Information, Liquidity, Asset Prices and Monetary Policy*

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Abstract

We study economies with multiple assets that are valued both for their return and liquidity. Liquidity is modeled by having some trade occur in markets where a medium of exchange is essential. Certain assets are more liquid, or more likely to be accepted in trade. This is modeled using information frictions: while everyone recognizes currency, say, they are less sure about and hence less likely to accept other assets. Recognizability is endogenized by letting agents invest in information, potentially generating multiple equilibria with different liquidity properties. We discuss implications for asset pricing, and monetary policy. We show in particular that what may look like a cash-in-advance constraint is *not* invariant to policy. We also use our theory to briefly discuss recent financial market events. It is not hard to see how the model can generate big negative responses to even relatively small reductions in the amount of, or in the cost of, information concerning asset quality.

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Aringosa walked back to his black briefcase, opened it, and removed one of the bearer bonds. He handed it to the pilot. "What's this?" the pilot demanded. "A ten-thousand-euro bearer bond drawn on the Vatican Bank." The pilot looked dubious. "It's the same as cash." "Only cash is cash," the pilot said, handing the bond back. Dan Brown, *The Da Vinci Code*.

1 Introduction

We study economies where currency as well as alternative assets may be valued both for their return and for their liquidity – by which we mean for their use as media of exchange.[1] In general, assets in our model are not perfect substitutes, and currency can be valued even if it has a low rate of return because it may be more liquid, or more likely to be accepted in exchange, due to a potential recognizability problem with alternatives. We model recognizability by assuming that agents have to invest in information to be able to distinguish high- from low-quality assets. We specify the environment so that agents who do not become informed will not accept assets they cannot recognize, for fear of getting a worthless lemon. This is technically convenient because it allows us to use straightforward bargaining theory to determine the terms of trade, since agents only exchange objects that they recognize. Hence, information frictions can drive liquidity differentials across assets, without overly complicating the analysis of price determination.

Although the model seems interesting even if information is exogenous – we show e.g. how inflation can affect individuals or markets that never use cash – the main substantive contribution here is to make recognizability and hence liquidity endogenous. We show the fraction of agents that invest in information depends on monetary policy: higher inflation makes them demand less real money balances and more alternative assets, raising the market value of these alternatives and thus the benefit to being able to transact in those assets, which makes agents

[1] It is by now well understood that it is both natural and efficient for some objects to emerge as media of exchange when barter is difficult due to a standard double coincidence problem, and credit is imperfect due to limited commitment and incomplete record keeping. See Kocherlakota [?], Wallace [?], Corbae et al. [?], Araujo [?], and Aliprantis et al. [?], [?] for extended discussions.

more willing to pay for information. We also highlight an important complementarity: if more agents become informed, assets become more liquid and hence more valuable, increasing the incentive to invest in information, and hence potentially generating multiple equilibria. One might say we show explicitly that the share of transactions that are apparently subject to a cash-in-advance constraint is endogenous, is not necessarily uniquely pinned down, and is not invariant to policy changes.

We think that endogenizing liquidly by requiring agents invest in information if they want to transact using assets other than currency has several interesting implications. For example, one that is consistent with much experience comes from interpreting the easily recognizable asset as a local currency, such as the peso in Latin American, and the alternative asset as the US dollar, which traditionally constitutes a better store of value. When peso inflation is not too high, locals are relatively happy using pesos as a means of payment, so dollars do not circulate widely and do not become universally recognized. If the peso inflation rate increases, transacting in local currency becomes more costly, and the economy begins to dollarize. Notice, however, that if the peso inflation rate later subsides, the dollar does not fall into disuse; once the locals learn to recognize it for transactions, they do not quickly forget. This imparts a natural hysteresis effect into dollarization, as has often been discussed in the literature, but never been formalized in this way.[2]

The framework can in principle be used to analyze any vector of assets, including local and foreign currency, but also stocks, bonds, or mortgage-backed securities. As our leading example, however, we often consider the case of two: money, and real claims like the claims to "trees" bearing "fruit" as dividends as in standard Lucas [?] asset-pricing theory. For example, while everyone understands what currency is and what it is worth, in the model, some people might

[2] See Uribe [?] for a discussion of the issues and literature. Uribe also provides a model of the phenomenon in question, but simply assumes exogenously that the cost of accepting foreign currency is decreasing in the fraction of other agents that accept it.

not be so sure about other claims. As we said above, sellers who do not recognize assets do not accept it (they won't give a buyer anything for it). The reason is that the seller is concerned a claim may be counterfeit or a worthless lemon – a "lemon tree," as it were.[3] In any case, this recognizability problem can give rise to a demand for both currency and real assets.

Although the way we model acceptability and recognizability seems new, aspects of the economic discussion go back to much earlier thought, including Tobin's [**?**] portfolio theory of money and Wallace's [**?**] overlapping generations models. As Wallace put it:

> Of course, in general, fiat money issue is not a tax on all saving. It is a tax on saving in the form of money. But it is important to emphasize that the equilibrium rate-of-return distribution on the equilibrium portfolio does depend on the magnitude of the fiat money-financed deficit ... the real rate-of-return distribution faced by individuals in equilibrium is less favorable the greater the fiat money-financed deficit. Many economists seem to ignore this aspect of inflation because of their unfounded attachment to Irving Fisher's theory of nominal interest rates. (According to this theory, (most?) real rates of return do not depend on the magnitude of anticipated inflation.) The attachment to Fisher's theory of nominal interest rates accounts for why economists seem to have a hard time describing the distortions created by anticipated inflation. The models under consideration here imply that the higher the fiat money-financed deficit, the less favorable the terms of trade – in general, a distribution – at which present income can be converted into future income. This seems to be what most citizens perceive to be the cost of anticipated inflation.

[3] More generally, a seller may worry the value of a claim is random and not know the distribution, even if the buyer does; the possibility that the claim may be totally worthless is simply a special case where its value can be 0. Also, we are well aware that an agent might accept an asset in exchange even if he does not recognize it, as in Williamson and Wright [**?**], but as we discuss below, one can specify the details carefully to be sure sellers reject outright things they do not recognize.

These words ring true, but many questions arise. How can the Fisher equation not hold? Why do different assets bear different returns in the first place? In the models Wallace mentions, it is *not* differences in liquidity – notice he talks about "saving" and defines returns in terms of the rate "at which present income can be converted into future income" but there is no mention of a transactions or medium of exchange role. This is where modern monetary theory comes in, with explicit descriptions of trading processes and liquidity. Early search-based models such as Kiyotaki and Wright [?] determine endogenously the acceptability of different objects in exchange, but are too crude to address the issues studied in this paper. Hence we use a multiple-asset version of the more recent model in Lagos and Wright [?]. While others have considered multiple assets in this environment, our focus is on differential liquidity and how this can be determined endogenously using information frictions.[4]

The rest of the paper is organized as follows. In Section 2 we lay out some basic assumptions and notation. In Section 3 we describe equilibrium in a simple version of the model in which the probability is taken as given that a random agent recognizes, and hence is able to accept, assets other than money. This is an important preliminary step to endogenizing information acquisition and hence the acceptability of alternative assets, since in order decide whether to invest in information one needs to know what happens for any given amount of information. Based on this, in Section 4, we let agents choose whether or not to acquire information, thus endogenizing recognizability and liquidity. We conclude in Section 5 with a summary, and a brief discussion of recent financial market events as seen through the lens of our theory. In particular, it is not hard to understand how one can generate big negative responses to even relatively small reductions in the amount of, or in the cost of, information concerning asset quality.[5]

[4]Lagos and Rocheteau [?] have two assets, money and capital, but they are equally liquid and hence bear the same return. Geromichalos et al. [?] have money and assets like our real claims, but again they are equally liquid. Differential liquidity was considered by Lagos [?], but this differential is exogenous. To reiterate, our goal is to make liquidity endogenous by incorporating recognizability and information acquisition.

[5]In terms of some additional literature, the idea in monetary economics goes back at least to Menger [?] that

2 The Basic Model

There is a continuum of infinitely-lived agents. As in Lagos and Wright [?], hereafter LW, in each period in discrete time, agents participate in two distinct markets: a frictionless centralized market CM, and a decentralized market DM where they meet bilaterally and anonymously. These meetings in the DM are characterized by a standard double coincidence problem, detailed below, which rules out barter. Since anonymity rules out credit, in the DM, some tangible medium of exchange is essential for trade (recall the references in footnote 1 for details). At each date in the CM there is a consumption good x that agents can produce using labor h according to $x = h$, and utility is quasi-linear, $U(x) - h$. In the DM there is another good q that all agents value according to $u(q)$ and can be produced at disutility cost $c(q)$. Define x^* and q^* by $U'(x^*) = 1$ and $u'(q^*) = c'(q^*)$. Assume $u' > 0$, $u'' < 0$, $c' > 0$, $c'' > 0$, $u(0) = c(0) = c'(0) = 0$, and $U'(0) = u'(0) = \infty$.

We assume that there are two assets (generalizing to n assets is straightforward). For now, the first is interpreted as fiat money, and the second is a real asset like the claims to "trees" in the standard Lucas [?] model, yielding a dividend δ in terms of "fruit" in the next CM. We introduce informational frictions as follows. Generally, a buyer might give you either a high- or low-quality asset, where we focus on the limiting case in which the latter is worthless – a pure lemon or counterfeit. For instance, consider a stock certificate in a profitable company. A counterfeit is a fake copy of the certificate which does not actually entitle the holder to anything.

recognizability is a key property media of exchange might have. Alchian [?], Brunner and Meltzer [?], Freeman [?], and Banerjee and Maskin [?] discuss the connection between money and information using varying degrees of formal modeling. Our approach is closer to search-based monetary theory, where informational frictions have been incorporated by Williamson and Wright [?], Trejos [?], [?], Li [?], Cuadras-Moreto [?], Kim [?], Velde et al. [?], Berentsen and Rocheteau [?], Ennis [?], Faig and Jerez [?], Nosal and Wallace [?], Cavalcanti and Nosal [?], Hu [?], Rocheteau [?], and Kim and Lee [?]. An alternative approach to modelling liquidity by Glosten and Milgrom [?] and Kyle [?] also considers bilateral transactions between asymmetrically informed agents; while similar in spirit, the models are different, and those papers have nothing to say about the substantive issues addressed here, including the role of money and monetary policy. Other work that has previously considered making the number of transactions that require cash endogenous include Shreft and Lacker [?], Ireland [?], and a recent paper by Dong [?], where one can find additional references.

Alternatively, a worthless lemon might be a legitimate certificate of stock in a different company that has zero profit. Here these two cases can be thought of as "bad claims to good trees" and "good claims to bad trees." We do not dwell on this distinction, but assume buyers can produce worthless assets whenever they like at no cost, and often describe them for convenience as claims to "trees" that yield no "fruit" and hence pay 0 dividends.

In the frictionless CM, all agents (or perhaps the market) can distinguish between high- and low-quality assets. But in the anonymous bilateral DM, only those seller who have acquired the relevant information can make this distinction. One story, stepping outside the formal model just a little, is that in the CM there are third parties available to certify quality, but they are not around in the DM. For ease of exposition we assume fiat money is universally recognized.[6] We emphasize that currency may be valued in equilibrium, but worthless claims cannot be valued, despite the superficial similarity that both pay 0 dividends. As discussed in a related context by Cavalcanti and Wallace [?] and [?], as long as individuals can produce their own worthless claims at no cost, there is no equilibrium where they value those produced by others. The key difference with currency is that you cannot produce it yourself.

Sellers who do not recognize an asset simply refuse to accept it, since buyers can produce at 0 cost worthless claims whenever they want. A seller who cannot verify whether a claim is genuine will therefore never accept it, since he knows the buyer can always slip him a lemon at the last minute. A seller without a technology for verifying a check, for instance, will not accept one if he knew you can always dupe him with a fake (say, sign someone else's name). This is different from some of the papers mentioned in the Introduction, where agents must make

[6] We can include a recognizability problem with currency without changing the basic results, but having universally recognized money seems a good benchmark. Currency and coins have always been designed exlcitly with recognizability in mind when they were stamped with the likeness of the monarch, and so on. Also, as Nobu Kiyotaki pointed out to us, in the olden days most people could recognize coins, although could not even read, let alone understand a piece of paper claiming some payoff in a future contingency. Literacy may have improved since then, but people still have trouble evaluating the worth of complicated financial instruments, including the recent example of mortgage-backed securities.

an ex ante choice to bring either good or bad assets to the market. In those models, agents always accept assets with positive probability, and sometimes probability 1, even if they do not recognize them. In a companion paper, Lester et al. [**?**], we analyze a class of related games to show in detail how the crucial assumption that assures sellers simply reject unrecognized assets is that buyers can costlessly produce worthless claims at any time.

It is these assumptions that make our framework extremely tractable, if not completely general. As suggested above, it simplifies the analysis dramatically to have sellers who do not recognize assets reject them outright. Given bilateral meetings in the DM, as is fairly standard, we assume the terms of trade are determined through bargaining (although there are other options, as discussed below). This is simple here, even though bargaining with information frictions is usually complicated, at best. In our set up, where a seller never accepts something he does not recognize, a buyer can only pay with assets the seller does recognize. Consequently, all bargaining occurs under full information. In this way informational frictions can be at the heart of determining acceptability and liquidity, but we completely avoid the usual problems with bargaining under asymmetric information. Again, we do not claim that we are studying the most general possible case, but that the gain in simplicity makes the strong assumptions reasonable.

To focus on steady state equilibria, assume there is a fixed supply of "trees" denoted A, while the supply of money M grows according to $\hat{M} = \gamma M$ (for any variable z, $\hat{z}$ denotes its value next period). Although M changes over time real balances will be constant in steady state just like the real supply of "trees." Changes in the money supply, $\hat{M} - M = (\gamma - 1)M$, are accomplished using lump sum transfers, or taxes if $\gamma < 1$ (although it is actually equivalent here to assume the government uses new money to buy good in the CM give our quasi-linear preference specification). In what follows, we assume $\gamma > \beta$ where β is the discount factor; we do consider the limit as $\gamma \to \beta$, which is the Friedman rule. Let ϕ be the CM price of money and ψ the CM price of the real asset, both in terms of x. In the CM all prices are taken parametrically; as mentioned above, in the DM agents bargain bilaterally.

It is well known (Lagos and Rocheteau [?]; Geromichalos et al. [?]) that, when the DM terms of trade are determined by bargaining, an agent who acquires a units of a real asset in the CM may not want to bring it all to the DM, since the bargaining solution generally depends on his asset holdings. Thus, an agent may acquire a large amount of a, because it is a good store of value, but not bring it all to the DM. There is no similar effect on m since it has no dividend; the *only* reason to acquire m is to use it as a medium of exchange. Thus, agents in the CM choose a portfolio comprised of m units of money taken to the DM, a_1 units of the asset not taken to the DM, and a_2 units taken to the DM. Note that in principle this detail could be avoided if buyers make take-it-or-leave it offers in the DM, since then the solution does not depend on what they bring to the bargaining table; this does not work for our purposes, however, since we want sellers to make ex ante investments in information, which they would never do if they get none of the ex post gains from trade.

Let $V(m, a_1, a_2)$ be the value function of an agent in the DM with portfolio (m, a_1, a_2), and $W(y)$ the value function in the CM with $y = \phi m + (\delta + \psi)(a_1 + a_2)$, since in the frictionless CM only the total value of a portfolio matters. The CM problem is

$$\begin{aligned} W(y) &= \max_{x,h,\hat{m},\hat{a}_1,\hat{a}_2} \{U(x) - h + \beta V(\hat{m}, \hat{a}_1, \hat{a}_2)\} \\ \text{s.t. } x &= h + y - \phi\hat{m} - \psi(\hat{a}_1 + \hat{a}_2) + T, \end{aligned}$$

where $T = (\gamma - 1)M$ is the transfer. Substituting for h, first order conditions are:

$$x \quad : \quad U'(x) = 1 \tag{1}$$

$$\hat{m} \quad : \quad \phi \geq \beta V_1(\hat{m}, \hat{a}_1, \hat{a}_2), \ = \ \text{if } \hat{m} > 0 \tag{2}$$

$$\hat{a}_1 \quad : \quad \psi \geq \beta V_2(\hat{m}, \hat{a}_1, \hat{a}_2), \ = \ \text{if } \hat{a_1} > 0 \tag{3}$$

$$\hat{a}_2 \quad : \quad \psi \geq \beta V_3(\hat{m}, \hat{a}_1, \hat{a}_2), \ = \ \text{if } \hat{a_2} > 0 \tag{4}$$

Notice that x and $(\hat{m}, \hat{a}_1, \hat{a}_2)$ do not depend on y, and that $W'(y) = 1$. We assume here a unique solution at least for $\hat{m}$ and $\hat{a}_2$, as is necessarily true under assumptions discussed below (we also assume an interior solution for h; see LW for assumptions that make this valid). What happens next, in the DM, depends on what we assume about information.

3 Exogenous Liquidity

In order to determine who invests in information, we obviously have to first determine the outcome for any given information structure. To this end, we now study in detail the DM with information fixed. First, we set up the double coincidence problem in a standard way: every agent in the DM is assumed to have an equal probability λ of a bilateral meeting where he wants to buy from his partner and a bilateral meeting where he wants to sell to his partner, as in the baseline LW model. It is possible to get the same substantive results in a version the model where there are two types, where one type can only be buyers in the DM and the other type can only be sellers in the DM, as in Rocheteau and Wright [**?**]. This actually may seem more natural, but since it involves slightly more notation, we stick to the representative agent version and leave the two-type model as an exercise.

The important new element of the structure is than when you are a buyer there are two subcases. With probability ρ you are a buyer in what we call a *type 2 meeting*, in which sellers accept either m or a_2 because they are informed about asset quality; and with probability $1 - \rho$

you are a buyer in what we call a *type 1 meeting*, in which sellers are uninformed about a and hence accept only m. As we said, we take $0 < \rho < 1$ as given in this section, where we think of ρ as the fraction of informed agents. However, for now, one can simply regard this as a random matching version of a cash-in-advance model, with currency required in some but not all meetings. Again, this is merely a first step toward endogenizing ρ in the next section by having agents explicitly invest in information.

We determine the DM terms of trade using the generalized Nash bargaining solution.[7] Consider a type j meeting between a buyer with (m, a_1, a_2) and a seller with $(\tilde{m}, \tilde{a}_1, \tilde{a}_2)$. The former pays p_j to the latter for q_j units of the good, determined by

$$\max \;\; [u(q_j) + W(y - p_j) - W(y)]^{\theta} [-c(q_j) + W(\tilde{y} + p_j) - W(\tilde{y})]^{1-\theta} \tag{5}$$

subject to the constraint $p_j \leq y_j$, where y and $\tilde{y}$ are total wealth of the buyer and seller, and y_j describes the wealth the buyer can use in that meeting: $y_1 = \phi m$ and $y_2 = \phi m + (\psi + \delta) a_2$. This is equivalent to the bargaining problem in LW, so we can use the known solution:

Lemma 1. *The solution to (5) is*

$$q_j = \min\left\{z^{-1}(y_j), q^*\right\} \;\; \text{and } p_j = \min\left\{y_j, y^*\right\},$$

where the function z is defined by

$$z(q) \equiv \frac{\theta u'(q)c(q) + (1-\theta)u(q)c'(q)}{\theta u'(q) + (1-\theta)c'(q)},$$

while q^* given by $u'(q^*) = c'(q^*)$ and $y^* = z(q^*)$.

[7] There are alternative approaches for determining the terms of trade in the DM. With symmetric information, the orginal LW model uses generalized Nash bargaining; Aruoba et a. [?] consider different axiomatic bargaining solutions; Rocheteau and Wright [?] analyze price taking and price posting; Galenianos and Kircher [?] and Dutu et al. [?] use auctions in versions with some multilateral meetings. In models with private information, Ennis [?] and Faig and Jerez [?] use posting, and Guerrieri [?] uses price taking. We like bargaining because it is natural, and very easy in this model.

The DM value function satisfies

$$V(m, a_1, a_2) = \lambda_1 \left[u(q_1) + W\left(y - p_1\right)\right] + \lambda_2 \left[u(q_2) + W\left(y - p_2\right)\right] + (1-\lambda)W(y) + k, \quad (6)$$

where $\lambda_1 = \lambda(1-\rho)$, $\lambda_2 = \lambda\rho$, and k is a constant unimportant for what follows. There are three relevant events described in (6): you are a buyer in a type 1 meeting; you are a buyer in a type 2 meeting; and you are not a buyer. In the third case, you may be a seller or you may not trade at all, and this effects your continuation value, but since Lemma 1 implies the terms of trade do not depend on the seller's state and $W(y)$ is linear, conveniently, we need not know what happens when you are a seller in order to determine your portfolio.

Differentiating V and substituting the derivatives of q_j wrt (m, a_1, a_2),

$$V_1(m, a_1, a_2) = \phi \left[\lambda_1 \ell(q_1) \mathbf{1}\{y_1 < y^*\} + \lambda_2 \ell(q_2) \mathbf{1}\{y_2 < y^*\} + 1\right] \quad (7)$$

$$V_2(m, a_1, a_2) = \psi + \delta \quad (8)$$

$$V_3(m, a_1, a_2) = (\psi + \delta) \left[\lambda_2 \ell(q_2) \mathbf{1}\{y_2 < y^*\} + 1\right] \quad (9)$$

where $\mathbf{1}\{z\}$ is an indicator function equaling 1 iff z is true and $\ell(q) \equiv \frac{u'(q)}{z'(q)} - 1$. Notice that $\ell(q)$ is a *liquidity premium* – the value of an additional unit of wealth available in a type j meeting, over and above its return if it were simply carried to the next CM. We assume $\ell'(q) < 0$, which is true under known conditions.[8] Combining (7)-(9) and (2)-(4), we arrive at the conditions determining portfolio demand:

$$m \ : \ \phi \geq \beta\hat{\phi} \left[\lambda_1 \ell(\hat{q}_1) \mathbf{1}\{\hat{y}_1 < y^*\} + \lambda_2 \ell(\hat{q}_2) \mathbf{1}\{\hat{y}_2 < y^*\} + 1\right], \ = \text{ if } \hat{m} > 0 \quad (10)$$

$$a_1 \ : \ \psi \geq \beta(\hat{\psi} + \delta), \ = \text{ if } \hat{a}_1 > 0 \quad (11)$$

$$a_2 \ : \ \psi \geq \beta(\hat{\psi} + \delta) \left[\lambda_2 \ell(\hat{q}_2) \mathbf{1}\{\hat{y}_2 < y^*\} + 1\right], \ = \text{ if } \hat{a}_2 > 0 \quad (12)$$

[8] As in LW, $\ell' < 0$ if θ is close to 1, or if c is linear and u displays decreasing absolute risk aversion. These conditions also guarantee a unique solution to the CM problem. The method in Wright [**?**] can be used to dispense with these side conditions and establish generic uniqueness of the CM solution even if ℓ is nonmonotone.

An equilibrium is defined in terms of time paths for asset holdings (m, a_1, a_2), asset prices (ϕ, ψ), the DM terms of trade (p_j, q_j), $j = 1, 2$, and the CM allocation (x, h), for every agent, satisfying the utility maximization conditions derived above, the bargaining solution, and market clearing. Given the other variables, we know that $x = x^*$ from (1) and can determine h from the budget equation, hence the CM allocation will be ignored in what follows. We focus on steady states. A steady state is an equilibrium in which the real variables (q_1, q_2) are constant over time, which implies ϕm and ψa_2 are constant, and hence, $\phi/\hat{\phi} = \hat{M}/M = \gamma$. We also focus on monetary steady states, where $\phi > 0$, $\hat{m} > 0$, $q_1 > 0$ and (10) holds with equality. We now characterize steady state monetary equilibrium.

To begin, notice from Lemma 1 that q_j is an increasing function of y_j and that $q_1 \leq q_2 \leq q^*$. It is easy to show $q_j \leq \bar{q}$, where $\bar{q}$ maximizes the buyer's surplus $u(q) - p = u(q) - z(q)$, and $\bar{q} \leq q^*$, with strict inequality unless $\theta = 1$ (Geromichalos et al. [**?**]). Also, notice that $\ell(\bar{q}) = 0$. We next claim that $a_2 > 0$ for all $\lambda_2 > 0$. The proof is in Appendix A; intuitively, because it is costly to carry cash, agents do not bring enough to buy $\bar{q}$, and so want to bring at least some a_2 to the DM.

Lemma 2. $a_2 > 0$ *for all* $\lambda_2 > 0$.

Thus, $a_2 > 0$ and $m > 0$ in monetary equilibrium. It remains to determine $a_1 = 0$ or $a_1 > 0$. To answer this, let $\tilde{q} < \bar{q}$ be defined by $\ell(\tilde{q}) = (\gamma - \beta)/\beta\lambda_1$, and let

$$\bar{A} = [z(\bar{q}) - z(\tilde{q})]\,(1 - \beta)/\delta > 0.$$

The next result, the proof of which is in Appendix B, demonstrates that $A \leq \bar{A}$ (the real asset is relatively scarce) implies $a_1 = 0$, and $A > \bar{A}$ (the real asset is plentiful) implies $a_1 > 0$.

Proposition 1. *(i) If* $A \leq \bar{A}$ *there exists a unique steady state monetary equilibrium, and in* *this equilibrium,* (q_1, q_2) *solves*

$$A\delta = [z(q_2) - z(q_1)] \{1 - \beta[\lambda_2 \ell(q_2) + 1]\} \tag{13}$$

$$\gamma = \beta [\lambda_1 \ell(q_1) + \lambda_2 \ell(q_2) + 1], \tag{14}$$

prices are $\phi = z(q_1)/M$ *and* $\psi = [z(q_2) - z(q_1)]/A - \delta$*, and* $(m, a_1, a_2) = (M, 0, A)$*.*

(ii) If $A > \bar{A}$ *there exists a unique steady state equilibrium, and in this equilibrium,* $(q_1, q_2) = (\tilde{q}, \bar{q})$*, prices are* $\phi = z(\tilde{q})/M$ *and* $\psi = \beta\delta/(1-\beta)$*, and* $(m, a_1, a_2) = (M, A - \bar{A}, \bar{A})$*.*

The above result is needed as an input to the next section, where we endogenize information. But before that, we can discuss some economic that may be of some interest even with a fixed ρ. To facilitate the discussion, imagine a hypothetical asset that costs 1 unit of x in the current CM and pays $1 + r$ units in the next CM, but cannot be traded in the DM (say, it is merely a book entry, not a tangible asset). In equilibrium its real return satisfies $1 + r = 1/\beta$. Now imagine an asset that costs 1 dollar in the current CM and pays $1 + i$ dollars in the next CM, and similarly cannot be traded in the DM. Its return, the nominal interest rate, satisfies $1 + i = \phi/\hat{\phi}\beta$. Hence, $1 + i = (1 + r)\phi/\hat{\phi}$, which is a version of the Fisher equation that must hold – it is a no-arbitrage condition across assets that are illiquid in the sense that they cannot be traded in the DM. Given this, we can equivalently discuss monetary policy in terms of either the nominal interest rate i or the inflation rate $\phi/\hat{\phi} = \gamma$.

It is convenient to use the nominal rate i, and rewrite (13)-(14) as

$$(1 + r)A\delta = [z(q_2) - z(q_1)][r - \lambda_2 \ell(q_2)] \tag{15}$$

$$i = \lambda_1 \ell(q_1) + \lambda_2 \ell(q_2). \tag{16}$$

Let $q_1 = \mu(q_2)$ and $q_2 = \alpha(q_1)$ denote the implicit functions characterized by (16) and (15). It is routine to show that $\mu(\cdot)$ is decreasing and $\alpha(\cdot)$ is increasing, and that they intersect for some $q_1 \in [0, \bar{q}]$, as seen in the figures below. For $A \leq \bar{A}$, the intersection of α and μ determines the equilibrium $(q_1, q_2) \in [0, \bar{q}]^2$. For $A > \bar{A}$, the intersection of α and μ occurs at $q_2 > \bar{q}$, and the equilibrium is $(q_1, q_2) = (\tilde{q}, \bar{q})$. The unique steady state monetary equilibrium is therefore conveniently characterized by the intersection of $q_1 = \mu(q_2)$ and $q_2 = \bar{\alpha}(q_1) = \min\{\alpha(q_1), \bar{q}\}$. When $A \leq \bar{A}$, we have $q_2 < \bar{q}$, and a bears a liquidity premium $\ell(q_2) > 0$. In this case, (12) implies $\psi > \beta\delta/(1-\beta) = \delta/r$ and the price of the real asset a exceeds the present value of its dividend stream, because this price reflects not only fundamentals (dividends) but, in addition, the value of the asset as a means of payment in the DM.

INSERT FIGURES 1 AND 2 ABOUT HERE

To be precise, in steady state (12) at equality yields

$$\psi = \frac{\beta\delta\left[1 + \lambda_2 \ell(q_2)\right]}{1 - \beta\left[1 + \lambda_2 \ell(q_2)\right]} = \frac{\delta}{r}\left[1 + \frac{(1+r)\lambda_2 \ell(q_2)}{r - \lambda_2 \ell(q_2)}\right],$$

which exceeds the fundamental price, given $\lambda_2 > 0$ and $q_2 < \bar{q}$, which as we said is the relevant case when $A < \bar{A}$, as seen in Figure **??**. Figure **??** depicts the other case, $A > \bar{A}$, where $q_2 = \bar{q}$ so that a bears no liquidity premium and $\psi = \delta/r$. When $A > \bar{A}$, we have $a_1 > 0$, and agents at the margin are indifferent between holding a for its dividend stream alone or using as a medium of exchange. It is also true that different agents may choose different a_1 when $A > \bar{A}$, since they are indifferent about how much to hold as a store of value when the real asset is priced fundamentally, even if equilibrium pins down all the other variables.

$x =$	i	A	δ	λ	ρ
$\frac{\partial q_1}{\partial x}$	$-$	$-$	$-$	$+$	$-$
$\frac{\partial q_2}{\partial x}$	$-$	$+$	$+$	$+$	?
$\frac{\partial \phi}{\partial x}$	$-$	$-$	$-$	$+$	$-$
$\frac{\partial \psi}{\partial x}$	$+$	$+$	$+$	$+$	$+$

Table 1: Effects of parameters when $A < \bar{A}$

Table 1 shows the effects of parameter changes when $A < \bar{A}$ (see Appendix D for details). These results show up graphically as shifts in the α and μ curves. Thus, an increase in i shifts the μ curve southwest and leaves α unchanged, reducing q_1 and q_2. Intuitively, as i increases agents try to economize on m, reducing its CM price ϕ and DM value $q_1 = z^{-1}(\phi M)$. Given this, agents want to hold more a, raising its CM price ψ but on net lowering $q_2 = z^{-1}(\phi M + \psi A)$. Notice the observed (accounting) return on a between meetings of the CM, $1 + \delta/\psi$, decreases with i. Hence the Fisher equation *apparently* does not hold for a, since the observed return on a is not independent of inflation or the nominal rate. This would not happen if a were never traded in the DM; only when a bears a liquidity premium does its observed return depend on i. So the Fisher equation holds in some circumstances, or for some assets, but not others.[9]

Finally, before moving to determine recognizability and liquidity endogenously, to close this section we mention one more application that seems interesting even with ρ fixed. This is to show how a change in i affects not only those agents or markets that use cash directly, but even those that never touch money. In Appendix C we sketch a simple extension with two distinct decentralized markets, call them markets B and C, where a fraction b and $1-b$ of the agents go between meetings of the CM. In market B, sellers always accept *both* a and m, and so everyone takes $m = 0$, while in market C a fraction ρ of transactions require *cash* as in the baseline model. We show in market B that $\partial q_2/\partial i < 0$. Why? As i increases, agents going to market C shift out of m and into a, driving up ψ. Hence, agents in market B enjoy lower utility when i increases,

[9] In terms of other parameters, an increase in δ or A shifts the μ curve northwest but leaves μ unchanged, leading to a fall in q_1 and rise in q_2, with resulting changes in ϕ and ψ. Intuitively, as dividends increase agents substitute into a out of m, which affects both their DM and CM values. Increasing λ shifts μ right and α left, but one can show the net effects on q_1 and q_2 are positive. Increasing ρ decreases q_1 but the effect on q_2 is ambiguous. One can show $\partial\phi/\partial\rho < 0$ and $\partial\psi/\partial\rho > 0$. All these results are for $A < \bar{A}$. When $A > \bar{A}$, we have $q_2 = \bar{q}$ and $q_1 = \tilde{q}$ where $\tilde{q}$ solves $\ell(\tilde{q}) = i/\lambda(1-\rho)$. In this case, $\partial\tilde{q}/\partial i < 0$, $\partial\tilde{q}/\partial\lambda > 0$, and $\partial\tilde{q}/\partial\rho < 0$, while neither A nor δ affect q_1, and none of these variables affects q_2. Also, when $A > \bar{A}$, ϕ is decreasing in i and ρ and increasing in λ, while as we already have remarked $\psi = \beta\delta/(1-\beta)$ is pinned down by fundamentals. Also, notice that $\tilde{q} < \bar{q}$ if $i > 0$, but $\tilde{q} \to \bar{q}$ as $i \to 0$. In fact, as $i \to 0$, $\bar{A} \to 0$, which means equilibrium entails $a_1 > 0$. This says that at the Friedman rule $i = 0$ we have $q_1 = q_2 = \bar{q}$ and all assets bear the same return $1 + r = 1/\beta$. Although the focus here is not on welfare, for completeness we mention that $i = 0$ is the optimal policy, but it does not give the first best $q = q^*$ unless $\theta = 1$.

even though they themselves never use cash.[10]

4 Endogenous Liquidity

Now that we have determined how the economy operates for any given value for ρ, the probability a give seller recognizes and hence accepts real claims in the DM, we can think about determining it endogenously. Suppose that agent $j \in [0, 1]$ has the ex ante choice whether or not to acquire at cost $\kappa(j)$ the information, or perhaps a technology, that allows him to recognize asset quality. We arrange agents so that $\kappa'(j) \geq 0$, assuming for simplicity that $\kappa(j)$ is differentiable. An agent accepts a in the DM if and only if he pays this cost, since this is the only way to distinguish genuine from worthless claims, and any buyer that meets an uninformed seller accepting a will clearly give him a worthless claim. The fraction of agents that incur the cost thus determines the fraction ρ that accept these assets in the DM in equilibrium.

One can imagine several interpretations of $\kappa(j)$. It is typically thought to be costly to learn how to use a new medium of exchange, for a variety of reasons, as has been documented in episodes of dollarization (Uribe [?]; Guidotti and Rodriguez [?]; Dornbusch et al. [?]). Stepping outside the formal model, a financial institution that wants to accept a pool of asset-backed securities in payment must hire analysts to ascertain their value. Other costs may be technological, as in the case of debit or credit cards, where sellers must buy a machine to verify buyers' credit or transfer funds from one institution to another. Agents choose whether to make an investment allowing them to accept certain types of assets. Naturally, one might think that coordination will be central in determining equilibrium, as is commonly discussed in the literature on payment networks, even though their models are quite different (see Hunt [?] and Rochet

[10] One interpretation is that even if all US dollars were abroad and all domestic payments were made with real assets, inflation can still affect equilibrium asset prices, consumption, and welfare at home, as long as it leads whoever it is holding the dollars to adjust their portfolios. Thus monetary policy can still be relevant despite more and more transactions taking place without the use of currency.

and Tirole [?] for surveys).

Conditional on a fraction $\rho \in [0,1]$ of other agents becoming informed, the benefit to a given agents of becoming informed is that he can now accept real assets in the DM. Thus,

$$\Pi(\rho) \equiv \beta\lambda \{z[q_2(\rho)] - c[q_2(\rho)]\} - \beta\lambda \{z[q_1(\rho)] - c[q_1(\rho)]\}, \tag{17}$$

is the benefit – the expected discounted surplus a seller gets in a type 2 meeting, over and above what he gets in a type 1 meeting. All the work in the previous section allows us to say that $q_1(\rho)$ and $q_2(\rho)$ are well-defined objects in (17), since Proposition 1 fully characterizes (the unique) equilibrium for any given ρ. The best response condition is obviously for agent j to acquire the relevant information if $\Pi(\rho) \geq \kappa(j)$. An equilibrium is a fixed point ρ^* of this best response condition. Existence of ρ^* follows immediately from the standard fixed-point theorems. Given ρ^* all of the other endogenous variables follow as in the previous section.

In terms of the kind of equilibria that may exist, there is always a trivial equilibrium with $\rho = 0$, in which no one invests and no one brings a to the DM, although sometimes this can be ruled out by assuming that some agents are exogenously informed. Of course, if the buyer has all the bargaining power, $\theta = 1$, then $\rho = 0$ is the only equilibrium, since there is no way a seller will invest when he gets none of the gains from trade. Similarly, if the costs were prohibitive – say, $\kappa(j) > u(q^*) - c(q^*)$ for all j – then $\rho = 0$ is obviously again the unique equilibrium. Whenever $\rho = 0$ the outcome in some sense looks like a pure cash-in-advance specification. And if $\kappa(j)$ is low enough for all j, the natural outcome is that everyone invests, so $\rho = 1$, and there is no role for money as long as A is sufficiently big.[11] We are interested also in interior equilibria, $\rho^* \in (0,1)$, in which case both m and a are used in some transactions.

Obviously, such an equilibrium exists when some agents have sufficiently low and others

[11] The case $\rho = 1$ is the one analyzed in Lagos and Rocheteau [?] and Geromichalos et al. [?]. Exactly as in those models, there is no essential role for money when $\rho = 1$ and A is large – but this does not conflict with our earlier results on monetary equilibrium since these results assume $\rho < 1$.

sufficiently high costs of information acquisition. To make this more precise, consider the situation where all agents carry $m = 0$ and just enough a to purchase $\bar{q}$. Clearly $\Pi(1) \leq \bar{\Pi} = \beta\lambda[z(\bar{q}) - c(\bar{q})]$. Also, consider

$$\begin{aligned}\lim_{\rho\to 0} q_1(\rho) &\equiv \hat{q}_1 = \ell^{-1}(i/\lambda) \\ \lim_{\rho\to 0} q_2(\rho) &\equiv \hat{q}_2 = z^{-1}\left[(1+r)A\delta/r + z(\hat{q}_1)\right] > \hat{q}_1,\end{aligned}$$

and let

$$\underline{\Pi} = \lim_{\rho\to 0} \Pi_1(\rho) = \beta\lambda\left[z(\hat{q}_2) - c(\hat{q}_2)\right] - \beta\lambda\left[z(\hat{q}_1) - c(\hat{q}_1)\right] > 0.$$

This provides us with parameter conditions that guarantee some but not all agents with invest in information.

Proposition 2. *Equilibrium with endogenous ρ^* always exists. If $\kappa(0) < \underline{\Pi}$ and $\kappa(1) > \bar{\Pi}$ then there exists an equilibrium with $\rho^* \in (0,1)$, where real assets are accepted in some but not all DM meetings..*

Clearly, there may easily exist multiple interior equilibria, or there may coexist interior and other equilibria, as one might expect given the network nature of the game. The various cases are illustrated in Figures **??**-**??**. The economics is straightforward, although perhaps somewhat more interesting than what one sees in simple network or coordination games meant to illustrate similar points, because here the result works through a general equilibrium asset market effect. Thus, when ρ is bigger it is easier to spend a in the DM, or its liquidity is greater. When this is the case, there is an increase in the demand for this asset. This bids up the CM price ψ, which makes agents more willing to pay the cost of information allowing them to trade a in the DM. Again, the strategic complementarity works through the asset market.

INSERT FIGURES 3-6 ABOUT HERE

Obviously policy can have a large impact here, since i affects the value of real assets and hence information acquisition and liquidity. The implications of Table 1 and the surrounding analysis permit us to determine how $\Pi(\rho)$ and thus the (set of) equilibrium values for ρ^* vary with parameters. Again, the meetings that require money are determined endogenously, and are certainly not invariant to changes in monetary policy. Consider for the sake of illustration an example with $u(q) = \sqrt{q}$, $c(q) = q$, and $\kappa(i) = ki$.[12] In Figure **??**, we graph $\Pi(\cdot)$ when $i_1 = 0.01$ and $i_2 = 0.06$. In this example there is a unique equilibrium ρ^* in either case, although as illustrated in Figure **??** monetary policy also could cause a shift to a region of parameter space with multiple equilibria. Higher inflation causes the price of a to increase and m to decrease, shifting Π up. Thus, inflation increases ρ^* and the acceptability of assets in this case, although, as always, when there is multiplicity the effects go in opposite directions in alternate equilibria.

INSERT FIGURES 7 AND 8 ABOUT HERE

This kind of prediction is consistent with experience in a variety of episodes. In many Latin American countries e.g. inflation has at times induced the adoption of an alternative medium of exchange with a better rate of return – namely, the US dollar (see Guidotti and Rodriguez [**?**] for a discussion of this in countries such as Bolivia, Mexico, Peru, and Uruguay). Similar episodes of currency substitution have been observed in Eastern Europe and the Middle East (Feige [**?**]). Our model also generates the phenomenon of hysteresis in dollarization: when inflation goes up agents make an investment in information that entails increase in its use for payments; when inflation goes back down they can continue to use dollars as a medium of exchange, because they have already paid the fixed cost and do not forget the information right away. Although this phenomenon has been discussed at length, we think our way of modeling dollarization and hysteresis will be useful for discussing the issues in greater depth in future work.

[12] For this example we use $\theta = 0.5$, $r = 0.01$, $A = 0.05$, $\delta = 0.01$, $\lambda = 0.2$, and $k = 0.025$.

Finally, although as we said above the primary focus here is not on welfare, we want to mention one feature of the model. There is a *double holdup problem* at work here, as in many other bargaining models with ex ante investments. Thus, the buyer must get the entire surplus in DM trade to encourage him to make an efficient investment by bringing the right amount of money, while the seller must get the entire surplus to encourage him to make an efficient investment in information. In particular, if buyers make take it or leave it offers, $\theta = 1$, then sellers get no surplus and have no incentive to invest in information. But at the Friedman rule they do not need to make any investment in information, since money is a perfect means of payment when $i = 0$. Thus, $i = 0$ combined with $\theta = 1$ actually gives the first best outcome: $q = q^*$, and no resources are wasted learning to recognize other assets. If for reasons not specified here, however, policy sets $i > 0$, then it would be socially valuable to have agents invest in information to facilitate exchange, but equilibrium will typically be inefficient.

5 Conclusion

We developed a tractable framework where assets potentially differ in terms of their liquidity based on recognizability. Given the information structure, the model generates a unique steady state equilibrium in which there may or may not be a liquidity premium on certain assets, depending on parameter values. Although the theory applies to any combination of assets, because recognizability has long been thought to be highly relevant for understanding monetary economics, we discussed in some detail the role of money and monetary policy. We showed how policy affects asset prices and equilibrium allocations, generally, and even those individuals or markets that never use cash. A difference between our model and many others in monetary economics is that we endogenize which assets are accepted in which transactions through investment in information. This not only seems natural, it highlights an interesting complementarity that can generate multiple equilibria with different liquidity properties.

Moreover, the decision to invest in information depends on policy. A clear message is that it is *not* generally appropriate to take cash-in-advance constraints as invariant. When the central bank inflates, the economy will respond, and typically substitute out of currency and into alternatives not only as stores of value but also as means of payment. This has many implications, including our results on dollarization and hysteresis. Still, much work remains on information frictions in models where exchange is modeled explicitly. In our set up, sellers who do not recognize an asset simply refuse to accept it. This is convenient, since among other things it allows a simple solution to the bargaining problem despite asymmetric information, but it is also extreme, and one might like to see generalizations or alternative specifications where agents sometimes trade for assets whose quality is unknown. This obviously would be interesting, but more difficult. We think our analysis provides some preliminary steps in the right direction.

We also think it is useful to think about recent financial market events in the context of even the relatively simple version of the theory presented here. Suppose we start in steady state where many people – i.e. a reasonable large fraction ρ_1 – are able to identify high- versus low-quality assets in circulation. Then exchange is relatively smooth. Now suppose that there is a shock and information gets worse – i.e. ρ falls to $\rho_0 < \rho_1$ – for whatever reason. Perhaps some new complicated assets are developed that are hard to evaluate, or perhaps there are real developments somewhere in the economy, maybe the mortgage market, that make existing assets harder to evaluate. This aggregate "liquidity shock" obviously hinders the exhange process. From Table 1 we see several effects: the market value of assets ψ drops; the value of more easily recognizable alternatives like currency ϕ increases as agents shift their portfolios in a "flight to liquidity;" q_1 increases as cash becomes more valuable, while q_2 can go either way; and in general we expect output and welfare take a hit.

This experiment describes what happens when there is an exogenous decline in the liquidity of assets. Since we have a model where liquidity is endogenized through a choice of becoming able to recognize asset quality, we can also ask what happens when there is a shock to, say, the cost of becoming or staying informed κ (generally, to the distribution of such costs across agents). Obviously, since we can have multiple equilibria, different outcomes are possible. But the natural prediction is that fewer agents invest in information, assets become endogenously less liquid, and from this point we can see what happens from the previous paragraph. However, it is important to note that there are potentially significant feedback effects: as fewer agents are willing to bear the cost of becoming or staying informed, in general equilibrium the value of assets falls, making more and more agents unwilling to bear the cost, and so on.

So the model highlights multiplier effects that can generate a large response to a relatively small shock, even if the equilibrium set does not change qualitaitvely. More generally, and more ominously, it is easy to see how a catastrophe (mathematical and economic) can result in the model from a small increase in the cost of information, as an equilibrium with a relatively high value of ρ might well vanish with a change in parameters even if the best response curve shifts only a little. This will force the economy to jump to a new and very different equilibrium, with a potentially much lower value ρ and, again, some of the unpleasant implications discussed above through the general equilibrium asset market effects. Although there is much to be done trying to understand recent financial events, we think that some version of an information-based theory with an emphasis on endogenous liquidity may be quite relevant, and that models like the one analyzed here may have something to contribute to the analysis.

Appendix

A. Proof of Lemma 2: Suppose $a_2 = 0$. Then $q_1 = q_2 \equiv q_0$. Given $\gamma > \beta$, we know $q_0 < \bar{q} \leq q^*$ by standard results (when $a_2 = 0$ there are no claims traded in the DM and the model is equivalent to the baseline LW model). Since $q_0 < q^*$, from (10) at equality we have

$$(\lambda_1 + \lambda_2)\ell(q_0) + 1 = \phi/\beta\hat{\phi} = \gamma/\beta > 1,$$

which implies $\ell(q_0) > 0$. Since $a_2 = 0$, market clearing implies $a_1 = A > 0$, and (11) holds at equality. Thus, $\psi = \beta(\hat{\psi} + \delta)$. Then (12) implies $\lambda_2\ell(q_0) \leq 0$, a contradiction. ∎

B. Proof of Proposition 1: Suppose $A \leq \bar{A}$. We first show that there exists a unique pair (q_1, q_2) that satisfy (13) and (14). We then show these conditions are equivalent to the necessary and sufficient conditions for equilibrium.

By the implicit function theorem:

$$\begin{aligned}
\mu'(q_1) &= -\frac{\beta\lambda_1\ell'(q_1)}{\lambda_2\ell'(q_2)} < 0 \\
\alpha'(q_1) &= \frac{-z'(q_1)\Big[1 - \beta[\lambda_2\ell(q_2) + 1]\Big]}{\beta\lambda_2 e'(q_2)(z(q_2) - z(q_1)) - z'(q_1)\Big[1 - \beta[\lambda_2\ell(q_2) + 1]\Big]} > 0
\end{aligned}$$

Let $\breve{q}$ satisfy $\ell(\breve{q}) = \frac{\gamma-\beta}{\beta\lambda_1} + \frac{\lambda_2}{\lambda_1}$, with $\breve{q} < \tilde{q} \leq \bar{q}$. Since $\ell'(q) < 0$ and $\lim_{q\to\infty}\ell(q) = -1$, it is easy to see that $\lim_{q_1\to\breve{q}^+}\mu(q_1) = \infty$. Moreover, we claim $\lim_{q_1\to\breve{q}^+}\alpha(q_1) < \infty$. Suppose not. That is, suppose $\lim_{q_1\to\breve{q}^+}\alpha(q_1) = \infty$. Then using (13) we have

$$A\delta = \lim_{q_1\to\breve{q}^+}[z\big(\alpha(q_1)\big) - z(q_1)][1 - \beta + \beta\lambda_2].$$

This implies $A\delta \geq [z(\bar{q}) - z(\breve{q})][1 - \beta + \beta\lambda_2]$, which implies $\frac{\delta}{1-\beta} > \frac{z(\bar{q}) - z(\breve{q})}{A}$, a contradiction. Therefore, $\lim_{q_1\to\breve{q}^+}\mu(q_1) > \lim_{q_1\to\breve{q}^+}\alpha(q_1)$.

Now consider (14) with $q_1 = \bar{q}$, so that $\frac{\gamma}{\beta} = \lambda_2\ell(q_2) + 1$. This implies $\ell(q_2) \leq 0$, so that $\mu(\bar{q}) \leq \bar{q}$. Now consider (13). If $q_2 = \bar{q}$ then $A\delta = [z(\bar{q}) - z(q_1)](1 - \beta)$. Since $\frac{A\delta}{1-\beta} > 0$, $\alpha^{-1}(\bar{q}) < \bar{q}$. Since $\alpha' > 0$, $\alpha(\bar{q}) > \bar{q} \geq \mu(\bar{q})$. Since μ and α are continuous, $\mu' < 0$ and $\alpha' > 0$, $\mu(q') > \alpha(q')$ for some $q' < \bar{q}$, and $\alpha(\bar{q}) \geq \mu(\bar{q})$, we conclude that there exists a unique pair (q_1, q_2) with $q_1 > 0$ and $q_2 \leq \bar{q}$ that satisfy (13) and (14).

It is left to show that (13) and (14) are equivalent to the necessary and sufficient conditions for an equilibrium with $m > 0$, $a_1 = 0$, and $a_2 > 0$. Since $m > 0$, (10) holds with equality in equilibrium. Since $\gamma = \phi/\phi'$, clearly (14) and (10) are equivalent. Since $a_2 > 0$, (12) must also hold with equality. We know that $a_1 = 0 \Rightarrow a_2 = A$. Also, $z(q_1) = \phi M$ and $z(q_2) = \phi M + (\psi + \delta)A$ implies the asset pricing equation

$$\psi = \frac{z(q_2) - z(q_1)}{A} - \delta. \tag{18}$$

Substituting this into (13) yields (12).

Now suppose $A > \bar{A}$. We claim that there does not exist a pair (q_1, q_2) with $q_2 < \bar{q}$ that satisfy (13) and (14). To see this, let $\tilde{\tilde{q}}$ be the value of q_1 such that $\alpha(q_1) = \bar{q}$. It is easy to show that $A > \bar{A} \Rightarrow \tilde{\tilde{q}} < \tilde{q}$, which implies that $\mu(\tilde{\tilde{q}}) > \bar{q}$, so there does not exist a $q_1 < \tilde{\tilde{q}}$ satisfying $\mu(q_1) = \alpha(q_1)$. Therefore, $q_2 = \bar{q}$. From (10), $q_1 = \tilde{q}$ and the corresponding prices follow immediately. ■

C. Results for the Cashless Market: Since market C is is identical to the DM in that model, the first order conditions are (10)-(12). To derive the first order conditions for agents going to the other market, for any variable z associated with market C write the analog for market B as z^B. Then

$$V^B(m^B, a_1^B, a_2^B) = (1 - \lambda^B)W^B(y^B) + \lambda^B \left[u(q_2^B) + W^B\left(y^B - p_2^B\right)\right],$$

since all market B meetings are type 2 meetings. Differentiation yields the analogs of (10)-(12). Let us focus on the case where q_2, $q_2^B < \bar{q}$. One can show a_2, $a_2^B > 0$, and $m^B = 0$. Also, the bargaining solutions are $z(q_1) = \phi m$, $z(q_2) = \phi m + (\psi + \delta)a_2$, and $z(q_2^B) = (\psi + \delta)a_2^B$, and market clearing implies $A = (1 - b)a_2 + ba_2^B$. Given all this, routine manipulation allows us to describe (q_1, q_2, q_2^B) by

$$\begin{aligned}
i &= (1-\rho)\lambda\ell(q_1) + \rho\lambda\ell(q_2) \\
(1+r)A\delta &= \left\{(1-b)[z(q_2) - z(q_1)] + bz(q_2^B)\right\}[r - \rho\lambda\ell(q_2)] \\
(1+r)A\delta &= \left\{(1-b)[z(q_2) - z(q_1)] + bz(q_2^B)\right\}[r - \lambda^B\ell(q_2^B)].
\end{aligned}$$

In fact, since $q_2 = h(q_2^B) \equiv \ell^{-1}\left[\frac{\lambda^B}{\rho\lambda}\ell(q_2^B)\right]$, these reduce to two equations in (q_1, q_2^B):

$$\begin{aligned} i &= (1-\rho)\lambda\ell(q_1) + \lambda^B\ell(q_2^B) \\ (1+r)A\delta &= \left\{(1-b)[z\left[h(q_2^B)\right] - z(q_1)] + bz(q_2^B)\right\}[r - \lambda^B\ell(q_2^B)]. \end{aligned}$$

We have

$$\begin{aligned} \frac{\partial q_1^B}{\partial i} &= \frac{(1-b)\Big\{z'[h(q_2^B)]h'(q_2^B)[r-\lambda^B\ell(q_2^B)]\Big\} - \Big\{(1-b)[z\big(h(q_2^B)\big) - z(q_1)] + bz(q_2^B)\Big\}\lambda^B\ell'(q_2^B)}{\Psi} \\ \frac{\partial q_2^B}{\partial i} &= \frac{(1-b)z'(q_1)[r-\lambda^B\ell(q_2^B)]}{\Psi} \end{aligned}$$

and so both take the sign of

$$\begin{aligned} \Psi &= (1-\rho)\lambda_1\ell'(q_1)\Big\{(1-b)\Big[z'[h(q_2^B)]h'(q_2^B)[r-\lambda^B\ell(q_2^B)]\Big] \\ &- \Big[(1-b)[z\big(h(q_2^B)\big) - z(q_1)] + bz(q_2^B)\Big]\lambda^B\ell'(q_2^B)\Big\} + (1-b)z'(q_1)[r-\lambda^B\ell(q_2^B)]\lambda^B\ell'(q_2^B) < 0. \end{aligned}$$

D. Results in Table 1: Let Δ denote the determinant of the following matrix:

$$\begin{bmatrix} \lambda_1\ell'(q_1) & \lambda_2\ell'(q_2) \\ [\lambda_2\ell(q_2) - r]\,z'(q_1) & [r-\lambda_2\ell(q_2)]z'(q_2) - [z(q_2) - z(q_1)]\lambda_2\ell'(q_2) \end{bmatrix}$$

From (15), an equilibrium with $q_2 \geq q_1$ requires $r - \lambda_2\ell(q_2) \geq 0$, so $\Delta < 0$. Then we have

$$\begin{aligned} \frac{\partial q_1}{\partial i} &= \frac{[r-\rho\lambda\ell(q_2)]z'(q_2) - [z(q_2)-z(q_1)]\rho\lambda\ell'(q_2)}{\Delta} < 0 \\ \frac{\partial q_2}{\partial i} &= \frac{[r-\rho\lambda\ell(q_2)]z'(q_1)}{\Delta} < 0 \\ \frac{\partial q_1}{\partial \delta} &= \frac{-(1+r)A\rho\lambda\ell'(q_2)}{\Delta} < 0 \\ \frac{\partial q_2}{\partial \delta} &= \frac{(1+r)A(1-\rho)\lambda\ell'(q_1)}{\Delta} > 0 \\ \frac{\partial q_1}{\partial A} &= \frac{-(1+r)\delta\rho\lambda\ell'(q_2)}{\Delta} < 0 \\ \frac{\partial q_2}{\partial A} &= \frac{(1+r)\delta(1-\rho)\lambda\ell'(q_1)}{\Delta} > 0 \\ \frac{\partial q_1}{\partial \rho} &= \frac{\lambda[\ell(q_1)-\ell(q_2)][r-\rho\lambda\ell(q_2)]z'(q_2) - [z(q_2)-z(q_1)]\rho\lambda\ell'(q_2)\lambda\ell(q_1)}{\Delta} < 0 \\ \frac{\partial q_2}{\partial \rho} &= \frac{\lambda[\ell(q_1)-\ell(q_2)][r-\rho\lambda\ell(q_2)]z'(q_1) + [z(q_2)-z(q_1)](1-\rho)\lambda\ell'(q_1)\lambda\ell(q_2)}{\Delta} \\ \frac{\partial q_1}{\partial \lambda} &= \frac{(1-\rho)\ell(q_1)[z(q_2)-z(q_1)]\rho\lambda\ell'(q_2) - [(1-\rho)\ell(q_1)+\rho\ell(q_2)][r-\rho\lambda\ell(q_2)]z'(q_2)}{\Delta} > 0 \\ \frac{\partial q_2}{\partial \lambda} &= \frac{(1-\rho)\ell(q_2)[z(q_2)-z(q_1)]\rho\lambda\ell'(q_1) - [(1-\rho)\ell(q_1)+\rho\ell(q_2)][r-\rho\lambda\ell(q_2)]z'(q_1)}{\Delta} > 0 \end{aligned}$$

Given $z(q_1) = \phi M$ and $z(q_2) - z(q_1) = (\psi + \delta)A$, we have

$$
\begin{aligned}
\frac{\partial \psi}{\partial i} &= \frac{[z(q_2) - z(q_1)]\rho\lambda\ell'(q_2)z'(q_1)}{A\Delta} > 0 \\
\frac{\partial \phi}{\partial i} &= \frac{z'(q_1)}{M}\frac{\partial q_1}{\partial i} < 0 \\
\frac{\partial \psi}{\partial \delta} &= \frac{[z(q_2) - z(q_1)](1-\rho)\lambda\ell'(q_1)\rho\lambda\ell'(q_2) + [1 + \rho\lambda\ell(q_2)][(1-\rho)\lambda\ell'(q_1)z'(q_2) + \rho\lambda\ell'(q_2)z'(q_1)]}{\Delta} \\
\frac{\partial \phi}{\partial \delta} &= \frac{z'(q_1)}{M}\frac{\partial q_1}{\partial \delta} < 0 \\
\frac{\partial \psi}{\partial A} &= \frac{[z(q_2) - z(q_1)](1-\rho)\lambda\ell'(q_1)\rho\lambda\ell'(q_2)}{A\Delta} < 0 \\
\frac{\partial \phi}{\partial A} &= \frac{z'(q_1)}{M}\frac{\partial q_1}{\partial A} < 0 \\
\frac{\partial \psi}{\partial \rho} &= \frac{[z(q_2) - z(q_1)][(1-\rho)\lambda\ell'(q_1)\lambda\ell(q_2)z'(q_2) + \rho\lambda\ell'(q_2)\lambda\ell(q_1)z'(q_1)]}{\Delta} > 0 \\
\frac{\partial \phi}{\partial \rho} &= \frac{z'(q_1)}{M}\frac{\partial q_1}{\partial \rho} < 0 \\
\frac{\partial \psi}{\partial \lambda} &= \frac{(1-\rho)[z(q_2) - z(q_1)][\ell(q_2)\ell'(q_1)z'(q_2) - \ell(q_1)\ell'(q_2)z'(q_1)]}{\Delta} \\
\frac{\partial \phi}{\partial \lambda} &= \frac{z'(q_1)}{M}\frac{\partial q_1}{\partial \lambda} > 0
\end{aligned}
$$