Money and Credit with Limited Commitment and Theft

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Symposium on Search Theory - Yale University and Cowles Foundation

September 2009
Introduction

- Understand the roles of money and credit in transactions, and the interaction between these alternative means of payment.
- Construct a model where money and credit are both “robust”.
- Show that, for robustness, recordkeeping must be imperfect and there must be costs to using money - consider theft.
- Given this, what are the implications for monetary policy?

Typical result: Efficient monetary policy drives out any transactions role for credit - no costs associated with monetary exchange.

Money and credit seem to be robust means of payment in the sense that we observe both monetary exchange and credit transactions under a wide array of technologies and monetary policy rules.

We show that limited commitment, imperfect recordkeeping, and theft are sufficient to account for the coexistence of money and credit as media of exchange.
Rocheteau and Wright (2005).

Time is discrete and continues forever.

Two subperiods: day and night.

Two types of agents: buyers and sellers.

Continuum of each type with measure one.

Buyers want to consume during the day but can produce only at night.

Sellers can produce during the day but want to consume at night.
Model

- Technology: one unit of labor produces one unit of the unique perishable consumption good.
- Day subperiod: each buyer is randomly matched with a seller.
- Night subperiod: Walrasian market.
Model

- Buyer has preferences given by:
  \[ u(q_t) - n_t. \]

- Assume \( u : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) is increasing, strictly concave, and continuously differentiable, with \( u(0) = 0 \) and \( u'(0) = \infty \).

- Seller has preferences given by:
  \[ -l_t + x_t. \]

- Common discount factor \( \beta \in (0, 1) \) over periods.
Planner’s Problem under Full Commitment

- Restrict attention to stationary allocations.
- A stationary allocation is a pair $(q, x)$, with $q$ being the seller’s production during the day and $x$ the buyer’s production at night.
- Agents can commit to the planner’s proposed allocation at date $t = 0$.
- Participation constraints:
  
  $$u(q) - x \geq 0$$

  and

  $$-q + x \geq 0.$$

- Efficient allocations satisfy these constraints and $q = q^*.$
Add incentive constraints:

\[-x + v \geq 0\]

and

\[-q + x + w \geq 0,\]

where \(v\) and \(w\) are the buyer’s and seller’s continuation values.

Simplifying, we obtain

\[q \leq x \leq \beta u(q).\]

\(q^*\) solves \(u'(q^*) = 1.\)

\(q^{**}\) solves \(q^{**} = \beta u(q^{**}).\)

If \(\beta\) is close to one, \(q^* \leq q^{**}.\) If \(\beta\) is small, \(q^{**} < q^*.\)

Limited commitment matters if and only if \(q^{**} < q^*.\)
Figure 1: Efficiency when $q^{**} > q^*$
Figure 2: Efficiency when $q^{**} < q^{*}$
Figure 3: Equilibrium with Credit

(a) $q^{**} > q^{*}$

(b) $q^{**} < q^{*}$
Equilibrium Allocations with Perfect Memory

- Bargaining protocol in the DM.
  - Buyer and seller announce willingness to trade.
  - Buyer makes a take-it-or-leave-it offer.

- No bargaining inefficiencies.
- Each buyer is endowed with one unit of fiat money at the beginning of the first day.
- Money stock grows at gross rate $\mu \geq \beta$.
- New money is injected through lump-sum transfers in the CM at night.
- The interactions of each buyer with the government is publicly observable.
Limited commitment with respect to tax liabilities as well as private liabilities (Andolfatto, 2008).

Default by any buyer triggers global autarky.

Bellman equation for the buyer:

\[ v = \max_{m,l} \left\{ -m + \beta \left[ u \left( \frac{m}{\mu} + l \right) - l + \gamma + v \right] \right\}, \]

subject to

\[-l + \gamma + v \geq 0\]

and

\[ l \geq 0.\]
Case $q^* \leq q^{**}$.

- If $\mu > \beta$, there is no monetary equilibrium.
- If $\mu = \beta$, there is a continuum of monetary equilibria indexed by $m$.

Case $q^* > q^{**}$.

- If $\mu > \beta u'(q^{**})$, there is no monetary equilibrium.
- If $\mu = \beta u'(q^{**})$, there is a continuum of monetary equilibria indexed by $m$.

Money is held only under the Friedman rule or Andolfatto rule.

Results consistent with Kocherlakota (1998):

- No social role for money under perfect memory.
- Limited commitment is insufficient to make money useful for society.
Fraction $\rho$ of sellers cannot be monitored.

Fraction $1 - \rho$ of sellers has monitoring potential.

Bargaining protocol.

- Agents announce willingness to trade.
- If there is a monitoring opportunity, the buyer chooses whether the interaction will be monitored.
- Seller announces willingness to trade.
- Buyer makes a take-it-or-leave-it offer.

The interaction of each agent with the government is publicly observable.
A monetary equilibrium is a pair \((x, y)\) satisfying (i) optimality,

\[ \rho u'(x) + (1 - \rho) u'(y) = \frac{\mu}{\beta}, \]

(ii) nonnegativity of consumption and loan quantity,

\[ 0 \leq x \leq y \leq q^*, \]

and (iii) incentive compatibility,

\[ \beta \rho [u(x) - x] + (1 - \rho) \beta u(y) - (1 - \beta \rho) y \geq (1 - \beta) \hat{v}, \]

where \(y = q^*\) if the IC does not bind.

Default on either tax or private liabilities triggers global autarky, so that \(\hat{v} = 0\).
Results:

- Money and credit coexist for $\mu > \beta u'[\min(q^*, q^{**})]$.
- $\mu = \beta u'[\min(q^*, q^{**})]$ is optimal and eliminates credit.
- Credit is not robust, and an efficient economy runs without it.
- At the optimum, money is memory - it recovers the solution to the planner’s problem under perfect memory.
Punishment equilibrium involves monetary exchange only.
Default triggers equilibrium in which each seller refuses trade if the buyer wants monitoring.
Punishment equilibrium must be sustainable - no incentive to default on tax liabilities.
The buyer’s continuation value $\hat{v}$ in the punishment equilibrium satisfies:

$$\hat{v} = -m(\mu) + \beta \left\{ u \left[ \frac{m(\mu)}{\mu} \right] + \left( 1 - \frac{1}{\mu} \right) m(\mu) + \hat{v} \right\},$$

where $m(\mu)$ is given by

$$u' \left[ \frac{m(\mu)}{\mu} \right] = \frac{\mu}{\beta}.$$

Sustainability requires

$$\left( 1 - \frac{1}{\mu} \right) m(\mu) + \hat{v} \geq \hat{v}.$$
As a result, we must have $\mu \geq 1$ in the punishment equilibrium.

Government cannot commit to its future promises.

If the punishment equilibrium is triggered, the government chooses $\mu$ optimally.

Results:

- Unique stationary equilibrium is the monetary equilibrium with $\mu = 1$ (in fact, incentive constraint is satisfied if and only if $\mu = 1$).
- No credit activity.
So far, there is no cost to monetary exchange other than the implicit foregone interest.

A sufficiently high rate of return on money eliminates the implicit opportunity cost, and money drives out credit.

In reality, there are costs to monetary exchange, such as counterfeiting, risk of loss, costs of maintaining the currency stock, and theft.

We choose to study theft.

\( \tau = \text{cost of theft for a seller in the DM} \) - a seller can only steal in a non-monitored transaction.

\( \alpha = \text{fraction of non-monitored transactions in which theft occurs.} \)
A monetary equilibrium is a list $(x, y, \alpha)$ satisfying (i) optimality,

$$\rho(1 - \alpha)u'(x) + (1 - \rho)u'(y) = \frac{\mu}{\beta},$$

(ii) nonnegativity of consumption and loan quantity,

$$0 \leq x \leq y \leq q^*,$$

(iii) incentive compatibility,

$$\beta[\rho(1 - \alpha)u(x) + (1 - \rho)u(y)] - \rho\beta x - (1 - \rho\beta)y \geq \hat{\nu}(1 - \beta),$$

with $y = q^*$ if IC does not bind, and (iv) optimal theft by sellers,

if $\alpha = 0$, then $x \leq \tau$,

if $0 < \alpha < 1$, then $x = \tau$,

if $\alpha = 1$, then $x \geq \tau$. 
The government chooses \((x, y, \alpha, \mu)\) to maximize welfare

\[ W = \rho(1 - \alpha)[u(x) - x] + (1 - \rho)[u(y) - y] - \rho\alpha \tau, \]

subject to the constraint that \((x, y, \alpha)\) is a monetary equilibrium given \(\mu\).

Solution has \(\alpha = 0\).

Equilibria in which theft occurs are suboptimal.

Efficient monetary policy must always drive out theft.
Theft matters if and only if $\tau < \min(q^*, q^{**})$.

Suppose theft matters.

Case $q^* \leq q^{**}$.

- $x = \tau$.
- $y \leq q^*$, with strict inequality if $\rho$ is close to one or $\tau$ is sufficiently small.
- $\mu = \beta \left[ \rho u' (\tau) + (1 - \rho) u' (y) \right] > \beta$.

Case $q^* > q^{**}$.

- $x = \tau$.
- $y < q^*$ always.
- $\mu = \beta \left[ \rho u' (\tau) + (1 - \rho) u' (y) \right] > \beta u' (q^{**})$. 
Welfare is increasing in $\tau$.

The optimal money growth rate rises as the cost of theft falls, as a lower cost of theft requires a higher $\mu$ to drive out theft.

Lower $\tau$ implies lower $x$ and $y$, but $l = y - x$ increases.

Less costly theft promotes the use of credit.
The theft with Non-Autarkic Punishment

- The punishment equilibrium involves only monetary exchange.
- Recall that the punishment equilibrium must be sustainable.
- Punishment equilibrium:
  - If $\tau \geq \hat{x}$, then $x = \hat{x}$, $\mu = 1$, and $(1 - \beta) \hat{v} = \beta u(\hat{x}) - \hat{x}$.
  - If $\tau < \hat{x}$, then $x = \tau$, $\mu = \beta u'(\tau) > 1$, and
    $$(1 - \beta) \hat{v} = \beta \{ -\tau [(1 - \beta)u'(\tau) + 1] + u(\tau) \}.$$ 
- $\hat{x}$ solves $u'(\hat{x}) = 1/\beta$. 
Theft with Non-Autarkic Punishment

- Equilibrium if $\tau \geq \hat{x}$: $x = y = \hat{x}$ and $\mu = 1$.
- Equilibrium if $\tau < \hat{x}$: $x = \tau$ and $y$ satisfies

$$\beta(1 - \rho)u(y) - (1 - \rho\beta)y \geq \beta(1 - \rho)[u(\tau) - \tau] - \beta(1 - \beta)\tau u'(\tau).$$
Theft with Non-Autarkic Punishment

- $y$ could increase with a decrease in $\tau$ - theft is a greater problem in the punishment equilibrium with valued money, which requires a higher money growth rate to drive out thieves (tends to promote credit).

- $y > \tau$ for $\tau < \hat{x}$, so that when theft matters, credit is supported at the optimum.

- Optimal money growth rate:

\[
\mu = \beta \left\{ \rho u'(\tau) + (1 - \rho) u'[y(\tau)] \right\}.
\]
Main Results

- When theft matters, the Friedman rule is suboptimal, and may not be feasible.
- The optimal money growth rate tends to rise as the cost of theft falls.
- An increase in the cost of theft increases consumption in non-monitored trades, and typically decreases the quantity of lending.
- With non-autarkic punishments, the money growth rate is higher in the punishment equilibrium, as the theft problem is more severe - theft acts to discipline the credit market.
Conclusion

Without costs of monetary exchange, Friedman rule or Andolfatto rule is optimal, and this drives credit out.

If theft is sufficiently low-cost:

- Friedman rule is never optimal.
- Theft can discipline credit market behavior.
- Money and credit always coexist at the optimum.
- The optimal money growth rate tends to rise as the cost of theft falls.