Block Recursive Equilibria for Stochastic Models of Search on the Job

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1. Motivation

On-the-job search (OJS) is natural explanation for:

- worker flows between jobs (large: 2.9% per month)
- wage-tenure and wage-quit relationships
  - generated partly by firms' responses to OJS
- wage dispersion among similar workers
  - endogenous heterogeneity in search histories

Popular models of OJS

	Burdett and Mortensen 98	Postel-Vinay and Robin 02	Burdett and Coles 03
shocks	No	match specific	No
workers	risk neutral	risk neutral	risk averse
offers	fixed wage	fixed wage with counter offers	wage-tenure contracts
eqm	steady state	steady state	steady state

Problem:

difficult to solve outside steady state

- OJS induces non-degenerate distribution of workers
- distribution is a state variable affecting
  - workers' acceptance decision
  - firms' retention rate and hence offer decisions
- aggregation of decisions affects next period's distribution
- two-way interaction is not tractable

So what?

Difficult for the model to address:

- business cycles
- dynamic effects of labor market policies
- consequences of growth and structural transformation
- good estimation with data

What do we do in this paper?

Formally establish existence of dynamic equilibrium with:

- on-the-job search (OJS)
- aggregate and idiosyncratic shocks to productivity
- various contracts and workers' preferences:
  - dynamic contracts
  - fixed-wage contracts
  - risk aversion or neutrality

How can all this be possibly done?

Directed search:

Workers know terms of trade before applications

$\Longrightarrow$ **Block Recursive Eqm (BRE)** as in Shi (09, ECMA)

search decisions, contracts, market tightness	$\Longrightarrow$ $\Longleftarrow\!\!\neq\neq\neq$	distribution of workers

2. The Model

Workers: utility $v(w)$

- unemployed worker can search with prob λ_u
- employed worker can search with prob λ_e

Worker flows:

- quits for other jobs
- endogenous destruction
- exogenous destruction: δ

Jobs or firms:

risk neutral, free entry

Productivity of a job: $y + z$

- aggregate productivity: $y \in Y$; transition: $\phi_{\hat{y}}(\hat{y}|y)$;
- idiosyncratic productivity: $z \in Z$; transition: $\phi_{\hat{z}}(\hat{z}|z)$ (all new matches have $z_0 \in Z$)

State of economy: $\psi \equiv (y, u, g) \in \Psi$

- u: measure of unemployed workers;

- $g(V, z)$: measure of workers employed at jobs with

  z: idiosyncratic productivity, and

  V: lifetime utility delivered by remaining contract

Directed (competitive) search:

- submarkets indexed by $x \in X = [\underline{x}, \bar{x}]$:

  x: lifetime expected utility of offer to a worker

  $\theta(x; \psi)$: vacancy/applicant (tightness)

- matching prob in submarket with tightness θ:

  worker: $p(\theta)$

  firm: $q(\theta) = p(\theta)/\theta$

$$0 < p'(\theta) < \frac{p(\theta)}{\theta}, \quad p(\theta) \text{ concave}$$

Directed search (continued):

- search decision of worker V:
  given functions $\theta(x,\psi)$ and $p(\theta)$, chooses submarket x:

$$R(V,\psi) = \max_{x \in X} \; p(\theta(x,\psi))\,(x - V)$$

- search policy function: $x^* = m(V,\psi)$
- implied quit prob: $\tilde{p}(V,\psi) = p\left(\theta(m(V,\psi),\psi)\right)$

Two contractual environments:

- Dynamic contracts: for each t, specify
  - wage w
  - separation rate next period: d
  - lottery for next period

  all contingent on: histories of z, ψ, and lotteries

- Fixed-wage contracts:
  - w is fixed throughout the relationship;
  - d and lottery can still be contingent on histories

Special cases:

- with fixed-wage contract environment:
  - risk neutrality and no shocks: BM 98
  - risk aversion and no shocks: Burdett and Coles 03

- with dynamic-contract environment:
  - risk aversion and no shocks: Shi 09
  - risk neutrality and shocks: Shi and Menzio 09

Recursive formulation of dynamic contracts

- summarize history of the match by V:
  worker's lifetime utility from remaining contract

- state: (V, s), $\quad s = (\psi, z) \quad$ (recall: $\psi \equiv (y, u, g)$)

- contract: $c = (\pi_i, w_i, d_i, \hat{V}_i)_{i=1}^{2}$

  $\hat{V}$: promised value from next period onward

  π_i and w_i are functions of (V, s)

  d_i and $\hat{V}_i$ are functions of $(V, s, \hat{s})$

Recursive formulation of contracts (continued):

$$J(V,s) = \max \sum_{i=1}^{2} \pi_i \left\{ \begin{array}{l} y + z - w_i + \beta \mathbb{E}_{\hat{s}}[(1 - d_i(\hat{s}))\times \\ \left(1 - \lambda_e \tilde{p}(\hat{V}_i(\hat{s}), \hat{\psi})\right) J(\hat{V}_i(\hat{s}), \hat{s})] \end{array} \right\}$$

Constraints:

$$\pi_i \in [0,1], \qquad \sum_{i=1}^{2} \pi_i = 1$$

$$d_i : \Psi \times Z \to [\delta, 1], \qquad \hat{V}_i : \Psi \times Z \to X$$

Constraints (continued):

individual rationality on separation:

$$d_i(\hat{s}) = \begin{cases} \delta, \text{ if } U(\hat{\psi}) \leq \hat{V}_i(\hat{s}) + \lambda_e R(\hat{V}_i(\hat{s}), \hat{\psi}) \\ \\ 1, \text{ else} \end{cases}$$

promise-keeping:

$$\textstyle\sum_{i=1}^{2} \pi_i \{ \upsilon(w_i) + \beta \mathbb{E}_{\hat{s}} [d_i(\hat{s}) U(\hat{\psi})$$

$$+(1 - d_i(\hat{s}))(\hat{V}_i(\hat{s}) + \lambda_e R(\hat{V}_i(\hat{s}), \hat{\psi}))]\} = V$$

Market tightness: $\theta(x, \psi)$

Free entry $\Longrightarrow$ zero expected profit for vacancy

- for all x such that $J(x, \psi, z_0) \geq k$:

$$k = q(\theta(x, \psi))J(x, \psi, z_0)$$

- for all x such that $J(x, \psi, z_0) < k$:

$$\theta(x, \psi) = 0$$

Recursive equilibrium:

- market tightness $\theta : X \times \Psi \rightarrow \mathbb{R}_+$ satisfies free entry

- search policy function $m : X \times \Psi \rightarrow X$ is optimal;
  return to search $R : X \times \Psi \rightarrow \mathbb{R}$ is generated by m

- contract $c : X \times \Psi \times Z \rightarrow C$ is optimal;
  firm's value function $J : X \times \Psi \times Z \rightarrow \mathbb{R}$ is induced by c

- unemp value function $U : \Psi \rightarrow \mathbb{R}$ satisfies Bellman eq

- aggregate state obeys $\Phi_{\hat{\psi}} : \Psi \times \Psi \rightarrow [0, 1]$ induced by (m, c).

Block Recursive Equilibrium (BRE):

A BRE is a recursive equilibrium in which
all elements below are independent of (u, g):

- workers' optimal decisions and value functions: θ, m, R

- firms' optimal contract offers and value functions: c, J

- unemployment value function: U

- market tightness: θ

Why can a BRE exist?

- directed search $\Longrightarrow$ workers sort into submarkets by V (different marginal tradeoff between value and prob)

- firms entering a submarket x
  - cater to ONE group of applicants $V = m^{-1}(x, y)$
  - know workers' future target if hired, $m(x, y)$

- Free entry of firms into each submarket $\Longrightarrow$ tightness independent of other submarkets

Why does NOT a BRE exist under random match?

Free-entry condition in submarket x is:

$$k = q\left(\theta(x,\psi)\right) \times \underbrace{\mathbb{I}\left(x,\psi\right)}_{\text{prob that offer will be accepted}} \times J(x,\psi,z_0)$$

offer is received by a random applicant $\Longrightarrow$

$\mathbb{I}\left(x,\psi\right)$ depends on where applicant is in distribution

$\Longrightarrow$ θ and/or J must depend on distribution

$\Longrightarrow$ equilibrium is not block recursive

3. Existence of BRE under dynamic contracts

The object of fixed point: J

- $\mathcal{J}(X \times Y \times Z)$ (set in which J lies) is defined by:

  (J0) independent of (u, g)

  (J1) bi-Lipschitz: for all (y, z), all V_1, V_2 with $V_1 \leq V_2$

$$-\bar{B}_J(V_2 - V_1) \leq J(V_2, y, z) - J(V_1, y, z) \leq -\underline{B}_J(V_2 - V_1)$$

$$\text{with } \bar{B}_J \geq \underline{B}_J > 0$$

  and (J2), (J3).

3. Existence of BRE under dynamic contracts

The object of fixed point: J

- $\mathcal{J}(X \times Y \times Z)$ (the set in which J lies) is defined by:

  (J0) independent of (u, g)

  (J1) bi-Lipschitz in V

  (J2) bounded: $\underline{J} \leq J(V, y, z) \leq \bar{J}$ for all (V, y, z)

  (J3) concavity: $J(V, y, z)$ is concave in V for all (y, z)

- Claim: $\mathcal{J}$ is non-empty, bounded, closed and convex

Construct eqm mapping T for J:

Taking arbitrary $J \in \mathcal{J}$, solve:

- market tightness θ (using free entry):

$$k \geq p(\theta(x,y))J(x,y,z_0) \quad \text{and } \theta(x,y) \geq 0$$

- search policy, m, and return/value to search, R:

$$R(V,\psi) = \max_{x \in X} \; p(\theta(x,\psi))\,[x - V]$$

and more on next page.

Taking arbitrary $J \in \mathcal{J}$, solve:

- unemployment value function U
- optimal contract c (using firm's problem)
- updated value function $\tilde{J} = TJ$

Theorem 1:

There exists a BRE under dynamic contracts.

(i.e., T has a fixed point in $\mathcal{J}$.)

Sketch of Proof:

Apply Schauder fixed point theorem

(1) $T : \mathcal{J} \to \mathcal{J}$, i.e., $\tilde{J} = TJ$ satisfies:

- (J0) independent of (u, g)

- (J1) bi-Lipschitz (all policy functions are Lipschitz)

- (J2) bounded in $[\underline{J}, \bar{J}]$

- (J3) concave in V (role of lottery)

Sketch of Proof (continued):

(2) T is continuous (in sup norm):

• tightness θ is continuous in J: for all $\rho > 0$,

$$\|J_n - J_r\| < \rho \quad \Longrightarrow \quad \|\theta_n - \theta_r\| < \rho \, \alpha_\theta$$

• quitting prob, $p(\theta(m(V, y), y))$, is continuous in J

• separation policy d is continuous in J

• other policy functions continuous in J

Sketch of Proof (continued):

(2) T is continuous (in sup norm):

- all policy functions are continuous in J
- payoff function is uniformly continuous in policy and J

(3) $T\mathcal{J}$ is equicontinuous:

- use bi-Lipschitz property of TJ

4. Existence of BRE under fixed-wage contracts

- For any fixed wage w, lifetime utility is $H(w, \psi)$;
  define wage function $w = h(V, \psi)$ by $H(h, \psi) = V$

- separation decision d may not be continuous in J

- prove existence of proxy BRE
  by restricting that workers cannot quit into unemp

- find condition under which restriction does not bind

5. Some properties of a BRE

Mobility:

- search target $m(V, y)$ strictly increases in V
  $\Longrightarrow$ sorting by current contract's value V

- tightness $\theta(x, y)$ strictly decreases in offer x
  - workers with higher V move less: $p(\theta(m(V, y), y))$
    $\Longrightarrow$ negative wage-quit relationship, given y
    $\Longrightarrow$ positive wage-tenure relationship, given y
  - firms with higher V more successful in recruiting

Business cycles:

- tightness $\theta(x, y)$ increases in y for all x:
  - job finding prob increases
  - job-to-job flows increase

- search target $m(V, y)$ increases in y

Numerical example.

6. Conclusion

- formally established existence of dynamic eqm with:
  - on-the-job search (OJS)
  - aggregate and idiosyncratic shocks to productivity
  - dynamic contracts and fixed-wage contracts

- **block recursivity**: directed search + free entry $\Longrightarrow$ decisions and tightness independent of distribution

- proof of existence is generally useful (e.g., monetary search with non-degenerate distribution)