre-specified position limits, can prevent agents from promising the same asset to multiple counterparties and then defaulting.

From a pedagogical standpoint our paper studies competitive equilibria of economies with moral hazard, where the moral hazard is induced by the agents' default decisions. In particular, in the terminology of this literature, we compare competitive equilibria in *exclusive contractual environments* (in the case of economies with a centralized clearing mechanism or with a centralized exchange) with competitive equilibria in *non-exclusive contractual environments* (in the case of economies with OTC markets, with and without netting). Exclusive contractual environments in this literature are by definition those in which one party in a contractual relationship can constrain all of the counterparty's trades with third parties. Therefore, in exclusive contractual environments counterparty risks play no role, as in our economies with a centralized clearing mechanism or with a centralized exchange. In

non-exclusive contractual environments, on the contrary, agents cannot be restricted from engaging in multilateral contractual obligations which are not observable by the counterparties, as in the case of OTC markets. The distinction between exclusive and non-exclusive contracts is central in the theory of competitive economies with moral hazard; see e.g., Bisin and Gottardi (1999) and Bisin, Geanakoplos, Gottardi, Minelli and Polemarchakis (2001).²³ In finance, several papers have exploited the distinction between exclusive and non-exclusive markets in different contexts: e.g., Bizer and DeMarzo (1992) in a sequential model of banking, Parlour and Rajan (2001) in a model of credit card loans, and Bisin and Rampini (2006a) in a model of bankruptcy.

7 Conclusion

We formalized an important market failure arising due to opacity of over-the-counter (OTC) markets, in particular that the payoff on each position depends in default on other positions sold by the defaulting party, but there is no way for market participants to condition their trades or prices based on knowledge of these other positions. We showed that this counterparty risk externality can lead to excessive default and production of aggregate risk, and more generally, inefficient risk-sharing. Centralized clearing, by enabling transparency of trades, and exchanges, by creating a centralized counterparty to all trades, can help agents fully internalize the counterparty risk externality. Our model provides one explanation for the substantial buildup of OTC positions in credit default swaps in the period leading up to the crisis of 2007-09, their likely contribution to over-extension of credit in the economy, and possible remedies for avoiding this excess in future.

Our analysis focused on competitive markets. In the future, we plan to consider bilateral OTC markets in the presence of a “large” individual agent that effectively observes the trades of all others but whose trades are not seen by others. Such an agent would enjoy monopoly rents in the OTC setting, which in turn would reduce private incentives in the economy to coordinate on a centralized trading platform and achieve Pareto improvement. Furthermore, our model suggests that excessive leverage and excessive production arising due to the OTC nature of trading can lead to a “bubble” in the market for goods (e.g., the housing stock), a subsequent crash upon realiza-

²³In the context of principal agent models, see the early work of Arnott and Stiglitz (1993) and Hellwig (1983).

tion of adverse shocks, and a breakdown of risk transfer (credit or insurance markets) in those states. Our analysis also suggests that the possibility of regulatory forbearance of “too big to fail” positions can result in ex-ante inefficiencies even with centralized exchange trading. These phenomena are worthy of detailed modeling and study in extensions of our basic setup.

Finally, we focused on symmetric information about states of the world in our analysis. However, the model can be consider adverse selection, e.g., in the form of unobservable probability distributions over S , the uncertain state at time 1. The model would combine features of Rothschild and Stiglitz (1976) and Akerlof (1970), with separating equilibria in the economy with centralized exchanges, and excessive *lemons* trading (in the form of risky short positions) in the case of OTC markets.²⁴ It remains an important exercise for the future to confirm that the inefficiency of OTC markets relative to centralized markets persists in this setting. Indeed, we conjecture that the inefficiency of OTC markets will be exacerbated in a setting with adverse selection.

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²⁴Santos and Scheinkman (2001) have adverse selection as well in their model of competition of exchanges.

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Appendix

Proof of Proposition 1. The proof proceeds by contradiction. Let $(x_0^i, x_1^i, z_+^{ij}, z_-^i)_{i,j}$ denote an equilibrium of the economy with centralized clearing mechanism and let $(\widehat{x}_0^i, \widehat{x}_1^i, \widehat{z}_+^{ij}, \widehat{z}_-^i)_{i,j}$ denote a constrained Pareto optimal allocation which dominates the equilibrium allocation. Both allocations satisfy netting, for any i and j :

$$z_+^{ij} z_-^i = 0, \quad \widehat{z}_+^{ij} \widehat{z}_-^i = 0.$$

Recall that $z^{ii} = 0$, for any i , by construction. The allocation $(\widehat{x}_0^i, \widehat{x}_1^i, \widehat{z}_+^{ij}, \widehat{z}_-^i)_{i,j}$ must not have been budget feasible at equilibrium prices. That is,

$$\begin{aligned} \widehat{x}_0^i + \sum_j q^j \widehat{z}_+^{ij} - q_-^i(\widehat{z}_-^i) \widehat{z}_-^i - w_0^i &\geq \\ x_0^i + \sum_j q^j z_+^{ij} - q_-^i(z_-^i) z_-^i - w_0^i &< \end{aligned}$$

with $>$ for at least one agent i . Summing over i :

$$\sum_i (\widehat{x}_0^i - x_0^i) + \sum_i \left(\sum_j q^j \widehat{z}_+^{ij} - q_-^i(\widehat{z}_-^i) \widehat{z}_-^i \right) - \sum_i \left(\sum_j q^j z_+^{ij} - q_-^i(z_-^i) z_-^i \right) > 0$$

But, since market clearing must hold for both $(x_0^i, x_1^i, z_+^i, z_-^i)$ and $(\widehat{x}_0^i, \widehat{x}_1^i, \widehat{z}_+^i, \widehat{z}_-^i)$, $\sum_i (\widehat{x}_0^i - x_0^i) = 0$, and we obtain:

$$\sum_i \left(\sum_j q^j \widehat{z}_+^{ij} - q_-^i(\widehat{z}_-^i) \widehat{z}_-^i \right) - \sum_i \left(\sum_j q^j z_+^{ij} - q_-^i(z_-^i) z_-^i \right) > 0.$$

Furthermore, at equilibrium, $q^i = q_-^i(z_-^i)$. Hence,

$$\sum_j q_-^j(\widehat{z}_-^j) \sum_i (\widehat{z}_+^{ij} - \widehat{z}_-^i) - \sum_j q_-^j(z_-^j) \sum_i (z_+^{ij} - z_-^i) > 0.$$

But market clearing at equilibrium implies $\sum_i (z_+^{ij} - z_-^i) = 0$, for any j , and hence $\sum_j q_-^j(z_-^j) \sum_i (z_+^{ij} - z_-^i) = 0$ for non-negative prices $q_-^j(z_-^j)$. Therefore,

$$\sum_j q_-^j(\widehat{z}_-^j) \sum_i (\widehat{z}_+^{ij} - \widehat{z}_-^i) > 0,$$

a contradiction with feasibility of $(\widehat{x}_0^i, \widehat{x}_1^i, \widehat{z}_+^{ij}, \widehat{z}_-^i)_{i,j}$. ■

Proof of Proposition 2. We only prove the statement regarding economies with OTC markets and netting. The proof in the case of OTC markets without netting only requires straightforward modifications. We restrict attention to the case when ε is small so that there is default in the economy. Furthermore, assume to start with that $\varepsilon = 0$. Let $(z_+^{ij}, z_-^i)_j$ be the equilibrium portfolio for agent i in an economy with centralized clearing mechanism. Let $S(i) \subseteq S$ denote the subset of the states of uncertainty in which, at equilibrium, an agent i will default. Then, $S(i)$ is robustly non-empty. Furthermore, if $S(i)$ is non-empty, then $z_-^i > 0$. For any economy such that $S(i)$ is non-empty (and $z_-^i > 0$) for some i , at equilibrium of the centralized clearing mechanism, we must have

$$q_-^i(z_-^i) = \sum_{s \in S(i)} p_s m^i(s) \frac{\alpha w^i(s)}{z_-^i} + \sum_{s \in S \setminus S(i)} p_s m^i(s) R(s).$$

Suppose, by contradiction, that such a competitive equilibrium of the centralized exchange economy can be supported in an economy with OTC markets and netting. Then it is necessarily supported by price $q^i = \sum_{s \in S(i)} p_s m^i(s) \frac{\alpha w^i(s)}{z_-^i} + \sum_{s \in S \setminus S(i)} p_s m^i(s) R(s)$, such that $q^i = q_-^i(z_-^i)$ at the equilibrium portfolio $(z_+^{ij}, z_-^i)_j$.

It is straightforward to see that in this case, at price q^i agent i prefers a portfolio $(z_+^{ij}, z_-^i + dz)$, for some $dz > 0$. This is because the marginal valuation of the discounted repayment of a unitary extra short portfolio dz , $\sum_{s \in S(i)} p_s m^i(s) \frac{\alpha w^i(s)}{z_-^i} + \sum_{s \in S \setminus S(i)} p_s m^i(s) R(s)$, depends negatively on z_-^i ; while the price obtained at time 0 from the same unitary extra short portfolio, dz , q^i , does not. Since the portfolio $(z_+^{ij}, z_-^i + dz)$ is budget feasible, a contradiction is reached. This is the case for any equilibrium of the centralized exchange economy such that $S(i)$ is non-empty, for some i , and hence the contradiction holds robustly.

The proof extends by continuity to ε sufficiently small. ■

Proof of Proposition 3. Once again, we only prove the statement regarding economies with OTC markets and netting. The proof in the case of OTC markets without netting only requires straightforward modifications. Once again, assume $\varepsilon = 0$ and the proof below extends by continuity to ε sufficiently small. Finally, the “weakly greater” part of the statement is straightforward. We turn to prove the robustly “strictly greater” part.

Consider the robust subset of economies for which, with centralized clearing at equilibrium, $S(i)$ is non-empty. An argument analogous to the one in the proof of Proposition 2 guarantees that, for these economies, when ε is small enough, at an equilibrium of the economy with OTC markets and netting, $S(i) = S$. Agents i , in other words, default in all states $s \in S$. This proves that default is robustly strictly greater at equilibria of the economy with OTC markets and netting than with centralized clearing.

Consider such an equilibrium with OTC markets and netting, to study leverage. Consider now the general case in which $\varepsilon > 0$. At equilibrium it must be that $q^i > 0$. Suppose on the contrary that $q^i \leq 0$. In this case, we claim agents i would rather choose $z_-^i = 0$ and hence would trivially not default. In fact, if $S(i) = S$, and $q^i \leq 0$, agents i would consume

$$\begin{aligned} x_0^i &= w_0^i + q^i z_-^i \\ x_1^i(s) &= (1 - \alpha) w_1^i(s) - \varepsilon z_-^i, \end{aligned}$$

(recall that, because of netting, $\sum_j q^j z_+^{ij} = 0$). But then

$$\begin{aligned} x_0^i &\leq w_0^i + q^i z_-^i \leq w_0^i \\ x_1^i(s) &= (1 - \alpha) w_1^i(s) - \varepsilon z_-^i \end{aligned}$$

By resorting to autarchy, $z_-^i = z_+^{ij} = 0$, instead agents i would guarantee themselves

$$\begin{aligned} x_0^i &= w_0^i \\ x_1^i(s) &= w_1^i(s) \end{aligned}$$

which they prefer. Prices such that $q^i \leq 0$ therefore imply no default. This is the case for all agents of all types i . But then $R_+(s) = R(s)$, for all $s \in S$ and z_+^{ji} is robustly > 0 , for some j , a contradiction with market clearing. At an equilibrium of the economy with OTC markets and netting, therefore, it must be that $q^i > 0$. In this case z_-^i grows unbounded as $\varepsilon \rightarrow 0$. This proves that leverage is robustly strictly greater in the economy with OTC markets and netting than with centralized exchange for ε small enough. ■

Figure 1: The quantity of insurance sold (z^3) as a function of the deadweight cost of default (ϵ)

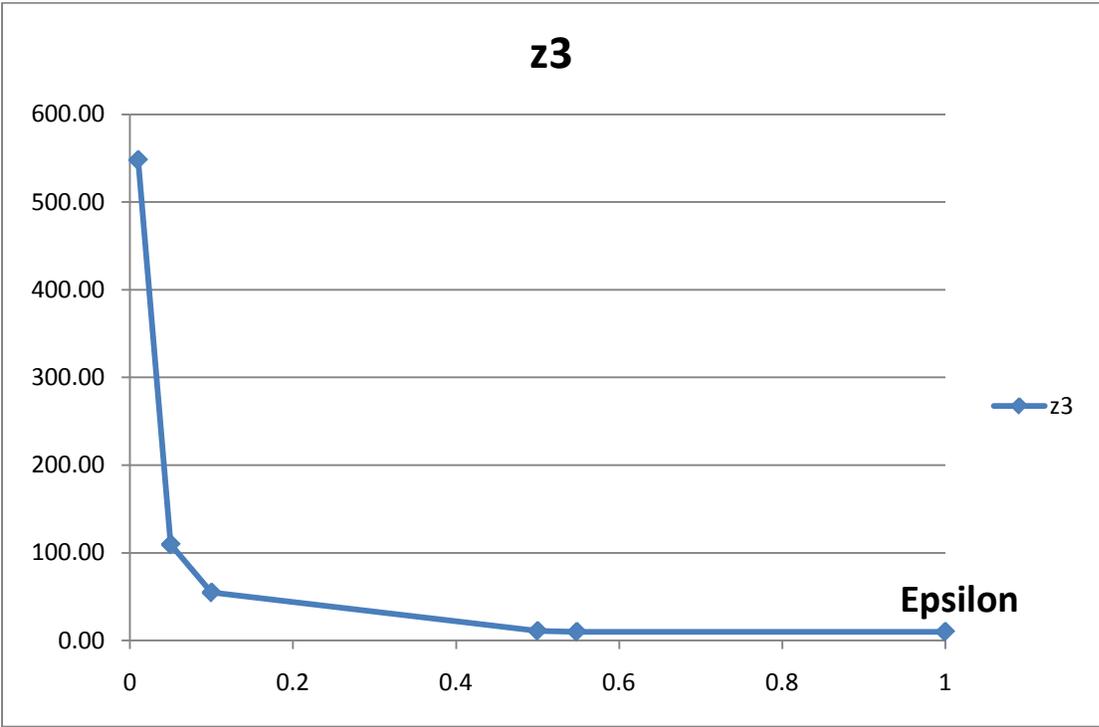


Figure 2: The realized payoff on the insurance (R^+) as a function of the deadweight cost of default (ϵ)

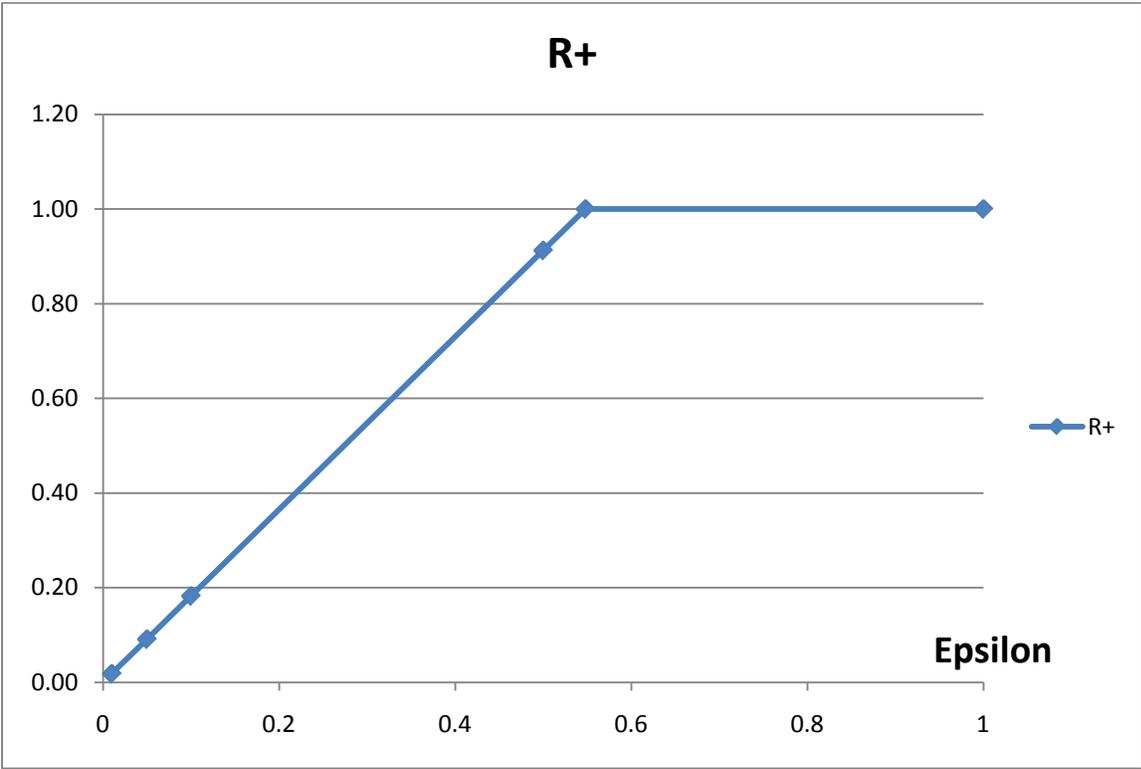


Figure 3: The equilibrium price of insurance (q) as a function of the deadweight cost of default (ϵ)

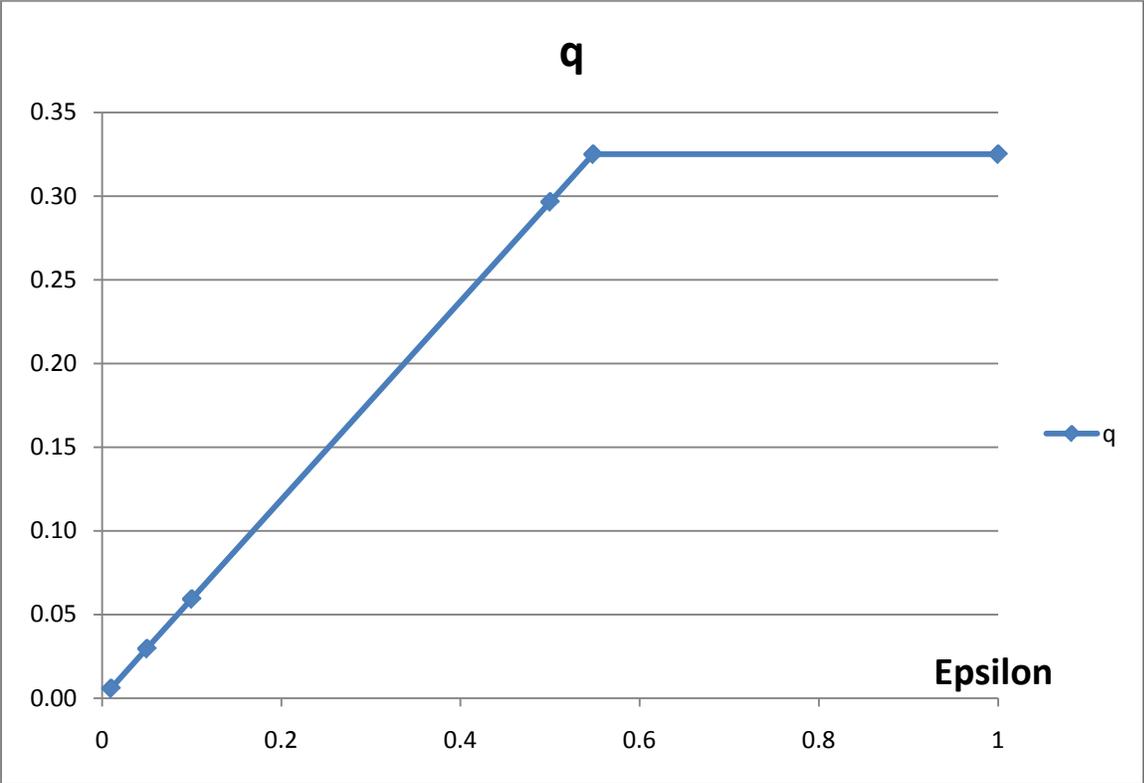


Figure 4: The equilibrium utilities as a function of the deadweight cost of default (ϵ)

