SORTING VS SCREENING -

PRICES AS OPTIMAL COMPETITIVE SALES MECHANISMS

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BROAD MOTIVATION

- price posting is pervasive

  (even eBay has half of its goods sold via a posted price)
- yet in many settings auctions are superior

  (standard auctions theory, McAffee '93, Peters '97)
- obvious explanations: transaction costs, no scarcity
- we propose an additional explanation based on the meeting technology:
  - are meetings a rival good for the buyers?

    (externalities in meeting)
- auctions screen ex post; prices induce sorting ex ante - and ex-ante sorting if meetings are a rival good

BROAD MOTIVATION

Matching

Bilateral Matching

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Meeting + Selection = Matching

Meeting Technology

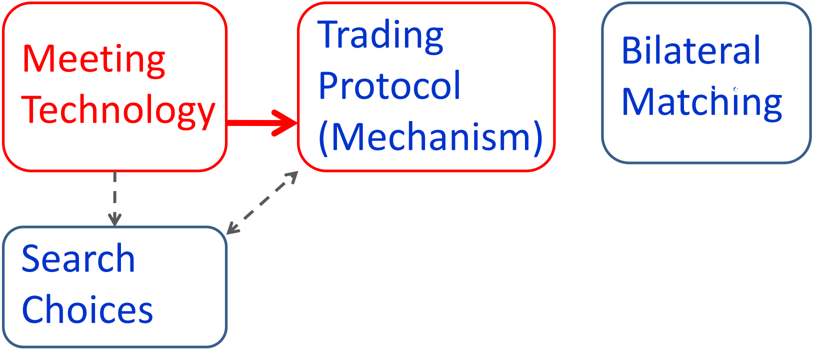
Trading Protocol (Mechanism)

Bilateral Matching

Search Choices

BROAD MOTIVATION

Meeting + Selection = Matching



Meeting Technology

Trading Protocol (Mechanism)

Bilateral Matching

Search Choices

How are goods/labor sold *depending on the frictions*? (fixed prices/auctions/bargaining)
How is competing mechanism design *affected by the meeting process*?

BROAD MOTIVATION

MEETING FUNCTION EXAMPLE

Example Meeting Technology:

- urnball application process
- N applications can be opened

Broad Motivation

Meeting Function Example

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N = ∞

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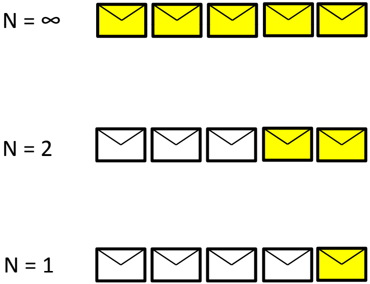

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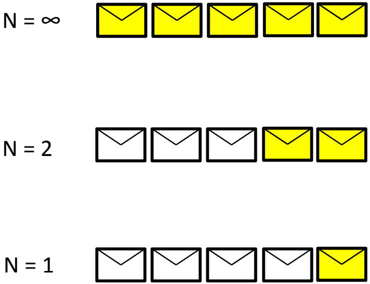

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Why does it matter: (1) Types of feasible mechanisms. (2) Interaction among types!

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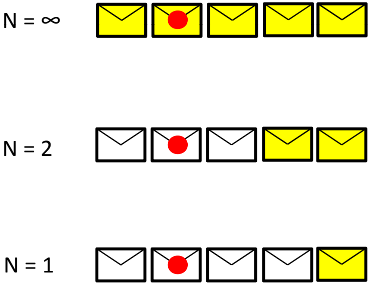

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● good type

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(meetings are non-rival)

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“**Directed Search**”:
Peters (1984, 1991, 1997a,b, …);
McAffee (1993); Burdett, Shi, Wright (2001);
Shi (2002); Shimer (2005); Albrecht, Gautier, Vroman (2006); Galenianos, Kircher (2009)

((Moscarini 2001,…))

“**Competitive Search**”:
Shi (2001); Rocheteau, Wright (2005);
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(Moen 1997; Mortensen, Wright 2003...)

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How does the nature of the frictions affect the type of trading protocol (mechanism)?

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THIS PAPER'S APPROACH

The approach in this paper:

- Lay out (potentially) multilateral meeting function
- Specify mechanism space
- Analyze which mechanisms (homogeneous, risk-neutral) sellers use to attract (risk-neutral) buyers
  - homogeneous buyers
  - heterogeneous buyers with private values $(\bar{v}, \underline{v})$

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The game:

1. each buyer draws private value (if heterogeneous).
2. each seller posts mechanisms *m*.
3. each buyer decides which mechanisms *m* to seek.
4. this gives buyer-seller ratios at each mechanism.
5. meeting function: how many buyers of each type arrive at seller.
6. mechanisms are being played.

RESEARCH QUESTIONS

Questions (focused on price posting relative to auctions, bargaining...):

- When is price posting profitable/equilibrium? When is price-posting outcome socially efficient?
- What is the relationship to random search?
- What is the relationship to the meeting technology?
  [Difference: "competitive" vs "directed" search]

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Open questions even for standard urnball ($N = \infty$):

- McAffee (1993) shows that auctions are always a best reply, and strictly preferred to price posting if individual seller is uncertain about buyer types.
- But under price posting each seller only faces one buyer type (no uncertainty), and allocation satisfy some planner problem.
- Are auctions only a weak best reply; are prices equally good?

RESULTS

homogeneous buyers:

Price Posting

a) equilibrium is constrained efficient

b) search is random in equilibrium (and this is efficient)

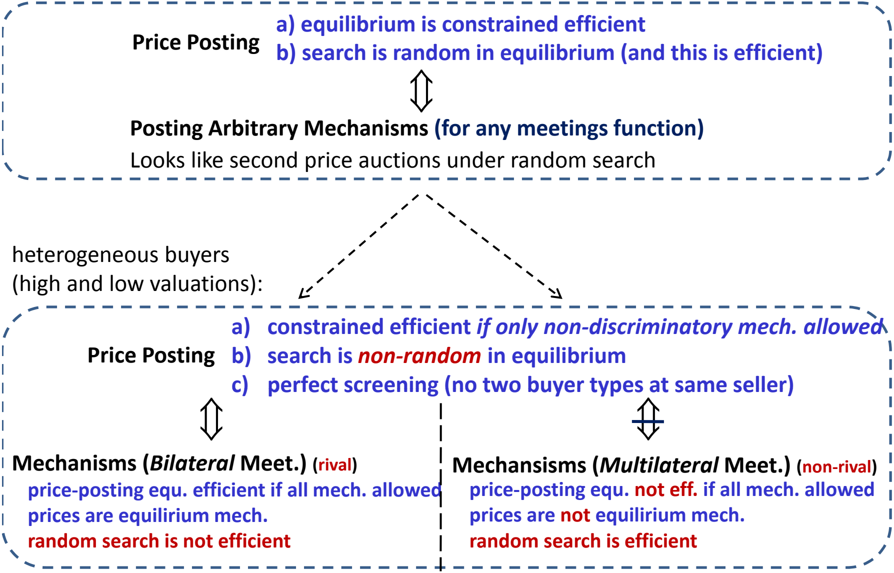
$\Updownarrow$

Posting Arbitrary Mechanisms (for any meetings function)

Looks like second price auctions under random search

Results

homogeneous buyers:



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heterogeneous buyers
(high and low valuations):

Price Posting

a) constrained efficient *if only non-discriminatory mech. allowed*

b) search is *non-random* in equilibrium

c) perfect screening (no two buyer types at same seller)

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Mechanisms (*Bilateral* Meet.) (rival)

price-posting equ. efficient if all mech. allowed

prices are equilirium mech.

random search is not efficient

$\not\Updownarrow$

Mechansisms (*Multilateral* Meet.) (non-rival)

price-posting equ. not eff. if all mech. allowed

prices are not equilirium mech.

random search is efficient

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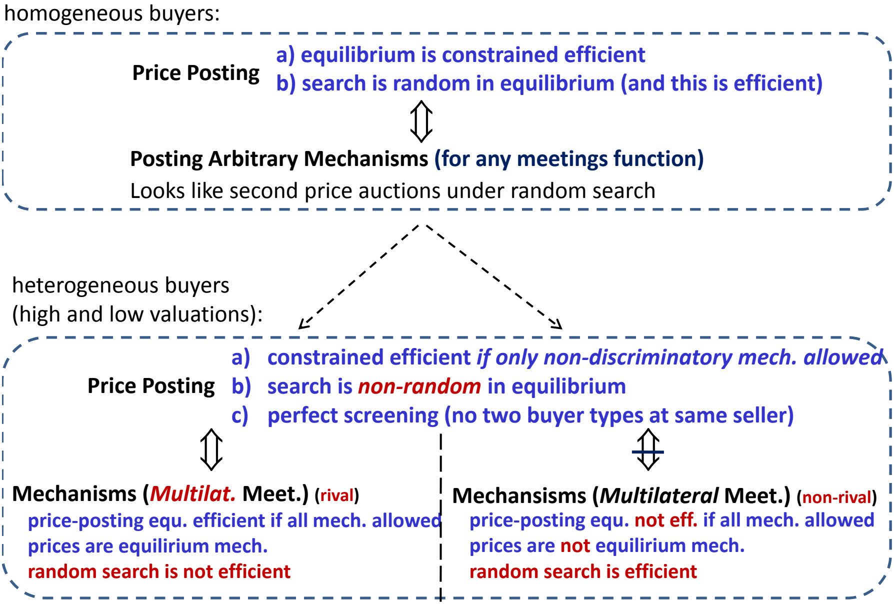

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THE MODEL

MEETING FUNCTION - MECHANISMS - PLAYERS AND COMPETITION

Consider “island” m with

- a measure 1 of homogenous sellers
- a measure λ of (possibly heterogeneous) buyers

The meeting function (depending on λ):

- $P_n(\lambda)$: Prob. that seller meets n buyers
- $Q_n(\lambda)$: Prob. that buyer meets a seller with n buyers.

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Assumptions on meeting function:

- Consistency: $nP_n(\lambda) = \lambda Q_n(\lambda), \quad \forall n \geq 1$
- Monotonicity (FOSD): $\sum_n^{\infty} P_n(\lambda)$ *increasing*, $\forall n \geq 1$.
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MEETING FUNCTION - MECHANISMS - PLAYERS AND COMPETITION

Examples:

- urnball: $P_n(\lambda) = \lambda^n e^{-\lambda}/n!$
- Kiyotaki-Wright: $P_1(\lambda) = \alpha\lambda/(1+\lambda) = 1 - P_0(\lambda)$

Definition 1: Directed Search (meetings are not rival)

- $1 - Q_0(\lambda)$ constant
- example: urnball $1 - Q_0(\lambda) = 1$

Definition 2: Competitive Search (meetings are fully rival)

- $\lambda(1 - Q_0(\lambda)) = 1 - P_0(\lambda)$ [if and only if bilateral meeting]
- example: Kiyotaki-Wright

Definition 3: Intermediate (meetings are partially rival)

- neither 1 nor 3
- example: prob. γ open all envelopes, prob.$1 - \gamma$ open one
- example: prob. γ urnball, prob. $1 - \gamma$ Kiyotaki-Wright

THE MODEL

MEETING FUNCTION - MECHANISMS - PLAYERS AND COMPETITION

Mechanism Space $\mathcal{M}$ is a subspace of anonymous extensive form games with representation: Given $\underline{n}$ low buyers

- seller's exp payoff: $\pi_{\underline{n}}$
- low buyer exp utility: $\underline{u}_{\underline{n}}$ trade probability $\underline{x}_{\underline{n}}$

Resource Constraint:

$$\pi_{\underline{n}} \quad + \ \underline{n}\,\underline{u}_{\underline{n}} \qquad \leq \ \underline{n}\,\underline{x}_{\underline{n}} \quad \underline{v}$$

Examples: posting price p with representation $\pi_{\underline{n}} = p$ and $\underline{u}_{\underline{n}} = (\underline{v} - p)/\ \underline{n}$, bargaining, auctions,....

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Ex-Ante Incentive Compatibility:

$$LOW: \sum_{\underline{n}=0}^{\infty}\sum_{\overline{n}=0}^{\infty}\tilde{Q}_{\underline{n},\overline{n}}(\underline{\lambda},\overline{\lambda})\underline{u}_{\underline{n}+1,\overline{n}} \geq \sum_{\underline{n}=0}^{\infty}\sum_{\overline{n}=0}^{\infty}\tilde{Q}_{\underline{n},\overline{n}}(\underline{\lambda},\overline{\lambda})\left[\overline{u}_{\underline{n},\overline{n}+1} + \overline{x}_{\underline{n},\overline{n}+1}(\underline{v} - \overline{v})\right]$$

$$HIGH: \sum_{\underline{n}=0}^{\infty}\sum_{\overline{n}=0}^{\infty}\tilde{Q}_{\underline{n},\overline{n}}(\underline{\lambda},\overline{\lambda})\left[\underline{u}_{\underline{n}+1,\overline{n}} + \underline{x}_{\underline{n}+1,\overline{n}}(\overline{v} - \underline{v})\right] \leq \sum_{\underline{n}=0}^{\infty}\sum_{\overline{n}=0}^{\infty}\tilde{Q}_{\underline{n},\overline{n}}(\underline{\lambda},\overline{\lambda})\overline{u}_{\underline{n},\overline{n}+1}$$

Examples: posting price p with representation $\pi_{\underline{n},\overline{n}} = p$ and $\underline{u}_{\underline{n},\overline{n}} = (\underline{v} - p)/(\underline{n} + \overline{n})$, bargaining, auctions,....

THE MODEL

MEETING FUNCTION - MECHANISMS - PLAYERS AND GAME

Large Game (Mas-Colell 1984). Players:

- Measure 1 of homogeneous sellers
- Measure $\underline{b}$ ($\overline{b}$) of buyers with private value $\underline{v}$ ($\overline{v}$)

The Market Interaction:

- Sellers post mechanisms $m \in \mathcal{M}$ according to measure μ
- Buyers choose mechanisms $m \in Supp(\mu)$ according to measure $\underline{\nu}$ ($\overline{\nu}$)
- On $Supp(\mu)$: buyer-seller-ratio $\underline{\lambda} = d\underline{\nu}/d\mu$ ($\overline{\lambda} = d\overline{\nu}/d\mu$) .

Overall expected payoffs from choosing m:

- seller: $\Pi(m|\mu, \underline{\nu}, \overline{\nu}) = \sum_{\underline{n}=0}^{\infty} \sum_{\overline{n}=0}^{\infty} P_{\underline{n},\overline{n}}(\underline{\lambda}(m), \overline{\lambda}(m))\pi_{\underline{n},\overline{n}}^{m}$
- low buyer: $\underline{U}(m|\mu, \underline{\nu}, \overline{\nu}) = \sum_{\underline{n}=0}^{\infty} \sum_{\overline{n}=0}^{\infty} \tilde{Q}_{\underline{n},\overline{n}}(\underline{\lambda}(m), \overline{\lambda}(m))\underline{u}_{\underline{n}+1,\overline{n}}^{m}$
- high buyer: $\overline{U}(m|\mu, \underline{\nu}, \overline{\nu}) = \sum_{\underline{n}=0}^{\infty} \sum_{\overline{n}=0}^{\infty} \tilde{Q}_{\underline{n},\overline{n}}(\underline{\lambda}(m), \overline{\lambda}(m))\overline{u}_{\underline{n},\overline{n}+1}^{m}$

THE MODEL

MEETING FUNCTION - MECHANISMS - PLAYERS AND GAME

- For low buyers *m* is in the support of $\underline{\nu}$ only if

$$\underline{U}(m|\mu,\underline{\nu},\overline{\nu}) = \underline{U} \equiv \max_{\tilde{m}\in Supp(\mu)} \underline{U}(\tilde{m}|\mu,\underline{\nu},\overline{\nu}) \tag{1}$$

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- At off-equilibrium mech. the queue $\overline{\lambda}(m), \underline{\lambda}(m)$ s.t.
  - $\underline{U} \geq \sum_{\underline{n}=0}^{\infty} \sum_{\overline{n}=0}^{\infty} \tilde{Q}_{\underline{n},\overline{n}}(\underline{\lambda}(m), \overline{\lambda}(m))\underline{u}^{m}_{\underline{n}+1,\overline{n}};\ \text{``}=\text{''}\ \ iff\ \underline{\lambda}(m) > 0$
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(for general mech. not necessarily unique; assumption: seller coordinates buyers (McAffee '93))

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(for general mech. not necessarily unique; assumption: seller coordinates buyers (McAffee '93))

- Sellers post *m* in the support of μ only if

$$\Pi(m|\mu,\overline{\nu},\overline{\nu}) = \Pi \equiv \max_{\tilde{m}\in\mathcal{M}} \Pi(\tilde{m}|\mu,\underline{\nu},\overline{\nu}) \qquad (3)$$

THE MODEL

- For low buyers *m* is in the support of $\underline{\nu}$ only if

$$\underline{U}(m|\mu,\underline{\nu},\overline{\nu}) = \underline{U} \equiv \max_{\tilde{m}\in Supp(\mu)} \underline{U}(\tilde{m}|\mu,\underline{\nu},\overline{\nu}) \tag{1}$$

- For high buyers *m* is in the support of $\overline{\nu}$ only if

$$\overline{U}(m|\mu,\overline{\nu},\overline{\nu}) = \overline{U} \equiv \max_{\tilde{m}\in Supp(\mu)} \overline{U}(\tilde{m}|\mu,\underline{\nu},\overline{\nu}) \tag{2}$$

- At off-equilibrium mech. the queue $\overline{\lambda}(m), \underline{\lambda}(m)$ s.t.
  - $\underline{U} \geq \sum_{\underline{n}=0}^{\infty} \sum_{\overline{n}=0}^{\infty} \tilde{Q}_{\underline{n},\overline{n}}(\underline{\lambda}(m), \overline{\lambda}(m))\underline{u}^{m}_{\underline{n}+1,\overline{n}};$ "$=$" *iff* $\underline{\lambda}(m) > 0$
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DEFINITION (EQUILIBRIUM)

An equilibrium is a tuple $(\mu,\underline{\nu},\overline{\nu})$ s.t. (1), (2) and (3) hold.

MECHANISMS - THE SELLER'S PROBLEM

In equilibrium with $\underline{U}^*$ and $\overline{U}^*$ the seller chooses mechanism m and $(\underline{\lambda}, \overline{\lambda})$ such that

$$\max_{(\underline{\lambda},\overline{\lambda})\in\mathbb{R}^2_+,\ m\in M} \sum_{\underline{n}=0}^{\infty} \sum_{\overline{n}=0}^{\infty} P_{\underline{n},\overline{n}}(\underline{\lambda}, \overline{\lambda}) {\pi_{\underline{n},\overline{n}}}^m \tag{4}$$

such that

$$\sum_{\underline{n}=0}^{\infty} \sum_{\overline{n}=0}^{\infty} \tilde{Q}_{\underline{n},\overline{n}}(\underline{\lambda}, \overline{\lambda}) {\underline{u}_{\underline{n}+1,\overline{n}}}^m \quad = \quad \underline{U}^* \qquad \texttt{if } \underline{\lambda} > 0, \tag{5}$$

$$\sum_{\underline{n}=0}^{\infty} \sum_{\overline{n}=0}^{\infty} \tilde{Q}_{\underline{n},\overline{n}}(\underline{\lambda}, \overline{\lambda}) \overline{u}^m_{\underline{n},\overline{n}+1} \quad = \quad \overline{U}^*. \qquad \texttt{if } \overline{\lambda} > 0, \tag{6}$$

(fulfilling incentive compatibility and the resource constraint).

MECHANISMS - THE SELLER'S PROBLEM

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$$\max_{(\underline{\lambda}\quad)\in\mathbb{R}_+,\ m\in M} \sum_{\underline{n}=0}^{\infty} P_{\underline{n}}(\underline{\lambda}\quad)\pi_{\underline{n}} \tag{4}$$

such that

$$\sum_{\underline{n}=0}^{\infty} \tilde{Q}_{\underline{n}}(\underline{\lambda}\quad)\underline{u}_{\underline{n}+1} = \underline{U}^* \quad \texttt{if}\ \underline{\lambda} > 0, \tag{5}$$

$$\tag{6}$$

Mechanisms - The seller's problem

In equilibrium with $\underline{U}^*$ the seller chooses mechanism m and $(\underline{\lambda}\quad)$ such that

$$\max_{(\underline{\lambda}\quad)\in\mathbb{R}_+,\ m\in M}\sum_{\underline{n}=0}^{\infty} P_{\underline{n}}\ (\underline{\lambda}\quad)\pi_{\underline{n}} \tag{4}$$

such that

$$\sum_{\underline{n}=0}^{\infty}\tilde{Q}_{\underline{n}}\ (\underline{\lambda}\quad)\underline{u}_{\underline{n}+1} = \underline{U}^* \quad \texttt{if}\ \underline{\lambda}>0, \tag{5}$$

(6)

Under full trade $\pi_n^m + n\underline{u}_n^m = \underline{v} \Rightarrow \Pi(m,\underline{\lambda}) = 1 - P_0(\underline{\lambda}) - \underline{\lambda}\underline{U}^*$

PRICE POSTING AND MECHANISMS

1.1) HOMOGENEOUS BUYERS

PROPOSITION (PRICE POSTING W/ HOMOG. BUYERS)

Under price posting, in equilibrium one price is offered, buyers select at random, and randomness is efficient

Def.: A class of mechanisms is *payoff complete* if it has some dimension (like the reserve price in an auction) to shift payoffs between buyers and sellers.

PROPOSITION (EQUIVALENCE)

In any class of pay-off complete (full-trade) mechanisms

- *an equilibrium mechanism exists*
- *remains equilibrium mech. when other mech. are added*
- *equilibrium payoffs are identical as under price posting*
- *search is (essentially) random.*

Generalized Camera and Selcuk (2009), Kultti (1999). Second price auctions: $r^* = 1 + \lambda^* P_0'(\lambda^*)/P_1(\lambda^*)$

PRICE POSTING

1.2) HETEROGENEOUS BUYERS

PROPOSITION (PRICE POSTING W/ HETEROG. BUYERS)

Price Posting leads in equilibrium to

- *two prices, one for each type*
- *buyers separate by "voting with their feet"*
- *constrained efficient given frictions and within the class of non-discriminatory mechanisms (Hosios' Condition).*

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Seller's maximization problem

$$\begin{array}{lll} \max\limits_{p,\underline{\lambda},\overline{\lambda}} & [1 - P_0(\underline{\lambda} + \overline{\lambda})]p & \\ \texttt{s.t.} & \sum\limits_{n\geq 1} Q_n(\underline{\lambda} + \overline{\lambda}) \frac{[\underline{v} - p]}{n} = \underline{U}^* & \texttt{if } \underline{\lambda} > 0 \\ \texttt{s.t.} & \sum\limits_{n\geq 1} Q_n(\underline{\lambda} + \overline{\lambda}) \frac{[\overline{v} - p]}{n} = \overline{U}^* & \texttt{if } \overline{\lambda} > 0 \end{array}$$

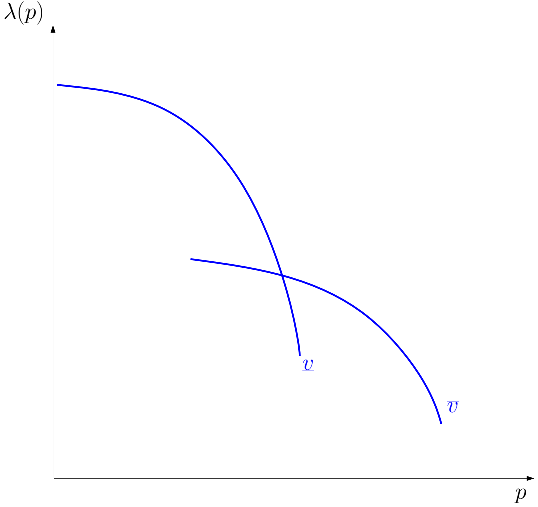
Sketch of separation argument:

- low types want low price more (single crossing property)
- pricing effectively separates the types.

PRICE POSTING

1.2) SINGLE CROSSING - SEPARATION "BY FEET"

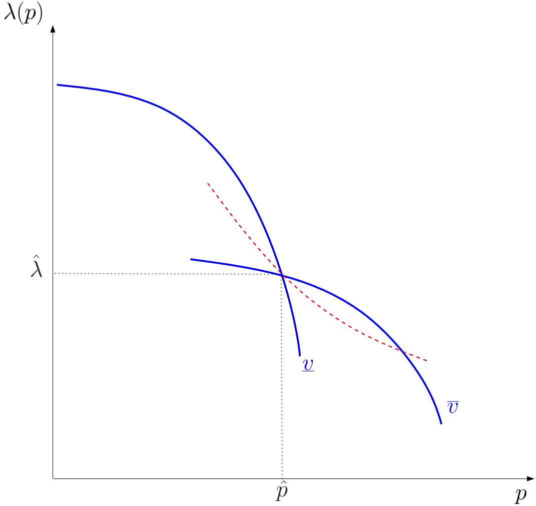
Buyer’s indifference curves



PRICE POSTING

1.2) SINGLE CROSSING - SEPARATION "BY FEET"

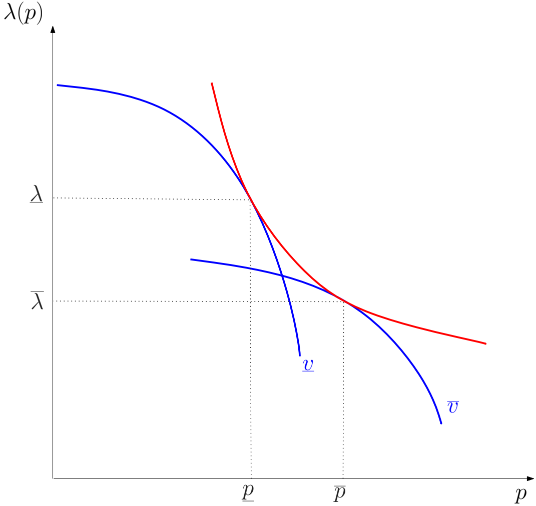
Iso-profit curve at a single market price



PRICE POSTING

1.2) SINGLE CROSSING - SEPARATION "BY FEET"

Equilibrium with two prices



MECHANISM POSTING

2.1) "DIRECTED" (MULTILATERAL MATCHING - NON-RIVAL)

PROPOSITION (MECHANISM POSTING - NON-RIVAL MEETINGS)

If meetings are non-rival

- *Identical auctions are more efficient than price posting*
- *Price posting is not an equ. when auction are available.*

Sketch of Proof:

- Random search yields most matches [$1 - P_0$ concave]
- More matches with identical auctions than w/ price posting
- High types choose randomly and get the object fist
- $\Rightarrow$ most matches for high types.
- Most matches & most matches for high types $\Rightarrow$ efficiency.
- Individual deviation to auction mechanisms is profitable.

MECHANISM POSTING

2.2.) "COMPETITIVE" (BILATERAL MEETING - FULLY RIVAL)

PROPOSITION (MECHANISM POSTING - FULLY RIVAL)

If meetings are fully rival (bilateral meetings)

- *Price posting is constrained efficient.*
- *Price posting is an equilibrium.*
- *Random search is never constrained efficient.*

Sketch of Proof:

- The presence of low types precludes high types from meeting.
- Sellers never see high types when a low type is present.
- All "selection" before the seller can intervene.
- Best not to mix types.
- Under separation: prices do a good job.

MECHANISM POSTING

2.3) THE INTERMEDIATE CASE - SOME EXTERNALITIES

Definition: Full information surplus $S^F(\underline{\lambda}, \overline{\lambda}) = P^H(\underline{\lambda}, \overline{\lambda})\overline{v} + P^L(\underline{\lambda}, \overline{\lambda})\underline{v}$

PROPOSITION (MECHANISM POSTING - PARTIALLY RIVAL)

Assume for all $\overline{\lambda}, \underline{\lambda} > 0$ *there exists* $\alpha \in [0, 1]$ *s.t.*

$$S^F(\underline{\lambda}, \overline{\lambda}) \gtrless \alpha S^F(\underline{\lambda}/\alpha, 0) + (1-\alpha)S^F(0, \overline{\lambda}/(1-\alpha))$$

- *"$<$" then price posting is an equilibrium*
- *"$>$" then price posting is not an equilibrium*

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- *"$<$" then price posting is an equilibrium*
- *"$>$" then price posting is not an equilibrium*

Lemma: For given $\overline{v}, \underline{v}$ and meetings $P = \gamma P^C + (1-\gamma) P^D$

- γ close to zero: "$<$"
- γ close to one: "$>$"

Lemma: For given $\underline{v}$ and any P with partial crowding

- $\overline{v}$ small: "$<$"
- $\overline{v}$ large: "$>$"

MECHANISM POSTING

2.3) THE INTERMEDIATE CASE - SOME EXTERNALITIES

Proof: Consider deviant seller attracting $\underline{\lambda}, \overline{\lambda} > 0$. Resource constraint:

$$\pi_{\underline{n},\overline{n}} + \underline{n}\,\underline{u}_{\underline{n},\overline{n}} + \overline{n}\,\overline{u}_{\underline{n},\overline{n}} \leq \begin{cases} \overline{v} & \text{if } \overline{n} > 0 \\ \underline{v} & \text{if } \overline{n} = 0, \underline{n} > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow \sum_{\underline{n}} \sum_{\overline{n}} P_{\underline{n},\overline{n}}(\underline{\lambda}, \overline{\lambda})[\pi_{\underline{n},\overline{n}} + \underline{n}\,\underline{u}_{\underline{n},\overline{n}} + \overline{n}\,\overline{u}_{\underline{n},\overline{n}}] \leq S^F(\underline{\lambda}, \overline{\lambda})$$

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Recall $nP_n = \lambda Q_n$ and $\sum \sum Q_n\, u = U$, then

$$\Pi + \underline{\lambda}\,\underline{U} + \overline{\lambda}\,\overline{U} \leq S^F(\underline{\lambda}, \overline{\lambda})$$

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MECHANISM POSTING

2.3) THE INTERMEDIATE CASE - SOME EXTERNALITIES

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Consider playing lottery over prices:

- $\underline{p}$ attracting $\underline{\lambda}/\alpha$ low buyers at $\underline{U}$: $S^F(\frac{\underline{\lambda}}{\alpha}, 0) = \Pi(\underline{p}) + \frac{\underline{\lambda}}{\alpha}\underline{U}$
- $\overline{p}$ attracting $\frac{\overline{\lambda}}{1-\alpha}$ high buyers at $\overline{U}$: $S^F(0, \frac{\overline{\lambda}}{1-\alpha}) = \Pi(\overline{p}) + \frac{\overline{\lambda}}{1-\alpha}\overline{U}$

MECHANISM POSTING

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$$\begin{aligned} \Pi + \underline{\lambda}\,\underline{U} + \overline{\lambda}\,\overline{U} &\leq \alpha[\Pi(\underline{p}) + (\underline{\lambda}/\alpha)\underline{U}] + (1-\alpha)[\Pi(\overline{p}) + (\overline{\lambda}/(1-\alpha))\overline{U}] \\ \Leftrightarrow \Pi &< \alpha \Pi(\underline{p}) + (1-\alpha)\Pi(\overline{p}) \end{aligned}$$

MECHANISM POSTING

2.3) THE INTERMEDIATE CASE - SOME EXTERNALITIES

Proof: Consider deviant seller attracting $\underline{\lambda}, \overline{\lambda} > 0$. Resource constraint:

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Sellers prefer to use prices. And there is a unique price posting equ.

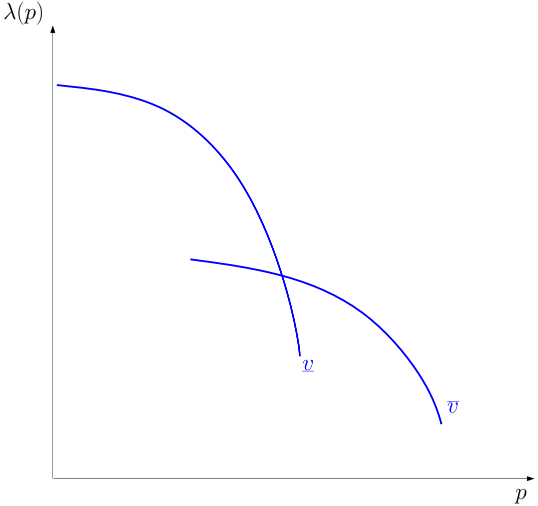
CONCLUSION

- Bilatateral (rival) meetings ("Competitive") or homog.:
  - Prices are equilibrium, and allocation is constrained efficient
    (other mechanisms only replicate the pricing outcome)
  - Random search is not efficient under buyer heterogeneity.
- Multilateral non-rival meetings ("Directed"):
  - Prices are not equilibrium. Posting allocation not constrained efficient
    (when auction mechanisms are available).
  - Random search is efficient
    (when auction mechanisms are available – Caveat: only when sellers are homogeneous).
- Larger relevance:
  - Clarifies when prices do a "good job".
  - Highlights the relevance of the meeting technology for competing mechanisms design.
  - Highlights when we can focus on one buyer type (even under additional problems such as moral hazard ect.)

PRICE POSTING

1.2) SINGLE CROSSING - SEPARATION "BY FEET"

Buyer's indifference curves



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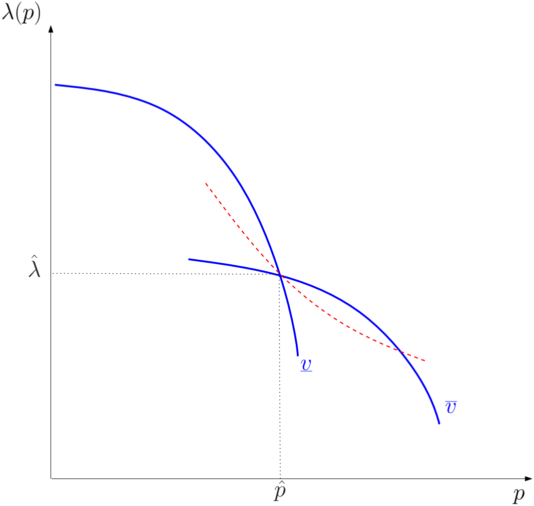
Iso-profit curve at a single market price



PRICE POSTING

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Equilibrium with two prices