Sorting by Search Intensity

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Study of match allocation in a standard on-the-job search model with endogenous search intensity.

- Worker skill ($h$) and firm productivity ($p$) heterogeneity.
- Multi-worker, non-discriminatory firms.
- Wage bargaining as in Cahuc, Postel-Vinay, and Robin (2006).
- Sorting driven by differential search intensities across worker skill levels.
- Assortative matching results tied to match production function characteristics,
  - Supermodular $\Rightarrow$ positive sorting.
  - Submodular $\Rightarrow$ negative sorting.
  - Modular $\Rightarrow$ no sorting.
- Worker skill heterogeneity, $h \sim \Psi(\cdot)$ on support $[0, 1]$.
- Firm productivity distribution, $p \sim \Phi(\cdot)$ on support $[0, 1]$.
- Match product: $f(h, p)$
- By assumption $f_h(h, p) \geq 0$ and $f_p(h, p) \geq 0$ for all $(h, p)$. Absolute advantage on both sides regardless of type of match.
Workers are income maximizers. Infinitely lived.

Choose search intensity $s$ at increasing, convex search cost $c(s)$.

Discount rate $r$.

Employment offers drawn from vacancy CDF $\Gamma(p)$ with support $[0, 1]$.

Unemployed worker:
- Instantaneous income $f(h, 0)$.
- Employment opportunities arrive with Poisson arrival rate $\kappa s \lambda$.

Employed worker:
- Outside employment opportunities arrive with Poisson arrival rate $s \lambda$.
- At rate $\delta$, the match is destroyed and the worker is laid off.
Shimer and Smith (2000) partnership model application to labor markets.

- Each position in the firm has its own hiring process.
- Any meeting can be applied only to the position in question.
- If the position is filled, the value of the hiring process is reduced to zero until the position become open again.
- Consequently, the firm is discriminatory in filling each position.

In this model: A central hiring process that allocates workers to open positions.

- The firm is discriminatory if the value of the hiring process is reduced by filling a given position.
- In the constant returns case, the value of the hiring process is not reduced and so the firm is not discriminatory.

The model in this paper and the partnership application occupy two extremes.
- Constant returns to scale technology.
- Decision to match with a worker does not affect future vacancy posting payoff.
- Consequently, firms match with any worker.
- All firms recruit with identical intensity \( \Gamma(p) = \Phi(p) \).
Employment contract and bargaining protocol

- Employment contract specifies a wage $w$ and a search intensity $s$ until renegotiation.

- Bargaining outcome same as Dey and Flinn (2005) and Cahuc, Postel-Vinay, and Robin (2006).

- Dey and Flinn (2005):
  - Nash-bargaining with worker bargaining power $\beta$.
  - If unemployed, bargain with outside option of unemployment.
  - If employed, bargain with most productive firm given outside option of full surplus extraction with less productive firm.

- Cahuc, Postel-Vinay, and Robin (2006):
  - Repeated offers game in artificial time with breakdown risk. Worker makes offer with probability $\beta$. Zero disagreement value.
  - If unemployed, breakdown value is unemployment.
  - If employed, firms submit offers subject to threat of subsequent back-and-forth bargaining where breakdown value is current offer in hand.
● $V(h, p, p)$, joint match value.

● Search intensity $s(h, p)$ maximizes joint match surplus.

● $V(h, q, p)$ is worker’s valuation of an $(h, p)$ match given outside offer $q \leq p$,

$$V(h, q, p) = \beta V(h, p, p) + (1 - \beta) V(h, q, q).$$

● Firm’s valuation of an $(h, q, p)$ match,

$$J(h, q, p) = V(h, p, p) - V(h, q, p) = (1 - \beta) [V(h, p, p) - V(h, q, q)].$$

● If hired out of unemployment, $q = R(h)$, where $R(h)$ is a skill $h$ worker’s reservation level.
• Unemployment flow value,

\[ rV_0(h) = \max_{s \geq 0} \left\{ f(h, 0) - c(s) + \kappa s \lambda \int_{R(h)}^{1} [V(h, R(h), p') - V_0(h)] d\Gamma(p') \right\}. \]

• Employment flow value,

\[ rV(h, q, p) = w(h, q, p) - c(s(h, p)) + \delta [V_0(h) - V(h, q, p)] + s(h, p) \lambda \left[ \int_{p}^{1} [V(h, p, p') - V(h, q, p)] d\Gamma(p') + \int_{p}^{q} [V(h, p', p) - V(h, q, p)] d\Gamma(p') \right]. \]
The search intensity first order conditions are,

\[ c'(s_0(h)) = \kappa \beta \lambda \int_{R(h)}^{1} \frac{f_p(h, p')(1 - \Gamma(p')) dp'}{r + \delta + \beta \lambda s(h, p')(1 - \Gamma(p'))} \]

\[ c'(s(h, p)) = \beta \lambda \int_{p}^{1} \frac{f_p(h, p')(1 - \Gamma(p')) dp'}{r + \delta + \beta \lambda s(h, p')(1 - \Gamma(p'))}. \]

Straightforward to show that \( s(h, p) \) is decreasing in \( p \).

Lemma 1:

- Supermodular \( (f_{hp} > 0) \Rightarrow s_h(h, p) \geq 0. \)
- Submodular \( (f_{hp} < 0) \Rightarrow s_h(h, p) \leq 0. \)
- Modular \( (f_{hp} = 0) \Rightarrow s(h, p) = s(p) \forall h. \)
If $\kappa \leq 1$ then $R(h) = b$ for all $h$.

If $\kappa > 1$ then $R(h) \in (b, \bar{p})$.

It is straightforward to produce examples with supermodularity where $R(h)$ can be somewhere decreasing in $h$. 
Steady state

- Joint CDF of matches by skill \((h)\), productivity \((p)\), and outside option \((q)\), \(G(h, q, p)\).

In steady state: Flow out equals flow in,

\[
(1 - u)\delta G(h, q, p) + (1 - u)\lambda \int_0^h \int_{R(h')}^q \left\{ (1 - \Gamma(p)) \int_{q'}^q s(h', p') dG(h', q', p') \right. \\
\left. + (1 - \Gamma(q)) \int_q^{p'} s(h', p') dG(h', q', p') \right\} = \\
u\lambda \int_0^h I(R(h') \leq q) [\Gamma(p) - \Gamma(R(h'))] \kappa s_0(h') d\Upsilon(h'),
\]

for all \((h, q, p)\).

- Straightforward and fast to solve numerically given \(s(h, p)\) and \(\Upsilon(h)\).
The skill distribution of unemployed workers must satisfy,
\[ \Psi(h) = (1-u)G(h, \bar{p}, \bar{p}) + u \Upsilon(h), \] implying
\[
\Upsilon(h) = \frac{\int_{h}^{h'} \left[ \delta + \left[ 1 - \Gamma(R(h')) \right] \kappa s_0(h') \lambda \right]^{-1} d\Psi(h')}{\int_{h}^{h'} \left[ \delta + \left[ 1 - \Gamma(R(h')) \right] \kappa s_0(h') \lambda \right]^{-1} d\Psi(h')}. 
\]

Definition: A steady state equilibrium is a tuple
\[ \{ G(\cdot), \Upsilon(\cdot), u, s(h, p), s_0(h), R(h), w(h, q, p) \} \] that satisfies

- Optimal choice conditions for \( s(h, p), s_0(h), \) and \( R(h). \)
- Bargaining condition for \( w(h, q, p). \)
- steady state conditions for \( G(\cdot) \) and \( u. \)
- Equilibrium condition for \( \Upsilon(\cdot). \)

Proposition 1: A unique steady state equilibrium exists.
The worker skill conditional firm productivity distribution is given by,

\[ \Omega(p|h) = \frac{\int_b^p g(h, p') dp'}{\int_b^p g(h, p') dp'}. \]

Proposition 2: For \( \kappa = 1 \) and any \( (h_0, h_1) \in [h, \bar{h}] \) such that \( h_0 < h_1 \),

- if \( f_{hp} > 0 \) then \( \Omega(p|h_0) \geq \Omega(p|h_1) \), \( \forall p \in [b, \bar{p}] \) with strict inequality for all \( p \in (b, \bar{p}) \).
- if \( f_{hp} < 0 \) then \( \Omega(p|h_0) \leq \Omega(p|h_1) \), \( \forall p \in [b, \bar{p}] \) with strict inequality for all \( p \in (b, \bar{p}) \).
- if \( f_{hp} = 0 \) then \( \Omega(p|h_0) = \Omega(p|h_1) \), \( \forall p \in [b, \bar{p}] \).

The result generalizes to any \( \kappa > 0 \) as long as \( R(h) \) is weakly increasing in \( h \).

Implies \( E[p|h] \) increasing (decreasing) in \( h \) if \( f_{hp} > 0 \) (\( f_{hp} < 0 \)).
The reverse conditioning does not necessarily produce an everywhere stochastically increasing relationship.

Define,

\[
\Omega(h|p) = \frac{\int_{h}^{h'} g(h', p) \, dh'}{\int_{h}^{\bar{h'}} g(h', p) \, dh'}. 
\]

Take some \( p_0 < p_1 \). It need not be that \( \Omega(h|p_0) \geq \Omega(h|p_1) \) for all \( h \) if \( f(h, p) \) is supermodular.
The model assumes unemployed income flow $f(h, 0)$. Consider a type independent income flow $b$,

- Would tend to make the reservation productivity level $R(h)$ a negative function of $h$. Related to the negative assortative matching pull in Shimer and Smith (2000) and Eeckhout and Kircher (2009) for the modular case.

Search costs are type-independent. In the stopping problem, discounting makes the cost of offer rejection worker skill dependent.
Specify

\[ f(h, p) = f_0 (\alpha h^\rho + (1 - \alpha) p^\rho)^{\frac{1}{\rho}}, \]

and

\[ c(s) = c_0 \frac{s^{1+1/c_1}}{1 + 1/c_1}. \]

Hence,

- Positive sorting if \( \rho < 1 \).
- Negative sorting if \( \rho > 1 \).
- No sorting if \( \rho = 1 \).
- Set parameters,

\[
\begin{align*}
\beta &= 0.50 \\
c_0 &= 1.00 \\
c_1 &= 1.00 \\
r &= 0.05 \\
\delta &= 0.10 \\
f_0 &= 1.00 \\
\alpha &= 0.50
\end{align*}
\]

- For any value of \( \rho \), \( \lambda \) is set such that \( u = 0.06 \).
- Assume that \( \Psi(\cdot) \) is uniform on support \([0, 1]\).
- Assume \( \Phi(\cdot) \) is uniform on support \([0, 1]\).
Type conditional search intensity

\[ \rho = -2 \]

\[ \rho = 1 \]

\[ \rho = 10 \]

\[ s(h, p) \]

\[ h = 1.0 \]

\[ h = 0.5 \]

\[ h = 0.0 \]
Skill conditional productivity distribution

\[ \Omega(p|h) = \rho = -2 \]

\[ \Omega(p|h) = \rho = 1 \]

\[ \Omega(p|h) = \rho = 10 \]

- Red: \( h = 1.0 \)
- Blue: \( h = 0.5 \)
- Green: \( h = 0.0 \)
\[ \Omega(h|p) \]

\( \rho = -2 \)

\( \rho = 1 \)

\( \rho = 10 \)

- \( p = 1.0 \)
- \( p = 0.5 \)
- \( p = 0.0 \)
Skill conditional productivity expectation

\[ E[p|h] \]

\( \rho = -2 \)

\[ \rho = 1 \]

\[ \rho = 10 \]
Productivity conditional skill expectation

\[ E[h|p] \]

\[ \rho = -2 \]

\[ E[h|p] \]

\[ \rho = 1 \]

\[ E[h|p] \]

\[ \rho = 10 \]
Concluding remarks

For empirical purposes, Bagger and Lentz (2008) extend the model in a number of directions to allow a richer worker flow and equilibrium feedback structure. In particular, the extension includes the choice of recruitment intensity by firms.

Equilibrium in this case involves a fixed point search in the vacancy offer distribution and labor market tightness.

The sorting mechanism and results are unaffected by these extensions.