Dynamics and Stability of Constitutions, Coalitions, and Clubs

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Motivation

- What makes certain social arrangements (political regimes, constitutions, coalitions, clubs, firms, international peace) emerge in equilibrium and persist?
- Fundamental question for political economy and organizational economics, and pertinent to the study of coalition formation.
- Each specific applied instance leads to different sets of issues.
- Are there general insights?
- How can we apply these results to important political economy problems?
- *This talk:* general framework and applications.

Toward General Insights

- This research project attempts to investigate such general insights.
- Many similarities between different instances (in political economy, organizational economics, club theory etc.).
- In particular:
  - **Payoffs:** different arrangements imply different payoffs and individuals care about payoffs.
  - **Power:** different arrangements reallocate decision-making (political) power and thus affect future evolution of payoffs.
- *Strategy:* Formulate a general dynamic framework to investigate the interplay of these two factors in a relatively "detail-free" manner.
  - Details useful to go beyond general insights.

Simple Example

- Consider a simple extension of franchise story
- Three states: absolutism a, constitutional monarchy c, full democracy d
- Two agents: elite E, middle class M

$$
\begin{aligned}
w_E(d) &< w_E(a) < w_E(c) \\
w_M(a) &< w_M(c) < w_M(d)
\end{aligned}
$$

- E rules in a, M rules in c and d.
- Myopic elite: starting from a, move to c
- Farsighted elite: stay in a: move to c will lead to M moving to d

Naïve and Dynamic Insights

- *Naïve insight:* a social arrangement will emerge and persist if a "sufficiently powerful group" prefers it to alternatives.
- Simple example illustrates: power to change towards a more preferred outcome is not enough to implement change
  - because of further dynamics
- Social arrangements might be stable even if there are powerful groups that prefer change in the short run.
- **Key:** social arrangements change the distribution of political power (decision-making capacity).
- **Dynamic decision-making:** future changes also matter (especially if discounting is limited)

Other Examples

- Members of a club decide whether to admit additional members by majority voting (Roberts 1999)
- Society decides by voting, what degree of (super)majority is needed to start a reform (Barbera and Jackson 2005)
- EU members decide whether to admit new countries to the union (Alesina, Angeloni, and Etro 2005)
- Inhabitants of a jurisdiction determine migration policy (Jehiel and Scotchmer 2005)
- Extension of franchise (Acemoglu and Robinson 2000, 2006, Lizzeri and Persico 2004)
- Participant of (civil) war decides whether to make concessions to another party (Fearon 1998, Schwarz and Sonin 2008)
- Dynamic political coalition formation: Junta (or Politburo) members decide whether to eliminate some of them politically or physically (Acemoglu, Egorov, and Sonin 2008)

Voting in Clubs or Dynamic Franchise Extension

- Suppose that individuals $\{1, ..., M\}$ have a vote and they can extend the franchise and include any subset of individuals $\{M+1, ..., N\}$.
- Instantaneous payoff of individual i a function of the set of individuals with the vote (because this influences economic actions, redistribution, or other policies)
- Political protocol: majority voting.
- $\{1, ..., M\}$ vote over alternative proposals.
- If next period the franchise is $\{1, ..., M'\}$, then this new franchise votes (by majority rule) on the following period's franchise etc.
- Difficult dynamic game to analyze.
- But once we understand the common element between this game and a more general class of games, a tight and insightful characterization becomes possible.

Model and Approach

- Model:
  - Finite number of individuals.
  - Finite number of states (characterized by economic relations and political regimes)
  - Payoff functions determine instantaneous utility of each individual as a function of state
  - Political rules determine the distribution of political power and protocols for decision-making within each state.
  - A dynamic game where "politically powerful groups" can induce a transition from one state to another at any date.
- Question: what is the **dynamically stable state** as a function of the initial state?

Main Results of General Framework

- An axiomatic characterization of "outcome mappings" corresponding to dynamic game (based on a simple *stability* axiom incorporating the notion of forward-looking decisions).
- Equivalence between the MPE of the dynamic game (with high discount factor) and the axiomatic characterization
- Full characterization: *recursive* and *simple*
- Under slightly stronger conditions, the stable outcome (dynamically stable state) is unique given the initial state
  - but depends on the initial state
- Model general enough to nest specific examples in the literature.
- In particular, main theorems directly applicable to situations in which states can be ordered and static payoffs satisfy single crossing or single peakedness.

Simple Implications

- A particular social arrangement is made stable by the instability of alternative arrangements that are preferred by sufficiently many members of the society.
    - stability of a constitution does not require absence of powerful groups opposing it, but the absence of an alternative stable constitution favored by powerful groups.
- Efficiency-enhancing changes are often resisted because of further social changes that they will engender.
    - Pareto inefficient social arrangements often emerge as stable outcomes.

Applications

- Voting in clubs.
- Dynamic taxation with endogenous franchise.
- Stability of constitutions.
- Political eliminations.
- Froma follow-up paper: dynamics of political selection
  - a small amount of incumbency advantage can lead to the emergence and persistence of very incompetent/inefficient governments (without asymmetric information)
  - a greater degree of democracy does not necessarily ensure better governments
  - **but**, a greater degree of democracy leads to greater **flexibility** and to better governments in the long run in stochastic environments.

Related Literature

- Papers mentioned above as applications or specific instances of the general results here.
- Dynamic coalition formation—Ray (2008).
- Dynamic political reform—Lagunoff (2006).
- Farsighted coalitional stability—Chwe (1994).
- Dynamic economic interactions with transferable utility—Gomes and Jehiel (2005).
- Dynamic inefficiencies with citizen candidates—Besley and Coate (1999).

Model: Basics

- Finite set of individuals $\mathcal{I}$ ($|\mathcal{I}|$ total)
  - Set of coalitions $\mathcal{C}$ (non-empty subsets $X \subset \mathcal{I}$)
- Each individual maximizes discounted sum of playoffs with discount factor $\beta \in [0, 1)$.
- Finite set of states $\mathcal{S}$ ($|\mathcal{S}|$ total)
- Discrete time $t \geq 1$
- State s_t is determined in period t; s_0 is given
- Each state $s \in \mathcal{S}$ is characterized by
  - Payoff $w_i(s)$ of individual $i \in \mathcal{I}$ (normalize $w_i(s) > 0$)
  - Set of winning coalitions $\mathcal{W}_s \subset \mathcal{C}$ capable of implementing a change
  - Protocol $\pi_s(k)$, $1 \leq k \leq K_s$: sequence of agenda-setters or proposals ($\pi_s(k) \in \mathcal{I} \cup \mathcal{S}$)

Winning Coalitions

Assumption

(Winning Coalitions) *For any state* $s \in \mathcal{S}$, $\mathcal{W}_s \subset \mathcal{C}$ *satisfies two properties:*
(a) If $X, Y \in \mathcal{C}$, $X \subset Y$, *and* $X \in \mathcal{W}_s$ *then* $Y \in \mathcal{W}_s$.
(b) If $X, Y \in \mathcal{W}_s$, *then* $X \cap Y \neq \varnothing$.

- (a) says that a superset of a winning coalition is winning in each state
- (b) says that there are no two disjoint winning coalitions in any state
- $\mathcal{W}_s = \varnothing$ is allowed (exogenously stable state)
- Example:
  - Three players $1, 2, 3$
  - $\mathcal{W}_s = \{\{1\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}\}$ is valid (1 is dictator)
  - $\mathcal{W}_s = \{\{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$ is valid (majority voting)
  - $\mathcal{W}_s = \{\{1\}, \{2, 3\}\}$ is not valid (both properties are violated)

Dynamic Game

1. Period t begins with state s_{t-1} from the previous period.
2. For $k = 1, \ldots, K_{s_{t-1}}$, the kth proposal $P_{k,t}$ is determined as follows. If $\pi_{s_{t-1}}(k) \in \mathcal{S}$, then $P_{k,t} = \pi_{s_{t-1}}(k)$. If $\pi_{s_{t-1}}(k) \in \mathcal{I}$, then player $\pi_{s_{t-1}}(k)$ chooses $P_{k,t} \in \mathcal{S}$.
3. If $P_{k,t} \neq s_{t-1}$, each player votes (sequentially) *yes* (for $P_{k,t}$) or *no* (for s_{t-1}). Let $Y_{k,t}$ denote the set of players who voted *yes*. If $Y_{k,t} \in \mathcal{W}_{t-1}$, then $P_{k,t}$ is accepted, otherwise it is rejected.
4. If $P_{k,t}$ is accepted, then $s_t = P_{k,t}$. If $P_{k,t}$ is rejected, then the game moves to step 2 with $k \mapsto k+1$ if $k < K_{s_{t-1}}$. If $k = K_{s_{t-1}}$, $s_t = s_{t-1}$.
5. At the end of each period (once s_t is determined), each player receives instantaneous utility $u_i(t)$:

$$u_i(t) = \begin{cases} w_i(s) & \text{if } s_t = s_{t-1} = s \\ 0 & \text{if } s_t \neq s_{t-1} \end{cases}$$

Key Notation and Concepts

- Define binary relations:
  - states x and y are payoff-equivalent

$$x \sim y \iff \forall i \in \mathcal{I} : w_x(i) = w_y(i)$$

  - y is weakly preferred to x in z

$$y \succeq_z x \iff \{i \in \mathcal{I} : w_y(i) \geq w_x(i)\} \in \mathcal{W}_z$$

  - y is strictly preferred to x in z

$$y \succ_z x \iff \{i \in \mathcal{I} : w_y(i) > w_x(i)\} \in \mathcal{W}_z$$

- Notice that these binary relations are **not** simply preference relations
  - *they encode information about preferences and political power.*

Preferences and Acyclicity

Assumption

(Payoffs) *Payoff functions* $\{w_i(\cdot)\}_{i\in\mathcal{I}}$ *satisfy:*
(a) For any sequence of states $s_1, \ldots, s_k$ *in* $\mathcal{S}$*,*

$$s_{j+1} \succ_{s_j} s_j \text{ for all } 1 \leq j \leq k-1 \Longrightarrow s_0 \not\succ_{s_k} s_k.$$

(b) For any sequence of states $s, s_1, \ldots, s_k$ *in* $\mathcal{S}$ *with* $s_j \succ_s s$ *(for all* $1 \leq j \leq k$*)*

$$s_{j+1} \succ_s s_j \text{ for all } 1 \leq j \leq k-1 \Longrightarrow s_0 \not\succ_s s_k.$$

- (a) rules out cycles of the form $y \succ_z z$, $x \succ_y y$, $z \succ_x x$
- (b) rules out cycles of the form $y \succ_s z$, $x \succ_s y$, $z \succ_s x$
- Weaker than transitivity of $\succ_s$.
- These assumptions cannot be dispensed with in the context of a general treatment because otherwise Condorcet-type cycles emerge.

Preferences and Acyclicity (continued)

- We will also strengthen our results under:

Assumption

(Comparability) *For* $x, y, z \in \mathcal{S}$ *such that* $x \succ_z z$, $y \succ_z z$, *and* $x \nsim y$, *either* $y \succ_z x$ *or* $x \succ_z y$.

- This condition sufficient (and "necessary") for uniqueness.

Approach and Motivation

- **Key economic insight:** *with sufficiently forward-looking behavior, an individual should not wish to transition to a state that will ultimately lead to another lower utility state.*
- Characterize the set of allocations that are consistent with this insight—without specifying the details of the dynamic game.
  - Introduce three simple and intuitive axioms.
  - Characterize set of mappings Φ such that for any $\phi \in \Phi$, $\phi : \mathcal{S} \to \mathcal{S}$ satisfies these axioms and assigns an **axiomatically stable state** $s^{\infty} \in \mathcal{S}$ to each initial state $s_0 \in \mathcal{S}$ (i.e., $\phi(s) = s^{\infty} \in \mathcal{S}$ loosely corresponding to $s_t = s^{\infty}$ for all $t \geq T$ for some T).
- Interesting in its own right, but the main utility of this axiomatic approach is as an input into the characterization of the (two-strategy) MPE of the dynamic game.

Axiom 1

(Desirability) If $x, y \in \mathcal{S}$ are such that $y = \phi(x)$, then either $y = x$ or $y \succ_x x$.

- A winning coalition can always stay in x (even a blocking coalition can)
- A winning coalition can move to y
- If there is a transition to y, a winning coalition must have voted for that

Axiom 2

(Stability) If $x, y \in \mathcal{S}$ are such that $y = \phi(x)$, then $y = \phi(y)$.

- Holds "by definition" of $\phi(\cdot)$: $\exists T : s_t = \phi(s)$ for all $t \geq T$; when $\phi(s)$ is reached, there are no more transitions
- If y were unstable ($y \neq \phi(y)$), then why not move to $\phi(y)$ instead of y

Axiom 3

(Rationality) If $x, y, z \in \mathcal{S}$ are such that $z \succ_x x$, $z = \phi(z)$, and $z \succ_x y$, then $y \neq \phi(x)$.

- A winning coalition can move to y and to z
- A winning coalition can stay in x
- When will a transition to y be blocked?
  - If there is another z preferred by some winning coalition
  - If this z is also preferred to x by some winning coalition (so blocking y will lead to z, not to x)
  - If transition to z is credible in the sense that this will not lead to some other state in perpetuity

Stable States

- State $s \in \mathcal{S}$ is ϕ-stable if $\phi(s) = s$ for $\phi \in \Phi$
- Set of ϕ-stable states: $\mathcal{D}_\phi = \{s \in \mathcal{S}\colon \phi(s) = s \text{ for } \phi \in \Phi\}$
- We will show that if ϕ_1 and ϕ_2 satisfy the Axioms, then $\mathcal{D}_{\phi_1} = \mathcal{D}_{\phi_2} = \mathcal{D}$
    - Even if ϕ is non-unique, notion of stable state is well-defined
    - But $\phi_1(s)$ and $\phi_2(s)$ may be different elements of $\mathcal{D}$

Axiomatic Characterization of Stable States

Theorem

Suppose Assumptions on Winning Coalitions and Payoffs hold. Then:

1. *There exists mapping ϕ satisfying Axioms 1–3.*
2. *This mapping ϕ may be obtained through a recursive procedure (next slide)*
3. *For any two mappings ϕ_1 and ϕ_2 that satisfy Axioms 1–3 the the sets of stable states of these mappings coincide (i.e., $\mathcal{D}_{\phi_1} = \mathcal{D}_{\phi_2} = \mathcal{D}$).*
4. *If, in addition, the Comparability Assumption holds, then the mapping that satisfies Axioms 1–3 is "payoff-unique" in the sense that for any two mappings ϕ_1 and ϕ_2 that satisfy Axioms 1–3 and for any $s \in \mathcal{S}$, $\phi_1(s) \sim \phi_2(s)$.*

Recursive Procedure

Theorem (continued)

Any ϕ that satisfies Axioms 1–3 can be recursively computed as follows. Construct the sequence of states $\left\{\mu_1, ..., \mu_{|\mathcal{S}|}\right\}$ with the property that if for any $l \in (j, |\mathcal{S}|]$, $\mu_l \nsucc_{\mu_j} \mu_j$. Let $\mu_1 \in \mathcal{S}$ be such that $\phi(\mu_1) = \mu_1$. For $k = 2, ..., |\mathcal{S}|$, let

$$\mathcal{M}_k = \left\{s \in \left\{\mu_1, \ldots, \mu_{k-1}\right\} : s \succ_{\mu_k} \mu_k \text{ and } \phi(s) = s\right\}.$$

Define, for $k = 2, ..., |\mathcal{S}|$,

$$\phi(\mu_k) = \begin{cases} \mu_k & \text{if } \mathcal{M}_k = \varnothing \\ z \in \mathcal{M}_k \text{: } \nexists x \in \mathcal{M}_k \text{ with } x \succ_{\mu_k} z & \text{if } \mathcal{M}_k \neq \varnothing \end{cases}.$$

(If there exist more than one $s \in \mathcal{M}_k$: $\nexists z \in \mathcal{M}_k$ with $z \succ_{\mu_k} s$, we pick any of these; this corresponds to multiple ϕ functions).

Extension of Franchise Example

- Get back to the simple extension of franchise story
- Three states: absolutism a, constitutional monarchy c, full democracy d
- Two agents: elite E, middle class M

$$\begin{aligned} w_E(d) &< w_E(a) < w_E(c) \\ w_M(a) &< w_M(c) < w_M(d) \end{aligned}$$

- $\mathcal{W}_a = \{\{E\}, \{E, M\}\}$, $\mathcal{W}_c = \{\{M\}, \{E, M\}\}$, $\mathcal{W}_d = \{\{M\}, \{E, M\}\}$
- Then: $\phi(d) = d$, $\phi(c) = d$, therefore, $\phi(a) = a$
  - Indeed, c is unstable, and among a and d player E, who is part of any winning coalition, prefers a
  - Intuitively, if limited franchise immediately leads to full democracy, elite will not undertake it

Example (continued)

- Assume $\mathcal{W}_c = \{\{E, M\}\}$ instead of $\mathcal{W}_c = \{\{M\}, \{E, M\}\}$
- Then: $\phi(d) = d$, $\phi(c) = c$, and, $\phi(a) = c$
- a became unstable because c became stable

- Now assume $\mathcal{W}_a = \mathcal{W}_c = \mathcal{W}_d = \{\{E, M\}\}$ and

$$\begin{aligned} w_E(a) &< w_E(d) < w_E(c) \\ w_M(a) &< w_M(c) < w_M(d) \end{aligned}$$

  - a is disliked by everyone, but otherwise preferences differ
- Then: $\phi(d) = d$, $\phi(c) = c$, and $\phi(a)$ may be c or d
- In any case, $\mathcal{D} = \{c, d\}$ is the same

Proof of Theorem (1)

- Take sequence $\left\{\mu_1, ..., \mu_{|\mathcal{S}|}\right\}$ such that

$$\text{if } 1 < j < l < |\mathcal{S}|\text{, then } \mu_l \not\succ_{\mu_j} \mu_j.$$

  - Assumption (Payoffs)a implies "acyclicity". Thus, for any nonempty collection of states $\mathcal{Q} \subset \mathcal{S}$, there exists state $z \in \mathcal{Q}$ such that for any $x \in \mathcal{Q}$, $x \not\succ_z z$.
  - Apply to $\mathcal{Q} = \mathcal{S}$; obtain $\mu_1 = z$
  - Apply to $\mathcal{Q} = \mathcal{S} \setminus \{\mu_1\}$; obtain μ_2...

Proof of Theorem (2)

- Let $\phi(\mu_1) = \mu_1$
  - This is the only possibility given Axiom 1 (desirability)
- For $k \geq 2$, let

$$\mathcal{M}_k = \left\{ s \in \{\mu_1, \ldots, \mu_{k-1}\} : s \succ_{\mu_k} \mu_k \text{ and } \phi(s) = s \right\}$$

  - Set of "higher" states which are preferred to μ_k and s
  - If $\phi(\mu_k) \neq \mu_k$, then $\phi(\mu_k) \in \mathcal{M}_k$ by desirability and stability axioms
- Define

$$\phi(\mu_k) = \begin{cases} \mu_k & \text{if } \mathcal{M}_k = \varnothing \\ z \in \mathcal{M}_k \text{: } \nexists x \in \mathcal{M}_k \text{ with } x \succ_{\mu_k} z & \text{if } \mathcal{M}_k \neq \varnothing \end{cases}.$$

  - Such z exists by Assumption (Payoffs)b applied to $\mathcal{M}_k$
  - This ensures that rationality axiom holds

Proof of Theorem (3)

- We have constructed mapping ϕ using the recursive procedure
- By construction, ϕ satisfies Axioms 1–3
- Any such mapping could be obtained by this procedure
  - The only degree of freedom we had was choosing $z \in \mathcal{M}_k$
  - Different choice of $\phi(\mu_k)$ does not affecl $\mathcal{M}_l$ for $l > k$
  - Any choice of sequence $\left\{\mu_1, ..., \mu_{|\mathcal{S}|}\right\}$ such that $\mu_l \not\succ_{\mu_j} \mu_j$ whenever $1 < j < l < |\mathcal{S}|$ would lead to the same (comprehensive) set of mappings
- This proves parts 1 and 2

Proof of Theorem (4)

- Part 3: For any two mappings ϕ_1 and ϕ_2 that satisfy Axioms 1–3 the sets of stable states of these mappings coincide.
  - This holds because $\phi(\mu_k) = \mu_k$ if and only if $\mathcal{M}_k = \varnothing$, and this does not depend on ϕ
- Part 4: If, in addition, Assumption (Comparability) holds, then for any two mappings ϕ_1 and ϕ_2 that satisfy Axioms 1–3 and for any $s \in \mathcal{S}$, $\phi_1(s) \sim \phi_2(s)$.
  - Assumption: For $x, y, z \in \mathcal{S}$ such that $x \succ_z z$, $y \succ_z z$, and $x \nsim y$, either $y \succ_z x$ or $x \succ_z y$.
  - If $\phi_1(s) \nsim \phi_2(s)$, then either $\phi_1(s) \succ_s \phi_2(s)$ or $\phi_2(s) \succ_s \phi_1(s)$
  - Both options would violate Rationality axiom.

Back to Dynamic Game

Assumption

(Agenda-Setting and Proposals) *For every state $s \in \mathcal{S}$, one (or both) of the following two conditions is satisfied:*
(a) For any state $q \in \mathcal{S} \setminus \{s\}$, there is an element $k : 1 \leq k \leq K_s$ of sequence π_s such that $\pi_s(k) = q$.
(b) For any player $i \in \mathcal{I}$ there is an element $k : 1 \leq k \leq K_s$ of sequence π_s such that $\pi_s(k) = i$.

- Exogenous agenda, sequence of agenda-setters, or mixture.
- This assumption ensures that all proposals will be considered (or all agenda-setters will have a chance to propose)

Definition

(Dynamically Stable States) State $s^\infty \in \mathcal{S}$ is a **dynamically stable state** if there exist a protocol $\{\pi_s\}_{s \in \mathcal{S}}$, a MPE strategy profile σ (for a game starting with initial state s_0) and $T < \infty$, such that in MPE $s_t = s^\infty$

Slightly Stronger Acyclicty Assumption

Assumption (Stronger Acyclicity) For any sequence of states $s, s_1, \ldots, s_k$ in $\mathcal{S}$ such that $s_j \nsim s_l$ (for any $1 \leq j < l \leq k$) and $s_j \succ_s s$ (for any $1 \leq j \leq k$)

$$s_{j+1} \succsim_s s_j \text{ for all } 1 \leq j < k-1 \Longrightarrow s_1 \not\succsim_s s_k.$$

Moreover, if for x, y, s in $\mathcal{S}$, we have $x \succ_s s$ and $y \not\succ_s s$, then $y \not\succ_s x$.

- Stronger version of part (b) of Payoffs Assumption.
- First part: $\succeq$-acyclicity as opposed $\succ$-acyclicity
- Second part: slightly stronger than acyclicity
  - but weaker than transitivity within states, i.e., $x \succ_s s$, $y \not\succ_s s$, then $y \not\succ_s x$, whereas transitivity would require $x \succ_s s$, $s \succ_s y$, then $x \succ_s y$, which implies our condition, but is much stronger.
- Alternative (with equivalent results): voting *yes* has a small cost.

Noncooperative Characterization

Theorem

(Noncooperative Characterization) *Suppose Assumptions on Winning Coalitions and Payoffs hold. Then there exists* $\beta_0 \in [0, 1)$ *such that for all* $\beta \geq \beta_0$*, the following results hold.*

1. *For any mapping* ϕ *satisfying Axioms* 1–3 *there is a protocol* $\{\pi_s\}_{s \in \mathcal{S}}$ *and a MPE* σ *of the game such that* $s_t = \phi(s_0)$ *for any* $t \geq 1$*; that is, the game reaches* $\phi(s_0)$ *after one period and stays in this state thereafter. Therefore,* $s = \phi(s_0)$ *is a dynamically stable state.*

Noncooperative Characterization (continued)

Theorem

... Moreover, suppose that Stronger Acyclicity Assumption holds. Then:

2. *For any protocol* $\{\pi_s\}_{s\in\mathcal{S}}$ *there exists a MPE in pure strategies. Any such MPE* σ *has the property that for any initial state* $s_0 \in \mathcal{S}$*, it reaches some state,* s^∞ *by* $t = 1$ *and thus for* $t \geq 1$*,* $s_t = s^\infty$*. Moreover, there exists mapping* $\phi : \mathcal{S} \to \mathcal{S}$ *that satisfies Axioms* 1–3 *such that* $s^\infty = \phi(s_0)$*. Therefore, all dynamically stable states are axiomatically stable.*
3. *If, in addition, Assumption (Comparability) holds, then the MPE is essentially unique in the sense that for any protocol* $\{\pi_s\}_{s\in\mathcal{S}}$*, any MPE strategy profile in pure strategies* σ *induces* $s_t \sim \phi(s_0)$ *for all* $t \geq 1$*, where* ϕ *satisfies Axioms* 1–3.

Proof of Theorem (I)

Part 1

- For any state s, choose $\pi_s(\cdot)$ such that $\pi_s(K_s) = \phi(s)$ (last proposal is $\phi(s)$)
- Let everyone who weakly prefers $\phi(s)$ to s $(w_i(\phi(s)) \geq w_i(s))$ vote for $\phi(s)$
  - These constitute a winning coalition in s
- In previous votings these block any transition to any other state as no other state is preferred by a winning coalition
- As a result, $\phi(s)$ will be implemented.
- A similar argument for any $\pi_s(\cdot)$ establishes existence.
  - In particular, truncate the game with continuation payoffs given by the axiomatic characterization from a certain point onwards, use backward induction and verify that strategies are Markovian.

Proof of Theorem (II)

Part 2

- Suppose that s leads to $\chi(s)$
- Let $\psi(s) = \chi^{|\mathcal{S}|}(s)$
- Suppose that in an equilibrium, if state is s, then proposals at stages $k_1, k_2, \ldots, k_j$ are accepted
- Let these proposals be $P_{k_1}, P_{k_2}, \ldots, P_{k_j}$
- We immediately have $\psi(P_{k_1}) \succeq_s \psi(P_{k_2}) \succeq_s \cdots \succeq_s \psi(P_{k_j}) \succeq_s \psi(P_{k_1})$
- Stronger acyclicity assumption implies $\psi(P_{k_1}) \sim \psi(P_{k_2}) \sim \cdots \sim \psi(P_{k_j})$

Proof of Theorem (III)

Part 2 (continued)

- Stability Axiom for ψ: satisfied by definition
- Desirability Axiom: otherwise players would wait until next period instead of accepting $\psi\left(P_{k_j}\right)$
- Rationality Axiom: otherwise some other proposal would be accepted in between
- Transition in one step: if this proposal is made, it is accepted

Part 3

- Follows from Part 2 and Axiomatic Characterization

Dynamic vs. Myopic Stability

Definition

State $s^m \in \mathcal{S}$ is *myopically stable* if there does not exist $s \in \mathcal{S}$ with $s \succ_{s^m} s^m$.

Corollary

1. *State* $s^\infty \in S$ *is a (dynamically and axiomatically) stable state only if for any* $s' \in \mathcal{S}$ *with* $s' \succ_{s^\infty} s^\infty$, *and any* ϕ *satisfying Axioms* 1–3, $s' \neq \phi(s')$.
2. *A myopically stable state* s^m *is a stable state.*
3. *A stable state* s^∞ *is not necessarily myopically stable.*

- E.g., state a in extension of franchise story

Inefficiency

Definition

(Infficiency) State $s \in S$ is *(strictly) Pareto inefficient* if there exists $s' \in S$ such that $w_i(s') > w_i(s)$ for all $i \in \mathcal{I}$.
State $s \in S$ is *(strictly) winning coalition inefficient* if there exists a winning coalition $\mathcal{W}_s \subset \mathcal{I}$ in s and $s' \in S$ such that $w_i(s') > w_i(s)$ for all $i \in \mathcal{W}_s$.

- Clearly, if a state s is Pareto inefficient, it is winning coalition inefficient, but not vice versa.

Corollary

1. *A stable state $s^\infty \in S$ can be (strictly) winning coalition inefficient and Pareto inefficient.*
2. *Whenever s^∞ is not myopically stable, it is winning coalition inefficient.*

Applying the Theorems in Ordered Spaces

- The characterization theorems provided so far are easily applicable in a wide variety of settings.
- In particular, if the set of states is ordered and static preferences satisfy single crossing or single peakedness, all the results provided so far can be applied directly.
- Here, for simplicity, suppose that $\mathcal{I} \subset \mathbb{R}$ and $\mathcal{S} \subset \mathbb{R}$ (more generally, other orders on the set of individuals and the set of states would work as well)

Single Crossing and Single Peakedness

Definition

Take set of individuals $\mathcal{I} \subset \mathbb{R}$, set of states $\mathcal{S} \subset \mathbb{R}$, and payoff functions $w_{\cdot}(\cdot)$. Then, single crossing condition holds if whenever for any $i, j \in \mathcal{I}$ and $x, y \in \mathcal{S}$ such that $i < j$ and $x < y$, $w_i(y) > w_i(x)$ implies $w_j(y) > w_j(x)$ and $w_j(y) < w_j(x)$ implies $w_i(y) < w_i(x)$.

Definition

Take set of individuals $\mathcal{I} \subset \mathbb{R}$, set of states $\mathcal{S} \subset \mathbb{R}$, and payoff functions $w_{\cdot}(\cdot)$. Then, single-peaked preferences assumption holds if for any $i \in \mathcal{I}$ there exists state x such that for any $y, z \in \mathcal{S}$, if $y < z \leq x$ or $x \geq z > y$, then $w_i(y) \leq w_i(z)$.

Generalizations of Majority Rule and Median Voter

Definition

Take set of individuals $\mathcal{I} \subset \mathbb{R}$, state $s \in \mathcal{S}$, and set of winning coalitions $\mathcal{W}_s$ that satisfies Assumption on Winning Coalitions. Player $i \in \mathcal{I}$ is called quasi-median voter (in state s) if $i \in X$ for any $X \in \mathcal{W}_s$ such that $X = \{j \in \mathcal{I} : a \leq j \leq b\}$ for some $a, b \in \mathbb{R}$.

- That is, quasi-median voter is a player who belongs to any "connected" winning coalition.
- Denote the set of quasi-median voters in state s by M_s (it will be nonempty)

Definition

Take set of individuals $\mathcal{I} \subset \mathbb{R}$, set of states $\mathcal{S} \subset \mathbb{R}$. The sets of winning coalitions $\{\mathcal{W}_s\}_{s \in \mathcal{S}}$ has monotonic quasi-median voter property if for each $x, y \in \mathcal{S}$ satisfying $x < y$ there exist $i \in M_x$, $j \in M_y$ such that $i \leq j$.

A Weak Genericity Assumption

- Let us say that preferences $w_{\cdot}(\cdot)$, given the set of winning coalitions $\{\mathcal{W}_s\}_{s\in\mathcal{S}}$, are *generic* if for all $x, y, z \in \mathcal{S}$, $x \succeq_z y$ implies $x \succ_z y$ or $x \sim y$.
- This is (much) weaker than the comparability assumption used for uniqueness above.
  - In particular, it holds generically.

Theorem on Single Crossing and Single Peakedness

Theorem

Suppose the Assumption on Winning Coalitions holds.

1. *If preferences satisfy single crossing and the monotonic quasi-median voter property holds, then Assumptions on Payoffs above are satisfied and the axiomatic characterization (Theorem 1) applies.*
2. *If preferences are single peaked and all winning coalitions intersect (i.e., $X \in \mathcal{W}_x$ and $Y \in \mathcal{W}_y$ imply $X \cap Y \neq \varnothing$), then Assumptions on Payoffs are satisfied and Theorem 1 applies.*
3. *If, in addition, in part 1 or 2, preferences are generic, then the Stronger Acyclicity Assumption is satisfied and the noncooperative characterization (Theorem 2) applies.*

- Note monotonic median voter property is weaker than the assumption that $X \in \mathcal{W}_x \wedge Y \in \mathcal{W}_y \Longrightarrow X \cap Y \neq \varnothing$.

Voting in Clubs

- N individuals, $\mathcal{I} = \{1, \ldots, N\}$
- N states (clubs), $s_k = \{1, \ldots, k\}$
- Assume single-crossing condition

$$\text{for all } l > k \text{ and } j > i,\ w_j(s_l) - w_j(s_k) > w_i(s_l) - w_i(s_k)$$

- Assume "genericity":

$$\text{for all } l > k,\ w_j(s_l) \neq w_j(s_k)$$

- Then, the theorem for ordered spaces applies and shows existence of MPE in pure strategies for any majority or supermajority rule.
- It also provides a full characterization of these equilibria.

Voting in Clubs

- If in addition only odd-sized clubs are allowed, unique dynamically stable state.
- Equilibria can easily be Pareto inefficient.
- If "genericity" is relaxed, so that $w_j(s_l) = w_j(s_k)$, then the theorem for ordered spaces no longer applies, but both the axiomatic characterization and the noncooperative theorems can still be applied from first principles.
- Comparison to Roberts (1999): much simpler analysis under weaker conditions, and more general results (existence of pure-strategy equilibrium, results for supermajority rules etc.)
- Also can be extended to more general structure of clubs
  - e.g., clubs on the form $\{k - n, ..., k, ..., k + n\} \cap \mathcal{I}$ for a fixed n (and different values of k).

An Example of Elite Clubs

- Specific example: suppose that preferences are such that

$$w_j(s_n) > w_j(s_{n'}) > w_j(s_{k'}) = w_j(s_{k''})$$

for all $n' > n \geq j$ and $k', k'' < j$

  - individuals always prefer to be part of the club
  - individuals always prefer smaller clubs.

- Winning coalitions need to have a strict majority (e.g., two out of three, three out of four etc.).
- Then,
  - $\{1\}$ is a stable club (no wish to expand)
  - $\{1, 2\}$ is a stable club (no wish to expand and no majority to contract)
  - $\{1, 2, 3\}$ is not a stable club (3 can be eliminated)
  - $\{1, 2, 3, 4\}$ is a stable club
- More generally, clubs of size 2^k for $k = 0, 1, ...$ are stable.
- Starting with the club of size n, the equilibrium involves the largest club of size $2^k \leq n$.

Example: Taxation

- Suppose there are k individuals $1, 2, \ldots, k$, and k states $s_1, s_2, \ldots, s_k$, where $s_j = \{1, 2, \ldots, j\}$.
- Suppose winning coalition is a simple majority rule of players who are enfranchised:

$$\mathcal{W}_{s_j} = \{X \in \mathcal{C} : \# (X \cap s_j) > j/2\}.$$

- Suppose player i's payoff is

$$w_i (s_j) = (1 - \tau_{s_j}) A_i + G_{s_j}$$

where A_i is player i's productivity; G_{s_j} and τ_{s_j} are the public good and the tax rate voting franchise is s_j.
- Assume $A_i > A_j$ for $i < j$, so the first players are the most productive ones

Example: Taxation (continued)

- τ_{s_j} is the tax rate determined by the median voter in the club s_j (or by one of the two median voters with equal probability in case of even-sized club)
- The technology for the production of the public good is

$$G_{s_j} = H\left(\sum_{i=1}^{k} \tau_{s_j} A_i\right),$$

where H is strictly increasing and concave.

Example: Taxation (continued)

- In light of the previous theorem, to apply our results, it suffices to show that if $i, j \in s_k, s_{k+1}$, then

$$w_j\left(s_{k+1}\right) - w_j\left(s_k\right) > w_i\left(s_{k+1}\right) - w_i\left(s_{k+1}\right)$$

  whenever $i < j$.

- This is equivalent to

$$\left(1 - \tau_{s_{k+1}}\right) A_j - \left(1 - \tau_{s_k}\right) A_j \geq \left(1 - \tau_{s_{k+1}}\right) A_i - \left(1 - \tau_{s_k}\right) A_i,$$

- Since $A_j < A_i$, this is in turn is equivalent to

$$\tau_{s_{k+1}} \geq \tau_{s_k}.$$

- This can be verified easily, so the theorem for order spaces can be applied.

Stable Constitutions

- N individuals, $\mathcal{I} = \{1, \ldots, N\}$
- In period 2, they decide whether to implement a reform (a votes are needed)
- a is determined in period 1
- Two cases:
  - Voting rule a: stable if in period 1 no other rule is supported by a voters
  - Constitution (a, b): stable if in period 1 no other constitution is supported by b voters
- Preferences over reforms translate into preferences over a
  - Barbera and Jackson assume a structure where these preferences are single-crossing and single-peaked
  - Motivated by this, let us assume that they are strictly single-crossing
- Stable voting rules correspond to myopically (and dynamically) stable states
- Stable constitutions correspond to dynamically stable states

Political Eliminations

- The characterization results apply even when states do not form an ordered set.
- Set of states $\mathcal{S}$ coincides with set of coalitions $\mathcal{C}$
- Each agent $i \in \mathcal{I}$ is endowed with political influence γ_i
- Payoffs are given by proportional rule

$$w_i(X) = \begin{cases} \gamma_i / \gamma_X & \text{if } i \in X \\ 0 & \text{if } i \notin X \end{cases} \quad \text{where } \gamma_X = \sum_{j \in X} \gamma_j$$

and X is the "*ruling coalition*".

  - this payoff function can be generalized to any function where payoffs are increasing in relative power of the individual in the ruling coalition

Political Eliminations (continued)

- Winning coalitions are determined by weighted (super)majority rule $\alpha \in [1/2, 1)$

$$\mathcal{W}_X = \left\{ Y : \sum_{j \in Y \cap X} \gamma_j > \alpha \sum_{j \in X} \gamma_j \right\}$$

- Genericity: $\gamma_X = \gamma_Y$ only if $X = Y$
- Assumption on Payoffs is satisfied and the axiomatic characterization applies exactly.
- If players who are not part of the ruling coalition have a slight preference for larger ruling coalitions, then Stronger Acyclicity Assumption is also satisfied.

Other Examples

- Inefficient inertia
- The role of the middle class in democratization
- Coalition formation in democratic systems
- Commitment, (civil or international) conflict and peace

Concluding Remarks

- A class of dynamic games potentially representing choice of constitutions, dynamic voting, club formation, dynamic coalition formation, organizational choice, dynamic legislative bargaining, international or civil conflict.
- Common themes in disparate situations.
- This paper: a framework for general analysis and tight characterization results.
- *Simple implications: social arrangements* are unstable **not** when some winning coalition (e.g., majority) prefers another social arrangement, but when it preferes another **stable** social arrangement
- We show that this gives rise to inefficiencies: a Pareto dominated state may be stable, even if discount factor is close to 1

Open Questions

- Beyond acyclicity...
- More on stochastic shocks.
- Different winning coalitions to implement different social arrangements/political changes
- **Most important:** model dynamic games with more limited foresight and richer dynamics so as to understand the dynamic evolution of constitutions, coalitions, clubs, governments, organizations,...
    - gradual extension of the franchise, constitutional cycles, slow improvements in the quality of governance...