PRELIMINARY AND INCOMPLETE

Reserve Requirements and Default*

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Abstract

We argue that contractual provisions should exist in defaultable contracts that require the seller of the contract to keep a certain amount of wealth for the future. We show that the presence of reserve requirements generate trade in asset markets which would otherwise be inactive and lead to higher volumes of trade. Finally, we suggest that they are welfare improving and decentralized trade chooses the optimal requirements.

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1 Introduction

When an individual seeks credit, he enters into a contractual agreement for repayment. If the debtor does not fulfill his obligation then the contract must stipulate either recourse for the creditor to the debtors assets or a punishment enforced on the debtor. Historically bondage, corporal punishment and debtors' prison were used, each being a mechanism to force a debtor to reveal assets assumed to be hidden. The emphasis on punishment changed in the mid-nineteenth century when bankruptcy started to be seen as an economic rather than moral failure. Coincident with this change was a shift towards collateralised lending[1] that now dominates credit markets. Although more humane, collateralised lending provides a similar market function as punishment as it results in a cost on the debtor in case of default as well as providing recourse to the debtors assets. However, there are many situations when neither collateral nor punishment alone is enough to guarantee liquid capital markets.

This paper proposes that reserve requirements attached to contractual debt obligations will reduce default and improve welfare even in situations where both traditional default punishment as well as the modern collateral system fails. Furthermore, we show that introducing reserve requirements on contracts will be optimally chosen by agents, provided the legal and institutional framework exists for them, and will result in a constrained Pareto optimal outcome.

In the classic Arrow-Debreu model, an implicit assumption is that all agents honour their obligations, and thus there is no possibility of default. Nevertheless, default is an old phenomenon in all societies with freedom of transactions. If agents are not accountable for their repayments, they will rationally choose not to repay any of their debts. Thus, the introduction of default penalties becomes natural. Shubik and Wilson (1977) introduce default penalties in strategic market games and treat default continuously, i.e. they allow for partial default in equilibrium. Dubey *et al.* (2005) formally analyse default in a General Equilibrium framework with real default penalties and derive conditions under which assets in the economy are traded.

A possible limitation of this apporach is that default penalties are modeled as utility punishments. Severe penalties for default, such as prison, are no longer in use in most developed countries. This is consistent with Dubey *et al.* (2005) who show that infinite default penalties are welfare worsening. However, these default penalties can be seen as proxies for other pecuniary or non-pecuniary economic penalties, such as deterioration of the credit rating, loss of access to the markets in the future and loss of reputation. An alternative view is to model these penalties as the time lost by an agent who defaults in court, completing related documentation and attending committees. In such a model agents are not only endowed with goods but also with leisure which they can use to receive a wage. If they default they have to spend some of their time, which decreases their leisure and imposes an opportunity cost from the foregone wages. By choosing to default on a contract agents can

[1] Traditionally found in the agricultural sector of medieval europe where land was used as collateral by farmers.

transform a unit of the numraire on which the contract's payoff are defined into time. The transformation ratio will be detrmined by the default penalty. Since time is money, this is equivalent to a pecuniary default penalty, which enters directly the budget constraint.

The more fundamental question we have to address is whether institutional arrangements can exist to reduce or eliminate the role of default penalties. Such an arrangement is the use of collateral to secure loans. Hart and Moore (1989,1998) provide a game-theoretic model for the use of collateral, whereas Geanakoplos (2003) and Geanakoplos and Zame (1997) formally treat collateral in a general equilibrium setting. As stated in their papers, collateral requirements both limit borrowing, since collateral is scarce, and distort consumption decisions, since it may not be held by agents who most wish to consume it.

We analyse an alternative institutional arrangement, that of reserve requirements allowing for low default penalties. The possibility of reserve requirements relies on the existence of a storage technology, which can be either a storable good or money (we introduce money in our model when we discuss the collapse of the bond market in section **??**). It differs from collateral since lenders have no claim on these reserves. In Section 2, we present our benchmark example. We show that for most choices of default penalties there is no trade in bond market. We also show that for an intermediate region of default penalties there will be trade in both the risky asset as well as storage of the storable consumable good. In Section 4, we show that if a reserve requirement is imposed then trade in the bond is possible for a larger set of default penalties. Section 5, presents the trade and welfare properties of allowing for reserve requirements. In Section 6, we argue that if reserve requirements are contractible then agents would voluntarily commit to holding the reserve requirement under which full delivery is guaranteed and constrained Pareto optimality is achieved. Section **??** discusses the implications for banking and capital regulation and section **??** applies the argument to a case where the bond market in a monetary economy collapses. Finally Section 8 concludes.

2 Trade with a Bond

2.1 The Model

This paper considers a two-period economy where agents know the present but face an uncertain future. Period 0 (the present) is defined as state 0 and the two agents trade a single commodity and 1 bond. In period 1 nature selects one of S states. Commodity trades take place again, assets pay off with agents choosing how much to deliver on each asset. In particular:

- $s \in S = \{1,2\}$ = set of states in period 1;
- $S^* = \{0\} \cup S$= set of all states;
- $h \in \{1,2\}$ set of agents;

- A single storable consumable good. The endowment vector is $e^1 = (e^1_0, e^1_1, e^1_2) = (1,1,0)$ for agent 1 and $e^2 = (e^2_0, e^2_1, e^2_2) = (1,0,1)$ for agent 2;

The endogenous variables consist of three macro variables which agents take as fixed and four individual choice variables:

- $\pi \in \mathbb{R}_+ =$ asset price;
- $K \in [0,1]^S$ is the expected delivery of the asset;
- $x^h \in \mathbb{R}_+$ is the consumption by agent h;
- $\sigma^h \in \mathbb{R}_+$ is the amount of storage for agent h;
- $\theta^h \in \mathbb{R}_+$ is the amount of asset J bought by agent h;
- $\phi^h \in \mathbb{R}_+$ is the amount of asset J sold by agent h;
- $D^h \in \mathbb{R}^S_+$ is the amount that agent h chooses to deliver at state $s \in S$

Assume that only a (defaultable) bond is available for trade, i.e. $R = (1,1)$, and that there is only one good in the economy which is storable.

2.1.1 Maximization Problem

The payoff of (x, θ, ϕ, D) to agent 1 is

$$w = \ln x_0 + \pi_1 \ln x_1 + \pi_2 \ln x_2 - \lambda \sum_s \pi_s \max[\phi - D_s, 0]$$

where $\pi_1 = \pi_2 = 1/2$.

The *budget set* $B^1(\pi, K)$ of agent 1 given by (Lagrange multipliers are in brackets)[2]:

$$x^1_0 + \sigma^1 + q(\theta^1 - \phi^1) = 1 \quad (\mu^1_0)$$

$$x^1_1 + D^1_1 = \theta^1 K_1 + \sigma^1 + 1 \quad (\mu^1_1)$$

$$x^1_2 + D^1_2 = \theta^1 K_2 + \sigma^1 \quad (\mu^1_2)$$

[2]Note that by symmetry all the variables for agent 1 in period 0 will be the same for agent 2 in period 0 while in state 1 the variables for agent 1 will be identical to the variables for agent 2 in state 2 and vice versa.

$$x_0^2+\sigma^2+q(\theta^2-\phi^2)=1 \quad (\mu_0^2)$$

$$x_1^2+D_1^2=\theta^2 K_1+\sigma^2+1 \quad (\mu_1^2)$$

$$x_2^2+D_2^2=\theta^2 K_2+\sigma^2 \quad (\mu_2^2)$$

From the first order conditions we obtain: $\frac{1}{x_0^1}-\mu_0^1=0$, $\frac{\pi_s}{x_s^1}-\mu_s^1=0$. If agents default partially in state s they equate marginal utility by $\pi_s\lambda-\mu_s^1=0$ or if they deliver fully $(\pi_s\lambda>\mu_s^1)$ or default completely $(\pi_s\lambda<\mu_s^1)$ we do not have this condition. As a consequence, the first order conditions for assets sold is $-\sum_{s\in S}\min[1/2\lambda,\mu_s^1]+q\mu_0^{1h}=0$. The condition for assets bought is $\sum_{s\in S}K_s\mu_s^1-q\mu_0^1=0$.

Note that the budget set is convex, and the payoff function w^1 is concave, in the agent choice variables (x,θ,ϕ,D).

We also have four markets:

Asset Market

$$\phi=\theta$$

i.e., aggregate bond sales are equal to aggregate bond purchases (the equilibrium will by essentially symmetric, thus both agents will sell (and buy) the same amount of the bond).

Goods Market

$$x_0^1+x_0^2=2-\sigma^1-\sigma^2, \quad x_1^1+x_1^2=1+\sigma^1+\sigma^2, \quad x_2^1+x_2^2=1+\sigma^1+\sigma^2$$

i.e., aggregate consumption is equal to the aggregate quantity of the good available in each state of the world. Since the equilibrium is symmetric, $\sigma^1=\sigma^2=\sigma$.

2.2 Equilibrium

We now define a $GE(R,\lambda,Q)$ The equilibrium is a list $(\pi,K,(x^h,\phi^h,\theta^h,D^h)_{h\in H})$ such that the following hold:

1. For $h\in H$, $(x^h,\theta^h,\phi^h,D^h)\in argmax\ \mathrm{w^h}(\mathrm{x},\theta,\phi,\mathrm{D})$, over $B^h(\pi,K)$
2. $\sum_{h\in H}(x_0^h+\sigma^h-e_0^h)=0$,

3. $\sum_{h\in H}(x_s^h - \sigma^h - e_s^h) = 0,$

4. $\sum_{h\in H}(\theta^h - \phi^h) = 0,$

5. $K_{sj} = \begin{cases} \dfrac{\sum_{h\in H} D_{sj}}{\sum_{h\in H}\phi_j^h} \text{ if } \sum_{\mathrm{h}\in\mathrm{H}} \phi_{\mathrm{j}}^{\mathrm{h}} > 0 \\ \text{or arbitrary if } \sum_{\mathrm{h}\in\mathrm{H}} \phi_{\mathrm{j}}^{\mathrm{h}} = 0 \end{cases}$

3 Characterisation of Equilibrium

Here we discuss the properties of an equilibrium without reserve requirements. Note that due to symmetry we present only the variables for agent 1.

We first present the method used by Dubey *et al.* (2005) to determine if new assets will be traded at a given equilibrium:

On the verge condition

The marginal utility to agent h of purchasing the bond is

$$MU^h = \sum_{s\in S} \mu_s^h K_s$$

and the marginal disutility of selling it is

$$MDU^h = \sum_{s\in S} \min[1/2\lambda, \mu_s^h]$$

At any equilibrium one of the following conditions will hold for the bond:

(a) $\dfrac{MU_j^{buyer}}{\mu_0^{buyer}} = \dfrac{MDU_j^{seller}}{\mu_0^{seller}}$

(b) $\dfrac{MU_j^{buyer}}{\mu_0^{buyer}} < \dfrac{MDU_j^{seller}}{\mu_0^{seller}}$

$\dfrac{MU_j^{buyer}}{\mu_0^{buyer}}$ is the price that the buyer bids to purchase the bond, while $\dfrac{MDU_j^{seller}}{\mu_0^{seller}}$ is the price that the seller asks to sell the bond. If at a given equilibrium the two prices are equal then the bond is traded (condition (a)). If the ask price is less than the bid price then the bond will not be traded (condition (b)). Nevertheless, when examining possible equilibria we might find that the ask price is lower than the bid price, i.e. $\frac{MU_j^{buyer}}{\mu_0^{buyer}} > \frac{MDU_j^{seller}}{\mu_0^{seller}}$. When this occurs there is excess demand for the bond and we use the t$\hat{a}$tonnement process to determine the final equilibrium.

Proposition 1: ***For*** $1 \leq \lambda$ ***the equilibrium will be autarkic***

Proof. Assume that the equilibrium for this region of λ is autarkic, i.e. the bond is not traded and agents optimize only over σ.

Under autarky we obtain the following FOC: $\sum_{s\in S} \mu_s - \mu_0^h = 0$. Solving for σ here we find $\sigma = \frac{-1+\sqrt{17}}{8}$. In this equilibrium we need to check whether the bond should be traded or not.

On the verge condition assuming autarky

> The marginal utilities for this equilibrium are $\mu_1 = 0.72$ and $\mu_2 = 2.56$ in states 1 and 2 respectively. For $\lambda < 0.72$ the bond would definetely not be traded, since agents would default completely in both states. Thus, we consider $\lambda > 0.72$ for our analysis. The price offered by the on the verge seller here is $\frac{\min[\frac{1}{2}\lambda,\mu_1]+\min[\frac{1}{2}\lambda,\mu_2]}{\mu_0}$. For $1 \leq \lambda$ this price would be at least 0.524 and increasing on λ. The price offered by the on the verge buyer would be $\frac{K_1\mu_1+K_2\mu_2}{\mu_0}$. However at $1 \leq \lambda < 4/3$ the seller would deliver fully in his good state as his marginal utility is strictly less than $\frac{1}{2}\lambda$ and delivery nothing in the other state. Given that there are two agents the delivery rate would be $\frac{1}{2}$. In this range of λ the price offered by the buyer would be $\frac{1}{2}$ which is less than that demanded by the buyer.

We now have to check whether there exists an equilibrium for this region of λ which involves trade in the bond *and* agents optimally choose $\sigma = 0$. The budget set for agent 1 will be

$$x_1 + D_1 = \theta K_1 + 1 \quad \text{and} \quad x_2 = \theta K_2$$

Using the market clearing condition for the delivery rate and using the properties of a symmetric allocation we have:

$$x_1 = 1 - \theta \frac{D}{2\phi} \quad \text{and} \quad x_2 = \theta \frac{D}{2\phi}$$

Using the market clearing condition for the asset we get:

$$x_1 = 1 - \frac{D}{2} \quad \text{and} \quad x_2 = \frac{D}{2}$$

First, take the case where agents default only partially in the state in which they have their endowment. This implies that for agent 1 $x_1 = \frac{1}{\lambda}$ will hold which gives us:

$$\frac{1}{\lambda} + D = \frac{D}{2} + \sigma + 1 \quad \Rightarrow D = 2 - \frac{2}{\lambda}$$

The allocation for each agent is then $x_1^1 = x_2^2 = \frac{1}{\lambda}$ and $x_2^1 = x_1^2 = 1 - \frac{1}{\lambda}$. Using the first order equations we get we get $q\mu_0 = \lambda$ and $q\mu_0 = K\sum_s \mu_s$ and hence $K = \frac{\lambda}{\sum_s \mu_s}$ which implies $K = 2\frac{\lambda - 1}{\lambda}$. From this we can see that the market will fail when $\lambda < 1$ and full delivery in the good state will occur when $K = .5$ or $\lambda = \frac{4}{3}$.

However it is still not clear that such an equilibrium will exist. We need to determine if the assumption of $\sigma = 0$ is correct. We can test this by using the on-the-verge condition to see if there is excess demand for the untraded storage technology:

On the verge condition assuming $\sigma = 0$ and partial default

> The marginal disutility in storing the commodity is μ_0 and the marginal utility is $\mu_1 + \mu_2$. If the marginal disutility is greater than the marginal utility then the assumption is correct. Otherwise it is not. The marginal disutility will be 1. The marginal utility will be $\frac{1}{2}\lambda + \frac{1}{2}\frac{1}{1 - \frac{1}{\lambda}}$ which simplifies to $\frac{1}{2}\frac{\lambda^2}{\lambda - 1}$. For $\lambda > 1$ the marginal utility will always be greater than the marginal disutility. Agents would have excess demand for σ, meaning that an equilibrium with $\sigma = 0$ is not correct.

Let us assume here that only the bond is traded and agents repay fully in their good state hence $4/3 \leq \lambda$. Agent 1's budget constraints become $x_1 = 1 - \frac{D}{2} + \sigma$ and $x_2 = \frac{D}{2} + \sigma$ but as they delivery fully $\mu_s + \frac{1}{2}\lambda - q\mu_0^h = 0$ which implies $\sum_{s \in S} K_s \mu_s - q\mu_0^h = 0$.

As they will default fully in the bad state (null endowment) then K=.5. Hence $\mu_1 + \frac{1}{2}\lambda = \frac{1}{2}\mu_1 + \frac{1}{2}\mu_2 \Rightarrow \frac{1}{2}\lambda = \frac{1}{2}\mu_2 - \frac{1}{2}\mu_1$ Which gives us $\lambda = \frac{1}{2}\frac{1}{\frac{D}{2}+\sigma} - \frac{1}{2}\frac{1}{1-\frac{D}{2}+\sigma}$. If we assume that $\sigma = 0$ then we have D as a quadratic and a function of λ. We can see numerically that $D \Rightarrow 0$ as $\lambda \Rightarrow \infty$.

On the verge condition assuming $\sigma = 0$ and full delivery in the good state

> We need to check whether the assumption that $\sigma = 0$ is valid. Using the equilibrium from the above and setting $\sigma = 0$ the equilibrium delivery will be $\frac{\lambda + 1 - \sqrt{\lambda^2+1}}{\lambda}$. At this equilibrium the marginal utility from storing is greater than the marginal disulity, thus this cannot be an equilibrium.

Hence we show that for $1 \leq \lambda$ the defaultable loan will not be traded. Therefore the autarkic equilibrium exists for this range of λ.

□

Proposition 2: ***For*** $.72 \leq \lambda < 0.92$[3] ***agents will both store and trade the bond in equilibrium***

Proof. Let us now consider the range $0.72 \leq \lambda < 0.92$ and assume the autarkic equilibrium. The marginal utilities are μ_1 and μ_2 in the good and the bad state respectively. The price offered by a *on the verge* seller would be $\frac{\mu_1 + \frac{1}{2}\lambda}{\mu_0}$, since $\lambda > \mu_1$ and agents would rationally expect the seller to deliver fully in their good state and nothing in their bad state. Therefore the price offered by a *on the verge* buyer is $\frac{\frac{1}{2}\mu_1 + \frac{1}{2}\mu_2}{\mu_0} > \frac{\mu_1 + \frac{1}{2}\lambda}{\mu_0}$ indicating that agents have an excess demand for the bond in this region. Moreover, trade in the bond alone is not possible, since the marginal utility from defaulting completely in the good state is 1 and the default penalty is less than 1 thus agents would default completely in all states. The only other possible candidate equilibrium, then, is one where both agents trade both the bond and partake in storage.

Let us now examine the equilibrium where agents optimize both over the bond and σ, the storage of the good. We first look at the case that agents partially default on the bond and next the one that they deliver fully.

[3] For $.92 \leq \lambda < 1$ the bond will not be traded and the equilibrium will be autarkic. The proof is the same as for Proposition 1.

The first order equation we get when agents optimize with respect to σ is:

$$\frac{1}{1-\sigma} = \frac{1}{2}\lambda + \frac{1}{2}\frac{2}{2\sigma + D}$$

Simplifying the above and recallng that $D = 2\sigma + 2 - \frac{2}{\lambda}$, the above expression then becomes:

$$4\lambda^2\sigma^2 + (8\lambda - 2\lambda^2)\sigma - (2\lambda^2 - 4\lambda + 4) = 0$$

$\Delta = 64\lambda^2 - 32\lambda^3 + 4\lambda^4 + 32\lambda^4 - 64\lambda^3 + 64\lambda^2 = 4\lambda^2(9\lambda^2 - 24\lambda + 32)$.

$9\lambda^2 - 24\lambda + 32 = 0$ has no real roots, since $\Delta < 0$. For $\lambda = 1$ its value is $9 \cdot 1 - 24 \cdot 1 + 32 > 0$. Hence, from Bolzano Theorem it is always positive.

Consequently, $\sigma = \frac{-4 + \lambda + \sqrt{9\lambda^2 - 24\lambda + 32}}{4\lambda}$, since the other root is negative.

The next step is to find the delivery rate from the first order equations of buying and selling the bond:

$$\lambda + \lambda = K\lambda + K\frac{1}{2\sigma + 1 - \frac{1}{\lambda}}$$

$$K = \frac{4\sigma\lambda + 2\lambda - 2}{2\sigma\lambda + \lambda}$$

Let us consider the delivery rate to be less than $\frac{1}{2}$ i.e.

$$\Rightarrow \sigma \leq \frac{4 - 3\lambda}{6\lambda}$$

Checking the above with the expression for σ we get

$$\frac{-4 + \lambda + \sqrt{9\lambda^2 - 24\lambda + 32}}{4\lambda} \leq \frac{4 - 3\lambda}{6\lambda}$$

$$\lambda \leq \frac{7}{9}$$

Up to $\lambda = \frac{7}{9}$ we get trade in both assets and agents partially deliver on the bond in their good states while completely defaulting in their bad.

After this λ we have to consider full delivery of the bond and solve again to check whether such an equilibrium exists.

Optimizing with respect to σ we get

$$\frac{1}{1-\sigma} = \frac{1}{2}\frac{2}{2\sigma+2-D} + \frac{1}{2}\frac{2}{2\sigma+D}$$

Equating the first order conditions of buying and selling the bond and assuming $K = \frac{1}{2}$

$$\frac{2}{2\sigma+2-D} + \lambda = \frac{1}{2}\frac{2}{2\sigma+2-D} + \frac{1}{2}\frac{2}{2\sigma+D}$$

when above equation yields positive σ and D satisfying the other constraints equilibrium exists where both are traded. It turns out that this is the case for $7/9 \leq \lambda < 0.92$. After that the equilibrium is autarkic. □

Proposition 3: ***For*** $0 < \lambda < 0.72$ ***the equilibrium will be autarkic***

Proof. Consider the range $0 \leq \lambda < 0.72$ and assume the autarkic equilibrium. In this region the expected delivery rate is 0 given that the marginal utility in the good state under autarky is less than the default penalty, and hence the price offered by the on-the-verge buyer is also 0 hence so autarky prevails here as wel. □

We have shown that the only region that the bond will be traded is when it is used in conjunction with storage. How is it possible that a redundant asset is traded in equilibrium? The answer to this again comes from the intuition that default is an externality. An agent defaulting on the bond would ideally like to default more than 100% in his bad state and hence improve his marginal utility there. Since he cannot do so, he must also restrict the quantity of bonds he is able to sell. This changes both the delivery rate of the bond and the relative attractiveness of trading other assets, in this simple example the storage of the good. Put differently, both the span and the *dimensions* of the span vary because of default. In such a setting the equilibrium which the market will pick will be inferior to the incomplete markets (GEI) equilibrium[4] and therefore the scope for government internvention in the markets increases. The question then is, how can the government determine the allocation with neither directly taxing agents nor altering their portfolio holdings. The culprit behind the suboptimality in the GED[5] equilibrium is default.

In the following section we show how introducing reserve requirements, that is forcing agents to hold more goods in storage than they would optimally like to do, reduces default and therefore improves welfare in this economy.

[4]see Geanakoplos and Polemarchakis (1986) for a discussion

[5]General Equilibrium with Default

4 Reserve Requirements

We argue that reserve requirements indeed reduce default and improve welfare. We consider a $GE(R,\lambda,\rho)$ where assets come with a requirement that an amount of at least ρ has to be held in storage if they were sold. This economy is a superset of the $GE(R,\lambda)$ economy considered previously. The calculations in this section are made on an economy with the same data as before with the added restriction on trade of assets.

There are two condition that ρ must satisfy for trade in the bond to occur. The first condition guarantees that agents do not default completely in their good state[6], while the second guarantees that there is no excess demand for storage under the reserve requirement. Take for example the case for which $\lambda < 0.72$. Agents would default completely on their obligations and the bond would not be traded as shown in Proposition 3. Alternatively, consider the area of λs greater than 1. For those levels of default penalties agents would not default completely on their obligations, but still, as shown in Proposition 1, would not be willing to trade the bond. We formalize these two arguments in the following Lemma.

Lemma
An equilibrium with trade in the bond exists only if the two following conditions hold.

Condition 1: The bond will be traded if the marginal utility of agents is lower than the default penalty in their good state. The level of reserve requirement then guarantees this is ρ^*. The condition becomes $\lambda > \frac{1}{1+\rho^*} \Rightarrow \rho^* > \frac{1}{\lambda} - 1$.

Condition 2: Let ρ^{**} be the minimum reserve requirement such that there is no excess demand for storage.

Condition 2a: *Agents sell the bond and deliver partially*

At this equilibrium agents would not prefer to store as long as:

$$\frac{1}{1-\rho^{**}} \geq \frac{1}{2}\lambda + \frac{1}{2}\frac{1}{1+2\rho^{**}-\frac{1}{\lambda}}$$
$$\rho^{**} \geq \frac{\lambda - 4 + \sqrt{9\lambda^2 - 24\lambda + 32}}{4\lambda}$$

Condition 2b: *Agents sell the bond and deliver fully in their good states*

[6] As shown by Dubey *et al.* (2005) if the delivery rate for an asset in state s is less than one in a one good economy, then there is an agent who defaults completely. Thus, we only have to guarantee that agents do not default completely in their good states.

At this equilibrium agents would not prefer to store as long as:

$$\frac{1}{1-\rho^{**}} \geq \frac{1}{2}\frac{2}{2\rho^{**}+2-D}+\frac{1}{2}\frac{2}{2\rho^{**}+D}$$
$$8\rho^{**2}+2D-D^2+2\rho^{**}-2 \geq 0$$

To guarantee trade in the bond agents have to hold in reserves at least the maximum between ρ^* and ρ^{**}, i.e. $\rho = \max\{\rho^*, \rho^{**}\}$.

Corollary
Under conditions 1 and 2 of the Lemma $\rho \geq \sigma$ *for all default penalties*

If there is no trade in the bond, then clearly $\rho > \sigma$. If trade in the bond already exists, then $\rho \geq \sigma$. For proof see propositions below.

Proposition 4: ***For*** $0.66 \leq \lambda < 0.72$ ***the bond will be traded in equilibrium under reserve requirements***

Proof. Under partial default the marginal utility for the good state will be $\frac{1}{2}\lambda$ and for the bad $\frac{1}{2}\frac{1}{1+2\rho^*-\frac{1}{\lambda}}$. The delivery rate will be $K = \frac{2\lambda}{\lambda+\frac{1}{1+2\rho^*-\frac{1}{\lambda}}}$.

For this area of default penalties $\rho^* > \rho^{**}$. Thus agents have to hold at least ρ^*. In addition, both D and K have to be positive and $K < 1/2$. This is satisfied for $0.66 \leq \lambda < 0.72$.

Without the reserve requirement the bond would not be traded for this area of $\lambda's$ as it has been shown in Propostition 4. Thus, we have managed to generate trade in the bond by forcing agents to hold a reserve requirement which is higher than their equilibrium level of storage. □

Proposition 5: ***For*** $0.72 \leq \lambda < 0.92$ ***the bond will be traded in equilibrium under reserve requirements***

Proof. For this area of default penalties ρ^* is always less than ρ^{**}. Thus, agents have to hold at least ρ^{**} to quarantee trade in the bond. Up to $\lambda = \frac{7}{9}$ agents default partially and the delivery rate as well as the actual delivery are given by the formulas above. Note that this is also the level of default penalty after which agents deliver fully in their good state when there are no default penalties. By holding more the required reserve requirement agents can increase their delivery up to the point that they start delivering fully.

For $\lambda > \frac{7}{9}$ agents deliver fully in their good states even if they are required to hold the minimum required, i.e. ρ^{**}. Thus, the final delivery for the asset is given by:

$$4\lambda\rho^{**2}+4\lambda\rho^{**}+2\lambda D-\lambda D^2+2D-2=0$$

As long as it yields a positive D under the relevant constraints, the equilibrium in which agents sell the bond and deliver fully in their good states exists. It turns out that the maximum level of λ for which the bond will be traded under reserve requirements is 0.92. □

Proposition 6: ***For*** $\lambda > 1$ ***the equilibrium will be autarkic under reserve requirements***

Proof. Assume that the default penalty is higher than 1. The bond should be traded, since agents would not default completely in their good states. Nevertheless, we showed in Propositions 1& 2 that agents would not actually prefer to trade the bond and would rather store some of their initial wealth. In Proposition 5 we showed that for default penalties greater than a level which depends on the endowment at t=0 (in the case we are examining $\lambda > 0.66$), agents would prefer to trade the bond, as long as they are required to put a certain amount as reserve requirements. The methodology is the same for this case as well, though the intuition behind is a bit different. Agents would not be required to keep a portion of their initial wealth in reserve requirements in order not to default completely in their good states, but to increase their delivery. Thus, the bond would become more appealling and agents would trade it. As before the following two relations have to hold:

$$4\lambda\rho^{**2} + 4\lambda\rho^{**} + 2\lambda D - \lambda D^2 + 2D - 2 = 0$$

and

$$8\rho^{**2} + 2D - D^2 + 2\rho^{**} - 2 \geq 0$$

As long as these two relations yield positive D and ρ^{**} equilibrium exist. Manipulating and adding them we get

$$-2\lambda D - 6\lambda\rho^{**} + 2\lambda D^2 - 4D + 4 - \lambda D^2 - 2\lambda \geq 0$$

Assume that $\rho^{**} > 0$ and $D > 0$, then the last equation becomes

$$\lambda D^2 - 4D + 4 > K > 0$$

where K is a positive number. The above relation gives no real solution for D for $\lambda > 1$, i.e. a contradiction.

Combining this with Propostition 6 we get that for $\lambda > 0.92$ the equilibrium is autarkic. This level of l is exactly the maximum one until which agents will both store and trade the bond in the absence of a reserve requirement. This will become important when we will examine the effect of reserve requirements on welfare. □

5 Properties of Equilibrium with Reserve Requirement

5.1 Trade

Proposition 7: ***For*** $.66 \leq \lambda < 0.72$ ***the bond starts being traded under reserve requirements.***

Proof. This is straightforward from Proposition 5.

Proposition 8: ***For*** $.72 \leq \lambda < \frac{7}{9}$ ***trade increases with the reserve requirement.***

Consider an equilibrium with partial default and some storage (optimised or not). The delivery is given by $2(1+\rho-1/\lambda)$ and the deliver rate by $2-\frac{2}{\lambda(1+2\rho)}$. The quantity traded is $\phi = D/(2K) = \frac{1+\rho}{2}\frac{\lambda-\frac{1}{1+\rho}}{\lambda-\frac{1}{1+2\rho}}$. As agents are delivering partially, $\lambda-\frac{1}{1+\rho}>0$. Differentiating, we find that the quantity of trade will increase with the reserve requirement for $\lambda < 1$. □

Proposition 9: ***For*** $\frac{7}{9} \leq \lambda < .92$ ***trade goes down when the reserve requirement increases.***

Proof. With full delivery we can take the derivative of the *on-the-verge* for trading the bond which is

$$\lambda = \frac{1}{2}\frac{1}{\frac{D}{2}+\rho} - \frac{1}{2}\frac{1}{1-\frac{D}{2}+\rho}.$$

Solving for $\frac{\partial D}{\partial \rho} = 2\frac{\mu_1^2-\mu_2^2}{\mu_1^2+\mu_2^2}$ we can see that the marginal utility in the state 1 is strictly less than that in state 2 and as the denominator is positive, then the delivery of agents is falling on ρ. When there is full delivery, then the quantity of bonds sold will equal the delivery on the bonds, and hence the trade in bonds falls on ρ for this region of λ. □

5.2 Welfare

Consider an equilibrium where there is partial delivery on the bond. Using $K = \frac{\lambda}{1/2\lambda+\mu_2}$, $D/2 = c_2-\rho = 1+\rho-1/\lambda$ and $\phi = \frac{D}{2K}$ we can take the derivative of a symmetric agents welfare with respect to ρ. Welfare is given by

$$w = ln(1-\rho)+1/2ln(1/\lambda)+1/2ln(1+2\rho-1/\lambda)-\lambda[\phi-D/2].$$

Taking the derivative and rearranging we find

$$\frac{\partial w}{\partial \rho} = [-\mu_0+\mu_2+\lambda/2]-4\mu_2^2[1/\lambda-(1+\rho)].$$

Proposition 10: ***For*** $\lambda < .72$ ***and*** $\lambda \geq .92$ ***welfare falls when the reserve requirment is introduced.***

Proof. The first expression is less than or equal to zero (depending on whether agents are optimising over the storage technology). The second expression is the difference between actual consumption of the agent and his original endowment plus the reserve. If the equilibrium is autarkic then the second expression is positive but with the negative coefficient the sign changes. Hence as ρ increases above the autarkic value, the welfare will fall but a falling rate. □

Proposition 11: ***For*** $.72 \leq \lambda < \frac{7}{9}$ ***welfare increases with the reserve requirement.***

Proof. Consider a region where agents optimise over both the bond and the storable good, and deliver partially in theis good states. In this region, the first expression is zero as it is the first order equation for the purchase of the bond. The second expression will be negative (positive with the coefficient) as consumption will be less than the endowment as they are delivering something. Now if we increase ρ slightly, the second term will dominate and hence the directional derivative will be positive. Thus, welfare will worsen with ρ in the region where we originally had autarky and it will improve in the region where the bond is traded initially. □

Proposition 12: ***For*** $\frac{7}{9} \leq \lambda < .92$ ***welfare goes down when the reserve requirement increases.***

Proof. Consider an equilibrium where there is full delivery on the bond and some storage. The consumption in the good state for each agent is now $1+\rho-D/2$ and in the bad state $\rho+D/2$. The delivery rate is given by $K = \dfrac{\mu_1 + 1/2\lambda}{\mu_1+\mu_2} = 1/2$ where we take state 1 to be the good state and state 2 to be the bad state. Rearranging this expression we get $\mu_1 + \lambda = \mu_2$. Now take the derivative of this equation with respect to ρ and we get that $\dfrac{\partial D}{\partial \rho} = 2\dfrac{\mu_1^2-\mu_2^2}{\mu_1^2+\mu_2^2}$ which is negative as the marginal utility in the state 1 is strictly less than that in state 2.

The welfare of this agent is given by

$$ln(1-\rho)+1/2ln(1+\rho-D/2)+1/2ln(\rho+D/2)-\lambda/2D.$$

since $\phi = D$. Taking the derivative of this expression with respect to ρ we get

$$-\mu_0+\mu_1+\mu_2+1/2\frac{\partial D}{\partial \rho}[\mu_2-\mu_1-\lambda].$$

We know that the last part of this expression is zero. The first part of the expression is zero when agents optimise over the bond and negative when they are forced to hold more liquidity than this. Under a reserve requirements the whole expression is negative Hence when there is already full delivery, welfare is falling when the reserve requirement is increasing. □

5.3 Numerical Representation

In this section we numerically determine the characteristics of different equilibria, differing by default penalty and reserve requirement. Figure 1 charts the net gain in welfare per agent of different equilibria when reserve requirements are allowed to emerge for the two agent, two state example discussed previously. It can be seen that for a given λ as ρ increases welfare increases and as λ increases we find pareto improvement by allowing for liquidity reserves.

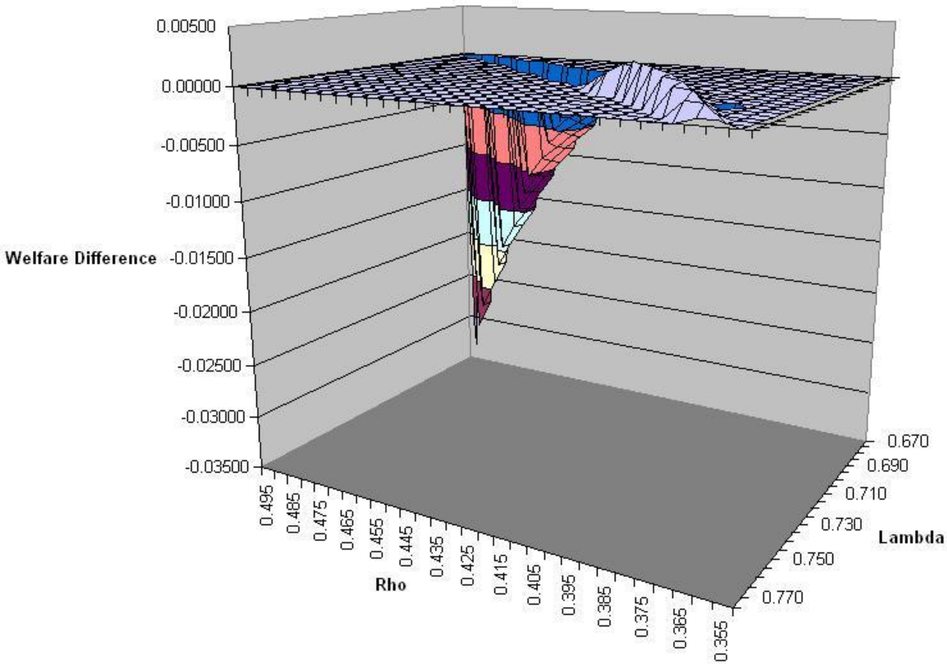


Figure 1: Net gain in welfare of Reserve Requirement Equilibrium over GED Equilibrium

6 Endogeneous Emergence of ρ and Constrained Optimality

In Section 6.1 we show that given a λ in the economy and a continuum of contracts with different reserve requirements, the market will choose the one that guarantees full delivery. In the Section 6.2 we show that if a continuum of contracts exist with different ρ and λ then the market will have a set of assets to choose from each which would result in full delivery in the good state, with the one that it chooses being random. In this setting there is a positive role for governments to play in choosing the optimal default penalty.

6.1 Multiple ρ

In determining which asset the market will choose in equilibrium, we need to recall that ρ's enforceability affects all assets traded by the seller. In order to determine whether a contract will be traded or not we consider an initial equilibrium and we assess whether a new contract with a per contract reserve requirement will be traded. Put differently, if we start from an initial equilibrium with $\hat{\rho} < \rho^*$, then by introducing per unit higher reserve requirements we reach the constrained optimal allocation. Note that this monotonicity argument rests on the fact that when we perturb the asset span slightly by introducing a new asset a previously traded asset will no longer be traded and welfare will improve.

To make matters concrete, consider the above example with a level of ρ picked such that an equilibrium with trade in the bond occurs with price q. Now, consider another contract with the same characteristics but agents must set aside a $\hat{\rho} \to 0$ per unit of contract traded ($\hat{\phi}$). From the first order conditions we can quickly see that the *on-the-verge* seller will require a price of at least

$$\hat{q} = \frac{\hat{\rho}[\mu_0 - \sum_{s \in S} \mu_s] + \lambda}{\mu_0}.$$

Now the current traded price is

$$q = \frac{\lambda}{\mu_0}.$$

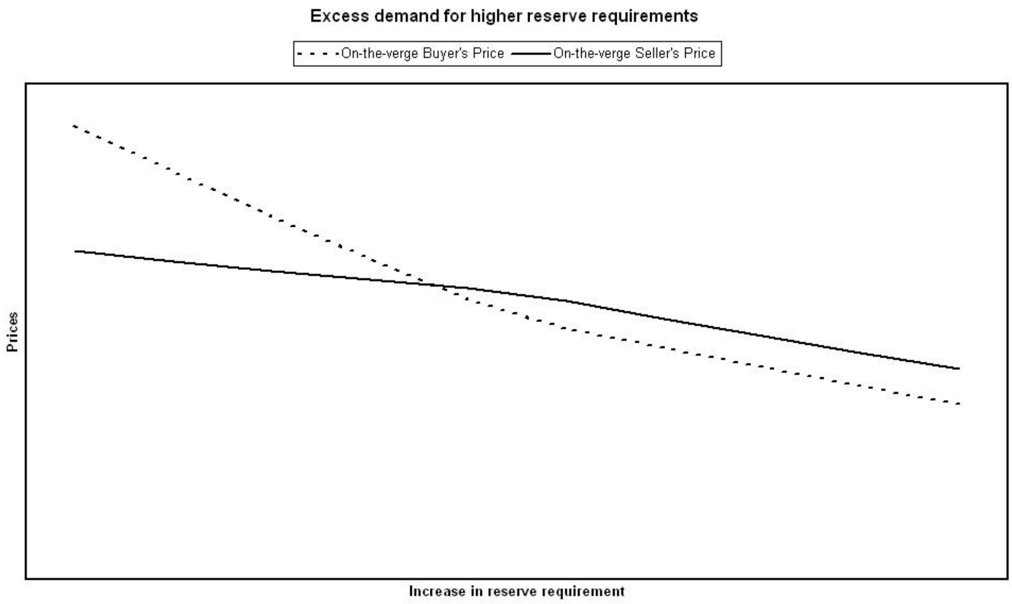
and so

$$\hat{q} - q = \frac{\hat{\rho}}{\mu_0}[\mu_0 - \sum_s \mu_s] > 0.$$

We know that $\mu_0 - \sum_s \mu_s > 0$ as agents are holding a higher amount of liquid wealth than they would if they optimised over it and $\hat{q} \to q$ as $\hat{\rho} \to 0$. Hence the *on the verge* seller of the new contract will demand a price marginally higher than the current traded price.

The *on the verge* buyer would rationally expect the seller to deliver fully in their good state (as their marginal utility is now strictly less than the default penalty) and hence would be willing to pay up to $.5\frac{\sum_{s \in S} \mu_s}{\mu_0}$. If this is greater than the *on-the-verge* seller's price, then the contract with the variable requirement would be traded. Since the price for marginal buyer jumps to the full delivery price and the price for the marginal seller gradually increases to that level, the variable reserve requirement will be traded. However an equilibrium with the variable requirement and the original requirement would collapse into an equilibrium with a higher flat reserve requirement, hence the market is able to distinguish between contracts with different reserve requirements. Furthermore it would pick the contract which results in agents delivering fully in their good state (i.e. agents do not default partially any more). Given that default cannot be reduced any further, the scope for government intervention here is limited.

To illustrate our argument assume a default penalty of 0.7. The required reserve requirement to get trade in the bond for this level of λ is 0.43. Solving for the equilibrium we get partial delivery is the good state and a delivery rate less than 0.5. Assume now that a contract with a higher reserve requirement, say 0.435 is available in that equilibrium. If it were to be traded the price that the marginal buyer would be willing to offer would be 0.43. The price that the marginal seller request at this equilibrium is 0.3997. Thus, there is excess demand for this contract and it will be traded in the initial equilibrium. This holds for any reserve requirement higher than the minimum required one until we reach full delivery and the price of the marginal buyer is equal to the price of the marginal seller. After this level of ρ the latter would request a higher price than the former is willing to pay, thus such a contract would not be traded. We present the above in the following diagram.



6.2 Multiple ρ and λ

If there exists a continuum of contracts with varying ρ and λ then using the argument above, as well as the argument in Dubey et al it is straightforward that multiple possible contracts exist for which agents are indifferent. Figure 2 shows different contracts with varying ρ and λ. The arc represents all the contracts in this space of contracts which would give rise to full delivery in the good state of the world for each agent and thereby allocations are contrained optimal. Although the market is indifferent among the contracts on the arc, they result in different welfare outcomes. In this setting the government, given its welfare criteria, may choose the λ which would result in the highest welfare[7].

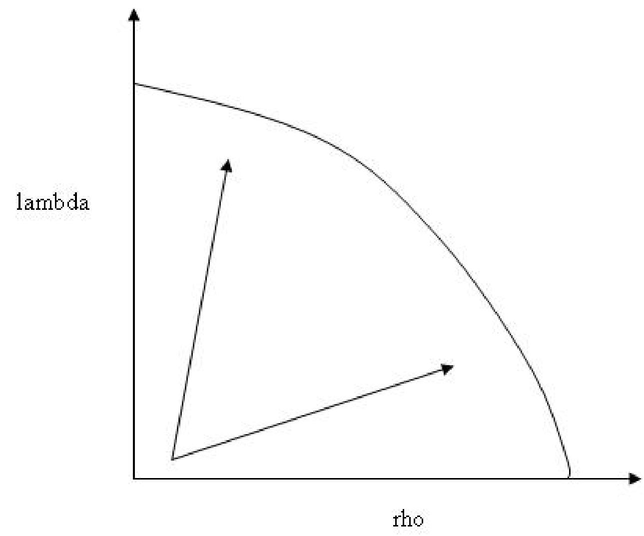


Figure 2: Contracts Chosen when a continuum of contracts exist over both ρ and λ

7 Application: A Monetary Economy

In the previous section we studied the effect of reserve requirements in an economy with a single storable consumable good. Now we study a (fiat) money economy and the effect of monetary reserve requirements in restoring trade in the bond market when it would otherwise collapse. To do so we will introduce fiat money into the model and consider two perishable consumable goods. We consider a two-period economy where agents know the present (period 0) but are uncertain about the future where one of $s \in \{1,2\}$ states may occur both with probability $1/2$. There are two perishable goods in the economy and agents have utility for both in all periods. Trade takes place between two agents $\{\alpha, \beta\}$ and each transaction is facilitated in money. There is also a bond market through which agents can borrow money at t=0 and repay at t=1. The market is risky (and hence default) since agents choose how much to repay after uncertainty is resolved. The exogenous parameters in the model are given as follows:

- $l \in L = \{1,2\}$ commodities.
- $s \in S = \{1,2\}$ states of nature.
- $s^* \in S^* = \{0\} \cup S$ set of all states.
- $t \in T = \{0,1\}$ where $s = 0$ when $t = 0$ and $s \in S = \{1,2\}$ when $t = 1$.

[7]The counter argument of the government fixing a reserve requirement and allowing the market to choose a default penalty is identical in this environment but unrealistic.

- $h \in H = \{\alpha, \beta\}$ set of agents in the monetary economy.
- $e^h_{s^*} = (e^h_{s^*}) \in R^{S^*L}_+$ endowment for agent $h \in H$ in state s of perishable goods.
- The private monetary endowment in state s^* belonging to agent h is $m^h_{s^*}$.
- ρ^h the reserve requirement for agent $h \in H$.
- $u^h : R^{S^* \times L}_+ \rightarrow R$ utility function of agent $h \in H$.

7.1 Governments and Central Banks

There is a central bank which has the authority to act on markets on behalf of its government. The actions of the central bank will be taken as exogenous.

In the short term (intra period) money market the central bank will fix the amount of money lent to agents $\{M_{s^*}\}$ with the interest rate being endogenously determined ($\{r_{s^*}\}$).

7.2 The Time Structure of Markets

In each period $t \in T$, four markets meet: first the short-term (intraperiod) loan market followed by the bond market and the commodity markets. Finally, short-term bonds come due at the end of the period. The bond market settles in the last period.

The first period thus has four transaction moments: short bonds, long (defaultable) bonds, commodities, short-bond deliveries while the second period has the following transaction moments: short bonds, long bond deliveries, commodities, short-bond deliveries. In the first period there is no delivery on the bond market while in the last period there is no market for bonds.

Figure A indicates our time line, including the moments at which the various loans and bonds come due. We make the sequence precise when we formally describe the budget set.

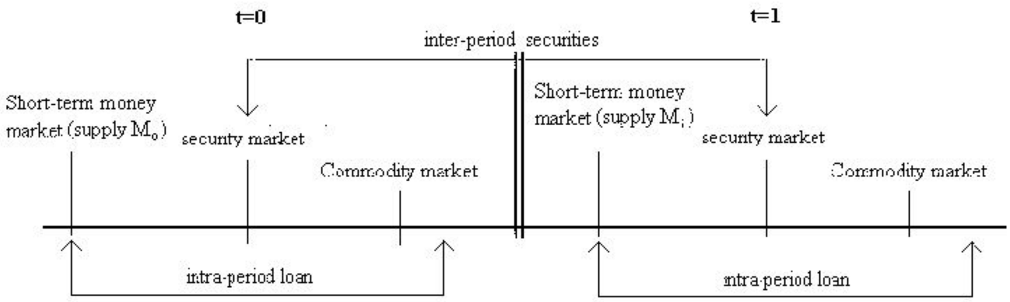


Figure 3: Time Line of Monetary Economy

7.3 Structure of the Model

Agent $h \in (\alpha, \beta)$ sells good $l \in L$ in each period with quantities given by q_{s^*l}. Agents α is endowed only with the first good while agent β only with the second.

Prices of goods are determined in equilibrium and are taken as fixed by agents p_{s^*l}. The money traded for a good in state s^* by agent h is $b^h_{s^*l}$.

The short-term money repaid by agent h in state s^* is $\mu^h_{s^*}$.

Agents can act in the bond market to transfer wealth intertemporally but are subject to a reserve requirement. We denote by $\bar{\mu}^h$ and $\bar{d}^h$ the amounts that agent h chooses to borrow and deposit in the bond market, respectively, at the ex-ante interest rate $\bar{r}$. The amount that each agent chooses to repay is state-contigent and is denoted by D^h_s. The bond market is an anomymous market and there is pooling of default, i.e. each agent that deposits in this market receives the effective delivery rate on each unit of deposit, which is given by K_s.

7.4 Market Clearing Conditions

7.4.1 Goods Market

The goods market clears when the amount of money offered for goods is exchanged for the quantity of goods offered for sale.

In state $s^* \in S^*$ for good $l \in L$.

$$p_{s^*l} = \frac{\sum_{h \in H} b^h_{s^*l}}{q^{h'}_{s^*l}}$$

That is, the price of a good sold by agent h' will be determined by the ratio of the money offerd to purchase this good by other agents divided by the quantity of the good sold.

7.4.2 Money Market

The intraperiod money market clears when the amount of money which agents offer to repay at the end of the period ($\mu^h_{s^*}$ by agent $h \in H$ for all $s^* \in S^*$) is exchanged for the (exogenously determined) amount of money lent by the central bank at the (endogenously determined) interest rate at the beginning of each period. In state $s^* \in S^*$ the central bank lends M_{s^*}. In each period and state the interest rate is determined endogenously:

For Period 0

$$(1 + r_0) = \frac{\sum_{h \in H} \mu^h_{0k}}{M_{0k}}$$

For $s \in S$

$$(1 + r_s) = \frac{\sum_{h \in H} \mu^h_{sk}}{M_{sk}}$$

We assume no default in the money market, and rationalise this by the ability of the government to impose harsh default penalities when loans extended by them are defaulted upon, or by the borrowers being of sufficiently high credit worthiness that default will not occur.

7.4.3 Bond Market

The bond market clears when the volume of money borrowed $\sum_{h \in H} \frac{\bar{\mu}^h}{1+\bar{r}}$ equals the amount of deposits $\sum_{h \in H} \bar{d}^h$.

Agents are promised a nominal repayment of $1+\bar{r}$ for each unit they deposit in the bond market in state s but receive K_s - the delivery rate on the bond - and is given by:

$$K_{sj} = \begin{cases} \frac{\sum_{h \in H} D_{sj}}{\sum_{h \in H} \bar{\mu}^h} & \text{if } \sum_{\mathrm{h} \in \mathrm{H}} \bar{\mu}^{\mathrm{h}} > 0 \\ \text{or arbitrary if } \sum_{\mathrm{h} \in \mathrm{H}} \bar{\mu}^{\mathrm{h}} = 0 \end{cases}$$

The price of the bond, $\frac{1}{1+\bar{r}}$ is determined endogenously in equilibrium and will be the total quantity of money offered to purchase the bond divided by the volume of deposits and is equivalent to adding $\frac{1}{1+\bar{r}}$ infront of the equation above, i.e. $1+\bar{r} = \frac{\sum_h \bar{\mu}^h}{\sum_h \bar{d}^h}$.

The variables determined in equilibrium and taken by agents as fixed is given by $\eta = \{p, r, \bar{r}\}$. The choices by agent $h \in (\alpha, \beta)$ is given by $\square^h = \{q^h, b^h, \mu^h, \bar{d}^h, \bar{\mu}^h, D^h\}$.

7.5 Budget Sets for Agents

Agent α maximimises intertemporal consumption with a discount factor of 1 and subjective belief .5 of each state ocurring. Therefore, considering agent α's endowments of goods and money, preferences and subjective probabilities, his optimization problem is (Lagrangian multipliers are in brackets):

$$\max_{q_{s1}^{\alpha}, b_{s2}^{\alpha}, \mu_s^{\alpha}, \bar{\mu}^{\alpha}, \bar{d}^{\alpha}, D_s^{\alpha}} \Pi^{\alpha} = u(e_{01}^{\alpha} - q_{01}^{\alpha}) + u\left(\frac{b_{02}^{\alpha}}{p_{02}}\right) + \sum_{s \in S} .5u(e_{s1}^{\alpha} - q_{s1}^{\alpha}) + \sum_{s \in S} .5u\left(\frac{b_{s2}^{\alpha}}{p_{s2}}\right)$$
$$- .5\lambda \sum_{\mathrm{s} \in \mathrm{S}} \max\left(\bar{\mu}^{\alpha} - \mathrm{D}_{\mathrm{s}}^{\alpha}\right), 0$$

$$s.t. \quad b_{02}^{\alpha} + \bar{d}^{\alpha} + \rho^{\alpha} \leq \frac{\mu_0^{\alpha}}{1+r_0} + \frac{\bar{\mu}^{\alpha}}{1+\bar{r}} + m_0^{\alpha} \qquad (\psi_{01}^{\alpha})$$

($i.e.$, expenditure for good 2 at t = 0 + bond deposits + reserve requirement ≤ amount borrowed short – term at t = 0 + bond borrowing + initial private monetary endowment)

$$\mu_0^{\alpha} \leq p_{01} q_{01}^{\alpha} \qquad (\psi_{02}^{\alpha})$$

($i.e.$, short – term loan repayment ≤ good 1 sales at t = 0)

$$b_{s2}^{\alpha} + D_s^{\alpha} \leq \frac{\mu_s^{\alpha}}{1+r_s} + \bar{d}^{\alpha} \cdot K_s \cdot (1+\bar{r}) + \rho^{\alpha} + m_s^{\alpha} \qquad (\psi_{s1}^{\alpha})$$

($i.e.$, expenditure for good 2 in state s ∈ S + repayments in the bond market in state s ∈ S ≤ amount borrowed short – term in state s ∈ S + bond deposits and interest payment in state s ∈ S + reserve requirement + private monetary endowment)

$$\mu_s^{\alpha} \leq p_{s1} q_{s1}^{\alpha} \qquad (\psi_{s2}^{\alpha})$$

($i.e.$, short – term loan repayment ≤ good 1 sales in state s ∈ S)

The default penalty is λ and $\bar{\mu} - D_s$ is the amount of his promise that the agent defaults.

The optimization problem is analagous for agent β as well, but instead of buying good 2 and selling good 1 he is doing the opposite:

$$\max_{q^{\beta}_{s2}, b^{\beta}_{s1}, \mu^{\beta}_{s}, \overline{\mu}^{\beta}, \bar{d}^{\beta}, D^{\beta}_{s}} \Pi^{\beta} = u(e^{\beta}_{02} - q^{\beta}_{02}) + u\left(\frac{b^{\beta}_{01}}{p_{01}}\right) + \sum_{s \in S} \theta_s u(e^{\beta}_{s2} - q^{\beta}_{s2}) + \sum_{s \in S} \theta_s u\left(\frac{b^{\beta}_{s1}}{p_{s1}}\right)$$
$$- .5\lambda \sum_{\mathrm{s} \in \mathrm{S}} \max(\overline{\mu}^{\beta} - \mathrm{D}^{\beta}_{\mathrm{s}}), 0$$

$$s.t. \quad b^{\beta}_{01} + \bar{d}^{\beta} + \rho^{\beta} \leq \frac{\mu^{\beta}_{0}}{1 + r_0} + \frac{\overline{\mu}^{\beta}}{1 + \bar{r}} + m^{\beta}_{0} \quad (\psi^{\beta}_{01})$$

(*i.e.*, expenditure for good 1 at t = 0 + bond deposits + reserve requirement ≤ amount borrowed short – term at t = 0 + bond borrowing + initial private monetary endowment)

$$\mu^{\beta}_{0} \leq p_{02} q^{\beta}_{02} \quad (\psi^{\beta}_{02})$$

(*i.e.*, short – term loan repayment ≤ good 2 sales at t = 0)

$$b^{\beta}_{s1} + D^{\beta}_{s} \leq \frac{\mu^{\beta}_{s}}{1 + r_s} + \bar{d}^{\beta} \cdot K_s \cdot (1 + \bar{r}) + \rho^{\beta} + m^{\beta}_{s} \quad (\psi^{\beta}_{s1})$$

(*i.e.*, expenditure for good 1 in state s ∈ S + repayments in the bond market in state s ∈ S ≤ amount borrowed short – term in state s ∈ S + bond deposits and interest payment in state s ∈ S + reserve requirement + private monetary endowment)

$$\mu^{\beta}_{s} \leq p_{s2} q^{\beta}_{s2} \quad (\psi^{\beta}_{s2})$$

(*i.e.*, short – term loan repayment ≤ good 2 sales in state s ∈ S)

7.6 Equilibrium

We say that $(\eta, (\square^h)_{h \in H})$ is an **Monetary Equilibrium with Default** and denote it MED for the economy $E = (u^h, e^h, m^h)_{h \in H}$ if and only if:

1. $(\square^h) \in Argmax_{\square^h \in B(\eta)} \; U(x^h)$
2. $p_{s^* l} \sum_{h \in H} q^h_{s^* l} = \sum_{h \in H} b^h_{s^* l}$
3. $\dfrac{\sum_{h \in H} \mu^h_{s^*}}{(1 + r_{s^*})} = M_{s^*}$

4. $\dfrac{\sum_{h\in H}\bar{\mu}^h}{(1+\bar{r})} = \sum_{h\in H}\bar{d}^h$

5. $K^{j^*}_{s^*} = \left\{ \begin{array}{ll} \dfrac{\sum_{h\in H} D^{hj}_s}{\sum_{h\in H}\bar{\mu}^h} & \text{if} \sum_{h\in H}\bar{\mu}^h > 0 \\ \text{arbitrary} & \text{if} \sum_{h\in H}\bar{\mu}^h = 0 \end{array} \right\}$

for agents $\forall s^* \in S^*$ and $\forall \gamma \in C$, $\forall \alpha, \beta \in C$, $\forall j^* \in J^*$ and $h \in H$.

Condition 1 says that all agents optimise; 2 says that all commodity markets clear, or equivalently that price expectations are correct, 3 and 4 says that all credit markets clear, or equivalently, that predictions of interest rates are correct, 5, together with the budget set, says that each potential buyer of an asset is correct in his expectation about the fraction of promises that get delivered.

7.7 Simulation

In this section we will parametrize our model and show that for default penalties for which the bond market collapses, i.e. both agents would default completely in both states if the bond were to be traded, introducing a monetary reserve requirement removes this problem and the bond market becomes active.

Table 1: Data of Economy

m_0^α	0.100	M_0	1.000	e_0^β	10.000
m_1^α	0.200	M_1	1.000	e_1^β	10.000
m_2^β	0.100	M_2	1.000	e_2^β	10.000
m_0^β	0.100	e_0^α	10.000	λ	1.145
m_1^β	0.100	e_1^α	10.000		
m_2^β	0.200	e_2^α	10.000		

The table below shows what occurs when reserve requirements are introduced. The first column shows the equilibrium without reserve requirements and the second column shows what occurs when agents who borrow are required to place .05 of their period 0 outside money in storage till period 1. We can see that this stimulates trade in the bond market and improves welfare[8][9].

[8]Rounding is made after the difference is calculated.

[9]Note that agents only deliver in their good state i.e. they default completely in their bad state.

Table 2: Endogenous variables

Endogenous variables	Equilibrium **without** reserve requirements	Equilibrium **with** reserve requirements	Change
$\bar{\mu}^{\alpha}$		0.002	0.002
$\bar{d}^{\alpha}$		0.001	0.001
$\bar{\mu}^{\beta}$		0.002	0.002
$\bar{d}^{\beta}$		0.001	0.001
D_1^{α}		0.002	0.002
D_2^{α}		0.000	-
D_1^{β}		0.000	-
D_2^{β}		0.002	0.002
$\bar{r}$		0.437	0.437
Π^{α}	2.984	2.986	0.003
Π^{β}	2.984	2.986	0.003

8 Concluding Remarks

This paper proposes a market-based mechanism to stimulate trade in assets which would otherwise remain untraded. If the asset is already traded, introducing reserve requirements reduces default, increases liquidity in the asset and results in a constrained optimal outcome. Finally, we show that this mechanism is readily adaptable to a (fiat) monetary setting with similar results.

9 References

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10 Appendix

Table 3: Endogenous variables

Endogenous variables	Equilibrium **without** reserve requirements	Equilibrium **with** reserve requirements	Change
μ_0^α	0.600	0.550	-0.050
μ_1^α	0.867	0.873	0.007
μ_2^α	0.433	0.528	0.094
μ_0^β	0.600	0.550	-0.050
μ_1^β	0.433	0.528	0.094
μ_2^β	0.867	0.873	0.007
$\bar{\mu}^\alpha$		0.002	0.002
$\bar{d}^\alpha$		0.001	0.001
$\bar{\mu}^\beta$		0.002	0.002
$\bar{d}^\beta$		0.001	0.001
q_{01}^α	4.546	4.762	0.216
q_{11}^α	4.348	4.165	-0.183
q_{21}^α	4.348	4.165	-0.183
b_{02}^α	0.600	0.550	-0.050
b_{12}^α	0.867	0.873	0.007
b_{22}^α	0.433	0.528	0.094
q_{02}^β	4.546	4.762	0.216
q_{12}^β	4.348	4.165	-0.183
q_{22}^β	4.348	4.165	-0.183
b_{01}^β	0.600	0.550	-0.050
b_{11}^β	0.433	0.528	0.094
b_{21}^β	0.867	0.873	0.007
D_1^α		0.002	0.002
D_2^α		0.000	-
D_1^β		0.000	-
D_2^β		0.002	0.002
r_0	0.200	0.100	-0.100
r_1	0.300	0.401	0.101
r_2	0.300	0.401	0.101
$\bar{r}$		0.437	0.437
K_1		0.331	0.331
K_2		0.331	0.331
p_{01}	0.132	0.116	-0.017
p_{11}	0.199	0.210	0.010
p_{21}	0.100	0.127	0.027
p_{02}	0.132	0.116	-0.017
p_{12}	0.100	0.127	0.027
p_{22}	0.199	0.210	0.010
Ψ_{01}^α	1.667	1.818	0.152
Ψ_{11}^α	0.577	0.572	-0.581
Ψ_{21}^α	1.154	0.948	-0.206
Ψ_{01}^β	1.667	1.818	0.152
Ψ_{11}^β	1.154	0.948	-0.206
Ψ_{21}^β	0.577	0.572	-0.004
Π^α	2.984	2.986	0.003
Π^β	2.984	2.986	0.003