

# FRÉCHET-DIFFERENTIATION OF FUNCTIONS OF OPERATORS WITH APPLICATION TO FUNCTIONAL REGRESSION

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Analytic functions of operators (in the sense of functional calculus) can be differentiated and an ensuing delta-method yields the asymptotic distribution of functions of the sample covariance operator in particular. Such functions include for instance regularized inverses of the sample covariance operator and their square roots. Whenever the asymptotic distribution of a function of the sample covariance operator is known, it automatically entails the asymptotics of its eigenvalues and functions under certain conditions. In this way one can obtain the asymptotic distribution of a version of the functional canonical correlation, and test statistics for certain functional hypotheses. In this paper we focus on application of the above in functional regression. The Fréchet derivative leads to a straight-forward calculus for obtaining an upper bound for the integrated squared error and the asymptotic distribution of the estimator of the (smoothed) regression function. The latter result can be exploited for testing a linear hypothesis regarding the regression function.