Financial Constraints and the Franchising Decision

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Abstract

We develop a theory relating the financial constraints of potential franchisees to a franchisor’s decision on a corporate outlet versus a franchised outlet when an opportunity to open an outlet arrives. The model predicts that the probability of opening a franchised outlet conditional on opening any outlet (franchisor or franchisee owned) should increase when franchisee collateralizable wealth is greater. Based on this theory, we set up a dynamic empirical model where a chain decides on whether to pay a sunk cost to start franchising at the beginning of each year. It also decides on what to do when an opportunity to open an outlet arrives during a year. Given an opportunity, a chain can set up a company owned outlet or pass up the opportunity. It can also open a franchised outlet if it has paid the sunk cost already. We estimate the determinants of the sunk cost and the profitability of a company owner outlet and a franchised outlet. In particular, we examine how changes in local collateralizable housing value, which we use to proxy for changes in franchisee wealth, affect chains’ decisions. The model is estimated using data on more than 2000 franchisors in the U.S. To estimate the model, we also develop computational approaches to deal with the fact that we observe net rather than gross yearly changes in number of outlets in the data.

1 Introduction

The recent collapse of debt financing has also greatly affected the franchising industry. In fact, the issue of franchisees obtaining financing to invest in a franchise outlet has always been a major concern for participants in the industry. For example, being highly leveraged may considerably undermine the incentives for franchisees that are at the core of the franchising idea. This is even explicitly acknowledged in many franchise chains where the franchisor puts explicit lower bounds on the capital that a franchisee must have. At the same time, the inability of franchisees to raise their own capital may also considerably constrain the ability of franchise chains to expand in times of tight credit. Despite the central nature of financing arrangements in franchising, this issue has not been studied either in the theoretical or empirical literature.

In this paper, we develop a simple theoretical model with financially constrained franchisees where franchisee effort and the profitability of franchised outlets will depend on how much collateral a franchisee is able

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to put up. If the franchisor is not able to find a potential franchisee with sufficient collateralizable assets so that the franchisee will exert more effort than a manager, a company owned outlet will generate higher profits. Based on this theory, we can test the hypothesis that financial constraints have a significant impact on the franchising decision on the basis of macroeconomic data.

To this end, we set up an empirical model of entry and expansion into franchising that explicitly takes into account financial constraints of potential franchisees in the franchising decision. In the model, chain stores are randomly and continuously presented with opportunities to expand by establishing new outlets. Every period these chains have to decide whether to pay a sunk cost to become a franchise chain and thus create the possibility of opening either franchisee owned or company owned outlets if they take advantage of the opportunity.

We estimate the determinants of the sunk cost of entry into franchising and the determinants of the relative profitability a company owned outlet and a franchised outlet. In particular, we are interested in examining how financial constraints influence a chain’s decision. We use “Gross Equity Extraction”, which is obtained from Greenspan and Kennedy (2007) and adjusted for local conditions using a local housing price index, as a measure of average collateralizable housing wealth. If this measure declines, it is harder to find a franchisee with sufficient collateralizable wealth so that it is optimal to open a franchised outlet. In this sense, the probability of opening such an outlet conditional on a marginal expansion should go down if financial constraints are important for such decisions.

The empirical model is estimated using data on more than 2000 chain stores in the U.S. In our data, we observe only the net change in the number company owned and franchisee owned outlets, which can be positive or negative. We therefore expand the standard count data approach to estimate both rates at which new outlets are opened and existing outlets are closed. This extension leads to computational challenges as a net change can be consistent with a large number of combinations of new outlets and exits. This combinatorial problem is exacerbated when the data is not on a yearly basis and there is a gap two consecutive observations. We develop an algorithm to deal with this complication.

While our results are of interest for the study of entry into and growth of franchising, they are also of interest to the general literature on the relevance of financial constraints for entrepreneurship. There is a substantial macroeconomic literature that has attempted to identify the presence of financial constraints from the decision of entry into self-employment. However, there is an important issue of endogeneity in this literature. It is not clear whether a positive correlation between self-employment and financial constraints is explained by the hypothesis that self-employed individuals are less financially constrained or they are more likely to accumulate wealth. Our paper avoids such problems because it makes inferences not from a franchisee’s decision itself, but from the fact that the marginal choice between opening a company owned outlet and a franchised outlet by the franchisor should depend on the collateralizable wealth distribution in a region if financial constraints matter and not otherwise. We are therefore providing a novel way of testing for the role of financial constraints for the transition to self-employment.

The rest of the paper is organized as follows. Section 2 presents the theoretical model that relates financial constraints to a franchisor’s decisions. Section 3 presents the empirical model for estimation. Section 4 describes the data. The estimation results will be presented in Section 5. Finally, Section 6 concludes.
2 A Theory of Franchising with Financially Constrained Franchisees

In our empirical mode, a franchisor (or “chain”)\(^1\) decides on when to enter franchising and whether to open a company owned outlet or a franchised outlet or pass on an opportunity to open an outlet. In this section, we develop a theoretical model explaining how financial constraints of potential franchisees affect a franchisor’s decision regarding an opportunity (i.e. the second decision), which influences the decision on entry (i.e. the first decision) because it determines the profitability of an opportunity and hence the value of entry.

Suppose that revenue for a specific chain outlet is given by \(\theta + e\). The variable \(\theta\) captures both the quality of the idea of the chain, the local conditions for that specific opportunity and a profit shock. It is random and drawn from some distribution \(F(\theta)\) with decreasing hazard rate \(\frac{f(\theta)}{F(\theta)}\). Let \(e\) be the effort level of the manager of the outlet. We assume that a minimum level of effort \(e_0\) can be induced even for an employed manager. This could be thought of as an observable component of effort or a minimum standard that can be monitored at low cost. The cost of effort is given by the cost function \(\Psi(e)\), which is increasing and strictly convex with \(\lim_{e \to \infty} \Psi'(e) = \infty\) and \(\Psi(e) > 0\) for any \(e > 0\).

The startup of an outlet requires capital of \(I\). This capital can either come from internal funds or borrowed from a bank in form of a debt contract. Banks compete a la Bertrand in the terms of the contract, \((R, C)\), where \(R\) denotes the repayment and \(C\) is the collateral that the bank receives in case a debtor defaults on the payment of \(R\). For simplicity of exposition we assume that the bank does not receive any other payment beyond \(C\) in the case of default.\(^2\) There are at least two banks.

Consider first a managerial outlet, i.e. a company owned outlet. Suppose an employed manager can only be paid a fixed wage so that it is impossible to induce a higher effort level than \(e_0\) for a company owned outlet. We normalize the outside option of an employed manager to zero, so that the wage would be set at \(\Psi(e_0)\). Then, this outlet makes positive profits if and only if \(E\{\theta\} + e_0 - \Psi(e_0) > I\). Since Bertrand competition drives the cost of the investment down to \(I\) and there is no incentive issue, a company owned outlet is profitable to open whenever it is efficient to open it. Therefore, there is no difference in this case whether there is internal or external financing.

Now consider a franchised outlet. We assume that \(1 > \Psi'(e_0)\), so that there is value to inducing more effort than in a managerial unit, so that opening a franchisee owned outlet would generally be efficient. The potential franchisee has collateralizable wealth \(W\) which we assume to be mostly housing, so that \(C \leq W\). We assume that this wealth is generally not sufficient to fully collateralize the debt so that a bank will face default risk. The franchisee and the banks sign a contract specifying the repayment and the collateral. Based on the contract, the franchisee decides on the amount of effort exerted. For simplicity we assume that there is no discounting. Let \(s\) be the revenue share of the franchisor, which is fixed at the time of entry into franchising. The franchisee will default if and only if what she earns from the outlet is smaller than the

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1Throughout the paper, we use “franchisor” and “chain” interchangeably because only chains started franchising by the last year of our data is in our sample and studied in this paper.

2Note that this is not quite as in standard debt contracts because the bank does not obtain the residual revenue in the case of default. We assume that the franchisee can walk away with these revenues before the bank can get hold on these assets. (We discuss the effect that the alternative assumption has in an Appendix to be added to the paper).
repayment, i.e.

$$(1 - s) (\theta + e) < R$$  \hspace{1cm} (1)$$

The critical state of the world below which default occurs is $\theta^* = \frac{R}{1-s} - e$. Under a standard debt contract with collateral, the payoff of the franchisee can be written as:

$$\int_{-\infty}^{\infty} [(1 - s) (\theta + e) - R] dF(\theta) + F(\theta^*) [R - C] - \Psi(e)$$  \hspace{1cm} (2)$$

For any given $R$ and $s$, the franchisee maximizes (2) with respect to $e$. It turns out that for equilibrium analysis it is easier to use the change of variable $e = \frac{R}{1-s} - \theta^*$ and write the franchisee’s maximization problem equivalently as:

$$\pi^*(s, C) \equiv \max_{\theta^*} \int_{-\infty}^{\infty} [(1 - s) (\theta - \theta^*)] dF(\theta) + F(\theta^*) [R - C] - \Psi\left(\frac{R}{1-s} - \theta^*\right)$$  \hspace{1cm} (3)$$

The franchisee earns the surplus above the critical $\theta^*$ in all of the states. With probability $F(\theta^*)$ he is bankrupt and does not have to pay $R$ but has to give up the collateral $C$. This maximization problem yields the first order condition:

$$- (1 - s) + f(\theta^*) [R - C] + \Psi'(\frac{R}{1-s} - \theta^*) = 0$$  \hspace{1cm} (4)$$

Since $f(\theta^*) + \Psi'(\frac{R}{1-s} - \theta^*)/(1-s) > 0$, standard comparative statics results imply that the schedule $\theta^*_F(R)$ that is defined by (4) is strictly upward sloping and continuous up to, possibly, a number of upward jumps. Intuitively, the higher $R$ the greater the likelihood that the franchisee goes bankrupt.

Bertrand competition implies that banks have to just break even in equilibrium:

$$R - F(\theta^*) [R - C] = I$$  \hspace{1cm} (5)$$

Since $C \leq W < I$, it follows that $R > C$. Note that by the implicit function theorem (5) generates a continuous and monotonically increasing schedule $R_B(\theta^*)$ for any $C$. This simply reflects that the higher the probability of default, the more adverse the terms of the contract need to be for given collateral for a bank to break even.

**Proposition 1** There exists a unique equilibrium of the game. Either there is no contracting in equilibrium or the equilibrium contract has $C = W$.

**Proof.** Fix $C$ at its equilibrium value. Now note that the break even constraint for the bank can only be satisfied for $R > I$. However, for $\theta^*$ small enough, (4) can only be satisfied if $R < 0$. Hence, there exists some $\hat{\theta} > -\infty$ such that for all $\theta^* < \hat{\theta}$ the (4) schedule is strictly below the (5) schedule, i.e. $R_F(\theta^*) = \theta^*_F^{-1}(R) < R_B(\theta^*)$ for all $\theta^* < \hat{\theta}$ where an inverse exists. We now show that $\theta^{**} = \sup \hat{\theta}$ is the unique equilibrium critical default state. By the fact that $R_F(\theta^*) = \theta^*_F^{-1}(R) < R_B(\theta^*)$ and that both schedules are strictly increasing it follows that the lowest $\theta^*$ at which the schedules intersect is $\theta^{**}$. Furthermore $R_B(\theta^*)$ intersects $R_F(\theta^*)$ from above. Hence there exists some $\theta^+ > \theta^{**}$ such that $R_F(\theta^*) - F(\theta^*) [R_F(\theta^*) - C] > I$ for all $\theta^* \in (\theta^{**}, \theta^+)$. Suppose there would exist an equilibrium with $R > R_F(\theta^{**})$. Then a bank would make zero profits in that equilibrium. However, it could reduce the repayment to $R_F(\theta^{**}) + \varepsilon$ and make strictly positive profits, a contradiction.
To complete the proof we need to show that $C = W$. Note that the schedule $R_B(\theta^*)$ shifts down as $C$ is increased and the schedule $R_F(\theta^*)$ shifts up as well. Both shifts lead to a lower intersection for both $\theta^{**}$ and $R$, so that $\theta^{**}(C)$ and $R(C)$ are both strictly decreasing in $C$. Now note that from (5)

$$\frac{dR}{dC} = \frac{d}{dC} \{F(\theta^*) [R - C]\}$$

Rewriting (4) as

$$(1 - s) - \frac{f (\theta^{**}(C))}{F(\theta^{**}(C))} F(\theta^*) [R(C) - C] - \Psi'(e(C)) = 0$$

it follows that

$$\frac{d}{d\theta^*} \frac{f (\theta^{**}(C))}{d\theta^*} F(\theta^*) [R(C) - C] - \Psi''(e(C)) \frac{de}{dC} = 0$$

Now note that the first term is positive by the assumption on the hazard rate and the result that $\frac{d\theta^*}{dC} < 0$. The second term is positive by (6) and the fact that $\frac{dR}{dC} < 0$. Since $\Psi''(e) > 0$ it follows that $\frac{de}{dC} > 0$. But then

$$\frac{\partial \pi^*}{dC} = [(1 - s) - \Psi'(e)] \frac{de}{dC} > 0$$

where the last inequality follows from (4).

¿From this analysis almost directly the comparative statics in a number of variables:

**Proposition 2** The effort and profits of the franchisee strictly increase with an increase in her wealth $W$, her revenue share $(1 - s)$, a first order dominant shift in the distribution of $\theta$, and a decrease in the capital requirement to open an outlet $I$.

**Proof.** The first part of the proof follows from the second half of the proof of proposition 1. An increase in $(1 - s)$ directly increases the incentive for effort and indirectly increases it by reducing $R$, which will increase effort further. A first order dominant shift in the distribution of $\theta$ directly decreases the probability of default and through this raises effort and reduces $R$, which raises effort further. A decrease in $I$, reduces $R$ and through this effect increases effort.

Combining these results we get:

**Proposition 3** For every triple $(F(\cdot), (1 - s), I)$, there exists a unique $\bar{W}$, such that the franchisor makes strictly greater profits opening a franchisee owned outlet than opening a company owned outlet if and only if $W > \bar{W}$.

**Proof.** Omitted.

For empirical purposes we do not observe the wealth constraints of the particular potential franchisee that the franchisor will have access to for a specific opportunity of opening an outlet. In fact, we only observe aggregate data on collateralizable wealth. If a shock in a given region generally reduces housing wealth and thus collateralizable wealth, the franchisor should be less likely to find a franchisee with sufficient collateralizable wealth to make it worthwhile to open up a franchisee owned outlet. While we will not observe the collateral available for any given outlet opening, we know that if the distribution is shifted we should see an impact on the conditional likelihood of a new outlet being franchisee owned. This is the main prediction that comes from our model.
3 The Empirical Model

A chain makes two decisions in the model. First, at the beginning of each year, it decides on whether to pay a sunk cost to start franchising or not. Second, during a year, it decides on what to do when an opportunity to open an outlet arrives. For example, an opportunity can be when a store in a mall goes out of business and makes the site available. Given an opportunity, a chain can set up a company owned outlet or pass up the opportunity. It can also open a franchised outlet if it has paid the sunk cost already. Since a chain has different choices before and after it pays the sunk cost to start franchising, we divide the description of the model into the “pre-entry” decision and the “post-entry” decision. Here “entry” refers to entering the franchising market.

3.1 Post-entry Decision

Suppose the opportunity arrives continuously and exogenously. We use $t$ to represent a year and $\tau$ to indicate the continuous opportunity arrival time. When an opportunity to open an outlet arrives after chain $i$ pays the sunk cost to start franchising, it can open a company owned outlet, a franchised outlet or pass on the opportunity. The relative attractiveness of these choices depends on the characteristics of the chain, such as its age and industry, the local conditions of the site of the opportunity, such as the tightness of credit constraints. Since we do not have data on the locations of outlets and most chains start expansion from their home state, we use the information from the home state of a chain as the local conditions. Specifically, let $x_{it}$ be a vector of the characteristics of chain $i$ and the local conditions of the home state of chain $i$. Let the profit of a company owned outlet and that of a franchised outlet at time $\tau$ in year $t$ be

$$\pi_o(x_{it}, \epsilon_{o\tau}; \beta_o) = x_{it}\beta_o + \epsilon_{o\tau}$$
$$\pi_f(x_{it}, \epsilon_{f\tau}; \beta_f) = x_{it}\beta_f + \epsilon_{f\tau},$$

where the error terms $\epsilon_{o\tau}$ and $\epsilon_{f\tau}$ capture the unobservable factors that affect the profitability of an outlet, and $(\beta_o, \beta_f)$ are parameters to be estimated. We assume that $\epsilon_{o\tau} = \epsilon_{o\tau} - \epsilon_{0\tau}$ and $\epsilon_{f\tau} = \epsilon_{f\tau} - \epsilon_{0\tau}$, and that $(\epsilon_{o\tau}, \epsilon_{f\tau}, \epsilon_{0\tau})$ are drawn i.i.d. from a type-1 extreme value distribution. Therefore, the probability that chain $i$ opens a company owned outlet conditional on the arrival of an opportunity is

$$p_{ao}(x_{it}) = \frac{\exp(x_{it}\beta_o)}{\exp(x_{it}\beta_f) + \exp(x_{it}\beta_o) + 1},$$

where the subscript $a$ stands for “after” (after entering into the franchising market) and the subscript $o$ represents “company owned”. Similarly, the conditional probability of opening a franchised outlet is

$$p_{af}(x_{it}) = \frac{\exp(x_{it}\beta_f)}{\exp(x_{it}\beta_f) + \exp(x_{it}\beta_o) + 1}.$$  

We assume that the opportunity arrival process follows a Poisson process with rate $\lambda$ and is independent across chains. Therefore, the processes for the appearance of a new franchised outlet and a company owned outlets after entry are Poisson processes with parameter $\lambda p_{ao}(x_{it})$ and $\lambda p(x_{it})$, respectively.

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3 As a result, our model cannot describe the decisions of large international chains, which are excluded from the study.
3.2 Pre-entry Decision

At the beginning of each year, chain \( i \) decides on whether to enter the franchising market or not. If it decides to enter, it has to pay a sunk cost to start franchising, such as the consulting fee for setting up the franchising contract. This decision therefore depends on the value of entry subtracted by the setup cost and the value of not entering.

The value of entry is the expected present value of the future profit flow after entry. It obviously depends on characteristics and local conditions \( x_{it} \). We assume that these variables follow a Markov process. Therefore, the value of entry \( VE(x_{it}) \) satisfies

\[
VE(x_{it}) = \lambda E_{(ε_{oit}, ε_{ftr})} \max \{ \pi_o(x_{it}, ε_{oit}; β_o), \pi_f(x_{it}, ε_{ftr}; β_f), 0 \} \\
+ \delta E_{x_{it+1}|x_{it}} VE(x_{it+1}),
\]

where the first summand is the expected profit in the current year \( t \) and \( \delta \) is the discount factor. Note that the expected profit of an opportunity is \( E_{(ε_{oit}, ε_{ftr})} \max \{ \pi_o(x_{it}, ε_{oit}; β_o), \pi_f(x_{it}, ε_{ftr}; β_f), 0 \} \) and the expected number of opportunities in a year is \( \lambda \).

If chain \( i \) decides not to enter the franchising market at the beginning of year \( t \), it can only open an own-run outlet when an opportunity arrives in year \( t \). The probability of opening an outlet conditional on the arrival of an opportunity is therefore

\[
p_{bo}(x_{it}) = \frac{\exp(x_{it}β_o)}{\exp(x_{it}β_o) + 1},
\]

where the subscript \( b \) represents “before”, i.e., before entering the franchising market. Thus, the expected profit in year \( t \) is \( \lambda E_{ε_{oit}} \max \{ \pi_o(x_{it}, ε_{oit}; β_o), 0 \} \). If chain \( i \) did not start franchising in year \( t \), it faces the same tradeoff at the beginning of year \( t + 1 \): If it pays the sunk cost to enter, it gets the value of entry \( VE(x_{it+1}) \). If it does not enter in year \( t + 1 \), it gets the value of not entering \( VN(x_{it+1}) \). Suppose the sunk cost is \( x_{it}φ + ω_{it} \), where \( φ \) is a vector of parameters to be estimated and \( ω_{it} \) is the unobservable determinant of the cost. In summary, the value of not entering is

\[
VN(x_{it}) = \lambda E_{ε_{oit}} \max \{ \pi_o(x_{it}, ε_{oit}; β_o), 0 \} \\
+ \delta E_{x_{it+1}|x_{it}} E_{ω_{it+1}} \max \{ VE(x_{it+1}) - x_{it+1}φ - ω_{it+1}, VN(x_{it+1}) \}.
\]

Therefore, chain \( i \) enters the franchising market at the beginning of year \( t \) if and only if \( VE(x_{it}) - x_{it}φ - ω_{it} \geq VN(x_{it}) \), where the value of entry and the value of not entering are determined by (13) and (15). We assume that \( ω_{it} \) follows a normal distribution with mean 0 and standard deviation \( σ \). Let \( Φ(·, σ) \) be the distribution function. Then, the probability of entry is given by

\[
Pr_e(x_{it}) = Φ(VE(x_{it}) - x_{it}φ - VN(x_{it}), σ),
\]

where the subscript \( e \) represents “entry”, meaning starting franchising.

3.3 Outlet Exit and the Transition Probability

To explain negative net changes of the number of outlets in the data, we allow exit. Specifically, we assume that the exit of an outlet is exogenous and independent across chains and outlets and the exit rate is a constant: \( μ \). Thus, the number of pre-entry company owned outlets follow an immigration-death process
with immigration rate $\lambda p_{bo}(x_i)$ and death rate $\mu$. The probability of $n'$ pre-entry company owned outlets conditional on there being $n$ company owned outlets in year $t - 1$ is given by

$$g_{bo}(n, n'; x_{it}) = \sum_{k=\max\{n'-n,0\}}^{n'} \frac{n!}{(n'-k)!} \gamma^{n'-k} (1 - \gamma)^{n-k} e^{\frac{-\rho_{bo}(x_{it})}{\mu}} \rho_{bo}(x_{it})^k,$$  \hspace{1cm} (17)

where $\gamma = 1 - e^{-\mu}$ is the exiting probability for an outlet and $\rho_{bo}(x_i) = p_{bo}(x_i) \lambda (1 - e^{-\mu}) / \mu$ is the modified net arrival rate of a new company owned outlet. Intuitively, a change from $n$ outlets to $n'$ outlets can be generated by a combination of $k$ new outlets and $n' - k$ exits where $k$ is in the interval $[\max\{n'-n,0\}, n']$.

Similarly, the post-entry transition probabilities for franchised outlets and company owned outlets, $g_{ao}(n, n'; x_{it})$ and $g_{af}(n, n'; x_{it})$, are defined analogously by replacing $\rho_{bo}(x_i)$ by $\rho_{ao}(x_i)$ and $\rho_{af}(x_i)$, respectively.

With a slight abuse of notations, we replace $\lambda (1 - e^{-\mu}) / \mu$ by $\lambda$. The parameters to be estimated are the net opportunity arrival rate $\lambda$, the exit probability $\gamma$, parameters in the profit functions $(\beta_f, \beta_o)$, parameters in the sunk cost $\varphi$, and the standard deviation of the normal distribution of the sunk cost shock $\sigma$. It is obvious that we cannot separately identify the opportunity arrival rate $\lambda$ and the overall quality of a chain’s project (i.e., the constant term in $x_i \beta_f$ and $x_i \beta_o$) from data on outlets. We therefore normalize the constant term in $x_i \beta_o$ to be 0. Even though we only observe the net change, the distribution of negative net changes in outlets identifies the probability of exit $\gamma$. For example, the probability of all outlets exiting $g_{bo}(n, 0; x_{it})$ depends on the exit probability parameter only. The empirical counterpart of this probability tells us about the probability of exit.

### 3.4 Estimation

#### 3.4.1 Log-Likelihood Function

For each chain $i$, let $t_{iB}, t_{iF}, t_{i1}$ be the year when chain $i$ starts its business, the year when it opens the first franchised outlet and the first year of chain $i$ in the data. Note that a chain is in our data only after it starts franchising, i.e., $t_{i1} \geq t_{iF}$. Let $n_{iot}$ and $n_{iFt}$ be the number of company owned outlets and franchised outlets of chain $i$ at the end of year $t$. We observe $n_{iFt}$ and $n_{iot}$ for the years in $T_i$. Note that $t_{i1} \leq t$ for any $t \in T_i$. Let $\theta_1 = (\varphi, \sigma)$ be the collection of parameters related to the entry cost and $\theta_2 = (\beta_f, \beta_o, \lambda, \gamma)$ be the collection of all other parameters. The log likelihood function can be written as

$$L(\theta_1, \theta_2) = \sum_i \left[ \sum_{t=t_{iB}}^{t_{iF}-1} \log (1 - H_{it}(\theta_1, \theta_2)) + \log H_{it,F}(\theta_1, \theta_2) + \sum_{t \in T_i} \log G_{oit}(\theta_2) + \sum_{t \in T_i} \log G_{fit}(\theta_2) \right],$$  \hspace{1cm} (18)

where $\sum_{t=t_{iB}}^{t_{iF}-1} \log (1 - H_{it}(\theta_1, \theta_2))$ is the log-likelihood of chain $i$ does not enter the franchising market in years between $t_{iB}$ and $t_{iF} - 1$, $\log H_{it,F}(\theta_1, \theta_2)$ is the log-likelihood of chain $i$ entering the franchising market in year $t_{iF}$, $\sum_{t \in T_i} \log G_{oit}(\theta_2)$ is the log-likelihood of the observations of chain $i$’s company owned outlets, i.e., $(n_{iot}, t \in T_i)$ and $\sum_{t \in T_i} \log G_{fit}(\theta_2)$ is that of chain $i$’s franchised outlets, i.e., $(n_{iFt}, t \in T_i)$.

The likelihood of entry is defined by the entry probability in (16):

$$H_{it}(\theta_1, \theta_2) = \Phi (VE(x_{it}, \theta_1) - x_{it} \varphi - VN(x_{it}, \theta_1, \theta_2), \sigma).$$  \hspace{1cm} (19)

\footnote{See Grimmett and Stirzaker (2008) (page 270) for an introduction of the immigration-death process and the derivative of the probability.}

\footnote{Here, we add the corresponding parameters into $VE$ and $VN$ functions to emphasize their dependence on the parameters.}
We now derive \( G_{\text{oit}}(\theta_2) \) and \( G_{\text{fit}}(\theta_2) \) based on the one-year transition probability (17).

When \( t = t_{i1} \), \( G_{\text{oit}}(\theta_2) \) is the likelihood of chain \( i \)'s company owned outlets growing from 0 to \( n_{i(t)} \) in \( t_{i1} - t_{iB} + 1 \) years.\(^6\) Let \( M(n, n', T) = \{m = (m_0, ..., m_T) \in N^{T+1} : m_0 = n, m_T = n' \} \) be the collection of all transition paths that are consistent with the change from \( n \) to \( n' \) in \( T \) years, i.e., the number of outlets is \( n \) in a year and \( n' \) after \( T \) years. Then, the likelihood of \( n_{i(t)} \) is the sum of the probabilities of all possible paths in \( M(0, n_{i(t)}, t_{i1} - t_{iB} + 1) \):

\[
G_{\text{oit},i1}(\theta_2) = \sum_{m \in M(0, n_{i(t)}, t_{i1} - t_{iB} + 1)} \prod_{t = t_{iB}}^{t_{i1} - 1} g_{bo}(m_{t-1}, m_t; x_{it'}, \theta_2) \prod_{t' = t_{iF}} g_{ao}(m_{t-1}, m_{t'}; x_{it'}, \theta_2), \quad (20a)
\]

where the pre-entry transition probability \( g_{bo}(m_{t-1}, m_t; x_{it'}, \theta_2) \) is given by (17)\(^7\) and the post-entry probability \( g_{ao} \) is similarly defined. Note that the likelihood depends on both pre-entry probability and post-entry probability because only chains with a franchised outlet are in our data, i.e., \( t_{i1} \geq t_{iF} \).

When \( t > t_{i1} \), the likelihood depends on the post-entry probability only. Suppose that for any observation of chain \( i \) in year \( t \), the last observation of chain \( i \) is \( l(\it) \). Then, the likelihood of the observation of chain \( i \) at \( t \) is determined by the transition from \( n_{i(l)(\it)} \) to \( n_{it} \) in \( t - l(\it) \) years. Therefore,

\[
G_{\text{oit}}(\theta_2) = \sum_{M(n_{i(l)(\it)}, n_{it}, t - l(\it))} \prod_{t' = l(\it) + 1}^t g_{ao}(m_{t'-1}, m_{t'}; x_{it'}, \theta_2) \quad \text{for } t > t_{i1}. \quad (20b)
\]

Similarly, the likelihood of the franchised outlets at \( t = t_{i1} \) is

\[
G_{\text{fit},i1}(\theta_2) = \sum_{M(0, n_{i(t)}, t_{i1} - t_{iF} + 1)} \prod_{t' = t_{iF}}^t g_{af}(m_{t'-1}, m_{t'}; x_{it'}, \theta_2), \quad (21a)
\]

and that for \( t > t_{i1} \) is (20b) with the post-entry transition probability for company owned outlets \( g_{ao} \) replaced by that of franchised outlet \( g_{af} \), i.e.,

\[
G_{\text{fit}}(\theta_2) = \sum_{M(n_{i(l)(\it)}, n_{it}, t - l(\it))} \prod_{t' = l(\it) + 1}^t g_{af}(m_{t'-1}, m_{t'}; x_{it'}, \theta_2) \quad \text{for } t > t_{i1}. \quad (21b)
\]

Different from the likelihood of the observations of company owned outlets in (20a), the likelihood of the observations of franchised outlets in (21a) obviously depends on the post-entry probability only.

Plugging (19), (20a), (20b), (21a) and (21b) into (18) gives us the log-likelihood function for estimation. Note that each likelihood \( G(\theta_2) \) is a sum of the one-year transition probability \( g(n, n'; x_{it}) \) over all possible paths of the number of outlets in \( M(n, n', T) \). Each transition probability itself is a sum over all possible combinations of new outlets and exits. Therefore, the log-likelihood is determined by all possible paths of new outlets and exits in each year. The number of such paths can be very large or infinite, which makes estimation prohibitively costly. In the next section, we explain how we solve this computational problem.

---

\(^6\)The value of entry \( VE \) obviously does not depend on the entry cost parameters \( \theta_2 \).

\(^7\)If a chain is in the data in the year it starts business, i.e., \( t_{i1} = t_{iB} \), we consider that the number of outlets grow from 0 to \( n_{i(t)} \) in one year.

\(^7\)Again, we add \( \theta_2 \) into the function to emphasize its dependence on the parameters in \( \theta_2 \).
3.4.2 Estimation Procedure

When there is no gap between two observations, the number of exits is bounded by the number of outlets last year and the number of new outlets is bounded by the number of outlets this year. However, when there is a gap between two observations, the bounds do not exist. For example, if we observe that a chain has 100 company owned outlets in year 2000 and 100 company owned outlets in year 2002. One possibility is that there are 100, 101, 100 outlets in years 2000, 2001, 2002, respectively. Another possibility is (100, 10000, 100). We think the latter is unlikely because it involves at least 10000 – 100 exits in year 2002. Moreover, in the former scenario of (100, 101, 100), the change from 100 outlets in 2000 to 101 outlets in 2001 can be generated by 1 new outlet and no exit in year 2001. It is also consistent with 101 new outlets and 100 exits. Again, the latter is probably less likely than the latter. Therefore, for computational feasibility, we need a criterion of choosing likely paths. Different from the paths in \( M(n, n', T) \), which are paths of the number of outlets each year, a path means a path of new outlets and exits each year from now on.

As it is clear from this example, the criterion should be on the number of exits. The criterion itself should depend on the probability of exit. For example, the criterion should not rule out paths involving a substantial amount of exits when exiting is quite likely. Since the probability of exit \( \gamma \) is identified by negative net changes of the number of outlets, we divide the observations into two groups: “first observation” and “growth”. An observation \( it \) is in the group of “first observation” if \( t = t_{i1} \). Otherwise, it is in the group of “growth”. We never observe negative changes in the “first observation” group because observations in this group give information on changes from 0 to the first observation of a chain. The observations in “growth”, however, can be used to estimate the exit probability \( \gamma \). Note that the cardinality of the set of possible paths \( M(n, n', T) \) increases rapidly with \( T \). But since the gap between observations in the “growth” group is generally small, we can impose a relatively loose criterion in this estimation. The estimate of \( \gamma \) can then be used to set a criterion for selecting paths for computing the likelihood of the observations in “first observation”. Specifically, the estimation is carried out in the following steps.

**Step 1.** We estimate \( \theta_2 \), which includes the exit probability \( \gamma \), using the observations in the “growth” group only. The log-likelihood function in this step is

\[
L_1(\theta) = \sum_{t} \left[ \sum_{t_{i1} < t < T} \log G_{oit}(\theta_2) + \sum_{t_{i1} < t < T} \log G_{fit}(\theta_2) \right].
\]

When computing the above log-likelihood function, we restrict the maximum of exits to be \( \bar{X} \). Denote the estimate by \( \hat{\theta}_2^{(1)} = (\hat{\beta}_f^{(1)}, \hat{\beta}_o^{(1)}, \hat{\lambda}^{(1)}, \hat{\gamma}^{(1)}) \).

**Step 2.** Based on the estimate \( \hat{\gamma}^{(1)} \), we compute the upper bound of the probability of each possible path and only include the paths whose upper bound is no less than \( Pr \) in computing the log-likelihood function \( L(\theta_1, \theta_2) \) in (18). In other words, in this step, we estimate both sets of parameters \( \theta_1 \) and \( \theta_2 \) using all observations. Denote the estimate by \( \left( \hat{\theta}_1^{(2)}, \hat{\theta}_2^{(2)} \right) \).

We can only rule out paths according to the upper bound of their probability because the likelihood of the observations in “first observation” depends on the pre-entry probability, which is further dependent on the entry cost parameters in \( \theta_1 \) that are not estimated in Step 1. The upper bound is computed as follows.

Let \( (y_t, z_t)_{t=1}^T \) be a typical path consistent with a change from \( n \) to \( n' \) in \( T \) years, where \( y_t \) and \( z_t \) represent the number of new outlets and exits. Therefore, \( n' = n + \sum_{t=1}^T (y_t - z_t) \). Let \( m_t \) be the number of outlets at the end of a period: \( m_0 = n, m_t = n + \sum_{t'=1}^t (y_{t'} - z_{t'}) \), \( m_T = n' \). According to (17), the probability of
a path is given by

$$
\prod_{t=1}^{T} \left( \frac{m_t}{z_t} \right) \gamma_z t (1 - \gamma)^{m_t - z_t} \exp \left( -\rho_{bo}(x_{it}; \theta_1, \theta_2) / y_t \right) \rho_{bo}(x_{it}; \theta_1, \theta_2)^{y_t} < \prod_{t=1}^{T} \left( \frac{m_t}{z_t} \right) \gamma_z t (1 - \gamma)^{m_t - z_t}.
$$

(22)

Intuitively, this step identifies the entry cost parameters $\theta_1$ and the profit parameters $\theta_2$, which affects the probability of opening an outlet $\exp(-\rho_{bo}(x_{it}; \theta_1, \theta_2))$. The upper bound above can be considered as a weight on this probability. We therefore rule out paths with extremely small weights. The upper bounds for the paths for post-entry company owned observations (i.e. $ao$ observations) and post-entry franchised observations (i.e. $af$ observations) are similarly defined.

See Appendix for an algorithm for searching for paths whose upper bound is no less than $Pr$ without computing the bound for each possible path, which is computationally infeasible as there are an infinite number of paths.

**Step 3.** Based on $\hat{\gamma}^{(2)}$ in $\hat{\theta}_2^{(2)}$, repeat step 2 and obtain estimate $(\hat{\theta}_1^{(3)}, \hat{\theta}_2^{(3)})$. Continue until the estimate of $(\theta_1, \theta_2)$ converges.

## 4 Data

Our data sources provide information concerning 800 to 900 U.S. franchised chains each year from 1980 through 2006, except for 1999 and 2002, when the data were not collected. In all, we have 22,762 observations on 4344 different U.S. franchisors.

For our analyses, we want to measure the macroeconomic environment in which the chain operated each year, starting from the moment it starts in business. We use several macroeconomic variables, most at the state/year, but some at the national level. Because some of our required macroeconomic variables are not available prior to 1975, we focus on the 2843 franchisors that started in business from that year on. In addition, as our theory presumes that franchising was an option at each point in time since these chains started their business, we eliminate those few chains (49 of them, with 192 observations) that take more than 15 years to start franchising. We expect that these did not seriously consider franchising early in their history, but rather learned about franchising only later. In that sense, they do not satisfy the conditions that our model presumes.

We also eliminate some observations directly. In particular, we exclude from our analyses consecutive observations that are separated by more than 5 years (40 of these). Our concern with these is that major unobserved changes might have occurred for these chains during these five years. We also eliminate observations when chains become very large – namely those that have more than 2000 franchised outlets (95 observations) or more than 500 company outlets (30 observations). We do this because our model focuses on local economic conditions. However, information on outlets’ locations is not available. We therefore use the local conditions in the home state (the state of headquarter) of a chain in our study as most chains expand first within and then outward from their home states. It is unlikely that the macroeconomic conditions in the home state are still important by the time the chain operates in all 50 states or so, and beyond. Because some of these observations are all we have for some franchisors, our final sample is comprised of 12249 observations on 2780 franchisors.

Table 1 below shows the average number of years during which franchisors operate their business before starting to franchise, by sector. There is an average lag of more than four years on average between the time
a franchisor begins their business and when they start franchising. There is much variance in this average across sector, however, as the mean goes from a low of about two years, in the hotel and motels sector, to a high of almost five and a half years, in the full-service restaurant sector. There is also much variance within sectors, however, as shown in the last column.

<table>
<thead>
<tr>
<th>Sector</th>
<th>Number of Franchisors</th>
<th>Mean (in years)</th>
<th>Std. Deviation (in years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Automotive Prod &amp; Services</td>
<td>193</td>
<td>3.75</td>
<td>2.93</td>
</tr>
<tr>
<td>Business Services</td>
<td>300</td>
<td>4.2</td>
<td>3.4</td>
</tr>
<tr>
<td>Business Supplies</td>
<td>41</td>
<td>3.66</td>
<td>2.54</td>
</tr>
<tr>
<td>Contractors</td>
<td>114</td>
<td>4.85</td>
<td>3.54</td>
</tr>
<tr>
<td>Cosmetic Prod. &amp; Services</td>
<td>68</td>
<td>4.16</td>
<td>2.69</td>
</tr>
<tr>
<td>Eating Places - Full Service</td>
<td>129</td>
<td>5.43</td>
<td>3.91</td>
</tr>
<tr>
<td>Eating Places - Limited Service</td>
<td>491</td>
<td>4.31</td>
<td>3.21</td>
</tr>
<tr>
<td>Education</td>
<td>67</td>
<td>4.72</td>
<td>3.12</td>
</tr>
<tr>
<td>Health/Fitness Prod. &amp; Services</td>
<td>103</td>
<td>3.71</td>
<td>2.88</td>
</tr>
<tr>
<td>Hotels &amp; Motels</td>
<td>29</td>
<td>2.03</td>
<td>2.01</td>
</tr>
<tr>
<td>Maintenance</td>
<td>148</td>
<td>4.64</td>
<td>3.89</td>
</tr>
<tr>
<td>Personal Services</td>
<td>227</td>
<td>4.3</td>
<td>3.39</td>
</tr>
<tr>
<td>Real Estate</td>
<td>59</td>
<td>3.15</td>
<td>2.38</td>
</tr>
<tr>
<td>Recreation</td>
<td>79</td>
<td>4.2</td>
<td>3.13</td>
</tr>
<tr>
<td>Rental</td>
<td>92</td>
<td>3.17</td>
<td>2.27</td>
</tr>
<tr>
<td>Repair</td>
<td>25</td>
<td>3.32</td>
<td>2.21</td>
</tr>
<tr>
<td>Retail - Build Materials</td>
<td>45</td>
<td>3.44</td>
<td>2.99</td>
</tr>
<tr>
<td>Retail - Clothing</td>
<td>65</td>
<td>4.17</td>
<td>2.95</td>
</tr>
<tr>
<td>Retail - Food</td>
<td>104</td>
<td>3.89</td>
<td>3.18</td>
</tr>
<tr>
<td>Retail - Home Furnishings</td>
<td>59</td>
<td>4.41</td>
<td>3.1</td>
</tr>
<tr>
<td>Retail - Other</td>
<td>287</td>
<td>3.97</td>
<td>2.94</td>
</tr>
<tr>
<td>Retail - Used Products</td>
<td>25</td>
<td>4.08</td>
<td>3.33</td>
</tr>
<tr>
<td>Travel</td>
<td>30</td>
<td>2.8</td>
<td>2.16</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>2,780</strong></td>
<td><strong>4.14</strong></td>
<td><strong>3.21</strong></td>
</tr>
</tbody>
</table>

In terms of macroeconomic variables, we measure the state and national housing price indexes using the “OFHEO House Price Indexes”. The data, which begin in 1975, are quarterly so we use mean yearly values. We measure the national annual interest rate using the “Federal funds effective rate”. The ”Gross Equity Extraction” is obtained from Greenspan and Kennedy (2007). These authors define this variable as ”the discretionary initiatives of home owners to convert equity in their homes into cash by borrowing in the residential mortgage market”. The article gives data from 1991 onward only, but the authors made historical
data, to 1968, available to us.\textsuperscript{8} We calculate available collateral as (Constant Dollar Gross Equity Extraction * Local Housing Index)/(National Housing index*Capital Required). We also estimate Leveraged Housing Value as the (Gross Equity Extraction*Deflated Local Housing Index/National Housing Index).

We measure the local level of economic activity using the Gross State Product (GSP) from the Bureau of Economic Analysis. We obtained CPI data from the Bureau of Labor Statistics and measure population using the “State Population Estimates” series. We use these to calculate annual Per Capita GSP, in constant dollars.

When a franchisor is present in the data in a given year, we observe, for that year: the number of franchised outlets, the number of company owned outlets, state of headquarter (which can change if they move), sector of business activity, capital required (in constant $000). From 1993 onward, we also observe number of employees required.\textsuperscript{9} Because certain franchisor characteristics, notably Capital Required and Number of Employees needed to run the business that the franchisor has developed, should be stable over time as they are intrinsically determined by the nature of the business, which itself is intrinsically connected to the brand name and, as such, basically does not change. Thus we use the average of all the data we have for these variables for each franchised chain under the presumption that most of the differences reflect noise in the data. We then calculate the labor to capital ratio for a franchisor as the ratio of the Number of Employees to the amount of Capital Required. Based on agency arguments, we expect this variable to affect the value of franchising – i.e. the value, to the chain, of making the local manager a residual claimant. Because we are missing data on Number of Employees for many franchisors, reliance on this variable in our analyses below reduces our sample sizes further.

5 Estimation Results

[To be completed]

6 Conclusions

[To be completed]

Appendix

A The Algorithm for Searching for Likely Paths

The goal of the algorithm is to search for paths of new outlets and exits in each period that satisfy two conditions: (1) they are consistent with a change of the number of outlets from $n$ to $n'$ in $T$ periods; (2) the upper bound of their probabilities given in (22) is not less than $Pr$.

The upper bound is the product of the probabilities of a certain number of exits in each period. Let $B(z; m, \gamma) = \binom{m}{z} \gamma^z (1 - \gamma)^{m-z}$ be the probability of $z$ out of $m$ outlets exiting given that each outlet exits with probability $\gamma$. The algorithm is as follows.

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\textsuperscript{8}The authors note, however, that the data quality is not as good prior to 1991.

\textsuperscript{9}We count part-time employees as equivalent to 0.5 of a full-time employee.
• We start from year $T$. Let $m_{T-1}$ be the number of outlets at the end of year $T - 1$. Note that when $m_{T-1}$ is large, there might be a large number of exits in year $T$ so that the number of outlets at the end of year $T$ is $n'$. Depending on the exiting probability of an outlet $\gamma$, such events may be unlikely. Therefore, we need to find the upper bound of $m_{T-1}$ and then the corresponding combinations of the number of new outlets and the number of exits in year $T$.

For any $m_{T-1} > n'$, the lower bound for the number of exits $z_{T-1}$ is $m_{T-1} - n'$. Define $\bar{B}(m, n', \gamma)$ as the maximum of the binomial probability mass function $B(z; m, \gamma)$ with this constraint that $z \geq m - n'$. Specifically,

$$\bar{B}(m, n', \gamma) = \begin{cases} B\left(\lfloor (m + 1)\gamma \rfloor; m, \gamma \right) & \text{if } m - n' \leq \lfloor (m + 1)\gamma \rfloor, \\ B(m - n'; m, \gamma) & \text{otherwise} \end{cases},$$

where $\lfloor (m + 1)\gamma \rfloor$ is the largest integer not greater than $(m + 1)\gamma$, which is the mode of a binomial distribution.

It is easy to show that for any given $(n', \gamma)$, $\bar{B}(m, n', \gamma)$ is decreasing in $m$ for $m > n'$. Therefore, there exist $\bar{m}_{T-1}$ such that $\bar{B}(m_{T-1}, n', \gamma) < \Pr$ for any $m_{T-1} > \bar{m}_{T-1}$. Therefore, we only need to consider $m_{T-1}$ in $[0, \bar{m}_{T-1}]$.

For any $m_{T-1} \in [0, \bar{m}_{T-1}]$, the lower bound of exits in year $T$ is $\max(0, m_{T-1} - n')$ and the upper bound is $m_{T-1}$. We can then compute $B(z_T; m_{T-1}, \gamma)$ for each possible number of exits $z_T$ and find the combinations of the number of new outlets and exits, i.e., $(n' - m_{T-1} + z_T, z_T)$, such that the upper bound of the probability is no less than $\Pr$.

• We now move on to year $T - 1$. In this year, we follow the above procedure for each one of the aforementioned combinations in year $T$, $(n' - m_{T-1} + z_T, z_T)$, with the following two modifications: $n'$ is replaced by corresponding $m_{T-1}$ and $\Pr$ is replaced by $\Pr \cdot B(z_T; m_{T-1}, \gamma)$. This gives us all likely combinations in year $T - 1$. Combined with the combinations in year $T$, we have the likely paths of new outlets and exits in years $T - 1$ and $T$.

• The search for year $T - 2$ is similar to that for year $T - 1$: $n'$ is replaced by corresponding $m_{T-2}$ and $\Pr$ is replaced by $\Pr \cdot B(z_{T-1}; m_{T-2}, \gamma) \cdot B(z_T; m_{T-1}, \gamma)$. We continue this procedure until year 2.

• In year 1, the number of outlets at the beginning of this year is $n$ and the possible number of outlets at the end of this year $m_2$ is determined by the last step. We can therefore easily find likely combinations of new outlets and exits in year 1 corresponding to each likely paths from year 2 to year $T$ determined by the above steps. This gives us all likely paths of the number of new outlets and exits in all years.