Asset Pricing in Large Information Networks

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Motivation

- In real-world financial markets, investors constantly communicate and learn from each other. Yet, standard asset pricing models assume away such interaction among investors.
- Empirically, social networks, or more generally **information networks**, have been shown to be important in explaining investors' trading decisions and portfolio performance.
  - Hong, Kubik, and Stein (2004) provide evidence that fund managers' portfolio choices are influenced by word-of-mouth communication.
  - Ivkovic and Weisbenner (2007) attribute more than a quarter of the correlation between households' stock purchases and stock purchases made by their neighbors to word-of-mouth communication.
  - Cohen, Frazzini, and Malloy (2007) posit that there is communication via shared education networks between fund managers and corporate board members.

Networks and asset pricing

- The theoretical literature on networks and asset pricing is quite limited (e.g., Ozsoylev (2005) and Colla and Mele (2008)).
- One limitation of current theoretical models is the absence of closed form solutions.
- This limitation is not surprising given the complexity arising from the combination of networks, rational agents and endogenous price formation.
- If one is willing to drop the assumption of rationality, then the analysis is significantly simplified.
  - DeMarzo, Vayanos, and Zwiebel (2003) propose a boundedly-rational model of opinion formation in social networks, and show that agents, who are "well-connected", may have more influence in the overall formation of opinions regardless of their information accuracies.

Networks in sociology

- Several studies have shown remarkable similarities between different large-scale networks that arise when humans interact, like friendship networks, networks of co-authorship and networks of e-mail correspondence.
- E.g., Milgram (1967), Barabasi and Albert (1999), Watts and Strogatz (1998), Chung and Li (2006).
- Specifically,
  - these networks tend to be **sparse**:
    # of connections between nodes are of the same order as # of nodes
  - they have **small effective diameter**:
    the so-called small world property
  - **power laws** govern their degree distributions:
    # of connections associated with a node is power law distributed

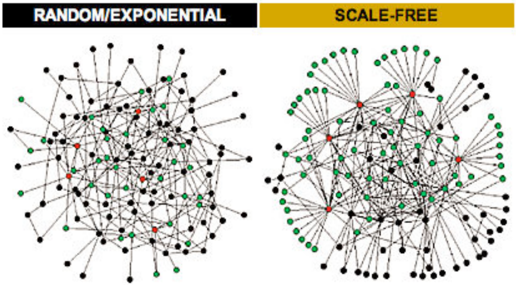
Random networks vs power-law networks

- If we were to generate a social network in a random manner by creating links between people independently with some probability p, then the fraction of people with k many links would decrease exponentially in k.
- However, most large social networks do not fit into the random network framework.
- Instead, in these social networks, the fraction of people with k many links decreases only polynomially in k. In other words, the degree distributions of many large social networks satisfy power-laws.
  - Our focus is on how information disseminates in social networks.
  - Recent studies show that information flow in social groups also exhibit a pattern which is consistent with an underlying network with a power-law degree distribution (e.g., Wu, Huberman, Adamic, and Tyler (2004)).

Power-law networks

Comparing Random and Scale-Free Distribution

In the random network, the five nodes with the most links (in red) are connected to only 27% of all nodes (green). In the scale-free network, the five most connected nodes (red) are connected to 60% of all nodes (green).



Source: the journal Nature

- Power law distributions characterize what are known as “scale-free” networks. In ”scale-free” networks,
  - some have “many” links and others have ”few” links.

Our approach

- Focus on studying a subclass of the general class of large-scale networks that satisfy these properties, and focus on asset pricing implications for this subclass of networks.
- The number of agents in the stock market's investor network is very large. A large economy approximation to the economy with a finite number of investors therefore seems to be in place.
- Theoretically, such an approximation is helpful, since we know, e.g., from the study of noisy rational expectations equilibria, that tractable solutions often can be found in large economies (Hellwig (1980) and Admati (1985).

Noisy Rational Expectations Equilibrium + Large-scale agent network

Our contribution

- We study asset pricing in a static, large, NREE economy in which agents communicate according to a network.
- We mostly focus on networks that are sparse and have power law degree distributions, in line with empirical studies of large scale human networks.
- Our contribution is two-fold:
  - the general existence theorem and the subsequent analysis of asset pricing implications
    - e.g., price volatility is a non-monotone function of network connectedness, as is average expected profit
    - profit distribution among investors is intimately linked to the properties of the information network
  - welfare implications and examination of what types of networks are "stable"
    - if agents are ex ante identical, strong conditions are needed to allow for non-degenerate network structures to arise
    - if agents face different costs of forming links, power-law distributed networks can arise naturally

Our setup - I

- **Assets**
  - risky asset in fixed supply $\tilde{Z} \sim N(\bar{Z}, \Delta^2)$, with value $\tilde{X} \sim N(\bar{X}, \sigma^2)$
  - risk-free asset in elastic supply with zero return
- **Network** of N agents, $\mathcal{G} = (\overbrace{\{1, \ldots, N\}}^{\text{agents}}, \overbrace{\mathcal{E}}^{\text{network relationship}})$.
  - $(i, j) \in \mathcal{E} \Leftrightarrow$ agent i connected to j
  - $\mathcal{E}$ is reflexive and symmetric.
  - $\mathbf{W} \in \mathbb{R}^{N \times N}$ is *neighborhood matrix*
  - $(\mathbf{W})_{ij}$ is number of common neighbors for agent i and j
  - $(\mathbf{W})_{ii}$ gives agent i's connectedness
  - Degree distribution, $d : \{1, \ldots, N\} \to \{1, \ldots, N\}$, $d(i) = |\{j : (\mathbf{W})_{j,j} = i\}|/N$
  - Network connectedness is given by $B = s^{-2} \sum_i i \times d(i)$
- **Agents**
  - price-taking CARA expected utility maximizers, with risk aversion equal to unity

Our setup - II

- **Information**
  - Each agent receives signal, $\tilde{x}_i = \tilde{X} + \eta_i$, such that
    - ★ agents with more neighbors get more precise signals
    - ★ error terms of signals of two agents who have no common neighbors are independent
    - ★ signals of two agents who have the same neighbors are identical
    - ★ agents with more common neighbors have more similar signals
  - *One (convenient) way:*
    $\tilde{x}_i := \tilde{X} + \frac{\sum_{j \in R(i)} \tilde{\epsilon}_j}{\mathbf{W}_{ii}}$, where $\tilde{\epsilon}_j \sim N(0, s^2)$ i.i.d.
  - Covariance matrix of signal:
    $\mathbf{S} = s^2 \mathbf{D}^{-1} \mathbf{W} \mathbf{D}^{-1}$, where $\mathbf{D} = diag(\mathbf{W}_{11}, \ldots, \mathbf{W}_{NN})$
  - Agent i's information set is $\{x_i, p\}$ and her demand is $\psi_i(x_i, p)$

Equilibrium

- **Linear NREE in finite economy with n agents:**

  Price function

  $$\tilde{p} = \pi_0 + \sum_{i=1}^{n} \pi_i \tilde{x}_i - \gamma \tilde{Z}_n$$

  such that

  - *Markets always clear*:
    $\tilde{Z}_n = \sum_{i=1}^{n} \psi_i(\tilde{x}_i, \tilde{p})$ for all realizations of $\{\tilde{x}_i\}_i$, $\tilde{X}$, and $\tilde{Z}_n$
  - *Agents optimize rationally*:
    each agent maximizes expected utility under rational expectations conditional on his information

  $$\psi_i(\tilde{x}_i, p) = \frac{E[\tilde{X}|\mathcal{I}_i] - p}{Var[\tilde{X}|\mathcal{I}_i]}$$

Existence theorem

Theorem 1 *Assume a sequence of n-agent markets, $\mathcal{M}^n$, $n = 1, 2, \ldots$, in which agents' information sets are defined by (7), the covariance matrix $\mathbf{S}^n$ of market $\mathcal{M}^n$ is defined via equation (6), where each matrix $\mathbf{W}^n$ satisfies equations (1)-(3), and also*

$$\|\mathbf{W}^n\|_\infty = o_p(n), \tag{10}$$

$$\lim_{n\to\infty} \frac{\sum_{i=1}^n (\mathbf{W}^n)_{ii}}{s^2 n} = B + o_p(1) > 0. \tag{11}$$

Then, with probability one, the equilibrium price converges to

$$\tilde{p} = \pi_0^* + \pi^* \tilde{X} - \gamma^* \tilde{Z}, \tag{12}$$

where

$$\pi^* = \gamma^* B, \tag{13}$$

$$\gamma^* = \frac{\sigma^2\Delta^2 + \sigma^2 B}{B\sigma^2\Delta^2 + \Delta^2 + B^2\sigma^2}, \tag{14}$$

$$\pi_0^* = \gamma^* \frac{\bar{X}\Delta^2 + \bar{Z}B\sigma^2}{\sigma^2\Delta^2 + \sigma^2 B}. \tag{15}$$

- Condition (11) ensures that the average number of connections for agents in the network is well defined as the economy grows.
- Condition (10) imposes a restriction on the asymptotic behavior of agents' degrees.

Socially plausible networks

What is socially plausible?

Networks that satisfy

$$d(i) \sim i^{-\alpha},$$

are said to have *power-law* distributed degree distributions, with *tail exponent* α.

Empirically, it has been argued that α is typically larger than 2 but smaller than 3 in power-law networks.
(see, e.g., Grossman et. al. (2007) and Barabasi and Albert (1999))

- One may question the plausibility of network topologies that arise in our large-economy equilibrium (Theorem 1).
- Under some technical conditions, we show that Theorem 1 holds for power-law networks with α larger than 2.

Interpreting parameters

- Power-law networks with low α's are said to be **heavy-tailed**.
- Recall from Theorem 1 that B is the average number of links in the large-economy equilibrium.
- Under some technical conditions, we can relate B to α:

Power-law networks in the large-economy equilibrium

$B(\alpha)$ is a decreasing, strictly convex function of α, such that $\lim_{\alpha\to\infty} B(\alpha) = 1$, $\lim_{\alpha\searrow 2} B(\alpha) = \infty$.

Financial relevance of networks

- Examine asset pricing implications of information networks:
    - volatility
    - market efficiency
    - trading profits
    - trading volume
    - risk-return trade-off
    - portfolio holdings
- Do we have trivial implications? How relevant are networks?

Volatility and market efficiency - I

- Variance decomposition:

$$var(\tilde{p}) = \underbrace{(\pi^*)^2\sigma^2}_{\text{information driven}} + \underbrace{(\gamma^*)^2\Delta^2}_{liquidity(supply)driven} .$$

1. The information driven volatility component increases as network connectedness increases.
2. The liquidity driven volatility component is a non-monotonic function of network connectedness. In particular,

$$\frac{\partial\,(\gamma^*)^2\Delta^2}{\partial B} < 0, \qquad \text{if } \; B > \frac{\Delta}{\sigma} - \Delta^2,$$
$$\frac{\partial\,(\gamma^*)^2\Delta^2}{\partial B} \geq 0, \qquad otherwise.$$

3. The price volatility is a non-monotonic function of network connectedness.

Volatility and market efficiency - II

- Excess (price) volatility corresponds to price being more volatile than the payoff, i.e.

$$var(\tilde{p}) > var(\tilde{X}) = \sigma^2.$$

1. There can be excess (price) volatility. There is excess volatility if and only if $B < \Delta^2$ and $\sigma > \sqrt{\frac{\Delta^2}{\Delta^4 - B^2}}$.
2. Also, there can be excess return volatility.
3. Market efficiency increases as the network's connectedness increases. That is,

$$\frac{\partial \; Var\left(\tilde{X} | \tilde{p}\right)}{\partial B} < 0.$$

Trading profits - I

For simplicity, assume $\bar{X} = \bar{Z} = 0$.

- Agent i's ex-ante trading profit, Π_i, is linear in the agent's connectedness, W_i. In particular,

$$\Pi_i = \underbrace{-\frac{\Delta^2}{\sigma^2(\Delta^2 + B)} E\left[p(\tilde{X} - p)\right]}_{\text{information-free}} + \underbrace{\frac{W_i}{s^2} E\left[(\tilde{X} - p)^2\right]}_{\text{information-related}}.$$

- Average trading profit: $\Pi = \lim_{n\to\infty} \frac{\sum_{i=1}^n E\left[\left(\tilde{X} - \tilde{p}^n\right)\psi_i^n(\tilde{x}_i^n, \tilde{p}^n)\right]}{n}$.
- Decomposition: $\Pi^I = \Pi^F + \Pi^I$.

  Here, Π^F is the information-free average trading profit and Π_i^I is the information-related average trading profit.

- The distribution of agents' ex-ante trading profits is an affine transformation of the network's degree distribution.

Trading profits - II

- The average ex-ante trading profit is a non-monotonic function of network connectedness, B:

1. The average ex-ante trading profit is a non-monotonic function of network connectedness. In particular,

$$\begin{aligned}
\frac{\partial \Pi}{\partial B} &> 0, \qquad \text{if } \; \sigma < \frac{1}{\Delta} \; \text{ and } \; B < \frac{\Delta}{\sigma} - \Delta^2, \\
\frac{\partial \Pi}{\partial B} &\leq 0, \qquad \textit{otherwise}.
\end{aligned}$$

2. Π^F is positive, decreasing in B, and approaches 0 as B tends to ∞.
3. Π^I is positive, non-monotonic in B, and approaches 0 as B tends to ∞.
4. As B tends to ∞, Π approaches 0.

Trading volume

For simplicity, assume $\bar{X} = \bar{Z} = 0$.

1. Trading volume of agent i is increasing in the agent's connectedness, W_i, with a higher slope for low degrees of connectedness.
2. An agent's expected trading profit is a convex, increasing function of expected trading volume.

Risk-return trade-off

In the computation of the Sharpe ratio, we use ex-ante expected (dollar) return and ex-ante standard deviation of return:

$$S = \frac{E\left[\tilde{X} - \tilde{p}\right]}{\sqrt{Var\left(\tilde{X} - \tilde{p}\right)}}.$$

1. Ex-ante expected return decreases as the network's connectedness increases provided that $\bar{Z} \neq 0$.
2. Ex-ante return volatility decreases as the network's connectedness increases.
3. The Sharpe ratio is a decreasing function of network connectedness, provided that $\bar{Z} \neq 0$.

Portfolio holdings

- Unsurprisingly, the demand correlation of agents i and j increases as the number of their common neighbors increases.
- This is consistent with recent empirical findings:
  - Hong, Kubik, and Stein (2004) show that the trades of any given fund manager respond more sensitively to the trades of other managers in the same city than to the trades of managers in other cities.
  - Using account-level data from PRC, Feng and Seasholes (2004) find that trades are highly correlated when investors are divided geographically.
  - We find a positive relationship between informational proximity and correlated trading. Geographical proximity is expected to encourage communication, therefore, arguably, these empirical studies lend support to our model.

Welfare and stability in networks

- Examine welfare implications of information networks.
- Basis of analysis: ex ante utilities (or certainty equivalents)
- Ex ante certainty equivalent of participating in the market for an agent is CE(W), where W is the connectedness of the agent.

1. For agent i, the certainty equivalent is

$$CE(W_i) \quad = \quad \frac{1}{2}\log\left(\frac{\left(\Delta^2+(B+\Delta^2)^2\sigma^2\right)\left(B^2s^2\sigma^2+\Delta^2s^2+W_i\Delta^2\sigma^2\right)}{s^2(B^2\sigma^2+\Delta^2+\Delta^2B\sigma^2)^2}\right).$$

2. The average ex ante certainty equivalent of trading across agents is

$$\overline{CE} = \sum_k CE(k)d(k).$$

Optimal networks

- If connections are costly to form and wealth transfers between agents are possible, which type of network topologies are the most efficient from a welfare perspective?
- We analyze this question under the specific assumption of a constant cost per link, no entry costs, and no variation of agents' skills.

Optimal network topology

Solution to the central planner's problem has connectedness $B < \infty$ and network degree distribution d with $supp[d] \subset \{\lfloor \mathrm{B} \rfloor, \lceil \mathrm{B} \rceil\}$ for $B \geq 1$.

Stable networks with constant link formation cost

An agent is said to be *content*, if she has no incentive to change her number of connections. A network is *stable* if all of its agents are content.

Stable network topology with constant cost

There exists a stable network in which the degree distribution has $supp[d] \subset \{\lfloor \mathrm{B} \rfloor, \lceil \mathrm{B} \rceil\}$, for some $B > 1$. Moreover, any stable network will have a degree distribution with $supp[d] \subset \{i, i+1\}$ for some $i \in \mathbb{N}$.

If the central planner's problem has a solution with connectivity B_S, then there is a stable network with connectivity B, where $B \geq \lceil B_S \rceil - 1$.

- In a stable network, agents overinvest in connectedness.
- This is reminiscent of Hirshleifer's (1971) proposition that competitive markets need not reflect the social value of information.

Stable networks with skill dispersion - I

We now study the case in which the cost of link formation, f, is linear in the number of connections, but may vary with agent type. The type distribution is characterized by the c.d.f., G.

Stable network topology with skill dispersion

Assume a large network, with a power-law degree distribution with tail exponent $\alpha > 2$, and that the cost function is of the form $f(t, W) = t(W - 1)$.

(i) There is a type distribution, G, that satisfies $G \sim t^{\alpha-1}$ for small t, for which the network is stable and all agents participate.

(ii) Any type distribution, G that is twice continuously differentiable, for which the network is stable, must have $G \sim t^{\alpha-1}$.

Stable networks with skill dispersion - II

- If the link formation cost function is agent-dependent, then power-law distributed networks, which have tail exponents larger than 2, arise naturally.
  - As we mentioned before, empirically, networks often have tail exponents between 2 and 3.
- The functional form of the type/talent distribution is in line with Gabaix and Landier (2008).
  - They use an assignment model to study executive payment, and find that a talent distribution which satisfies a power-law relation fits the data well.
  - The analogy, of course, can not be taken too far, since the talent in Gabaix and Landier (2008) is interpreted as a CEO's skill to increase the revenue of a firm.

Concluding remarks

- We have introduced a parsimonious model in which the relationship between network properties and asset pricing can be conveniently analyzed.
- We have found information networks to be financially relevant:
  - non-trivial relationships between network topology and financial features, such as volatility, trading profit, portfolio correlation, etc.
- We have examined which networks are "stable":
  - if agents are ex ante identical, then strong conditions are needed to allow for non-degenerate network structures
  - if agents face different costs of forming links, then power-law distributed networks do arise
- Potential extensions: multiple stocks, endogenous network formation.