On Dynamic Compromise*

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PRELIMINARY AND INCOMPLETE

Abstract

What prevents majorities from extracting surplus from minorities in legislatures? We study an infinite horizon game where a legislative body votes to determine distributive policy each period. Proposals accepted by a simple majority are implemented, otherwise the status quo allocation prevails. We construct symmetric Markov perfect equilibria that exhibit compromise in the following sense: if the initial status quo allocation is "not too unequal", then the Markov process is absorbed into allocations in which more than a minimum winning majority receives a positive share of the social surplus with positive probability. The compromise is only sustainable if, starting from the "unequal" allocations, the Markov process is absorbed into allocations in which there is a complete absence of compromise. These compromise equilibria exist when discounting is neither too small nor too large. We find that, contrary to intuition, the range of discount factors for which these equilibria exist increases as the number of legislators increases. In this sense, compromise is easier in larger legislatures.

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1 Introduction

Many legislatures require the consent of only a simple majority for policies to be implemented. It follows that when distributive policy is being determined, a simple majority of legislators can split the surplus among themselves and freeze out the remaining legislators. What prevents them from doing so?

In fact, observed outcomes in which resources are distributed beyond a minimal majority are common. Examples such as medicare, military bases, and transportation all share this property to varying degrees.[1] This paper examines the possibility that *compromise* — agreed upon outcomes in which more than a minimum winning majority receives a positive share of the social surplus — is sustainable in legislatures.

We posit a model of legislative bargaining and construct equilibria of that model in which compromise may occur. In each period, a law is proposed by a randomly selected legislator and is passed by a majority vote.[2] If the current period's proposal fails to achieve a majority vote, the previous period's allocation is implemented. The basic idea is that compromise arises in response to smoothing motives in an explicitly dynamic setting. Each member of today's majority group must be concerned with the possibility that he might belong to a part of tomorrow's minority.

The setup resembles the legislative process that distributes benefits under many federal spending programs in the US. The distribution of benefits is determined by a formula that is enacted by the legislature and written into law. The formula remains effective until new legislation is passed that alters that formula.[3] When benefits are distributed by this kind of process, laws made today can potentially be overturned tomorrow, so legislators must consider the trade-offs between appropriating political spoils in the short term or equitably sharing benefits in the long term. These decisions require strategic calculations because legislators with proposal power cannot be certain they will have proposal power in the future.

[1]For instance, based on data from U.S. Department of Health and Human Services on Medicare allocations, after adjusting for differences in population size we observe that all states receive shares that are roughly equal and consistent across time. In any given year the share allocated to each state is about 2% of total medicare expenditures (with the exception of Arkansas and the District of Columbia). A striking fact is that the variance across states is no larger than .01% in any year.

[2]This stylized legislative process is common in the literature on legislative bargaining. It was introduced by Baron and Ferejohn (1989) who argue that, with a large number of legislators, each seeking to put forward his own policy, a legislative process that does not favor a particular legislator will result in a randomly selected proposer each period.

[3]This setup clearly ignores other explanations for an even distribution of benefits such as reputation or institutional effects.

One could, of course, apply standard repeated game arguments to explain the existence of compromise in such a setting. If legislators condition on the entire history of play, simple trigger strategies could yield the compromise we observe. The problem with this argument is that trigger strategies admit so many outcomes the theory offers very little explanatory power. Moreover, there are good reasons why trigger strategies would not be feasible in our setting. Most legislatures are characterized by periodic turnover. For example, the United States House of Representatives consists of 435 members each serving a two-year term, in Israel there are 120 members of the Knesset each serving 4-year terms, and the Mexican Chamber of Deputies comprises 500 deputies each serving three-year terms with no re-election. Even with powerful incumbency advantages, Matland and Studlar (2004) estimate 10% average annual legislative turnover for 25 countries, or 32% turnover per election. This periodic turnover can result in a lack of institutional memory. Each new legislature will be aware of the previous division of resources, but how this division was arrived at may not be known.

To address the memory problems that could arise, we restrict attention to Markov perfect equilibria. Markov perfect equilibria are subgame perfect equilibria in which players choose strategies that require no memory of past play beyond that which is relevant to today's payoffs. The Markov assumption does not imply that individual legislators have no memory. However, equilibrium "memory" captured by a legislature with turnover can be appropriately modeled as depending only on the current, payoff relevant information.

In this game the payoff-relevant information, or the state variable, is the status quo allocation and the current period's proposer. The stage game is a game of pure conflict. Since the state variable does not embody any information that is mutually beneficial to legislators they have no reason to be more cooperative at certain times than others.[4] Note that the one shot Nash equilibrium of this game is for the proposer to offer a minimum winning coalition their status quo allocation, and extract the remainder of the surplus for himself. The difficulty with finding compromise in Markov equilibria is in giving the proposing legislator sufficient incentive to overcome his natural desire to extract short term gains, without requiring our equilibrium to

[4]This is in contrast to games that are studied in Battaglini and Coate (2006) where the state variable is public debt. A high level of public debt is mutually disadvantageous to all legislators. This echoes a result by Dutta (1995), where he finds that sustaining efficient Markov prefect equilibria requires some amount of "state symmetry". This condition is clearly violated in our model.

use a punishment scheme that singles out a deviant legislator over an extended period (which would not be consistent with the Markov structure). The previous literature has so far been unable to show how this may be possible.

We construct symmetric Markov perfect equilibria such that if the initial status quo allocation is not too unequal, the equilibrium Markov process is absorbed into a closed class of proposals that exhibit compromise.[5] Critically, these equilibria are synergistic. The compromise is only sustainable if, starting from the remaining, unequal allocations, the Markov process is absorbed into a closed class of proposals in which there is a *complete absence* of compromise. If initial allocations are unevenly distributed, legislators cherry-pick minimum-winning coalitions and eventually the equilibrium transitions into this class.

To sustain these equilibria legislators are required to be neither too patient, nor too impatient. To understand why legislators need to be *impatient*, suppose they were not, and suppose someone deviated from the compromise class. Then, as a punishment, the process would have to spiral towards the no-compromise class. However, an excessively patient legislator would unilaterally make an offer to return to the compromise class, that would in turn be accepted, thereby voiding the punishment.[6]

We find that as the number of legislators increases, the range of discount factors that sustains these equilibria increases. Since more legislators implies greater uncertainty over the agenda-setter, continuations that involve no compromise become less attractive, while compromise continuations become more attractive. This is in contrast to the conventional wisdom due to Olson (1965) that suggests cooperation diminishes as group size increases.[7]

We also find that as the utility function becomes more concave the upper bound and the lower bound on the range of admissible discount factors both decrease.[8] The

[5] We use the term *closed class* as defined in Norris (1997). It is similar to the notion of an absorbing state, but refers to a set of states. Once a state in the closed class has been reached, the Markov process will not transition to a state outside that class.

[6] Markov equilibria of the type of game we analyze are notoriously difficult to characterize because of the infinite multi-dimensional state space. As such, we characterize here one equilibrium with the feature that compromise is a possible outcome, and it obtains when discount factors are within a certain range. Other equilibria of this model with risk averse legislators have not yet been found in the current literature, but work has been done by Kalandrakis (2003), Kalandrakis (2007) and Duggan and Kalandrakis (2006) in closely related models. These are discussed at the end of the Introduction.

[7] Other authors that have also shown a converse result are Pecorino (1999), Haag and Lagunoff (2005), and Esteban and Ray (2001).

[8] This result is found for a parametrization of the utility function.

intuition is that as concavity increases, the compromise continuation becomes more attractive, hence less-patient legislators are willing to offer it, but at the same time, less patience is needed for the no-compromise "threat" to be enforceable.

The fact that we find equilibria that exhibit compromise even when restricted to Markovian strategies implies that even institutions without a mechanism for conveying history can result in a compromise outcome. The sensitivity to initial conditions suggests that distributive policies that have a balanced allocation have their origins in conditions that were already somewhat equitable, while the reverse is true for unevenly distributed policies.

Our work is related to that of Dixit, Grossman and Gul (2000) who investigate political compromise based on tacit cooperation in a two-party framework. They consider efficient subgame perfect equilibria for a similar model with only two players and employ trigger strategies to sustain compromise. They find that the possibility of compromise between parties diminishes with the length of time any single party retains power. They do not, however, say if or when compromise is actually achieved. This is what we show in a general framework for an arbitrary number of legislators, and without relying on trigger strategies. The model analyzed by Dixit et al. (2000) is relevant when considering compromise between political parties, in contrast we consider compromise among many competing legislators, each of whom is interested solely in distribution of benefits towards their district.[9] Lagunoff (2001) looks at the question of civil liberties and the formation of legal standards through majority voting. He finds that due to imprecise signals and the possibility of making mistakes, in equilibrium, groups choose standards that are not too severe. This argument parallels what we find. The lack of severe standards can be viewed as a compromise outcome and is driven by the noise introduced by imprecise signalling. Similarly in our model, compromise is driven by the uncertainty over the identity of future agenda-setters.[10]

Our game closely resembles that analyzed by Kalandrakis (2004) who considers three risk neutral legislators. He finds that equilibrium outcomes are absorbed in a closed class in which no compromise is achieved. Kalandrakis (2007) extends this result to the case of five or more legislators and with arbitrary recognition proba-

[9] We do not factor in ideological bias along party lines when considering payoffs, hence coalitions form independent of party affiliation. This basis of coalition formation is supported in empirical work by Lee (2000).

[10] Battaglini and Coate (2006) also include a bargaining game similar to ours as part of their model, but the status quo is not endogenous.

bilities. He also shows that with equal recognition probabilities, and for a subset of state variables where a half or more legislators have a zero status quo allocation, the equilibrium strategies satisfy incentive constraints for utility functions that exhibit some concavity. He provides sufficient conditions on the utility functions to ensure this incentive compatibility. The equilibrium we have constructed is complementary in the sense that we have similar strategies leading to the no-compromise class for the same set of state variables. The derived restrictions on concavity in Kalandrakis (2007) are satisfied by the derived upper bound on the discount factor in our model. The upper bound on the discount factor in our model ensures that legislators are impatient enough to implement a no-compromise proposal if they have an immediate opportunity to do so regardless of how risk averse they are. Duggan and Kalandrakis (2006) prove a general existence result for this class of games, but the result does not extend to equilibria with more than a minimum winning majority obtaining a positive allocation. The equilibria found in the papers discussed above result in the proposer being able to extract the entire surplus. This provides a counterpoint to our paper, as we show that a sharing outcome is possible in equilibrium.

Earlier work on political compromise was done by Baron and Ferejohn (1989), Baron (1996) and Gerber and Ortuno-Ortin (1998). Baron and Ferejohn (1989) was one of the earliest works to look at the dynamics of political compromise. Subsequent to that Baron (1996) and Gerber and Ortuno-Ortin (1998) looked at the question for the case of a public good and a single dimensional policy space. Baron (1996) looked at a dynamic policy setting game while Gerber and Ortuno-Ortin (1998) looked at a static game and considered the endogenous formation of coalitions. Both these papers obtained a version of the Median Voter Theorem, with some form of compromise occurring in equilibrium, but in both papers the status quo was exogenous and constant each period.

The remainder of the paper is organized as follows. In Section 2 we present the general model with $n + 1$ legislators. In Section 3 we define a symmetric Markov perfect equilibrium of the model. In Section 3.1 we give a brief illustration of the equilibrium we characterize, and some intuition for the strategies. In Section 3.2 we formally characterize the set of equilibria. Section 4 provides some comparative statics results and Section 5 concludes.

2 The Model

We present here a stylized version of the legislative process. Let $I = \{1, \ldots, n+1\}$ be a set of $n+1$ symmetric legislators, where $n \geq 4$ and n even.[11] They play a policy setting game over an infinite number of periods $t = 1, 2, \ldots$. Each period a surplus of unit size is divided among the $n+1$ legislators' districts, and each split of the surplus is an element of the n-dimensional simplex, Δ^n. Let the vector $s^t \in \Delta^n$ denote the division of the surplus in period t, where $s^t = (s_1^t, s_2^t, \ldots, s_{n+1}^t)$ and s_i^t is the share to legislator i's district. Legislator i is concerned only about the welfare of his district. The payoff to legislator i in period t, is given by $u(s_i^t)$, where $u(\cdot)$ is increasing and strictly concave, allowing for inter-temporal gains from smoothing. All legislators discount the future with a common discount factor, δ, and legislator i seeks to maximize average expected lifetime utility for his own district,

$$E\left[\sum_{t=1}^{\infty} \delta^{t-1}(1-\delta)u(s_i^t)\right].$$

At the beginning of each period a legislator, $x^t \in I$, is randomly recognized to make a proposal for the division of the surplus for that period. Legislators are recognized with equal probability in each period. This proposer selection process was first motivated by Baron and Ferejohn (1989) as discussed in the introduction. The recognized legislator, x^t, then makes a proposal, $\mathbf{p} \in \Delta^n$, which is voted on by all legislators, each legislator having a single vote. A simple majority of votes is required for a proposal to be implemented, hence the proposer requires $\frac{n}{2}$ legislators besides himself to be in agreement. If the proposal fails to achieve $\frac{n}{2}$ other legislators' vote, the status quo allocation, s^{t-1}, prevails. The persistence of the status quo allocation reflects the fact that the allocation schemes of the policies we consider remain intact if no new legislation is passed to alter it.

We ask whether or not the physical payoff relevant information is enough to allow coordination on an equitable outcome as is reflected in the data. As argued in the introduction, legislatures that are characterized by a large number of members with periodic turnover may embody little institutional memory. We therefore focus on the class of subgame perfect equilibria consisting of Markovian strategies, i.e. Markov perfect equilibria. Below we carefully define our notion of a symmetric

[11] In many legislatures, district representation is allocated by population size hence the assumption of symmetry is entirely justified.

Markov perfect equilibrium, define an equilibrium that exhibits compromise, and characterize such an equilibrium in the remaining sections of the paper.

3 Markov Perfect Equilibrium

With Markov perfect equilibria players' strategies condition only on information that is relevant to current period payoffs. The payoff relevant variables in this model are the status quo allocation and the identity of the proposing legislator, $(s^{t-1}, x^t) \in \Delta^n \times I$. A complete history of the state is therefore defined as $h^t = (s^0, x^1, \ldots, s^{t-1}, x^t)$.

Each legislator's strategy is a pair (α_i, σ_i) such that α_i is legislator i's acceptance strategy and σ_i is legislator i's mixed proposal strategy. A mixed proposal strategy for legislator i, is a probability function $\sigma_i(\cdot; h^t)$. Given a history of the state h^t and a proposal $\mathbf{p}$, $\sigma_i(\mathbf{p}; h^t)$ will be the probability legislator i assigns to proposal $\mathbf{p}$. An acceptance strategy for legislator i is a binary function $\alpha_i(\cdot; h^t)$ such that

$$\alpha_i(\mathbf{p}; h^t) = \begin{cases} 1 & \text{if legislator } i \text{ accepts proposal } \mathbf{p}, \\ 0 & \text{if legislator } i \text{ rejects proposal } \mathbf{p}. \end{cases}$$

A strategy profile is given by (α, σ) where α is a vector of acceptance strategies for all legislators, and σ is a vector of proposal strategies. Note that these acceptance and proposal strategies can potentially condition on the entire history of the state, h^t. We restrict our attention to Markovian strategies for the reasons explain above hence we consider only proposal and acceptance strategies that condition on (s^{t-1}, x^t). That is, we focus on a strategy pair $[\alpha_i(\cdot; s^{t-1}, x^t), \sigma_i(\cdot; s^{t-1}, x^t)]$.[12]

We seek a notion of symmetry for the legislators' strategies reflecting the fact that any legislator i will be expected to behave in the same manner as legislator j if he was in legislator j's position. More concretely, define the one-to-one operator, $\Phi : I \to I$ that represents any permutation of the identity of the legislators. Given a proposal vector, $\mathbf{p} = (p_1, \ldots, p_{n+1})$, and permutation $\Phi(\cdot)$, we denote the resulting permuted proposal as $\mathbf{p}_\Phi = (p_{\Phi(1)}, \ldots, p_{\Phi(n+1)})$. A permutation of the state variable (s^{t-1}, x^t) is therefore denoted $(s_\Phi^{t-1}, \Phi(x^t))$, and a symmetric strategy profile is given by the following definition.

[12]Although we restrict attention to Markov strategies as our equilibrium concept, in Definition 2 below we ensure that equilibrium strategies are robust to any history contingent strategy, $[\alpha_i(\cdot; h^t), \sigma_i(\cdot; h^t)]$.

Definition 1. A strategy profile (α, σ) is *symmetric* if for any permutation of the identities of legislators, Φ,

$$\alpha_i(\mathbf{p}; s^{t-1}, x^t) = \alpha_{\Phi(i)}(\mathbf{p}_\Phi; s_\Phi^{t-1}, \Phi(x^t)), \text{ and}$$
$$\sigma_i(\mathbf{p}; s^{t-1}, x^t) = \sigma_{\Phi(i)}(\mathbf{p}_\Phi; s_\Phi^{t-1}, \Phi(x^t)).$$

Given a proposal, $\mathbf{p}$, and an acceptance strategy profile α the law of motion for the period's allocation is given by

$$s^t = \begin{cases} \mathbf{p} & \text{if } \sum\limits_{i \neq x^t} \alpha_i(\mathbf{p}; s^{t-1}, x^t) \geq \frac{n}{2}, \\ s^{t-1} & \text{otherwise.} \end{cases}$$

This simply says that if the proposal receives the required majority of votes it is implemented, otherwise the policy reverts to the status quo. The expected dynamic payoff for any legislator i, given a strategy profile, (α, σ), and a state (s^{t-1}, x^t), $V_i(\alpha, \sigma; s^{t-1}, x^t)$ is given by

$$V_i(\alpha, \sigma; s^{t-1}, x^t) = \int_{\Delta^n} \left\{(1-\delta)u(s_i^t) + \delta E_{x^{t+1}}[V_i(\alpha, \sigma; s^t, x^{t+1})]\right\} \sigma_{x^t}[\mathbf{p}; s^{t-1}, x^t] d\mathbf{p}.$$

A Markov perfect equilibrium strategy profile must maximize this dynamic payoff for all legislators, for all possible states and must be a best response among *any* history contingent strategy. This leads to Definition 2.[13]

Definition 2. A symmetric *Markov Perfect Equilibrium (MPE)* is a symmetric strategy profile, (α^*, σ^*), such that for all $(s^{t-1}, x^t) \in \Delta^n \times I$, for all $[\alpha_i(\cdot; h^t), \sigma_i(\cdot; h^t)]$, for all h^t, and for all i,

$$V_i(\alpha^*, \sigma^*; s^{t-1}, x^t) \geq V_i(\alpha_i(\cdot; h^t), \alpha^*_{-i}, \sigma_i(\cdot; h^t), \sigma^*_{-i}; s^{t-1}, x^t).$$

[13] In the case of two legislators it is easy to see that there is no payoff relevant state. Since there are only two legislators, the proposer is automatically a majority, hence, effectively a dictator. There is no inter-temporal decision so the unique solution to the single period maximization results in the proposing legislator extracting the entire surplus. This is in contrast to the results of Dixit et al. (2000).

We seek to identify a symmetric MPE in which more than a minimum winning majority of legislators receive a positive allocation in each period. We define a compromise outcome as one is which this occurs. Formally

Definition 3. An allocation, s^t, exhibits *compromise* if $|\{i : s_i^t > 0\}| > \frac{n}{2} + 1$.

Proposition 1 is the main result of the paper. It states that a symmetric MPE exists for an intermediate range of discount factors where equilibrium allocations exhibit compromise. Before stating proposition 1 we must introduce some useful notation. Following Kalandrakis (2004), let Δ_θ, $\theta = 0, \ldots, n$ be a collection of subsets of Δ^n such that a number, θ, of the legislators receive zero allocation. That is

$$\Delta_\theta \equiv \left\{ s^{t-1} \in \Delta^n : |\{i : s_i^{t-1} = 0\}| = \theta \right\}$$

For example, letting $\mathcal{P}$ denote the set of all permutations, Φ, of a vector, we have

$$\Delta_n = \mathcal{P}(1, \underbrace{0, \ldots, 0}_{n}).$$

The set Δ_n is the set of proposals in which one legislator receives the entire share, hence we will call Δ_n our *no-compromise* class. Define also the set $\overline{\Delta}_1$ where one legislator receives zero, and the remaining legislators receive an equal share. That is, $\overline{\Delta}_1 \equiv \mathcal{P}(0, \frac{1}{n}, \ldots, \frac{1}{n})$. Since these proposals exhibit compromise, we refer to the set $\overline{\Delta}_1$ as our *compromise* class of proposals. Clearly $\overline{\Delta}_1 \subset \Delta_1$. In section 3.6 we discuss how the equilibrium extends to other compromise classes.

Proposition 1. *There exists a non-degenerate interval $[\underline{\delta}, \overline{\delta}]$, and a set of allocations, $\Gamma_i \subseteq \Delta_{\theta \leq \frac{n}{2}}$ for every $i \in I$, such that for every $\delta \in [\underline{\delta}, \overline{\delta}]$ a symmetric MPE exists with the following property: If $(s^0, x^1) \in \cup_i(\Gamma_i \times \{i\})$ then $\omega^t \in \overline{\Delta}_1 \times I$ for all $t \geq 1$.*

The proof is constructive. In the sections that follow we first illustrate the equilibrium strategies we find and then provide a formal characterization. To preview the result, we will show that the equilibrium outcome is sensitive to initial conditions. There are two possible outcomes: a compromise outcome, where all but a single legislator shares the surplus evenly, and a no-compromise outcome, where only the proposer takes the surplus each period. The upper bound on the discount factor guarantees that as long as the proposing legislator has an immediate opportunity to take the entire surplus, he will have an incentive to do this rather than propose the compromise outcome. This of course means that the expected payoff to the no-compromise outcome is higher for the proposing legislator than the expected payoff to the compromise outcome. Why will a compromise outcome therefore ever be implemented? If the proposer does not have an immediate opportunity to extract the entire surplus he will have to cherry-pick a minimum winning coalition to offer some allocation and extract the remainder for himself. When the cost of buying-off this coalition is too high, the proposer will seek a compromise instead. The same intuition *sustains* the compromise: once in the compromise, the demands of the minimum winning coalition are too high to justify leaving the compromise. This is guaranteed by the lower bound on the discount factor.

3.1 Equilibrium Illustration

Proposition 1 states that we characterize a symmetric MPE that induces a Markov process for which there is a closed class of proposals that exhibit compromise. The model is specified for five or more legislators, but we can illustrate the essential elements of the equilibrium for the three-legislator case. Although the equilibrium strategies are inconsistent with this case, the intuition for the results remain intact. [14]

With three legislators, all policy proposals lie in the two dimensional simplex. The set of proposals where a single legislator receives a zero share and the remaining legislators split the surplus evenly are the points that lie half-way along each face of the simplex as illustrated in simplex on the left in Figure 1. This represents the closed class of proposals that exhibit compromise in the $n \geq 4$ case. The fact that the class is closed means that if any of these proposals is reached in equilibrium, all proposals subsequently implemented lie in this set.

As mentioned, the equilibrium we characterize is subject to initial conditions. There exists another closed class of proposals that exhibit no-compromise. These points, represented by the vertices of the simplex on the right in Figure 1, indicate

[14] One reason the prescribed strategies are not an equilibrium in the case of three legislators is that the compromise, generally speaking, involves one legislator receiving zero and the remaining legislators splitting the surplus evenly. In the case of three legislators, this proposal takes the form $(\frac{1}{2}, \frac{1}{2}, 0)$. However a cherry-picking proposal which can lead to the no-compromise, also takes this form in the case of three legislators, hence the strategies are inconsistent.

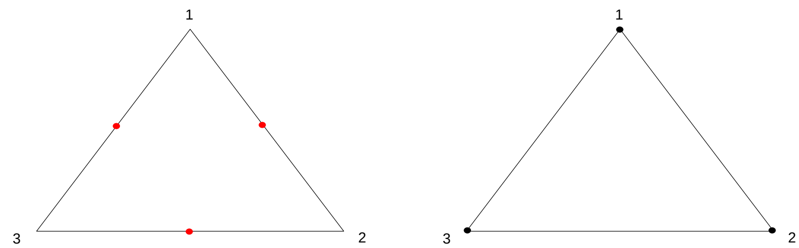

Figure 1: Closed Classes

allocations where a single legislator has captured the entire surplus.

If initial allocations are "well distributed" among non-proposing legislators, then the proposer finds the compromise proposals to be most attractive in equilibrium. If, however, the initial allocations are closer to the vertices, then a no-compromise or cherry-picking proposal becomes more attractive. What we mean by "well distributed" will be made precise in the characterization below, but, assuming legislator 1 is the proposer, these well distributed allocations lie within a boundary illustrated by the shaded area in Figure 2.

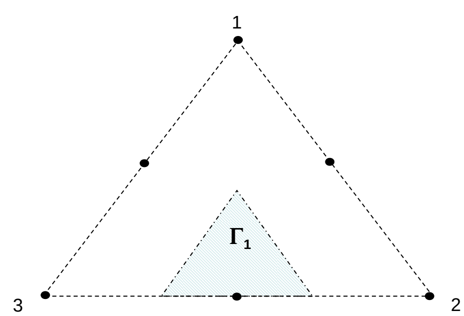

Figure 2: Equilibrium Illustration

Impatient legislators always have an incentive to capture as much of the surplus as possible for their district at the expense of the other legislative districts. This is reflected in the equilibrium strategies. Starting from an interior allocation (that is, not on the face of the simplex), but outside the shaded area, a proposing legislator identifies a minimum winning coalition of legislators that demand the least, and includes them in a coalition by offering them just enough to make them indifferent between the status quo allocation and the current proposal. In the three-legislator case, this minimum winning coalition is simply the non-proposing legislator with the lower demand. This legislator will be "cherry-picked" to form a minimum winning coalition and the other legislator will be frozen out and given no allocation. In the very next period the proposing legislator has the opportunity to extract the entire surplus, which leads to a sustained no-compromise outcome where each legislator grabs the entire surplus when he is the proposer.

What prevents this spiral towards no-compromise from happening in the shaded region? These allocations are well-distributed making the demands of the minimum winning coalition relatively high, so the proposer makes an offer to split the surplus between himself and one other legislator, knowing that in the next period the proposer will have an incentive to sustain this sharing. Once legislators have the expectation that sharing will occur, it is too costly to buy them off with a cherry-picking strategy, so the compromise is maintained.

3.2 Equilibrium Characterization

Given an allocation $s^t = (s_1^t, \ldots, s_{n+1}^t)$ and strategies (α, σ) define the dynamic payoff to player i as

$$U_i(s^t; \alpha, \sigma) = (1 - \delta)u(s_i^t) + \delta E_{x^{t+1}}[V_i(\alpha, \sigma; s^t, x^{t+1})]$$

The equilibrium acceptance strategy for any legislator i is α_i^* such that he accepts proposals that give a dynamic payoff that is at least as great as the payoff to the status quo. That is, given proposal $\mathbf{p}$,

$$\alpha_i^*(\mathbf{p}; s^{t-1}, x^t) = \begin{cases} 1 & \text{if } U_i(\mathbf{p}; \alpha^*, \sigma^*) \geq U_i(s^{t-1}; \alpha^*; \sigma^*) \\ 0 & \text{otherwise.} \end{cases}$$

Under the equilibrium proposal strategies, $\sigma^*(\cdot; s^{t-1}, x^t)$, if more than $\frac{n}{2}$ non-proposing legislators have a zero status quo allocation, the proposing legislator will extract the entire surplus and offer a *no-compromise* proposal. If less than $\frac{n}{2}$ non-proposing legislators have a zero status quo allocation, and the status quo is not one of the compromise allocations, (what we call *interior allocations*), the proposer will either offer a *compromise* proposal or extract as much surplus as possible by using a *cherry-picking* strategy. Once a compromise proposal has been implemented, the compromise is sustained.

Specifically, first consider all (s^{t-1}, x^t), such that $s^{t-1} \in \Delta_n$. These are status quo allocations in the no-compromise class. The equilibrium proposal strategy is to assign probability one to the no-compromise proposal, $\underline{\mathbf{p}}$, where

$$\underline{p}_i = \begin{cases} 1 & \text{for } i = x^t \\ 0 & \text{for } i \neq x^t \end{cases}.$$

Notice that this proposal is also an element of the set Δ_n this means that the no-compromise class, Δ_n, represents a closed class of proposals. Now consider $s^{t-1} \in \Delta_{\theta > \frac{n}{2}}$, or $s^{t-1} \in \Delta_{\frac{n}{2}}$ and $s^{t-1}_{x^t} \neq 0$. These are allocations where fewer than $\frac{n}{2}$ non-proposing legislators have a zero status quo allocation. The equilibrium proposal is therefore again the no-compromise proposal, with probability 1, so $\sigma_i^*(\underline{\mathbf{p}}; s^{t-1}, x^{t-1}) = 1$

Define the compromise proposal, $\overline{\mathbf{p}}^j$, such that legislator j receives zero and the remaining legislators split the surplus evenly. So

$$\overline{p}_i^j = \begin{cases} 0 & \text{if } i = j, \\ \frac{1}{n} & \text{otherwise.} \end{cases}$$

Now consider all (s^{t-1}, x^t), such that $s^{t-1} \in \overline{\Delta}_1$ and $s^{t-1}_{x^t} = \frac{1}{n}$. These are status quo allocations in the compromise class. In this case one legislator (excluding the proposer) has a zero status quo allocation and the remaining n legislators split the surplus evenly. The equilibrium proposal strategy assigns probability 1 to the compromise proposal $\overline{\mathbf{p}}^j$, for j such that $s^{t-1}_j = 0$. So the legislator that had a zero status quo allocation is given zero again and all other legislators split the surplus evenly.

If $s^{t-1} \in \overline{\Delta}_1$ and $s^{t-1}_{x^t} = 0$, so the proposer's status quo allocation is zero, the proposer takes a legislator at random to give no allocation and splits the surplus evenly among himself and the remaining legislators. So $\sigma_i^*(\overline{\mathbf{p}}^j; s^{t-1}, x^t) = \frac{1}{n}$ for all $j \in I/\{x^t\}$. Notice again that once a proposal in the compromise class, $\overline{\Delta}_1$, has been implemented the equilibrium strategies dictate that all subsequent proposals lie in this set. Hence the set of proposals $\overline{\Delta}_1$ represents a closed class of proposals.

Now consider $s^{t-1} \in \Delta_{\frac{n}{2}}$ with $s^{t-1}_{x^t} = 0$. This is considered an interior allocation. As Kalandrakis (2007) notes, it is not necessarily an equilibrium strategy to offer a positive allocation to the legislator with the lowest positive status quo an allocation, while freezing out the others. In equilibrium a cherry-picking strategy is used where, with some probability, a share $A(b^*)$ is offered to one of b^* legislators. Following Kalandrakis (2007), without loss of generality suppose $s^{t-1} = (s^{t-1}_1, \ldots, s^{t-1}_{\frac{n}{2}+1}, 0, \ldots, 0)$ with $0 < s^{t-1}_i \leq s^{t-1}_{i+1}$ and $s^{t-1}_{x^t} = 0$. Given a value $b \in \{1, \ldots, \frac{n}{2} + 1\}$, define the allocation

$$A(b) = u^{-1}\left[\frac{\sum_{i=1}^{b} u(s_i^{t-1})}{b - \frac{\delta n}{2(n+1)}}\right]. \tag{1}$$

This is the value that is demanded by one of b legislators in equilibrium. Define the value b^* as[15]

$$b^* = \begin{cases} \min b \in \{1, \ldots, \frac{n}{2}\} & \text{s.t. } A(b) \leq A(b+1), \\ \frac{n}{2} + 1 & \text{otherwise.} \end{cases}$$

Now define the cherry-picking proposal, $\mathbf{p}^j(A(b^*))$, where legislator j receives $A(b^*)$, the proposer receives $1 - A(b^*)$, and all other legislators receive zero. So

$$p_i^j(A(b^*)) = \begin{cases} A(b^*) & \text{for } i = j, \\ 1 - A(b^*) & \text{for } i = x^t, \\ 0 & \text{otherwise.} \end{cases}$$

The equilibrium proposal strategy is to assign probability $\mu_j(b^*)$ to the cherry-picking proposal $\mathbf{p}^j(A(b^*))$ for all $j \leq b^*$, where $\mu_j(b^*)$ is given by

$$\mu_j(b^*) = \frac{u(A(b^*)) - u(s_j^{t-1})}{u(A(b^*))\frac{\delta n}{2(n+1)}}. \tag{2}$$

Now consider $s^{t-1} \in \Delta_{\theta < \frac{n}{2}} \backslash \overline{\Delta}_1$. This is the remainder of the interior allocations. Let C_i be the set of legislators that are a part of legislator i's coalition. Note that that $|C_i| = \frac{n}{2}$. Define the vector of demands for legislator i's coalition member's as $A^{*i} = (A_1^{*i}, \ldots, A_{\frac{n}{2}}^{*i})$, where $0 \leq A_j^{*i} \leq 1$. This is the set of allocations that makes each of legislator i's coalition members indifferent between the current proposal and the status quo, given that legislator i is the proposer. Given a proposer i and demands A^{*i} the implemented proposal is either a cherry picking proposal, $\mathbf{p}(A^{*i})$, or the compromise proposal. In this case the proposal $\mathbf{p}(A^{*i})$ is given by

$$p_j(A^{*i}) = \begin{cases} A_j^{*i} & \text{for } j \in C_i \\ 1 - \sum_{j \in C_i} A_j^{*i} & \text{for } j = x^t \\ 0 & \text{otherwise .} \end{cases}$$

Denote the set of all permutations of cherry-picking proposals where legislator i is

[15] Kalandrakis (2007) shows that b^* is unique.

the proposer, given a vector of demands, A^{*i}, as $\mathbf{P}(A^{*i})$. The equilibrium strategy is a probability distribution $\mu^{*i}(\cdot)$ over all $\mathbf{p}(A^{*i}) \in \mathbf{P}(A^{*i})$, and over all compromise proposals $\overline{\mathbf{p}}^j \in \overline{\Delta}_1$.

Given a status quo allocation, $s^{t-1} = (s_1^{t-1}, \ldots, s_{n+1}^{t-1})$, the vectors A^{*i} and the distributions $\mu^{*i}(\cdot)$ are determined by the fixed point of a map, $\mathbf{B}$, that is discussed in the following sections. The dynamics of the equilibrium are illustrated in Figure 3. (For a more detailed illustration of the dynamics see figure 4 in the Appendix.)

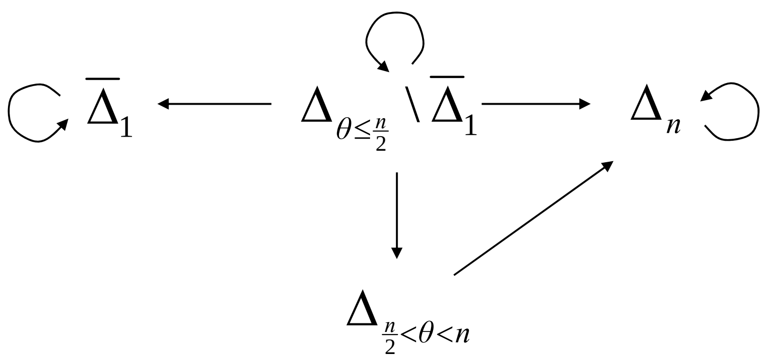

Figure 3: Equilibrium Dynamics

3.3 Equilibrium Payoffs

Before defining the map that generates the equilibrium demands and associated distributions over cherry-picking proposals, it is necessary to derive the equilibrium payoffs for all other equilibrium proposals.

The dynamic payoff to the no-compromise proposal, $\underline{\mathbf{p}}$, is

$$U_i(\underline{\mathbf{p}}; \alpha^*, \sigma^*) = \begin{cases} (1-\delta)u(1) + \delta E_{x^{t+1}}[V_i(\alpha^*, \sigma^*; \underline{\mathbf{p}}, x^{t+1})] & \text{if } i = x^t, \\ (1-\delta)u(0) + \delta E_{x^{t+1}}[V_i(\alpha^*, \sigma^*; \underline{\mathbf{p}}, x^{t+1})] & \text{if } i \neq x^t. \end{cases}$$

In the next period, since the status quo is an element of the no-compromise class, the equilibrium strategy is the no-compromise proposal again so we can define the recursive payoffs when the status quo is in the no compromise class as $\underline{V}_x$ for the proposer and $\underline{V}_z$ for everyone else. With probability $\frac{1}{n+1}$ each legislator is the proposer in the next period, hence any legislator's continuation value is $\underline{V}_x$ with probability $\frac{1}{n+1}$ and $\underline{V}_z$ with probability $\frac{n}{n+1}$. This gives

$$\underline{V}_x = (1-\delta)u(1) + \tfrac{\delta}{n+1}\left(\underline{V}_x + n\underline{V}_z\right),$$
$$\underline{V}_z = (1-\delta)u(0) + \tfrac{\delta}{n+1}\left(\underline{V}_x + n\underline{V}_z\right).$$

Normalizing $u(0) = 0$ and $u(1) = 1$, and solving gives

$$\underline{V}_x = \tfrac{n+1-\delta n}{n+1} \tag{3}$$

and

$$\underline{V}_z = \tfrac{\delta}{n+1}. \tag{4}$$

The dynamic payoff to the compromise proposal, $\overline{\mathbf{p}}^j$, is

$$U_i(\overline{\mathbf{p}}^j;\alpha^*,\sigma^*) = \begin{cases} (1-\delta)u(\frac{1}{n}) + \delta E_{x^{t+1}}[V_i(\alpha^*,\sigma^*;\overline{\mathbf{p}}^j,x^{t+1})] & \text{if } i \neq j, \\ (1-\delta)u(0) + \delta E_{x^{t+1}}[V_i(\alpha^*,\sigma^*;\overline{\mathbf{p}}^j,x^{t+1})] & \text{if } i = j. \end{cases}$$

If the status quo is an element of the compromise class, the equilibrium strategy is the compromise proposal again, so we can define the recursive payoff when the status quo is in the compromise class as γ for the legislators receiving $\frac{1}{n}$ and ζ for the legislator that is frozen out. With probability $\frac{n}{n+1}$ each legislator receives the same payoff as he did in the previous period, and with probability $\frac{1}{n+1}$ the current loser becomes the proposer, and a new legislator is randomly selected to be frozen out. These payoffs are given by

$$\begin{aligned} \gamma = &\ (1-\delta)u(\tfrac{1}{n}) + \tfrac{\delta}{n+1}\left(n\gamma + [\tfrac{1}{n}\zeta + \tfrac{n-1}{n}\gamma]\right), \\ \zeta = &\ (1-\delta)u(0) + \tfrac{\delta}{n+1}\left(\gamma + n\zeta\right). \end{aligned}$$

Solving for ζ and γ gives

$$\gamma = \tfrac{n(n+1-\delta n)}{(n+1)(n+\delta-\delta n)}u(\tfrac{1}{n}), \tag{5}$$

$$\zeta = \tfrac{n\delta}{(n+1)(n+\delta-\delta n)}u(\tfrac{1}{n})\ . \tag{6}$$

Now consider the dynamic payoff to the cherry-picking proposal $\mathbf{p}^j(A(b^*))$. This proposal is an element of the set Δ_2 so the equilibrium continuation strategies dictate that the no-compromise proposal, $\underline{\mathbf{p}}$, is implemented. The continuation payoffs are therefore $\underline{V}_x$ if legislator i is the proposer and $\underline{V}_z$ otherwise. Hence we can simplify these dynamic payoffs as

$$U_i(\mathbf{p}^j(A(b^*)); \alpha^*, \sigma^*)) = \begin{cases} (1-\delta)u(1-A(b^*)) + \frac{\delta}{n+1} & \text{if } i = x^t, \\ (1-\delta)u(A(b^*)) + \frac{\delta}{n+1} & \text{if } i = j, \\ \frac{\delta}{n+1} & \text{otherwise.} \end{cases}$$

The probability assigned to legislator $i \leq b^*$ being allocated a positive share is μ_i, so define the expected dynamic payoffs to the cherry-picking strategy $\sigma_i^*(\mathbf{p}^j(A(b^*)); s^{t-1}, x^t)$ as $V_i(\Delta_{\frac{n}{2}}; s_{x^t}^{t-1} = 0)$. These are given by

$$V_i(\Delta_{\frac{n}{2}}; s_{x^t}^{t-1} = 0) = \begin{cases} (1-\delta)u\left(1-A(b^*)\right) + \frac{\delta}{n+1} & \text{if } i = x^t \\ (1-\delta)u\left(A(b^*)\right)\mu_i + \frac{\delta}{n+1} & \text{if } i \leq b^*, \\ \frac{\delta}{n+1} & \text{otherwise} \end{cases} \tag{7}$$

Now consider the dynamic payoff to a cherry-picking proposal, $\mathbf{p}(A^{*j})$. The cherry-picking proposal, $\mathbf{p}(A^{*j})$, is in the set $\Delta_{\frac{n}{2}}$. The equilibrium continuation strategies will be the no-compromise proposal, $\underline{\mathbf{p}}$, if the period $t+1$ proposer has a positive status quo allocation. If the period $t+1$ proposer has a zero status quo allocation the continuation payoffs are given by $V_i(\Delta_{\frac{n}{2}}; s_{x^t}^{t-1} = 0)$, so the expected continuation payoffs for the cherry-picking proposal, $\mathbf{p}(A^{*j})$, are given by

$$V_i(A^{*j}) = \begin{cases} \frac{1}{n+1}(\underline{V}_x + \frac{n}{2}\underline{V}_z + \frac{n}{2}V_i(\Delta_{\frac{n}{2}}; s_{x^t}^{t-1} = 0)) & \text{for } p_i(A^{*j}) > 0, \\ \frac{1}{n+1}((\frac{n}{2}+1)\underline{V}_z + \frac{n}{2}V_i(\Delta_{\frac{n}{2}}; s_{x^t}^{t-1} = 0)) & \text{for } p_i(A^{*j}) = 0. \end{cases} \tag{8}$$

Below I remove the conditioning on (α^*, σ^*) to conserve space. So the dynamic payoffs to the cherry-picking proposal are

$$U_i(\mathbf{p}(A^{*j})) = \begin{cases} (1-\delta)u(A_i^{*j}) + \delta V_i(A^{*j}) & \text{if } i \in C_j, \\ (1-\delta)u(1-\sum_{i\in C_j} A_i^{*j}) + \delta V_i(A^{*j}) & \text{if } i = j \\ \delta V_i(A^{*j}) & \text{otherwise.} \end{cases}$$

3.4 Derivation of $\mu^{*i}(\cdot)$ and A^{*i}

This section outlines the derivation of $\mu^{*i}(\cdot)$ and A^{*i} for all i. Denote an arbitrary cherry-picking proposal for legislator j as $\mathbf{p}(A^j)$, and, as before, let the set $\mathbf{P}(A^j)$

be all permutations of this cherry-picking proposal, given demands A^j. Now denote for legislator j an arbitrary probability distribution over cherry-picking proposals in $\mathbf{P}(A^j)$ and over all compromise proposals $\overline{\mathbf{p}}^j$ as $\mu^j(\cdot)$, and let $\mu = (\mu^1(\cdot), \dots, \mu^{n+1}(\cdot))$. Finally, denote the matrix of demands for legislators $i = 1, \dots, n+1$ as $\mathbf{A} = (A^1, \dots, A^{n+1})$.

Denote the expected continuation payoff given demands, $\mathbf{A}$, and probability distributions, μ, as $V_i(\mathbf{A}, \mu)$. This is

$$V_i(\mathbf{A}, \mu) = \frac{1}{n+1} \sum_{j=1}^{n+1} \left[\sum_{\mathbf{p}(A^j) \in \mathbf{P}(A^j)} U_i(\mathbf{p}(A^j)) \mu^j(\mathbf{p}(A^j)) + \gamma \sum_{h \in I/\{i\}} \mu^j(\overline{\mathbf{p}}^h) + \zeta \mu^j(\overline{\mathbf{p}}^i) \right].$$

Hence given a status quo, an arbitrary vector of demands, and probability distributions, the dynamic payoff to the status quo is

$$U_i(s^{t-1}; \mathbf{A}, \mu) = (1-\delta) u(s_i^{t-1}) + \delta V_i(\mathbf{A}, \mu). \tag{9}$$

Assume $p_i(A^j) > 0$, then calculate for all i and all j the value $\hat{A}_i^j(s^{t-1}; \mathbf{A}, \mu)$ as

$$\hat{A}_i^j(s^{t-1}; \mathbf{A}, \mu) = u^{-1} \left[\min\{\max\{0, \frac{1}{1-\delta} \left(U_i(s^{t-1}; \mathbf{A}, \mu) - \delta V_i(A^j) \right)\}, 1\} \right].$$

Define a vector of these values as $A^j(s^{t-1}; \mathbf{A}, \mu)$. Last define the set of all distributions over cherry-picking proposals in $\mathbf{P}(A^j)$ and all compromise proposals as $\mathbf{M}(\mathbf{P}(A^j))$. Now let us pick new demands and distributions, $(A^{j\prime}, \mu^{j\prime}) \in \mathbf{B}_j(\mathbf{A}, \mu; s^{t-1})$ where

$$\begin{aligned} \mathbf{B}_j(\mathbf{A}, \mu; s^{t-1}) = \quad & \{(A^{j\prime}, \mu^{j\prime}) : \\ & (A^{j\prime}, \hat{\mu}^{j\prime}) \in \arg\max_{\hat{A}^j, \hat{\mu}^j} U_j(s^{t-1}; \hat{A}^j, A^{-j}, \hat{\mu}^j, \mu^{-j}) \\ & \text{s.t.} \quad \hat{A}^j = \hat{A}^j(s^{t-1}; \mathbf{A}, \hat{\mu}^j, \mu^{-j}) \\ & \qquad\; \hat{\mu}^j \in \mathbf{M}(\mathbf{P}(A^j)) \\ & \text{and} \quad \mu^{j\prime}(\mathbf{p}(A^{j\prime})) = \hat{\mu}^{j\prime}(\mathbf{p}(A^j))\}. \end{aligned}$$

Define

$$\mathbf{B}(\mathbf{A}, \mu; s^{t-1}) = \times_{j=1}^{n+1} \mathbf{B}_j(\mathbf{A}, \mu; s^{t-1}).$$

Lemma 1. *The map* $\mathbf{B}(\mathbf{A}, \mu; s^{t-1})$ *has a fixed point* $(\mathbf{A}^*, \mu^*)$ *such that* $(\mathbf{A}^*, \mu^*) \in \mathbf{B}(\mathbf{A}^*, \mu^*; s^{t-1})$.

Proof. See Appendix. ■

We can now formally define the set Γ_j to be the set of status quo allocations where less than $\frac{n}{2}$ legislators have a zero allocation, and from which legislator j will choose to go to compromise. This will be where $\mu^{*j}(\overline{\mathbf{p}}^i) = 1$ for some i. So

$$\Gamma_j = \left\{ s^{t-1} \in \Delta_{\theta < \frac{n}{2}} : \mu^{*j}(\overline{\mathbf{p}}^i) = 1 \text{ for some } i \right\}.$$

Now define the set Γ to be the set of all status quo allocations for which there is some positive probability that some legislator j will choose compromise. That is

$$\Gamma = \left\{ s^{t-1} \in \Delta_{\theta < \frac{n}{2}} : \mu^{*j}(\overline{\mathbf{p}}^i) > 0 \text{ for some } i, \text{ for some } j \right\}.$$

Note that $\Gamma_j \subset \Gamma$. Last define all other allocations in $\Delta_{\theta < \frac{n}{2}}$ excluding the compromise class to be $\Delta^c_{\theta < \frac{n}{2}}$. This is

$$\Delta^c_{\theta < \frac{n}{2}} = \Delta_{\theta < \frac{n}{2}} / (\Gamma \cup \overline{\Delta}_1).$$

We can now provide a more detailed illustration of the equilibrium dynamics. This is given in figure 4 below.

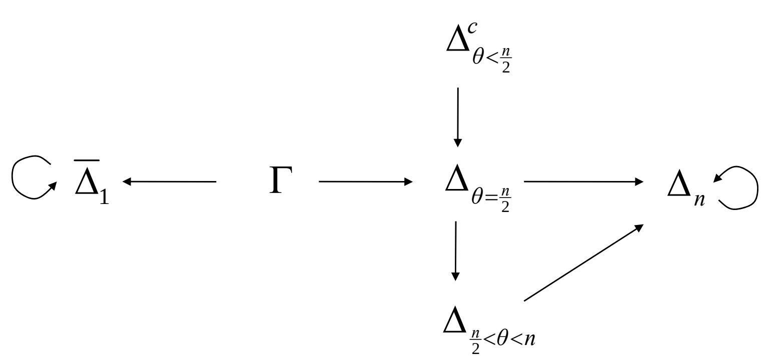

Figure 4: Detailed Equilibrium Dynamics

3.5 Incentives

Now to show that (σ^*, α^*) is an equilibrium we observe that the one-shot deviation principle applies and it suffices to check that legislators' one-shot incentives hold for all possible values of the state variable. We focus on the incentives that define the admissible range of the discount factor, and relegate a complete incentives analysis to the Appendix.

We define the lower bound on the discount factor as follows. We consider a deviation from the compromise class in which the proposing legislator employs a cherry-picking strategy. He will attempt to buy-off a minimum winning coalition and extract the remainder of the surplus for himself. This implies a payoff for the proposer of $(1-\delta)u(1-\sum_{i\in C_j} A_i^j)+\frac{\delta}{(n+1)}$. This payoff is no greater than the payoff to remaining in the compromise as long as $(1-\delta)u(1-\sum_{i\in C_j} A_i^j)+\frac{\delta}{(n+1)} \leq \gamma$. Defining the variable $\phi \equiv \frac{1}{1-\delta}\left[\gamma - \frac{\delta}{(n+1)}\right]$ and rearranging gives us

$$\textstyle\sum_{i\in C_j} A_i^j \geq 1 - u^{-1}(\phi)\,. \quad [16] \tag{10}$$

The coalition in this case will consist of the legislator who was receiving zero under the status quo, and $\frac{n}{2}-1$ randomly selected legislators who were each receiving $\frac{1}{n}$. The legislator who is receiving zero, will accept zero, because $\underline{V}_z \geq \xi$ and the remaining $\frac{n}{2}-1$ will be offered the same A_i^j that makes them indifferent between staying in the compromise or going towards no compromise. Let's call this value that makes them indifferent A. So equation (10) implies

$$(\tfrac{n}{2}-1)A \geq 1 - u^{-1}(\phi). \tag{11}$$

Here A is determined by the equality of $(1-\delta)u(A)+\frac{\delta}{n+1}$ and γ. We have $(1-\delta)u(A)+\frac{\delta}{n+1} = \gamma$. So $A = u^{-1}(\phi)$. Substituting this value of A into (11) implies $(\frac{n}{2}-1)u^{-1}(\phi) \geq 1-u^{-1}(\phi)$ for the proposer to not want to propose such a deviation. Rearranging gives $u(\frac{2}{n}) \leq \phi$ or

$$u(\tfrac{2}{n}) \leq \tfrac{1}{1-\delta}(\gamma - \tfrac{\delta}{n+1}). \tag{12}$$

As $\delta \to 0$ this inequality becomes $u(\frac{2}{n}) \leq u(\frac{1}{n})$ which is a contradiction, so the proposer would want to propose this, but as $\delta \to 1$ this becomes $1 \leq nu(\frac{1}{n})$, which

[16] Note that in addition to concavity and monotonicity of $u(\cdot)$, by the inverse function theorem, we need to assume it is continuously differentiable to allow it to be invertible.

holds by concavity of $u(\cdot)$. Hence the proposer does not want to propose such a deviation as long as legislators are *patient* enough. So there is a lower bound on the discount factor, $\underline{\delta}$, such that for all $\delta > \underline{\delta}$ the inequality in (12) is satisfied.

We define the upper bound on the discount factor as follows. Consider a deviation from the compromise class that is $\tilde{\mathbf{p}}$ within Γ_j for all j. For the three-legislator case the set Γ_1 is illustrated by the shaded area in Figure 2. The outer boundary of the triangle implies a lower bound on the acceptor's status quo allocation.

Once the compromise class has been reached, at least one legislator must be singled out every period to receive the zero allocation. If the proposer's status quo is non-zero, he gives the zero to the legislator who already had zero, if not, he must select someone at random to receive the zero. This means that within the compromise, there is an element of uncertainty, and legislators may have incentives to "game" the system by proposing a share for themselves that will give a certain continuation payoff of γ. A proposer can do this by making a proposal that is within Γ_i for all i, while making sure he does not have the highest share. This guarantees a continuation payoff of γ.

The only way for there to be no such incentive, is if the allocations that are attainable within Γ_i are so small that they offset any gain in continuation payoff. A deviation $\tilde{\mathbf{p}}$ within Γ_i for all i implies, at best, a continuation payoff equal to γ, so we must have that $(1-\delta)u(\tilde{p}_j) + \delta\gamma \leq \gamma$. So all allocations where legislator j does not have the highest share, $\tilde{\mathbf{p}}$, within Γ_j must satisfy

$$u(\tilde{p}_j) \leq \gamma.$$

This boundary cannot be arbitrarily imposed since the boundaries of Γ_j in the next period are determined by the incentives of the next period's proposer (legislator j). So this upper bound on the allocations attainable in Γ_j must be induced by the lower bound on the next period's acceptors' allocations. Figure 5 illustrates this point for the case with three legislators. The horizontal dashed line in figure 5 represents the upper bound on the deviation allocation for legislator 1 that is required to prevent a deviation from the compromise class. The diagonal lines represent the lower bounds on the acceptors' allocations implied by condition (i) of Γ_1.

The intersection of the diagonal lines, indicated by the black dot, represents the maximum allocation the proposing legislator (legislator 1) can take while still remaining in Γ_1, based on the lower bound on the acceptors' allocations. This

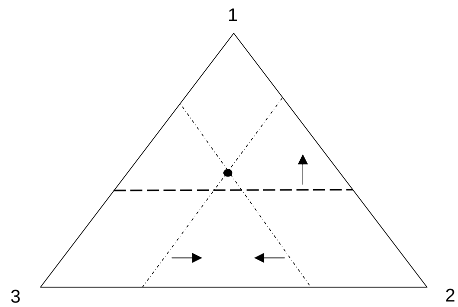

Figure 5: Decreasing δ

maximum must lie below the dashed line in equilibrium. The arrows indicate the direction in which these lines move as δ decreases, so for δ low enough, we achieve the situation depicted in figure 6. This defines the upper bound on the discount factor, $\overline{\delta}$.

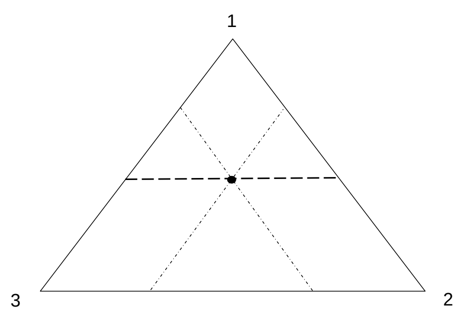

Figure 6: Condition for $\overline{\delta}_2$

The derivation of $\overline{\delta}$ is given in Section 6.2 in the Appendix. We have $\overline{\delta}$ such that for all $\delta \leq \overline{\delta}_2$

$$u\left(\frac{1-2u^{-1}(\gamma)}{2(n-1)}\right) \leq \frac{n+1}{n+1-\delta}u\left(\frac{2}{n}\left[1-u^{-1}(\phi)\right]\right) + \frac{\delta}{1-\delta}\left(\frac{1}{n+1}-\gamma\right). \tag{13}$$

It remains to be verified that $\underline{\delta} \leq \overline{\delta}$ to guarantee a non-degenerate range between $\underline{\delta}$ and $\overline{\delta}$. Lemma 2 shows that this is the case.

Lemma 2. *There exists a non-degenerate range* $[\underline{\delta}, \overline{\delta}]$.

Proof. In the appendix. ■

The equilibrium strategies, together with this incentive analysis and lemmas 1-2 complete the proof of proposition 1.

3.6 Other Compromise Classes

The above characterization focused on the specific compromise class where a single legislator received zero and the remaining legislators split the surplus evenly. However the equilibrium is not restricted to this compromise class. The significance of the compromise class is that it is a distinct set of allocations that signals the compromise, and there must be an incentive compatible algorithm to transition between states in the compromise class, once the compromise class has been reached.

For example, consider that there are five legislators and the compromise class is $\widehat{\Delta} = \mathcal{P}(\widehat{s}_1, \widehat{s}_2, \widehat{s}_3, \widehat{s}_4, \widehat{s}_5)$, such that $\widehat{s}_1 \geq \widehat{s}_2 \geq \widehat{s}_3 \geq \widehat{s}_4 \geq \widehat{s}_5 > 0$. Now we define a compromise proposal, $\overline{\mathbf{p}}$, that essentially rotates all payoffs among the five legislators with equal probability, as

$$\overline{p}_{x^t+i-I(x^t+i>5)} = \widehat{s}_{1+i},$$

for $i = 0, \dots, 4$. Here $I(x^t+i > 5)$ is an indicator function that returns 1 if $x^t+i > 5$ and returns 0 otherwise.

For this compromise class there will be *a* set of initial allocations, Γ_j, (different from the previous Γ_j), such that starting from allocations within Γ_j the proposer will have an incentive to propose the compromise and the acceptors will have an incentive to accept. This is as opposed to the strategies spiralling towards no compromise. The complete set of proposal strategies can therefore be written as before.

Clearly, not every compromise allocation can be sustained in this way. The payoffs to the compromise class must allow a non-degenerate set of allocations, Γ_j, and must allow a non-degenerate range of discount factors $[\underline{\delta}, \overline{\delta}]$. This is to say that payoffs must be incentive compatible.

A notable compromise that cannot be sustained in this set of equilibria is the "perfect" compromise class $\overline{\Delta}_0 = \mathcal{P}(\frac{1}{n+1}, \dots, \frac{1}{n+1})$. This would imply a payoff in the compromise as $\gamma = u(\frac{1}{n+1})$. We leave exploring the limits of compromise to future work.

4 Comparative Statics

The intuition that more legislators make it easier for compromise to be sustained is reflected in the comparative statics for the range of discount factors for which this equilibrium holds.

Lemma 3. *As the number of legislators, n, becomes large, the lower bound on the discount factor, $\underline{\delta}$, is decreasing in n.*

Proof. The lower bound on the discount factor is such that for all $\delta \geq \underline{\delta}$

$$u(\tfrac{2}{n}) \leq \tfrac{1}{1-\delta}[\gamma - \tfrac{\delta}{n+1}]. \tag{14}$$

Define the function

$$f(\delta, n) = u(\tfrac{2}{n}) - \tfrac{1}{1-\delta}[\gamma - \tfrac{\delta}{n+1}].$$

and define the function $g(n) = \underline{\delta}$ such that $f(\underline{\delta}, n) = 0$. By the implicit function theorem we know that

$$\tfrac{\partial}{\partial n} g(n) = -\tfrac{\partial_n f(\delta,n)}{\partial_\delta f(\delta,n)}.$$

So taking limits as $n \to \infty$ we have

$$\tfrac{\partial}{\partial n} g(n) = -\tfrac{1}{(1-\delta)^2}.$$

■

Lemma 4. *As the number of legislators, n, becomes large, the upper bound on the discount factor, $\overline{\delta}$, is increasing in n.*

Proof. See Appendix. ■

Proposition 2. *For a large number of legislators, n, the range of values $[\underline{\delta}, \overline{\delta}]$ for which (α^*, σ^*) is an equilibrium is increasing in n.*

By lemma 3 we show that the lower bound on the discount factor is decreasing as n increases. By lemma 4 we show that the upper bound on the discount factor is increasing as n increases. The proof of proposition 2 follows naturally. Proposition 2 is illustrated in figure 7.

Figure 7: Bounds on δ as n increases

The intuition for this result is simple. In this model legislators' incentives to compromise is driven in part by the uncertainty over their future agenda-setting power. As the number of legislators becomes large, this uncertainty increases, thereby increasing legislator's willingness to compromise. This explains the decrease in the lower bound as n becomes large. In addition, as the number of legislators increase, the difference in current period payoff between a no-compromise proposal and a compromise proposal gets larger, making the no-compromise proposal more attractive, which drives up the upper bound.

The following comparative statics results were derived for a parameterization of $u(\cdot)$. Let

$$u(x) = x^{\frac{1}{b}}$$

where $b \geq 1$. The value, b, represents the concavity of this function. We consider the behavior of the range of discount factors as the stage payoff becomes more concave.

Proposition 3. *As $b \to \infty$, (and hence the utility function $u(\cdot)$ becomes more concave), $\underline{\delta}$ decreases.*

Proof. Again, by the implicit function theorem we can define a function $F(\cdot)$ such that

$$F(\delta, b) = (\tfrac{2}{n})^{\frac{1}{b}} - \tfrac{1}{1-\delta}[\gamma - \tfrac{\delta}{n+1}].$$

and $G(b) = \underline{\delta}$ such that $F(\underline{\delta}, b) = 0$. By the implicit function theorem we know that

$$\tfrac{\partial}{\partial b} G(b) = -\tfrac{\partial_b F(\delta,b)}{\partial_\delta F(\delta,b)}.$$

So we have

$$\frac{\partial}{\partial b} G(b) = -\frac{(1-\delta)(n+\delta-\delta n)\left[-n(n+1-\delta n)\left[\left(\frac{1}{n}\right)^{\frac{1}{b}} \ln\left(\frac{1}{n}\right) - (1-\delta)\left(\frac{2}{n}\right)^{\frac{1}{b}} \ln\left(\frac{2}{n}\right)\right] + \delta(1-\delta)\left(\frac{2}{n}\right)^{\frac{1}{b}} \ln\left(\frac{2}{n}\right)\right]}{n\left(\frac{1}{n}\right)^{\frac{1}{b}}[(1-\delta)^2 n^2 + (1-\delta^2)n + 2\delta - 1] - (n+\delta-\delta n)^2}.$$

We are interested in the sign of this expression. We evaluate it in the neighborhood of $\underline{\delta}$, which is the value of $\underline{\delta}$ that sets $F(\underline{\delta}, b) = 0$. We substitute in the expression for $(\frac{2}{n})^{\frac{1}{b}} = \frac{1}{1-\delta}[\gamma - \frac{\delta}{n+1}]$. This yields

$$\frac{\partial}{\partial b}G(b) = -\frac{(1-\delta)\left[\frac{n(n+1-\delta n)+\delta}{(n+1)}\left[n(n+1-\delta n)\left(\frac{1}{n}\right)^{\frac{1}{b}}\ln(2)-(n(n+1-\delta n)+\delta)\left(\frac{\delta}{n}+1\right)\ln\left(\frac{2}{n}\right)\right]\right]}{n\left(\frac{1}{n}\right)^{\frac{1}{b}}[(1-\delta)^2n^2+(1-\delta^2)n+2\delta-1]-(n+\delta-\delta n)^2}.$$

which is a negative expression (given that the denominator is positive). Hence $\underline{\delta}$ is decreasing as $u(\cdot)$ becomes more concave. ■

The intuition for this results is that for more concave utility functions, the compromise becomes more attractive. So legislators do not need to be as patient to want to sustain the compromise.

Proposition 4. *As $b \to \infty$, (and hence the utility function $u(\cdot)$ becomes more concave), $\overline{\delta}$ decreases.*

Proof. See Appendix. ■

Again, as the utility function becomes more concave, the compromise outcome becomes more attractive so legislators need to be even more impatient to want to maintain no compromise. Propositions 3 and 4 are illustrated in figure 8.[17]

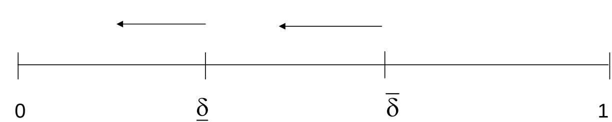

Figure 8: Bounds on δ as $u(\cdot)$ more concave

5 Conclusion

Casual observation indicates that almost all legislative districts in the United States participate in benefits from distributive policies. The theoretical literature on political compromise shows no consensus on whether or not political compromise will be achieved in equilibrium in a general setting [Dixit et al. (2000), Lagunoff (2001), and Kalandrakis (2004)]. We provide a general framework that predicts conditions under which political compromise is achieved, and when compromise is not achieved.

We model legislators with concave utilities that condition strategies only on information that is payoff relevant. We show existence of a set of equilibria that induces

[17]Note that by lemma **??** there always exists a non-degenerate range between $\underline{\delta}$ and $\overline{\delta}$.

a Markov process with two closed classes of proposals: one in which no-compromise is the outcome and the other in which more than a minimum winning majority share in the surplus. We refer to the latter as the compromise outcome.

The question is what determines the outcome in equilibrium. The simple premise is that if a legislator has the opportunity to take the entire surplus for himself he will. Legislators are always therefore somewhat impatient. If the legislator does not have the opportunity to take the entire surplus, he weighs the future cost of a short term strategy that involves taking as much as he can, against the current cost of a long term strategy that involves compromise. If initial allocations are well distributed among non-proposing legislators the proposer will choose a compromise outcome. If on the other hand non-proposing legislators' shares are not well distributed, then the proposing legislator can find enough legislators to buy off cheaply.

We show that this equilibrium holds for an intermediate range of discount factors. This range becomes larger as the number of legislators increases, indicating that compromise is *easier* to sustain with a larger number of legislators, contrary to intuition. In addition, as the utility function becomes more concave, both end-points of the range decrease.

6 Appendix

6.1 Proof of Lemma 1

To prove the map **B** has a fixed point I will employ Kakutani's Fixed Point Theorem. The space of **A** and μ are $[0,1]^{\frac{n(n+1)}{2}}$ and $[0,1]^{(n+1)}$ respectively. These spaces are non-empty, compact and convex. The correspondence **B** is non-empty since $U_j(s^{t-1}; \hat{A}^j, A^{-j}, \hat{\mu}^j, \mu^{-j})$ is continuous and the space of $\hat{A}^j$ and $\hat{\mu}^j$ are compact so a maximizer exists. The map **B** must also be convex valued since $U_j(s^{t-1}; \hat{A}^j, A^{-j}, \hat{\mu}^j, \mu^{-j})$ is linear in $\mu^{j\prime}$, and any $A^{j\prime}$ that maximizes $U_j(s^{t-1}; \hat{A}^j, A^{-j}, \hat{\mu}^j, \mu^{-j})$ must result in the same $\sum_{i \in C_j} A_i^{j\prime}$. By Theorem of the Maximum we can show that **B** is upper hemicontinuous.

For Theorem of the Maximum we already know that $U_j(s^{t-1}; \hat{A}^j, A^{-j}, \hat{\mu}^j, \mu^{-j})$ is continuous, we need only show $\mathbf{M}(\mathbf{P}_j(A^j))$ compact, and $\mathbf{M}(\mathbf{P}_j(A^j))$ continuous. To show $\mathbf{M}(\mathbf{P}_j(A^j))$ continuous and compact note that there are a finite number of elements of $\mathbf{P}_j(A^j)$, specifically, $|\mathbf{P}_j(A^j)| = ^n P_{\frac{n}{2}}$. So $\mathbf{M}(\mathbf{P}_j(A^j))$ is essentially the m-dimensional simplex, where $m = ^n P_{\frac{n}{2}}$. This space is compact and continuous.

6.2 Derivation of $\overline{\delta}$

We would like to ensure that once in the compromise class, the proposer does not have an incentive to deviate to an allocation outside the compromise class, but that would lead to the compromise with certainty. So we are interested in the maximum that the proposer, legislator, j, can allocate to himself under deviation $\tilde{\mathbf{p}}$ while remaining in Γ_i for all i.

All allocations where legislator j does not have the highest share, $\tilde{\mathbf{p}}$, within Γ_j must satisfy

$$u(\tilde{p}_j) \leq \gamma.$$

For $\tilde{\mathbf{p}}$ to be in Γ_i for all i, the payoff to the proposer under $\tilde{\mathbf{p}}$ must not exceed the payoff to a cherry-picking proposal. Letting $\tilde{A}_i^j$ denote the demands of the coalition members given an allocation $\tilde{\mathbf{p}}$ we know that these must satisfy

$$(1-\delta)u\left(1 - \sum_{i \in C_j} \tilde{A}_i^j\right) + \frac{\delta}{n+1} \leq \gamma$$
$$\Rightarrow \sum_{i \in C_j} \tilde{A}_i^j \geq 1 - u^{-1}(\phi).$$

For an optimal deviation these $\tilde{A}_i^j = \tilde{A}$ for all $i \in C_j$. This implies $\tilde{p}_i = \tilde{p}$ for all $i \in C_j$. The maximum deviation for the proposer will put all the acceptors at the lower bound hence for an optimal deviation we will have

$$\tilde{A} = \frac{2}{n}\left[1 - u^{-1}(\phi)\right] \tag{15}$$

We are interested in the deviation allocations, $\tilde{p}_i$, that results in these $\tilde{A}$'s for the coalition members. Assuming all continuation strategies lead to the compromise class, the payoff to $\tilde{\mathbf{p}}$ for a coalition member is given by

$$U_i(\tilde{\mathbf{p}}) = (1-\delta)u(\tilde{p}) + \delta\gamma. \tag{16}$$

Now to calculate their demands under a cherry-picking proposal, this must be set equal to $U(\mathbf{p}(\tilde{A}))$. Given that all coalition members are symmetric, they face equal probability of being in the coalition of a proposer with a zero status quo allocation in the next period. Hence $U(\mathbf{p}(\tilde{A}))$ is given by

$$U(\mathbf{p}(\tilde{A})) = \frac{(1-\delta)(n+1)}{n+1-\delta}u(\tilde{A}) + \frac{\delta}{n+1}. \tag{17}$$

Now $\tilde{A}$ is determined by equality of $U_i(\tilde{\mathbf{p}})$ and $U(\mathbf{p}(\tilde{A}))$ so after some algebra we have

$$\tilde{p} = u^{-1}\left[\tfrac{n+1}{n+1-\delta}u(\tilde{A}) + \tfrac{\delta}{1-\delta}\left(\tfrac{1}{n+1} - \gamma\right)\right].$$

Substituting for $\tilde{A}$ from (15) gives.

$$\tilde{p} = u^{-1}\left[\frac{n+1}{n+1-\delta}u\left(\frac{2}{n}\left[1-u^{-1}(\phi)\right]\right) + \frac{\delta}{1-\delta}\left(\frac{1}{n+1}-\gamma\right)\right].$$

For this deviation to not be profitable, recall that $u(\tilde{p}_j) \leq \gamma$. Since $\tilde{p}_j$ cannot be the largest allocation, it must be smaller than $\frac{1}{2}$, minus the allocations of all other legislators. Hence $\tilde{p}_j \leq \frac{1}{2} - (n-1)\tilde{p}$. Hence $\frac{1}{2} - (n-1)\tilde{p} \leq u^{-1}(\gamma)$ to prevent a deviation, or

$$\frac{1-2u^{-1}(\gamma)}{2(n-1)} \leq \tilde{p}.$$

Combining this with equation (6.2) gives the upper bound on the discount factor.

$$u\left(\frac{1-2u^{-1}(\gamma)}{2(n-1)}\right) \leq \frac{n+1}{n+1-\delta}u\left(\frac{2}{n}\left[1-u^{-1}(\phi)\right]\right) + \frac{\delta}{1-\delta}\left(\frac{1}{n+1}-\gamma\right). \tag{18}$$

As $\delta \to 1$ this inequality becomes $u\left(\frac{1-2u^{-1}\left(\frac{n}{n+1}u\left(\frac{1}{n}\right)\right)}{2(n-1)}\right) \leq -\infty$ which is a contradiction since $1 > u^{-1}(\gamma)$, but as $\delta \to 0$ this inequality becomes $u\left(\frac{n-2}{2n(n-1)}\right) \leq u\left(\frac{2(n-1)}{n^2}\right)$ hence it is satisfied. Given that the expressions on the left hand side and right hand side are both continuous in δ, this implies that there exists some δ for which this expression holds with equality and below which the equality is always satisfied. This gives the upper bound on δ.

6.3 Proof of Lemma 2

Now to show that $\underline{\delta} \leq \overline{\delta}$ we have the following proof.

Proof. First note that the condition for $\underline{\delta}$ gives the lowest possible value for ϕ while still satisfying the lower bound on the discount factor. So at this value of ϕ inequality (13) must be satisfied to guarantee a range between $\underline{\delta}$ and $\overline{\delta}$. Substituting for $\phi = u(\frac{2}{n})$ gives

$$u\left(\frac{1-2u^{-1}\left[(1-\delta)u(\frac{2}{n})+\frac{\delta}{n+1}\right]}{2(n-1)}\right) \leq \frac{n+1}{n+1-\delta}u\left(\frac{2}{n}\left[1-\frac{2}{n}\right]\right) - \delta\left(u\left(\frac{2}{n}\right) - \frac{1}{n+1}\right). \tag{19}$$

Numerical simulations show that this inequality holds for large enough values of n.

■

6.4 Complete Incentives

A complete incentives analysis ensures that for each set indicated in figure 4 there is no incentive to transition to any other set than what the equilibrium strategies dictate. So we proceed by considering status quo allocations in each set.

6.4.1 $s^{t-1} \in \Delta_{\frac{n}{2}<\theta\leq n}$

Consider a status-quo $s^{t-1} \in \Delta_{\frac{n}{2}<\theta\leq n}$. The equilibrium strategies dictate a no-compromise proposal $\underline{\mathbf{p}}$ such that the proposer, x^t, receives payoff $\underline{V}_x$ from equation (3), and all other legislators receive payoffs $\underline{V}_z$ from equation (4). These are

$$\underline{V}_x = \tfrac{n+1-\delta n}{n+1}$$

and

$$\underline{V}_z = \tfrac{\delta}{n+1}.$$

Consider the incentives of legislators to accept the equilibrium proposal. Under the status quo at least $\frac{n}{2}$ non-proposing legislators are receiving payoff $\underline{V}_z$ hence there at least $\frac{n}{2}$ legislators who will accept a payoff of $\underline{V}_z$, hence achieving a majority of votes.

Now consider incentives of the proposer to propose this allocation rather than any other deviation allocation. Consider deviation allocations in the following sets.

Deviation into $\overline{\Delta}_1$

Consider a deviation such that the proposer proposes a compromise allocation $\overline{\mathbf{p}}^i$ for some i. In the compromise class the best available dynamic payoff is γ given by (5). We check that $\gamma \leq \underline{V}_x$. We have

$$\begin{aligned}\frac{n(n+1-\delta n)}{(n+1)(n+\delta-\delta n)} u\left(\frac{1}{n}\right) &\leq \frac{n+1-\delta n}{n+1}, \\ &\Leftrightarrow \delta \leq \frac{1-u(\frac{1}{n})}{1-\frac{1}{n}}.\end{aligned}$$

Let $\overline{\delta}_1 = \frac{1-u\left(\frac{1}{n}\right)}{1-\frac{1}{n}}$. To show that this is not profitable deviation it suffices to show that $\overline{\delta} \leq \overline{\delta}_1$ where defined by condition (13). Plugging δ_1 into (13) we obtain:

$$\begin{aligned}LHS &= u\left(\frac{1-2u^{-1}\left(\frac{n^2u\left(\frac{1}{n}\right)-1}{n^2-1}\right)}{2(n-1)}\right) \\ RHS &= -\frac{n^2(1-u\left(\frac{1}{n}\right))}{n^2-1}\end{aligned}$$

Since the RHS of this inequality is negative and the LHS is positive, the condition is violated meaning that $\overline{\delta} < \delta_1$. So $\overline{\delta}$ implies that $\gamma \leq \underline{V}_x$.

Deviation into $\Delta_{\frac{n}{2}<\theta<n}$

Consider a deviation to an arbitrary allocation in $\Delta_{\frac{n}{2}<\theta<n}$, once in this set equilibrium dynamics dictate proceeding to the no compromise set Δ_n. Hence the continuation values of the deviation into $\Delta_{\frac{n}{2}<\theta<n}$ and playing the equilibrium outlined for Δ_n are identical. It thus suffices to compare the stage allocations from the deviation to the equilibrium allocation where the proposer, legislator j, receives $1-\sum_{i\neq j} s_i^t$. Clearly

$$1-\sum_{i\neq j} s_i^t < 1,$$

so this is not a profitable deviation.

Deviation into $\Delta_{\theta=\frac{n}{2}}$

Consider a deviation to an allocation in $\Delta_{\theta=\frac{n}{2}}$, once in this set equilibrium dynamics dictate proceeding to the no-compromise set, Δ_n, or if the proposer, x^{t+1}, possesses a zero status quo allocation then proceed to $\Delta_{\frac{n}{2}<\theta<n}$. Under the deviation proposal the proposer receives

$$(1-\delta)u\left(1-\sum_{i\neq j} s_i^t\right)+\delta V_j(s^t). \tag{20}$$

Substituting for the different values of $V_j(\Delta_{\frac{n}{2}}; s_{x^t}^t = 0)$ in equation (7) we obtain the following. If $V_j(\Delta_{\frac{n}{2}}; s_{x^t}^t = 0) = \frac{\delta}{n+1}$, we have that $V_j(s^t) = \frac{1}{n+1}$. As shown above, the continuation value is the same for the proposer but the stage payoff is lower hence this is not a profitable deviation. Now consider if $V_i(\Delta_{\frac{n}{2}}; s_{x^t}^t = 0) = (1-\delta)u\left(A(b^*)\right)+\frac{\delta}{n+1}$, so that he ensures he is in the coalition in the next period. The subsequent proposal strategies are such that the probability assigned to him being in the coalition ensures that his dynamic payoff is exactly equal to $(1-\delta)u\left(A(b^*)\right)+\frac{\delta}{n+1}$, hence this cannot be a profitable deviation. Last, for the case $V_j(\Delta_{\frac{n}{2}}; s_{x^t}^{t-1} = 0) = (1-\delta)u\left(1-A(b^*)\right)+\frac{\delta}{n+1}$. For this to be the continuation from the deviation, it must be that the proposer allocates a zero share to himself. Substituting this into (20), we show that the deviation payoff is equal to

$$\frac{\delta}{n+1}(1-\delta)u(1-A(b^*))+\frac{\delta}{n+1}.$$

This is clearly less than $\underline{V}_x$ since $\frac{\delta}{n+1}u\left(1-A(b^*)\right) < 1$.

Deviation into $\Delta^c_{\theta<\frac{n}{2}}$

Consider a deviation to an allocation in $\Delta^c_{\theta<\frac{n}{2}}$. The payoff from this deviation is given by equation (9). This is

$$U_j(s^t; \mathbf{A}^*, \mu^*) = (1-\delta)u(s_j^t)+\delta V_j(\mathbf{A}^*, \mu^*).$$

Let us consider the continuation payoff $V_j(\mathbf{A}^*, \mu^*)$. Notice that if legislator j is included in any coalition in the next period, equilibrium proposal strategies, μ^{*i}, are such that his payoff from the allocation s^t are equal to or less than his payoff from the next period's cherry-picking allocation, hence

$$U_j(s^t; \mathbf{A}^*, \mu^*) \leq (1-\delta)u(A^{*i}) + \delta V(A^{*i}).$$

This payoff we just showed is always lower than $\underline{V}_x$, hence this is not a profitable deviation.

Now consider if he is not included in a coalition in the next period. As shown below in section 6.4.4 this continuation payoff is strictly less than $\frac{1}{n+1}$, and since $1 - \sum_{i \neq j} s_j^t < 1$ this deviation is also not profitable.

Deviation into Γ

A deviation into the set Γ may imply a deviation into any arbitrary intersection of Γ_i. Note that if it is any intersection that includes Γ_j, by $\overline{\delta}$ the status quo payoff to being in Γ_j is never greater than γ, and we know again by $\overline{\delta}$ that $\gamma < \underline{V}_x$ hence this is not a profitable deviation. Now consider a deviation into some arbitrary intersection that does not include Γ_j. If the deviation is such that legislator j is included in any other legislator's coalition, by the same arguments above the payoff from the deviation is $U_j(s^t; \mathbf{A}^*, \mu^*) = (1-\delta)u(A^{*i}) + \delta V(A^{*i})$, which we showed is always lower than $\underline{V}_x$.

So we must consider when the deviation allocation is not in Γ_j, and the legislator is not included in anyone else's coalition. This implies a continuation payoff of

$$V_i(\mathbf{A}, \mu) = \frac{1}{n+1} \sum_{j=1}^{n+1} \left[\sum_{\mathbf{p}(A^j) \in \mathbf{P}(A^j)} U_i(\mathbf{p}(A^j)) \mu^j(\mathbf{p}(A^j)) + \gamma \sum_{h \in I/\{i\}} \mu^j(\overline{\mathbf{p}}^h) + \zeta \mu^j(\overline{\mathbf{p}}^i) \right].$$

6.4.2 $s^{t-1} \in \overline{\Delta}_1$

Consider a status-quo $s^{t-1} \in \overline{\Delta}_1$. The equilibrium strategies dictate a compromise proposal $\overline{\mathbf{p}}^i$ such that all legislators except legislator i receives payoff γ and legislator i receives payoff ξ. Consider the incentives of legislators to accept. Under the status quo at least $n-1$ non-proposing legislators are receiving payoff γ hence there at least $n-1$ legislators who will accept a payoff of γ, thereby achieving a majority of votes.

Now consider incentives of the proposer to propose this allocation rather than any other deviation allocation. Consider deviation allocations in the following sets.

Deviation into Δ_n

As shown above the payoff from proposing $\underline{\mathbf{p}}$ compared to $\overline{\mathbf{p}}$ would represent a profitable deviation for the proposer $\overline{\Delta}_1$. So we need to check the incentives for coalition members to accept such a proposal. The payoff for coalition members from the deviation would be $\underline{V}_z$ whereas the payoff from the status quo is γ. Clearly

$$\underline{V}_z \leq \gamma$$
$$\Leftrightarrow \delta(n + \delta - \delta n) \leq nu\left(\frac{1}{n}\right)(n + 1 - \delta n)$$

Hence the proposer will not be able to form a coalition that accepts such a proposal, and the deviation is not possible.

Deviation into Γ

First, as shown in section 3.5 in condition (13), $\overline{\delta}$ prevents any deviation from the compromise class into $\bigcap_{j=1}^{n+1} \Gamma_j$. Hence this deviation is not possible by construction.

Second, we prove that a deviation from $\overline{\Delta}_1$ into Γ/Γ^j is not profitable. A deviation into Γ entails a continuation payoff that is a convex combination of a deviation into $\bigcap_{j=1}^{n+1} \Gamma_j$ and a deviation into $\Delta^c_{\theta<\frac{n}{2}}$, both of which we have shown to be non-viable deviations. It follows that a deviation into Γ/Γ^j is not a profitable deviation.

Deviation into $\Delta_{\frac{n}{2}<\theta<n}$

Consider a deviation into $\Delta_{\frac{n}{2}<\theta<n}$. Legislators that receive zero under this allocation receive a dynamic payoff of $\underline{V}_z$, which we have already shown is less than γ. However, the legislator receiving zero under the compromise proposal has a dynamic payoff of ξ which is lower than $\underline{V}_z$ since

$$\xi \leq \underline{V}_z$$
$$\Leftrightarrow \delta \leq \frac{1 - u(\frac{1}{n})}{1 - \frac{1}{n}}$$

We know this holds by $\overline{\delta}$. So the minimum number of players to give a positive share under the deviation is $\frac{n}{2} - 1$. For this deviation to profitable we must have

$$u\left(1 - \sum_{i \neq j} s_j^t\right) + \frac{\delta}{n+1} \geq \gamma$$

We have already shown in section 3.5 that by $\underline{\delta}$ this is not possible.

Deviation into $\Delta_{\frac{n}{2}} \cup \Delta^c_{\theta<\frac{n}{2}}$

In section 3.5, $\underline{\delta}$ implied by condition 12 ensures that a deviation from the compromise class to an allocation with cherry picking in $\Delta_{\frac{n}{2}<\theta<n}$ is not possible. Attempting to deviate into $\Delta_{\frac{n}{2}}$, requires compensating an extra player over and above the $\frac{n}{2} - 1$ in deviating into $\Delta_{\frac{n}{2}<\theta<n}$. If compensating $\frac{n}{2} - 1$ players is not profitable for any proposer, then it follows that compensating $\frac{n}{2}$ players will be even less profitable, hence this deviation is also ruled out.

6.4.3 $s^{t-1} \in \Delta_{\frac{n}{2}}$

When the status quo is in $\Delta_{\frac{n}{2}}$ equilibrium strategies specify either a no compromise proposal if the proposer has a positive status-quo allocation, or a cherry-picking proposal, $\mathbf{p}^j(A(b^*))$, if the proposer has a status quo allocation of zero. For $s_{x^t}^{t-1} > 0$, where the the equilibrium proposal made is in Δ_n, the incentives analysis is identical to status quo allocations in $\Delta_{\frac{n}{2}<\theta\leq n}$. Below we will show that for $s_{x^t}^{t-1} = 0$, there are no profitable deviations.

Let us first check the incentives of the accepting legislators. The payoff to the legislator receiving zero allocation is $\frac{\delta}{n+1}$, and the payoff under the status quo allocation is $\delta V_i(A^{*j})$, where

$$\delta V_i(A^{*j}) = \frac{\delta}{n+1}\left[\delta + (1-\delta)u(1-A(b^*))\right].$$

Simplifying shows that $\frac{\delta}{n+1} > \delta V_i(A^{*j})$ since $1 > u(1-A(b^*))$. The legislator that is offered a positive status quo allocation is offered $A(b^*)$ with probability μ_i, where $A(b^*)$ and μ_i are calculated to make the accepting legislator exactly indifferent to the status quo.

Now we consider possible deviation proposals by the proposer. Under the equilibrium strategies the proposer receives the payoff $(1-\delta)u(1-A(b^*)) + \frac{\delta}{n+1}$. For deviations in $\Delta_{\frac{n}{2}\leq\theta\leq n}$, we rely on the proof in Kalandrakis (2007) but restate his concavity restrictions.

Deviation into $\overline{\Delta}_1$

Consider a deviation where the proposer x^t proposes a compromise allocation, $\overline{\mathbf{p}}^i$. From equation (5) we have the payoffs from such a deviation: $\gamma = \frac{n(n+1-\delta n)}{(n+1)(n+\delta-\delta n)} u\left(\frac{1}{n}\right)$ whereas the payoff of following equilibrium strategies for the proposer are given by equation (7) where the proposer assigns $1-A(b^*)$ to himself. This requires a restriction on the discount factor which is satisfied by $\overline{\delta}$.

Deviation into $\Delta^c_{\theta<\frac{n}{2}}$

This deviation involves allocating more than $\frac{n}{2}$ players a positive share in the proposal. Equilibrium strategies after the deviation dictates an allocation in $\Delta_{\frac{n}{2}}$. Similar to the cases above the proposer will have to allocate a positive share to more than one player that possesses a positive share in the status quo, and some $\tilde{s}_j = \epsilon_j > 0$ to the other $\frac{n}{2}-1$ players with a zero status quo allocation. This lowers the stage payoff to proposer while not improving the continuation payoff, hence this is not a profitable deviation.

6.4.4 $s^{t-1} \in \Delta^c_{\theta<\frac{n}{2}}$

Consider incentives when the status quo is an element of $\Delta^c_{\theta<\frac{n}{2}}$ and legislator j is the proposer. From these allocations legislator j makes a cherry-picking proposal such that coalition members $i \in C_j$ are offered the demands A_i^{j*}, where A^{j*} and μ^{j*} solve

$$(A^{j*}, \hat{\mu}^{j*}) \quad \in \arg\max_{\hat{A}^j, \hat{\mu}^j} U_j(s^{t-1}; \hat{A}^j, A^{-j*}, \hat{\mu}^j, \mu^{-j*})$$

$$\text{s.t.} \quad \hat{A}^j = \hat{A}^j(s^{t-1}; \mathbf{A}^*, \hat{\mu}^j, \mu^{-j*})$$
$$\hat{\mu}^j \in \mathbf{M}(\mathbf{P}(A^{j*}))$$

$$\text{and} \quad \mu^{j*}(\mathbf{p}(A^{j*})) = \hat{\mu}^j(\mathbf{p}(A^{j*}))\}.$$

So

$$A_i^{j*} = u^{-1}\left[\min\{\max\{0, \frac{1}{1-\delta}\left(U_i(s^{t-1}; \mathbf{A}^*, \mu^*) - \delta V_i(A^{j*})\right)\}, 1\}\right].$$

Let us first check the coalition member's incentives. We must verify that coalition members will prefer this allocation to the status quo. Note that if $A_i^{j*} < 1$, then $U_i(s^{t-1}; \mathbf{A}^*, \mu^*) \leq U_i(\mathbf{p}(A^{*j}))$. We know that if the proposer did not choose the compromise allocation then $\sum_{i \in C_j} A_i^{j*} < 1 - u^{-1}(\phi) < 1$ hence each coalition member's demand must also be less than 1 hence their incentives hold.

Now let us check the proposer's incentives. Given that this solves the maximization problem for the proposer we know the proposer would prefer this to any other cherry-picking allocation that would be accepted, or any compromise allocation. We must also verify that the proposer would prefer this to remaining at the status quo allocation. Under the status quo the proposer has the payoff

$$U_j(s^{t-1}; \mathbf{A}^*, \mu^*) = (1-\delta)u(s_j^{t-1}) + \delta V_j(\mathbf{A}^*, \mu^*).$$

Under the cherry-picking proposal, he has payoff

$$U_j(\mathbf{p}(A^{*j})) = (1-\delta)u\left(1 - \sum_{i \in C_j} A_i^{*j}\right) + \delta V_j(A^{*j}).$$

We will prove that $U_j(s^{t-1}; \mathbf{A}^*, \mu^*) \leq U_j(\mathbf{p}(A^{*j}))$ by first showing that $V_j(\mathbf{A}^*, \mu^*) \leq V_j(A^{*j})$ and then showing that $u(s_j^{t-1}) \leq u\left(1 - \sum_{i \in C_j} A_i^{*j}\right)$. We will first consider the lower bound on $V_j(A^{*j})$. This will be where legislator j is not included in the coalition when the status quo is in $\Delta_{\frac{n}{2}}$ and $s_{x^t}^{t-1} = 0$, so

$$V_j(A^{*j}) = \frac{1}{n+1}\left(\frac{n+1-\delta n}{n+1} + \frac{n}{2}\frac{\delta}{n+1} + \frac{n}{2}\frac{\delta}{n+1}\right) = \frac{1}{n+1}$$

Now let us consider the upper bound on $V_j(\mathbf{A}^*, \mu^*)$. Here $V_j(\mathbf{A}^*, \mu^*)$ is given by

$$V_j(\mathbf{A}^*, \mu^*) = \frac{1}{n+1}\left[U_j(\mathbf{p}(A^{*j})) + \sum_{i \neq j}\left\{\sum_{\mathbf{p}(A^{i*}) \in \mathbf{P}(A^{i*})} U_j(\mathbf{p}(A^{i*}))\mu^{i*}(\mathbf{p}(A^{i*}))\right\}\right]. \quad (21)$$

When legislator j is not the proposer he is in another legislator's coalition if $U_j(s^{t-1};\mathbf{A}^*,\mu^*)$ is sufficiently low. So at a maximum value of $U_j(s^{t-1};\mathbf{A}^*,\mu^*)$ legislator j is not included in the other legislator's coalition so $U_j(\mathbf{p}(A^{i*}))=\delta V_j(A^{i*})$ for all $i\neq j$. Substituting this into (21) gives

$$V_j(\mathbf{A}^*,\mu^*)=\frac{1}{n+1}\left[(1-\delta)u\left(1-\sum_{i\in C_j}A_i^{*j}\right)+\delta V_j(A^{*j})+\sum_{i\neq j}\delta V_j(A^{i*})\right]. \tag{22}$$

We showed above that $V_j(A^{j*})=\frac{1}{n+1}$. When any other legislator $i\neq j$ proposes in period t, $V_j(A^{i*})$ is given by

$$V_j(A^{i*})=\frac{1}{n+1}\left(\left(\frac{n}{2}+1\right)\underline{V}_z+\frac{n}{2}V_j(\Delta_{\frac{n}{2}};s^t_{x^{t+1}}=0)\right). \tag{23}$$

Here $V_j(\Delta_{\frac{n}{2}};s^t_{x^{t+1}}=0)=(1-\delta)u(1-A(b^*))+\frac{\delta}{n+1}$ when legislator j is the proposer in period $t+1$ and is $\frac{\delta}{n+1}$ when all other legislators propose in period $t+1$. Substituting into (23) and simplifying gives

$$V_j(A^{i*})=\frac{(1-\delta)}{n+1}u(1-A(b^*))+\frac{\delta}{n+1}. \tag{24}$$

Substituting these values into (21) gives

$$\begin{aligned}V_j(\mathbf{A}^*,\mu^*)&=\frac{1}{n+1}\left[(1-\delta)u\left(1-\sum_{i\in C_j}A_i^{*j}\right)+\delta\left(\frac{1}{n+1}+\sum_{i\neq j}\left\{\frac{(1-\delta)}{n+1}u(1-A(b^*))+\frac{\delta}{n+1}\right\}\right)\right]\\&=\frac{1}{n+1}\left[(1-\delta)u\left(1-\sum_{i\in C_j}A_i^{*j}\right)+\delta\left(\frac{1+\delta n}{n+1}+\frac{1-\delta}{n+1}\sum_{i\neq j}u(1-A(b^*))\right)\right].\end{aligned}$$

Notice that the value in square brackets is the convex combination of two values that are less than 1, hence the value in brackets is less than 1 making $V_j(\mathbf{A}^*,\mu^*)\leq\frac{1}{n+1}=V_j(A^{*j})$.

Now we must prove $u(s_j^{t-1})\leq u\left(1-\sum_{i\in C_j}A_i^{*j}\right)$, or $s_j^{t-1}\leq 1-\sum_{i\in C_j}A_i^{*j}$. First observe that if s_i^{t-1} are equal for all $i\in C_j$, this implies the minimum value of $\sum_{i\in C_j}A^{*j}$ for any given s_j^{t-1}. Now observe that $s_j^{t-1}\leq 1-ns_i^{t-1}$ since all other legislators must have at least as high a status quo allocation as the coalition members if we assume the legislators with the lowest status quo allocations are always in the coalition. To express $\sum_{i\in C_j}A_i^{*j}$ in terms of s_i^{t-1} we must calculate the value of A_i^{*j} as a function of s_i^{t-1}. Recall that

$$A_i^{j*}=u^{-1}\left[\min\{\max\{0,\frac{1}{1-\delta}\left(U_i(s^{t-1};\mathbf{A}^*,\mu^*)-\delta V_i(A^{j*})\right)\},1\}\right]. \tag{25}$$

Here $U_i(s^{t-1};\mathbf{A}^*,\mu^*)=(1-\delta)u(s_i^{t-1})+\delta V_i(\mathbf{A}^*,\mu^*)$. We wish to find the maximum value of $U_i(s^{t-1};\mathbf{A}^*,\mu^*)$, let's call this value $\overline{U}_i$. This is where legislator i is included in n other legislator's coalition in period t, and recall that if he is included in the coalition then he receives at least $\overline{U}_i$. So

$$\overline{U}_i = (1-\delta)u(s_i^{t-1}) + \frac{\delta}{n+1}\left[U_i(\mathbf{p}(A^{*i})) + n\overline{U}_i\right].$$

The maximum value $U_i(\mathbf{p}(A^{*i}))$ can take is $\underline{V}_x$. Substituting in 26 and simplifying gives

$$\overline{U}_i = \frac{(1-\delta)(n+1)}{n+1-\delta n}u(s_i^{t-1}) + \frac{\delta}{n+1}. \tag{26}$$

Now (25) is equivalent to setting $\overline{U}_i = U_i(\mathbf{p}(A^{*j}))$ and solving for A_i^{*j} when $A_i^{*j} > 0$. So we conjecture that $A_i^{*j} > 0$ and find the minimum value of $U_i(\mathbf{p}(A^{*j}))$. Let's call this minimum value $\underline{U}_i$. Given that we are considering coalition members with equal status quos and hence equal values of A_i^{*j}, they will face equal probability, $\frac{1}{n+1}$, of being in the coalition of the period $t+1$ legislator if his status quo allocation was zero. So

$$\underline{U}_i = (1-\delta)u(A_i^{*j}) + \frac{\delta}{n+1}\left[\underline{V}_x + (n-1)\underline{V}_z + \underline{U}_i\right].$$

Substituting and simplifying gives

$$\underline{U}_i = \frac{(1-\delta)(n+1)}{n+1-\delta}u(A_i^{*j}) + \frac{\delta}{n+1}.$$

Setting $\overline{U}_i = \underline{U}_i$ yields

$$A_i^{*j} = u^{-1}\left[\frac{n+1-\delta}{n+1-\delta n}u(s_i^{t-1})\right]. \tag{27}$$

Notice that $s_i^{t-1} > 0$ by assumption, hence the conjecture that $A_i^{*j} > 0$ is proved true. We wish to have $1 - ns_i^{t-1} \leq 1 - \frac{n}{2}A_i^{*j}$. Substituting from (27) for A_i^{*j} and simplifying gives

$$\frac{n+1-\delta}{n+1-\delta n}u(s_i^{t-1}) \leq u(2s_i^{t-1}).$$

This represents a restriction on the concavity of the function.

6.4.5 $s^{t-1} \in \Gamma$

Let us first consider s^{t-1} in $\bigcap_{j=1}^{n+1}\Gamma_j$, and the incentives of the proposing legislator. Since $\mu^{*j}(\overline{\mathbf{p}}^i) = 1$ is the solution to the fixed point of the map $\mathbf{B}$ we know that $\overline{\mathbf{p}}^i$ is optimal for the proposer among cherry-picking strategies and among compromise proposals. We also know by the upper bound on the discount factor, $\overline{\delta}$, that the proposer will prefer the compromise to remaining at the the status quo allocation.

Now let us consider the incentives of the accepting legislators. Since the status quo is an element of $\bigcap_{j=1}^{n+1}\Gamma_j$ and all legislators are symmetric, we know by the upper bound on the discount factor, $\overline{\delta}$, that the highest status quo payoff available to any legislator satisfies $(1-\delta)u(s^{t-1}) + \delta\gamma \leq \gamma$. Hence no legislator has an incentive to deviate from accepting the compromise allocation.

Now consider $s^{t-1} \in \Gamma$ where $0 < \mu^{*j}(\overline{\mathbf{p}}^i) < 1$ for some proposing legislator j and some i. Consider the proposing legislator's incentives. Again, we know since $\mu^{*j}(\overline{\mathbf{p}}^i)$ is the solution to the fixed point of the map $\mathbf{B}$ the proposer is indifferent between the compromise proposal $\overline{p}^i$ and some cherry-picking strategy, so the payoff to legislator j when he is the proposer is γ regardless, and this is optimal among cherry-picking proposals and compromise proposals. We must check that it is preferred to the status quo. If legislator j is not the proposer, other legislators will choose either a cherry-picking strategy or a compromise proposal. Under another legislator's cherry-picking strategy, his payoff is highest when he is included in the coalition, and then he is given at most his status quo payoff. Let us call this status quo payoff, U_j, and let's say with probability μ he receives this payoff, and with probability $1 - \mu$ he receives γ under a compromise. His status quo payoff can therefore be written down as

$$U_j = (1 - \delta)u(s_i^{t-1}) + \delta\left[(1 - \mu)\gamma + \mu U_j\right]$$
$$\Leftrightarrow U_j = \frac{1 - \delta}{1 - \delta\mu}u(s_i^{t-1}) + \frac{(1 - \delta)\mu}{1 - \delta\mu}\gamma.$$

To show that the proposer has no incentive to deviate from the compromise proposal to the status-quo, we must show that $\gamma \geq U_j$. This is true, by the upper bound on the discount factor. Since we know that they highest status quo payoff to any proposer j when the status quo is in Γ_j satisfies $u(s_i^{t-1}) \leq \gamma$.

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