Money and Credit with Limited Commitment and Theft

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Introduction

- Understand the roles of money and credit in transactions, and the interaction between these alternative means of payment.
- Construct a model where money and credit are both "robust".
- Show that, for robustness, recordkeeping must be imperfect and there must be costs to using money - consider theft.
- Given this, what are the implications for monetary policy?

Introduction

- Models of money and credit: Wallace (1980), Lucas and Stokey (1987), Ireland (1994), Aiyagari and Williamson (2000), Williamson (2008), and He, Huang, and Wright (2006).
- Typical result: Efficient monetary policy drives out any transactions role for credit - no costs associated with monetary exchange.
- Money and credit seem to be robust means of payment in the sense that we observe both monetary exchange and credit transactions under a wide array of technologies and monetary policy rules.
- We show that limited commitment, imperfect recordkeeping, and theft are sufficient to account for the coexistence of money and credit as media of exchange.

Model

- Rocheteau and Wright (2005).
- Time is discrete and continues forever.
- Two subperiods: day and night.
- Two types of agents: buyers and sellers.
- Continuum of each type with measure one.
- Buyers want to consume during the day but can produce only at night.
- Sellers can produce during the day but want to consume at night.

Model

- Technology: one unit of labor produces one unit of the unique perishable consumption good.
- Day subperiod: each buyer is randomly matched with a seller.
- Night subperiod: Walrasian market.

Model

- Buyer has preferences given by:

$$u(q_t) - n_t.$$

- Assume $u : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is increasing, strictly concave, and continuously differentiable, with $u(0) = 0$ and $u'(0) = \infty$.
- Seller has preferences given by:

$$-l_t + x_t.$$

- Common discount factor $\beta \in (0, 1)$ over periods.

Planner's Problem under Full Commitment

- Restrict attention to stationary allocations.
- A stationary allocation is a pair (q, x), with q being the seller's production during the day and x the buyer's production at night.
- Agents can commit to the planner's proposed allocation at date $t = 0$.
- Participation constraints:

$$u(q) - x \geq 0$$

and

$$-q + x \geq 0.$$

- Efficient allocations satisfy these constraints and $q = q^*$.

Planner's Problem under Limited Commitment

- Add incentive constraints:

$$-x + v \geq 0$$

  and

$$-q + x + w \geq 0,$$

  where v and w are the buyer's and seller's continuation values.

- Simplifying, we obtain

$$q \leq x \leq \beta u(q).$$

- q^* solves $u'(q^*) = 1$.
- q^{**} solves $q^{**} = \beta u(q^{**})$.
- If β is close to one, $q^* \leq q^{**}$. If β is small, $q^{**} < q^*$.
- Limited commitment matters if and only if $q^{**} < q^*$.

Figure 1: Efficiency when q** > q*

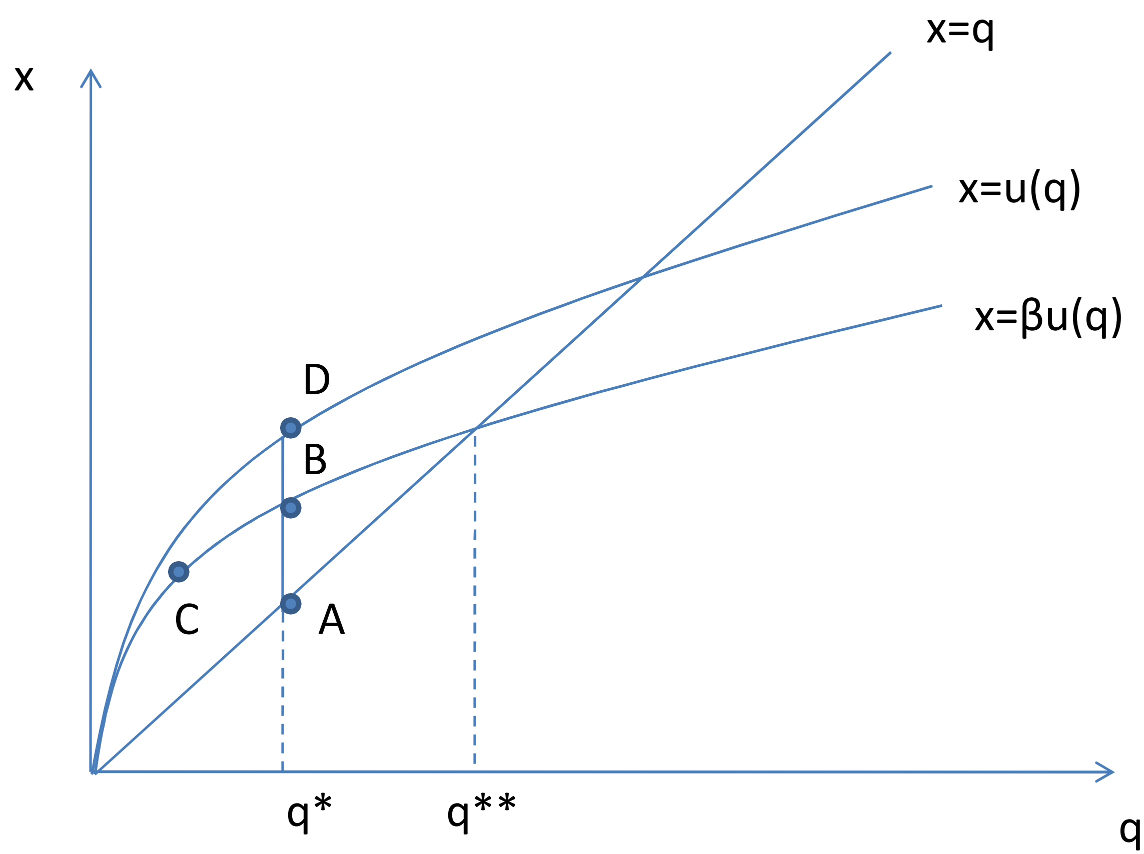


Figure 2: Efficiency when q** < q*

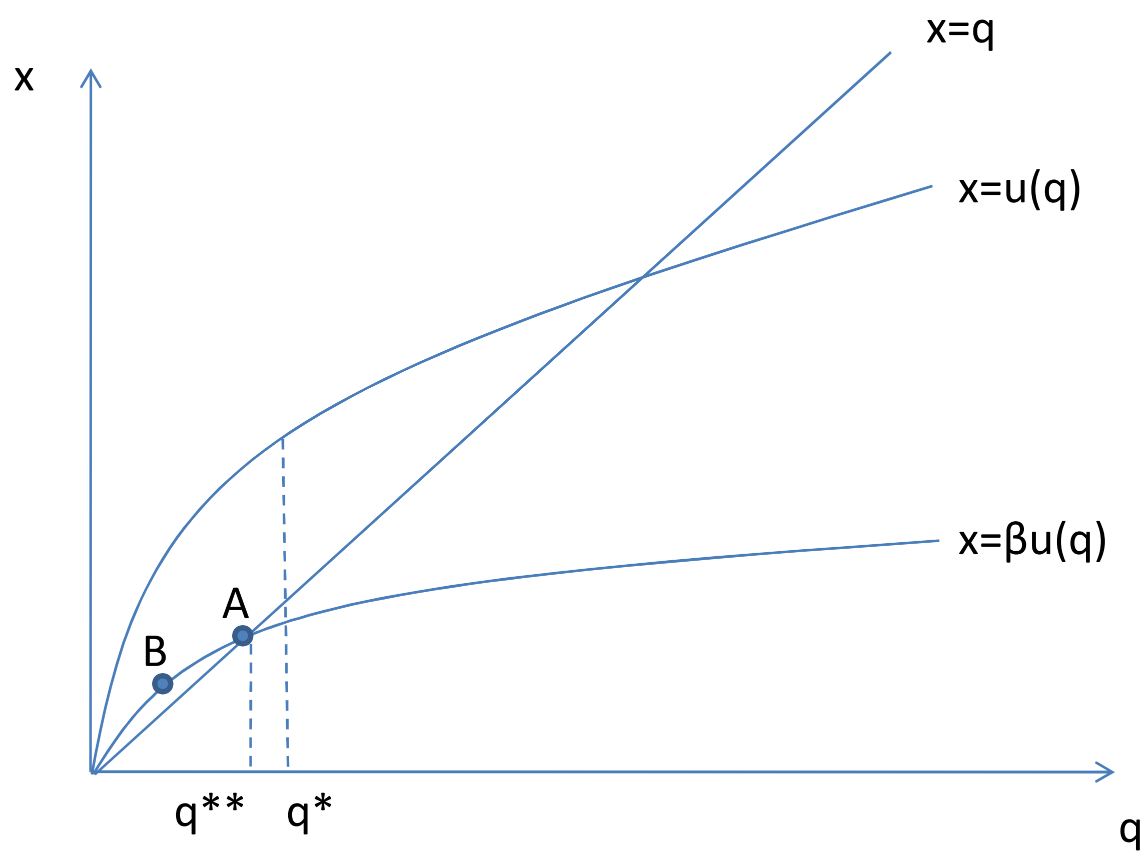
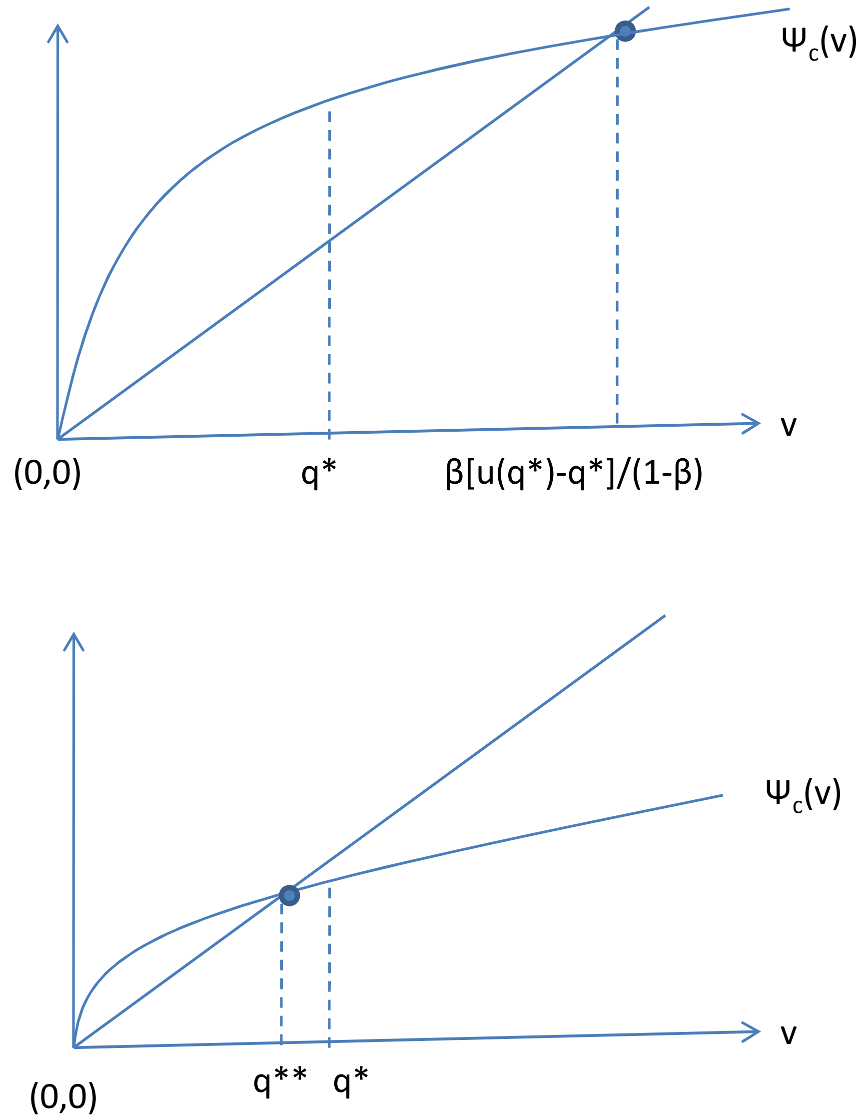


Figure 3: Equilibrium with Credit

(a) q** > q*

(b) q** < q*



Equilibrium Allocations with Perfect Memory

- Bargaining protocol in the DM.
  - Buyer and seller announce willingness to trade.
  - Buyer makes a take-it-or-leave-it offer.
- No bargaining inefficiencies.
- Each buyer is endowed with one unit of fiat money at the beginning of the first day.
- Money stock grows at gross rate $\mu \geq \beta$.
- New money is injected through lump-sum transfers in the CM at night.
- The interactions of each buyer with the government is publicly observable.

Equilibrium Allocations with Perfect Memory

- Limited commitment with respect to tax liabilities as well as private liabilities (Andolfatto, 2008).
- Default by any buyer triggers global autarky.
- Bellman equation for the buyer:

$$v = \max_{m,l} \left\{ -m + \beta \left[u\left(\frac{m}{\mu} + l \right) - l + \gamma + v \right] \right\},$$

subject to

$$-l + \gamma + v \geq 0$$

and

$$l \geq 0.$$

Equilibrium Allocations with Perfect Memory

- Case $q^* \leq q^{**}$.
  - If $\mu > \beta$, there is no monetary equilibrium.
  - If $\mu = \beta$, there is a continuum of monetary equilibria indexed by m.
- Case $q^* > q^{**}$.
  - If $\mu > \beta u'(q^{**})$, there is no monetary equilibrium.
  - If $\mu = \beta u'(q^{**})$, there is a continuum of monetary equilibria indexed by m.
- Money is held only under the Friedman rule or Andolfatto rule.
- Results consistent with Kocherlakota (1998):
  - No social role for money under perfect memory.
  - Limited commitment is insufficient to make money useful for society.

Imperfect Memory and Autarkic Punishment

- Fraction ρ of sellers cannot be monitored.
- Fraction $1 - \rho$ of sellers has monitoring potential.
- Bargaining protocol.
  - Agents announce willingness to trade.
  - If there is a monitoring opportunity, the buyer chooses whether the interaction will be monitored.
  - Seller announces willingness to trade.
  - Buyer makes a take-it-or-leave-it offer.
- The interaction of each agent with the government is publicly observable.

Imperfect Memory and Autarkic Punishment

- A *monetary equilibrium* is a pair (x, y) satisfying **(i)** optimality,

$$\rho u'(x) + (1-\rho) u'(y) = \frac{\mu}{\beta},$$

(ii) nonnegativity of consumption and loan quantity,

$$0 \leq x \leq y \leq q^*,$$

and **(iii)** incentive compatibility,

$$\beta\rho [u(x) - x] + (1-\rho) \beta u(y) - (1-\beta\rho) y \geq (1-\beta) \hat{v},$$

where $y = q^*$ if the IC does not bind.

- Default on either tax or private liabilities triggers global autarky, so that $\hat{v} = 0$.

Imperfect Memory and Autarkic Punishment

- Results:
  - Money and credit coexist for $\mu > \beta u' [\min (q^*, q^{**})]$.
  - $\mu = \beta u' [\min (q^*, q^{**})]$ is optimal and eliminates credit.
- Credit is not robust, and an efficient economy runs without it.
- At the optimum, money is memory - it recovers the solution to the planner's problem under perfect memory.

Imperfect Memory and Non-Autarkic Punishment

- Punishment equilibrium involves monetary exchange only.
- Default triggers equilibrium in which each seller refuses trade if the buyer wants monitoring.
- Punishment equilibrium must be sustainable - no incentive to default on tax liabilities.
- The buyer's continuation value $\hat{v}$ in the punishment equilibrium satisifies:

$$\hat{v} = -m(\mu) + \beta \left\{ u\left[\frac{m(\mu)}{\mu}\right] + \left(1 - \frac{1}{\mu}\right) m(\mu) + \hat{v} \right\},$$

where $m(\mu)$ is given by

$$u'\left[\frac{m(\mu)}{\mu}\right] = \frac{\mu}{\beta}.$$

- Sustainability requires

$$\left(1 - \frac{1}{\mu}\right) m(\mu) + \hat{v} \geq \hat{v}.$$

Imperfect Memory and Non-Autarkic Punishment

- As a result, we must have $\mu \geq 1$ in the punishment equilibrium.
- Government cannot commit to its future promises.
- If the punishment equilibrium is triggered, the government chooses μ optimally.
- Results:
  - Unique stationary equilibrium is the monetary equilibrium with $\mu = 1$ (in fact, incentive constraint is satisfied if and only if $\mu = 1$).
  - No credit activity.

Theft

- So far, there is no cost to monetary exchange other than the implicit foregone interest.
- A sufficiently high rate of return on money eliminates the implicit opportunity cost, and money drives out credit.
- In reality, there are costs to monetary exchange, such as counterfeiting, risk of loss, costs of maintaining the currency stock, and theft.
- We choose to study theft.
- $\tau =$ cost of theft for a seller in the DM - a seller can only steal in a non-monitored transaction.
- $\alpha =$ fraction of non-monitored transactions in which theft occurs.

Theft

- A *monetary equilibrium* is a list (x, y, α) satisfying **(i)** optimality,

$$\rho(1-\alpha)u'(x) + (1-\rho)u'(y) = \frac{\mu}{\beta},$$

(ii) nonnegativity of consumption and loan quantity,

$$0 \leq x \leq y \leq q^*,$$

(iii) incentive compatibility,

$$\beta[\rho(1-\alpha)u(x) + (1-\rho)u(y)] - \rho\beta x - (1-\rho\beta)y \geq \hat{v}(1-\beta),$$

with $y = q^*$ if *IC* does not bind, and **(iv)** optimal theft by sellers,

$$\text{if } \alpha = 0, \text{ then } x \leq \tau,$$

$$\text{if } 0 < \alpha < 1, \text{ then } x = \tau,$$

$$\text{if } \alpha = 1, \text{ then } x \geq \tau.$$

Theft

- The government chooses (x, y, α, μ) to maximize welfare

$$W = \rho(1-\alpha)[u(x) - x] + (1-\rho)[u(y) - y] - \rho\alpha\tau,$$

  subject to the constraint that (x, y, α) is a monetary equilibrium given μ.
- Solution has $\alpha = 0$.
- Equilibria in which theft occurs are suboptimal.
- Efficient monetary policy must always drive out theft.

Theft with Autarkic Punishment

- Theft matters if and only if $\tau < \min(q^*, q^{**})$.
- Suppose theft matters.
- Case $q^* \leq q^{**}$.
  - $x = \tau$.
  - $y \leq q^*$, with strict inequality if ρ is close to one or τ is sufficiently small.
  - $\mu = \beta[\rho u'(\tau) + (1-\rho) u'(y)] > \beta$.
- Case $q^* > q^{**}$.
  - $x = \tau$.
  - $y < q^*$ always.
  - $\mu = \beta[\rho u'(\tau) + (1-\rho) u'(y)] > \beta u'(q^{**})$.

Theft with Autarkic Punishment

- Welfare is increasing in τ.
- The optimal money growth rate rises as the cost of theft falls, as a lower cost of theft requires a higher μ to drive out theft.
- Lower τ implies lower x and y, but $l = y - x$ increases.
- Less costly theft promotes the use of credit.

Theft with Non-Autarkic Punishment

- The punishment equilibrium involves only monetary exchange.
- Recall that the punishment equilibrium must be sustainable.
- Punishment equilibrium:
  - If $\tau \geq \hat{x}$, then $x = \hat{x}$, $\mu = 1$, and $(1-\beta)\,\hat{v} = \beta u\,(\hat{x}) - \hat{x}$.
  - If $\tau < \hat{x}$, then $x = \tau$, $\mu = \beta u'\,(\tau) > 1$, and $(1-\beta)\,\hat{v} = \beta\left\{-\tau\left[(1-\beta)u'(\tau)+1\right]+u(\tau)\right\}$.
- $\hat{x}$ solves $u'\,(\hat{x}) = 1/\beta$.

Theft with Non-Autarkic Punishment

- Equilibrium if $\tau \geq \hat{x}$: $x = y = \hat{x}$ and $\mu = 1$.
- Equilibrium if $\tau < \hat{x}$: $x = \tau$ and y satisfies

$$\beta(1-\rho)u(y) - (1-\rho\beta)y \geq \beta(1-\rho)[u(\tau) - \tau] - \beta(1-\beta)\tau u'(\tau).$$

Theft with Non-Autarkic Punishment

- y could increase with a decrease in τ - theft is a greater problem in the punishment equilibrium with valued money, which requires a higher money growth rate to drive out thieves (tends to promote credit).
- $y > \tau$ for $\tau < \hat{x}$, so that when theft matters, credit is supported at the optimum.
- Optimal money growth rate:

$$\mu = \beta \left\{ \rho u' \left(\tau \right) + \left(1 - \rho \right) u' \left[y \left(\tau \right) \right] \right\}.$$

Main Results

- When theft matters, the Friedman rule is suboptimal, and may not be feasible.
- The optimal money growth rate tends to rise as the cost of theft falls.
- An increase in the cost of theft increases consumption in non-monitored trades, and typically decreases the quantity of lending.
- With non-autarkic punishments, the money growth rate is higher in the punishment equilibrium, as the theft problem is more severe - theft acts to discipline the credit market.

Conclusion

- Without costs of monetary exchange, Friedman rule or Andolfatto rule is optimal, and this drives credit out.
- If theft is sufficiently low-cost:
    - Friedman rule is never optimal.
    - Theft can discipline credit market behavior.
    - Money and credit always coexist at the optimum.
    - The optimal money growth rate tends to rise as the cost of theft falls.