Search by Committee

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September 2009

Introduction

- We modify the standard sequential search problem by assuming that the decision to stop searching is made by a committee with N members.
- Specifically, a group of agents must choose one option from an exogenous sequence. The committee continues to search until M members vote to stop.

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Related Work

- **Single Agent Search:** McCall (1970).
- **Private Values Voting:** Condorcet (1785), Hotelling (1929), Black (1948), Börgers (2004),
- **Collective Search:** Compte and Jehiel (2008).

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Preview: Comparison to Solo Search

- Committee members are less “picky”. Negative externalities lead to lower acceptance thresholds.
- Patient *and* impatient committees conclude search faster.
- Committees are more “conservative”. Negative externalities induce non convexities $\Rightarrow$ mean preserving spreads may lower acceptance thresholds.

Preview: Comparative Statics

Committee Size (Plurality Fraction Constant):

- Average welfare falls in committee size.
- When unanimity is required, larger committees expect to take longer to conclude search.
- If unanimity is not required, larger committees expect to conclude search faster when the discount rate is sufficiently low.

Preview: Comparative Statics

Plurality Rule:

- Expected Search duration rises in the plurality rule.
- Equilibrium acceptance thresholds are “hump shaped” in the plurality rule, initially rising (more picky) and then falling (less picky).
- The welfare-maximizing plurality rule increases in the discount rate.
- Unanimity is optimal for (boundedly) high patience.

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Outline of Talk

- Description of the problem
- Existence/uniqueness of equilibrium
- Comparison to single agent search results
- Comparative statics in size and plurality
- Optimal Plurality Rule

Description of the Problem

- Committee: A pair (N, M): N members and search ends iff at least $M \leq N$ vote to stop.
- Timing: Discrete time with common discount factor δ. Within each period, values for the currently available option are drawn and votes are cast.
- Values: Each committee member has a **private** value $0 \leq x_i \leq 1$ for the currently available option, which is an iid draw from distribution F with density f.

Perfect correlation corresponds to the single agent problem. Correlation in values would not change the basic results. Expected values rise in correlation.

Symmetric Stationary Equilibrium

- Stationary Equilibrium:
  - Sequential search with no recall.
  - Markovian Strategies: Conditioning on current draws.
- Best Responses: Given continuation value $V(\cdot)$, a **pivotal** voter with draw x solves: $\max\{x, \delta V(\cdot)\}$.
- Undominated Strategies: We rule out weakly dominated strategies, by assuming voters always condition strategies on being pivotal.

$\Rightarrow$ Agents use acceptance thresholds: y, i.e., stop iff $x \geq y$.

- Symmetry: We restrict attention to Symmetric Equilibria in which x^* is the best response to all other committee members setting acceptance threshold x^*.

Equilibrium Characterization

- Continuation Value: Let $V(y, M, N, \delta)$ be the expected value for each member just before values are drawn, given all members set acceptance threshold y:

$$V(y,\cdot) = P(y,N,M)\delta V(y,\cdot)+(1-P(y,N,M))\Omega(y,N,M)$$

$P(y, N, M) =$ the continuation probability

$\Omega(y, N, M) =$ the expected payoff conditional on stopping.

- Equilibrium Condition: $x^* = \delta V(x^*, M, N, \delta)$

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Existence and Uniqueness

Proposition: A symmetric stationary equilibrium exists. If f is log-concave then this equilibrium is unique.

1. Existence: There exists at least one solution to $y = \delta V(y, \cdot)$ since V is continuous in y and we have boundary conditions $\delta V(0, \cdot) > 0$ and $\delta V(1, \cdot) = 0$.
2. Uniqueness: Log-concavity of f yields $V_y \leq 1$. Thus, there is a unique solution: $x^* = \delta V(x^*, \cdot)$.

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Log-Concavity?

Log-Concavity Assumption:

- Fundamental Implication:

$$x - E[X'|X' \geq x] \quad \text{and} \quad x - E[X'|X' \leq x] \quad \uparrow \text{ in } x.$$

- Used to sign comparative statics: not just uniqueness!
- Examples: uniform, normal, exponential, logistic, power, gamma, weibull, etc.
- See Bagnoli and Bergstrom (2005) for an excellent discussion.

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Less Picky

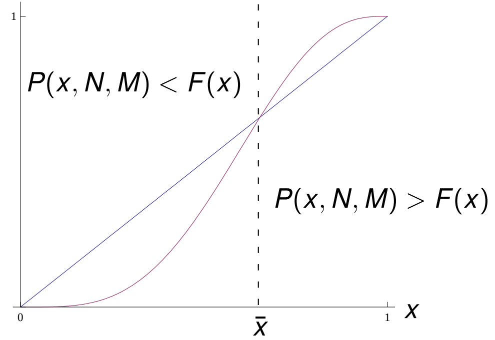
Proposition [Less Picky]: In equilibrium each committee member sets a lower acceptance threshold than would a single agent.

Intuition: One may not be pivotal in the future, which lowers continuation values relative to single agent searchers.

Search Duration: Preliminaries

- Lower acceptance thresholds do not immediately imply lower expected search duration.
- Equilibrium example in which a committee searches longer: F uniform and $(M, N, \delta) = (4, 5, 0.8)$.
- Competing Effects:
- Threshold Effect: Holding $P(\cdot)$ fixed, decreasing the acceptance threshold lowers search duration.
- Aggregation Effect: Holding the acceptance threshold fixed, downward shifts in $P(\cdot)$ lower search duration.

Search Duration: Result



Prop: Committees with $M < N$ conclude search faster than individuals for sufficiently low *and* high rates of patience.

- Low $\delta \Rightarrow$ low x^*. When x is low, $P(x, N, M) < F(x)$.
- As $\delta \to 1$ the single agent threshold goes to 1, while the committee threshold does not (via the negative ext.).

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MPS in Single Agent Search

In a single agent search problem, mean preserving spreads in the distribution over rewards increase expected values and thus acceptance thresholds.

Intuition: Single decision makers like mean preserving spreads, since the fact that good draws can now be even better is good; the fact that bad draws can be even worse is irrelevant.

MPS in Committee Search

Proposition [More Conservative]: The equilibrium threshold of committee members can fall with mean preserving spreads in the common distribution over values.

Why the single agent intuition fails: On a committee, fellow members can force you to stop at a bad draw or continue with a good draw.

- Let $N = 2$ and fix a symmetric cutoff z, then define

$$v(x,y) = \begin{array}{ll} x + y & \text{on stopping set} \\ 2\delta V(z,\cdot) & \text{on continuation set} \end{array}$$

- Then we can write:

$$V(z,\cdot) = \frac{1}{2}\int\int v(x,y)f(x)f(y)dxdy,$$

Graphical Intuition: $N = 2$

Let $v(x, y)$ be the joint payoff when one member draws x and the other $y < x^*$ ($=$ continuation value).

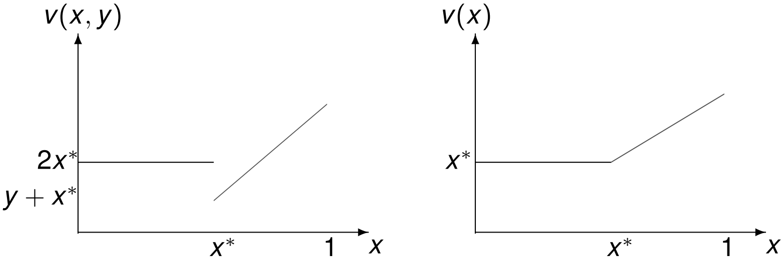

Figure: Left Graph: $v(x, y)$ given $M = 1$. Right Graph: Single agent payoff.

"Less picky" follows from the negative externality and "more conservative" from the non-convexity.

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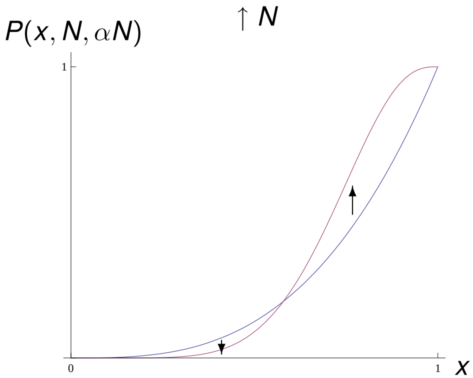
Changes in N: Complications

What happens to acceptance thresholds and search duration when N increases holding $\alpha \equiv M/N$ constant?

Why the impact on search duration is not a slam dunk:

- Changes in N induce changes in two relevant variables:
  1. The equilibrium acceptance threshold, x^*.
  2. The probability of stopping, $1 - P(x^*, N, \alpha N)$.
- As N rises with α constant, x^* falls.
- As N rises with α constant, the change in $P(x^*, N, \alpha N)$ satisfies single crossing: falling for low x and rising for high x.
- These effects may be opposed for search duration.

Changes in N: Results



Proposition [Size and Duration]: Fix $\alpha = M/N$, x^* falls in N. If $\alpha < 1$, expected search duration falls for sufficiently low discount rates. If $\alpha = 1$, expected duration rises in N.

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Changes in M: Results

Proposition: Expected search duration increases in M. There exists $k \in (0, N]$, such that:

- for all $M < k$, x^* increases in M
- for all $M > k$, x^* decreases in M

Intuition for $\uparrow x^*$ when M low:

- Forced stopping is relatively more costly than forced continuation.
- So increasing M raises expected welfare.

That search duration rises in M is intuitive, but not immediate (as x^* may fall in M).

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Welfare Maximizing Plurality: Result

Objective: Maximize the expected payoff (prior to any draw), $V(x^*, \cdot)$.

Simplified Objective: Since $x^* = \delta V(x^*, \cdot)$, this is equivalent to maximizing $x^*(M)$.

Proposition [Optimal M]: The welfare maximizing plurality rule, M^*, weakly increases in patience. If committee members are sufficiently patient, then unanimity is optimal.

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Welfare Maximizing Plurality: Intuition

- There are two types of negative externalities:
  1. Forced stopping
  2. Forced continuation
- The optimal M balances these two externalities.
- As $\delta \uparrow$ forced stopping becomes more costly relative to forced continuation $\Rightarrow M^* \uparrow$ in δ.
- The integer constraint on M implies that unanimity is optimal for high δ bounded away from 1.

Summary

- Committees vs. Single Searchers:
  1. Negative externalities lower acceptance thresholds.
  2. Committees stop sooner for low *and* high δ.
  3. Non convexities in values may reverse comparative statics in the MPS order.
- Changes in N: Average welfare falls in committee size. If unanimity is not required, larger committees search faster when δ is sufficiently low. With unanimity, larger committees search longer.
- Changes in M: Acceptance thresholds are hump shaped in M.
- Optimal M: As δ increases the welfare maximizing plurality rule rises. Unanimity is optimal for high δ.