Sorting by Search Intensity

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Sorting by search intensity

- Study of match allocation in a standard on-the-job search model with endogenous search intensity.
  - Worker skill (h) and firm productivity (p) heterogeneity.
  - Multi-worker, non-discriminatory firms.
  - Wage bargaining as in Cahuc, Postel-Vinay, and Robin (2006).
  - Sorting driven by differential search intensities across worker skill levels.
  - Assortative matching results tied to match production function characteristics,
    - Supermodular $\Rightarrow$ positive sorting.
    - Submodular $\Rightarrow$ negative sorting.
    - Modular $\Rightarrow$ no sorting.

Heterogeneity and match product

- Worker skill heterogeneity, $h \sim \Psi(\cdot)$ on support $[0, 1]$.
- Firm productivity distribution, $p \sim \Phi(\cdot)$ on support $[0, 1]$.
- Match product: $f(h, p)$
- By assumption $f_h(h, p) \geq 0$ and $f_p(h, p) \geq 0$ for all (h, p). Absolute advantage on both sides regardless of type of match.

Worker Side

- Workers are income maximizers. Infinitely lived.
- Choose search intensity s at increasing, convex search cost $c(s)$.
- Discount rate r.
- Employment offers drawn from vacancy CDF $\Gamma(p)$ with support $[0, 1]$.
- Unemployed worker:
  - Instantaneous income $f(h, 0)$.
  - Employment opportunities arrive with Poisson arrival rate $\kappa s \lambda$
- Employed worker:
  - Outside employment opportunities arrive with Poisson arrival rate $s\lambda$.
  - At rate δ, the match is destroyed and the worker is laid off.

Are multi-worker firms discriminatory?

- Shimer and Smith (2000) partnership model application to labor markets.
  - Lise, Meghir and Robin (2008) and Melo (2008).
  - Each position in the firm has its own hiring process.
  - Any meeting can be applied only to the position in question.
  - If the position is filled, the value of the hiring process is reduced to zero until the position become open again.
  - Consequently, the firm is discriminatory in filling each position.
- In this model: A central hiring process that allocates workers to open positions.
  - The firm is discriminatory if the value of the hiring process is reduced by filling a given position.
  - In the constant returns case, the value of the hiring process is not reduced and so the firm is not discriminatory.
- The model in this paper and the partnership application occupy two extremes.

Firm side

- Constant returns to scale technology.
- Decision to match with a worker does not affect future vacancy posting payoff.
- Consequently, firms match with any worker.
- All firms recruit with identical intensity $\Rightarrow \Gamma(p) = \Phi(p)$.

Employment contract and bargaining protocol

- Employment contract specifies a wage w and a search intensity s until renegotiation.
- Bargaining outcome same as Dey and Flinn (2005) and Cahuc, Postel-Vinay, and Robin (2006).
- Dey and Flinn (2005):
  - Nash-bargaining with worker bargaining power β.
  - If unemployed, bargain with outside option of unemployment.
  - If employed, bargain with most productive firm given outside option of full surplus extraction with less productive firm.
- Cahuc, Postel-Vinay, and Robin (2006):
  - Repeated offers game in artificial time with breakdown risk. Worker makes offer with probability β. Zero disagreement value.
  - If unemployed, breakdown value is unemployment.
  - If employed, firms submit offers subject to threat of subsequent back-and-forth bargaining where breakdown value is current offer in hand.

- $V(h,p,p)$, joint match value.
- Search intensity $s(h,p)$ maximizes joint match surplus.
- $V(h,q,p)$ is worker's valuation of an (h,p) match given outside offer $q \leq p$,
$$V(h,q,p) = \beta V(h,p,p) + (1-\beta)V(h,q,q).$$
- Firm's valuation of an (h,q,p) match,
$$J(h,q,p) = V(h,p,p) - V(h,q,p) = (1-\beta)\big[V(h,p,p) - V(h,q,q)\big].$$
- If hired out of unemployment, $q = R(h)$, where $R(h)$ is a skill h worker's reservation level.

Asset equations

- Unemployment flow value,

$$rV_0(h) \quad = \quad \max_{s\geq 0}\left\{ f(h,0) - c(s) + \kappa s \lambda \int_{R(h)}^{1} \left[V(h,R(h),p') - V_0(h)\right] d\Gamma(p') \right\}.$$

- Employment flow value,

$$\begin{aligned} rV(h,q,p) \quad = \quad & w(h,q,p) - c\big(s(h,p)\big) + \delta\big[V_0(h) - V(h,q,p)\big] + \\ & s(h,p)\lambda\left[\int_p^1 \big[V(h,p,p') - V(h,q,p)\big] d\Gamma(p') + \right. \\ & \left. \int_q^p \big[V(h,p',p) - V(h,q,p)\big] d\Gamma(p')\right]. \end{aligned}$$

Search intensities

- The search intensity first order conditions are,

$$\begin{aligned} c'(s_0(h)) &= \kappa\beta\lambda \int_{R(h)}^{1} \frac{f_p(h,p')\big(1-\Gamma(p')\big)dp'}{r+\delta+\beta\lambda s(h,p')\big(1-\Gamma(p')\big)} \\ c'\big(s(h,p)\big) &= \beta\lambda \int_{p}^{1} \frac{f_p(h,p')\big(1-\Gamma(p')\big)dp'}{r+\delta+\beta\lambda s(h,p')\big(1-\Gamma(p')\big)}. \end{aligned}$$

- Straightforward to show that $s(h,p)$ is decreasing in p.
- Lemma 1:
  - Supermodular ($f_{hp} > 0$) $\Rightarrow s_h(h,p) \geq 0$.
  - Submodular ($f_{hp} < 0$) $\Rightarrow s_h(h,p) \leq 0$.
  - Modular ($f_{hp} = 0$) $\Rightarrow s(h,p) = s(p)\ \forall h$.

Reservation productivity level

- If $\kappa \leq 1$ then $R(h) = b$ for all h.
- If $\kappa > 1$ then $R(h) \in (b, \bar{p})$.
- It is straightforward to produce examples with supermodularity where $R(h)$ can be somewhere decreasing in h.

Steady state

- Joint CDF of matches by skill (h), productivity (p), and outside option (q), $G(h,q,p)$. In steady state: Flow out equals flow in,

$$(1-u)\delta G(h,q,p) + (1-u)\lambda \int_0^h \int_{R(h')}^q \Big\{ \big(1-\Gamma(p)\big) \int_{q'}^q s\big(h',p'\big) dG\big(h',q',p'\big) + \big(1-\Gamma(q)\big) \int_q^p s\big(h',p'\big) dG\big(h',q',p'\big) \Big\} = u\lambda \int_0^h I(R(h') \leq q)\big[\Gamma(p) - \Gamma(R(h'))\big]\kappa s_0(h') d\Upsilon(h'),$$

for all (h,q,p).

- Straightforward and fast to solve numerically given $s(h,p)$ and $\Upsilon(h)$.

- The skill distribution of unemployed workers must satisfy, $\Psi(h) = (1-u)G(h,\bar{p},\bar{p}) + u\Upsilon(h)$, implying

$$\Upsilon(h) = \frac{\int_{\underline{h}}^{h} \left[\delta + [1-\Gamma(R(h'))]\kappa s_0(h')\lambda\right]^{-1} d\Psi(h')}{\int_{\underline{h}}^{\bar{h}} \left[\delta + [1-\Gamma(R(h'))]\kappa s_0(h')\lambda\right]^{-1} d\Psi(h')}.$$

- Definition: A steady state equilibrium is a tuple $\{G(\cdot),\Upsilon(\cdot),u,s(h,p),s_0(h),R(h),w(h,q,p)\}$ that satisfies
  - Optimal choice conditions for $s(h,p)$, $s_0(h)$, and $R(h)$.
  - Bargaining condition for $w(h,q,p)$.
  - steady state conditions for $G(\cdot)$ and u.
  - Equilibrium condition for $\Upsilon(\cdot)$.

- Proposition 1: A unique steady state equilibrium exists.

Sorting in steady state equilibrium

- The worker skill conditional firm productivity distribution is given by,

$$\Omega(p|h) = \frac{\int_b^p g(h,p')dp'}{\int_b^{\bar{p}} g(h,p')dp'}.$$

- Proposition 2: For $\kappa = 1$ and any $(h_0, h_1) \in [\underline{h}, \bar{h}]$ such that $h_0 < h_1$,
  - if $f_{hp} > 0$ then $\Omega(p|h_0) \geq \Omega(p|h_1), \ \forall p \in [b, \bar{p}]$ with strict inequality for all $p \in \left(b, \bar{p}\right)$.
  - if $f_{hp} < 0$ then $\Omega(p|h_0) \leq \Omega(p|h_1), \ \forall p \in [b, \bar{p}]$ with strict inequality for all $p \in \left(b, \bar{p}\right)$.
  - if $f_{hp} = 0$ then $\Omega(p|h_0) = \Omega(p|h_1), \ \forall p \in [b, \bar{p}]$.
- The result generalizes to any $\kappa > 0$ as long as $R(h)$ is weakly increasing in h.
- Implies $E[p|h]$ increasing (decreasing) in h if $f_{hp} > 0$ ($f_{hp} < 0$).

Productivity conditional distributions

- The reverse conditioning does not necessarily produce an everywhere stochastically increasing relationship.
- Define,

$$\Omega(h|p) = \frac{\int_{\underline{h}}^{h} g(h', p) dh'}{\int_{\underline{h}}^{\bar{h}} g(h', p) dh'}.$$

- Take some $p_0 < p_1$. It need not be that $\Omega(h|p_0) \geq \Omega(h|p_1)$ for all h if $f(h, p)$ is supermodular.

Sorting comments

- The model assumes unemployed income flow $f(h,0)$. Consider a type independent income flow b,
  - Would tend to make the reservation productivity level $R(h)$ a negative function of h. Related to the negative assortative matching pull in Shimer and Smith (2000) and Eeckhout and Kircher (2009) for the modular case.
- Search costs are type-independent. In the stopping problem, discounting makes the cost of offer rejection worker skill dependent.

- Specify

$$f(h,p) = f_0 \left(\alpha h^\rho + (1-\alpha) p^\rho\right)^{\frac{1}{\rho}},$$

and

$$c(s) = c_0 \frac{s^{1+1/c_1}}{1+1/c_1}.$$

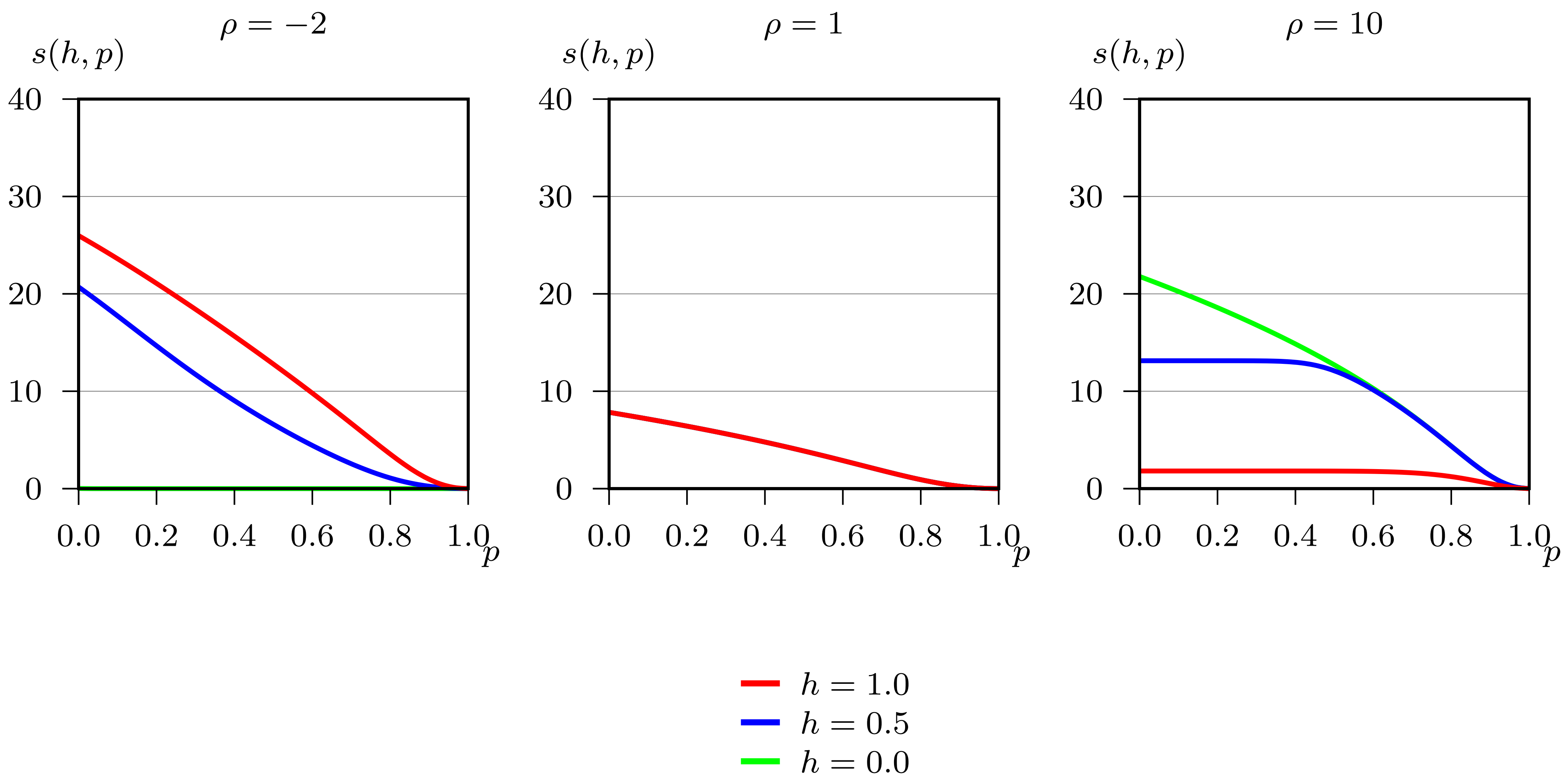
- Hence,
  - Positive sorting if $\rho < 1$.
  - Negative sorting if $\rho > 1$.
  - No sorting if $\rho = 1$.

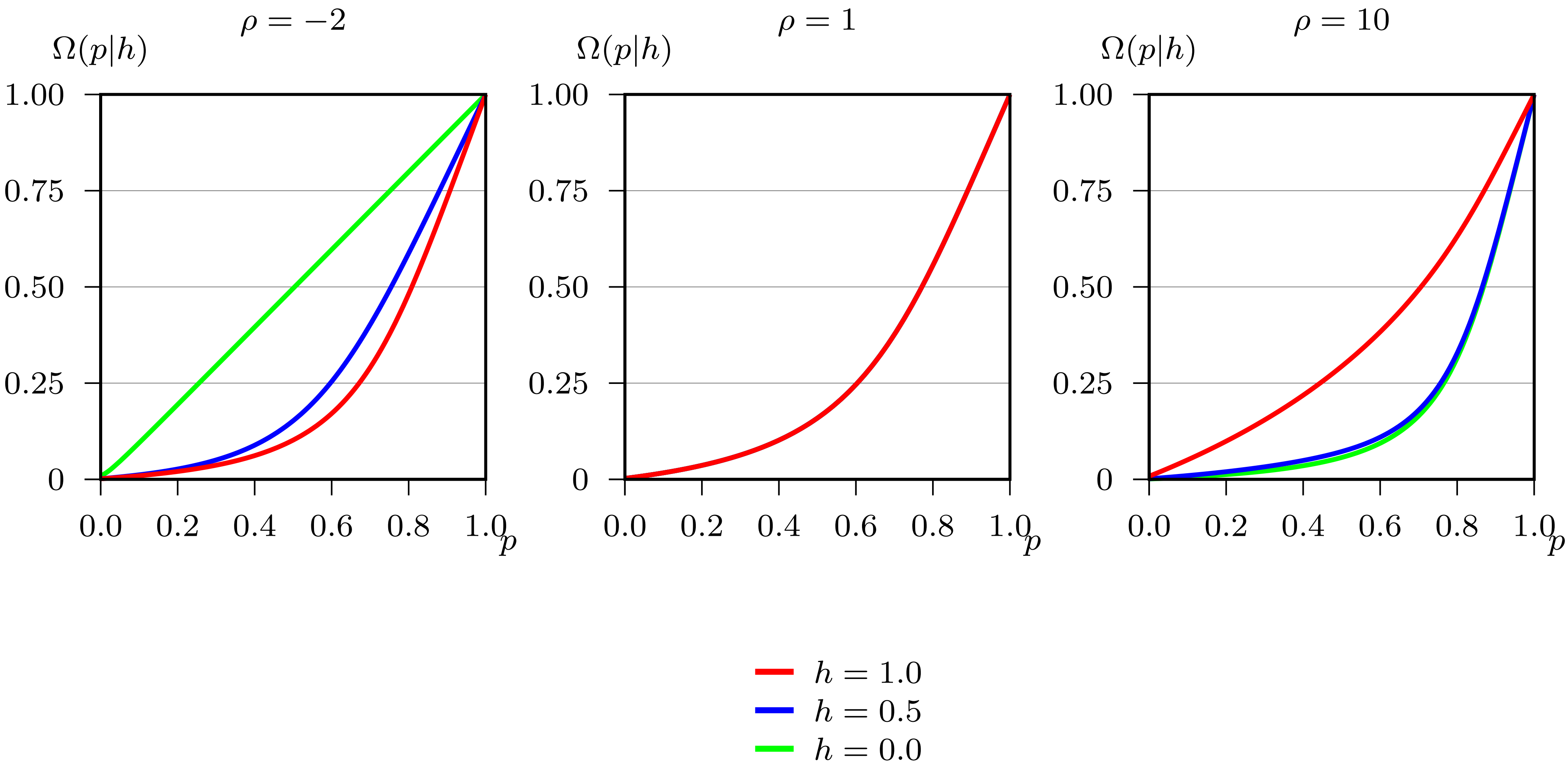
- Set parameters,

$$\begin{aligned} \beta &= 0.50 \\ c_0 &= 1.00 \\ c_1 &= 1.00 \\ r &= 0.05 \\ \delta &= 0.10 \\ f_0 &= 1.00 \\ \alpha &= 0.50 \end{aligned}$$

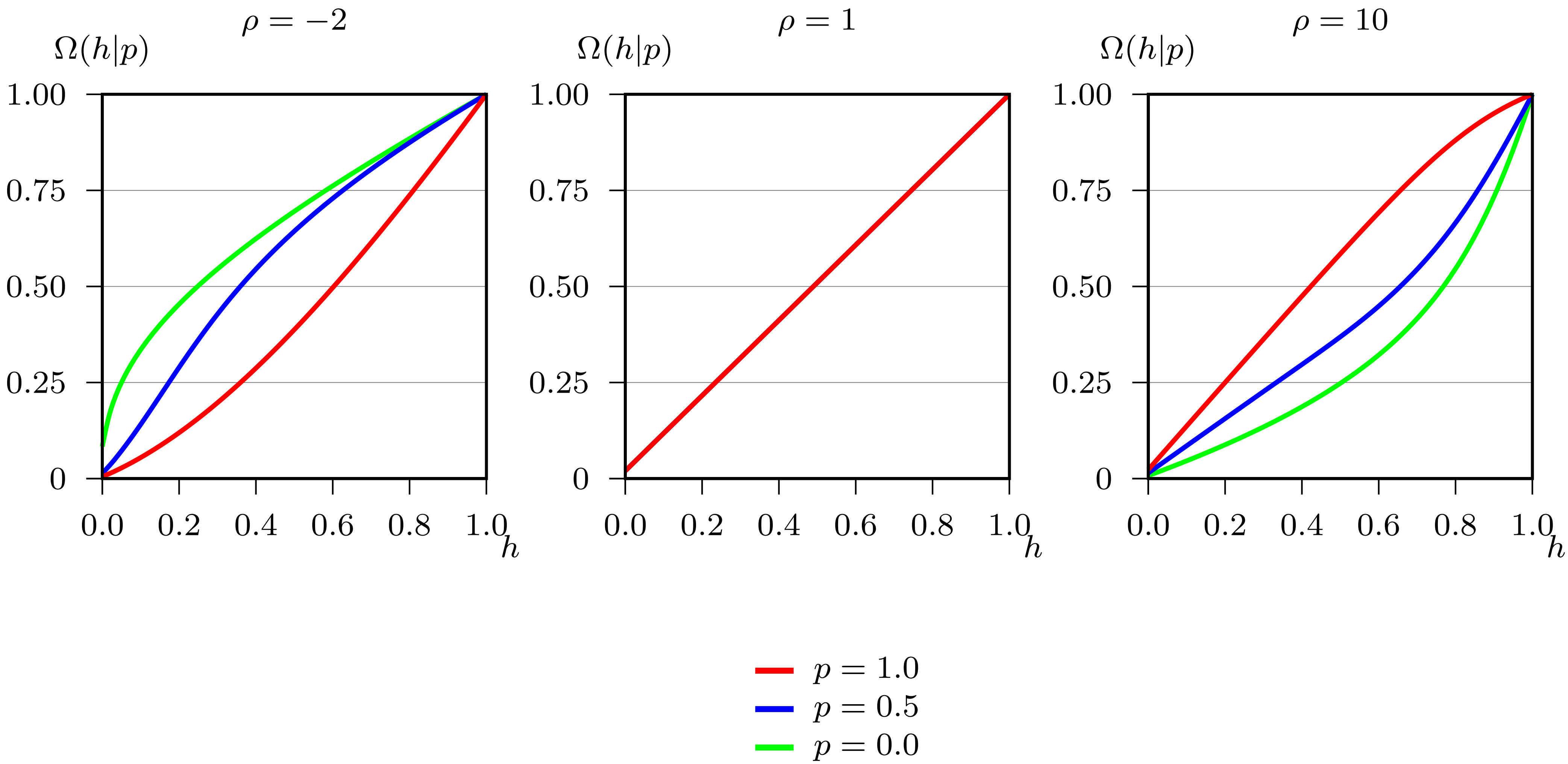
- For any value of ρ, λ is set such that $u = 0.06$.
- Assume that $\Psi(\cdot)$ is uniform on support $[0, 1]$.
- Assume $\Phi(\cdot)$ is uniform on support $[0, 1]$.

Type conditional search intensity

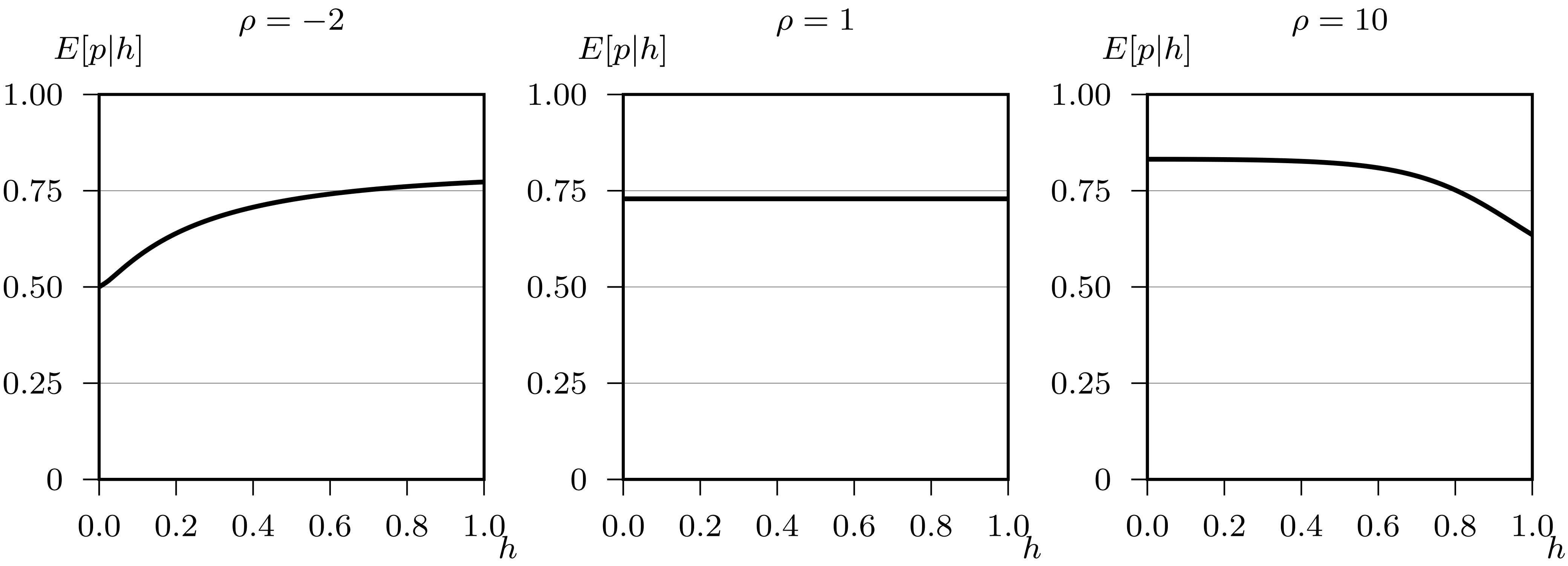



$\rho = -2$
$\rho = 1$
$\rho = 10$
$\Omega(p|h)$
1.00
0.75
0.50
0.25
0
0.0
0.2
0.4
0.6
0.8
1.0
p
$h = 1.0$
$h = 0.5$
$h = 0.0$

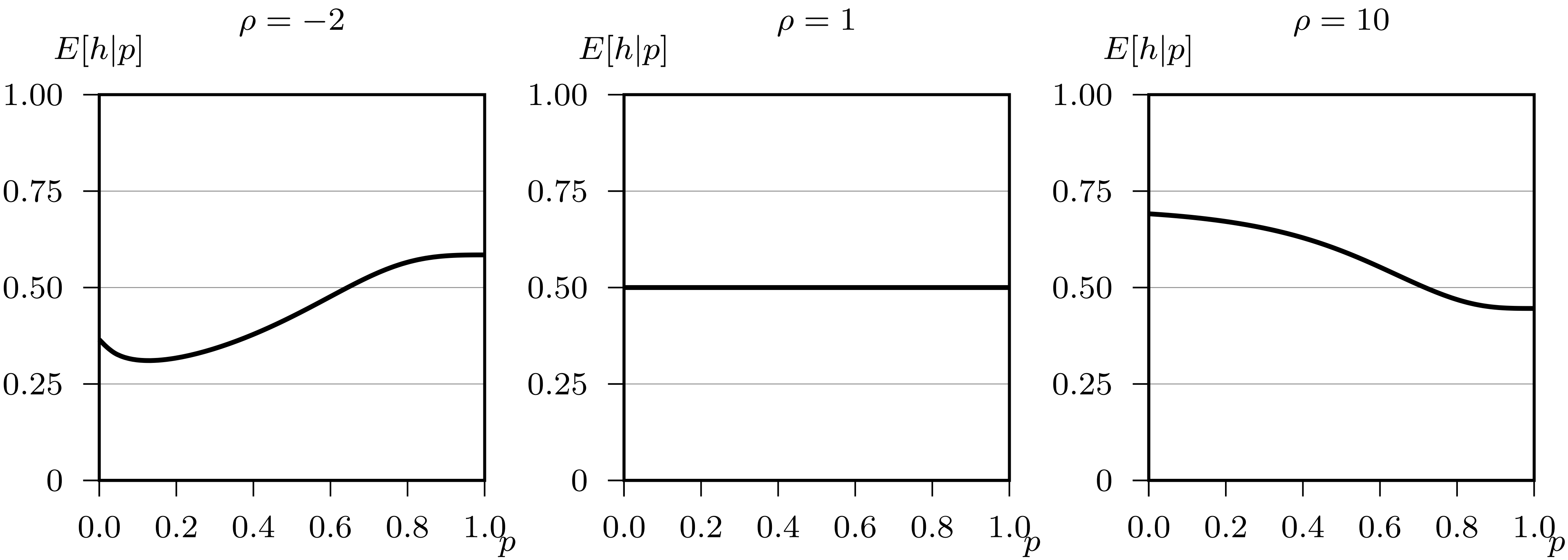
Productivity conditional skill distribution



Skill conditional productivity expectation



Productivity conditional skill expectation



Concluding remarks

- For empirical purposes, Bagger and Lentz (2008) extend the model in a number of directions to allow a richer worker flow and equilibrium feedback structure. In particular, the extension includes the choice of recruitment intensity by firms.
- Equilibrium in this case involves a fixed point search in the vacancy offer distribution and labor market tightness.
- The sorting mechanism and results are unaffected by these extensions.