Diverse Beliefs and Time Variability of Risk Premia

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Defining A Risk Premium on An Asset

- **Realized Premium** $$\pi_{t+1} = \frac{p_{t+1} + D_{t+1} - R_t p_t}{p_t}.$$

<u>Assume</u>: Data available to compute empirical moments

No one knows true probability distributions

- **m = probability implied by the empirical distribution**
- **All agree on m**
- **"The" Premium is the conditional expectations under m**

(1) $$E_t^m[\pi_{t+1} \mid I_t] = \frac{1}{p_t} E_t^m[p_{t+1} + D_{t+1} - R_t p_t \mid I_t]$$

<u>Problem</u>:

What causes the Premium to fluctuate over time?

Some Answers: Macroeconomic Variables

Fama and Bliss (1987) – Internal dynamics of past yields
Cambbpell and Shiller (1991) – Unexplained shocks to the bond market
Bernanke and Kuttner (2003) – Federal Reserve Policy shocks
Cocharane Piazzesi (2005) – Past yields and Business Cycles
Piazzesi and Swanson (2004) – Recessions forecasts, i.e. Non Farm Payroll..
In empirical finance focus is mostly on predictability of returns by dispersion of earning forecasts e.g. Miller (1977), Diether et al (2002). But also Lee et al (2002), Park (2005), Baker and Wurgler (2006), Campbell and Diebold (2007)

This Paper:

- **Effects of diverse beliefs on the risk premium**
- **Restrictions diverse rationality imposes**
- **Measuring market belief and testing hypotheses**

A Sketch of the Basic Model of Belief

- **Risky asset payoff $\{ D_t , t = 1, 2,...\}$ with true probability Π**
- **Non stationary with structural breaks. Π unknown**
- **True process is Stable:**

  Relative frequencies converge

  Has empirical distribution
- **Substantial past data: all know empirical probability m**

Assume: under m process is Markov with mean μ and transition F

(2) $$d_{t+1} = \lambda_d d_t + \rho^d_{t+1} \quad , \quad \rho^d_{t+1} \sim N(0, \sigma^2_d) \quad \text{where} \quad d_t = D_t - \mu .$$

- **The truth is [I tell you, agents do not know that]**

$$d_{t+1} = \lambda_d d_t + b_t + \xi^d_{t+1} \quad , \quad \xi^d_{t+1} \sim N(0, \sigma^2_\xi)$$ **b_t sequence of regimes.**

A Sketch of the Basic Model of Belief (cont.)

Note $\Pi \neq m$. <u>Theorem</u>: m is stationary and unique

- **A belief Q is rational if data generated under it reproduces m**
- **Here: Agents' beliefs Q^i are Markov**

- **Disagreement persists:**
  Data shows diverse forecasts. Must hold diverse transitions F_t^i

Rationality implies (minimally)

(A) A Rational agent cannot hold a constant $F^i \neq F$

(B) F_t^i fluctuate over time with $\frac{1}{N}\sum_{t=1}^{N} (F_t^i - F) \Rightarrow 0$, for all i

[Diversity and Rationality] $\Rightarrow$ Dynamics

Rational Agents are right on average.

A Sketch of the Basic Model of Belief (cont.)

Aim is to manageably describe diversity, not explain it.

Definition: A Belief State g_t^i pins down i's perceived transition of all state variables. In the case of dividend, it takes the form

(3) $$d_{t+1}^i = \lambda_d d_t + \lambda_d^g g_t^i + \rho_t^{id} \quad , \quad \rho_t^{id} \sim N(0, \hat{\sigma}_d^2)$$

- **g_t^i is observable since**

$$E^i[d_{t+1}^i | I_t, g_t^i] - E^m[d_{t+1} | I_t] = \lambda_d^g g_t^i.$$

Assume anonymity: only the distribution is observable

A Sketch of the Basic Model of Belief (cont.)

In Sum: if rational, g_t^i must (1) fluctuate, and (2) have a 0 mean

We represent state of belief with:

(4) $$g_{t+1}^i = \lambda_Z g_t^i + \lambda_Z^d d_t + \rho_{t+1}^{ig} \quad , \quad \rho_{t+1}^{ig} \sim N(0, \sigma_g^2) \quad , \quad \lambda_Z^d > 0$$

Why:

(I) It is Compatible with the data

(II) Can give an analytic- Bayesian justification.

This also provides an explanation for diversity, if you want it!

Belief Dynamics: A Bayesian Approach

- Two sources: Quantitative d_t's and others Ψ_t^i

Agent believes

- $d_t - \lambda_d d_{t-1} = b_{t-1} + \rho_t^d \quad , \quad \rho_t^d \sim N(0, 1/\beta)$
- At t-1 (=0), a prior distribution on $b_{t-1} \quad N(b, 1/\alpha))$ (d_{t-1} is known)
- At t observe data d_t and update. The posterior on b_{t-1} is

$$E_t^i(b_{t-1}|d_t) = \frac{\alpha b + \beta[d_t - \lambda_d d_{t-1}]}{\alpha + \beta} \quad , \quad b_{t-1}(d_t) \sim N[E_t^i(b_{t-1}|d_t), \frac{1}{\alpha+\beta}]$$

- At t you forecast d_{t+1} and needs estimate of b_t!
- Uses additional sources to assess regime in place
- End up with an alternate estimate of mean value.
- Qualitative information; subjective estimate: Ψ_t^i with uncertainty

$$RG_t^i \sim N(\Psi_t^i, \frac{1}{\gamma})$$

Belief Dynamics: A Bayesian Approach (cont.)

Assumption: The adjusted posterior distribution at t is then

$$b_t = \mu\, b_{t-1}(d_t) + (1-\mu)\, RG_t^{\,i}$$

Hence we have

$$E_t^{\,i}(b_t | d_t, \Psi_t^i) = \mu E_t^{\,i}(b_{t-1} | d_t) + (1-\mu)\Psi_t^i \qquad 0 < \mu < 1$$

$$Var(b_t | d_t, \Psi_t^i) = [\mu^2/(\alpha+\beta)] + [(1-\mu)^2/\gamma]\ .$$ **Let** $\zeta = 1/\mu^2$, $\xi = 1/(1-\mu)^2$

precision $$\Gamma(b_t) = \frac{\zeta(\alpha+\beta)\xi\gamma}{\zeta(\alpha+\beta)+\xi\gamma}.$$

Now observe d_{t+1}.

- **Followed with (i) a Bayesian update and (ii) a new Ψ_{t+1}^i**
- **Deduce a new prior for b_{t+1} as above**

Iterate forward.

Belief Dynamics: A Bayesian Approach (cont.)

Theorem 1: The precision of the posterior evolves according to

$$\Gamma_{t+1} = \frac{\zeta(\Gamma_t + \beta)\xi\gamma}{\zeta(\Gamma_t + \beta) + \xi\gamma}.$$

It converges to a unique positive bounded solution $\Gamma \Rightarrow \Gamma^*$. Defining

$$g_t^i = E_t^i(b_t | d_t, \Psi_t^i) - \frac{\mu\beta}{\Gamma^* + \beta} d_t ,$$

then the law of motion of g_t^i converges to a Markov transition

$$g_{t+1}^i = \lambda_Z g_t^i + \lambda_Z^d d_t + \rho_{t+1}^{ig}$$

with $\lambda_Z = \dfrac{\mu\Gamma^*}{\Gamma^* + \beta}$, $\lambda_Z^d = \dfrac{\mu\beta}{\Gamma^* + \beta}[\lambda_Z - \lambda_d]$, $\rho_{t+1}^{ig} = (1-\mu)\Psi_{t+1}^i$,

Dynamics of Market Belief

<u>Definition</u>: Market belief is the distribution $(g_t^1, g_t^2, \ldots g_t^N)$ observed by sampling hence with known moments.

- **Define: Mean market state of belief is** $Z_t = \frac{1}{N}\sum_{i=1}^{N} g_t^i$
- **Average (4)** $\frac{1}{N}\sum_{i=1}^{N} g_{t+1}^i = \lambda_Z \frac{1}{N}\sum_{i=1}^{N} g_t^i + \lambda_Z^d d_t + \frac{1}{N}\sum_{i=1}^{N} \rho_{t+1}^{ig}$
- **Key condition: ρ_t^{ig} are correlated hence** $\frac{1}{N}\sum_{i=1}^{N} \rho_t^{ig} = \rho_{t+1}^Z \neq 0.$
- **Z_t is a state variable with empirical distribution**

$$Z_{t+1} = \lambda_Z Z_t + \lambda_Z^d d_t + \rho_{t+1}^Z \quad , \quad \rho_{t+1}^Z \sim N(0, \sigma_Z^2).$$

<u>In all models</u>: this correlation is the crucial factor

The Structure of Beliefs: Perception Models

- **We thus *expand the empirical distribution* to $\{(d_{t+1}, Z_{t+1}), t = 1, 2, \ldots\}$.**

(5a) $d_{t+1} = \lambda_d d_t + \rho^d_{t+1}$

(5b) $Z_{t+1} = \lambda_Z Z_t + \lambda^d_Z d_t + \rho^Z_{t+1}$

$$\begin{pmatrix} \rho^d_{t+1} \\ \rho^Z_{t+1} \end{pmatrix} \sim N\left(\begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} \sigma^2_d, & 0, \\ 0, & \sigma^2_Z \end{bmatrix} = \Sigma \right), \quad \text{i.i.d.}$$

- **Individual i's *perception model* (together with (4))**

(6a) $d^i_{t+1} = \lambda_d d_t + \lambda^g_d g^i_t + \rho^{id}_{t+1}$

(6b) $Z^i_{t+1} = \lambda_Z Z_t + \lambda^d_Z d_t + \lambda^g_Z g^i_t + \rho^{iZ}_{t+1}$

$$\begin{pmatrix} \rho^{id}_{t+1} \\ \rho^{iZ}_{t+1} \end{pmatrix} \sim N\left(\begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} \hat{\sigma}^2_d, & \hat{\sigma}_{Zd} \\ \hat{\sigma}_{Zd}, & \hat{\sigma}^2_Z \end{bmatrix} = \Sigma^i \right)$$

Parameter sign $\lambda^g_d \geq 0$ and $\lambda^g_Z \geq 0$ orient the model: When $g^i_t > 0$, i believes t+1 dividend and market belief will persist <u>above normal</u> .

An Illustrative Infinite Horizon Model

- **Many agents and a single commodity -- "consumption"**
- **Riskless technology: 1 input at t produces R > 1 at t+1**
- **One risky asset with supply S = 1**
- **Risky asset payoff process $\{D_t, t = 1, 2, ...\}$ with *unknown* probability Π**
- **<u>Under m</u> $\{D_t, t = 1, 2, ...\}$ is Markov with transition**

$$d_{t+1} = \lambda_d d_t + \rho^d_{t+1} \quad , \quad \rho^d_{t+1} \sim N(0, \sigma^2_d) \quad \text{where} \quad d_t = D_t - \mu .$$

<u>Notation</u>

- θ^i_t **= date t asset purchases of i.**
- B_t^i **= date t investment of i in riskless technology**
- p_t **= date t price of the risky asset. Think of it as the S&P 500**

An Infinite Horizon Model (Cont.)

(7) **Maximize** $\underset{(\theta^i, B^i)}{\text{Max}} \; E_t^i\left[\sum_{k=t}^{\infty} -\beta^{k-1} e^{-\left(\frac{c_k^i}{\tau}\right)} \,\middle|\, I_t\right]$

Subject to (i) $c_t^i = \theta_{t-1}^i[p_t + d_t + \mu] + B_{t-1}^i R - \theta_t^i p_t - B_t^i$,

(ii) i's belief as specified in (6a)-(6b)

(iii) Initial values (θ_0^i, B_0^i)

Advantage: only mean market belief matters.

State variables in individual optimization:

$\psi_t^i = (1, d_t, Z_t, g_t^i)$, $u = (u_0, u_1, u_2, u_3)$ **constants**

Equilibrium Asset Price and Risk Premium

Stability Conditions:

$$R = 1 + r > 1\ , \quad 0 < \lambda_d < 1 \quad , \quad 0 < R - (\lambda_Z + \lambda_Z^g) < 1\ .$$

Theorem 3: There is a unique equilibrium price function of the form $p_t = a_d d_t + a_Z Z_t + P_0$ **with parameters**

$$a_d = \frac{a_Z \lambda_Z^d + (\lambda_d + u_1)}{R - \lambda_d}\ , \quad a_Z = \frac{(a_d + 1)\lambda_d^g + (u_2 + u_3)}{R - (\lambda_Z + \lambda_Z^g)} > 0 \quad P_0 = \frac{(\mu + u_0)}{r} - \frac{\hat{\sigma}_V^2}{R\tau}\ .$$

- **Price exhibits "excess" volatility due to beliefs**

Now we can compute the premium

$$E_t^m[\pi_{t+1} | I_t] = \frac{1}{p_t} E_t^m[p_{t+1} + d_{t+1} + \mu - Rp_t | I_t]$$

Structure of the Risk Premium: Main Result

(9) $$E_t^m[\pi_{t+1}|I_t] = \frac{1}{p_t}[(\frac{r\hat{\sigma}_V^2}{R\tau} - u_1 d_t - u_0) - a_Z(R - \lambda_Z)Z_t]$$

<u>Main Theorem</u>: Since $a_Z(R - \lambda_Z) > 0$, the Risk Premium increases in $\hat{\sigma}_V^2$ and falls with mean market belief Z_t. Also,

$$\hat{\sigma}_V^2 \simeq (a_d + 1)^2 \hat{\sigma}_d^2 + a_Z^2 \hat{\sigma}_Z^2$$

That is

- **In an optimistic market the long position is less m risky**
- **In a pessimistic market the long position is more m risky**

Effects of Belief on Premium: What Does It Not Say

- **Agents are on their demand functions**
- **It is not "optimal" to be a contrarian**
- **A "contrarian" may be short when wants to be long and long when wants to be short**
- **Analogous to why we do not adopt a log utility**

Comments

- **<u>With Non-Exponential Utility</u>: entire distribution** $(g_t^1, \ldots, g_t^N)$ **matters but <u>Main Result</u> holds.**

Empirical work considers first two moments:

- **Mean market belief** $Z_t = \frac{1}{N}\sum_{i=1}^{N} g_t^i$**. Diversity** $\sigma_t^Z = \sqrt{\frac{1}{N}\sum_{i=1}^{N} (g_t^i - Z_t)^2}$

- **Hypothesis: effect of cross sectional diversity** σ_t^Z **on risk premium is negative (the "Narrow Door Hypothesis")**

Empirical Test of the Main Result

- **Estimate premia on holding returns of long positions in**

Fed Funds futures over	**1988:10 - 2003:11,**
3 month T Bills over	**1987:12 - 2003:11,**
6 month T Bills over	**1987:12 - 2003:11 .**

- **Dependent variable = realized excess holding returns**
- **1 - 6 months holdings of Fed Funds futures and 1 - 12 months for T Bills**
- **Thus: 30 models.**

(1) T Bill and Federal Funds Futures Interest Rate Forecasts:

- **Forecast data of interest rates by Blue Chip Financial Forecasts (BLUF)**
- **Use forecasts of interest rates to construct beliefs as in theory**
- **Construct $Z_t^{(k,h)}$ for maturity k and holding period h**
- **Notation: $Z_t^{(k,h)} > 0$ means market believes interest rate on maturity k will be higher than normal at t + h.**
- **To be beneficial to long position, orientation of $-Z_t^{(k,h)}$ is as in theory.**

Measuring Market Belief Distribution

- **For any X, we defined**

$$E^{i}[X^{i}_{t+h}|I_t, g_t^{i}] - E^{m}[X_{t+h}|I_t] = \lambda^{g}_{d} g_t^{iXh}$$

- $E^{i}[X^{i}_{t+h}|I_t, g_t^{i}]$ **is provided in BLUF data, The Problem is** $E^{m}[X_{t+h}|I_t]$
- **Stock and Watson's diffusion indices method explained in paper**
- **Method uses 215 monthly time series integrated via factor analysis**
- **Stock and Watson (2002) show the best stationary forecast of X uses a combination of factors and lagged variables of X**
- **Hence, we forecast inflation and interest rates by using the appropriate factors and past inflation and interest rates.**
- **<u>Data on realized returns</u>: CBOT and the familiar Fama Bliss (1987) files, updated to 2003:11 by using Bliss's program. (see Bliss (1997) for details of the method used).**

Some Statistic on Market Beliefs

Table 1A: Summary Statistics of Market Beliefs

6 Months or 2 Quarters Ahead	Time Ave.	Std Dev	Autocorrelation
Fed Fund rate	0.273	0.528	0.700
1 year T-bill rate	0.238	0.429	0.735
GDP deflator	0.365	0.595	0.674
12 Months or 4 Quarters Ahead			
Fed Fund rate	0.267	0.763	0.632
1 year T-bill rate	0.385	0.681	0.841
GDP deflator	0.398	0.798	0.740

Market Belief and Cross Sectional Standard Deviations

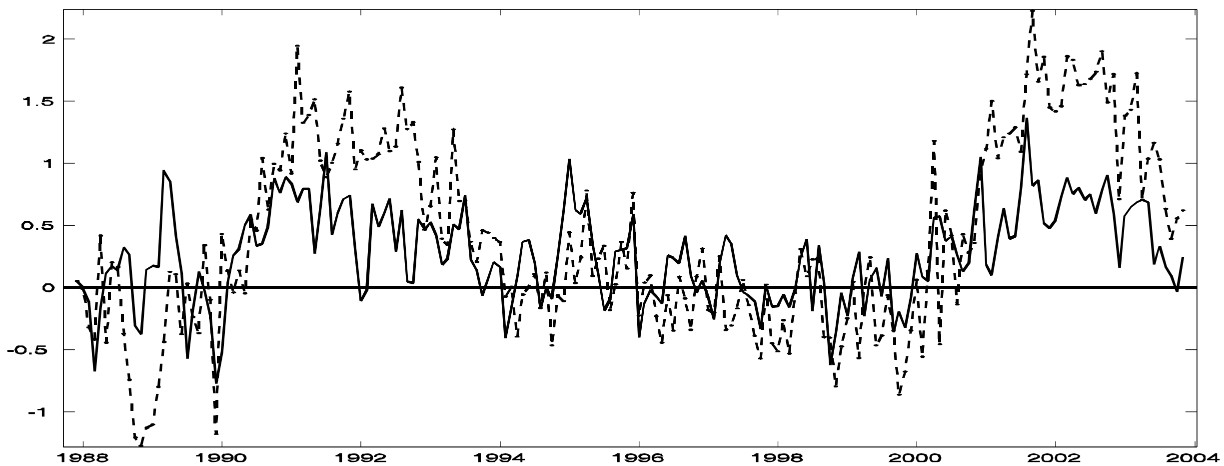

Figure 1: 6-month Treasury Bill rate: 4 and 12(dashed line) month ahead Market Belief $Z_t^{(6)}$

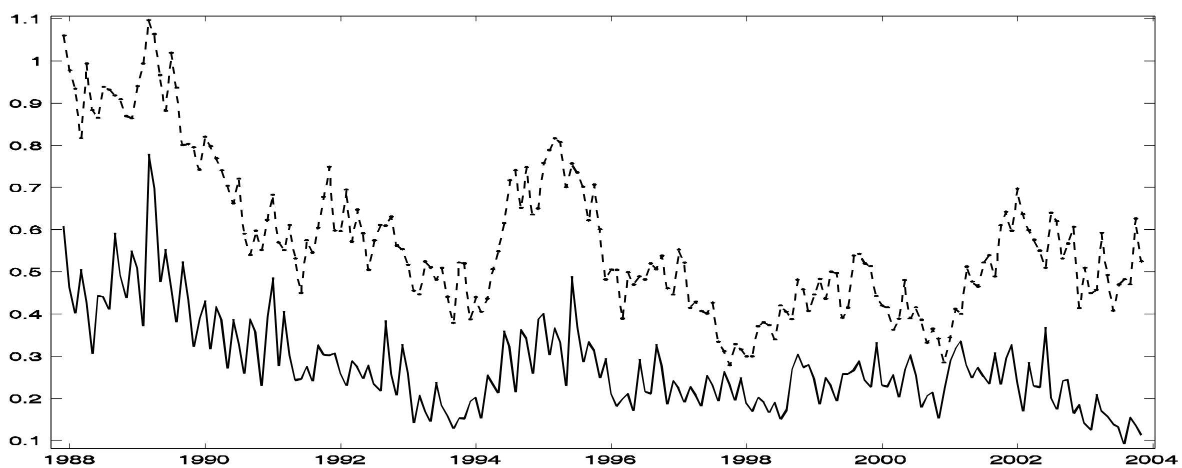

Figure 2: 6-month Treasury Bill rate: 4 and 12(dashed line) month ahead standard deviation of Market Belief $Z_t^{(6)}$

Estimating Risk Premium Functions

Estimated Function $\pi_{t+h}^{(X,h)} = \alpha_0^{(X,h)} + \alpha_1^{(X,h)} B_t + \alpha_2^{(X,h)} M_t + \varepsilon_{t+h}^{(X,h)}$

B_t = vector of market belief variables

M_t = standard macroeconomic variables [models of Cocharane Piazzesi (2005), Piazzesi and Swanson (2004)]

B_t : Two Moments of the Distribution of Beliefs

- **$Z_t^{(k,h)}$ – belief in date t+h interest rate of maturity k at t+h**
- **$\sigma_t^{(k,h)}$ – cross sectional standard deviation of individual beliefs: + or - ?**

M_t : Traditional Variables: Recession, Monetary Policy and Past Yields

- **NFP_{t-1} – year over year growth rate of Non Farm Payroll at t-1**
- **CPI_{t-1} – year over year rate of inflation at t-1**
- **F_t – Federal Funds rate measuring state of monetary policy**
- **R_{t-1}^{Fj} – principal components of past interest rates j = 1, 2, 3.**

Results: Table 2A: Federal Fund Futures Market - Time Variability of Excess Returns

	Constant	NFP_{t-1}	CPI_{t-1}	F_t	$\sigma_t^{(F,h)}$	$-Z_t^{(F,h)}$	R^2
h=2	-0.000 (0.056)	-0.016 (0.020)	-0.003 (0.026)	0.006 (0.018)	0.354 * (0.204)	-0.294 † (0.051)	0.313
h=4	-0.047 (0.101)	-0.142 † (0.040)	-0.037 (0.045)	0.122 † (0.032)	-0.752 * (0.438)	-0.419 † (0.066)	0.380
h=6	-0.199 (0.136)	-0.284 † (0.047)	-0.056 (0.085)	0.233 † (0.042)	-0.413 (0.455)	-0.397 † (0.130)	0.407

Table 2B: 3 Months Treasury Bills Market - Time Variability of Excess Returns

	Constant	NFP_{t-1}	CPI_{t-1}	F_t	R_{t-1}^{F1}	R_{t-1}^{F2}	R_{t-1}^{F3}	$\sigma_t^{(3,h)}$	$-Z_t^{(3,h)}$	R^2
h=2	0.724 (0.676)	-0.042 (0.044)	0.005 (0.042)	-0.033 (0.095)	0.203 (0.308)	-0.181 † (0.067)	-0.010 (0.060)	0.065 (0.397)	-0.919 † (0.091)	0.422
h=4	0.939 * (0.567)	-0.136 † (0.033)	-0.021 (0.045)	0.013 (0.086)	0.398 (0.273)	-0.036 (0.063)	-0.062 (0.052)	-0.721 † (0.321)	-0.542 † (0.082)	0.416
h=6	1.329 † (0.552)	-0.191 † (0.023)	-0.000 (0.043)	-0.033 (0.079)	0.603 † (0.259)	0.014 (0.057)	-0.062 (0.051)	-0.397 (0.259)	-0.276 † (0.080)	0.434
h=8	1.964 † (0.535)	-0.181 † (0.025)	-0.003 (0.034)	-0.119 * (0.070)	0.915 † (0.243)	0.028 (0.054)	-0.030 (0.045)	-0.183 (0.193)	-0.250 † (0.035)	0.537
h=10	1.781 † (0.543)	-0.178 † (0.026)	0.001 (0.023)	-0.042 (0.071)	0.796 † (0.261)	-0.053 (0.035)	-0.023 (0.037)	-0.447 † (0.152)	-0.223 † (0.042)	0.640
h=12	1.684 † (0.593)	-0.200 † (0.021)	0.004 (0.026)	-0.029 (0.079)	0.749 † (0.295)	-0.068 (0.038)	-0.012 (0.026)	-0.403 † (0.097)	-0.180 † (0.027)	0.673

† indicates significance at 5% or better while * indicates significance at 10%

Results (Cont.)

Table 2C: 6 Months Treasury Bills Market - Time Variability of Excess Returns

	Constant	NFP_{t-1}	CPI_{t-1}	F_t	R_{t-1}^{F1}	R_{t-1}^{F2}	R_{t-1}^{F3}	$\sigma_t^{(6,h)}$	$-Z_t^{(6,h)}$	R^2
h=2	1.027 (1.414)	-0.111 (0.084)	-0.029 (0.089)	-0.017 (0.204)	0.225 (0.646)	-0.426 † (0.136)	0.095 (0.112)	-0.127 (0.952)	-1.985 † (0.179)	0.444
h=4	1.713 (1.293)	-0.316 † (0.071)	-0.076 (0.100)	0.056 (0.182)	0.788 (0.594)	-0.062 (0.118)	-0.075 (0.093)	-1.200 (0.747)	-1.182 † (0.148)	0.451
h=6	2.703 † (1.239)	-0.391 † (0.069)	-0.051 (0.089)	-0.015 (0.161)	1.268 † (0.568)	0.009 (0.116)	-0.081 (0.119)	-1.246 † (0.467)	-0.799 † (0.147)	0.494
h=8	3.261 † (1.207)	-0.394 † (0.073)	-0.075 (0.082)	-0.052 (0.164)	1.514 † (0.587)	-0.074 (0.106)	-0.028 (0.096)	-0.752 * (0.412)	-0.622 † (0.082)	0.596
h=10	4.174 † (1.319)	-0.387 † (0.075)	0.005 (0.051)	-0.187 (0.175)	1.852 † (0.640)	-0.170 † (0.084)	-0.020 (0.074)	-0.763 † (0.359)	-0.451 † (0.124)	0.638
h=12	4.069 † (1.253)	-0.401 † (0.059)	-0.004 (0.050)	-0.173 (0.162)	1.812 † (0.619)	-0.176 † (0.085)	0.024 (0.054)	-0.683 † (0.212)	-0.388 † (0.098)	0.664

Explaining the Premium: Contribution of Market Belief

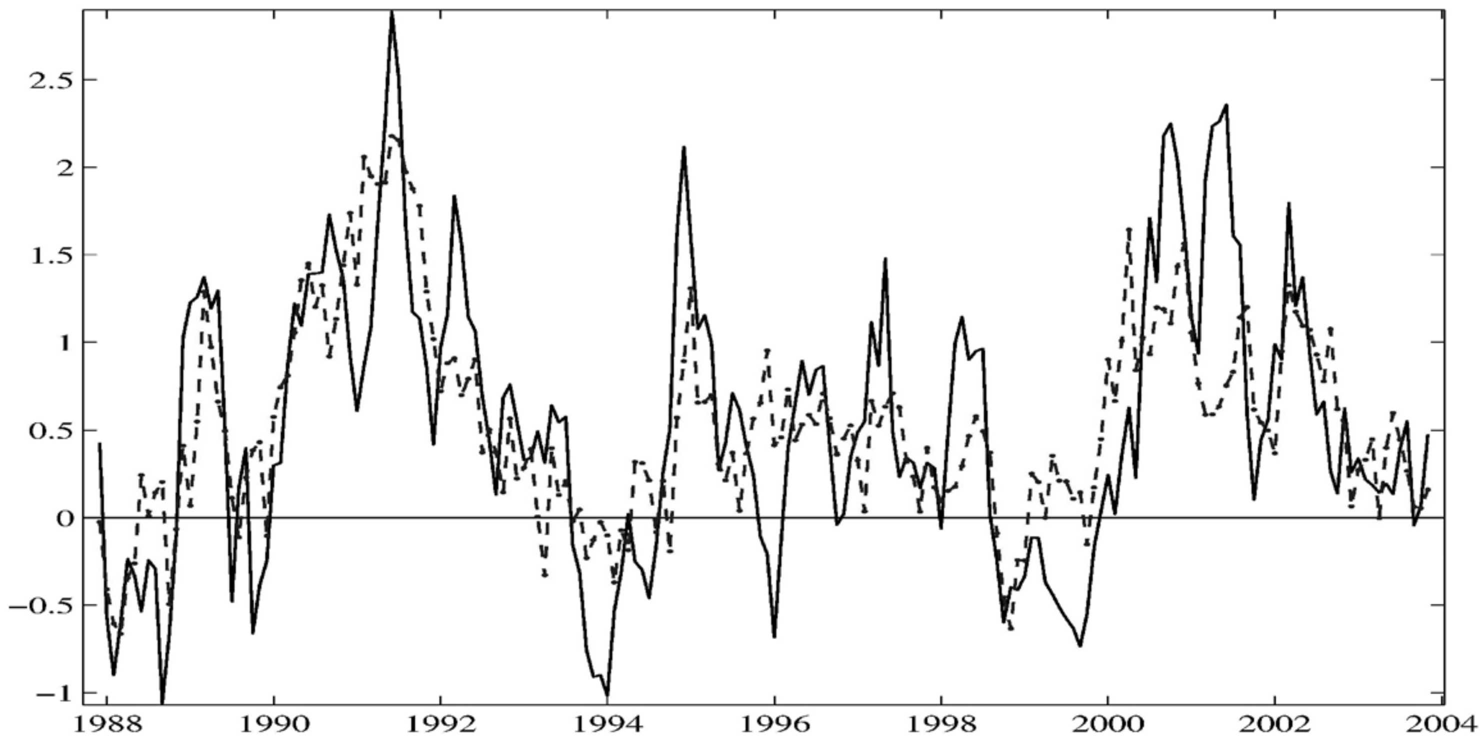

Figure 5: Excess returns on 6 Months TBill 6 months ahead. The dashed line represents the fitted values from regression (24)

Statistical Evaluation

Table 3: Contribution of Belief to Excess Returns Predictability

Asset	Horizon	Without Beliefs		With Beliefs		Diebold Mariano Statistic
		R^2	Std. Errors of the regression	R^2	Std. Errors of the regression	
Fed Fund Futures	h=1	-0.005	0.119	0.237	0.103	-1.922 *
	h=3	0.139	0.280	0.311	0.250	-2.226 †
	h=6	0.345	0.479	0.407	0.455	-0.848
3 Months T-Bill	h=1	0.062	0.671	0.482	0.499	-6.125 †
	h=3	0.208	0.368	0.450	0.307	-3.794 †
	h=5	0.309	0.306	0.391	0.287	-2.349 †
	h=7	0.425	0.254	0.466	0.245	-1.834 *
	h=9	0.489	0.247	0.607	0.216	-3.096 †
	h=12	0.595	0.214	0.673	0.192	-3.300 †
6 Months T-Bill	h=1	0.077	1.371	0.444	0.930	-6.078 †
	h=3	0.219	0.836	0.451	0.689	-3.757 †
	h=5	0.349	0.657	0.494	0.601	-2.710 †
	h=7	0.462	0.551	0.596	0.510	-2.537 †
	h=9	0.541	0.492	0.638	0.430	-2.639 †
	h=12	0.600	0.431	0.664	0.395	-2.642 †

Economic Evaluation of the Effects of Belief

JOINT EFFECT: Both belief variables $B_t^{(X,h)} = (\sigma_t^{(X,h)}, Z_t^{(X,h)})$.

- **$J_t^{(X,h)} = \hat{\alpha}_2^{(X,h)} \bullet B_t^{(X,h)}$ - belief component is Positive or Negative.**
- **Look at $|J_t^{(X,h)}|$ with a mean of $\overline{|J^{(X,h)}|}$.**

Table 5: Component of Belief in the Premium (b. points, annualized)

	Fed Funds Futures		3 Months Treasury Bills		6 Months Treasury Bills	
	Mean Premium	$\overline{\lvert J^{(F,h)}\rvert}$	Mean Premium	$\overline{\lvert J^{(3,h)}\rvert}$	Mean Premium	$\overline{\lvert J^{(6,h)}\rvert}$
h=2	44.4	61.4	37.2	24.6	60.4	59.2
h=4	57.6	51.3	29.4	17.9	54.2	37.8
h=6	68.8	34.0	27.2	11.5	54.2	34.9
h=8	----	----	27.5	11.1	70.8	32.0
h=10	----	----	40.6	19.7	69.5	35.8
h=12	----	----	38.9	18.5	66.1	33.2

Economic Evaluation (Cont.)

(1) $\overline{|J^{(X,h)}|}$ is often larger than 50% of mean premium
(2) $\overline{|J^{(X,h)}|}$ declines to about 50% at h=12

Conclude that

- Risk premia dominated by beliefs for short h
- The impact of beliefs declines with duration of holding period
- Pure belief effect accounts for 50% of mean premia for h = 12