# Setting the Right Prices for the Wrong Reasons*

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**Abstract**

We consider a model of nominal price adjustment with firm-specific and aggregate shocks to economic fundamentals and incomplete, dispersed information. Firms update their expectations about fundamentals based on their own cash flows (revenues and wages). We show that in a model with realistic levels of product-level price dispersion, the firms' inference about aggregate shocks is very gradual, yet in the aggregate prices adjust rapidly in response to aggregate nominal shocks. When an aggregate shock occurs, firms mistakenly attribute it to firm-specific shocks, but adjust prices nevertheless, since the exact nature of the shock matters little for its optimal pricing decision.

## 1 Introduction

Understanding the relation between monetary policy, monetary aggregates, prices and income at various horizons is a central issue in monetary economics. One potential explanation of monetary non-neutrality at short- to medium-run horizons, going back to Phelps (1970) and Lucas (1972), is based on lack of information. Without real time access to all relevant information, economic agents face uncertainty about the current economic conditions, and their decisions will reflect changing economic fundamentals only gradually, as the information becomes available.

A drawback of the original incomplete information models was the lack of an internal amplification mechanism. In these models, the speed of price adjustment and the degree of non-neutrality

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was directly and exclusively related to the arrival rate of new information, and prices fully incorporated all new information as it became available. Concerns of this kind prompted the literature on monetary policy to by and large abandon incomplete information as a source of non-neutrality, and instead focus on rigidities and adjustment costs in price-setting.

Recently, the incomplete information hypothesis has seen a revival with the works of Woodford (2001) and Mankiw and Reis (2002), who argued that heterogeneity in beliefs, in combination with pricing complementaries across firms, may amplify the effects of incomplete information. With heterogeneous information firms face uncertainty not only about aggregate fundamentals, but also about the pricing decisions of other firms. When pricing decisions are complementary, firms become more reluctant to incorporate new information, because they can no longer be sure that other firms share the same assessment of fundamentals, and hence will also adjust their prices.[1]

In this paper, we ask whether heterogeneous information can provide substantial amplification and persistence of aggregate shocks in a calibrated, dynamic stochastic general equilibrium model with incomplete, heterogeneous information. We enrich the model in Woodford (2001) to allow for firm-specific as well as aggregate shocks to economic fundamentals. The addition of large, persistent firm-specific shocks into the model is motivated by the observation of large idiosyncratic price fluctuations. In addition, we micro-found the information structure in a way that is consistent with the firms' market activities by postulating that at a minimum, firms update their expectations based on the information generated by their own transactions in input and output markets, i.e. firm-level revenues and costs. These observables are impacted by both aggregate as well as firm-specific fundamental shocks.

We calibrate our model to match observations about price fluctuations at the product level, in particular the observed level of cross-sectional price dispersion, and the large transitory price movements. By allowing for only a very limited set of information sources, we have, in principle, given the model the best possible chance of generating large adjustment delays from information heterogeneity. Nevertheless, price adjustment in our model is remarkably close to a benchmark economy with complete information, and heterogeneous information is unable to generate any

[1]While multiple authors have followed up on this with different interpretations for the sources of disagreement (e.g. Inattentiveness or infrequent, staggered information updating in Mankiw and Reis, (2002), rational inattention or information processing constraints in Sims, (2003) and Woodford, (2001), and Mackowiak and Wiederholt, (2008), or spatial separation in an island structure as in Lorenzoni (2008)), the basic insight that disagreement in expectations increases persistence, when coupled with decision complementarities, is robust across these models. See Hellwig (2008b) for an overview of the recent literature on incomplete information business cycle models.

quantitatively meaningful short-run real effects from nominal shocks. More specifically, in response to an aggregate monetary shock, prices absorb anywhere up to 90% of the long-run adjustment within a very short time horizon. This is followed by gradual convergence to full adjustment in the long run.

From a theoretical perspective, we offer two explanations for this striking result, which complement each other by highlighting different aspects of the environment. Our first explanation is based on how the firms' updating of beliefs and optimal pricing are affected by the addition of large and persistent idiosyncratic shocks and market-generated signals. As in Woodford (2001), it is still the case that heterogeneous beliefs dampen the firms' willingness to respond to aggregate shocks if they can't be sure that other firms' whose prices are complementary to theirs will adjust. However, in addition, firms use their market signals to update expectations about firm-specific shocks and adjust prices accordingly. As a result, firms will attribute part of the variation resulting from an aggregate monetary shock to idiosyncratic increases in nominal costs and demand, which causes them to increase their prices. Since firm-specific shocks in the data are large, this second channel becomes the overwhelming force for price adjustment in the short-run, explaining the large response almost on impact - in other words, firms make the right adjustments, but for the wrong reasons. That said, complete long run adjustment still requires firms to fully separate idiosyncratic from aggregate shocks. Given the magnitudes of these shocks, this learning is very gradual and leads to slow long-run adjustment, and is further slowed by the firms' tendency to discount their private information for the same reasons as in Woodford (2001). Thus, although the model generates only small real effects, these tend to be very highly persistent. Obviously this implication is not robust to the inclusion of additional sources of information about aggregate shocks.

Our second explanation departs from a simple game-theoretic insight, and explains why it shouldn't come as a surprise that aggregate adjustment is so close to the full information benchmark. In particular, notice that, in order to sustain the full information prices as equilibrium pricing strategies in an environment with incomplete information, firms only need to have access to information that enables them to compute these optimal full information prices. These prices are a function of marginal cost and marginal revenue and correspond to a particular linear combination of idiosyncratic and aggregate shocks. The information conveyed by their market activities enables the firms to accurately infer their marginal revenue from the demand for their product and their marginal costs from their wage bill - in other words, despite being very uninformative about idiosyncratic vs. aggregate shocks, these signals provide a parsimonious, yet reasonably accurate indicator of the firms' optimal pricing decisions. Meaningful departures from the full information benchmark arise only if there is delay in the incorporation of new information into prices, and idiosyncratic and/or aggregate shocks are not fully persistent.

Our paper is related to several branches of the literature. For our calibration, we draw on the recent literature documenting price adjustment at the micro level using large scale data sets of individual price quotes. Three observations about product-level fluctuations are particularly relevant for our quantitative conclusions: First, we draw on Klenow and Kryvtsov (2008) and Burstein and Hellwig (2007) for measures of price dispersion and the large idiosyncratic fluctuations in prices. Although these fluctuations are transient, they are too large to be consistent with a model that, like Woodford (2001), focuses on information as the only source of heterogeneity, which suggests a role for firm-specific fundamental shocks in product-level price adjustment. Second, we draw on Midrigan (2007) for a measure of serial correlation in prices at a monthly frequency. This measure in turn is consistent with the serial correlation typically assumed in calibration of menu cost models to micro data and plays an important role in pinning down the importance of the idiosyncratic pricing motive. Finally, we draw on Burstein and Hellwig (2007) and Eichenbaum, Jaimovich and Rebelo (2008) for observations about quantity fluctuations and correlations between prices and quantities. These authors both document that prices and quantities are at best weakly negatively correlated, and they often move in the same direction, suggesting that prices respond to cost as well as demand fluctuations. This in turn suggests that both cost and demand shocks must be sufficiently persistent to generate large pricing responses.

Our conclusion of a high degree of price flexibility is in stark contrast to the results obtained by Mackowiak and Wiederholt (2008), and it is therefore worthwhile discussing the differences behind these results. In their benchmark environment, Mackowiak and Wiederholt (2008) study a model of price adjustment with rational inattention, in which firms have to divide a limited information processing capacity between tending to idiosyncratic technology and aggregate demand shocks. Since idiosyncratic technology shocks are an order of magnitude larger than aggregate demand shocks, firms tend to pay very little attention to the latter, so that nominal shocks can have large real effects. As a result, prices are sticky in response to aggregate demand shocks, yet very responsive to firm-specific technology shocks.

The key distinction between our model and theirs is the assumption of a complete separation between learning about aggregate and idiosyncratic shocks. This eliminates the response to aggregate prices stemming from the 'wrong' reasons. In particular, our model delivers results similar to theirs if we (i) make the idiosyncratic shocks affecting the wage and demand signals completely transitory (ii) add idiosyncratic technology shocks and technology signals that are completely separated from the aggregate monetary shocks. The assumption in (i) eliminates the 'wrong' reason to change prices - an aggregate shock still leaves firms confused as to nature of the shock but since the idiosyncratic shocks are transitory, information about the past realizations of these shocks does not have any bearing on the firms' optimal price. These shocks then serve purely as informational noise and delay the learning/adjustment process. Since the idiosyncratic technology shocks in (ii) are observed separately, their stochastic process can be chosen to match the dispersion and persistence of prices without any implication for the learning process. However, as we will see, such a calibration of our model has the counter-factual implication that prices and quantities are highly negatively correlated.[2]

On the methodological side, our paper contributes to the solution of general equilibrium models with heterogeneous information. Using techniques first suggested in Hellwig (2002) and fully developed in Hellwig (2008a), we propose a simple and computationally efficient algorithm for solving our model. A key problem in heterogeneous information models is that the model does not easily admit a recursive equilibrium characterization on a finite-dimensional state space. In equilibrium, firms need to make forecasts about the forecasts and pricing decisions of other firms, which in turn depend on their forecasts of others, and so on... (see Townsend (1983)). This problem of forecasting the forecasts of others results because of (i) the fact that the firms' best response pricing strategies depend on their forecasts of what other firms are doing, and (ii) firms will then need to form forecasts of the other firm strategies in order to interpret the market-generated information, if the latter is endogenously generated from market transactions, and thus dependent on equilibrium pricing strategies. This renders the equilibrium filtering problem intractable, and typically deprives the model of a finite-dimensional recursive equilibrium structure.

Instead of looking for a recursive solution to the model, we assume that shocks become common knowledge after a finite, arbitrarily large delay, and recast the equilibrium characterization as a static, finite-dimensional signal extraction problem. The resulting filtering problem turns out to have a very tractable structure, with an easily interpretable solution that is almost in closed form and easy to implement computationally. Although we develop the solution for a specific model, the method can be generalized to almost arbitrary information structures and requires little to no

[2]In an extension, Mackowiak and Wiederholt (2008) relax the assumption of independent signals and consider composite signals of idiosyncratic and aggregate demand shocks. However, their static analysis only focuses on the response on impact and does not consider the dynamic nature of the learning problem, which is the focus in our paper. In our benchmark calibration, the initial price adjustment in response to an aggregate nominal shocks is quite muted, but this period of sluggish adjustment does not last very long.

assumptions about the economic model other than a log-linearization around a steady-state.

In section 2, we show by means of a simple example, how the addition of idiosyncratic reasons for price changes alters aggregate implications of heterogeneous information and price adjustment, mitigating in particular the effects of pricing complementarity. The example also illustrates the tension that arises within the model between generating substantial amounts of price dispersion, and generating large aggregate output effects from nominal shocks. In section 3, we formulate our general model, and develop methods for equilibrium characterization, along with some analytical results. In section 4, we discuss our calibration strategy and the quantitative implications of our model. Section 5 presents extensions of our benchmark model, which are followed by a brief conclusion.

## 2 A simple example

We illustrate our main insights using a stylized example. Consider an economy with a continuum of firms, whose optimal pricing decisions are characterized by the following best-response relation[3] :

$$p_i = \mathbb{E}_i\left(p_i^*\right) + r\mathbb{E}_i\left(p - p^*\right) \tag{1}$$

where $p_i$ denotes firm $i$'s optimal price, $p = \int p_i di$ denotes the average price of all firms, $p_i^*$ and $p^* = \int p_i^* di$ denote exogenous, stochastic 'target' levels for firm $i$'s price and for the aggregate price level, respectively, and $\mathbb{E}_i\left(\cdot\right)$ denotes the firm's expectations conditional on its available information. If the firms had complete information of the targets $p_i^*$ and $p^*$, there would be full adjustment of all prices to the target levels in equilibrium: $p_i = p_i^*$ and $p = p^*$; $p_i^*$ and $p^*$ can thus be interpreted as the full information equilibrium levels. The parameter $r \in (-1, 1)$ measures the degree of complementarity or substitutability in the firms' pricing decisions, i.e. the extent to which an individual firm's optimal price is increasing or decreasing in the average price of the other firms.

Firm $i$'s target is given by

$$p_i^* = \gamma z_i + m, \text{ and hence } p^* = m, \tag{2}$$

where $m \sim \mathcal{N}\left(0, \sigma_m^2\right)$ denotes an aggregate shock to the target level, and $z_i \sim \mathcal{N}\left(0, \sigma_z^2\right)$ denotes an idiosyncratic shock. The targets are not directly observable. The firm instead observes a signal

[3]In this example, this is by assumption but in our full model, we will explicitly derive a similar best response relation from the FOC of the firms' problem.

$s_i$ that is a linear combination of these two shocks and idiosyncratic noise:

$$s_i = m + z_i + \zeta_i, \tag{3}$$

where $\zeta_i \sim \mathcal{N}\left(0, \sigma_\zeta^2\right)$ represents idiosyncratic signal noise. $\mathbb{E}_i\left(\cdot\right) = \mathbb{E}\left(\cdot|s_i\right)$ then denotes the firm's expectation conditional on its signal $s_i$. The interpretation of this information structure, which we will render explicit in our full-fledged model, is the following: the firms face idiosyncratic and aggregate fluctuations in their demand, cost or productivity levels. They extract information from the information that is generated by their market activities (such as their sales, their productivity level, or their wage bill). This information reflects fluctuations in both idiosyncratic and aggregate conditions, but does not enable the firms to fully separate the fluctuations due to idiosyncratic shocks from those due to aggregate shocks. The extent to which firms may want to respond to one type of shock but not the other is captured by the parameter $\gamma$.

The example incorporates a number of notable special cases. If $\gamma = 0$, the idiosyncratic shock does not affect payoffs, and the distinction between $z_i$ and $\zeta_i$ loses its meaning. In this case, the only source of heterogeneity is informational - loosely speaking, this corresponds to the class of heterogeneous information models that were initiated by Woodford (2001). If on the other hand, $\gamma = 1$, we have a special case where the agents observe a signal of their target price level. Moreover, when $\sigma_\zeta^2 = 0$, this signal becomes perfect. In that case, each firm would be able to set its price exactly equal to its target, and it will be willing to do so, if it can count on all other firms doing the same.

**Equilibrium Characterization:** We conjecture that the equilibrium of this simple pricing game takes the form of a linear pricing rule , we conjecture a linear pricing rule $p_i = ks_i$, implying an aggregate adjustment of prices to the aggregate shock that is given by $p = km$. Given this conjecture, we can write firm $i$'s best response as

$$p_i = \mathbb{E}_i[\gamma z_i + m] + r\mathbb{E}_i[p - m] = \gamma \mathbb{E}_i\left(z_i\right) + (1 - r + rk)\mathbb{E}_i\left(m\right). \tag{4}$$

By Bayesian updating, $\mathbb{E}_i\left(m\right) = \sigma_m^2 / \left(\sigma_m^2 + \sigma_z^2 + \sigma_\zeta^2\right) \cdot s_i$ and $\mathbb{E}_i\left(z_i\right) = \sigma_z^2 / \left(\sigma_m^2 + \sigma_z^2 + \sigma_\zeta^2\right) \cdot s_i$. Substituting this into the previous equation, we can write the firm's best response as a linear function of its signal i.e.

$$p_i = \left[\frac{\gamma \sigma_z^2}{\sigma_m^2 + \sigma_z^2 + \sigma_\zeta^2} + (1 - r + rk)\frac{\sigma_m^2}{\sigma_m^2 + \sigma_z^2 + \sigma_\zeta^2}\right] s_i,$$

which verifies the conjecture with

$$k=\frac{\gamma\sigma_z^2}{\sigma_m^2+\sigma_z^2+\sigma_\zeta^2}+(1-r+rk)\frac{\sigma_m^2}{\sigma_m^2+\sigma_z^2+\sigma_\zeta^2} \tag{5}$$

Solving (5) for $k$, we find

$$\begin{aligned} k &= \frac{(1-r)\,\sigma_m^2}{(1-r)\,\sigma_m^2+\sigma_z^2+\sigma_\zeta^2}+\gamma\frac{\sigma_z^2}{(1-r)\,\sigma_m^2+\sigma_z^2+\sigma_\zeta^2} && (6)\\ &= k_W+R(1-k_W), && (7)\\ \text{where } k_W &= \frac{(1-r)\,\sigma_m^2}{(1-r)\,\sigma_m^2+\sigma_z^2+\sigma_\zeta^2} \text{ and } R=\frac{\gamma\sigma_z^2}{\sigma_z^2+\sigma_\zeta^2}. && (8)\end{aligned}$$

This coefficient $k$ summarizes both the response of individual firms' prices to their signals, and the response of the aggregate price level to aggregate shocks. Note that an aggregate shock $m$ causes a price response in 2 ways. First, it affects the firms' expectations of the aggregate shock and aggregate prices, i.e. $\mathbb{E}_i[m]$ and $\mathbb{E}_i[p]$. This is captured by $k_W$. We will refer to this component as the 'right' reason for prices to respond to aggregate shocks. Second, the aggregate shock also affects the firms' expectations of their idiosyncratic shocks ($\mathbb{E}_i[z_i]$) and therefore, induces a price response for the 'wrong' reason. The second term $R(1-k_W)$ captures this effect. The parameter $R$, which multiplies the parameter $\gamma$ with the signal-to-noise ratio for the idiosyncratic shock, measures the importance of the idiosyncratic motives for price adjustment.

**Implications for aggregate price adjustment:** With this characterization, we can note a number of results, both old and new. We begin with the case when $R=0$, in which case the only source of heterogeneity is informational. In this case, $k=k_W$, i.e. only the 'right' reasons affect aggregate price adjustment.

**Proposition 1** *When $R=0$, a higher degree of complementarity ($\frac{\partial k}{\partial r}<0$), or a lower precision of the idiosyncratic shock reduce the response of prices to aggregate shocks.*

This proposition summarizes the main idea of the literature on heterogeneous information, following Woodford (2003): In the presence of heterogeneous information, pricing complementarities amplify the adjustment delay of prices to aggregate shocks: when $r=0$, $k=k_W=\sigma_m^2/\left(\sigma_m^2+\sigma_z^2+\sigma_\zeta^2\right)$, so prices respond to the aggregate shocks with a rate that just reflects the signal to noise ratio $\sigma_m^2/\left(\sigma_z^2+\sigma_\zeta^2\right)$. When instead pricing decisions are complementary ($r>0$), strategies discount the private signals relative to their information content in a way that is directly proportional to the strength of the complementarity. In this case, firms attempt to forecast not only the common target price level, but also the other firms' forecasts of the target. While the former relies on the firms' private signals just in proportion to their information content, the firms higher-order forecasts of the other firms expectations puts higher weights on the firm's priors or other common sources of information, as these are more directly informative of the other firms' beliefs. In equilibrium, the firms' optimal pricing response thus discounts the information contained in the private signals, and amplifies the weight attributed to the prior, as the latter is disproportionally more useful in forecasting the other firms' prices. As $r \to 1$, this discount can become arbitrarily large, and hence in the limit, firms may completely discount private signals, even if they are very precise, because they do not expect other firms to respond to their private information.

The next proposition summarizes how this finding is affected when the firms' optimal pricing decisions also depend on the idiosyncratic shocks $z_i$, i.e. when $R > 0$. In this case, prices adjust for the right as well as the wrong reasons. This proposition immediately follows from the comparative statics w.r.t. $R$ that are implied by equation (7).

**Proposition 2** *$(i)$ An increase in $R$ increases the response of prices to aggregate shocks ($\frac{\partial k}{\partial R} > 0$).*

*$(ii)$ When $R < 1$, the response of prices to aggregate shocks is bounded below by $R$. Pricing complementarities still delay adjustment ($\frac{\partial k}{\partial r} < 0$), but an increase in $R$ mitigates their effects ($\frac{\partial^2 k}{\partial R \partial r} > 0$).*

*$(iii)$ When $R = 1$, the equilibrium generates full adjustment to aggregate shocks i.e. $k = 1$. Complementarities and Heterogeneity in information have no effect on the equilibrium. Moreover, if $\gamma = 1$ and $\sigma_\zeta^2 = 0$, the targets are directly observable ($s_i = p_i^*$), and the equilibrium replicates the full information equilibrium.*

*$(iv)$ When $R > 1$, prices respond more than under full information ($k > 1$), and the effects of pricing complementarities are overturned: complementarities amplify the response to aggregate shocks ($\frac{\partial k}{\partial r} > 0$), and their effects are reinforced if $R$ increases.*

The addition of idiosyncratic shocks to the firm's pricing target thus significantly alters the conclusions of Proposition 1. When $R < 1$, the effects of pricing complementarities still hold, but are weakened by the fundamental shocks. The parameter $R$ serves as a lower bound on the response to aggregate shocks - we can thus no longer obtain arbitrary amplification effects from complementarities. To understand these results, suppose first that the firms thought that $\sigma_m^2$ was zero, i.e. that they never faced any aggregate shocks, and suppose now that there was a one-time increase in $m$, which appears as an increase in the firms' signals. The firms would mistakenly attribute this shock to a combination of the idiosyncratic state and noise, and would respond to this shock with a rate $R$, i.e. their optimal full information rate adjusted for the signal noise. Now, if the variance of an aggregate shock was not zero, the firms' inference and optimal pricing decisions will also allow for that, but this matters only for the response in excess of $R$, and it is only on this remainder that the pricing complementarity has its effect of delaying adjustment.

When $R = 1$, equation (7) implies $k = 1$ i.e. the adjustment resulting from an efficient response to idiosyncratic shocks then implies a full adjustment to the aggregate shock. In this case, firms maybe highly uncertain about whether changes in the signal result from idiosyncratic or aggregate shocks, but this uncertainty doesn't affect their decisions. For example, when $\gamma = 1$ and $\sigma_\zeta^2 = 0$, signals provide perfect information about the current period's target under full information. The resulting equilibrium displays full price adjustment, despite the fact that firms remain imperfectly informed about the magnitudes of idiosyncratic and aggregate shocks, and despite the fact that they face a motive to coordinate pricing decisions. Firms will then set exactly the right prices, albeit for the wrong reasons.

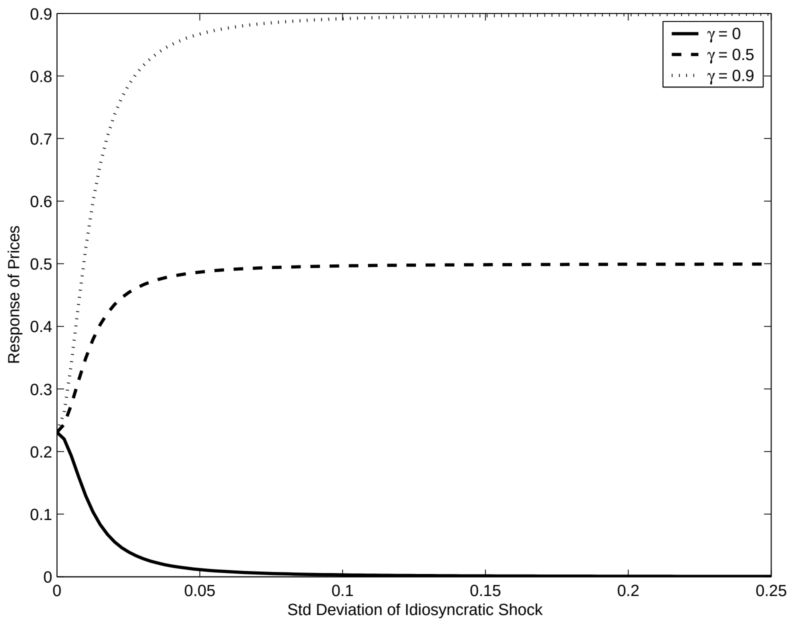


Figure 1: Effect of $\sigma_z^2$ on Price Response

Finally, we discuss comparative statics with respect to the magnitudes of idiosyncratic shocks. Clearly, idiosyncratic noise as measured by $\sigma_\zeta^2$ reduces aggregate adjustment, because it lowers both the 'right reasons' response in $k_W$ and the wrong reasons (through the signal to noise ratio in $R$). The effects of idiosyncratic payoff shocks, as measured by $\sigma_z^2$, on the other hand,are ambiguous, because bigger idiosyncratic shocks reduce $k_W$, but increase $R$, and hence the importance of the 'wrong reasons' adjustment. The latter effect dominates whenever $\gamma$ is sufficiently large. This is illustrated in figure 1.

**Implications for price dispersion:** As the previous discussion made clear, the extent to which idiosyncratic shocks influence the firms' pricing targets is key for the aggregate implications of heterogeneous information. A quantitative assessment of such an information channel thus requires imposing some discipline on the parameter choice of $\gamma$ and $\sigma_z$. To conclude the discussion of this simple example, we explore how the use of micro data, in particular on dispersion of prices at the micro level, can help us draw quantitative conclusions.

In this simple example, price dispersion, measured as the cross-sectional standard deviation of prices at the product level, is given by

$$\text{Disp} = \left\{ \int (p_i - p)^2 \, di \right\}^{1/2} = k \cdot \sqrt{\sigma_z^2 + \sigma_\zeta^2} = [k_W + R(1 - k_W)] \sqrt{\sigma_z^2 + \sigma_\zeta^2}$$

We begin again with the case when $\gamma = 0$ (which implies $R = 0$). In this case, dispersion is increasing for low levels of $\sigma_z^2 + \sigma_\zeta^2$, and decreasing for high levels, and reaches a maximal level when $(1 - r)\, \sigma_m^2 = \sigma_z^2 + \sigma_\zeta^2$. Dispersion is therefore bounded above by

$$Disp \leq \frac{\sqrt{1 - r}}{2} \sigma_m.$$

Thus, a model purely based on informational heterogeneity will imply a level of cross-sectional price dispersion that is necessary smaller in magnitude than the standard deviation of the aggregate shock to the target, and for commonly used degrees of pricing complementarity, the gap may be quite substantial (if $r = 0.84$, $Disp \leq 0.2\sigma_m$).[4] In practice, we see the opposite, i.e. cross-sectional price dispersion tends to be an order of magnitude larger than the standard deviation of aggregate shocks.

How is this conclusion affected by $\gamma$? When $\gamma > 0$, any level of price dispersion may be the result of sufficiently large idiosyncratic shocks. To see this, notice that $\lim_{\sigma_z^2 \to \infty} Disp = \infty$, for any $R > 0$. Price dispersion is then driven entirely by the fact that the equilibrium response to the signal is bounded away from zero, regardless of whether it is the result of an idiosyncratic or an aggregate shock. This is illustrated in figure 2.

To summarize, our example illustrates a tension for heterogeneous information models between generating significant levels of price dispersion and meaningful delays in aggregate price

[4]What is more, the computation of this bound allowed for the maximal level of dispersion, at which exactly $1/2$ of the aggregate shock gets absorbed by prices. Assuming any stronger degree of adjustment delays would further reduce cross-sectional price dispersion.

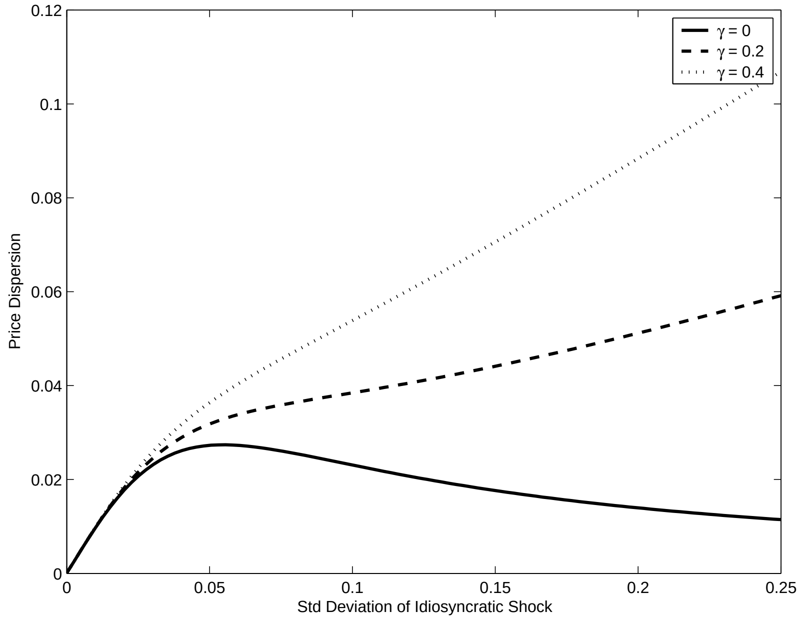


Figure 2: Effect of $\sigma_z^2$ on Price Dispersion

adjustment. Large delays of price adjustment are most likely to result from heterogeneous information, when complementarities are strong (i.e. $r$ close to 1), and optimal prices do not respond to idiosyncratic shocks ($\gamma$ is small). However, heterogeneity in information is, by itself, unable to generate quantitative meaningful levels of price dispersion, which points to an important role for additional idiosyncratic payoff shocks, requiring positive $\gamma$ and positive $\sigma_z^2$. Our model can achieve both significant price dispersion and large real effects only when $\gamma$ is low and $\sigma_z^2$ is high, so that the effects of $R$ on aggregate adjustment are small, yet the effects for price dispersion can be made arbitrarily large by scaling up the variance of idiosyncratic payoff shocks.

**The role of informational non-separability:** Another way around the tension between price dispersion and aggregate adjustment is to separate learning about aggregate variables from learning about idiosyncratic ones. This roughly corresponds to the analysis in Mackowiak and Wiederholt (2008), where firms must optimally allocate a fixed information processing capacity to learn about idiosyncratic and aggregate shocks. Since the former are much bigger than the latter, firms optimally choose to devote most of their attention to firm-specific factors. As a result, price response to the aggregate shocks is relatively muted while price dispersion is achieved by adjusting the variance of the idiosyncratic shock.

We can illustrate the difference between Mackowiak and Wiederholt's results and ours by considering a simple variation of our model, in which we assume that the firms' target price takes the form given by (2), but that there are two different signals, one about $m$, and one about the idiosyncratic shock $z_i$:

$$s_i^1 = m + u_i \text{ and } s_i^2 = z_i + \zeta_i$$

where $u_i \sim \mathcal{N}\left(0, \sigma_u^2\right)$ and $\zeta_i \sim \mathcal{N}\left(0, \sigma_\zeta^2\right)$. In this case, the firm's expectations of $m$ only depend on $s_i^1$, while expectations about $z_i$ only depend on $s_i^2$. As a result of this separation, aggregate price adjustment is only driven by the firms' expectations of $m$ and $p$. Equilibrium prices then are characterized by $p_i = ks_i^1 + ls_i^2$, where $k = (1-r)\sigma_m^2 / \left((1-r)\sigma_m^2 + \sigma_u^2\right)$ and $l = \gamma\sigma_z^2 / \left(\sigma_z^2 + \sigma_\zeta^2\right)$. Aggregate price adjustment is then given by $p = km$, while price dispersion is $Disp = k\sigma_u + l\sqrt{\sigma_z^2 + \sigma_\zeta^2} = k\sigma_u + \gamma\sigma_z^2 / \sqrt{\sigma_z^2 + \sigma_\zeta^2}$. Thus, Woodford's analysis fully applies to the response to aggregate shocks, while price dispersion is explained separately by the response to firm-specific idiosyncratic shocks. Mackowiak and Wiederholt (2008) show that within the rational inattention paradigm, this is reinforced by the endogenous 'choice' of $\sigma_\zeta$ and $\sigma_u$ under a capacity constraint of the form $\sigma_\zeta^{-2} + \sigma_u^{-2} \leq \kappa$: with the assumption that idiosyncratic shocks are much larger than aggregate shocks ($\sigma_z^2 >> \sigma_m^2$), firms will optimally choose to devote all their attention to the former and assign little processing capacity to the latter - formally this amounts to choosing a small value for $\sigma_\zeta$ and a large value for $\sigma_u$, which helps generate more price dispersion as well as less aggregate price adjustment.

**From the example to the full model:** Our primary motivation in introducing this simple example was to highlight the key forces and tensions in play. It is far too stylized to enable us to assess the reasonableness of parameter combinations or assumptions about signal structure, such as the informational non-separability. In the remainder of this paper, we use the additional structure imposed by a fully micro-founded, quantitative, dynamic general equilibrium model, which will not only attach a natural interpretation to the information structure but also enable us to use a richer set of micro moments (as opposed to using just the dispersion in prices) to inform our choice of parameters.

Many of the insights from our simple example will extend to the full model, but the latter will have a couple of additional features that go beyond the simple model and are worth discussing here. First, our dynamic general equilibrium model will allow for gradual learning about the underlying aggregate and idiosyncratic shocks over time. This leads to richer effects on price dynamics that are super-imposed within the present static framework - in particular, we will see how expectations about firm-specific shocks (i.e. the 'wrong' reasons) generate substantial short-term price adjustment, which is followed by a much more gradual convergence to full adjustment as a result of learning about aggregate shocks (i.e. the 'right' reasons).

Second, our signals will be interpreted with the general equilibrium model as resulting from firm-specific demand and input cost observations. These signals will depend also depend on aggregate prices and will therefore be endogenous.

We can illustrate the effects of this signal endogeneity with another simple variation on our example. Suppose that the signal observed by the firm was $s_i = \delta p + (1-\delta)m + z_i + \zeta_i$. Using our equilibrium conjecture that $p = km$, this signal can be rewritten as $s_i = (1 - \delta + \delta k)m + z_i + \zeta_i$. In the resulting equilibrium, the response coefficient $k$ is characterized as a solution to the following fixed point problem

$$k = K_W(k) + (1 - \delta + \delta k) R (1 - K_W(k)) \tag{9}$$

$$\text{where } K_W(k) = \frac{(1-r)(1-\delta+\delta k)^2 \sigma_m^2}{(1-r)(1-\delta+\delta k)^2 \sigma_m^2 + \sigma_z^2 + \sigma_\zeta^2}$$

We thus obtain an equilibrium characterization that is very similar to the original one, except for the fixed point nature of (9), which comes from the fact that the informativeness of the signal is now endogenous - a higher price response (a higher $k$) increases the informativeness of the signal. As before, the aggregate response is decomposed into a term resulting from the "right" reasons, measured by $K_W(k)$, and a term resulting from the wrong reasons, which is now scaled by $R(1 - \delta + \delta k)$.

We also recover our main results from the example with exogenous signals. When $R = 0$, price adjustment is driven entirely by expectations about aggregate shocks and prices, which are captured by $K_W(k)$. The main insights regarding the effects of complementarity and information heterogeneity (Prop. 1) go through, but they are reinforced by the fact that less price adjustment mutes the quality of the signal about $m$ ($\partial K_W / \partial k < 0$). When instead $R = 1$, there again is a full adjustment equilibrium with $k = 1$, irrespective of the degree of price endogeneity $\delta$, or the complementarity $r$. Finally, for the intermediate case with $R < 1$ and $k < 1$, the idiosyncratic adjustment motive mutes the effect of both complementary and signal endogeneity, as in the benchmark model. For these intermediate values, the right-hand side of the fixed point characterization (9) is decreasing in $\delta$, for given $k$, so the fixed point must be decreasing in $\delta$. In other words, the stronger the endogeneity of signals to prices, the less responsive firms become to their signals in equilibrium, because a lower rate of adjustment also lowers the informativeness of the signals about $m$.

# 3 The general model

In this section, we formulate a fully micro-founded dynamic stochastic general equilibrium model in which information heterogeneity emerges endogenously from the assumption that firms have only limited access to information. Our working assumption is that firms observe the information that is generated through their market transactions, which consist of the demand for their products (or equivalently, the firms' revenues), and of the wages paid to their workers. Wages and demand observations are subject to idiosyncratic and aggregate fluctuations, and do not allow the firms to fully infer the underlying states.

In order to focus on the role of pricing complementarities and information vs. payoff heterogeneity on the production side, the household side is kept as close to the New Keynesian benchmark as possible. In particular, we assume that the representative household has access to complete information about the aggregate and market-specific shocks, and has access to complete contingent claims markets.

## 3.1 Model description

**Representative Household:** The representative household maximizes its lifetime utility over consumption $C_t$ and real balances $M_t/P_t$, as well as disutility of effort over a measure $1$ continuum of labor types $N_{it}$,

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\psi}}{1-\psi} + \ln \frac{M_t}{P_t} - \frac{1}{1+\kappa} \int_0^1 Z_{it} N_{it}^{1+\kappa} di \right),$$

where $\beta \in (0,1)$, $\psi > 0$, $\kappa > 0$, and $N_{it}$ represents labor supply and $Z_{it}$ , an idiosyncratic preference shock for labor of type $i$. $\mathbb{E}_t(\cdot)$ denotes the representative household's expectations as of date $t$. Assuming that the household has access to a complete contingent claims market, we can write the household's life-time budget constraint as

$$M_0 \geq \mathbb{E}_0 \sum_t \lambda_t \{ C_t P_t + i_t M_t - \int_0^1 W_{it} N_{it} di - \Pi_t - T_t \}$$

where $\lambda_t$ denotes the economy's stochastic discount factor used to price nominal balances, $\Pi_t$ and $T_t$ denote aggregate corporate profits and taxes or transfers (in nominal terms), $W_{it}$ denotes the nominal wage for labor of type $i$, and the term $i_t M_t$ denotes the household's opportunity costs of holding monetary balances at date $t$. The first order conditions of this problem are

$$\lambda_t = \beta^t \frac{C_t^{-\psi}}{P_t} = \beta^t \frac{1}{i_t M_t} = \beta^t \frac{\eta Z_{it} N_{it}^{\kappa}}{W_{it}} = (1+i_t) \mathbb{E}_t \lambda_{t+1}$$

Along with the budget constraint and a law of motion for $i_t$ that is determined by monetary policy, these equations characterize the solution to the household's problem. Throughout this paper, we focus on the special case where $\ln M_t$ follows a random walk with drift $\mu$:

$$\ln M_{t+1} = \ln M_t + \mu + \sigma_u u_t$$

where $u_t$ is an iid random variable, distributed $N(0,1)$. This implies that interest rates are constant at a level $\hat{\imath}$, defined by $(1+\hat{\imath})^{-1} = \beta \mathbb{E}_t \left(M_t/M_{t+1}\right) = \beta exp\left(-\mu + \sigma_u^2/2\right)$, which we assume to be strictly positive. Along with the FOC, we can then write state prices, consumption, and wages as follows:

$$\lambda_t = \beta^t \frac{1}{M_t} \hat{\imath} \tag{10}$$

$$C_t = K_0 \left(\frac{M_t}{P_t}\right)^{\frac{1}{\psi}} \tag{11}$$

$$W_{it} = K_1 M_t Z_{it} N_{it}^{\kappa} \tag{12}$$

where $K_0$ and $K_1$ are time-independent constants. Besides this characterization of the household's equilibrium behavior through static equations for aggregate consumption and type-specific wages, the constant interest rate benchmark also eliminates the effects of nominal interest rates as a public signal of aggregate economic activity.[5]

**Production Side:** Consumption $C_t$ is a composite good, assembled by competitive firms using a standard Dixit-Stiglitz aggregator

$$C_t = \left(\int B_{it}^{\frac{1}{\theta}} C_{it}^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}$$

where $B_{it}$ is an idiosyncratic shock process and $C_{it}$ is the amount of output of sector $i$ used in the production of the final good. This leads to the standard expressions for demand functions for the output of sector $i$

$$C_{it} = B_{it} C_t \left(\frac{P_{it}}{P_t}\right)^{-\theta} \tag{13}$$

and the aggregate price index

$$P_t = \left(\int B_{it} P_{it}^{1-\theta} di\right)^{\frac{1}{1-\theta}}. \tag{14}$$

[5] It is conceptually straight-forward to extend our analysis to a more complete monetary policy framework. Moreover, the additional information conveyed by interest rates would reduce both the level of price dispersion and the degree of non-neutrality, which we conjecture would reinforce the results presented here.

Each intermediate good $i$ is produced by a single firm, using labor type $i$ as its unique input into production. We assume that production in intermediate sector $i$ is linear in the labor input, and is given by

$$Y_{it} = A_{it} N_{it} \tag{15}$$

For the moment, we assume that $A_{it} = 1$, i.e. that productivity is constant for all firms.[6] The intermediate producer in sector $i$ sets a price $P_{it}$ to solve

$$\max_{P_{it}} \mathbb{E}_{it} \left( \lambda_t \left( P_{it} C_{it} - N_{it} W_{it} \right) \right).$$

These prices are set before markets open, and the expectation $\mathbb{E}_{it}(\cdot)$ is conditional on the information available to the firm at the time of making its pricing decision. The firm's information set $\mathcal{I}_t^i$ will be defined below.

Substituting from the optimality conditions of the household (10)-(12) and the demand function (13), the problem can be written as

$$\max_{P_{it}} \mathbb{E}_{it} \left[ B_{it} \frac{C_t P_t}{M_t} \left( \frac{P_{it}}{P_t} \right)^{1-\theta} - K_1 Z_{it} \left( B_{it} C_t \left( \frac{P_{it}}{P_t} \right)^{-\theta} \right)^{1+\kappa} \right]$$

The resulting first order condition is

$$P_{it}^{1+\theta\kappa} = \frac{\theta (1+\kappa) K_1}{\theta - 1} \cdot \frac{\mathbb{E}_{it}[Z_{it} B_{it}^{1+\kappa} C_t^{1+\kappa} P_t^{\theta(1+\kappa)}]}{\mathbb{E}_{it}[B_{it} C_t P_t^{\theta} M_t^{-1}]}$$

Under the assumption that $(i)$ conditional on the firm's information, $Z_{it} b_{it}^{1+\kappa} C_t^{1+\kappa} P_t^{\theta(1+\kappa)}$ and $B_{it} C_t P_t^{\theta} M_t^{-1}$ are log-normally distributed, and $(ii)$ conditional on a realization of aggregate shocks, the cross-sectional distribution of prices across firms is also log-normal, we can take logs on both sides and find:[7]

$$p_{it} = Const + \frac{\kappa}{1+\theta\kappa} \mathbb{E}_{it}[b_{it} + c_t + \theta p_t] + \frac{1}{1+\theta\kappa} \mathbb{E}_{it}[z_{it} + m_t] \tag{16}$$

for some real-valued constant $Const$. Using the household FOC (11) to substitute for $\ln C_t$, this becomes

$$\begin{aligned} p_{it} &= Const + \frac{1}{1+\theta\kappa} \mathbb{E}_{it}[\kappa b_{it} + z_{it}] + \mathbb{E}_{it}[m_t] + \frac{\kappa \left( \theta - \psi^{-1} \right)}{1+\theta\kappa} \mathbb{E}_{it}[p_t - m_t], \qquad (17) \\ \text{where } p_t &= \ln P_t = \frac{1}{1-\theta} \ln \left( \int_0^1 B_{it} P_{it}^{1-\theta} di \right) = const + \int_0^1 p_{it} di. \end{aligned}$$

[6]In section 5, we will consider extensions with idiosyncratic and aggregate productivity shocks.

[7]We shall use small letters to denote the natural logs of capital-lettered variables, e.g. for any variable $X$, we write $x = \ln X$.

For a given information structure $\left\{\mathcal{I}_t^i, i \in [0,1]\right\}$, a price function $p\left(\mathcal{I}_t^i\right)$ then characterizes an *equilibrium*, if and only if it constitutes a fixed point of the firms' optimal pricing condition (17). Notice that (17) represents exactly the form of equilibrium pricing relation (1) that we used in the simple example of the previous section. In particular, $r \equiv \frac{\kappa\left(\theta-\psi^{-1}\right)}{1+\theta\kappa}$ determines the degree of pricing complementarity (corresponding to $r$ in the previous section's notation). The firms' willingness to respond to the idiosyncratic demand and cost processes is determined in part by $\frac{\kappa}{1+\theta\kappa}$ and $\frac{1}{1+\theta\kappa}$, respectively.

**Information structure, idiosyncratic and aggregate shocks:** To complete the model description, we describe the information structure $\left\{\mathcal{I}_t^i, i \in [0,1]\right\}$ and specify the stochastic processes for the various shocks. The model has a single aggregate shock to supply $m_t$, which follows a random walk with drift $\mu$ and variance $\sigma_u^2$, as well as product-specific shocks to demand $B_{it}$ and labor supply $Z_{it}$. We assume that the two product-specific shocks are each the sum of a persistent component that admits an AR(1) representation, and a transitory shock that is iid over time. That is, $B_{it}$ and $Z_{it}$ admit the following representations:

$$b_{it} = \sigma_b \sum_{s=0}^{\infty} \rho_b^s v_{i,t-s}^1 + \tilde{\sigma}_b \tilde{v}_t^1 \qquad\qquad z_{it} = \sigma_z \sum_{s=0}^{\infty} \rho_z^s v_{i,t-s}^2 + \tilde{\sigma}_z \tilde{v}_t^2 \tag{18}$$

where $v_t^1$, $\tilde{v}_t^1$, $v_t^2$, and $\tilde{v}_t^2$ are iid random variables distributed according to $\mathcal{N}(0,1)$, and $\rho_b$, $\sigma_b$, $\tilde{\sigma}_b$, $\rho_z$, $\sigma_z$ and $\tilde{\sigma}_z$ are all non-negative.

In each period, a firm observes the demand for its product $C_{it}$, as well as its wage bill $W_{it}N_{it}$. This is informationally equivalent to observing one demand signal $x_{it}$ and a wage signal $\omega_{it}$. Using the demand function (13) and the household optimality conditions (10)-(12), these signals can be written as a function of the exogenous shocks and the aggregate price level:

$$x_{it} = b_{it} + \psi^{-1} m_t + (\theta - \psi^{-1}) p_t \tag{19}$$

$$\omega_{it} = z_{it} + m_t \tag{20}$$

The idiosyncratic demand and wage shocks thus prevent the full revelation of aggregate shocks from demand and cost observations. Nevertheless, firms are able to infer their target prices within each period from these signals - in fact, if the firms had the information about the current signal realization in advance of setting their current prices, then by setting $p_{it} = \frac{\kappa}{1+\theta\kappa} \cdot x_{it} + \frac{1}{1+\theta\kappa} \cdot \omega_{it}$, they would be able to set exactly the price that is optimal for the current period.[8]

[8]Therefore, if we allowed firms to observe the current signals in real time while making decisions (as would be the case in a noisy rational expectations equilibrium), the demand and wage observations would be sufficient to exactly infer the current full information targets, and as a consequence the equilibrium would lead to full price adjustment.

In our model, this is prevented by the assumption that a firm has to decide on its price in advance of the realization of current market outcomes and current wage and demand signals. Its expectations are then based on realizations of the signals up until the preceding period. This timing assumption enables us to link the specification of idiosyncratic shocks back to our stylized example. The transitory components will play a role similar to the signal noise in the example, while the persistent component will play the role of the firm-specific shocks to price targets, with the degree of persistence determining the extent to which firms will want to respond to the information about firm-specific shocks contained in the previous period's signals, corresponding to our previous parameter $\gamma$. The rationale for the parametrization of idiosyncratic shocks in terms of transitory and persistent components will be discussed further, when we turn to our quantitative results.

Finally, we assume that, at time $t$, the shocks $(u_{t-T}, v^1_{i,t-T}, \tilde{v}^1_{t-T}, v^2_{i,t-T}, \tilde{v}^2_{t-T})$ become common knowledge (where $T$ is large, but finite). As we will see in the next section, this assumption will help us cast the problem as a finite-dimensional filtering problem, and thereby lead to simple and tractable characterization of equilibrium dynamics.

In summary, a firm's information set at the beginning period $t$ is defined as

$$\mathcal{I}^i_t = \left\{x_{it-s}, \omega_{it-s}, u_{t-T-s}, v^1_{i,t-T-s}, \tilde{v}^1_{t-T-s}, v^2_{i,t-T-s}, \tilde{v}^2_{t-T-s}\right\}^{\infty}_{s=1}.$$

Because states are fully revealed with a delay of $T$ periods, the information contained in the demand and wage signals that are more than $T$ periods old is redundant.

### 3.2 Solving the model

We solve this model using techniques and characterization results first suggested by Hellwig (2002) and further developed in Hellwig (2008a).[9] The central technical issue in this class of models is the infinite-regress problem that results from $(a)$ the firm's desire to forecast the other firms' prices due to the complementarity in price-setting, and $(b)$ their need to form a forecast of other firms' prices in order to interpret the demand signal $x_{it}$, when $\theta \neq \psi^{-1}$. In general, such higher-order filtering problems can quickly become intractable, since the model typically does not give rise to a recursive structure with a finite-dimensional state vector - in other words, Kalman filtering techniques, which in this class of models work well in single-agent filtering problems, are a lot less tractable for the higher-order forecasting problem that we face here. This problem is

[9]See also Hellwig and Veldkamp (2008) for similar equilibrium characterization results in a static model context.

compounded by the endogeneity of information to equilibrium strategies through the demand signals.[10]

By assuming that fundamental shocks become common knowledge after a delay of $T$ periods, we can recast the firms' decision problem as a filtering problem over the finite-dimensional vector of shocks, and thereby circumvent the tractability issues associated with the infinite regress problem. The resulting filtering problem is very tractable and admits a simple closed form solution, for a given information structure. The endogeneity of the demand signals to equilibrium prices is then resolved by finding a fixed point between the pricing conjecture entering the firm's filtering problem, and the resulting best-response prices - much like in the endogenous signal extension of our simple example.

**Vector Representation and Model Solution:** Let $U_t, V^1_{it}$, $\tilde{V}^1_{it}$ and $V^2_{it}$ and $\tilde{V}^2_{it}$ denote the vector of shocks that have occurred, but not yet been fully revealed at the time of choosing $p_{it}$; that is, $U_t' = (u_{t-1}, u_{t-2}, ...u_{t-T})$, $V^{1\prime}_{it} = (v^1_{it-1}, v^1_{it-2}, ...v^1_{it-T})$, $\tilde{V}^{1\prime}_{it} = (\tilde{v}^1_{it-1}, \tilde{v}^1_{it-2}, ...\tilde{v}^1_{it-T})$, $V^{2\prime}_{it} = (v^2_{it-1}, v^2_{it-2}, ...v^2_{it-T})$, and $\tilde{V}^{2\prime}_{it} = (\tilde{v}^2_{it-1}, \tilde{v}^2_{it-2}, ...\tilde{v}^2_{it-T})$.[11]

We separate the firms' optimal target prices and signals into two components - a 'common knowledge component' which consists of the contributions of all the shocks realized prior to date $t-T$, which are therefore common knowledge at date $t$, and a 'filtering component', which consists of the contributions of all the more recent shocks, i.e. the vectors $U_t, V^1_t$ and $V^2_t$. Formally, from the firms best response function (17), and the MA representation of the shocks (18), we define the common knowledge component of the firm's optimal price as

$$\hat{p}_{it} = \text{Const} + \frac{1}{1+\theta\kappa} \sum_{s=T+1}^{\infty} \left(\kappa\sigma_b\rho_b^s v^1_{i,t-s} + \sigma_z\rho_z^s v^2_{i,t-s}\right) + m_{t-T-1},$$

where we have already made use of the fact that the common knowledge component of the aggregate price, $\hat{p}_t$, equals $m_{t-T-1} + \text{Constant}$. The firms' optimal price is the sum of the commonly known component and the part that depends on signals from the last $T-1$ periods.

$$p_{it} = \hat{p}_{it} + (1-r)\,\sigma_u \mathbf{1}'\mathbb{E}_{it}[U_t] + r\left(\mathbb{E}_{it}[p_t] - \hat{p}_t\right) + \sigma_b \underbrace{\frac{\kappa\rho_b}{1+\theta\kappa}\boldsymbol{\Upsilon}_{\mathbf{b}}{}'}_{\gamma'_{\mathbf{b}}}\mathbb{E}_{it}[V^1_{it}] + \sigma_z \underbrace{\frac{\rho_z}{1+\theta\kappa}\boldsymbol{\Upsilon}_{\mathbf{z}}{}'}_{\gamma'_{\mathbf{z}}}\mathbb{E}_{it}[V^2_{it}] \quad (21)$$

[10] Woodford (2001) does solve his model using Kalman filtering techniques, but in his case, the information structure has no endogenous elements and admits a simple recursive representation of higher-order expectations.

[11] Notice that we do not include $u_t$, $v^1_{it}$, $\tilde{v}^1_{it}$, $v^2_{it}$ and $\tilde{v}^2_{it}$ in these vectors as these shocks have not yet been realized. They are certainly relevant for the firms' optimal prices when choosing $p_{it}$, but since the firms have no information about these shocks yet, they have an expected value of zero and therefore no effect on the optimal price.

where $\mathbf{\Upsilon_b}' \equiv (1, \rho_b, \rho_b^2, ..., \rho_b^{T-1})$, $\mathbf{\Upsilon_z}' \equiv (1, \rho_z, \rho_z^2, ..., \rho_z^{T-1})$ and $\mathbf{1} = (1, 1, ..., 1)'$, and $r = \frac{\kappa(\theta - \psi^{-1})}{1+\theta\kappa}$. Note that this is in exactly the same form as the optimal pricing rule (**??**) from the simple example in Section 2. The vectors $\gamma_\mathbf{b}'$ and $\gamma_\mathbf{z}'$ are analagous to the parameter $\gamma$ in the example.

Next, we conjecture that equilibrium prices will fully adjust to the shocks included in the common knowledge component, but the response to shocks included in the filtering component will be determined from the resulting equilibrium filtering problem. In other words, we conjecture that

$$p_t = \hat{p}_t + \sigma_u \phi' U_t \tag{22}$$

for some $T \times 1$ vector $\phi'$=$(\phi_1, ....\phi_T)$. With this conjecture, we arrange firm $i$'s optimal price as

$$p_{it} = \hat{p}_{it} + \sigma_u \left[ (1-r)\,\mathbf{1}' + r\phi' \right] \mathbb{E}_{it}[U_t] + \sigma_b \gamma_\mathbf{b}' \mathbb{E}_{it}[V_{it}^1] + \sigma_z \gamma_\mathbf{z}' \mathbb{E}_{it}[V_{it}^2] \tag{23}$$

The firms' optimal pricing decisions thus depend on its expectations about the aggregate component $U_t$, as well as its expectations about the firm-specific shocks $V_{it}^1$ and $V_{it}^2$. Averaging (23) across all $i$ and using the equilibrium conjecture $p_t = \hat{p}_t + \sigma_u \phi' U_t$, we find

$$\sigma_u \phi' U_t = \sigma_u \left[ (1-r)\,\mathbf{1}' + r\phi' \right] \bar{E}_t[U_t] + \sigma_b \gamma_\mathbf{b}' \bar{E}_t[V_{it}^1] + \sigma_z \gamma_\mathbf{z}' \bar{E}_t[V_t^2], \tag{24}$$

where the $\bar{E}_t(\cdot) = \int E_{it}[\cdot]di$ denotes the firm's average expectations at the time of making the period $t$ pricing decision. We thus obtain a representation for average prices as a function of average expectations about the underlying shocks. Average prices respond to $U_t$ for two reasons: $(i)$ the firms' average expectations about aggregate conditions (the first term), and $(ii)$ the firms' average expectations of firm-specific shocks (the second and third terms). Our equilibrium conjecture is confirmed if (as will be shown below) these average expectations are linear functions of $U_t$.

We make use of the underlying information structure to compute these average expectations. Let $X_{it}$ and $\Omega_{it}$ be the set of non-redundant signals at time $t$: $X_{it}' = (x_{i,t-1}, x_{i,t-2}, ...x_{t-T})$ and $\Omega_{it}' = (\omega_{i,t-1}, \omega_{i,t-2}, ...\omega_{t-T})$. Just like the target prices, we decompose the signal vectors $X_{it}$ and $\Omega_{it}$ into a common knowledge component and a filtering component. For any vector $d' = (d_1, d_2, ...d_T)$, let $B(d)$ denote the upper-dimensional $T \times T$ matrix with $d_i$ in its $k, k+i-1$-th entry:

$$B(d) \equiv \begin{pmatrix} d_1 & d_2 & . & d_{T-1} & d_T \\ 0 & d_1 & d_2 & . & d_{T-1} \\ . & . & . & . & . \\ 0 & . & 0 & d_1 & d_2 \\ 0 & 0 & . & 0 & d_1 \end{pmatrix}$$

With this notation, the signal vectors are written as

$$\begin{aligned} X_{it} &= \hat{X}_{it} + \sigma_u \left( \psi^{-1} B(\mathbf{1}) + \left(\theta - \psi^{-1}\right) B(\hat{\phi}) \right) U_t + \sigma_b B(\boldsymbol{\Upsilon}_\mathbf{b}) V_{it}^1 + \tilde{\sigma}_b \tilde{V}_{it}^1 \\ \Omega_{it} &= \hat{\Omega}_{it} + \sigma_u B(\mathbf{1}) U_t + \sigma_z B(\boldsymbol{\Upsilon}_\mathbf{z}) V_{it}^2 + \tilde{\sigma}_z \tilde{V}_{it}^2 \end{aligned}$$

where

$$\hat{\phi}' \equiv (0, \phi_1, ..., \phi_{T-1})$$

and the common knowledge components are $\hat{X}_{it}' = (\hat{x}_{it-1,t}, ..., \hat{x}_{it-T,t})$ and $\hat{\Omega}_{it}' = (\hat{\omega}_{it-1,t}, ..., \hat{\omega}_{it-T,t})$, with

$$\begin{aligned} \hat{x}_{it-k,t} &= \sigma_b \sum_{s=T+1}^{\infty} \rho_b^{s-k} v_{i,t-s}^1 + \theta m_{t-T-1} \\ \hat{\omega}_{it-k,t} &= \sigma_z \sum_{s=T+1}^{\infty} \rho_z^{s-k} v_{i,t-s}^2 + m_{t-T-1}. \end{aligned}$$

The next lemma computes the average expectations of $\left(U_t, V_{it}^1, V_{it}^2\right)$, using the solution to the firms' filtering problem for $\left(U_t, V_{it}^1, V_{it}^2\right)$ conditional on $(X_{it}, \Omega_{it})$:

**Lemma 1** *(Average expectations lemma): (i) $\bar{E}_t[U_t]$, $\bar{E}_t[V_{it}^1]$, and $\bar{E}_t[V_{it}^2]$ are given by*

$$\begin{aligned} \bar{E}_t[U_t] &= \Gamma' \left(\Gamma\Gamma' + \Delta\Delta'\right)^{-1} \Gamma U_t, \\ \bar{E}_t[V_{it}^1] &= \left[ \begin{array}{cc} \sigma_b B(\boldsymbol{\Upsilon}_\mathbf{b})' & 0 \end{array} \right] \left(\Gamma\Gamma' + \Delta\Delta'\right)^{-1} \Gamma U_t, \\ \bar{E}_t[V_{it}^2] &= \left[ \begin{array}{cc} 0 & \sigma_z B(\boldsymbol{\Upsilon}_\mathbf{z})' \end{array} \right] \left(\Gamma\Gamma' + \Delta\Delta'\right)^{-1} \Gamma U_t \end{aligned}$$

*where*

$$\Gamma = \sigma_u \left( \begin{array}{c} \psi^{-1} B(\mathbf{1}) + \left(\theta - \psi^{-1}\right) B(\hat{\phi}) \\ B(\mathbf{1}) \end{array} \right) \text{ and } \Delta = \left( \begin{array}{cccc} \sigma_b B(\boldsymbol{\Upsilon}_\mathbf{b}) & \tilde{\sigma}_b I & 0 & 0 \\ 0 & 0 & \sigma_z B(\boldsymbol{\Upsilon}_\mathbf{z}) & \tilde{\sigma}_z I \end{array} \right).$$

To interpret these expressions, notice that the firms' posterior variance-covariance matrix over $U_t$, conditional on $(X_{it}, \Omega_{it})$, is

$$\Sigma = I - \Gamma' \left(\Gamma\Gamma' + \Delta\Delta'\right)^{-1} \Gamma.$$

The average expectations $\bar{E}_t[U_t]$ can thus be rewritten as $\bar{E}_t[U_t] = [I - \Sigma] U_t$. Similarly, the two terms $\left[ \begin{array}{cc} \sigma_b B(\boldsymbol{\Upsilon}_\mathbf{b})' & 0 \end{array} \right] \left(\Gamma\Gamma' + \Delta\Delta'\right)^{-1} \Gamma$ and $\left[ \begin{array}{cc} 0 & \sigma_z B(\boldsymbol{\Upsilon}_\mathbf{z})' \end{array} \right] \left(\Gamma\Gamma' + \Delta\Delta'\right)^{-1} \Gamma$ denote the negative of the conditional covariance matrices of $V_{it}^1$ and $V_{it}^2$ with $U_t$, respectively.

The proof of this lemma is straight-forward: using standard projection arguments, we first derive expectations for $U_t$, $V_{it}^1$ and $V_{it}^2$ conditional on $(X_{it}, \Omega_{it})$. These are then averaged across all individuals to find average expectations of $U_t$ in terms of the signal averages - the signal averages in turn are just functions of the vector of aggregate shocks $U_t$. The matrix $\Gamma$ and variance-covariance matrix $\Sigma$ depend on $\hat{\phi}$, because the demand signals depend on aggregate price adjustment vector $\hat{\phi}$.

Substituting this characterization of average expectations into (24), we obtain, for given equilibrium conjecture $\hat{\phi}$ and the resulting posterior variance-covariance matrix $\Sigma$, the vector $\phi$ that result from (24). An equilibrium is then a fixed point of (24), given the characterization of average expectations in Lemma 1. For given parameters, the solution to this fixed point problem is easy to compute numerically, even for large values of $T$.[12]

**Analytical Results:** Before presenting numerical results, we can provide analytical results for some special cases. When $\rho_b = \rho_z = 0$, only the expectations about aggregate shocks matter for price adjustment - firm-specific shocks do not affect optimal prices. This is the case on which existing results about propagation and persistence are based. In this case, $\phi'$ is given by

$$\phi' = (1-r)\,\mathbf{1}'\,(I-\Sigma)\,[I-r\,(I-\Sigma)]^{-1}\,. \tag{25}$$

When $r = 0$, the firms' prices then adjust according to their expectations of $U_t$, i.e. $p_t = \mathbf{1}'\bar{E}_t[U_t]$. When $r > 0$, since $(1-r)\,[I-r\,(I-\Sigma)]^{-1} = (1-r)\sum_{s=0}^{\infty} r^s\,(I-\Sigma)^s << I$, real effects will be larger. Therefore, we have the standard effect that complementarities delay price adjustment. This is the generalization of proposition 1 from the example section, and corresponds to the main result of Woodford (2001).[13]

When instead the firm-specific shocks are persistent ($\rho_b > 0$ and/or $\rho_z > 0$), the firms' optimal prices respond to their expectations about idiosyncratic conditions, i.e. the second and third terms in (24) are positive. This will increase the firms' overall price adjustment, which is now characterized as

$$\begin{aligned} \phi' \;=\;& (1-r)\,\mathbf{1}'\,(I-\Sigma)\,[I-r\,(I-\Sigma)]^{-1} \\ &+\frac{1}{\sigma_u}\begin{bmatrix} \sigma_b^2\gamma_{\mathbf{b}}' B(\boldsymbol{\Upsilon}_{\mathbf{b}})' & \sigma_z^2\gamma_{\mathbf{z}}' B(\boldsymbol{\Upsilon}_{\mathbf{z}})' \end{bmatrix}\left(\Gamma\Gamma' + \Delta\Delta'\right)^{-1}\Gamma\,[I-r\,(I-\Sigma)]^{-1}\,. \end{aligned} \tag{26}$$

[12]Since the fixed point map is highly non-linear, we have not been able to prove analytically whether the resulting fixed point is unique. However, we checked numerically that we always found the same fixed point characterization, for a very wide range of starting guesses.

[13]In addition, the endogeneity of demand signals generates extra persistence, because monetary shocks are only gradually reflected in demand signals.

The relative sizes of aggregate, as well as persistent and transitory idiosyncratic shocks determines the magnitudes of the two components to overall price adjustment. When aggregate shocks are large relative to idiosyncratic ones, the posterior variance-covariance matrix $\Sigma$ will be small, and adjustment dynamics will mainly be driven by the first term which captures the firms' expectations about aggregate shocks - firms adjust prices for the 'right' reasons.

When instead aggregate shocks are small relative to idiosyncratic ones, $\Sigma$ is close to $I$, firms do not update much their beliefs about aggregates and instead attribute all changes in signals to idiosyncratic shocks. As a consequence, the first term in (26) is small. The response of prices to aggregate shocks is then driven mainly by the second term in (26), i.e. prices adjust 'for the wrong reasons', because firms mistakenly attribute changes in aggregate conditions to changes in idiosyncratic shocks. The size of this second term depends on the conditional co-variances between aggregate and persistent idiosyncratic shocks. The size of these covariances in turn depends on the relative importance of transitory and persistent idiosyncratic shocks - just like in the simple example, in which $R$ was the combination of the impact $\gamma$ of idiosyncratic shocks on optimal prices, and the relative weight of idiosyncratic fundamentals vs. idiosyncratic noise.

We can make these observations precise by considering the limiting case where $\sigma_u \to 0$, holding fixed the other parameters. In that case, $\Sigma \to I$, and $\phi$ converges to

$$\begin{aligned}\phi' \quad &= \quad \mathbf{e}_1' \rho_b \sigma_b^2 B(\mathbf{\Upsilon_b}) B(\mathbf{\Upsilon_b})' \left[\tilde{\sigma}_b^2 I + \sigma_b^2 B(\mathbf{\Upsilon_b}) B(\mathbf{\Upsilon_b})'\right]^{-1} \frac{\kappa}{1+\theta\kappa} \left[\psi^{-1} B(\mathbf{1}) + \left(\theta - \psi^{-1}\right) B(\hat{\phi})\right] \\ &\quad + \mathbf{e}_1' \rho_z \sigma_z^2 B(\mathbf{\Upsilon_z}) B(\mathbf{\Upsilon_z})' \left[\tilde{\sigma}_z^2 I + \sigma_z^2 B(\mathbf{\Upsilon_z}) B(\mathbf{\Upsilon_z})'\right]^{-1} \frac{1}{1+\theta\kappa} B(\mathbf{1}),\end{aligned}$$

where $\mathbf{e}_1' = (1, 0, ..., 0)$. In this limiting case, the firms' price response depends on $(i)$ the serial correlation of the persistent idiosyncratic shocks, as determined by $\rho_b$ and $\rho_z$, and $(ii)$ relative importance of the transitory vs. persistent idiosyncratic shocks to demand and costs, as determined by $\sigma_b^2 B(\mathbf{\Upsilon_b}) B(\mathbf{\Upsilon_b})'$ vs. $\tilde{\sigma}_b^2 I$ and $\sigma_z^2 B(\mathbf{\Upsilon_z}) B(\mathbf{\Upsilon_z})'$ vs. $\tilde{\sigma}_z^2 I$. Our general model thus replicates the findings of the example and confirms our earlier interpretation for the example's reduced form parameters.

As with the example, the resulting price adjustment 'for the wrong reasons' can be substantial, and in some cases, converge to complete adjustment. This occurs, in particular, if there are no transitory idiosyncratic shocks so that firms attribute all fluctuations in signals to the persistent firm-specific cost and demand conditions. The following proposition characterizes the adjustment dynamics if in addition to $\sigma_u \to 0$, we set $\tilde{\sigma}_b = \tilde{\sigma}_z = 0$:

**Proposition 3** *(Setting the right prices for the wrong reasons): In the limit as $\sigma_u \to 0$, and $\tilde{\sigma}_b = \tilde{\sigma}_z = 0$,* *the impulse response function to nominal shocks is given by the vector $\phi$, where*

$$\phi_s = \left(\frac{\kappa\psi^{-1}\rho_b + \rho_z}{1+\theta\kappa}\right)\frac{1-(r\rho_b)^s}{1-r\rho_b} \tag{27}$$

*denote the impact of $u_{t-s}$ on $p_t$.*

Thus, average prices adjust even though firms remain completely in the dark about the aggregate shocks. In this particular case, the adjustment takes the form of a geometric convergence at rate $r\rho_b$ to a permanent adjustment level given by

$$\lim_{s\to\infty}\phi_s = \left(\frac{\kappa\psi^{-1}\rho_b + \rho_z}{1+\theta\kappa}\right)\frac{1}{1-r\rho_b}.$$

The persistence in the adjustment in this particular case results from the fact that aggregate shocks affect demand observations only gradually, and with delay: Demand will respond by $(1-r)$ on impact, but prices do not change. The following period, prices respond to the original increase, which further raises the demand signal, and so on. Over time, aggregate prices then gradually increase to the permanent adjustment level, as the shock gets reflected in aggregate prices, and hence in demand. When $\rho_b = \rho_z = \rho$, we can rearrange this as

$$\phi_s = \frac{\rho\,(1-r)}{1-r\rho}\left(1-(r\rho)^s\right)$$

and we therefore observe that the permanent adjustment level $\rho\,(1-r)\,/\,(1-r\rho)$ is an increasing function of the degree of persistence, and a decreasing function of the complementarity $r$. When $r = 0$, demand observations are not affected by the aggregate price level, and the permanent adjustment level is reached after the first period. Notice that this finding exactly mirrors the additional delay generated by endogenous signals in the extension of our simple example.[14]

Moreover, when $\rho = 1$, prices eventually reach full adjustment, i.e. $\lim_{s\to\infty}\phi_s = 1$. In this particular case, firms will fully adjust to the idiosyncratic shocks over time. When an aggregate shock occurs, they will confuse it with an idiosyncratic one, but they nevertheless fully adjust their prices to these permanent shocks to their demand and costs.

## 4 Quantitative Results

In this section, we proceed to a quantitative evaluation of our model. In particular, we aim to explore the quantitative effects of information and payoff heterogeneity in our price-setting model.

[14]The issue here is not the complementarity per se, but the fact that the same parameters that determine $r$ also determine the effects of aggregate prices on the demand signal. Since prices respond to demand signal with a one period delay, demand signals also incorporate the aggregate shock only gradually, as it enters into prices.

We set the common knowledge horizon $T$ equal to 300. This is high enough that small changes in $T$ do not have any meaningful effect on our results.[15] The standard deviation of (monthly) innovations to money growth $\sigma_u$ is set to .0036, following Golosov and Lucas (2007). The parameters $\theta, \psi$ and $\kappa$ are borrowed from the literature and set to $\psi = 2, \theta = 4$ and $\kappa = 1$. These parameters imply that the pricing complementarity $r \equiv \kappa(\theta - \psi^{-1})/(1 + \theta\kappa)$ is equal to $0.7$.

The remaining parameters of the model relate to the stochastic process for the idiosyncratic shocks. Before explaining our calibration strategy, it will be useful to discuss how these parameters interact with each other. For this purpose, we shall, for the moment, set the variance of the two transitory idiosyncratic shocks to zero, and just focus on the case in which idiosyncratic demand and wages are AR(1) processes.

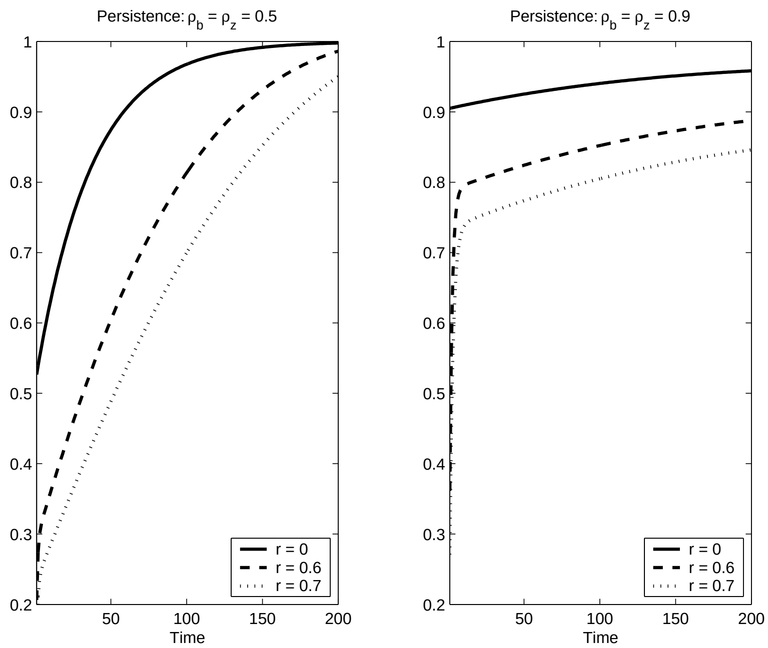


Figure 3: Impulse Responses to a monetary shock

Figure 3 illustrates the interaction between the persistence of idiosyncratic shocks and the degree of complementarity[16]. In both figures, prices rapidly adjust on impact from the firm-specific component, followed by slow adjustment as the firms fully separate idiosyncratic from aggregate shocks. The complementarity affects both the initial adjustment level and the subsequent speed

[15]To check the robustness of our results, we have also computed our model with values of $T$ up to 3500. This longer horizon does not alter any of our results about short-run adjustment, which confirms that the choice of $T$ is immaterial for our conclusions. It also confirms our result that there is slow convergence to full adjustment in the long run.

[16]The degree of complementarity is varied by changing the value of $\psi$, holding all the other parameters constant.

of convergence, but its effects are stronger when the firm-specific shocks are less persistent. In the panel on the right, $\rho_b$ and $\rho_z$ are close to 1, so the impulse response functions show a rapid adjustment to monetary shocks, almost independently of the degree of complementarity.

The remaining long-run adjustment is then governed by learning about the aggregate shocks. In the long run, higher persistence makes the filtering problem more challenging because it takes longer to disentangle permanent aggregate from persistent idiosyncratic shocks. Therefore, convergence to full adjustment takes much longer with persistent idiosyncratic shocks. Obviously, this conclusion is an artefact of our assumption that firms only have access to a very limited set of signals from market demand and wages, whose information content depends on the persistence parameters. While this may seem reasonable over the short-to-medium term, it seems much less attractive for studying long run price adjustment - where firms will also learn through publication of aggregate statistics, news reports, market reports or other sources of data that are independent from their own market transactions. Adding such exogenous signals with realistic levels of noise does not alter much the short-run implications of our model, but significantly increases the speed of price adjustment in the long run.

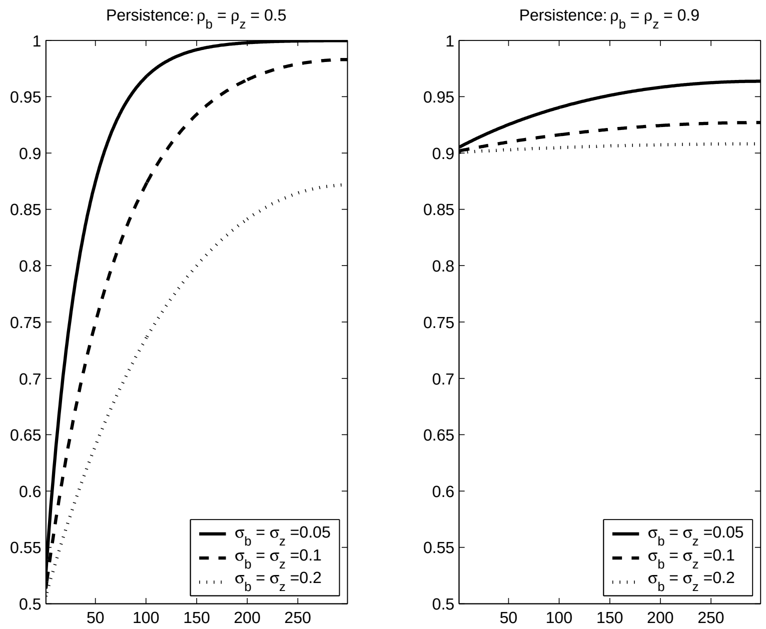


Figure 4: Impulse Responses to a monetary shock

Figure 4 highlights a similar interaction between the persistence parameters and the effect

| | $\sigma_b = \sigma_z = 0.02$ | $\sigma_b = \sigma_z = 0.05$ | $\sigma_b = \sigma_z = 0.10$ |
|---|---|---|---|
| $\rho_b = \rho_z = .5$ | 0.003 | 0.009 | 0.017 |
| $\rho_b = \rho_z = .7$ | 0.005 | 0.014 | 0.028 |
| $\rho_b = \rho_z = .9$ | 0.012 | 0.029 | 0.058 |

Table 1: Price dispersion for different levels of persistence and std. deviations of idiosyncratic shocks

of informational heterogeneity[17]. When $\rho_b$ and $\rho_z$ are close to 1, prices respond very quickly to monetary shocks. This is true even if informational heterogeneity is large i.e. even if the variance of the idiosyncratic shocks is high relative to the monetary shock. This is evident from the panel on the right, which plots the impulse response functions for various values of $\sigma_b^2$ and $\sigma_z^2$. In the other panel, shocks are much less persistent and changes in informational heterogeneity have much larger effects.

Finally, Table 1 shows the implications of the persistence and variance parameters for price dispersion. In particular, note that for a given level of the variances $\sigma_b^2$ and $\sigma_z^2$, higher persistence increases price dispersion. However, as we saw earlier, higher persistence also implies quicker adjustment to monetary shocks. Thus, the only way to have both reasonable price dispersion and persistent real effects of monetary shocks is to have high variances for the idiosyncratic shock processes.

### 4.1 Calibrating the remaining parameters

The earlier discussion illustrates the importance of the parameters governing the idiosyncratic processeses for the quantitative implications of the model. To discipline our choice of these parameters, we target four key moments

1. Price dispersion (the standard deviation of the log of relative prices) of 6-10%
2. Quantity dispersion (the standard deviation of log of quantities) of 25-30 %
3. Correlation between prices and quantities of roughly -0.20

[17] In order to isolate the effects of heterogeneity, we set the degree of complementarity to zero.

4. Autocorrelation (daily) of the log of relative prices of 0.98.

The dispersion and autocorrelation of relative prices are related to the persistence and variance of the idiosyncratic shock processes. Our target for price dispersion is derived from the statistics reported by Burstein and Hellwig (2007) for the Dominicks scanner price data.[18] The autocorrelation target is derived from the monthly number of 0.65 estimated by Midrigan (2007) for the same data. These targets enable us to infer the importance of idiosyncratic shocks for firm pricing.

The quantity moments targeted in the calibration then help us identify the demand shock parameters separately from the cost shock ones. To see how, note that, from the demand function of the firm

$$c_{it} - c_t = b_{it} - \theta(p_{it} - p_t) \tag{28}$$

Thus, both shocks affect relative quantities through their effect on relative prices, but quantities are also directly affected by the demand shock. In a model with only cost shocks, (28) implies that relative quantities and prices would would be perfectly negatively correlated. This is at odds with the data - Burstein and Hellwig (2007), for example, find that the correlation is only modestly negative in the Dominick's data. This feature of the data, they argue, suggests that both demand and cost shocks are necessary to simultaneously explain the observed price and quantity moments. Eichenbaum et al.(2008) support similar observations about demand fluctuations using a different, much wider data set.

One concern with this choice of moments might be that they result from data sources, in which, in contrast to our model, prices are usually changed only infrequently, on average every four to five months. For instance, one might wonder whether this would over-state the degree of price dispersion or serial correlation (especially if the latter is also results from rigidities). Numerical results however suggest that the main conclusions from our calibration are robust to even quite substantial changes in targets. As we will see below, the quantitative results are the most sensitive to our serial correlation target (or equivalently, the choice of a model period), but even substantial changes in this target do not alter the main qualitative conclusions of our model with regards to short-run adjustment and long-run convergence.

[18] Burstein and Hellwig find price dispersion measures of roughly $10\%$. We did not find direct measures of relative price dispersion in papers using other data sources, but this level seems consistent with the widely reported numbers on the magnitude of price changes (Klenow and Kryvtsov(2008), Bils and Klenow (2004), Nakamura and Steinsson (2008) ).

### 4.2 Results

In our benchmark calibration, we let a model period correspond to one day. The impulse response function of average prices for this benchmark daily model is shown in Figure 5. The addition of transitory noise to the signals increases the rigidity on impact (the aggregate price level reflects only 20 % of the aggregate shock on impact), but this effect dies out very quickly. In little over a month, prices have incorporated 90% of the shock. These results show that the tension highlighted in the simple example at the beginning of the paper - between generating both meaningful delays in price adjustment and realistic levels of price dispersion - survives even in a micro-founded model.

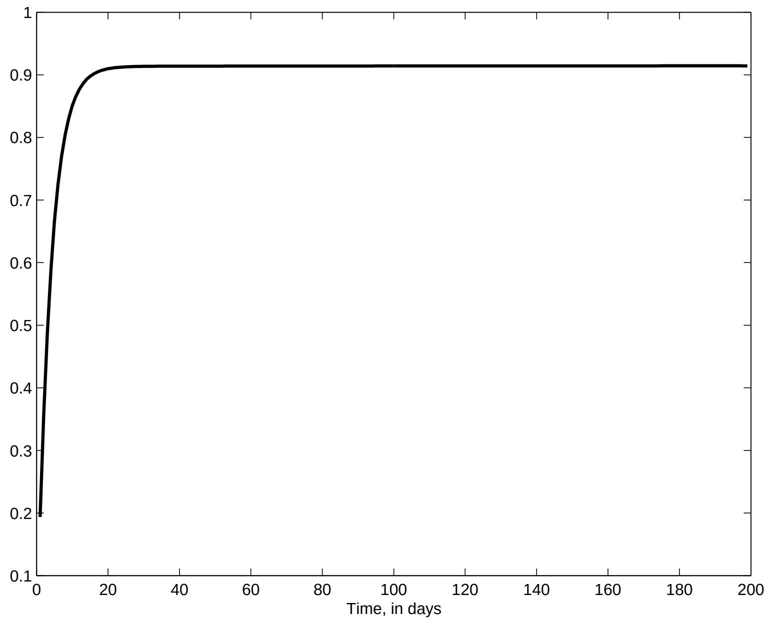


Figure 5: Impulse responses to a monetary shock in the benchmark daily model

That said, the graph also reveals that complete convergence takes a long time. In other words, the real effects of a monetary shock, albeit small in magnitude, have a highly persistent component. When both idiosyncratic and aggregate shocks are highly persistent, it takes the firms a very large number of periods to separate the two using the market-generated signals. However, since the desired responses to an aggregate vs. an idiosyncratic shock are only slightly different, the firms' optimal response is to make a fairly rapid adjustment to almost the right level and then wait for further information before full convergence.

### 4.3 Effect of Period Length

We conclude this section by discussing the need to take a stand on how a period in model translates into calendar time. In our computations, we consider different cases, with a period length representing a day, a week, or a month - when we attempted to calibrate to a quarterly period length, we were unable to do so because idiosyncratic shocks became too transient to have any impact on optimal pricing from one quarter to the next.

The choice of the length of a period has implications for 2 key aspects of the model. It affects both the rate of information flow as well as the ability of firms to respond to new information. The first effect acts through the sampling frequency and time aggregation. Firms in a daily version of our model draw signals more frequently and therefore receive more information than firms in the monthly model. The latter draw just one signal, which aggregates information about an entire month of shocks. This effect leads to greater responsiveness of prices in the model with higher sampling frequency (i.e. the daily model). We can partly address this problem by using the transitory components of the idiosyncratic shock process. By increasing the variance of these noise terms in a model with shorter period lengths, we can slow down the learning process. We follow this approach in our benchmark calibration for the daily model and set the standard deviations of this transitory component so that the speed of learning (as measured by the variance of the posterior distribution for the persistent component) is the same as in the weekly or monthly model.

Second, the choice of the period length also determines the time lapse between the information on which firms base their prices, and the moment their prices come into effect, and the longer the period length, the longer is the time lapse (and hence the stronger the mean reversion in idiosyncratic shocks). A longer period length thus artificially reduces the firms' ability to respond to idiosyncratic shocks by increasing their reaction time to new information. Notice that this effect is separate from the speed of learning effect that we described first, and would be present, for example, even if we considered a model in which firms obtained new information every day, but could adjust prices only once a month.

In Figure 6, we highlight the importance of these effects by calibrating 3 different versions of our model - a daily, weekly and monthly version. All 3 versions have the same rate of information flow. As the graph shows, prices in the daily and weekly versions respond much faster than in the monthly model. Note that this is not because firms learn faster in the daily or weekly model, but because the 'wrong reasons' to adjust prices are relatively stronger. This simple exercise suggests that as we vary the period length, the model can generate any degree of price adjustment, ranging

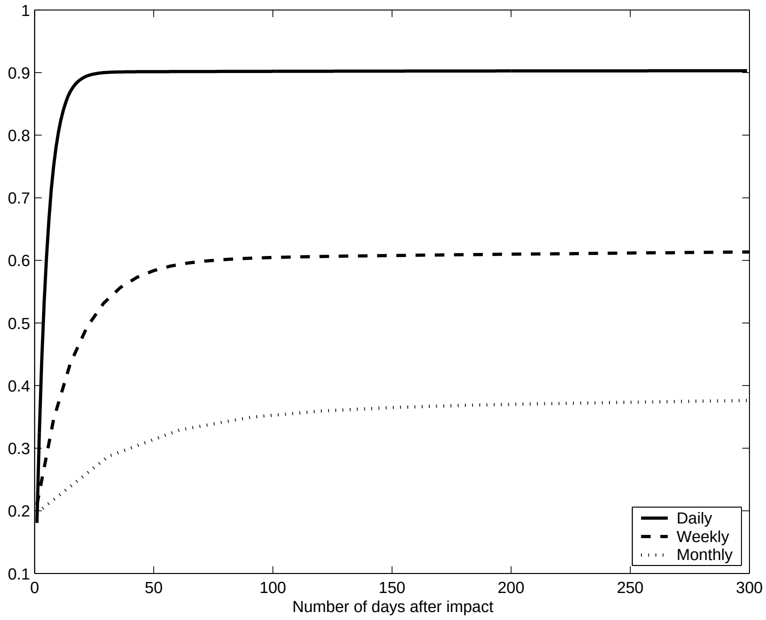


Figure 6: Impulse Responses to a monetary shock

from almost perfectly responsive (very short period lengths) to significant rigidity in long run adjustment (with very long periods). In all cases, however a substantial short-run adjustment results and is required to match the observed degrees of price dispersion.

Obviously this leads to the question of how one should pick the period length. One way to pick the period would be to simply set it equal to the average length of a price spell - between 4 and 5 months, to match the length for which a price remains in place. This however ignores our previous two observations about information flow, and firms reaction time to changes in economic conditions - in a menu cost model, for instance firms are constantly reevaluating their prices (and thereby reacting to new information), even if they change prices only infrequently. Alternatively, one could focus on weekly or monthly period lengths to take into account the firm's actual decision-making (the unit of price observations at Dominick's is a week, which corresponds to the minimum intervals at which prices are held fixed). However, this does not take into account the presence of other rigidities that might delay the firms' ability to react to new information as soon as it becomes available.

Thus, although it may seem unrealistic from the point of view of how decisions are made in practice, our focus on a short period length enables us to isolate the effects of incomplete information on nominal adjustment from other frictions that prevent firms from adjusting their prices in real time in response to the arrival of new information. Our model with longer period length on the other hand might provide a simple way to illustrate the effects of informational frictions on pricing when the firm's reaction time is not instantaneous. Of course, a complete discussion of this subject would eventually require a much richer model that analyzes how firms choose to incorporate internal and external news into their decision-making process.

# 5 Productivity Shocks

Our results in the benchmark model are driven by two key features of our environment: the need for large and persistent idiosyncratic shocks in order to explain the micro data and the composite nature of the signals. Here, we will present an extension of our model, which will both serve to highlight the role of these 2 key assumptions as well as explain why our results on real effects are starkly different from those in the existing literature. For example, Mackowiak and Wiederholt (2008) study a model of price adjustment with rational inattention, in which firms have to divide a limited information processing capacity between tending to idiosyncratic technology and aggregate demand shocks. Since idiosyncratic technology shocks are an order of magnitude larger than aggregate demand shocks, firms tend to pay very little attention to the latter[19], so that nominal shocks can have persistent real effects. As a result, prices are sticky in response to aggregate demand shocks, yet very responsive to firm-specific technology shocks.

The key distinction between our model and the benchmark model in Mackowiak and Wiederholt (2008) is that they assume a complete separation between learning about aggregate and idiosyncratic shocks. This eliminates the response to aggregate prices stemming from the 'wrong' reasons.[20]

We now show that our model delivers results very similar to theirs if we (i) make the idiosyncratic shocks affecting the wage and demand signals completely transitory (ii) add idiosyncratic technology shocks which are observed separately from the aggregate shocks. The assumption in (i) eliminates the 'wrong' reason to change prices - an aggregate shock still leaves firms confused as to nature of the shock but since the idiosyncratic shocks are transitory, information about the

[19]Our transitory demand and cost shocks can also be reinterpreted in terms of channel noise in a rational inattention model. In that case, the variances of the transitory shocks would be the result of a firms' optimal decision, subject to a capacity constraint on information processing.

[20]In an extension, Mackowiak and Wiederholt (2008) also consider composite demand signals with idiosyncratic and aggregate demand shocks, in a static environment, similar to the variant of our example with endogenous signals. This extension leads to a smaller adjustment on impact (due to the endogenous information feedback from prices to demand signals), but their analysis does not separate between adjustment for the 'right' or 'wrong' reasons, and it does not allow for the rapid adjustment at short horizons, as the firms separate noise from fundamental shocks.

past realizations of these shocks does not have any bearing on the firms' optimal price. These shocks then serve purely as informational noise and delay the learning/adjustment process. Since the idiosyncratic technology shocks in (ii) are observed separately, their stochastic process can be chosen to match the dispersion and persistence of prices without any implication for the learning process. However, such a calibration of our model has the counter-factual implication that prices and quantities are highly negatively correlated.

We incorporate productivity shocks by assuming that the log of the productivity term in (15) is the sum of an aggregate and an idiosyncratic component:

$$a_{it} = g_t + g_{it} \tag{29}$$

where $g_t$ is an aggregate term and $g_{it}$ is a firm-specific idiosyncratic factor. Both these components in turn are modeled as AR(1) processes:

$$g_t = \sigma_A \sum_{s=0}^{\infty} \rho_A^s u_{t-s}^2 \qquad \text{and} \qquad g_{it} = \sigma_a \sum_{s=0}^{\infty} \rho_a^s v_{i,t-s}^3 \tag{30}$$

where $u_t^2, v_{i,t}^3$ are distributed according to $\mathcal{N}(0,1)$ and $\rho_A, \rho_a, \sigma_a, \sigma_A$ are non-negative.[21]

Firms observe $a_{it}$ but not the aggregate and idiosyncratic components separately. Note that there are 2 sources of aggregate risk in this version - the innovations to the aggregate productivity index and money supply. However, only the former affects the firms' productivity signals. The innovation to the money supply $m_t$ has no effect on the firms signals about its own productivity. This separation plays a key role in allowing this version of our model to slow down the response of prices to these aggregate nominal shocks while matching the dispersion and autocorrelation of relative prices.

As before, firms make their choices in period $t$ before observing the shocks for that period. The other elements of the information structure are the same as in the benchmark model. In particular, productivity shocks also become common knowledge $T$ periods after they occur.

The FOC of the firm's price-setting problem takes the form

$$P_{it}^{1+\theta\kappa} = \frac{\theta(1+\kappa)K_1}{\theta-1} \cdot \frac{\mathbb{E}_{it}[Z_{it}B_{it}^{1+\kappa}A_{it}^{-1-\kappa}C_t^{1+\kappa}P_t^{\theta(1+\kappa)}]}{\mathbb{E}_{it}[B_{it}C_tP_t^{\theta}M_t^{-1}]}$$

Log-linearizing the FOC yields

$$p_{it} = Const + \frac{\kappa}{1+\theta\kappa}\mathbb{E}_{it}[b_{it} + c_t + \theta p_t] + \frac{1}{1+\theta\kappa}\mathbb{E}_{it}[z_{it} + m_t] - \frac{1+\kappa}{1+\theta\kappa}\mathbb{E}_{it}[a_{it}] \tag{31}$$

[21] It is straight-forward, but not necessary for our purposes, to include transitory as well as persistent idiosyncratic shocks to technology.

Now, using the household FOC (11) to substitute for $c_t$, we have

$$p_{it} = Const + \frac{1}{1+\theta\kappa}\mathbb{E}_{it}[\kappa b_{it} + z_{it}] + \frac{1+\kappa\psi^{-1}}{1+\theta\kappa}\mathbb{E}_{it}[m_t] + \frac{\kappa\left(\theta-\psi^{-1}\right)}{1+\theta\kappa}\mathbb{E}_{it}[p_t] - \frac{1+\kappa}{1+\theta\kappa}\mathbb{E}_{it}[a_{it}] \quad (32)$$

As before, denote by $U_t^2$ and $V_{it}^3$ the vector of 'relevant' aggregate and idiosyncratic productivity shocks i.e. those that have been realized but have not yet been fully revealed at the time of making period $t$ decisions. Then, as before, we write the optimal price in terms of a common knowledge component and the firms' conditional expectations of the shocks that havent yet been revealed:

$$\begin{aligned} p_{it} &= \hat{p_{it}} + \sigma_u\left(1-r\right)\mathbf{1}'\mathbb{E}_{it}[U_t] + r\left(\mathbb{E}_{it}[p_t] - \hat{p}_t\right) \\ &+ \sigma_b\gamma_b'\mathbb{E}_{it}[V_{it}^1] + \sigma_z\gamma_z'\mathbb{E}_{it}[V_{it}^2] \\ &- \sigma_A\underbrace{\rho_A\left(\frac{1+\kappa}{1+\theta\kappa}\right)\mathbf{\Upsilon}_{\mathbf{A}}'}_{\gamma_{\mathbf{A}}'}\mathbb{E}_{it}[U_t^2] - \sigma_a\underbrace{\rho_a\left(\frac{1+\kappa}{1+\theta\kappa}\right)\mathbf{\Upsilon}_{\mathbf{a}}'}_{\gamma_{\mathbf{a}}'}\mathbb{E}_{it}[V_{it}^3] \end{aligned} \quad (33)$$

where $\mathbf{\Upsilon}_{\mathbf{A}}' = (1, \rho_A, \rho_A^2, ..., \rho_A^{T-1})$, $\mathbf{\Upsilon}_{\mathbf{a}}' = (1, \rho_a, \rho_a^2, ..., \rho_a^{T-1})$.

The solution proceeds as in the benchmark model. We start with a conjecture about the aggregate price index

$$p_t = \hat{p}_t + \sigma_u\phi_1' U_t + \sigma_A\pi' U_t^2 \quad (34)$$

where $\phi' = (\phi_1, \phi_2, .....\phi_T)$ and $\pi' = (\pi_1, \pi_2, .....\pi_T)$ are real-valued vectors. Given this conjecture, the signal vectors are related to the realizations of the shocks as follows:

$$\begin{aligned} X_{it} &= \hat{X}_{it} + \sigma_u\left(\psi^{-1}B(\mathbf{1}) + \left(\theta-\psi^{-1}\right)B(\hat{\phi})\right)U_t + \sigma_A\left(\theta-\psi^{-1}\right)B(\hat{\pi})U_t^2 + \sigma_b B(\mathbf{\Upsilon}_{\mathbf{b}})V_{it}^1 \\ \Omega_{it} &= \hat{\Omega}_{it} + \sigma_u B(\mathbf{1})U_t + \sigma_z B(\mathbf{\Upsilon}_{\mathbf{z}})V_{it}^2 \\ A_{it} &= \hat{A}_{it} + \sigma_A B(\mathbf{\Upsilon}_{\mathbf{A}})U_t^2 + \sigma_a B(\mathbf{\Upsilon}_{\mathbf{a}})V_{it}^3, \end{aligned}$$

where $A_{it}$ denotes the vector of productivity observations and $\hat{A}_{it}$ its common knowledge component. Note that the aggregate productivity shocks enter the demand signals through their effect on the aggregate price. Thus, the variance-covariance matrices depend jointly on $\phi$ and $\pi$. Finally, using the conjecture and averaging the FOC across firms, we get

$$\begin{aligned} \sigma_u\phi' U_t + \sigma_A\pi' U_t^2 = \sigma_u\left[\left(1-r\right)\mathbf{1}' + r\hat{\phi}'\right]\bar{E}_t[U_t] - \sigma_A\left[\gamma_A' - r\hat{\pi}'\right]\bar{E}_t[U_t^2] \\ + \sigma_b\gamma_{\mathbf{b}}'\bar{E}_t[V_{it}^1] + \sigma_z\gamma_{\mathbf{z}}'\bar{E}_t[V_t^2] - \sigma_a\gamma_a'\bar{E}_t[V_{it}^3] \end{aligned} \quad (35)$$

As before, we can numerically solve the fixed point problem implicit in the above expression by characterizing the relevant average expectation terms in terms of the underlying aggregate shocks.

As mentioned before, we calibrate this version such that the firm-specific wage and demand shocks are completely transitory i.e. $\rho_b = \rho_z = 0$, so that $\gamma_b' = \gamma_z' = \mathbf{0}'$ The parameters for the idiosyncratic technology shocks are then chosen to match the observed price dispersion and persistence.

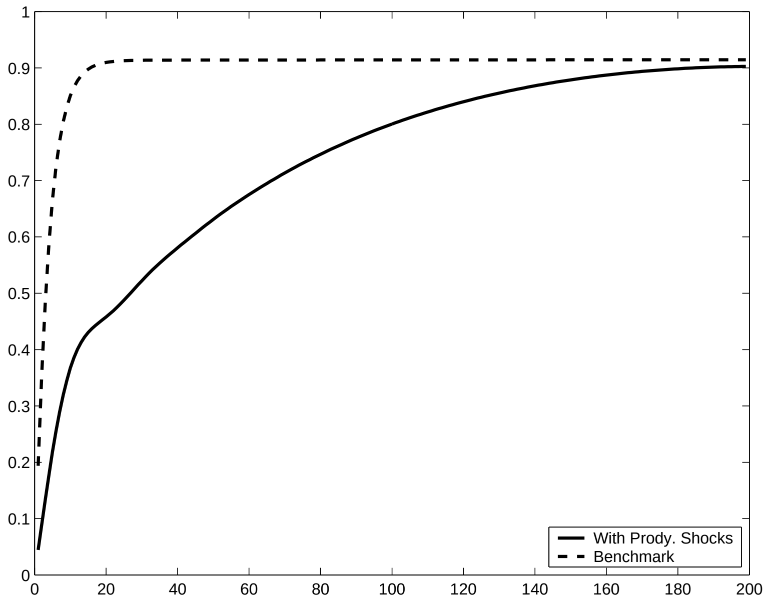


Figure 7: Impulse responses to a monetary shock with productivity shocks and transitory wage/demand shocks

Figure 7 plots the impulse response functions. As the graph shows, aggregate prices respond much more slowly to monetary shocks in this version compared to our benchmark model. Note that, as before, idiosyncratic cost and demand shocks are much larger than aggregate monetary shocks so firms still tend to attribute nominal shocks largely to idiosyncratic factors. However, in contrast to our benchmark model, firms do not want to adjust their prices in response to these transitory shocks. In other words, firms choose to wait for more draws to learn more precisely about the aggregate nominal shock. Since pricing complementarity is relatively high, firms also find it optimal to delay their responses to the perceived aggregate shock until other firms respond. As a result, aggregate prices respond rather sluggishly to small aggregate shocks. Yet at the same time, prices adjust rapidly to firm-specific technology shocks, which account for the large degree of price dispersion. This conclusion parallels the findings in the benchmark model in Mackowiak and Wiederholt (2008).

The row labeled Model I in Table 2 shows the relevant moments for this version of our model.

| | Prc Disp | Prc Autocor | Prc-Qty Corr | Qty Disp |
|---|---|---|---|---|
| Targets | 0.06-0.10 | 0.98 | -0.20 | 0.25-0.30 |
| | | | | |
| Model I | 0.08 | 0.98 | -0.99 | 0.32 |
| Model II | 0.08 | 0.98 | -0.35 | 1.05 |

Table 2: Relative Price and Quantity Moments with Productivity Shocks

Given our calibration, the model has the counter-factual implication that prices and quantities are highly negatively correlated. To see why this is so, recall from (28) that relative quantities are a linear combination of the demand shocks and a term involving relative price dispersion. In our calibration, demand shocks are assumed to be completely transitory, so their contribution to overall dispersion of quantities is very small, so the dispersion in relative quantities comes almost exclusively from the dispersion in prices, leading to an almost perfect negative correlation. Of course, we could reduce this correlation simply by increasing the variance of the demand shock process. The row labeled Model II in Table 2 is an example of such a calibration. While it fixes the correlation problem, it does so at the cost of counterfactually high quantity dispersion. Thus, as in many standard sticky price models, demand shocks must play an important role if we are to simultaneously match quantity and price moments. In our environment, that means increasing the persistence of the demand shock process. But, this will take us closer to our benchmark model, where price adjustment occurs relatively fast, because firms wish to adjust prices in response to idiosyncratic demand shocks. In conclusion, this exercise illustrates that the basic tension faced by the benchmark model is not significantly mitigated even if we add productivity shocks. It also highlights the importance of composite signals in the price adjustment process. If firms have access to signals that combine aggregate and idiosyncratic shocks, they have 2 complementary motives to respond to prices, leading to a quicker adjustment process.

## 6 Conclusion

In this paper, we have studied a model of price adjustment with heterogeneously informed firms, who face idiosyncratic as well as aggregate shocks. Firms seek to infer these shocks from the information generated by their market activities. The market-generated signals, specifically demand and wage histories, combine aggregate and idiosyncratic factors. Two important conclusions emerge from our analysis.

First, the presence of firm-specific cost and demand shocks significantly mutes the real effects of monetary shocks on impact. Although these idiosyncratic shocks must be large to account for observed levels of price dispersion, thus generating large uncertainty about aggregate conditions, they also render the signal extraction problem less relevant for the firms' pricing decisions. As long as positive adjustment on impact is optimal in response to either shock, the firm will adjust its price, despite its confusion about the exact nature of the shock, i.e. idiosyncratic vs. aggregate. In the case where the shocks are equally persistent, prices quickly adjust to aggregate shocks despite the fact that firms remain permanently confused about the idiosyncratic and the aggregate fluctuations they are exposed to. This result of almost complete neutrality is reminiscent of related flexibility results in the sticky price literature (e.g. Caplin and Spulber (1987), or Golosov and Lucas (2007)), although the underlying reasons are very different.

Second, at longer horizons, our minimalist information structure, which is meant to capture the large degree of product-level dispersion, leads to implausible large delays in learning. While the response on impact is sizeable and positive, it is not complete, and in the absence of additional information, it takes firms a long time to completely adjust prices in response to permanent nominal shocks. We view this result as an artifact of our assumption that rules out additional sources of information, in particular from asset values or nominal interest rates. A richer model that embeds these additional information sources would speed up the learning process and mitigate the small but persistent long-run effects without changing the models' significant short run implications.

Although these results represent a challenge for models with heterogeneous information as the only source of nominal non-neutrality, they also point to directions for future research: the large adjustment of prices on impact, due to the presence of firm specific shocks, is due in part to an assumption of complete flexibility of prices, along with the premise that firms update based on noisy information in real time. It is plausible to think that adding small amounts of nominal rigidities or menu costs would reduce the adjustment in the short run, and might also lead to more important real effects at medium to long horizons, i.e. once the emphasis of the firms' filtering problem shifts from the transient nature of firm-specific shocks to the problem of inference about aggregate variables.

Future work will have to explore these issues, which generate substantive as well as technical challenges. In particular, the infinite regress problem and the associated aggregation issues, which we circumvented with the finite-horizon assumption, along with the linearity of our flexible adjustment model, will come back in full force in the non-linear pricing model that would emerge from a combination of menu costs with heterogeneous information.[22]

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[22]See Gorodnichenko, (2008), for an attempt at tackling these issues.

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# 7 Appendix: Proofs

**Proof of Propositions 1 and 2:.** Follows immediately from the characterization of $k$ in (**??**). ■

**Proof of Lemma 1:.** Given the definition of $\Gamma$ and $\Delta$, the signal vector can be rewritten as

$$\begin{pmatrix} X_{it} - \hat{X}_{it} \\ \Omega_{it} - \hat{\Omega}_{it} \end{pmatrix} = \Gamma U_t + \Delta \begin{pmatrix} V_{it}^1 \\ \tilde{V}_{it}^1 \\ V_{it}^2 \\ \tilde{V}_{it}^2 \end{pmatrix}$$

Therefore, the vector $\left(U_t', V_{it}^{1\prime}, \tilde{V}_{it}^{1\prime}, V_{it}^2, \tilde{V}_{it}^{2\prime}; X_{it}' - \hat{X}_{it}', \Omega_{it}' - \hat{\Omega}_{it}'\right)'$ is normally distributed with mean zero and variance-covariance matrix:

$$\begin{bmatrix} \multicolumn{2}{c}{I} & \begin{matrix}\Gamma' \\ \Delta'\end{matrix} \\ \Gamma & \Delta & \Gamma\Gamma' + \Delta\Delta' \end{bmatrix}$$

and $\left(U_t', V_{it}^{1\prime}, \tilde{V}_{it}^{1\prime}, V_{it}^{2}, \tilde{V}_{it}^{2\prime}\right)'$, conditional on $\left(X_{it} - \hat{X}_{it}, \Omega_{it} - \hat{\Omega}_{it}\right)$ is normally distributed with mean

$$\begin{pmatrix} \Gamma' \\ \Delta' \end{pmatrix} \left(\Gamma\Gamma' + \Delta\Delta'\right)^{-1} \begin{pmatrix} X_{it} - \hat{X}_{it} \\ \Omega_{it} - \hat{\Omega}_{it} \end{pmatrix}.$$

Averaging over $i$ and using the above characterization of signals, we find that average expectations of $\left(U_t', V_{it}^{1\prime}, \tilde{V}_{it}^{1\prime}, V_{it}^{2}, \tilde{V}_{it}^{2\prime}\right)'$ are

$$\begin{pmatrix} \Gamma' \\ \Delta' \end{pmatrix} \left(\Gamma\Gamma' + \Delta\Delta'\right)^{-1} \Gamma U_t$$

From this, the characterization follows immediately. Moreover, the posterior variance-covariance matrix of $\left(U_t', V_{it}^{1\prime}, \tilde{V}_{it}^{1\prime}, V_{it}^{2}, \tilde{V}_{it}^{2\prime}\right)'$ is

$$I - \begin{pmatrix} \Gamma' \\ \Delta' \end{pmatrix} \left(\Gamma\Gamma' + \Delta\Delta'\right)^{-1} \begin{pmatrix} \Gamma & \Delta \end{pmatrix},$$

so that $I - \Gamma' \left(\Gamma\Gamma' + \Delta\Delta'\right)^{-1} \Gamma$ denotes the posterior variance covariance matrix of $U_t$. ■

**Proof of Proposition 3:.** When $\tilde{\sigma}_b^2 = \tilde{\sigma}_z^2 = 0$, $\phi'$ simplifies to

$$\phi = \frac{\kappa\psi^{-1}\rho_b + \rho_z}{1 + \theta\kappa}\mathbf{1} + r\rho_b\hat{\phi}.$$

Using the fact that $\hat{\phi} = (0, \phi_1, ..., \phi_{T-1})' = \Lambda\phi$, where

$$\Lambda = \begin{pmatrix} 0 & 0 \\ I_{T-1} & 0 \end{pmatrix},$$

we have

$$\phi = \frac{\kappa\psi^{-1}\rho_b + \rho_z}{1 + \theta\kappa} \left[I - r\rho_b\Lambda\right]^{-1} \cdot \mathbf{1}$$

The result then follows from

$$[I - r\rho_b\Lambda]^{-1} = \begin{pmatrix} 1 & 0 & \cdot & \cdot & 0 & 0 \\ -r\rho_b & 1 & \cdot & \cdot & \cdot & 0 \\ 0 & -r\rho_b & \cdot & \cdot & \cdot & \cdot \\ \cdot & 0 & \cdot & 1 & 0 & \cdot \\ 0 & \cdot & \cdot & -r\rho_b & 1 & 0 \\ 0 & 0 & \cdot & 0 & -r\rho_b & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 & \cdot & \cdot & 0 & 0 \\ r\rho_b & 1 & \cdot & \cdot & \cdot & 0 \\ (r\rho_b)^2 & r\rho_b & \cdot & \cdot & \cdot & \cdot \\ \cdot & (r\rho_b)^2 & \cdot & 1 & 0 & \cdot \\ (r\rho_b)^{T-2} & \cdot & \cdot & r\rho_b & 1 & 0 \\ (r\rho_b)^{T-1} & (r\rho_b)^{T-2} & \cdot & (r\rho_b)^2 & r\rho_b & 1 \end{pmatrix}$$

■