Money and Credit with Limited Commitment and Theft*

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Abstract

We study the interplay among imperfect memory, limited commitment, and theft, in an environment that can support monetary exchange and credit. Imperfect memory makes money useful, but it also permits theft to go undetected, and therefore provides lucrative opportunities for thieves. Limited commitment constrains credit arrangements, and the constraints tend to tighten with imperfect memory, as this mitigates punishment for bad behavior in the credit market. Theft matters for optimal monetary policy, but at the optimum theft will not be observed in the model. The Friedman rule is in general not optimal with theft, and the optimal money growth rate tends to rise as the cost of theft falls. *Journal of Economic Literature* Classification Numbers: E5, D8. *Keywords*: Money, Credit, Imperfect Memory, Theft, Optimal Monetary Policy.

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1 Introduction

It is hard to find examples of economies in which we do not observe the use of both money and credit in transactions. Thus, we should think of money and credit as robust, in the sense that we will observe transactions involving both money and credit under a wide array of technologies and monetary policy rules. One goal of this paper is to help us understand what is required to obtain robustness of money and credit in an economic model. Then, given robustness, we want to explore the implications for monetary policy. As well, this paper will serve to tie together some key ideas in monetary economics.

As is now well-known, barriers to the flow of information across locations and over time appear to be critical to the role that money plays in exchange. If there were no such barriers, in particular if there were perfect "memory", i.e. recordkeeping, then it would be possible to support efficient allocations in the absence of valued money - see [11]. One can think of the models of [7], or [5], as determining efficient allocations with credit arrangements under private information, where the memory of past transactions by economic agents supports incentive compatible intertemporal exchange. As [11] points out, spatial separation of the type encountered in turnpike models - such as [14] - or random matching models - such as [15] - also yields efficient credit arrangements under perfect memory. [1] study an environment with private information *and* random matching where credit arrangements are efficient. Thus, neither private information nor spatial separation is a sufficient friction to provide a socially useful role for monetary exchange. Both frictions mitigate credit arrangements, but not to the point where monetary exchange necessarily improves matters.

The work of [10] and [11] seems to suggest that limited commitment works much like private-information and spatial frictions, in that it in general implies less intertemporal exchange than would occur in its absence, but does not imply a welfare-improving role for money. However, in the case of limited commitment, this is not obvious. For example, suppose that two economic agents, A and B meet. Agent A can supply B with something that B wants, but all that B can offer in exchange is a promise to supply A with some object in the future. Agent B is unable to commit, and what A is willing to give to B will depend on $A's$ ability to punish B, or to have other economic agents punish B, if he or she fails to fulfil his or her promises in the future. The amount of credit that A is willing to extend to B will in general be limited. However, suppose that B has money to offer A in exchange. Possibly A and B can trade more efficiently using money, or by using money and credit, because monetary exchange is not subject to limited commitment.

First, we wish to construct a framework which can potentially permit monetary exchange, trade using credit under limited commitment, and the coexistence of valued money and credit. We build on the model of [13] and [12], which has quasilinear utility and alternating decentralized and centralized trading among economic agents. This lends tractability to our analysis, but we think that the basic ideas are quite general.

The first result is that, consistent with [11], limited commitment is in fact not sufficient to provide a social role for money in our model. The result hinges on the fact that lack of commitment applies to tax liabilities as well as private liabilities. If there is perfect memory and limited commitment matters, then limited commitment also may make the Friedman rule infeasible for the government. This is because agents may want to default on the tax liabilities that are required to support deflation at the Friedman rule rate. While it is possible in special cases for money to be valued in equilibrium with limited commitment and perfect memory, there is no welfare-improving role for money. This is similar to the flavor of some results in the monetary model without credit considered by [3].

As mentioned above, one of our aims in this paper is to determine a set of frictions under which money and credit are both robust as means of payment. Clearly, perfect memory does not provide conditions under which monetary exchange is robust, so we need to add imperfect memory to provide a role for money. However, we do not want to shut down memory entirely, as is typical in some monetary models (e.g. [12]), as this will also shut down credit. We use a hybrid approach, whereby decentralized meetings between buyers and sellers are either monitored, or are not. A monitored trade is subject to perfect memory, while there is no access to memory in non-monitored transactions (see also [6], where the monitoring structure is quite different from ours).

In this context, our results depend critically on the punishments that are triggered by default in the credit market. For tractability, we consider global punishments, whereby default by a borrower will imply that all would-be lenders refuse to extend credit. At the extreme, this can result in global autarky. With global autarky as an off-equilibrium path supporting valued money and credit in equilibrium, higher money growth lowers the rate of return on money, and there is substitution of credit for money. Efficient monetary policy is either a Friedman rule, if incentive constraints do not bind at the optimum, or else optimal money growth is greater than at the Friedman rule and incentive constraints bind at the optimum. In either case, efficient monetary policy drives out credit. Money works so well that if the government gives money a sufficiently high rate of return there will be no lending in equilibrium.

We also consider off-equilibrium punishments that are less severe than autarky. Much as in [2] or in [4], we allow punishment equilibria to include monetary exchange. That is, if a borrower defaults, this triggers a global punishment where there is no credit, but agents can trade money for goods. Here, the only equilibrium that can be supported is one with no credit, and with a fixed stock of money (also implying a constant price level in our model). This is quite different from results obtained in [2] or [4]. A key difference in our setup is that we take account of the fact that the government cannot commit to punishing private agents through monetary policy. When a default occurs, the government adjusts monetary policy so that it is a best response to the decision rules that private agents adopt as punishment behavior.

Imperfect memory provides a role for money, but in the context of imperfect memory alone, money in some sense works too well in our model, relative to what we see in reality. That is, optimal monetary policy always drives credit out of the system. This is a typical result, which is obtained for example in Ireland's cash-in-advance model of money and credit (see [9]). Ireland's model has the property that a Friedman rule is optimal and, at the Friedman rule, all transactions are conducted with cash, which eliminates the costs of using credit. The intuition for this is quite clear. All alternatives to using currency in transactions come at a cost, for example there are costs of operating a debit-card or credit-card system, there are costs to clearing checks, etc. If it is costless to produce currency and to carry it around, then if the government generates a deflation that induces a rate of return on money equivalent to that on the best safe asset, then this should be efficient, and it should also eliminate the use of all cash substitutes in transactions.

Of course, in practice it is costly for the government to operate a currency system. For example, maintaining the currency stock by printing new currency to replace worn-out notes and coins is costly, as is counterfeiting and the prevention of counterfeiting. As well, a key cost of holding currency is the risk of theft, which has been studied, for example, in [8]. We model theft differently here, and do this in the context of monitored credit transactions. That is, we assume that it is possible for sellers, at a cost, to steal currency in non-monitored transactions, but theft is not possible if the transaction is monitored. This changes our results dramatically. Now, monetary policy will affect the amount of theft in existence, and theft will matter for how borrowers are punished in the event of default. For example, if the off-equilibrium punishment path involves no credit market activity and only monetary exchange, then the risk of theft is higher on the off-equilibrium path, and this reduces welfare in the punishment equilibrium.

At the optimum it will always be optimal for the government to eliminate theft. Theft will matter for policy, but in the model theft will not be observed in equilibrium. Because theft is potentially more prevalent with off-equilibrium punishment, however, money and credit will in general coexist at the optimum. When theft matters, the Friedman rule is not optimal, and the optimal money growth rate tends to increase as the cost of theft falls.

In Section 2 we set up the environment, then in Section 3 we study a planner's problem in order to determine optimal allocations. In Sections 3-5 we then analyze the predictions of the model under, respectively, perfect memory, imperfect memory with autarkic punishment, and imperfect memory with non-autarkic punishment. In Section 6 we study theft, and Section 7 concludes.

2 The Environment

Time is discrete and each period is divided into two subperiods: day and night. There are two types of agents in the economy, buyers and sellers, and there is a continuum of each type with unit measure. There is a unique perishable consumption good which is produced and consumed within each subperiod. During the day, a seller can produce one unit of the consumption good with one unit of labor. At night, a buyer is able to produce one unit of the consumption good with one unit of labor.

A buyer has preferences given by

$$\sum_{t=0}^{\infty} \beta^t [u(q_t) - n_t] \tag{1}$$

where q_t is consumption during the day, and n_t is labor supply at night, with $\beta \in (0,1)$ the discount factor between night and day. Assume $u(\cdot)$ is strictly concave, strictly increasing, and twice continuously differentiable with $u(0) = 0$, $u'(0) = \infty$, and define q^* to be the solution to $u'(q^*) = 1$. A seller has preferences given by

$$\sum_{t=0}^{\infty} \beta^t (-l_t + x_t) \tag{2}$$

where l_t is labor supply during the day and x_t is consumption at night. Sellers and buyers discount at the same rate.

Agents are bilaterally and randomly matched during the day and at night trade is centralized.

3 Planner's Problem

3.1 Full Commitment

First, consider what a social planner could achieve in this economy in the absence of money. Ultimately, money will consist of perfectly divisible and durable objects that are portable at zero cost, and can be produced only by the government. In this section assume there is complete memory and that each agent can commit to the plan proposed by the social planner at $t = 0$. If the planner treats all sellers identically and all buyers identically, then an allocation $\{(q_t, x_t)\}_{t=0}^{\infty}$ satisfies the participation constraints

$$\sum_{t=0}^{\infty} \beta^t [u(q_t) - x_t] \geq 0, \tag{3}$$

and

$$\sum_{t=0}^{\infty} \beta^t (-q_t + x_t) \geq 0, \tag{4}$$

which state that a buyer and a seller, respectively, each prefer to participate in the plan at $t = 0$. Then, if we confine attention to stationary allocations with $q_t = q$ and $x_t = x$ for all t, we must have

$$u(q) - x \geq 0 \tag{5}$$

and

$$-q + x \geq 0 \tag{6}$$

The set of feasible stationary allocations is given by (5) and (6), and this set is non-empty given our assumptions. Further, the set of efficient allocations is also non-empty, satisfying (5), (6) and $q = q^*$.

3.2 Limited Commitment

Now, continue to assume complete memory, but now suppose that any agent can at any time opt out of the plan. The worst punishment that the planner can impose is zero consumption forever for an agent who deviates. Let v_t denote the utility of a buyer at the beginning of t, with w_t similarly denoting the utility of a seller. Then an allocation must satisfy the participation constraints (3) and (4) as before, as well as the incentive constraints

$$-x_t + \beta v_{t+1} \geq 0, \tag{7}$$

and

$$-q_t + x_t + \beta w_{t+1} \geq 0 \tag{8}$$

for $t = 0, 1, 2, ..., \infty$. Constraints (7) and (8) state that the buyer and seller, respectively, prefer to produce at each date rather than defecting from the plan.

Now, confining attention to stationary allocations, the planner's problem is then:

$$\max_{(q,x)\in\mathbb{R}^2_+} u(q) - x$$

subject to

$$x \leq \beta u(q),$$

and

$$-q + x \geq (1-\beta)\, w.$$

where w is the seller's lifetime utility. At the optimum, the seller's incentive constraint binds, so that $x = q + (1-\beta)\, w$. Now, let q^{**} denote the solution to $\beta u(q^{**}) = q^{**}$. We can then rewrite the buyer's incentive constraint as

$$\beta u(q) - q \geq (1-\beta)\, w. \tag{9}$$

Given that $w \geq 0$, we must have $q \in [0, q^{**}]$. Then, we can rewrite the planner's problem in the following way:

$$\max_{q\in[0,q^{**}]} u(q) - q - (1-\beta)\, w$$

subject to (9). The first-order conditions are

$$u'(q) - 1 - \lambda\left[\beta u'(q) - 1\right] \geq 0, \text{ with equality if } q < q^{**}, \tag{10}$$

and

$$\lambda\left[\beta u(q) - q - (1-\beta)\, w\right] = 0,$$

where $\lambda \geq 0$ is the Lagrange multiplier associated with the constraint (9).

Suppose that $q^{**} \geq q^*$. Then, (10) holds with equality. If $\lambda > 0$, then the buyer's incentive constraint (9) binds, and any $q \in [\hat{q}, q^*]$, together with $x = \beta u(q)$, is an efficient allocation. If $\lambda = 0$, then from (10) any $q = q^*$, together with $x \in [q^*, \beta u(q^*)]$, is an efficient allocation. In Figure 1, the set of efficient allocations with limited commitment for the case $q^{**} \geq q^*$ is ABC, and the set of efficient allocations under full commitment is AD. Notice that the incentive constraint binds for the efficient allocations BC.

[Figure 1 here.]

The perhaps more interesting case is when $q^{**} < q^*$, in which case the set of efficient allocations is $\{(q,x) : x = \beta u(q), q \in [\hat{q}, q^{**}], \beta u'(\hat{q}) = 1\}$. This case is depicted in Figure 2, where the set of efficient allocations is AB. That is, in this case the incentive constraint for the buyer always binds for the efficient stationary allocation, and an efficient allocation with limited commitment is not efficient under full commitment.

[Figure 2 here.]

4 Equilibrium Allocations with Perfect Memory

To establish a benchmark, we first assume that there is perfect memory. As we would expect from the work of [11], this will severely limit the role of money in this economy. An important element of the model will be the bargaining protocol carried out when a buyer and seller meet during the daytime. We assume that the seller first announces whether or not he or she is willing to trade with the buyer. If the seller is not willing to trade, then no exchange takes place. Otherwise, the buyer then makes a take-it-or-leave-it offer to the seller. This protocol in part allows us to focus on the limited commitment friction, as take-it-or-leave-it offers imply that there will be no bargaining inefficiencies.

When a buyer and seller are matched during the day, they continue to be matched at the beginning of the following night, after which all agents enter the nighttime Walrasian market.

4.1 Credit Equilibrium

Ultimately we will want to determine the role for valued money in this perfect-memory economy, but our first step will be to look at equilibria where money is not valued. Here, the daytime take-it-or-leave-it offers of buyers consist of credit contracts, with a loan made during the day and repayment at night. We will confine attention to stationary equilibria where sellers always choose to trade when they meet a buyer during the day. Let l denote the loan quantity offered by the buyer to the seller in the day. Then, letting v denote the lifetime continuation utility of the buyer after repaying the loan during the night, we have

$$v = \beta \max_{l} [u(l) - l + v] \tag{11}$$

subject to the incentive constraint

$$l \leq v - \hat{v},$$

and

$$l \geq 0.$$

Here, $\hat{v}$ is the buyer's continuation utility if he or she defaults on the loan, which triggers a punishment. In the equilibrium we consider here, $\hat{v} = 0$, so that the punishment for default is autarky for the defaulting buyer. On the off-equilibrium path, it is an equilibrium for no one to trade with an agent who has defaulted as, if agent A trades with agent B who has defaulted in the past, this triggers autarky for agent A. Here, note that the individual punishment for a buyer who defaults is identical to a global punishment whereby default by any buyer triggers global autarky. Letting $\psi_c(v)$ denote the right-hand side of (11), we get

$$\psi_c(v) = \beta u(v), \text{ for } 0 \leq v \leq q^*,$$

and

$$\psi_c(v) = \beta \left[u(q^*) - q^* + v \right], \text{ for } v \geq q^*.$$

An equilibrium is then a solution to $v = \psi_c(v)$. If $q^{**} < q^*$ then there are two equilibria. In the first, $v = 0$, and in the second $v > 0$, which are both solutions to $v = \beta u(v)$. Note that v is also the consumption of the buyer during the day, and of the seller during the night, with $v < q^*$. In this case, the incentive constraint for the buyer binds in either equilibrium. If $q^{**} \geq q^*$ then the $v = 0$ equilibrium still exists, and the equilibrium with $v > 0$ has

$$v = \frac{\beta}{1-\beta} \left[u(q^*) - q^* \right],$$

in which case consumption is q^* for any agent consuming at any date and the incentive constraint does not bind.

Note that, in equilibrium, a seller meeting a buyer during the daytime is always indifferent to trading or not. If he or she announces a willingness to trade, then the buyer makes an offer that leaves the seller with zero surplus, and utility is identical to what the seller would have achieved without trade. In the equilibrium we study, the seller always chooses to trade.

In Figure 3, panel (b) shows the case where the incentive constraint binds in equilibrium, and panel (a) the case where the incentive constraint does not bind. The nonmonetary credit equilibrium, by virtue of the bargaining solution we use, just picks out the efficient stationary allocation that gives all of the surplus to the buyer.

[Figure 3 here.]

4.2 Monetary Equilibrium

Assume that money is uniformly distributed across buyers at the beginning of the first day. Subsequently the government makes equal lump-sum transfers at the beginning of the night to buyers, so that the money stock grows at the gross rate μ. Confine attention to stationary monetary equilibria, and consider only cases where $\mu \geq \beta$, as otherwise a monetary equilibrium does not exist. Let m denote the real money balances acquired by a buyer in the night, and γ the real value of a lump-sum transfer received by a buyer from the government during the night. Suppose that the buyer receives the lump-sum transfer before acquiring money balances during the night, and continue to let v denote the continuation utility for the buyer after receiving the lump-sum transfer. As in [3], we treat the government symmetrically with the private sector, in that there is limited commitment with respect to tax liabilities as well as private liabilities.

Continue to assume complete memory and, as in the previous subsection, default by a buyer triggers autarky for that buyer. Since a seller will always be indifferent to trading with a buyer, sellers not only refuse to engage in credit contracts with a buyer who has defaulted; they also refuse to take his or her money. Note that the trigger to individual autarky is identical to a global punishment where, if an agent defaults, no seller will trade with any buyer. With global punishment, the value of money is zero on the off-equilibrium path.

In this case, we determine the continuation value v for the buyer by

$$v = \max_{l,m} \left\{ -m + \beta \left[u\left(\frac{1}{\mu} m + l \right) - l + \gamma + v \right] \right\} \tag{12}$$

subject to

$$l \leq \gamma + v - \hat{v}$$
$$l \geq 0.$$

Again, we have $\hat{v} = 0$. Here, note that we need to be careful about the lump-sum transfer the buyer receives. Should the buyer default on his or her debt, he or she will also not receive the transfer, or will default on current and future tax liabilities if $\gamma < 0$. In equilibrium, we have

$$\gamma = m \left(1 - \frac{1}{\mu} \right).$$

For $\mu > \beta$, the right-hand side of equation (12) is given by

$$\psi_m(v) = -v_1 + \beta u(v_1) + v, \text{ for } \max \left[0, m^* \left(\frac{1}{\mu} - 1 \right) \right] \leq v \leq v_1,$$

$$\psi_m(v) = \beta u(v), \text{ for } v_1 \leq v \leq q^*,$$
$$\psi_m(v) = \beta \left[u(q^*) - q^* + v \right], \text{ for } v \geq q^*.$$

Here, m^* solves

$$u' \left(\frac{m^*}{\mu} \right) = \frac{\mu}{\beta},$$

and v_1 satisfies

$$u'(v_1) = \frac{\mu}{\beta},$$

For $\mu = \beta$, we have

$$\psi_m(v) = \beta [u(q^*) - q^*] - m(1 - \beta) + \beta v, \text{ for } v \geq m^* \left(\frac{1}{\beta} - 1 \right),$$

where

$$m \in [q^* - \min(q^*, v), \beta q^*].$$

Proposition 1 *If $q^{**} \geq q^*$, then a monetary equilibrium does not exist if $\mu \neq \beta$.*

Proof. If $\mu < \beta$, a monetary equilibrium does not exist, for standard reasons. Suppose $\mu > \beta$. Define the function $\Gamma(v) = \Psi_m(v) - v$. Notice that $\Gamma(\cdot)$ is continuous and $\lim_{v\to\infty} \Gamma(v) = -\infty$. Moreover, $\Gamma(v) > 0$ for all $v \in [\max\{0, (1-\mu) v_1\}, q^*)$ and $\Gamma(\cdot)$ is strictly decreasing on (q^*, ∞). Hence, there exists a unique value $v \geq q^*$ such that $\Gamma(v) = 0$. However, money is not valued in this equilibrium. ■

Proposition 2 *If $q^{**} \geq q^*$ and $\mu = \beta$, then a continuum of monetary equilibria exists with $v \in [\frac{\beta[u(q^*)-q^*]}{1-\beta} - \min\left\{\beta q^*, \frac{\beta u(q^*)-q^*}{1-\beta}\right\}, \frac{\beta[u(q^*)-q^*]}{1-\beta})$. All of these equilibria yield expected utility for the buyer of $\frac{u(q^*)-q^*}{1-\beta}$.*

Proof. Suppose $\mu = \beta$. It follows that $\Gamma(\cdot)$ is continuous everywhere except possibly at $v = q^*$ and $\lim_{v\to\infty} \Gamma(v) = -\infty$. We have $\Gamma(v) = \beta u(q^*) - q^* > 0$ for all $v \in [(1-\beta) q^*, q^*)$. At $v = q^*$ we have $\Gamma(q^*) = \beta u(q^*) - q^* - (1-\beta) m$. A necessary condition for the existence of a monetary equilibrium is $\Gamma(q^*) \geq 0$, which requires

$$m \leq \frac{\beta u(q^*) - q^*}{1-\beta}.$$

Hence, a necessary and sufficient condition for the existence of a monetary equilibrium is

$$m \leq \min\left\{\beta q^*, \frac{\beta u(q^*) - q^*}{1-\beta}\right\}.$$

Given a positive value of m satisfying the inequality above, there exists a unique value $v \geq q^*$ such that $\Gamma(v) = 0$, in which case money is valued in equilibrium. Therefore, there exists a continuum of monetary equilibria with

$$v \in [\frac{\beta[u(q^*) - q^*]}{1-\beta} - \min\left\{\beta q^*, \frac{\beta u(q^*) - q^*}{1-\beta}\right\}, \frac{\beta[u(q^*) - q^*]}{1-\beta}).$$

All of these equilibria support the allocation $(q, x) = (q^*, q^*)$. ■

Proposition 3 *If $q^{**} < q^*$, then a monetary equilibrium does not exist if $\mu \neq \beta u'(q^{**})$.*

Proof. Suppose $\mu > \beta u'(q^{**})$. Notice that $\Gamma(v) = \beta u(v_1) - v_1 > 0$ for all $v \in [\max\{0, (1-\mu) v_1\}, v_1)$, $\Gamma(v) > 0$ for all $v \in [v_1, q^{**})$, and $\Gamma(q^*) < 0$. Since $\Gamma(\cdot)$ is continuous and strictly decreasing on (q^{**}, ∞), it follows that $v = q^{**}$ is the unique value satisfying $\Gamma(v) = 0$. Since $v_1 < q^{**}$, it follows that money is not valued in equilibrium. Suppose $\mu \in (\beta, \beta u'(q^{**}))$. In this case, $\Gamma(v) < 0$ for all $v \geq (1-\mu) v_1$, so that a monetary equilibrium does not exist. Finally, assume $\mu = \beta$. Again, we find that $\Gamma(v) < 0$ for all $v \geq (1-\beta) q^*$, so that a monetary equilibrium does not exist. ■

Proposition 4 *If $q^{**} < q^*$ and $\mu = \beta u'(q^{**})$, then a continuum of monetary equilibria exists with $v \in [q^{**}[1 - \beta u'(q^{**})], q^{**})$. All of these equilibria yield expected utility for the buyer of $\frac{u(q^{**}) - q^{**}}{1-\beta}$.*

Proof. Take $\mu = \beta u'(q^{**})$. Then, $\Gamma(v) = 0$ for all $v \in [q^{**}[1 - \beta u'(q^{**})], q^{**}]$ and $\Gamma(v) < 0$ for all $v > q^{**}$. Hence, a continuum of monetary equilibria exists with $v \in [q^{**}[1 - \beta u'(q^{**})], q^{**})$. All of these equilibria yield the allocation $(q, x) = (q^{**}, q^{**})$. ■

If the money growth rate is sufficiently high, that is if $\mu > \beta \max[1, u'(q^{**})]$, then the rate of return on money is sufficiently low that money is not held in equilibrium. If $q^* \leq q^{**}$, it certainly seems clear why a monetary equilibrium will not exist when the money growth rate is sufficiently high. In this case, when money is not valued a credit equilibrium exists which is efficient and incentive constraints do not bind. Thus, there is clearly no role for money in equilibrium in relaxing incentive constraints in decentralized trade. Why money is not valued even when $q^* > q^{**}$ and $\mu > \beta u'(q^{**})$ is perhaps less clear. In this case, the only stationary equilibria that exist are the two credit equilibria: one where $v = 0$ and one with $v > 0$ and binding incentive constraints, as in Figure 3. Money cannot relax the binding incentive constraints, as in order to support a money growth rate sufficiently low as to induce agents to hold money, the government would have to impose sufficiently high taxes that buyers would choose to default on their tax liabilities. Thus, there is no role for money in improving efficiency.

If $\mu = \beta \max[1, u'(q^{**})]$, then in equilibrium buyers are essentially indifferent between using money and credit in decentralized transactions with sellers, and there exist a continuum of equilibria with valued money. Each of these equilibria supports the same allocation as does the credit equilibrium with $v > 0$. The continuum of equilibria is indexed by the quantity of real money balances held by buyers. Across these equilibria, as the quantity of real balances rises, the quantity of lending falls.

Our results are consistent with the ideas in [11], as they should be. With perfect memory, money is not socially useful. At best, money can be held in equilibrium. This equilibrium is either one where incentive constraints do not bind and the monetary authority follows a Friedman rule, or incentive constraints bind and the money growth rule is similar to what [3] finds. In either case, money provides no efficiency improvement.

5 Imperfect Memory and Autarkic Punishment

As we have seen, with perfect memory there is essentially no social role for money, and it will only be held under special circumstances. As is well-known, particularly given the work of Kocherlakota, we need some imperfections in record-keeping in order for money to be useful and to help it survive as a valued object. We will start by assuming that, during the day, there is no memory in some bilateral meetings, and perfect memory in other meetings. In particular, a fraction ρ of sellers has no monitoring potential, while a fraction $1-\rho$ does. In any day, a given buyer has probability ρ of meeting a seller with no monitoring potential, in which case there is no memory in the interaction between the buyer and seller. That is, each agent in such a meeting has no knowledge of his or her trading partner's history, and nothing about the meeting will be recorded. With probability $1-\rho$ a buyer meets a seller with monitoring potential. In this case, the buyer has the opportunity to choose to have his or her interaction with the seller monitored. Here, $0<\rho\leq 1$. If the buyer chooses a monitored interaction in the day, then his or her history is observable to the seller, and the interaction between that buyer and seller will be publicly observed during the day and through the beginning of the following night. Otherwise, the buyer's and seller's actions are unobserved during the day and the following night.

Trade is carried out anonymously in the Walrasian market that opens in the latter part of each night, in the sense that all that can be observed in the Walrasian market is the market price. Individual actions are unobservable. Here, the case where $\rho=1$ is the standard one in monetary models with random matching such as [12]. However, even in the case with $\rho=1$, we deviate from the usual assumptions, in that there is lack of commitment with respect to tax liabilities. We assume that each agent can observe the interaction between the government and all other agents. That is, default on tax liabilities is publicly observable.

We change the bargaining protocol between a buyer and seller during the day as follows. The buyer first declares whether interactions with the seller during the period will be monitored or not. If monitoring is chosen, then the seller learns the buyer's history of publicly-recorded transactions. Then, the seller decides whether or not to transact with the buyer. If the seller is willing to transact, the buyer then makes a take-it-or-leave-it offer.

Given our setup, if a buyer defaults on a loan made in a monitored trade, this will be public information. As well, default by a buyer on tax liabilities is public information. However, suppose that a seller were to make a loan to a buyer during the day in a non-monitored trade, and then the buyer defaulted on the loan during the night. In this case, it is impossible for that seller to signal to anyone else that default has occurred. The interaction between the buyer and seller is private information, and the individual seller cannot affect prices in any subsequent nighttime Walrasian market. Further, in the equilibria we study, a seller in a non-monitored trade during the day will never have the opportunity to engage in a monitored trade during any subsequent day and so will be unable to signal that a default has occurred.

5.1 Credit Equilibrium

First consider stationary equilibria where money is not valued, so that all exchanges in the day market involve credit. Here, in the case where a buyer does not have the opportunity to engage in a monitored transaction, there will be no exchange between the buyer and the seller, as the buyer will be able to default and this will be private information. Thus if money is not valued, then trade takes place during the day only in monitored transactions, and the buyer will always weakly prefer to have the interaction with a seller monitored. Here, v is determined by

$$v = \beta \left\{ (1-\rho) \max_{l} \left[u(l) - l \right] + v \right\} \qquad (13)$$

subject to the incentive constraint

$$l \leq v - \hat{v},$$

and

$$l \geq 0.$$

As in the previous section, $\hat{v}$ is the continuation value when punishment occurs, and the punishment is autarky so $\hat{v} = 0$. Now, letting $\phi_c(v)$ denote the right-hand side of (13), we can rewrite (13) as

$$v = \phi_c(v),$$

with

$$\phi_c(v) = \beta \left[(1-\rho) u(v) + \rho v \right], \text{ for } 0 \leq v \leq q^*$$

and

$$\phi_c(v) = \beta \left\{ (1-\rho) [u(q^*) - q^*] + v \right\}, \text{ for } v \geq q^*.$$

Let q^{***} denote the solution to

$$\frac{\beta(1-\rho)}{1-\rho\beta} u(q^{***}) = q^{***}$$

Proposition 5 *If $q^{***} < q^*$ then there are two credit equilibria, one where $v = 0$, and one where the incentive constraint binds, $l < q^*$ and $v = q^{***}$.*

Proof. Define $\Gamma_c(v) = \phi_c(v) - v$. Note that $\Gamma_c(\cdot)$ is continuous and $\lim_{v \to \infty} \Gamma_c(v) = -\infty$. Since $q^{***} < q^*$, it follows that $\Gamma_c(v) > 0$ for all $v \in (0, q^{***})$, $\Gamma_c(q^{***}) = 0$, and $\Gamma_c(v) < 0$ for all $v \in (q^{***}, \infty)$. This implies that $v = q^{***}$ is the unique positive value satisfying $\Gamma_c(v) = 0$. Since the incentive constraint binds, it follows that $l = q^{***}$ in such equilibrium. ■

Proposition 6 *If $q^{***} \geq q^*$ then there are two credit equilibria, one where $v = 0$ and one where the incentive constraint does not bind, $l = q^*$, and*

$$v = \frac{\beta(1-\rho)[u(q^*) - q^*]}{1-\beta}.$$

Proof. In this case, $\Gamma_c(v) > 0$ for all $v \in (0, q^*)$ and $\Gamma_c(q^*) \geq 0$. Note that $\Gamma_c(\cdot)$ is strictly decreasing on (q^*, ∞), with $\lim_{v \to \infty} \Gamma_c(v) = -\infty$. Hence, there exists a unique positive value $v \geq q^*$ satisfying $\Gamma_c(v) = 0$. This means that

$$\beta \left\{ (1-\rho) \left[u(q^*) - q^* \right] + v \right\} - v = 0,$$

so that

$$v = \frac{\beta(1-\rho)}{1-\beta}\left[u(q^*) - q^*\right],$$

and $l = q^*$ is the amount consumed by the buyer in a monitored meeting during the day. ■

Now, since $q^{***} < q^{**}$ for $\rho > 0$, imperfect memory limits credit market activity, just as one might expect. Relative to the credit equilibrium with perfect memory there is in general less trade in a credit equilibrium with imperfect memory, and the quantity traded decreases as ρ increases. Of course, there is no credit market activity when $\rho = 1$.

5.2 Monetary Equilibrium

As in the previous section, publicly observable default triggers autarky for the agent who defaults. However, in this case autarkic punishment is carried out through a global punishment whereby, if a single buyer defaults, this triggers an equilibrium where no seller will trade during the day and therefore money is not valued.

Here, we solve for the equilibrium continuation value in a similar fashion to the previous section. That is,

$$v = \max_{m,l}\left(-m + \beta\left\{\rho u\left(\frac{m}{\mu}\right) + (1-\rho)\left[u\left(\frac{m}{\mu}+l\right) - l\right] + \gamma + v\right\}\right) \qquad (14)$$

subject to

$$l \leq \gamma + v - \hat{v},$$

$$l \geq 0.$$

Given autarkic punishment, $\hat{v} = 0$. In equilibrium, the real value of the government transfer is

$$\gamma = m\left(1 - \frac{1}{\mu}\right). \qquad (15)$$

Here, and in the rest of the paper, it will prove to be more straightforward to define an equilibrium and solve for it in terms of the consumption quantities for the buyer in non-monitored and monitored trades, rather than solving for the continuation value v. Therefore, let x be the daytime consumption of a buyer in the non-monitored state, and y the buyer's daytime consumption in the monitored state. Then in the problem (14) above, we have $m = \mu x$, $\gamma = x(\mu - 1)$, and $l = y - x$. Thus from (14), we can solve for v in terms of x and y to get

$$v = -\mu x + \frac{\beta\left\{\rho\left[u(x) - x\right] + (1-\rho)\left[u(y) - y\right]\right\}}{1-\beta}.$$

We can then define an equilibrium in terms of x and y as follows.

Definition 7 *A stationary monetary equilibrium is a pair* (x, y)*, where* x *and* y *are chosen optimally by the buyer,*

$$\rho u'(x) + (1-\rho) u'(y) = \frac{\mu}{\beta}, \tag{16}$$

x *and* y *have the property that consumptions and the loan quantity are nonnegative, and consumptions do not exceed the surplus-maximizing quantity,*

$$0 \leq x \leq y \leq q^*, \tag{17}$$

and (x, y) *is incentive compatible,*

$$\beta [\rho u(x) + (1-\rho) u(y)] - \rho\beta x - (1-\rho\beta) y \geq (1-\beta)\hat{v}, \tag{18}$$

where $y = q^*$ *if (18) does not bind.*

Proposition 8 *If* $q^{**} \geq q^*$ *then a unique stationary monetary equilibrium exists for* $\mu \geq \beta$*.*

Proof. Suppose $q^* \leq q^{***} \leq q^{**}$. In this case, we cannot have an equilibrium with a binding incentive constraint. Now, if the incentive constraint does not bind, then $y = q^*$ and

$$\rho u'(x) + 1 - \rho = \frac{\mu}{\beta}. \tag{19}$$

Note that $(1-\rho)\beta u(q^*) - (1-\rho\beta) q^* \geq 0$, so that the incentive constraint is always slack when $y = q^*$. Therefore, a unique stationary monetary equilibrium with a non-binding incentive constraint exists for any $\mu \geq \beta$, with x defined by (19) and $y = q^*$.

Suppose $q^{***} < q^* \leq q^{**}$. First, assume the incentive constraint does not bind. Then, there exists $\tilde{\mu} > \beta$ such that

$$\rho\beta [u(x) - x] \geq -(1-\rho)\beta u(q^*) + (1-\rho\beta) q^*$$

if and only if $\mu \in [\beta, \tilde{\mu}]$. Again, a unique stationary monetary equilibrium with a non-binding incentive constraint exists for $\mu \in [\beta, \tilde{\mu}]$. Let $\tilde{x}$ be the value of x satisfying (19) when $\mu = \tilde{\mu}$. For $\mu > \tilde{\mu}$, the incentive constraint binds, and a unique stationary monetary equilibrium exists with (x, y) satisfying

$$\rho\beta [u(x) - x] = -(1-\rho)\beta u(y) + (1-\rho\beta) y \tag{20}$$

and

$$\rho u'(x) + (1-\rho) u'(y) = \frac{\mu}{\beta}, \tag{21}$$

where $x < \tilde{x}$ and $q^{***} < y < q^*$. ■

Proposition 9 *If* $q^{**} < q^*$ *then a unique stationary monetary equilibrium exists for* $\mu \geq \beta u'(q^{**})$*.*

Proof. Note that we cannot have an equilibrium with a non-binding incentive constraint because

$$\rho\beta\left[u\left(x\right)-x\right]<-\left(1-\rho\right)\beta u\left(q^{*}\right)+\left(1-\rho\beta\right)q^{*}$$

when $q^{**}<q^{*}$. Then, (x,y) satisfy (20) and (21), with $0\leq x\leq y<q^{*}$. Note that (20) requires that $y\leq q^{**}$. Then, a unique stationary monetary equilibrium exists for any $\mu\geq\beta u'\left(q^{**}\right)$. ■

If $q^{***}\geq q^{*}$, which guarantees that $q^{**}>q^{*}$, as $q^{***}<q^{**}$, then the incentive constraint does not bind for all $\mu\geq\beta$. In this case, the buyer consumes q^{*} in all monitored trades where credit is used during the day, and consumes x in nonmonitored trades, where $x\leq q^{*}$ and x is decreasing in μ. Therefore, the welfare of the buyer is decreasing in μ, while the seller receives zero utility in each period for all μ. Further, when $\mu=\beta$, then $x=y=q^{*}$, in which case the loan quantity is $l=y-x=0$, and no credit is used. As μ increases, then, the quantity of credit rises, that is credit is substituted for money in transactions.

If $q^{**}\geq q^{*}>q^{***}$, then the incentive constraint binds for $\mu>\tilde{\mu}$, where

$$\rho\beta\left[u\left(\tilde{x}\right)-\tilde{x}\right]=-\beta(1-\rho)u(q^{*})+(1-\rho\beta)q^{*}$$

with $\tilde{x}$ the solution to

$$\rho u'\left(\tilde{x}\right)+1-\rho=\frac{\tilde{\mu}}{\beta}.$$

The incentive constraint does not bind for $\beta\leq\mu\leq\tilde{\mu}$. Here, $\mu=\beta$ implies that $x=y=q^{*}$ and there is no credit, just as in the previous case. However, if the money growth rate is sufficiently high, then the incentive constraint binds. If the incentive constraint does not bind, then just as in the previous case $y=q^{*}$ and x falls as μ rises, so that the welfare of buyers falls with an increase in μ and credit is substituted for money in transactions. If the incentive constraint binds, then it is straightforward to show that an increase in μ causes both x and y to fall, with the loan quantity $l=y-x$ increasing. Thus, as in the other cases, the welfare of buyers must fall as μ rises, and the use of credit rises with an increase in the money growth rate.

Finally, if $q^{**}<q^{*}$, then the incentive constraint will always bind in a stationary monetary equilibrium. Here, when $\mu=\beta u'(q^{**})$, then $x=y=q^{**}$ and there is no credit. Again, it is straightforward to show that x, y, and the welfare of buyers decrease with an increase in μ, and the quantity of lending rises.

Which case we get (the incentive constraint never binds; the incentive constraint binds only for large money growth rates; the incentive constraint always binds) depends on $q^{*}-q^{***}$. While q^{*} is independent of β and ρ, q^{***} is increasing in β and decreasing in ρ. Thus, the incentive constraint will tend to bind the lower is β and the higher is ρ. Higher β tends to relax incentive constraints for typical reasons. That is, as buyers care more about the future, potential punishment is more effective in enforcing good behavior. Higher ρ implies that the imperfect memory friction becomes more severe, and credit can be used with lower frequency. In general, monetary exchange will be less efficient than credit, and so a reduction in the frequency with which credit can be used will tend to reduce the utility of a buyer in equilibrium. This will therefore reduce the relative punishment to a buyer if he or she defaults and thus tighten incentive constraints.

Proposition 10 *If $q^{**} \geq q^*$, then $\mu = \beta$ is optimal, and this implies that $l = 0$, the incentive constraint does not bind, and the buyer consumes q^* in all trades during the day.*

Proof. Suppose the government treats buyers and sellers equally. Then, the government chooses a money growth rate $\mu \geq \beta$ to maximize $\rho\left[u\left(x\right) - x\right] + (1-\rho)\left[u\left(y\right) - y\right]$ subject to (16), (17), and (18). It follows that $\mu = \beta$ implies $x = y = q^*$, and the efficient allocation under full commitment is implemented. ■

Proposition 11 *If $q^{**} < q^*$, then $\mu = \beta u'(q^{**})$ is optimal, and this implies that $l = 0$, the incentive constraint binds, and the buyer consumes q^{**} in all trades during the day.*

Proof. If $q^{**} < q^*$, the incentive constraint requires that $y \leq q^{**}$. It follows from (20) and (21) that setting $\mu = \beta u'(q^{**})$ implements the efficient allocation (q^{**}, q^{**}). ■

Here, we have essentially generalized the results of [3] to the case where credit is permitted in some types of bilateral trades. If the discount factor is sufficiently small, then the Friedman rule is not feasible and the incentive constraint binds at the optimum. In terms of our goal of constructing a model with robust money and credit, an undesirable feature of this setup is that optimal monetary policy drives credit out of the economy. Here, the only inefficiency in monetary exchange is due to the fact that buyers in general hold too little real money balances in equilibrium, and this inefficiency can be corrected in the usual way, with the caveat that too much deflation can cause agents to default on their tax liabilities. Ultimately, at the optimum money is equivalent to memory, in that an appropriate monetary policy achieves the same allocation that could be achieved by a social planner with perfect record keeping.

6 Imperfect Memory and Non-Autarkic Punishment

In the previous section, given the limited commitment friction, optimal monetary policy will yield an equilibrium allocation where credit is not used. Credit seems to be more robust than this in practice, so we would like to study frictions that potentially imply that money and credit coexist, even when monetary policy is efficient.

Here, we will assume the same information technology and bargaining protocol as in the previous section. However, we will consider a different equilibrium, where default does not trigger autarky, but instead triggers an equilibrium where money is valued. That is, a default results in reversion to an equilibrium where sellers will not trade if a buyer announces that he or she wishes the interaction to be monitored, but will exchange goods for money if the buyer announces that the interaction will not be monitored. The government is not able to commit to a monetary policy, so the money growth rate that is chosen by the government when punishment occurs is chosen optimally at that date given the behavior of private sector agents.

We restrict attention to punishment equilibria that are stationary. Further, a punishment equilibrium must be sustainable, in that no agent would choose to default on his or her tax liabilities in such an equilibrium. Letting $\hat{v}$ denote the continuation value in the punishment equilibrium, after agents receive their lump-sum transfers from the government, we have

$$\hat{v} = -m(\mu) + \beta \left[u\left(\frac{m(\mu)}{\mu}\right) + m(\mu)\left(1 - \frac{1}{\mu}\right) + \hat{v} \right]$$

where $m(\mu)$ is the quantity of real balances acquired by the buyer during the night, which solves the first-order condition

$$u'\left(\frac{m(\mu)}{\mu}\right) = \frac{\mu}{\beta}. \tag{22}$$

Now, for the punishment equilibrium to be sustainable, we require that

$$m(\mu)\left(1 - \frac{1}{\mu}\right) + \hat{v} \geq \hat{v}, \tag{23}$$

i.e. the equilibrium is sustained in the sense that, if an agent chooses not to accept the transfer from the government, then the punishment is reversion to the punishment equilibrium. Clearly, condition (23) implies that punishment equilibria are sustainable if and only if $\mu \geq 1$. That is, private agents need to be bribed to enforce the punishment with positive transfers, otherwise they would default on the tax liabilities.

The government will choose μ optimally in the punishment equilibrium, and it must choose a sustainable money growth factor, i.e. $\mu \geq 1$. Assume that the government weights the utility of buyers and sellers equally, though since sellers receive zero utility in any punishment equilibrium, it is only the buyers that matter. Therefore, the government solves

$$\max_{\mu} \left[u\left(\frac{m(\mu)}{\mu}\right) - \frac{m(\mu)}{\mu} \right]$$

subject to (22) and $\mu \geq 1$. Clearly, the solution is $\mu = 1$, so we have

$$\hat{v} = \frac{-\hat{m} + \beta u(\hat{m})}{1 - \beta}, \tag{24}$$

where $\hat{m}$ solves

$$u'(\hat{m}) = \frac{1}{\beta} \tag{25}$$

When punishment occurs, the government would like to have been able to commit to an infinite growth rate of the money supply so as to make the punishment as severe as possible. However, given the government's inability to commit, once punishment is triggered the government chooses the sustainable money growth rate that maximizes welfare, consistent with the optimal punishment behavior of sellers in the credit market. Thus, the money growth rate is set as low as possible without inducing default on tax liabilities.

Now that we have determined the continuation value in a punishment equilibrium, we can work backward to determine what the equilibrium can be. For this purpose, we again define the stationary equilibrium in terms of (x, y), where x denotes the daytime consumption of a buyer in the non-monitored state, and y the buyer's daytime consumption in the monitored state. The definition of a stationary monetary equilibrium is the same as in the previous section, except now $\hat{v}$ is defined by (24) and (25).

Proposition 12 *The only monetary equilibrium is the punishment equilibrium.*

Proof. First, suppose that $y > x$ in equilibrium. Then, using Jensen's inequality,

$$\beta[\rho u(x)+(1-\rho)u(y)]-\rho\beta x-(1-\rho\beta)y < \beta u\left[\rho x + (1-\rho)y\right]-\rho x-(1-\rho)y \leq -\hat{m}+\beta u(\hat{m}),$$

by virtue of (25). Thus, given that an equilibrium must satisfy (18), we have a contradiction. Therefore, if an equilibrium exists, it must have $y = x$, in which case inequality (18) can be written, using (24),

$$-x + \beta u(x) \geq -\hat{m} + \beta u(\hat{m}),$$

but then by virtue of (25), (24) can only be satisfied, with equality, when $x = y = \hat{m}$, and this can be supported, from (16) only if the money growth factor is $\mu = 1$. ■

Therefore, the only monetary equilibrium with non-autarkic punishment is one where no credit is supported. The incentive constraint is satisfied with equality and no seller is willing to lend to a borrower, even if the interaction is monitored. The optimal money growth factor, indeed the only feasible money growth factor, is $\mu = 1$.

Intuition might tell us that, in line with some of the ideas in [2] and [4], that the possibility of being banned from credit markets but with punishment mitigated by the ability to trade money for goods, would tend to promote credit. That is, because the degree of punishment depends on money growth, the government might tend to produce inflation so as to increase the punishment for bad behavior in the credit market, thus reducing the payoff to holding money and causing buyers to substitute credit for money. In the context of this model, this intuition is wrong, in part because we take account here of the government's role as a strategic player, and its inability to commit to inflicting punishment.

Thus far, we have not arrived at a set of assumptions concerning the information structure under which credit is robust. Either efficient monetary policy will drive credit out of the system, or the only equilibrium that exists is one without credit. Thus, it appears that there must be another friction or frictions that are necessary to the coexistence of robust money and credit that we observe in reality.

7 Theft

One aspect of monetary exchange is that, due to anonymity, theft is easier in most respects than it is with exchange using credit. It seems useful to consider a framework where limited commitment makes credit arrangements difficult, and theft makes monetary exchange difficult. However, the fact that theft makes monetary exchange difficult may lessen the limited commitment friction in the credit market, as this will make default less enticing.

We will assume the same imperfect memory structure as in the previous section, but allow for a technology that permits the theft of cash. Suppose the following bargaining protocol. On meeting a seller in the daytime, the buyer first announces whether his or her interaction with the seller will be monitored or not. Recall that it is necessary that the seller have the potential for monitoring (occurring with probability $1-\rho$ from the buyer's point of view) in order for the interaction to be monitored. Then, the seller announces whether or not he or she is willing to trade. Following this, if the interaction is not monitored, the seller can pay a fixed cost τ to acquire a technology (a "gun"), which permits him or her to confiscate the buyer's money, if the buyer has any. Clearly, if the buyer's money is stolen in a non-monitored trade, the interaction with the seller ends there. Otherwise, the buyer makes a take-it-or-leave-it offer to the seller if the seller has agreed to trade.

With theft, an equilibrium can be characterized by (x, y, α) where, as before, x is consumption by the buyer in a non-monitored trade when theft does not occur, y is consumption when monitored, and α is the fraction of non-monitored daytime meetings where theft occurs, so that $\alpha \in [0,1]$. In general, given the continuation value $\hat{v}$ in the punishment equilibrium, we can define a monetary equilibrium as follows.

Definition 13 *A monetary equilibrium is a triple (x, y, α), where x and y are chosen optimally by the buyer,*

$$\rho(1-\alpha)u'(x) + (1-\rho)u'(y) = \frac{\mu}{\beta}, \tag{26}$$

x and y have the property that consumptions and the loan quantity are nonnegative, and consumptions do not exceed the surplus-maximizing quantity,

$$0 \leq x \leq y \leq q^*, \tag{27}$$

(x, y, α) is incentive compatible,

$$\beta[\rho(1-\alpha)u(x) + (1-\rho)u(y)] - \rho\beta x - (1-\rho\beta)y \geq \hat{v}(1-\beta), \tag{28}$$

where $y = q^*$ if (28) does not bind. Further, x and α must be consistent with optimal theft by sellers in non-monitored trades, that is

$$\text{if } \alpha = 0, \text{ then } x \leq \tau, \tag{29}$$

$$\text{if } 0 < \alpha < 1, \text{ then } x = \tau, \tag{30}$$

$$if \ \ \alpha = 1, \text{ then } x \geq \tau. \tag{31}$$

Conditions (29)-(31) state that in equilibrium there is either no theft, so sellers must weakly prefer not to steal in non-monitored trades, or sellers sometimes steal, so they must be indifferent to being honest, or sellers always steal, so they must weakly prefer theft.

Now, the government will choose μ so as to maximize welfare in equilibrium, where the utilities of sellers and buyers are weighted equally. Thus, in the stationary equilibria we study, the government wishes to maximize

$$W = \rho(1-\alpha)\left[u(x) - x\right] + (1-\rho)\left[u(y) - y\right] - \alpha\rho\tau$$

Lemma 14 *When the government chooses μ optimally, $\alpha = 0$.*

Proof. First, suppose that there exists an equilibrium with $\alpha = 1$, $y = \bar{y} < q^*$ and $x > \tau$, supported by $\mu = \bar{\mu}$. Then from the definition of equilibrium, we can construct another equilibrium with $\alpha < 1$, $y > \bar{y}$ and $x = \tau$, supported by some $\mu > \bar{\mu}$. In this other equilibrium, W must be larger. If there exists an equilibrium with $\alpha = 1$, $y < q^*$ and $x = \tau$ in equilibrium, we can accomplish the same thing except by holding x constant at τ. Similarly if $\alpha = 1$ and $y = q^*$ the same argument applies except that we do not increase y. Next, if $0 < \alpha < 1$ in equilibrium, we can construct another equilibrium with lower α, larger μ, and larger y if $y < q^*$ which achieves higher welfare. ■

A smaller amount of theft necessarily increases the continuation value for the buyer and relaxes the incentive constraint, while increasing welfare. A smaller amount of theft can be achieved in this fashion as an equilibrium outcome with a higher money growth rate. The higher money growth rate discourages the holding of currency, and therefore reduces the payoff from theft. Note that this is true no matter what $\hat{v}$ is. Irrespective of the punishment that is imposed when a buyer defaults, efficient monetary policy must always drive out theft.

7.1 Autarkic Punishment

First, consider the case where default triggers autarky. In determining what is optimal for the government in this context, we know from the above arguments that we can restrict attention to equilibria where $\alpha = 0$, and search among these equilibria for the one that yields the highest welfare. The government then solves the following problem:

$$\max_{x,y,\mu} \{\rho [u(x) - x] + (1 - \rho) [u(y) - y]\} \tag{32}$$

subject to

$$\rho u'(x) + (1 - \rho)u'(y) = \frac{\mu}{\beta}, \tag{33}$$

$$x \leq \tau \tag{34}$$

$$0 \leq x \leq y \leq q^* \tag{35}$$

$$\beta[\rho u(x) + (1 - \rho)u(y)] - \rho\beta x - (1 - \rho\beta)y \geq 0, \tag{36}$$

where $y = q^*$ if the last constraint does not bind. We first have the following results.

Proposition 15 *If $q^* \leq q^{**}$ and $\tau \geq q^*$, then a Friedman rule is optimal, and this supports an efficient allocation in equilibrium.*

Proof. Suppose that we ignore the constraint (34) in the government's optimization problem. If $q^* \leq q^{**}$, then the solution to the problem is $x = y = q^*$ and $\mu = \beta$, i.e. the solution is what we obtained when we studied non-autarkic punishment with the same setup and no theft technology. However, for the constraint (34) not to bind at the optimum then requires $\tau \geq q^*$. ■

Proposition 16 *If $q^* > q^{**}$ and $\tau \geq q^{**}$, then $\mu = \beta u'(q^{**})$ at the optimum, and this supports an efficient allocation in equilibrium.*

Proof. Again, suppose that we ignore the constraint (34) and solve the government's optimization problem in the case where $q^* > q^{**}$. Then the solution to the problem is $x = y = q^{**}$ and $\mu = \beta u'(q^{**})$, i.e. the solution is what we obtained when we studied non-autarkic punishment with the same setup and no theft technology. Now, for the constraint (34) not to bind at the optimum requires $\tau \geq q^{**}$. ■

Thus, as should be obvious, if the cost of theft is sufficiently large that theft does not take place in equilibrium given the efficient monetary policy rules we derived in the absence of theft, then theft is irrelevant for policy. Of course, our interest is in what happens when theft is sufficiently lucrative, i.e. when τ is sufficiently small that (34) binds at the optimum.

Now, since $x = \tau$ at the optimum when theft matters, this makes solving the government's optimization problem easy. First, suppose that $q^* \leq q^{***} \leq q^{**}$ in which case theft matters if and only if $\tau \leq q^*$. Then $(x, y) = (\tau, q^*)$ must be optimal, as this satisfies (36) as a strict inequality, (35) is satisfied, and we can recover the money growth factor that supports this as an equilibrium from (33), i.e.

$$\mu = \beta [\rho u'(\tau) + 1 - \rho]. \tag{37}$$

Note that the optimal money growth rate rises as the cost of theft falls, as a lower cost of theft requires a higher money growth rate to drive out theft. An interesting feature of the efficient equilibrium is that money and credit now coexist. Indeed, the loan quantity is $l = q^* - \tau$, which increases as the cost of theft decreases. Essentially, money and credit act as substitutes. As the theft friction gets more severe, money becomes more costly to hold at the optimum (the optimal money growth rate rises), and buyers use credit more intensively.

Now, suppose that $q^{***} < q^* \leq q^{**}$ in which case theft matters if and only if $\tau \leq q^*$. Let $\bar{\tau} < q^*$ be the unique value of τ satisfying

$$\rho\beta \left[u\left(\bar{\tau}\right) - \bar{\tau}\right] + \beta(1-\rho)u(q^*) - (1-\rho\beta)q^* = 0.$$

Then, for $\tau \in (0, \bar{\tau}]$ the incentive constraint binds, and the optimal equilibrium allocation is $(x, y) = (\tau, \bar{y})$, where $\bar{y}$ is the solution to

$$\rho\beta \left[u\left(\tau\right) - \tau\right] + \beta(1-\rho)u(\bar{y}) - (1-\rho\beta)\bar{y} = 0. \tag{38}$$

The optimal money growth factor in this case is

$$\mu = \beta \left[\rho u'(\tau) + (1-\rho)u'(\bar{y})\right]. \tag{39}$$

For $\tau \in [\bar{\tau}, q^*]$, the incentive constraint does not bind, and the optimal equilibrium allocation is $(x, y) = (\tau, q^*)$ with the optimal money growth factor given by

$$\mu = \beta \left[\rho u'(\tau) + 1 - \rho\right].$$

Clearly, given $q^{***} < q^* \leq q^{**}$, x and y both decrease as τ decreases, at the optimum, so that the welfare of buyers falls. Further, it is straightforward to show that the quantity of lending, $y - x$ increases as τ falls, at the optimum, so that less costly theft promotes credit. As well, the optimal money growth factor decreases as the cost of theft rises.

Finally, consider the case where $q^* > q^{**}$, in which case theft matters if and only if $\tau \leq q^{**}$. Here, the incentive constraint always binds, and $(x, y) = (\tau, \bar{y})$, where $\bar{y}$ is the solution to (38), and the optimal money growth factor is given by (39). Just as in the other cases, x and y fall as τ falls, at the optimum, and welfare decreases. As well, the quantity of lending rises as τ falls at the optimum.

7.2 Non-Autarkic Punishment

Recall that, with non-autarkic punishment we are looking for a sustainable punishment equilibrium in which, if a buyer meets a seller and announces that the interaction will be monitored, the seller will not trade. Money will be valued in the punishment equilibrium, but all transactions between buyers and sellers will be non-monitored ones. The government cannot commit to a monetary policy rule, so when default occurs the government will choose the money growth factor that maximizes welfare in the punishment equilibrium.

Through arguments identical to what we used previously when theft was not an issue, any sustainable punishment equilibrium must have $\mu \geq 1$, as buyers need to be bribed with a transfer to sustain the punishment. Note that we cannot have $\alpha = 1$ in the punishment equilibrium since, if all sellers steal, no buyer would accumulate money balances, but if no buyer accumulates money balances there will be no theft. Let x denote the buyer's daytime consumption in the punishment equilibrium. Then, the punishment equilibrium is the solution to the following problem.

$$\max_{x,\alpha,\mu}(1-\alpha)\left[u(x)-x\right]-\alpha\tau$$

subject to

$$(1-\alpha)u'(x)=\frac{\mu}{\beta}$$

$$0 \leq x \leq q^*$$

$$\alpha \in [0,1)$$

$$\mu \geq 1$$

$$\text{if } \alpha = 0, \text{ then } x \leq \tau$$

$$\text{if } \alpha > 0, \text{ then } x = \tau$$

Now, just as in the efficient equilibrium, it is straightforward to show that part of the solution to this problem is $\alpha = 0$. That is, if there is a sustainable equilibrium where $\alpha > 0$, then there is another equilibrium with a higher money growth factor, lower α, and higher welfare that is also sustainable. Given that $\alpha = 0$ is optimal (no theft in the punishment equilibrium), the government will choose the lowest money growth rate consistent with sustainability and no theft. Therefore, the solution to the above problem is

$$\text{If } \beta u'(\tau) \leq 1, \text{ then } x = \hat{m},\ \mu = 1, \text{ and } \hat{v} = \frac{\beta u(\hat{m}) - \hat{m}}{1-\beta}$$

$$\text{If } \beta u'(\tau) > 1, \text{ then } x = \tau,\ \mu = \beta u'(\tau), \text{ and } \hat{v} = \frac{\beta\left\{-\tau\left[(1-\beta)u'(\tau)+1\right]+u(\tau)\right\}}{1-\beta}$$

Here, recall that $u'(\hat{m}) = \frac{1}{\beta}$.

Now, suppose that $\beta u'(\tau) \leq 1$, that is $\tau \geq \hat{m}$. Then, given the same arguments as we used in the absence of the theft technology, the only incentive compatible equilibrium allocation is $x = y = \hat{m}$ and $\mu = 1$. Since $\tau \geq \hat{m}$, this is an equilibrium where there is only monetary exchange and no theft. It is identical to what we obtained when there was no theft technology.

The interesting case is the one where $\beta u'(\tau) > 1$, or $\tau < \hat{m}$. Here, in a manner similar to what we did in the last subsection, we are looking for an efficient equilibrium that is the solution to the government's problem

$$\max_{x,y,\mu}\left\{\rho\left[u(x)-x\right]+(1-\rho)\left[u(y)-y\right]\right\} \tag{40}$$

subject to

$$\rho u'(x) + (1-\rho)u'(y) = \frac{\mu}{\beta}, \tag{41}$$

$$x \leq \tau \tag{42}$$

$$0 \leq x \leq y \leq q^* \tag{43}$$

$$\rho\beta\left[u(x) - x\right] + \beta(1-\rho)u(y) - (1-\rho\beta)y \geq \beta\left\{-\tau\left[(1-\beta)u'(\tau) + 1\right] + u(\tau)\right\}, \tag{44}$$

Lemma 17 *If $\beta u'(\tau) > 1$, then with non-autarkic punishment, $x = \tau$ in an efficient equilibrium.*

Proof. Suppose not. Then, an increase in x will relax constraint (44), since $\tau < \hat{m}$. Therefore if there exists an equilibrium with $x < \tau$ and $y = q^*$, there exists another equilibrium with larger x and smaller μ such that the constraints in the above problem are all satisfied and the value of the objective function increases. Similarly, if there exists an equilibrium with $x < \tau$ and $y < q^*$, so that (44) holds with equality, then we can construct another equilibrium satisfying all of the constraints in the problem and increase the value of the objective function, simply because increasing x relaxes the incentive constraint and increases the value of the objective function, and we can find a value for μ that satisfies (41) and therefore supports this allocation as an equilibrium. ∎

Given the above lemma, we can write the incentive constraint (44) as

$$\beta(1-\rho)u(y) - (1-\rho\beta)y \geq \beta(1-\rho)[u(\tau) - \tau] - \beta(1-\beta)\tau u'(\tau) \tag{45}$$

Now, let $y(\tau)$ denote the value of y satisfying (45), given τ, where $y(\tau) = q^*$ if (45) does not bind. The function $y(\tau)$ is defined for $\tau \in [0, \hat{m}]$. We know that $y(0) = q^{***}$ and $y(\hat{m}) = \hat{m}$. Therefore, for example, if $q^{***} > \hat{m}$, then by continuity there are some values of τ for which a reduction in τ causes an increase in y. That is, a decrease in the cost of theft can increase the quantity of consumption in the monitored state, which makes this case much different from the one where the punishment equilibrium is autarky. It is straightforward to show that, if $\beta u'(\tau) > 1$, and $y = \tau$, then (45) is satisfied as a strict inequality, so that $y(\tau) > \tau$ for $\tau \in [0, \hat{m})$. Thus, as long as theft matters, an efficient equilibrium supports some credit, just as in the autarkic punishment case. Finally, the optimal money growth rate will be given by

$$\mu = \beta\left\{\rho u'(\tau) + (1-\rho)\, u'\left[y(\tau)\right]\right\} < \beta u'(\tau).$$

With non-autarkic punishment, theft acts as a disciplining device. The opportunities are greater for thieves in the punishment equilibrium, since all exchange is carried out using money. Thus, the government needs to inflate at a higher rate in order to drive out thieves, which makes the punishment more severe. The efficient money growth rate is always smaller than it is in the punishment equilibrium. Therefore, buyers who default not only give up access to credit markets, but they will have to face a higher inflation tax.

8 Conclusion

In determining the roles for money and monetary policy, it is important to analyze models with credit. Credit and outside money are typically substitutes in making transactions, and an important aspect of the effects of monetary policy may have to do with how central bank intervention works through credit market relationships. In the model studied in this paper, limited memory provides a role for money, as in much of the recent monetary theory literature, and does this by reducing the role for credit. This role for credit is further mitigated by limited commitment.

In this context, monetary policy works too well, in the sense that efficient monetary policy drives out credit. In reality, money and credit appear to be robust, in that it is hard to imagine an economy where there are not some transactions carried out with both money and credit. To obtain this robustness in our environment, it is necessary that there be some cost to operating the monetary system. The cost we choose to model is theft, as we think that theft, or the threat of theft, is likely an empirically significant cost associated with monetary exchange. If the cost of theft is small enough to matter, then money and credit always coexist under an optimal monetary policy, and a reduction in the cost of theft acts to increase lending in the economy, though this depends to some extent on how bad behavior in the credit market is punished. In general, the Friedman rule is not optimal given theft, and the optimal money growth rate tends to increase as the cost of theft falls.

For convenience, we have modeled monetary intervention by the central bank as occurring through lump-sum transfers. Though we have not shown this in the paper, we think that the results are robust to how money injections occur. For example, it should not matter if money is injected through central bank lending or open market purchases. In the latter case, of course, we would have to take a stand on why government bonds are not used in transactions.

This model should be useful for evaluating the performance of monetary policy in the context of aggregate shocks. As well we could easily consider other types of costs of operating a monetary system, including counterfeiting, the costs of deterring counterfeiting, or the costs of replacing worn currency.

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Figure 1: Efficiency when q** > q*

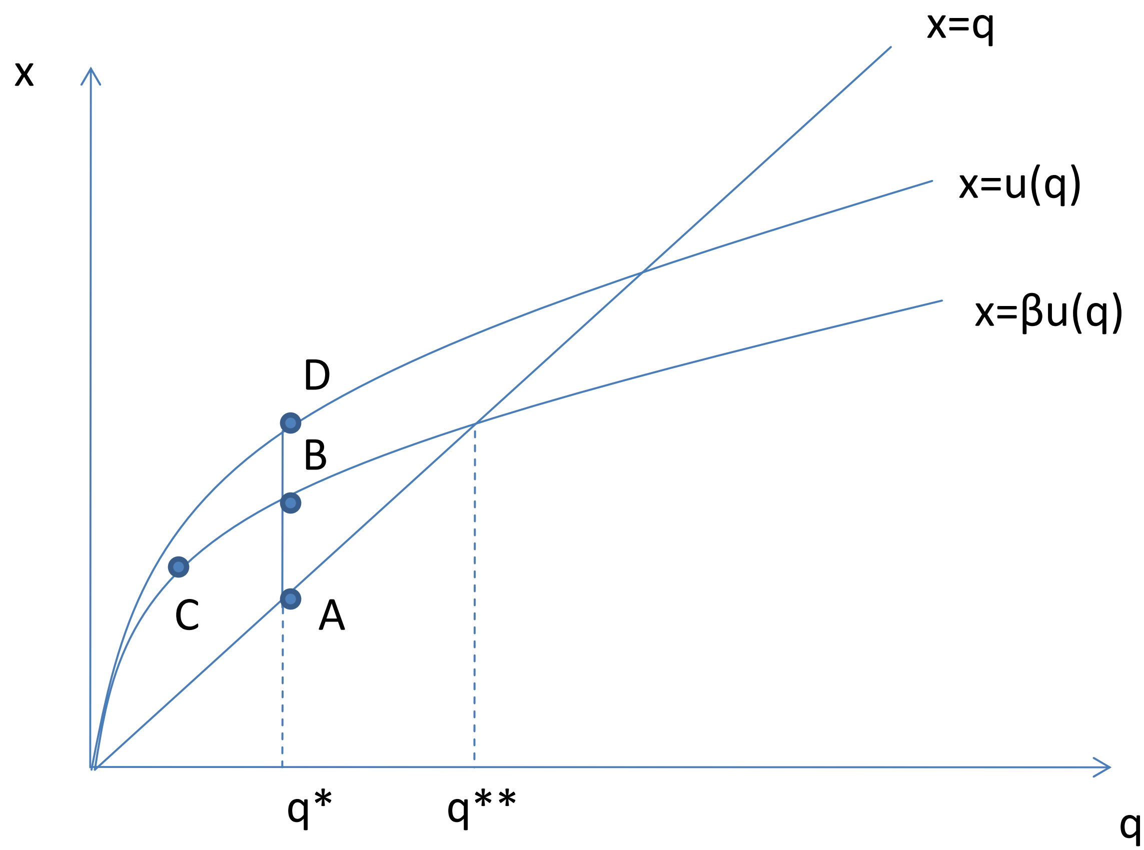


Figure 2: Efficiency when q** < q*

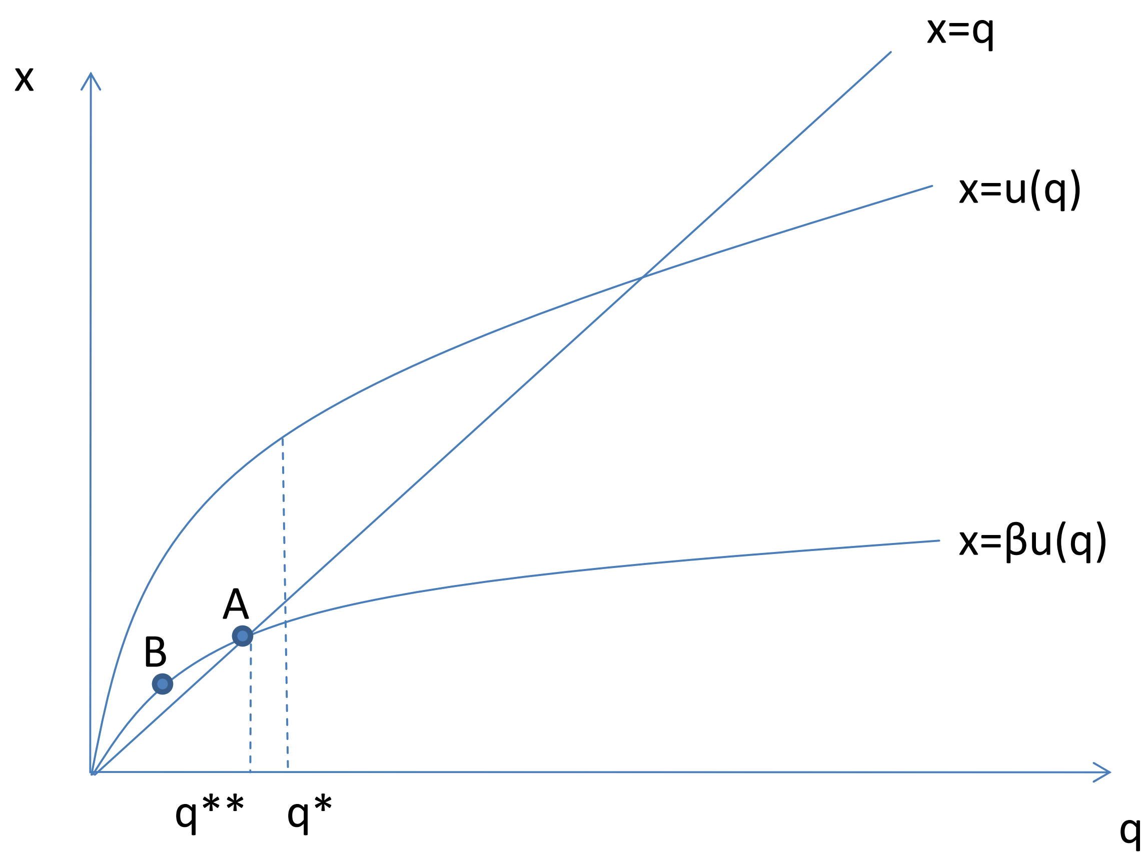
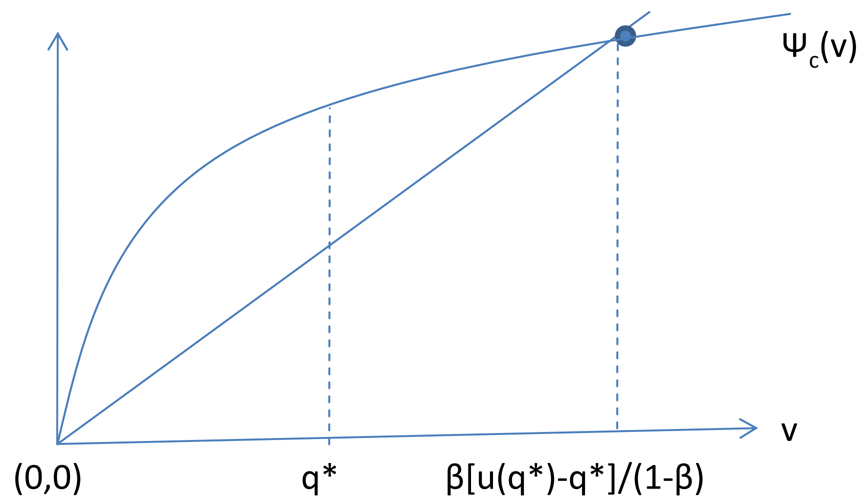


Figure 3: Equilibrium with Credit

(a) q** > q*



(b) q** < q*