Equilibrium Corporate Finance and Macroeconomics

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Introduction

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- But this is different, we are addressing an interesting economic question.... :-)

Real Introduction

- Macro with heterogeneous agents and incomplete markets goes a long way without production - with exogenous earning processes; e.g.,
  - J. Heathcote, K. Storesletten, and G. Violante (2008) - ”Bewley models”
  - O. Attanasio and N. Pavoni (2008) + F. Perri and D. Krueger (2006) - dynamic contracts
- **Production** in macro involves, either
  - complete markets; or
  - investor-entrepreneur models of dynamic contracting
- Prototypical examples are, e.g.,:
  - U. Jermann (2008) - production based asset pricing literature
  - R. Castro, G.L. Clementi, and G. MacDonald (2004); G. L. Clementi and H. Hopenhaym (2006); T. Piskorski (2008) - dynamic contracting.

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- This is a bit extreme: recent papers on the bridge include: R. Albuquerque and N. Wang (2005), *Agency conflicts, investments, and asset pricing*; J. Dow, G. Gorton, A. Krishnamurthy, AER (2005), *Equilibrium investment and asset pricing under imperfect corporate control*, G. Gorton and P. He (2006), *Agency-Based Asset Pricing*, J. Gomes, L. Kogan, and L. Zhang JPE (2003), *Equilibrium Cross Section of Returns*.

WHY?

- Problems with objective function of the firm in incomplete markets; see J. Dreze (1974), S. Grossman and O. Hart (1980), D. Duffie and W. Shafer (1987) *[Google Scholar has 101 related papers to D. Duffie and W. Shafer (1987)]*

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- Problems with competitive notions in economies with asymmetric information; see the whole strategic approach, from M. Rothschild and J. Stiglitz (1976), C. Wilson (1977), and more recently D. Gale (1993), P. Dubey, J. Geanakoplos, and M. Shubik (2005), M. Magill and M. Quinzii (2005).

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- We follow
  - L. Makowski (1980, 1983), L. Makowski and J. Ostroy (1991); and W. Pesendorfer (1995)
  - E. Prescott and R. Townsend (1984) and T. Kehoe and D. Levine (1993), A. Bisin and P. Gottardi (2005).

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- Perfect competition throughout!

- Bond has payoff:

$$\min\{1, \frac{f(k; s_1)}{B}\}$$

- Equity has payoff:

$$\max\{f(k; s_1) - B, 0\}$$

Equilibrium

- The firm's problem is:

$$V = \max_{k,B} -k + q(k,B) + p\ B \tag{1}$$

subject to the solvency constraint (ensuring that the bonds issued are riskfree):

$$f(k; s_1) \geq B \qquad \forall s_1 \tag{2}$$

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- Let $\bar{k}, \bar{B}$ denote the solutions to this problem.

Equilibrium; cont.ed

- The problem of agent i is:

$$\max_{\theta^i, b^i, c^i} u(c_0^i) + \beta E_{s_0} u(c^i(s_1)) \tag{3}$$

subject to

$$b^i \geq 0, \quad \theta^i \geq 0, \ \forall i \tag{4}$$

and

$$c_0^i = w_0^i + [-k + q + p\, B\,]\, \theta_0^i - q\, \theta^i - p\, b^i \tag{5}$$

$$c^i(s_1) = w^i(s_1) + [f(k; s_1) - B\,]\, \theta^i + b^i, \qquad \forall s_1, \tag{6}$$

where θ_0^i is some predetermined stock-holdings agent i enters the economy with (and $\sum_i \theta_0^i = 1$).

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- Let $\bar{\theta}^i, \bar{b}^i, \bar{c}_0^i, \left(\bar{c}^i(s_1)\right)_{s_1 \in S_1}$ denote the solutions of this problem.
- Let $MRS^i(s_1)$ denote the marginal rate of substitution between consumption in states s_0 and s_1 for consumer i evaluated at his optimal consumption choice $\bar{c}^i$.

Equilibrium; cont.ed

- In equilibrium, the equity price map faced by the firm must satisfy the following consistency condition:

$$q(k, B) = \max_i E_{s_0}\left[MRS^i(s_1)(f(k; s_1) - B)\right] \; for \; all \; k, B$$

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- In equilibrium, also,
  - $q = q(\bar{k}, \bar{B})$.
  - Markets clear:

$$\begin{aligned} \sum_i c_0^i + k &\leq \sum_i w^i \\ \sum_i c^i(s_1) &\leq \sum_i w^i(s_1) + f(k;s_1), \text{ for all } s_1 \end{aligned}$$

  or equivalently:

$$\begin{aligned} \sum_i \theta^i &\leq 1 \\ \sum_i b^i &\leq B \end{aligned}$$

Objective function of the firm

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- In this paper (following Makowski ...):

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Unanimity

- **Proposition:** At a competitive equilibrium, shareholders unanimous support the production and financial decisions of firms $\bar{k}, \bar{B}$. That is every agent i holding a positive initial amount θ_0^i of equity of the representative firm will be made -weakly - worse off by any other choice k', B' of a firm (for which, obviously the firm's value will be -weakly - lower).

Welfare

- **Definition:** A competitive equilibrium is *constrained Pareto efficient* if its allocations are:
  1. feasible: there exists a production plan k of firms such that:
$$\begin{aligned} \sum_i c_0^i + k &\leq \sum_i w^i \\ \sum_i c^i(s_1) &\leq \sum_i w^i(s_1) + f(k; s_1) \text{ for all } s_1 \end{aligned} \tag{7}$$
  2. attainable with the existing asset structure: that is there exists B and, for each consumer's type i, a pair θ^i, b^i such that:
$$c^i(s_1) = w^i(s_1) + [f(k; s_1) - B\,]\,\theta^i + b^i, \qquad \forall s_1 \tag{8}$$
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- **Proposition:** Competitive equilibria are constrained Pareto efficient.

Modigliani-Miller

- **Proposition:** At a competitive equilibrium, the capital structure of each individual firm is indeterminate, with the only, possible exception of the case where the optimal choice obtains at a point where the no default constraint binds. On the other hand, the equilibrium capital structure of all firms in the economy is, at least partly, determinate:
  - either B is such that all equityholders are also bondholders (in which case B lies in an interval, or
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- Why "all equityholders are also bondholders"? Otherwise equityholders is at constraint where he would go short on the bond, which he could achieve by leveraging the firm.

Parametric exercise

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- Production technology: $f(k; s_1) = a(s_1)k^{\alpha}$, with $\alpha = 0.75$.
- Two states with the following structure of endowment and productivity shocks:

	$\underline{s}$	$\overline{s}$
w^1	2	3
w^2	3	8
a	2.1429	4.7143

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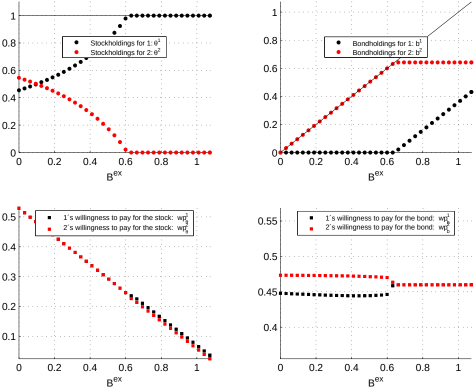
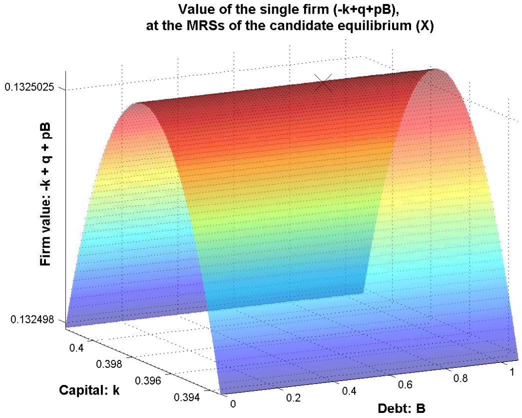
- We consider the case where at date 0 the state is also recession, i.e. $w_0^i = w^i(\underline{s})$ for all i, and $\pi(\underline{s}) = .8$, $\pi(\bar{s}) = .2$, or the persistence of the shocks is fairly high.

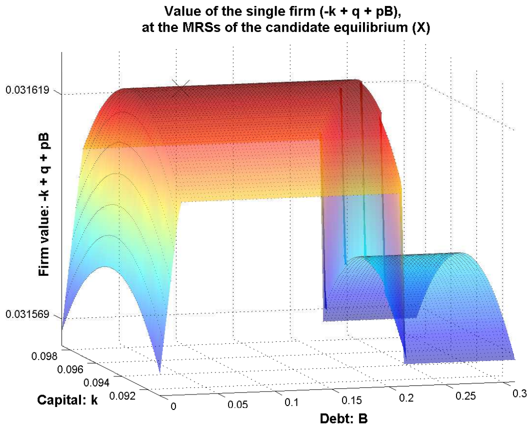
Parametric exercise; cont.ed



Parametric exercise; cont.ed



Risky Debt


Value of the single firm (-k + q + pB),
at the MRSs of the candidate equilibrium (X)
Firm value: -k + q + pB
0.031619
0.031569
Capital: k
0.098
0.096
0.094
0.092
Debt: B
0
0.05
0.1
0.15
0.2
0.25
0.3

Short-sales and intermediation

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  where γ is the amount of equity of the firm retained as collateral by the intermediary
- The intermediary chooses m, and γ, to maximize total revenue at date 0 :

$$\max_{m,\gamma} (q^{+} - q^{-})m - q\gamma$$

  subject to

$$m \leq m(1-\delta) + \gamma$$

Short-sales and intermediation; cont.ed

- The consumers' budget constraints are then in this set-up modified as follows:

$$c_0^i = w_0^i + [-k + q + p\,B\,]\,\theta_0^i - q\,\theta^i - p\,b^i - q^+\lambda_+^i - q^-\lambda_-^i$$

$$c^i(s_1) = w^i(s_1) + [f(k; s_1) - B\,]\,(\theta^i + \lambda_+^i - \lambda_-^i) + b^i, \qquad \forall s_1,$$

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- The consistency condition for $q(k, B)$ is:

$$q(k, B) = \max \left\{ \max_i E_{s_0} MRS^i(s_1) \, [f(k; s_1) - B] \, , \right.$$

$$\left. \frac{\max_i E_{s_0} MRS^i(s_1) \, [f(k; s_1) - B] - \min_i E_{s_0} MRS^i(s_1) \, [f(k; s_1) - B]}{\delta} \right\}$$

Short-sales and intermediation; cont.ed

- Market clearing of equity is:

$$\gamma + \sum_{i \in I} \theta^i = 1,$$

$$\sum_{i \in I} \lambda_+^i = \sum_{i \in I} \lambda_-^i = m,$$

Short-sales and intermediation; cont.ed

- For the firm issuing equity, two possible situations can arise at equilibrium:
  1. $q = (q^+ - q^-)/\delta > q^+$, which is in turn equivalent to $q^+ > q^-/(1-\delta)$: equity sells at a premium over the long positions on the derivative claim issued by the intermediary; all the amount of equity outstanding is purchased by the intermediary, who can bear the additional cost of equity thanks to the presence of a sufficiently high spread $q^+ - q^-$ between the cost of long and short positions on the derivative.
  2. $q = q^+$: there is a single price at which equity and long positions in the derivative can be traded; consumers are indifferent between buying long positions in equity and the derivative and some if not all the outstanding amount of equity is held by consumers; when consumers hold all the outstanding amount of equity, intermediaries are non active at equilibrium and the bid ask spread $q^+ - q^-$ is sufficiently low (in particular, it is less or equal than δq).

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- **Proposition:** Competitive equilibria with short-sales are constrained Pareto efficient.

Unobservable risk composition: moral hazard

- Production takes place according to the function $f(k, \phi; s)$, where ϕ represents a technological choice, affecting the stochastic structure of the firm's future output; e.g.,

$$f(k, \phi; s) = [a(s) + \phi\epsilon(s)]\, k^{\alpha},$$

ϕ is chosen by the firm's manager - agent i;
the manager is hired by initial shareholders;
ϕ, k and B are chosen at time $t = 0$, *before financial markets open*;
but, unlike the choice of B and k, ϕ is not observed by bond-holders nor by shareholders in financial market at time 0.

- The choice of ϕ affects the manager's disutility, $v^i(\phi)$.
- The manager's compensation package is observable and consists of a net payment x_0, in units of the consumption good at date 0, together with a portfolio of θ^m units of equity and b^m units of bonds.

- The cost to the firm of the manager i's compensation package, $W^i(\phi, k, B; q, p)$ is
  - the payment made to this agent at date 0, x_0^i,
  - plus the value of the portfolio $q(k,B,\phi)\left(\theta^{i,m}-\theta_0^{i,m}\right)+p(k,B,\phi)b^{i,m}$ attributed to him,
  - minus the amount of the dividends due to this agent on account of his initial endowment $\theta_0^{i,m}$ of equity, $\theta_0^{i,m}\begin{bmatrix} -k+p(k,B,\phi)B \\ -W^i(\phi,k,B;q,p) \end{bmatrix}$.

- The optimal choice problem of a firm who has a hired as manager a type i agent is then the following:

$$V^i = \max_{k,B,\phi,x_0^i,\theta^{i,m},b^{i,m}} -k+q(k,B,\phi)+p(k,B,\phi)B-W^i(\phi,k,B;q,p) \tag{9}$$

s.t.

$$\mathbb{E}u^i\left(w_0^i \ + x_0^i, w^i(s) + \left[\begin{array}{c} \max\{0, f(k,\phi;s) - B\}\theta^{i,m} \\ + \min\left\{1, \frac{f(k,\phi;s)}{B}\right\} b^{i,m}) - v^i(\phi) \end{array}\right]\right) \geq$$
$$\mathbb{E}u^i\left(w_0^i \ + x_0^i, w^i(s) + \left[\begin{array}{c} \max\{0, f(k,\phi';s) - B\}\theta^{i,m} \\ + \min\left\{1, \frac{f(k,\phi';s)}{B}\right\} b^{i,m}) - v^i(\phi') \end{array}\right]\right)$$
$$\text{for all } \phi' \in \Phi \tag{10}$$

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s.t.

$$\begin{aligned}&\mathbb{E}u^i\left(w_0^i \ + x_0^i, w^i(s) + \left[\begin{array}{c}\max\{0, f(k,\phi;s)-B\}\theta^{i,m}\\ +\min\left\{1, \frac{f(k,\phi;s)}{B}\right\} b^{i,m}) - v^i(\phi)\end{array}\right]\right) \geq \\ &\mathbb{E}u^i\left(w_0^i \ + x_0^i, w^i(s) + \left[\begin{array}{c}\max\{0, f(k,\phi';s)-B\}\theta^{i,m}\\ +\min\left\{1, \frac{f(k,\phi';s)}{B}\right\} b^{i,m}) - v^i(\phi')\end{array}\right]\right)\\ &\text{for all } \phi' \in \Phi\end{aligned} \tag{10}$$

$$\mathbb{E}u^i\left(w_0^i \ + x_0^i, w^i(s) + \left[\begin{array}{c}\max\{0, f(k,\phi;s)-B\}\theta^{i,m}\\ +\min\left\{1, \frac{f(k,\phi;s)}{B}\right\} b^{i,m}) - v^i(\phi)\end{array}\right]\right) \geq \bar{U}^i \tag{11}$$

- The reservation utility for a manager of type i, $\bar{U}^i$, is endogenously determined in equilibrium (see below).

- The type $\bar{\imath} \in I$ of agent to be hired as manager is then chosen by selecting the type which maximizes the firm's value:

$$\max_{i \in I} V^i, \tag{12}$$

for V^i as determined in (9).

- This determines $\bar{U}^i$

- The type $\bar{\imath} \in I$ of agent to be hired as manager is then chosen by selecting the type which maximizes the firm's value:

$$\max_{i \in I} V^i, \tag{12}$$

for V^i as determined in (9).

- Each consumer of a given type j not chosen as a manager chooses:

$$\max_{\theta^j, b^j, c^j} \mathbb{E}u^j(c_0^j, c^j(s)) \tag{13}$$

subject to

$$c_0^j = w_0^j + \left\{-k + q + pB - W^i(\phi, k, B; q, p)\right\} \theta_0^j - q\,\theta^j - p\,b^j \tag{14}$$

$$c^j(s) = w^j(s) + \left[\begin{array}{c} \max\{0, f(k, \phi; s) - B\}\theta^j \\ + \min\left\{1, \frac{f(k,\phi;s)}{B}\right\} b^j \end{array} \right], \qquad \forall s, \tag{15}$$

and

$$b^j \geq 0, \quad \theta^j \geq 0, \ \forall j \tag{16}$$

- This determines $\bar{U}^i$

- In equilibrium, the bond and equity price maps faced by the firm must satisfy the following consistency conditions for all k, B, ϕ:

[p]$p(k,B,\phi) = \max_i \mathbb{E} MRS^i(s) \min\left\{1, \frac{f(k,\phi;s)}{B}\right\}$

[q]$q(k,B,\phi) =$
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- These consistency conditions guarantee the following:

i) Investors correctly anticipate the payoff distribution of the risky bond and equity, given the observed levels of k and B and the manager's choice of the risk composition parameter ϕ given k, B and his compensation package. In particular, investors correctly anticipate that ϕ satisfies (10).
ii) The value of the bond and the equity price maps faced by each firm equal, for each k, B, ϕ, the highest marginal valuation across all consumers, evaluated at their equilibrium consumption choices, of the return on these assets.

Proposition: *At a competitive equilibrium of the economy with moral hazard, shareholders unanimously support the production and financial decisions of firms as well as the choice of management,* $\bar{k}, \bar{B}, \bar{\phi}, \bar{\imath}, x_0^{\bar{\imath}}, \theta^{\bar{\imath},m}, b^{\bar{\imath},m}$*; that is, every agent i holding a positive initial amount θ_0^i of equity of the representative firm will be made - weakly - worse off by any other admissible choice of a firm (that is, any* $k', B', \phi', i', x_0^{i\prime}, \theta^{i',m}, b^{i',m}$ *which satisfies (10) and (11)).*

- A consumption allocation $(c^i)_{i=1}^I$ is *admissible* if:
  1. It is *feasible*: there exists a production plan k and a risk composition choice ϕ of firms such that

$$\begin{aligned} \sum_i c_0^i + k &\leq \sum_i w_0^i \\ \sum_i c^i(s) &\leq \sum_i w^i(s) + f(k,\phi;s) \text{ for all } s \end{aligned} \tag{17}$$

  2. It is *attainable with the existing asset structure*: that is there exists B and, for each consumer's type i, a pair θ^i, b^i such that

$$c^i(s) = w^i(s) + \left[\begin{array}{c} \max\{0, f(k,\phi;s) - B\}\theta^i \\ + \min\left\{1, \frac{f(k,\phi;s)}{B}\right\} b^i \end{array} \right], \qquad \forall s \tag{18}$$

  3. It is *incentive compatible*: given the production plan k and the financing plan B, there exists $\bar{\imath}$ such that:

$$\begin{array}{c} \mathbb{E}u^i(c_0^{\bar{\imath}}, w^{\bar{\imath}}(s) + \left[\begin{array}{c} \max\{0, f(k,\phi;s) - B\}\theta^{\bar{\imath}} \\ + \min\left\{1, \frac{f(k,\phi;s)}{B}\right\} b^{\bar{\imath}}) - v^{\bar{\imath}}(\phi) \end{array} \right] \geq \\ \mathbb{E}u^i(c_0^{\bar{\imath}}, w^{\bar{\imath}}(s) + \left[\begin{array}{c} \max\{0, f(k,\phi';s) - B\}\theta^{\bar{\imath}} \\ + \min\left\{1, \frac{f(k,\phi';s)}{B}\right\} b^{\bar{\imath}}) - v^{\bar{\imath}}(\phi') \end{array} \right] . \\ \text{for all } \phi' \in \Phi \end{array}$$

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- And the First Welfare theorem readily applies. It can be established by an argument essentially analogous to the one used to establish the Pareto efficiency of competitive equilibria in Arrow Debreu economies.
  Proposition: *Competitive equilibria of the economy with moral hazard are constrained Pareto efficient.*

Unobservable risk composition: hidden information

- The risk composition ϕ is not an unobservable choice of the manager of a firm, but rather is private information of the agent who is hired as manager of the firm at time $t = 0$, before the level of the firm's capital k and financial structure B are chosen.

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- The risk composition ϕ is not an unobservable choice of the manager of a firm, but rather is private information of the agent who is hired as manager of the firm at time $t = 0$, before the level of the firm's capital k and financial structure B are chosen.
- The cost of the compensation package for a manager of type i is:

$$W^i_\phi(k_\phi, B_\phi; q, p) = x^i_{0\phi} + \left[\begin{array}{c} \frac{q(k_\phi, B_\phi; \phi)\left(\theta^{i,m}_\phi - \theta^{i,m}_0\right) + p(k_\phi, B_\phi; \phi) b^{i,m}_\phi}{\left(1 - \theta^{i,m}_0\right)} \\ -\frac{\theta^{i,m}_0}{1 - \theta^{i,m}_0}\left[p(k_\phi, B_\phi; \phi) B_\phi - k_\phi - x^i_{0\phi}\right] \end{array} \right]$$

- The firm's problem, given the type i of the agent hired as manager, is then the following:

$$V^i = \max_{\left(k_\phi, B_\phi, x^i_{0\phi}, \theta^{i,m}_\phi, b^{i,m}_\phi\right)_{\phi\in\Phi}} \sum_{\phi\in\Phi} \Pr(\phi) \left[\begin{array}{c} -k_\phi + q(k_\phi, B_\phi; \phi) + p(k_\phi, B_\phi; \phi \\ -W^i_\phi(k_\phi, B_\phi; q, p) \end{array} \right. \tag{19}$$

s.t.

$$\begin{array}{c} \mathbb{E}u^i(w^i_0 \ + x^i_{0\phi}, w^i(s) + \left[\begin{array}{c} \max\{0, f(k_\phi, \phi; s) - B_\phi\}\theta^{i,m}_\phi \\ + \min\left\{1, \frac{f(k_\phi, \phi; s)}{B_\phi}\right\} b^{i,m}_\phi) \end{array} \right] \geq \\ \mathbb{E}u^i(w^i_0 \ + x^i_{0\phi}, w^i(s) + \left[\begin{array}{c} \max\{0, f(k_{\phi'}, \phi; s) - B_{\phi'}\}\theta^{i,m}_{\phi'} \\ + \min\left\{1, \frac{f(k_{\phi'}, \phi; s)}{B_{\phi'}}\right\} b^{i,m}_{\phi'}) \end{array} \right] \\ \text{for all } \phi \text{ and all } \phi' \neq \phi \end{array} \tag{20}$$

$$\sum_{\phi\in\Phi} \Pr(\phi)\mathbb{E}u^i(w^i_0 + x^i_{0\phi}, w^i(s) + \left[\begin{array}{c} \max\{0, f(k_\phi, \phi; s) - B_\phi\}\theta^{i,m}_\phi \\ + \min\left\{1, \frac{f(k_\phi, \phi; s)}{B_\phi}\right\} b^{i,m}_\phi) \end{array} \right] \geq \bar{U}^i \tag{21}$$

- It is tedious (very!) but not difficult to show that competitive equilibria for this economy also satisfy unanimity and constrained efficiency.

Unobservable leverage

- Debt-holders of each firm do not observe the total amount of debt issued by the firm in the market, i.e., its leverage.

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- The production function is given by $y = f(k; s_1)$.
- The firm's problem becomes:

$$V = \max_{k,B} -k + q(k, B) + p(k, B)B$$

s.t.

$$q(k, B) + p(k, B)B \geq q(k, B) + p(k, B)B' \text{ for all } B' \neq B$$

- The firm's shareholders have an incentive to expand the amount of debt issued to increase the revenue from its sale.

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- Competitive equilibria for this economy continue to satisfy unanimity and constrained efficiency properties.

Unobservable manager's quality - adverse selection

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- The problem of the (shareholders) of the firm is choosing the production plan k and the financial structure B, as well as the type of agent serving as manager, were the type is now given by an observable component i and a second, unobservable component, ϕ, together with the associated compensation package.

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- The manager's compensation package consists of an amount x_0 of the consumption good at date 0, θ^m units of equity and b^m of bonds.

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- Redefine the size of the mass of firms and assume that it is less than χ^i_ϕ, the mass of agents of type i and quality ϕ.
- Assume that the firms' technology is such that some production and financing levels and a compensation package can always be found so as to separate managers of different unobservable types (*single crossing property*): The firms' technology is such that, for any tuple $v = (x_0, b, \theta, B, k) \in \mathbb{R}^5_+$ the vectors

$$D_v \mathbb{E} u^i(w^i_0 + x_0, w^i(s) + \left[\begin{array}{c} \varsigma_\phi + \max\{0, f(k,\phi;s) - \varsigma_\phi - B\}\theta \\ + \min\left\{1, \frac{f(k,\phi;s)-\varsigma_\phi}{B}\right\} b) \end{array} \right],$$

$$\phi \quad \in \quad \Phi$$

are linearly independent.

- The cost of the compensation package for a manager of type i and quality ϕ is

$$W^i(\phi, k, B; q, p,) = \left[\begin{array}{c} x_0^i + \frac{q(k,B,\phi)\left(\theta^{i,m} - \theta_0^{i,m}\right) + p(k,B,\phi) b^{i,m}}{\left(1-\theta_0^{i,m}\right)} \\ -\frac{\theta_0^{i,m}}{1-\theta_0^{i,m}} \left[p(k, B, \phi)B - k - x_0^i\right] \end{array} \right]$$

- The value maximization problem of a firm who is hiring as manager an agent of type i and unobservable quality ϕ takes the following form:

$$V^i(\phi) = \max_{k,B,x_0^i,\theta^{i,m},b^{i,m}} -k+q(k,B,\phi)+p(k,B,\phi)B-W^i(\phi,k,B;q,p)$$

s.t.

$$\bar{U}^i \geq \mathbb{E}u^i(w_0^i + x_0^i, w^i(s) + \left[\begin{array}{c} \varsigma_{\phi'} + \max\{0, f(k,\phi';s) - \varsigma_{\phi'} - B\}\theta^{i,m} \\ + \min\left\{1, \frac{f(k,\phi';s)-\varsigma_{\phi'}}{B}\right\} b^{i,m}) \end{array}\right] \text{ for all } \phi' \neq \phi \tag{22}$$

and

$$\bar{U}^i \leq \mathbb{E}u^i(w_0^i+x_0^i, w^i(s)+\left[\begin{array}{c} \varsigma_{\phi} + \max\{0, f(k,\phi;s) - \varsigma_{\phi} - B\}\theta^{i,m} \\ + \min\left\{1, \frac{f(k,\phi;s)-\varsigma_{\phi}}{B}\right\} b^{i,m}) \end{array}\right] \tag{23}$$

- If at equilibrium the optimal choice of the firm is to hire a single quality type $\bar{\phi}$ as manager, we call the equilibrium *separating*, following Rothschild-Stiglitz (1979). On the other hand, if the optimal choice is to hire a nonsingleton set $\Phi' \subseteq \Phi$ of quality types, we say the equilibrium is (partially) *pooling*, where agents of different quality become managers.

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- By a similar argument as in Bisin and Gottardi (2006), we can show that competitive equilibrium are necessarily separating and moreover that, differently from the economy with moral hazard, equilibrium allocations are not in general constrained Pareto efficient, in the sense of Diamond (1967) and Prescott-Townsend (1984). On the other hand, unanimity still holds in this environment.

Conclusions

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- Corporate financing decisions, in these economies, are therefore interesting and one can investigate their interaction with the properties of the equilibrium allocation and prices.
- The conceptual problems usually associated with modeling firm decisions when markets are incomplete or with asymmetric information can be overcome with *appropriate,* and *natural* modeling choices.
- **Corporate finance is thus ready to be passed on to macroeconomists.**