A Simple Model of Bankruptcy in General Equilibrium

5th Annual CARESS Cowles Conference

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April 19th, 2009

CARESS Rules

From: Dave Cass
Date: January 14, 2005

- Introduction, i.e. rubbish - or more bluntly, crap - at the beginning of the seminar, shall last no longer than 15 minutes (which is, from my own viewpoint, even too liberal)...**If there is not a serious equation, model, result,...(on the blackboard or a slide) by the end of 15 minutes, then anyone in the audience - including me - is perfectly free to simply leave: THE SEMINAR IS FORMALLY OVER!**
- Mention no more than 5 (or, **if there is a very good reason**, really stretching it, 10) references, preferably just those which the speaker thinks are the most substantively important (not the most politically correct or self-serving).

Introduction

- I remove the assumption that households, having made financial commitments today ($s = 0$), must fulfill these commitments in the realized state ($s > 0$) tomorrow.
- Not considered in this model is defaulting (choosing not to fulfill the financial commitment on an asset-by-asset basis).
- Rather, a household considers the value of its entire portfolio and faces a binary decision:
  - If the household fulfills its commitment (solvency).
  - If not (bankruptcy).

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Introduction (cont.)

- As a result of bankruptcy, financial payouts to creditors must be appropriately diluted.
- I assume assets are pooled (each asset with its own pool) and that all creditors receive the same fraction of the original asset payouts.
- For a nonbankrupt household h, the payout on asset j is

$$\rho_j(s) r_j(s) \ \text{ when } z_j^h \geq 0 \ \text{ and } \ r_j(s) \ \text{ when } z_j^h < 0$$

$\rho_j(s)-$ endogenous dilution factor

$$\rho_j(s) \sum_{\text{all h}} (\text{long positions}) = 1 - \sum_{\text{solvent}} (\text{short positions})$$

$$+ \sum_{\text{bankrupt}} \begin{pmatrix} \text{fraction of household's seized} \\ \text{assets paid back to asset pool } j \end{pmatrix}.$$

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Fewer than 5 references

1. Dubey, Geanakoplos, and Shubik. Econometrica (2005).
   - Model of household default of unsecured credit
   - Households choose "degree" of default
   - Penalty for default: utility loss
   - Chain reactions of default possible
   - Model with real assets and investment constraints (though constraints could be removed when dealing with numeraire assets)

1a. Zame. AER (1993).
   - Theorems describing the beneficial normative role that default *can* play

2. Geanakoplos and Zame (2002).
   - Model of household default of secured credit
   - Collateral requirements exogenously set
   - Investment constraints endogenously depend on collateral requirement and scarcity of collateral

Household bankruptcy in a pure exchange model

I model chapter 7 bankruptcy

-70% of individual bankruptcies are chapter 7
-with chapter 13, still liable for debt ($5 -$ year repayment)
-more potential for abuse with chapter 7

in the unsecured credit markets

-with secured credit, there is no anonymous interaction between the bankrupt household and its creditors (this is pre-contracted)

without investment constraints

-justification of constraints is as a policy tool to mitigate effects of bankruptcy (need to understand bankruptcy first)
-setting of such constraints is arbitrary

and with assets in unit net supply

-short-hand way to model a diverse asset structure .

The model

2− period model with S states of uncertainty in the second time period
households $h \in \mathcal{H}$ (continuum) with finite types $f \in \{1, .., F\}$
$\mathcal{H}_f = \{\text{households of type } f\}$ with $\int_{h \in \mathcal{H}_f} dh = \frac{1}{F} \quad \forall f$

real

- L physical commodities per state ($G = L(S+1)$)
- consumption $x^h \in \mathbb{R}^G_+$
- endowments $e^f >> 0$
- u^f is C^2, differentiably strictly increasing, differentiably strictly quasi-concave, and satisfies the boundary condition.
- spot prices $p >> 0$

financial

- J numeraire assets in unit net supply
- assets $z^h \in \mathbb{R}^J$
- initial assets $z^f(0) > 0$
- yields matrix $Y = [r_j(s)]_{\forall j, \forall s > 0}$ is strictly positive, has full rank, and is in general position.
- asset prices q

Incorporating bankruptcy

- Under chapter 7, a household forfeits its portfolio z^h (including those assets held in a long position with positive value) and walks away from its debt.
- In principle, this system legislates a "fresh start" with no wage garnishments and no confiscation of current period endowments.

However...

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Incorporating bankruptcy (cont.)

- The court appoints a case trustee to oversee the liquidation of a bankrupt household's assets to repay the creditors.
- The case trustee is endowed with "avoiding powers"; trustee can void any transfer of assets that occurred 90 days prior to the bankruptcy filing.
- For bankrupt households with short positions in all assets, there is no role for "avoiding powers" as such households would not have any assets to shelter from the state.

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Incorporating bankruptcy (cont.)

If a bankrupt household currently holds long positions in assets $\left(z_j^h\right)^+$,

- then its sale of assets in the last 90 days will likely be voided.
- having consumed the benefits from this asset sale in previous periods, there must be a corresponding cost (in terms of lost endowments) in the current period.

Cost of bankruptcy (in current period)

$$\sum_j \alpha_j r_j(s)(z_j^h)^+$$

Household wealth (in current period)

$$p(s)e^h(s) - \sum_j \alpha_j r_j(s)(z_j^h)^+$$

Key first assumption: $\alpha >> 0$

Incorporating bankruptcy (cont.)

The Bankruptcy Abuse Prevention and Consumer Protection Act of 2005 sought to do as its name indicates.

- Any household applying for chapter 7 bankruptcy must pass a "means test". Simply put, if a household's average monthly income over the past 6 months exceeds the state median, then it cannot declare bankruptcy.

Key second assumption: $\forall f \in \mathcal{F}$, $\exists s > 0$ s.t. $e_l^f(s) > \frac{1}{F} \sum_{\bar{f} \in \mathcal{F}} e_l^{\bar{f}}(s) \quad \forall l$.

- By making this assumption, then by law, $\forall h$, $\exists s > 0$ s.t. h cannot declare bankruptcy.

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Results leading to existence

Recall $r(s) >> 0$, so no Arrow securities.
Suppose a household declares bankruptcy in state $s = 1$.

"This household can use its portfolio to replicate an Arrow security. The following portfolio replicates the desired Arrow security:

$$z' = \begin{bmatrix} r_1(1) & .. & r_J(1) \\ : & & : \\ r_1(J) & .. & r_J(J) \end{bmatrix}^{-1} \begin{pmatrix} 1 \\ \vec{0} \end{pmatrix}.$$

The household could then short this replicated portfolio to ∞ and obtain an arbitrage profit."

To bound assets and obtain existence, such a "story" cannot be possible (notice that z' is a linear combination of both long and short asset positions and recall that $\alpha >> 0$).

Results leading to existence (cont.)

Lemma

With $\alpha >> 0$ and $r(s) >> 0$ $\forall s > 0$, then the assets z^h of bankrupt households are bounded.

Proof:

Household wealth in a bankrupt state

$$p(s)e^h(s) - \sum_j \alpha_j r_j(s) \left(z_j^h\right)^+$$

Household wealth in a nonbankrupt state (at least one such state exists)

$$p(s)e^h(s) + \sum_j \left\{ r_j(s) \left(z_j^h\right)^- + \rho_j(s) r_j(s) \left(z_j^h\right)^+ \right\}$$

- Suppose $\exists z_j^\nu \rightarrow +\infty$, then consumption is outside the consumption set in bankrupt state $(\Rightarrow\Leftarrow)$.
- Suppose $\exists z_j^\nu \rightarrow -\infty$, then with z^ν bounded above, consumption would be outside the consumption set in nonbankrupt state $(\Rightarrow\Leftarrow)$.

Definition of equilibrium terms

Define the household specific asset payouts for nonbankrupt households $r_j^h(s)$ as:

$$\begin{aligned} r_j^h(s) &= \rho_j(s) r_j(s) \quad \text{if } z_j^h \geq 0 \\ r_j^h(s) &= r_j(s) \quad \text{if } z_j^h < 0. \end{aligned}$$

Define $\mathcal{S}_f^* = \{s > 0 : p(s)e^f(s) > \frac{1}{F} \sum_{\bar{f} \in \mathcal{F}} p(s)e^{\bar{f}}(s)\}$.

Then a household will declare bankruptcy in state $s > 0$ if

$$\begin{aligned} s &\notin \mathcal{S}_f^* \text{ for } h \in \mathcal{H}_f \\ \textstyle\sum_j r_j^h(s) z_j^h &< -\textstyle\sum_j \alpha_j r_j(s)(z_j^h)^+. \end{aligned} \tag{1}$$

Define the set of bankrupt households in state $s > 0$:

$$\mathcal{H}'_s = \{h \in \mathcal{H} : (1) \text{ holds}\}.$$

Household budget set

The budget set for each household is given by:

$$B^h(p,q,\rho) = \{(x,z) \in X^h \times \mathbb{R}^J : p(0)(e^f(0) - x(0)) + qz^f(0) - qz \geq 0,$$

$$p(s)(e^f(s) - x(s)) + \max\left\{\textstyle\sum_j r_j^h(s) z_j, -\sum_j \alpha_j r_j(s)(z_j)^+\right\} \geq 0 \; \forall s \notin \mathcal{S}_f^*,$$

$$p(s)(e^f(s) - x(s)) + \textstyle\sum_j r_j^h(s) z_j \geq 0 \quad \forall s \in \mathcal{S}_f^*\} \text{ for } h \in \mathcal{H}_f.$$

Distribution rule

- Bankrupt households forfeit the value of their long positions $\sum_j \rho_j(s) r_j(s) \left(z_j^h\right)^+$ and must pay the bankruptcy cost $\sum_j \alpha_j r_j(s) \left(z_j^h\right)^+$.
- I assume that these total funds $\sum_j \left(\rho_j(s) + \alpha_j\right) r_j(s) \left(z_j^h\right)^+$ are divided among the vector of asset pools with the fraction $\frac{r_k(s)\left(z_k^h\right)^-}{\sum_k r_k(s)\left(z_k^h\right)^-}$ to each asset pool k.
- Defining

$$\delta^h(s) = \frac{-\sum_j \left(\rho_j(s) + \alpha_j\right) r_j(s) \left(z_j^h\right)^+}{\sum_k r_k(s) \left(z_k^h\right)^-},$$

then $-\delta^h(s) r_k(s) \left(z_k^h\right)^-$ is returned to asset pool k.
- By the definition of $h \in \mathcal{H}'_s$, the variable $0 \leq \delta^h(s) < 1$.

Bankruptcy equilibria

A *bankruptcy equilibrium* is $\left((x^h, z^h)_{h\in\mathcal{H}}, p, q, \rho\right)$ s.t.

$$\forall h \in \mathcal{H}, \text{ given } (p, q, \rho) \qquad (x^h, z^h) \in \underset{\text{subj to } (x,z)\in B^h(p,q,\rho)}{\arg\max}\, u^h(x). \tag{H}$$

$$\begin{aligned}
&\int_{h\in\mathcal{H}} z_j^h = 1 \quad \forall j. \\
&\int_{h\in\mathcal{H}} x_l^h(s) = \tfrac{1}{F}\textstyle\sum_{f\in\mathcal{F}} e_l^f(s) \quad \forall (l,s) \notin \{(L,1),\ldots,(L,S)\}. \\
&\int_{h\in\mathcal{H}} x_L^h(s) = \tfrac{1}{F}\textstyle\sum_{f\in\mathcal{F}} e_L^f(s) + \sum_j r_j(s) \quad \forall s > 0.
\end{aligned} \tag{M}$$

$$\rho_j(s)\int_{h\in\mathcal{H}}\left(z_j^h\right)^+ + \int_{h\notin\mathcal{H}'_s}\left(z_j^h\right)^- + \int_{h\in\mathcal{H}'_s}\delta^h(s)\left(z_j^h\right)^- = 1 \quad \forall j, s > 0. \tag{AC}$$

Implications of bankruptcy

Consider the aggregate consistency (AC) condition for some j, s :

$$\rho_j(s)\int_{h\in\mathcal{H}}\left(z_j^h\right)^+ + \int_{h\notin\mathcal{H}'_s}\left(z_j^h\right)^- + \int_{h\in\mathcal{H}'_s}\delta^h(s)\left(z_j^h\right)^- = 1.$$

The dilution factor $0 < \rho_j(s) \leq 1$.

Using the market clearing condition $\int_{h\in\mathcal{H}} z_j^h = 1$, (AC) is equivalent to:

$$\rho_j(s) = 1 + \frac{\int_{h\in\mathcal{H}'_s}\left(1-\delta^h(s)\right)\left(z_j^h\right)^-}{\int_{h\in\mathcal{H}}\left(z_j^h\right)^+}.$$

Recalling $0 \leq \delta^h(s) < 1$,

1. if $z_j^h \geq 0 \ \ \forall h \in \mathcal{H}'_s$ (or $\mathcal{H}'_s = \emptyset$), then $\rho_j(s) = 1$.
2. if $z_j^h < 0$ for some $h \in \mathcal{H}'_s$, then $\rho_j(s) < 1$.

Existence

Theorem

Under the smooth assumptions listed, with strictly positive Y, and with the two key assumptions on how bankruptcy is implemented, then a bankruptcy equilibrium $\left((x^h, z^h)_{h\in\mathcal{H}}, p, q, \rho\right)$ exists.

Boundedness of Assets (sketch)

- For nonbankrupt households $\left(h \notin \cup_{s>0}\mathcal{H}'_s\right)$, financial payout $\hat{Y}^h z^h$ is bounded where $\hat{Y}^h$ is the strictly positive matrix:

$$\hat{Y}^h = \begin{bmatrix} r_1^h(1) & .. & r_J^h(1) \\ \vdots & & \vdots \\ r_1^h(S) & .. & r_J^h(S) \end{bmatrix}.$$

- Impose an artificial investment constraint $z_j^h \geq -K_j^h$. Then existence is guaranteed, markets clear, and the (AC) conditions are met.
- Consider a sequence of economies as $K \longrightarrow \infty$. Then, either
  1. the investment constraints cease to bind (assets bounded)
  2. the constraints continue to bind
     1. the payout matrix $\hat{Y}^h \rightarrow Y^-$, a rank deficient payout matrix
     2. the payout matrix $\hat{Y}^h \rightarrow Y$, the original full rank payout matrix

Features of the equilibrium

- Chain reaction of bankruptcy is possible (aka contagion in finance). Suppose one household type $h \in \mathcal{H}_f$ declares bankruptcy. Then $\rho_j(s) < 1$ for all j s.t. $z_j^h < 0$. The value of all household portfolios are now

$$\sum_j \rho_j(s) r_j(s) \left(z_j^h\right)^+ + \sum_j r_j(s) \left(z_j^h\right)^- < r(s) \cdot z^h.$$

  This may lead to further bankruptcy declarations.
  Particularly vexing is that for bankrupt households with both short and long positions, if $\rho(s) << 1$, then the liquidated long positions will have less value and this reduces the dilution factor further.

- No distinction between complete markets and incomplete markets. With bankruptcy, the first basic welfare theorem fails in both cases. I need to characterize equilibria as resulting from a dynamic trading environment over both commodity and financial markets.

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Extension I: Real costs of bankruptcy

- Bankrupt households turn over $\sum_j \left(\rho_j(s) + \alpha_j\right) r_j(s) \left(z_j^h\right)^+$.
- Suppose during the liquidation process, only $\sum_j \beta\rho_j(s) r_j(s) \left(z_j^h\right)^+$ is made available to creditors, with $\beta \in [0,1]$.
- Define how the remaining funds are distributed:

$$\delta_{RC}^h(s) = \frac{-\sum_j \beta\rho_j(s) r_j(s) \left(z_j^h\right)^+}{\sum_k r_k(s) \left(z_k^h\right)^-}.$$

- Then the (AC) equation yields $\forall j, \forall s > 0$:

$$\rho_{j,RC}(s) = 1 + \frac{\int_{h \in \mathcal{H}_s'} \left(1 - \delta_{RC}^h(s)\right) \left(z_j^h\right)^-}{\int_{h \in \mathcal{H}} \left(z_j^h\right)^+}.$$

- $\rho_{j,RC}(s) \leq \rho_j(s)$, with $<$ if $\left(z_j^h\right)^- \cdot \sum_k \left(z_k^h\right)^+ < 0$ for some $h \in \mathcal{H}_s'$.

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Extension II: Firm bankruptcy

Introduce firms $i \in \mathcal{I} = \{1, .., I\}$.
Firms use the same set of J numeraire assets to finance production investment.
If a firm has negative profit $\pi(s)$ in some state, then it must declare bankruptcy and will only pay back the fraction $\delta^i(s) \sum_j r_j(s) \left(z_j^i\right)^-$ of its financial debt such that $\pi(s) = 0$ (limited liability of shareholders).
If a firm has positive profit, then obviously $\delta^i(s) = 1$.
The condition (AC) for this production model is the same as for the pure exchange model with the second line added:

$$\rho_j(s) \int_{h \in \mathcal{H}} \left(z_j^h\right)^+ + \int_{h \notin \mathcal{H}'_s} \left(z_j^h\right)^- + \int_{h \in \mathcal{H}'_s} \delta^h(s) \left(z_j^h\right)^-$$
$$+\rho_j(s) \sum_{i \in \mathcal{I}} \left(z_j^i\right)^+ + \sum_{i \in \mathcal{I}} \delta^i(s) \left(z_j^i\right)^- = 0 \ \forall j, s > 0.$$

Extension III: Infinite time horizon

2−period assumptions

- There is a cost of bankruptcy parameterized by α.
- Households must fail the "means test" in some state.
- Nonbankrupt households must repay their debt entirely.
- All households face the same asset prices when borrowing and these prices are independent of the size of the loan.

∞−time replacements

- Bankruptcy is harmful by lowering the credit score.
- Bankrupt households may not declare again for 8 years.
- Nonbankrupt households choose how much debt to hold.
- All households *with the same credit score* (as determined by bankruptcy history and debt held) *and requesting the same size loan* face the same asset prices.